Week 13: Balanced Tree

CSCI 2100 Data Structures

Fall 2019

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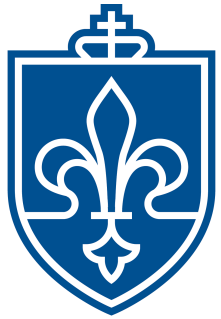

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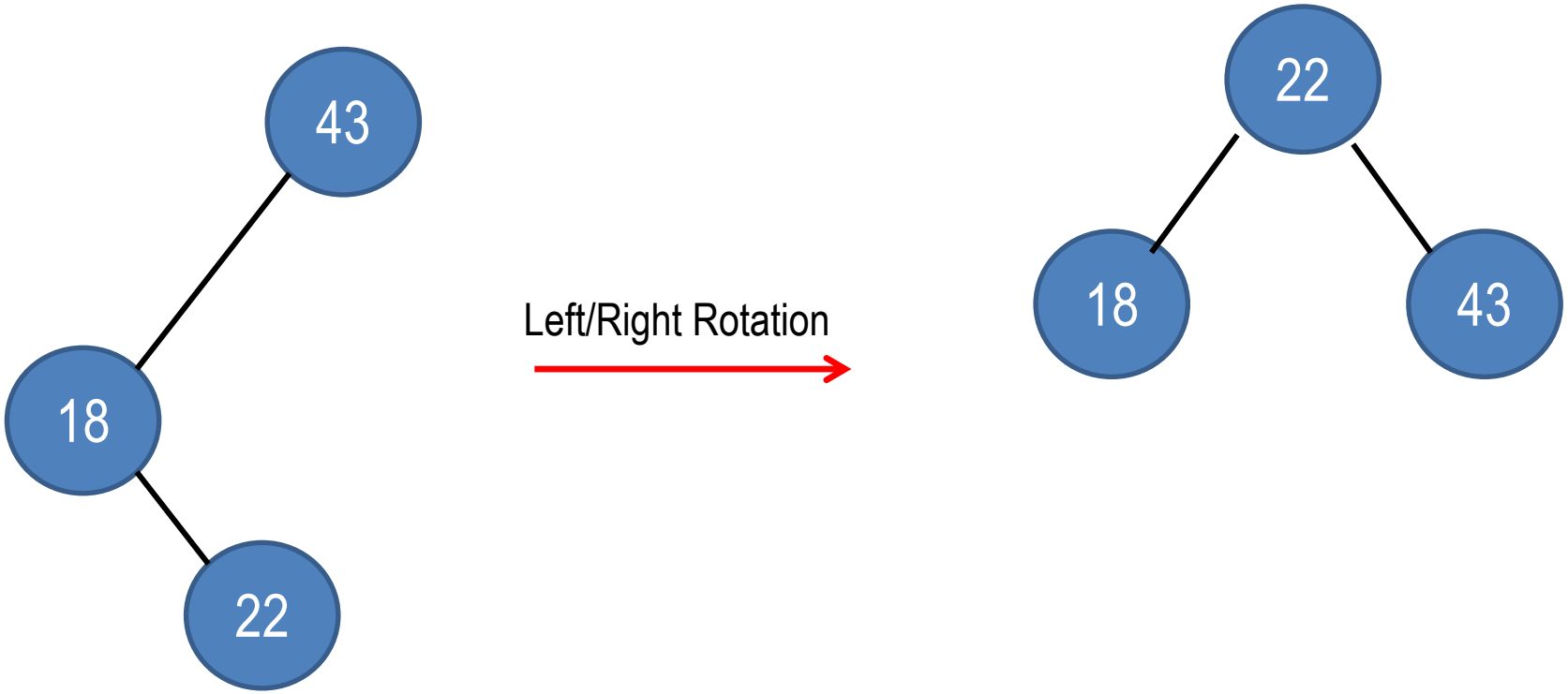
Learning Objectives

- How to keep balance by tree rotation?

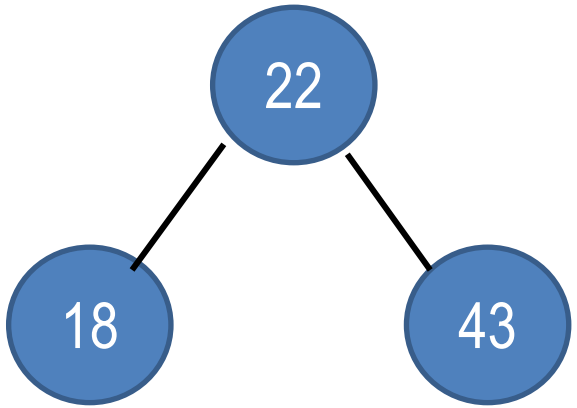
How to construct AVL Tree?

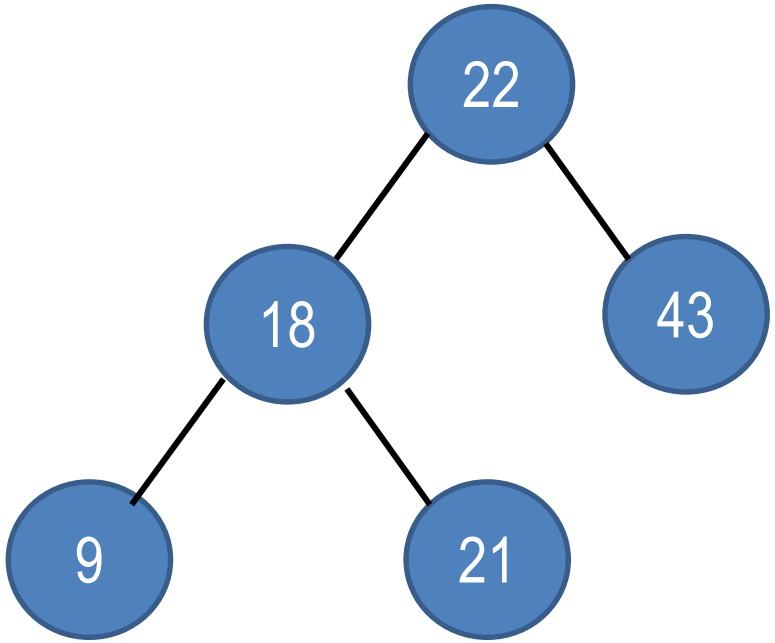
43, 18, 22, 9, 21, 6, 8



How to construct AVL Tree?

43, 18, 22, 9, 21, 6, 8





How to construct AVL Tree?

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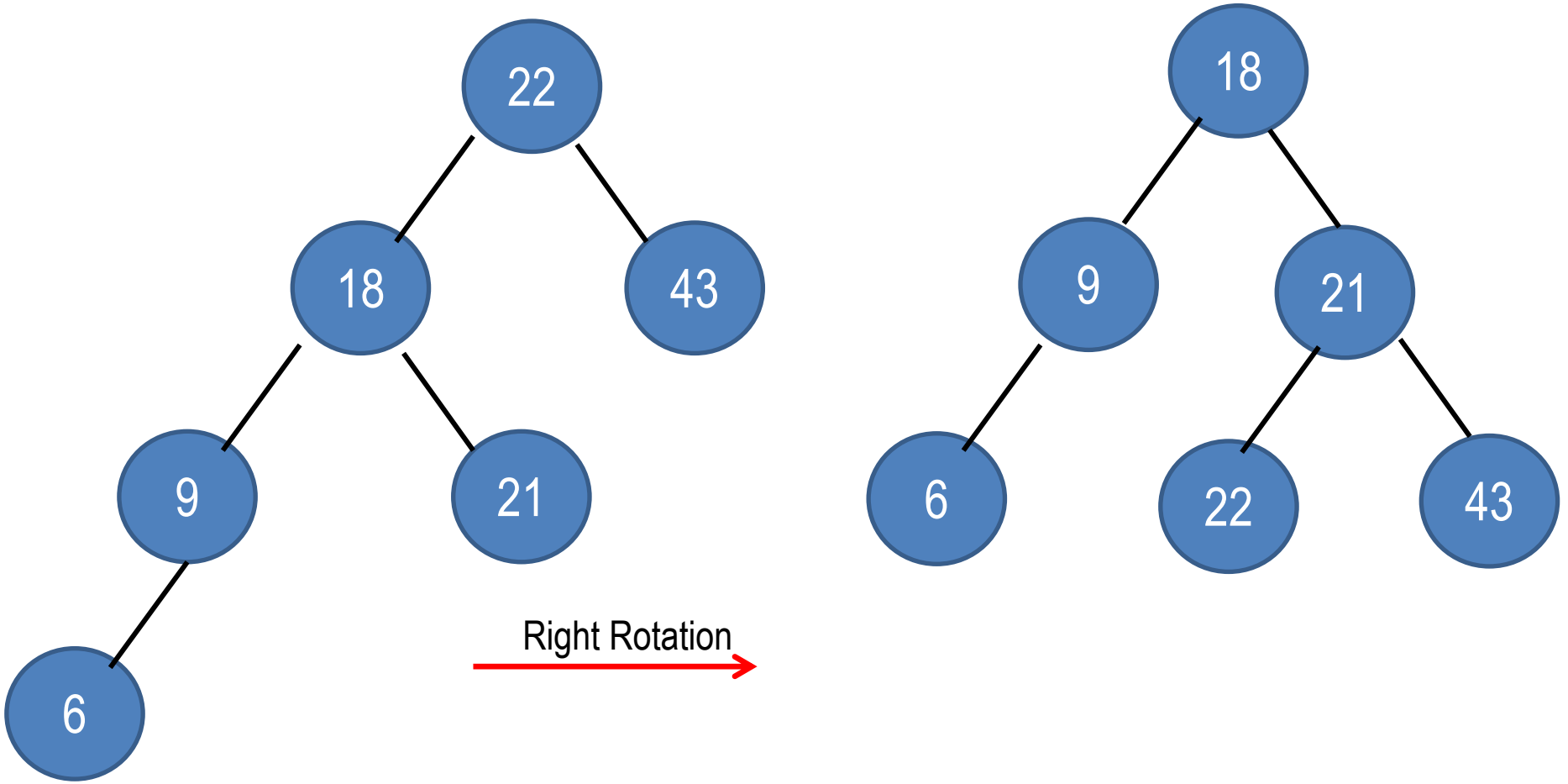

How to construct AVL Tree?

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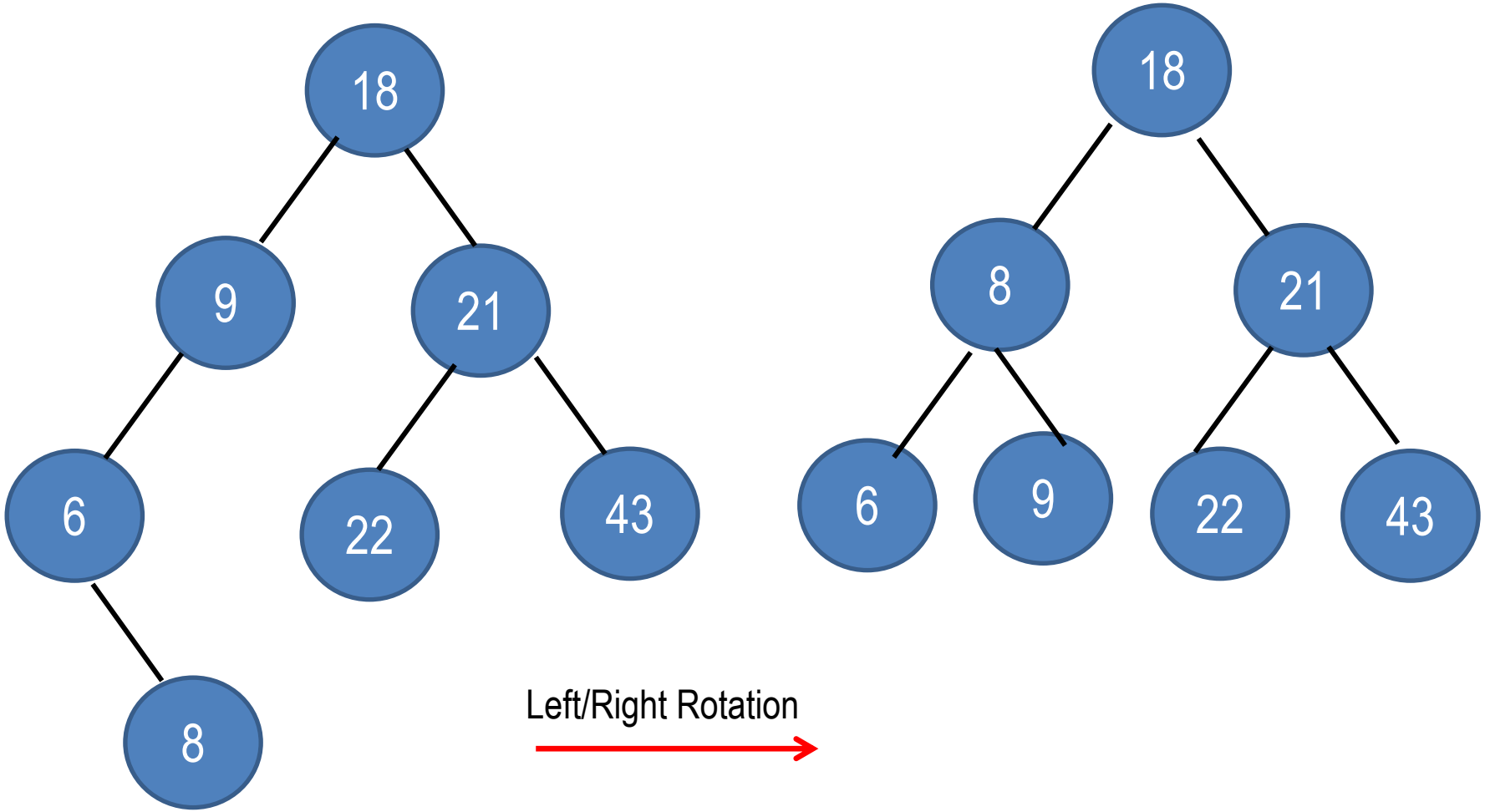
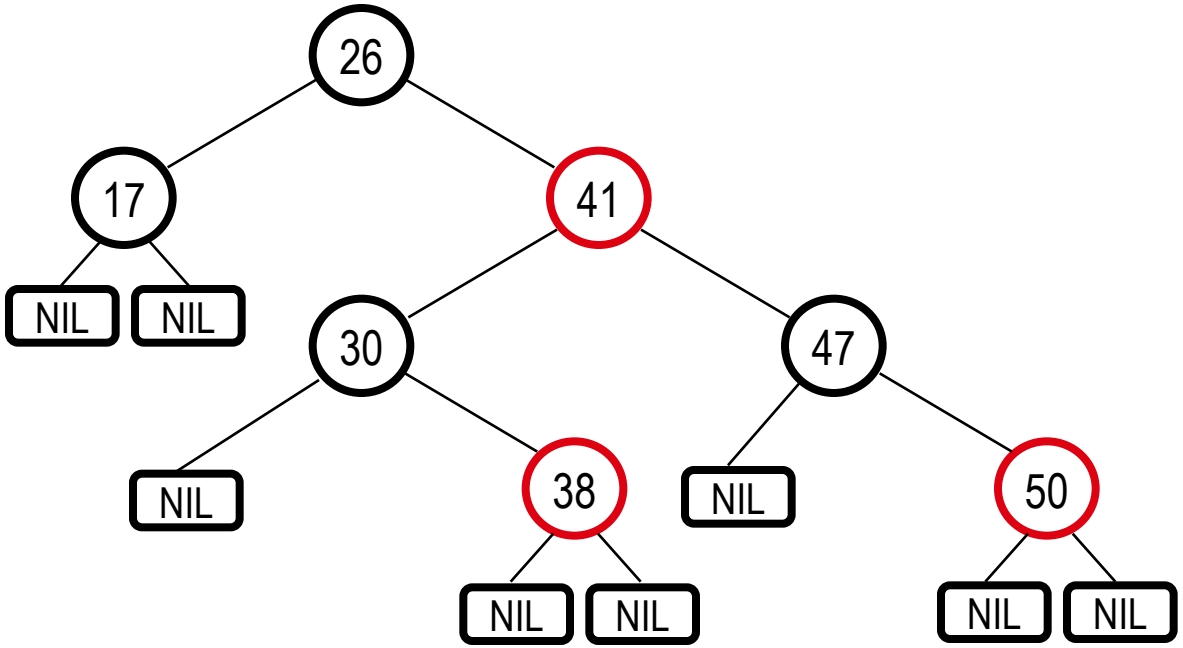


Red-Black Trees

- "Balanced" binary search trees guarantee an $O(\log n)$ running time
- Red-black-tree
  - Binary search tree with an additional attribute for its nodes: `color` which can be **red** or **black**
  - Constrains the way nodes can be colored on any path from the root to a leaf:

> Ensures that no path is more than twice as long as any other path
>
> ==> the tree is balanced

Example: RED-BLACK-TREE



Red-Black-Trees Properties

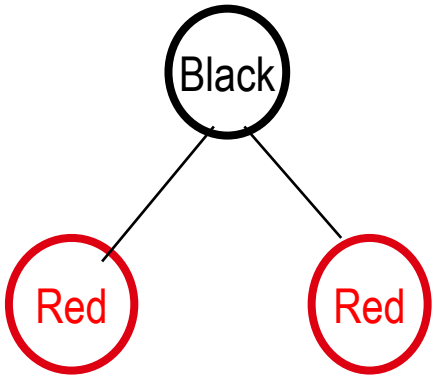
(**Satisfy the binary search tree property**)

1. Every node is either **red** or **black**
2. The root is **black**
3. New insertion are always **red**
4. Every leaf (NIL) is **black**
5. No path can have two consecutive **red** nodes (If a node is red, then its parent is black.)
6. All simple paths from any node x to a descendant leaf have the same number of **black** nodes (black-height(x))

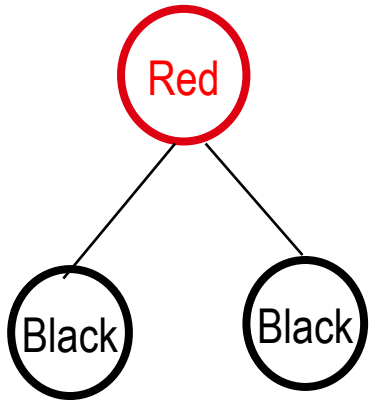
Two rules in Rebalance

- Black Uncle Rotation
- Red Uncle Color-Flip

After Rotation

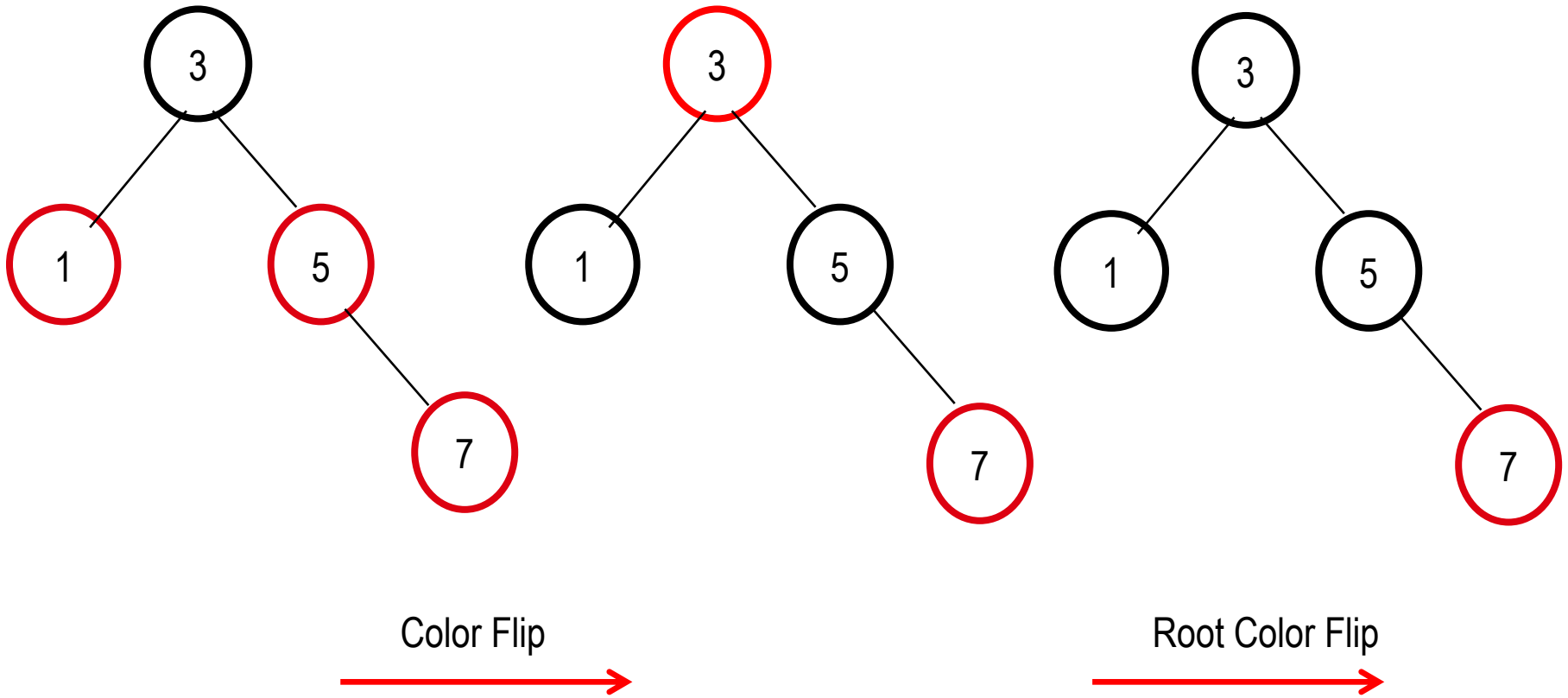


After Color-Flip



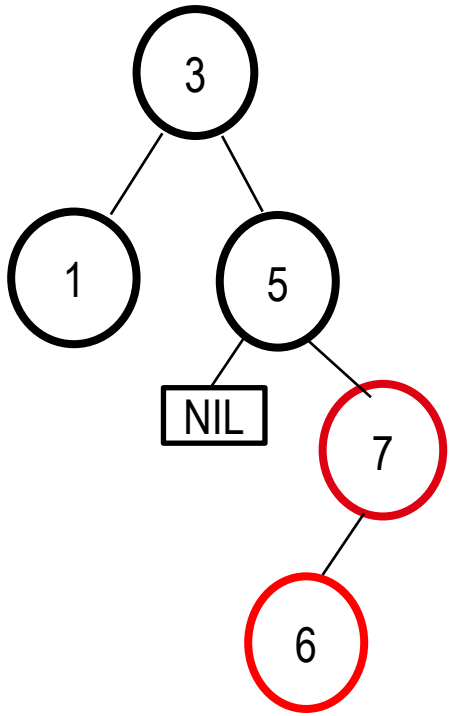
Red Black Tree Example

3, 1, 5, 7, 6

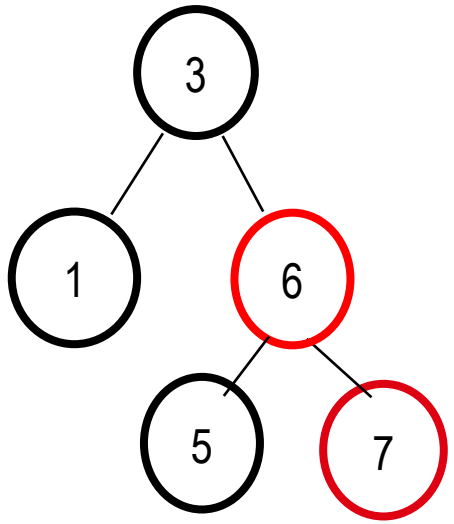


Red Black Tree Example

3, 1, 5, 7, 6



Right/Left Rotation

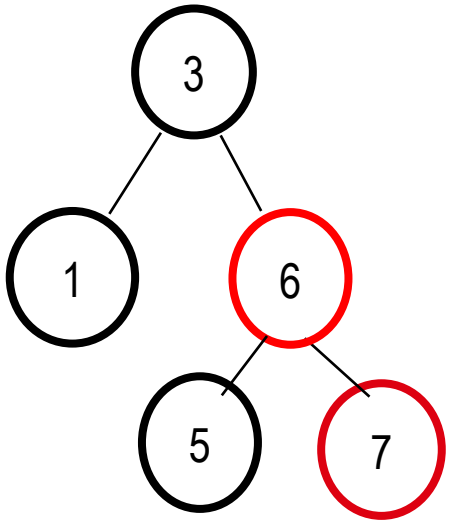


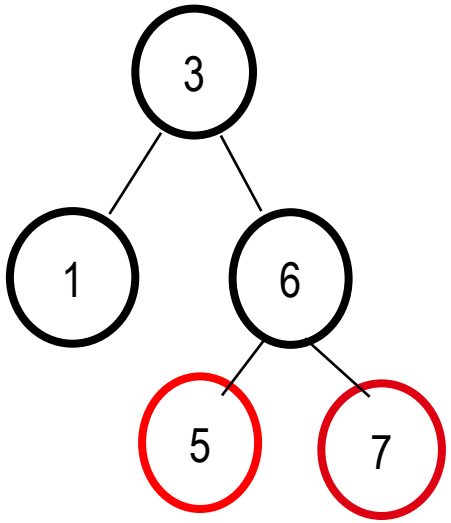
The three has black NIL at node 5.
In this case Black uncle at 6

This case, uncle of node 7 is black. So, black uncle rotation.

Red Black Tree Example

3, 1, 5, 7, 6





This case, uncle of node 7 is black. So, black uncle rotation.

Red-Black Tree Rebalance

- Introduction to Algorithms
  - https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-046j-introduction-to-algorithms-sma-5503-fall-2005/video-lectures/lecture-10-red-black-trees-rotations-insertions-deletions/lec10.pdf

ZyBooks RBTree Implementation

- Study all the pseudocodes and utility functions in ZyBooks Chap 6!!!