Coord definitions

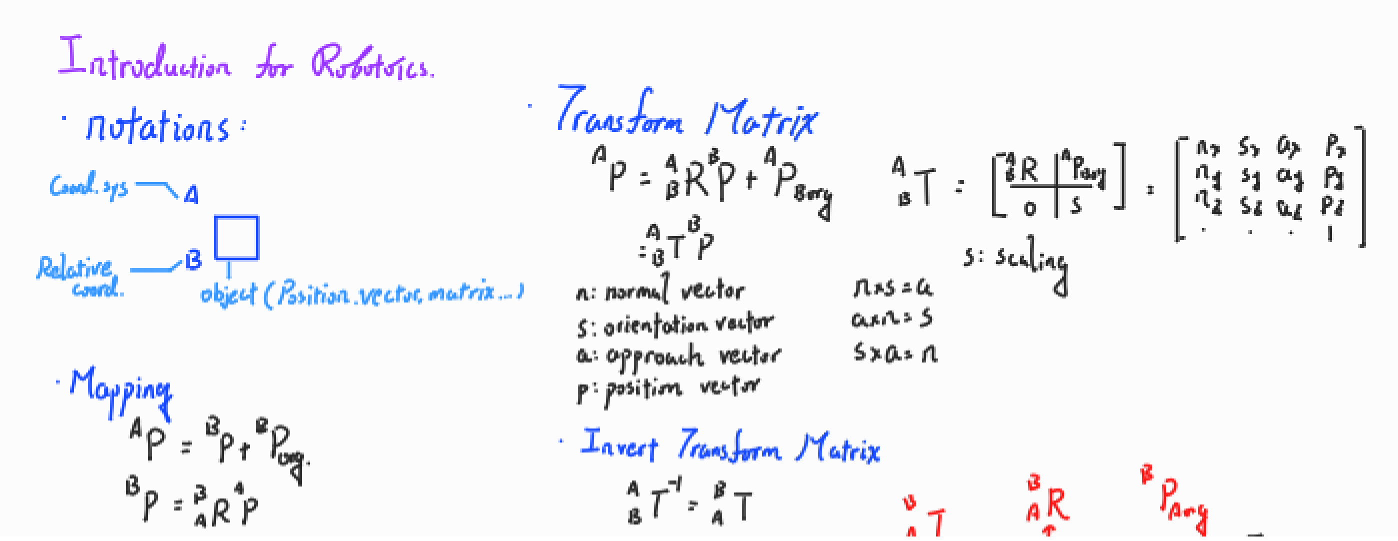
Coords and Kinematic Definitions

§ Coords and Kinematic

- Frames
  - i. represents in symbol {Frame name}
  - -list:
    - World Frame $\{W\}$
    - Robot Frame $\{R\}$
      - Origin $\langle 0,0,0 \rangle$
      - imu center.
      - $+X$: front. $+Y$: Left. $+Z$: up
    - Module Frame $\{M_i\}$
      - $(0,0,0)_{\{M_i\}} = (\pm l_{body}, \pm W_{body}, + d_{abad})_{\{R\}}$
      - $\left.\begin{aligned} \langle 0,0,1 \rangle_{\{M_i\}} &= \langle 1,0,0 \rangle_{\{R\}} \\ \langle 1,0,0 \rangle_{\{M_i\}} &= \langle 0,1,0 \rangle_{\{R\}} \end{aligned}\right\} i \in 0,3$
      - $\left.\begin{aligned} \langle 0,0,1 \rangle_{\{M_i\}} &= \langle -1,0,0 \rangle_{\{R\}} \\ \langle 1,0,0 \rangle_{\{M_i\}} &= \langle 0,-1,0 \rangle_{\{R\}} \end{aligned}\right\} i \in 1,2$
    - Leg Frame $\{L_i\}$ $\quad i \in \{ABCD\}$
      - $(0,0,0)_{\{L_i\}} = (d_{wheel}, 0, 0)_{\{M_i\}} \quad i \in \{0 \sim 3\}$
      - $\langle 0,0,1 \rangle_{\{L_i\}} = \langle 1,0,0 \rangle_{\{M_i\}} \quad i \in \{0,3\}$
      - $\langle 0,0,1 \rangle_{\{L_i\}} = \langle -1,0,0 \rangle_{\{M_i\}} \quad i \in \{1,2\}$
      - $\langle 0,1,0 \rangle_{\{L_i\}} = \langle 0,0,1 \rangle_{\{W\}}$

- F.K.
  - Known. F.K. In 矢状面. Rewrite in Func $P(\alpha, \theta, \beta)$
    - input: α, θ, β.
    - output: $\langle x, y \rangle$ in $\{L_i\}_{(i = {A\sim D \atop 0\sim 3})}$.
- Transfer to Module Frame:
  - add offset (wheel Axial offset) d_{wheel}
    - $\langle 0,0,0 \rangle_{\{L_i\}} = \langle d_{wheel}, 0, 0 \rangle$

0. Notation and Frame Representation



To maintain consistency with the derivation notes, we adopt the following symbolic conventions:

Point and Vector Representation

A physical entity is represented by its components relative to a reference frame $\{F\}$.

- **Position (Point):** Defined using square brackets.
  - Notation: $[x, y, z]^T_{\{F\}}$
- **Direction (Unit Vector/Axis):** Defined using angle brackets.
  - Notation: $\langle x, y, z \rangle_{\{F\}}$

Coordinate Transformation

The transformation of a point from frame $\{A\}$ to frame $\{B\}$ is described by a homogeneous transformation matrix $\mathbf{T}_A^B$.

$$[x, y, z, 1]^T_{\{B\}} = \mathbf{T}_A^B \cdot [x, y, z, 1]^T_{\{A\}}$$

Where $\mathbf{T}_A^B$ encapsulates the relative rotation and translation between the two frames.

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1. Coordinate Frames

In this system, each coordinate frame is represented by the symbol `{FrameName}`.

World Frame {W}

The global reference frame used for absolute positioning.

Robot Frame {R}

- **Origin:** Located at the IMU measurement center.
  - $O_R = (0, 0, 0)_R$
- **Axis Orientation:**
  - +X: Front
  - +Y: Left
  - +Z: Up

Module Frame {Mi}

The module frames define the base for each limb assembly.

Leg Mapping (i -> Leg)

Mapping the numerical index i to the physical leg location:

Index i	Module ID	Leg ID	Name	Group
0	A	**FL**	Front-Left	Left Side
1	B	**FR**	Front-Right	Right Side
2	C	**RR**	Rear-Right	Right Side
3	D	**RL**	Rear-Left	Left Side

Origin in {R}

Defining geometric placement based on quadrants (+X Front, +Y Left):

- **FL (0)**: $(+\frac{1}{2}L_{body}, +\frac{1}{2}W_{body}, +d_{abad})_R$
- **FR (1)**: $(+\frac{1}{2}L_{body}, -\frac{1}{2}W_{body}, +d_{abad})_R$
- **RL (3)**: $(-\frac{1}{2}L_{body}, +\frac{1}{2}W_{body}, +d_{abad})_R$
- **RR (2)**: $(-\frac{1}{2}L_{body}, -\frac{1}{2}W_{body}, +d_{abad})_R$

Orientation

Instead of defining explicit basis vectors, the orientation is defined by the mounting angle γ:

> **Definition:** γ is the angle between the **YZ plane** in frame $\{M_i\}$ and the **XZ plane** in frame $\{R\}$, measured along the z-axis of $\{M_i\}$.

- **Actuation:** The angle γ is actively controlled by the **ABAD actuators** for each leg, allowing the leg plane to rotate relative to the robot body.
- **Left Legs ($i \in \{0, 3\}$)**: $Z_{\{M_i\}}$ aligns with $+X_{\{R\}}$.
- **Right Legs ($i \in \{1, 2\}$)**: $Z_{\{M_i\}}$ aligns with $-X_{\{R\}}$.

Leg Frame $\{L_i\}$

Defined for each leg $i \in \{0, 1, 2, 3\}$.

- **Origin in $\{M_i\}$:**

$$(0, 0, 0)_{\{L_i\}} = (d_{wheel}, 0, 0)_{\{M_i\}}$$

- **Orientation relative to $\{M_i\}$:**
  The orientation of $\{L_i\}$ is coupled with $\{M_i\}$, and thus its absolute orientation in $\{R\}$ is a dynamic function of the ABAD joint angle γ_i.
    - **For Left Legs ($i \in \{0, 3\}$):**

$$\langle 0, 0, 1 \rangle_{\{L_i\}} = \langle 1, 0, 0 \rangle_{\{M_i\}}$$

$$\langle 1, 0, 0 \rangle_{\{L_i\}} = \langle 0, 0, -1 \rangle_{\{M_i\}}$$

- **For Right Legs ($i \in \{1, 2\}$):**

$$\langle 0, 0, 1 \rangle_{\{L_i\}} = \langle -1, 0, 0 \rangle_{\{M_i\}}$$

$$\langle 1, 0, 0 \rangle_{\{L_i\}} = \langle 0, 0, 1 \rangle_{\{M_i\}}$$
