

★ Member-only story

Time Series, Signals, & the Fourier Transform

An introduction and primer for future posts



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In my next three posts, I will introduce and discuss the Fourier and Wavelet Transforms, which extract information on how a quantity oscillates over time. These methods have almost endless applications, a few of which I describe here. *

Example code link can be found in the “Resources” section below.

Time Series, Signals, & the Fourier Transform | Introduction



Time Series

Everyone is undoubtedly familiar with time series, even if you have not heard the term. A **time series** is simply a **set of values ordered by time**. For instance, stock index prices are usually depicted as price vs. time plots on your favorite news network. Another example is a 7-day forecast, which shows temperature highs over several days. Time series are a natural way to represent data.

Market Summary > Nasdaq Composite

INDEXNASDAQ: .IXIC

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10,664.70 -128.58 (1.19%) ↓

Sep 21, 12:32 PM EDT · Disclaimer

1 day

5 days

1 month

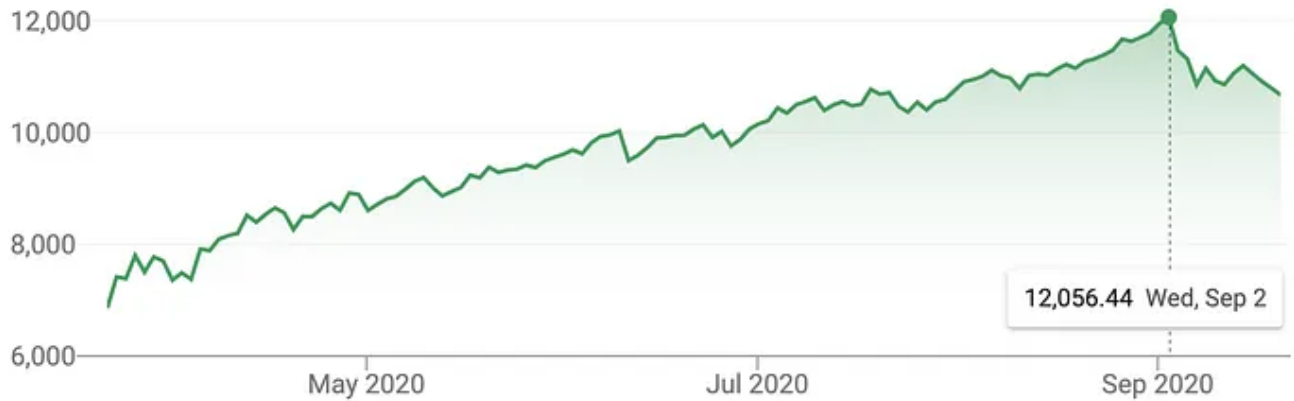
6 months

YTD

1 year

5 years

Max



Open
High

10,610.14
10,683.94

Low

10,519.49



[More about Nasdaq Composite](#)

NASDAQ index price over time is a familiar time series. Image captured from Google search of "NASDAQ".

Richardson, TX

Monday 11:00 AM

Cloudy



72°F | °C

Precipitation: 15%

Humidity: 72%

Wind: 12 mph

Temperature

Precipitation

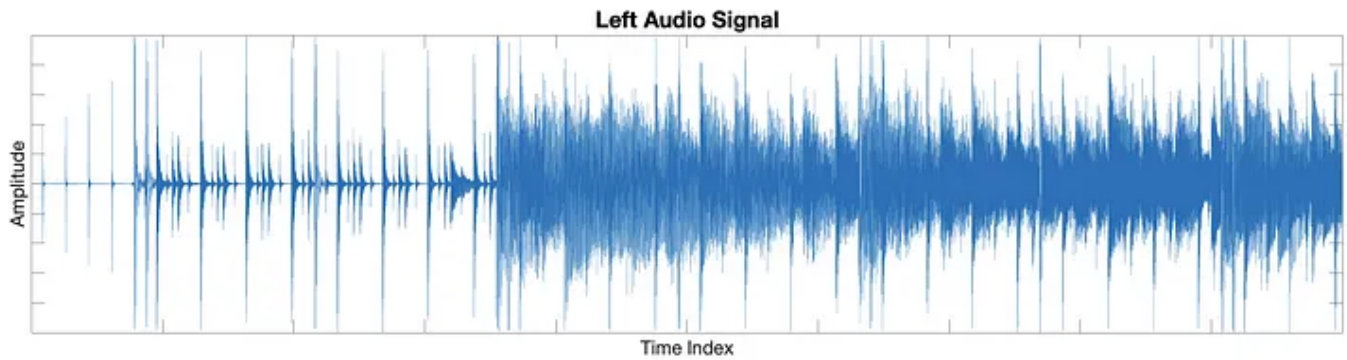
Wind



Temperature highs and lows in a 7-day forecast form a time series. Image captured from Google search of “weather”.

Signals

Signals are a type of time series. More specifically, **signals** are **time-varying quantities that represent physical events**. Two fundamental properties of signals are: amplitude and frequency. The amplitude of a signal is its magnitude e.g. the loudness of an audio signal. A signal’s frequency characterizes its oscillation in time e.g. the pitch emitted by a guitar.



Audio signal waveform of music. Signal is left part of stereo mix. Image by author.

Implicitly, when going from a continuous signal (an infinite set of quantities) to a discrete signal (a finite set of quantities) information is lost. Thus, a discrete signal is often an approximation of a continuous signal. For example, consider capturing temperature fluctuations in an office to run an HVAC system. At any moment, the office will have some average ambient temperature. Suppose a set of thermometers is used to measure this temperature every hour. This measurement would generate a discrete signal approximating the true temperature fluctuations in the office. It is reasonable to suspect that these hourly recordings of temperature would result in poor air conditioning since temperatures would likely change over time scales much smaller than an hour. If so, how frequently should the temperature be measured? This question is answered formally using the Nyquist Theorem. Which states to capture a continuous signal reliably, the rate at which information is recorded, called the sampling rate, must be twice the frequency of the signal of interest. This is why audio is typically sampled at 44,100 Hz, since the upper limit to human hearing is about 20,000 Hz.

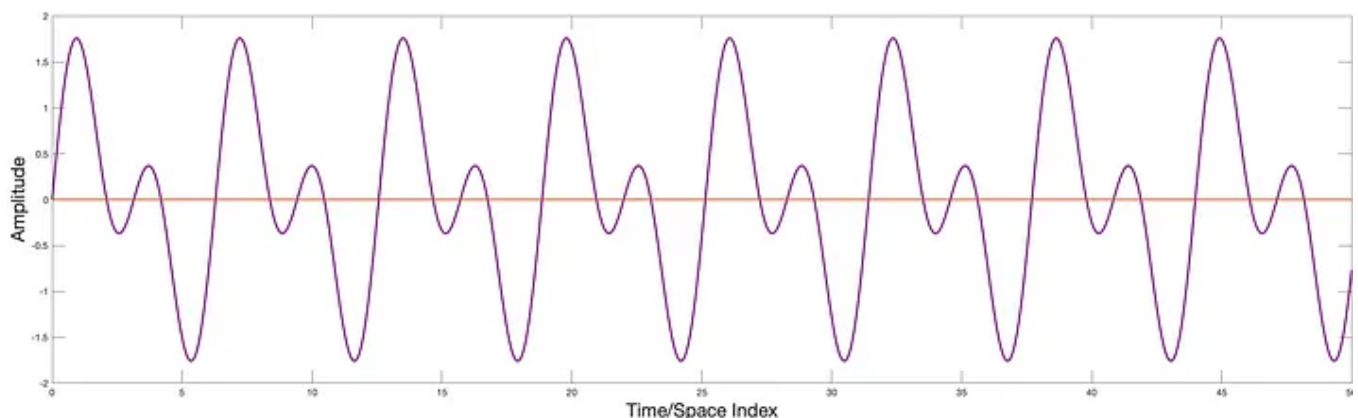
In practice, capturing meaningful information from an environment may not be so simple. Real-world signals are commonly aperiodic, noisy, and influenced by multiple sources. Extracting useful information from these signals is a fundamental goal of signal processing.

The Fourier Transform

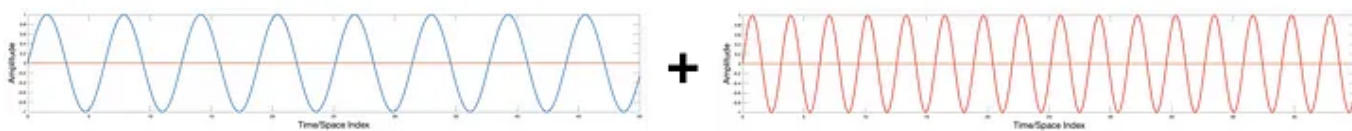
$$f(k) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i k x} dx$$

A definition of the Fourier Transform. Other conventions exist which differ by a prefactor. Image by author.

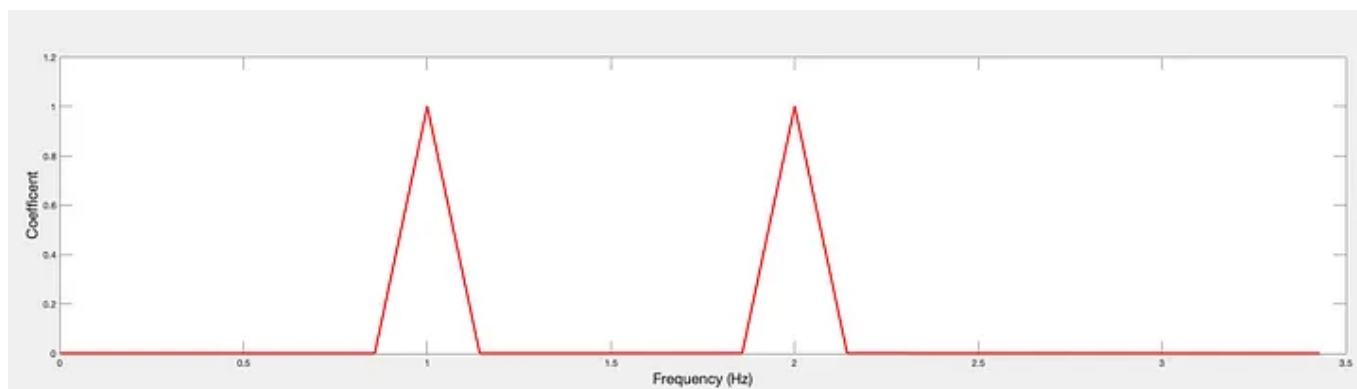
At the core of signal processing is the **Fourier Transform (FT)**. The FT **decomposes a function into sines and cosines** i.e. waves. In theory, any function can be represented in this way, that is, as a sum of (possibly infinite) sine and cosine functions of different amplitudes and frequencies. A toy example is given below. The code can be found at the accompanying [GitHub repo](#).



Toy example signal. Image by author.



Constituent frequencies of toy example: In words, the toy signal can be represented as the sum of 2 sine functions of frequencies 1 and 2 Hz. Image by author.



One sided power spectrum of toy signal: Two prominent peaks of equal magnitude are present at $f=1$ and 2 Hz corresponding to signal decomposition into two sine functions. Image by author.

Applications

The Fourier Transform has countless applications in spectral analysis, solving differential equations, and A/V production.

Audio Production

Equalization (EQ) is an essential part of audio production. The volume of specified frequency ranges can be turned up or down, which alters how the audio sounds. A basic use of EQ is removing background noise from an audio file.

Open in app ↗



Search



Write

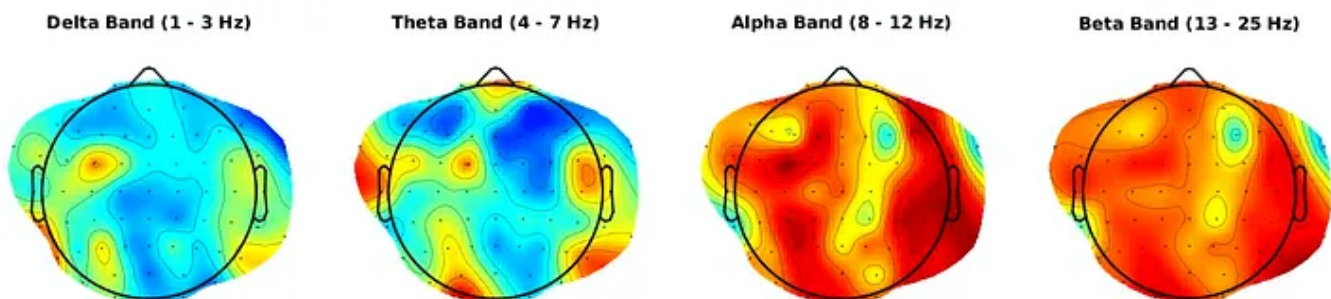




Top panel shows an audio signal. Bottom panel shows the EQ, which increases or decreases the gain of frequency ranges. Image by author.

EEG

EEG is a technique for recording electrical activity from the brain. Spectral analysis of EEG is widely used, that is, the examination of a signal via constituent frequency energies. A widely used convention of frequency bands in EEG is given below.



Conclusion

Time series and signals are natural ways to organize data. The Fourier Transform extracts frequency information embedded in data. There are countless use cases for this approach in fields such as: audio engineering, physics, and data science. The Fourier Transform is discretized and made computationally efficient for practical applications via the **Fast-Fourier Transform (FFT)** algorithm. The FFT will be discussed in my next post.

👉 More in the series: [FFT](#) | [Wavelet Transform](#)

The Fast-Fourier Transform (FFT)

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The Data Entrepreneurs