Comparing Means

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Online Workshop
Hands-on Biostatistics
July, 2020

Overview

- Introduction to hypotheses testing
- Student t-test
    - One-sample t-test
    - Paired samples t-test
    - Independent samples t-test
- One-way ANOVA

Statistical hypothesis testing

Statistical hypothesis testing is defined as:

Assessing evidence provided by the data against the null hypothesis.

Step 1	Step 2	Step 3	Step 4
• ***Formulate the hypotheses***: • H_0 – null and H_1- alternative	• Collect relevant data and summarize them.	• Test how likely it is to observe data we obtained, if null hypotheses is true. ***Compute test statistics***	• ***Compute p-value*** and make our decision.

Hypotheses testing: Type I and II errors

- The probability of a **type I error** is the probability of rejecting the null hypothesis when H_0 is true. Is denoted by **α** and is commonly referred to as the **significance level of a test**.

- The probability of a **type II error** is the probability of accepting the null hypothesis when H_1 is true. Is denoted by β and is highly affected by the sample size.

- The power of a test is defined as 1 - β

or 1 - probability of a type II error = $P(\text{rejecting } H_0 | H_1 \text{ true})$

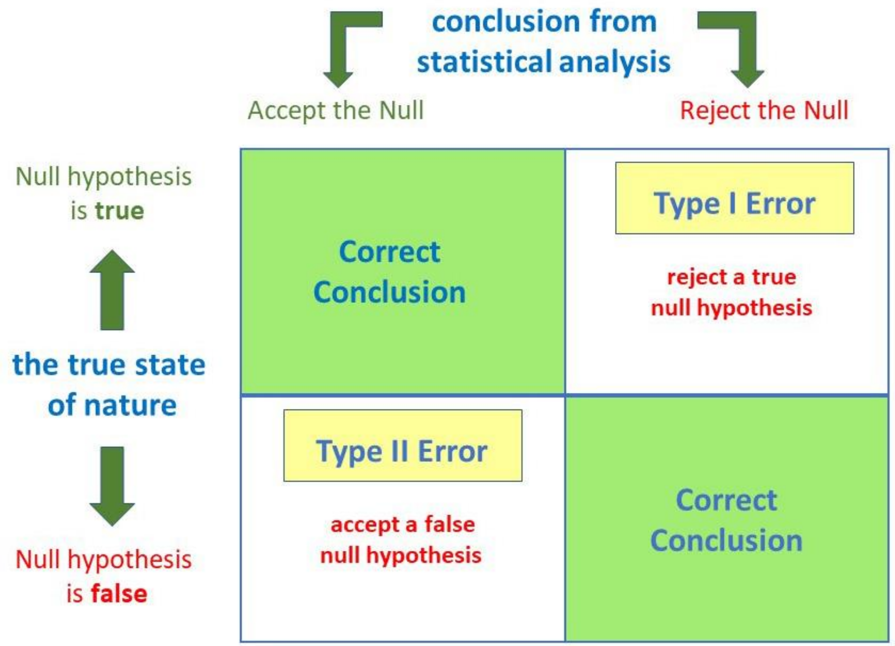


Image source: simplypsychology.org

Simple hypotheses testing: One-sample t-test

1. Hypotheses (two sided)

H_0: $\mu = \mu_0$ vs. H_1: $\mu \neq \mu_0$

2. Test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

3. Compute the p-value

$$p = \begin{cases} 2 \times Pr\left(t_{n-1} \leq t\right), \text{if } t \leq 0 \\ 2 \times \left[1 - Pr\left(t_{n-1} \leq t\right)\right], \text{if } t > 0 \end{cases}$$

4. Decision

p<0.05 Reject H_0

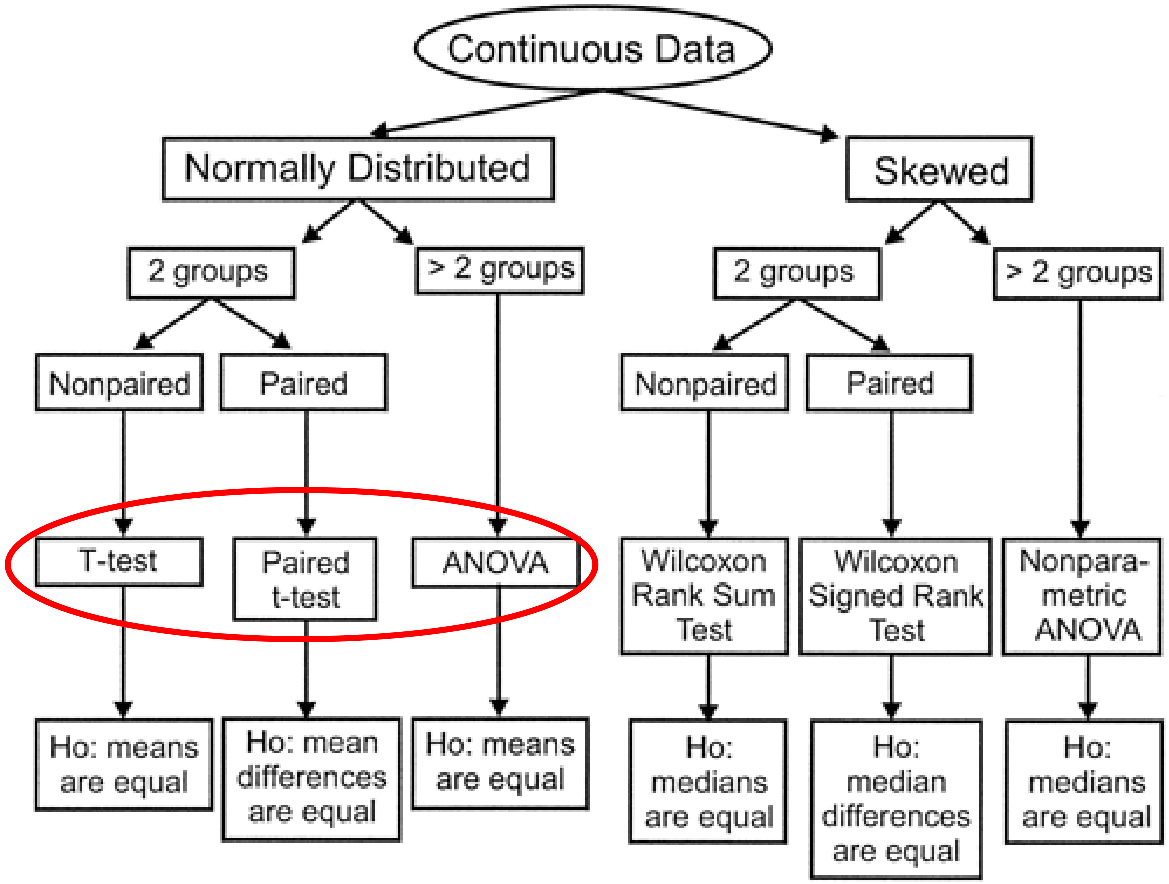
P>0.05 Accept H0

Guidelines for Judging the Significance of p-value

- **Guidelines for Judging the Significance of a p-Value**

    - If $0.01 < p < 0.05$, then the results are significant
    - If $p < 0.01$, then the results are highly significant
    - If $p > 0.05$, then the results are not significant

Which statistical test to perform?



Paired Samples t-test

- The paired t-test is used to determine whether the mean of a dependent variable is the same in two related groups of the independent variable:

- Paired/related groups mean that same individuals are measured at two different "time points" or under two different "conditions"

- **Assumptions:**

1. Your dependent variable should be continuous.
2. Your independent variable should consist of two categorical, related groups.
3. The differences between pairs should be normally distributed

Paired Samples t-test

1. Hypotheses

H_0: $\Delta = 0$ vs. H_1: $\Delta \neq 0$

2. Test statistics

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

- *Has a $t_{(n-1)}$ distribution*

3. Decision

p-value > 0.05 accept H_0

- The difference is not significantly different from zero.

P-value < 0.05 reject H_0

- The difference is significantly different from zero.

Test for Normality

- Shapiro Wilk or Kolmogorov Smirnov test
- Perform one of them in R and decide based on p-value.
  - If p-value > 0.05 the data is normally distributed
  - If p-value < 0.05 the data is skewed or not normally distributed.

Independent Samples t-test

- The independent-samples t-test compares the means between two unrelated groups on the same continuous, dependent variable.

- **Assumptions**

1. Your dependent variable should be measured on a continuous scale.
2. Your independent variable should consist of two categorical, independent groups (i.e., gender).
3. There should be no significant outliers.
4. Your dependent variable should be approximately normally distributed for each group of the independent variable.
5. There is need to test homogeneity of variances.

Test homogeneity of variances

- Levene's test
- Hypotheses: H_0: $\sigma_1^2 = \sigma_2^2$ vs. H_1 : $\sigma_1^2 \neq \sigma_2^2$
- *Test statistics:* $F = s_1^2 / s_2^2 \sim F_{(n1-1;n2-1)}$ distribution
- Decision

p-value > 0.05 accept Ho – equal variances

P-value < 0.05 reject H0 – unequal variances

Independent Samples t-test: Equal variances assumed

1. Hypotheses

H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 \neq \mu_2$.

1. Test statistics

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \sim t_{(n1+n2-2)} \textit{ distribution}$$

$$\text{where} \quad s = \sqrt{\left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right] / (n_1 + n_2 - 2)}$$

3. Decision

p-value > 0.05 accept H_0

- The populations means are not significantly different from each other.

P-value < 0.05 reject H_1

- The populations means are significantly different from each other.

Independent Samples t-test: Unequal variances assumed

1. Hypotheses

H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 \neq \mu_2$.

2. Test statistics

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$ ~ $t_{(d')}$ *distribution*

Compute the approximate degrees of freedom d', where

$$d' = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\left(s_1^2/n_1\right)^2/(n_1 - 1) + \left(s_2^2/n_2\right)^2/(n_2 - 1)}$$

3. Decision

p-value > 0.05 accept H_0

- The populations means are not significantly different from each other.

P-value < 0.05 reject H_1

- The populations means are significantly different from each other.

One-way ANOVA

- The one-way analysis of variance (ANOVA) is used to determine whether there are any statistically significant differences between the means of two or more independent (unrelated) groups (although you tend to only see it used when there are a minimum of three, rather than two groups).

- **Assumptions**

1. Your dependent variable should be measured on a continuous scale.
2. Your independent variable should consist of two or more categorical, independent groups.
3. There should be no significant outliers.
4. Your dependent variable should be approximately normally distributed for each group of the independent variable.
5. There is need to test homogeneity of variances.

One-way ANOVA

1. **Hypotheses**

H_0: $\mu_1 = \mu_2 = \ldots = \mu_a$

H_1: At least two means are different

2. **Test statistics**

$F = s_b^2 / s_w^2$

$\sim F_{(a-1;n-a)}$ distribution

3. Decision

- p-value > 0.05 accept H0

Means are statistically equal.

- p-value < 0.05

We can reject H_0, that all the means are equal, and can conclude that at least two of the means are significantly different. These results are displayed in an ANOVA table (we discuss it in R).

One-way ANOVA: Post-hoc analysis

- If H_0 is rejected, we should perform a post-hoc analysis to see which groups are different.

- We will discuss and interpret this in R

References/Useful links

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