# A Monte Carlo Analysis of Hypothetical Multi-Line Slot Machine Play

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**Abstract:** Behavioural research into slot machine gambling tends to focus on characteristics of the gambler or on qualitative aspects of the slot machine such as audiovisual displays and bonus features. In this paper we take a different approach by using Monte Carlo simulation to relate hypothetical slot machine gambling behaviour to the statistical characteristics of the slot machines themselves. The measures we use – expected monetary win, volatility of payouts, and the probability that any single play returns a winning result – have the advantage that they are mathematically precise and can be linked to psychological risk and return criteria that people may look to as they decide both whether to gamble or not and how to play.

## Introduction

Slot machines represent an enormously popular and profitable industry[1]. Yet despite their popularity and status as money-spinners there has been relatively little attempt outside of the gambling industry itself to investigate the statistical characteristics of slot machines, or to relate these statistical characteristics to the behaviour and preferences of those playing the slot machines. Thus, behavioural research into the roots of persistent slot machine play has tended to focus more on the characteristics of the gambler (e.g. Ladouceur et al., 2003; Rodgers, 1997) or on the qualitative aspects of the slot machine such as audiovisual displays (Loba et al., 2001) and bonus features (Moodie and Finnigan, 2005), than on the statistical characteristics of the slot machines themselves. Our main aim in this paper is to use a Monte Carlo simulation approach to evaluate slot machine performance in terms of three fundamental measures of risk and return. The three measures we use are the expected monetary value of any payouts, the volatility or uncertainty around those payouts, and the excitement or entertainment value of play, as operationalised by the probability that a single play results in a payout being won.

For a given slot machine, these three measures vary as a function of various aspects of game play that must be selected by the player – specifically, the number of lines played and the amount bet pet line. Each variety of slot machine can therefore be depicted by a three-dimensional risk-return curve representing the various trade-offs between risk and return that are presented to a player through his or her selection of a number of lines to play and a total amount to bet. Different slot machine varieties can be depicted by a set of three-dimensional risk-return curves, which allows a casino to describe various forms of hypothetical gambling behaviour that they are catering for with their portfolio of machines. It is also possible to use the risk-return framework to investigate different types of gambling behaviour that might theoretically occur.

The modelling of the slot machines themselves has also become increasingly complex as the slot machine has evolved from a mechanical device – the classic spinning reels activated by pulling a lever – to something more like a computer game, with a push of a button activating a random number generator that generates the graphical display that ultimately determines the payout. While this in itself does not complicate the statistical analysis of a machine, the evolution of graphical interfaces has led to two developments that do complicate the modelling: first, 'bonus' games have become increasingly complex, and now often encompass simple strategy elements that make even the calculation of an expected value difficult; second, the number of possible lines that one may play simultaneously i.e. within a single 'spin of the reels', has become increasingly large – whereas the norm used to be 3 or 5, it is now possible to play many more lines. Our second aim in this paper is to highlight the important implications that this second development has for modelling in the slot machine industry, by using the volatility[2] of payouts and the probability of a winning play. As we will discuss later, the recent trend towards having a large number of possible winning lines makes it difficult to analytically determine the variance of the payouts made by a machine. We again use a simulation approach to illustrate the effect of independence assumptions on risk and return.

The remainder of the paper is structured as follows. The next section describes the problems of defining 'risk' and 'return' in the case of slot machines, and identifies suitable measures of each. It also discusses the implication of the assumption of line independence that is commonly made in the casino industry when computing risk and return measures. Following that, the three measures are used to construct simulated risk-return curves for a set of 4 different hypothetical slot machines. A final section ends the paper with a discussion of some of the implications of the earlier findings and some ideas for future research.

**Quantifying Risk and Return for Slot Machine Players**

From a pure expected value perspective, slot machine gambling is something that should simply not occur. Long run expected values are exclusively negative, and this is common knowledge. Furthermore most of the money lost at slot machines is not by novice or first-time gamblers, but by 'people who have bet, and generally lost, in the past' (Gilovich, 1983). Explanations of why people persist in general gambling activities can be categorised into two groups: the first group of explanations center on the claim that the act of gambling is a form of entertainment, and that the excitement and action offered in most cases more than offsets the financial loss (e.g. Campbell, 1976; Herman, 1976; Moodie and Finnigan, 2005). Excitement is the most frequently given self-reported motivation for regular slot machine playing (Dumont and Ladouceur, 1990), and subsequent research has refined the basic entertainment claim by showing that autonomic arousal occurs when playing for money (Ladouceur et al., 2003), is higher in frequent than infrequent gamblers (Moodie and Finnigan, 2005), is higher for winners than for losers (Wulfert et al., 2005), and can also be triggered by specialist characteristics of a machine such as bonuses and special features (Moodie and Finnigan, 2005). Recent research has added neurophysiological support for the claim that arousal is a key feature in slot machine gambling by identifying particular neurons in the posterior cingulate cortex that respond to the riskiness i.e. variability, of a particular choice, independent of the expected return (McCoy and Platt, 2005).

The second group of explanations proposes that the persistence is due to cognitive biases in which gains are overvalued and remembered, and losses are explained away or at least discounted (e.g. Gilovich, 1983; Rodgers, 1997; Walker, 1992). The cognitive biases explanation is rooted in the broader heuristics and biases program initiated by Kahneman and Tversky (e.g. Kahneman et al., 1982), which subsequent research has applied to the special case of gambling judgements, and to subsets of gamblers such as heavy gamblers (Toneatto et al., 1997), adolescents (Delfabbro et al., 2006), and lottery players (Rodgers, 1997). This research essentially shows that gamblers make widespread use of cognitive distortions and biases, and that heavy gamblers in particular are prone to what is termed 'active illusory control', the notion that one can increase the odds of winning a chance event through perceived personal attributes like being lucky or skilful (Toneatto et al., 1997).

A fair summary of the field of gambling research might be that strong evidence exists in support of both explanations, and that there is no reason to believe that the two are mutually incompatible. In all likelihood, the cause of any particular player's persistent gambling would be some combination of the two. A related problem in the specific context of slot machine play which we address in the current paper is why people might choose to play different numbers of winning lines or more than the minimum possible bet per line. This provides a direct inroad into questions of preferences for risk and return.

*Risk and Return Under the Assumption of Line Independence*

If we let $W_{a,b}$ be a random variable representing the amount won from a single spin of *a* lines with a bet of *b* per line, net of the amount bet (so that if no payout is received then $W_{a,b} = -ab$), then $W_{a,b} = bW_{a,1}$. If we further assume that the winning lines are independent of one another i.e. that $W_{a,1}$ is equivalent to the sum of *a* independent realisations of $W_{1,1}$ then

$$W_{a,b} = b \sum_{k=1}^{a} W_{1,1}^{(k)} \qquad (1)$$

where we index the realisations with the superscript *k*. The expectation and variance of $W_{a,b}$ are then given by $E[W_{a,b}] = E[W_{1,1}]$ and $\sigma^2_{W_{a,b}} = b^2\, a\, \sigma_{W_{1,1}}$ respectively[3]. The standard deviation of payouts is therefore linear in the amount bet per line and follows the square root of the number of lines played *a*. As an example, consider a gambler who has decided to play for a certain length of time, say 100 spins. How might such a person choose to play? Firstly, they may choose to play a single-line game with the minimum bet of 1 unit per line. For this player, the expected return is $100\,E[\,W_{1,1}\,]$ with volatility of payout $10\,\sigma_{1,1}$ (i.e. $\sqrt{100} * 1 * 1^2 * \sigma_{1,1}$ for the standard deviation of *n*=100 independent outcomes). Secondly, they may play using 5 lines, but keep the minimum unit bet per line. This player has an expected return of $500\,E[W_{1,1}]$ with volatility of payout $10\sqrt{5}\,\sigma_{1,1}$. A third type of player may choose to play only a single line at a time, but to increase the bet per line, say to 5 units. The expected return for this player is the same as the second player, $500\,E[W_{1,1}]$, but the volatility of payout is now $50\,\sigma_{1,1}$, substantially higher than for the other two types of players. Of course other combinations of played lines and bets per line are possible, but the three cases described here suffice for a discussion of the main issues.

There is one important comment to make on the subject of modelling expected monetary return and volatility. The definition of volatility used above is the standard deviation of the payouts themselves, which is sensitive to the amount bet, as in the above example. However, it is perhaps more meaningful to scale the volatility of the payouts by the amount bet, and to use those adjusted volatilities as, for example, is commonly done in portfolio analysis. This is the approach that we generally take in the current paper, although raw volatilities can be easily obtained from the adjusted volatilities and vice versa. Increasing the number of lines played, or the bet per line, will naturally lead to an increase in raw volatilities, under the important assumption that the denomination or quantum of minimum bet remains the same (e.g. 1 token is \$1, or 1 token is 50*c*). Note that if denomination is *not* held constant, it is possible to reduce volatility by playing more lines, by reducing the bet per line to 1/*a* on a play of *a* lines, giving $\sigma^2_{W_{a,1/a}} = \sigma^2_{W_{1,1}}/a$. This would be the view of those advocating risk reduction in, say, roulette, where the quantum of bet is highly variable. Our view of slot machines is that the quantum of bet does not constitute a major influence on slot machine play, motivated by the predominance in both number and share of total revenues of a very limited set of low denominations. Barr and Kantor (2003) point out that on the Las Vegas strip in the US, 80% of all machines are either of 25*c* or \$1 denominations, with denominations of \$5 or more making up just 2% of all machines, while in South Africa the case is even more extreme, with 78% of machines being played with a unit bet of either 1 South African Rand or 50 South African cents (about 15 and 7 US cents respectively at the time of writing). An analysis using a conventional risk-return framework is difficult, because the strategy with the lowest raw volatility (single-line play with minimum bet) also has the highest expected return, since $E[W_{1,1}] < 0$. Yet previous research (Walker, 2004) as well as our own observations of gambling establishments make it clear that this type of behaviour is in the minority, and that most players play multiple lines with the minimum number of credits. As pointed out in Dowling et al. (2005), this suggests that 'players tend to prefer more frequent reinforcement rather than larger and less frequent reinforcement' (although there is also evidence of a "big win effect" (see Kassinove and Schare, 2001)). In fact, both the 'entertainment' and 'overvaluation of wins' explanations of the persistence of gambling behaviour suggest that a 'return' criterion that players might use to evaluate and choose between betting strategies might not be so much expected monetary return – though this is surely a consideration – as some sort of excitement or action that is generated by the act of winning. A simple way to operationalise expected entertainment is to use the probability of winning *any* kind of payout on a particular play, the so-called hit probability, denoted by $\phi_{hit}$ . The motivation behind the use of the hit probability is that it is the act of winning some kind of payout that contributes significantly to the excitement or action for the player – including audiovisual activities such as lights going off, sounds of coins dropping, and so on, which have been shown to be important components of the entertainment (Loba et al., 2001) – and that long periods of inactivity are likely to be construed as boring intervals between the exciting winning plays, and to detract from the entertainment experience. Payne (2005) has recently provided strong evidence that overall probabilities of winning or losing are important influences on risky choice in fairly simple (five-outcome) gambles, and while it may be difficult for a player to give any kind of accurate numerical estimate of the hit probability in the slot-machine case, particularly bearing in mind the complexity of the gamble and the tendency discussed earlier to bias the evaluation upwards in favour of winning (Gilovich, 1983), it would be fairly easy to notice that exciting events i.e. winning plays, tend to occur far more frequently when more lines are played. The calculation of a multiple-line hit probability is slightly more complicated because the probability of scatter wins does not increase with the number of lines played. Let $\phi_{sc}$ be the probability of a scatter win, and $\phi_{nsc}$ be the single-line probability of any non-scatter win (so that the two probabilities sum to the single-line hit probability $\phi_{hit}$ ). Under the assumption of line independence, winning events on multiple lines are independent of one another, so that the total number of successes in a play of $a$ lines is binomially distributed and the multiple-line hit probability can be calculated as

$$\phi_{hit}(a,b) = 1 - (1 - \phi_{nsc})^a + \phi_{sc} \qquad (2)$$

Previous research suggests that people are far more comfortable reasoning with raw frequencies than probabilities (e.g. Gigerenzer, 1996), it might be possible that players can even give quite accurate estimates of the relative differences in excitement that result from different line-and-bet combinations.

Finally, note that is also possible to define an exciting event as any success in which the payout exceeds a certain threshold. In this paper, we do not explore this option, but it is clear that players can easily adjust their choices so that *any* success is experienced as exciting by choosing a machine that uses an appropriately large bet denomination, or by increasing the bet per line (which has the same affect as choosing a higher-denomination machine).

*Risk and Return Without the Assumption of Line Independence*

The previous section introduced three measures of risk and return obtained for a single-line, minimum-bet play, and then adjusted these measures to obtain values for any number of played lines and amounts bet per line using the assumption that payouts on multiple winning lines are independent of one another. It should be emphasised that these adjustments are often used in practice, particularly regarding volatility, which in its single-line form is a standard output of slot machine manufacturers and a common term of reference within the casino industry (sometimes known as core volatility), and one for which single-line results are readily available. Though the adjustments rely crucially on the assumption of line independence, we are not aware of any previous work that evaluates the effect of violating the assumption. In this section, we consider the independence assumption more closely, show how it cannot hold when any substantial number of lines are played, and discuss some practical implications for the statistical modelling of slot machines. Before turning to mathematics, however, it may be worth first giving some intuition for these results by way of an example. Essentially, as long as the lines are completely independent, playing one machine on 9 lines is the same as playing 9 machines (of that same type) with one line simultaneously. It can be appreciated for these two cases that the expected payout, the volatility of the payout and the hit percentage will all be the same as long as the lines are independent for the one machine 9-line play. However, if the lines are dependent, (because they overlap to some extent) these two different cases would not yield the same results. The expected returns will be the same but the 9 machines played once with one line simultaneously will yield a lower volatility of total payout and a higher hit percentage than that for the 9 (dependent) lines played once on one machine. In the extreme case, where all 9 lines overlapped completely, playing 9 lines on one machine would be tantamount to playing one line with a bet of 9 on that one line. This simply gears up the bet and the payout by a factor of 9 for the one machine case and will give volatilities which are significantly more than the volatility of total payout for the 9 machine case, and hit percentages that are significantly less.

Figure 1 shows three different configurations of a game in which three lines are being played[4]. In the left-hand grid, the lines are independent and $\sigma_{W_{a,b}} = b\sqrt{a}\,\sigma_{W_{1,1}}$ in the center grid they are completely dependent and $\sigma_{W_{a,b}} = a\,b\;\sigma_{W_{1,1}}$, and in the right-hand grid they are neither completely dependent nor independent and $b\sqrt{a}\,\sigma_{W_{1,1}} \le \sigma_{W_{a,b}} \le a\,b\,\sigma_{W_{1,1}}$. Modern slot machines often make use of far more lines, and there is sometimes substantial dependence between some of these lines. For games such as these, using the assumption of line independence is likely to substantially understate volatility and overstate hit probability, though expected monetary returns would remain undisturbed. This is of some concern since volatility statistics are regularly used to monitor machines to identify when machine maintenance is required because a machine's payout is outside a specified confidence interval. When a machine's payout is outside these confidence intervals, the machine is assumed not to be playing "in spec"; understated volatilities calculated from an assumption of line independence will result in too-narrow confidence intervals and hence unnecessary maintenance.

INSERT FIGURE 1

Unfortunately, the dropping of the dependence assumption means that analytical results are considerably more complex. The reasons for the difficulty are briefly sketched here using the example shown in Figure 1(c), which clearly shows that line dependence is a matter of degree, the degree of dependence between line 1 and 2 in the third figure being greater than between lines 1 and 3. If any payouts are made for two adjacent symbols in the first two positions, lines 1 and 3 will effectively be completely dependent for those payouts only; if the sequence must be three symbols long, then lines 1 and 3 are not dependent, but lines 1 and 2 are. In fact, things are slightly more complicated than this because in the case of there needing to be at least a three-symbol sequence, winnings on lines 1 and 3 will be negatively correlated since a win on one of these lines precludes a win on the other. The presence of 'wild' symbols complicates matters even further: in this example, wild symbols would mean that winning sequences could appear on both lines 1 and 3, but the probability of these would depend on not only the number of wild symbols on each reel but also on their placement e.g. whether they were next to one of the relevant line-win symbols.

The difficulty of analytical results suggests that it may be more appropriate to consider a simulation approach, and in fact it is relatively straightforward to approximate the multiple-line volatilities and hit probabilities using simulation. Every slot machine is described by the placement of symbols on its reels, a schedule of payouts (the "paytable"), and a set of line configurations for the winning lines, so that a simulation of the main game relies only on these simple inputs. For each simulated play, a random number generator selects a position on each of the reels. The displayed symbols for each reel are then the symbol in the randomly selected position, as well as the two adjacent symbols on the reel. This constitutes the simulated grid that would be observed by the slot machine player. The combination of symbols appearing along each line configuration is checked against the schedule of payouts, and any winnings on different lines are added to a scatter win determined by the number of scatter-win symbols that are counted in the grid display. The simulation stores the total amount won on each play, which can later be used to calculate empirical return, volatility and hit probability statistics. In the next section, we discuss a Monte Carlo simulation constructed in order to quantify the effects of the line independence assumption on volatility and hit probability.

## Describing Slot Machine Play using Risk and Return

Table 1 shows the specifications of 4 hypothetical slot machine games. Each machine is described by the maximum number of lines that may be played, $a_{\max}$, expected win, volatility, probability of a scatter win $\phi_{sc}$, and probability of any non-scatter win $\phi_{nsc}$ (so that the two probabilities sum to the hit probability $\phi_{hit}$). The figures are given for a single-line play with a unit bet per line.

INSERT TABLE 1

Under the assumption of line independence, the single-line values can easily be adjusted to the case of $a$ played lines with a bet of $b$ units on each played line using $\sigma_{W_{a,b}} = b\sqrt{a}\,\sigma_{W_{1,1}}$ and $\phi_{hit}(a,b) = 1-(1-\phi_{nsc})^a + \phi_{sc}$ as before. Without the assumption of line independence, a Monte Carlo simulation approach can be used to determine the multiple-line volatilities and hit probabilities. The specifications of the simulated machines we have used are as close as possible to real-world slot machines – configurations of lines and payout schedules have been taken from actual slot machines, where this information is publicly displayed on the machines. The number of positions on reels and the positions of symbols are proprietary information; we have simply randomly generated some typical cases and adjusted these by trial-and-error until appropriate machine statistics (e.g. win percentage, volatilities, and hit probability) such as those in Table 1 were obtained. For each line-and-bet combination of each of the four slot machines, 5 000 000 plays were simulated. The expected monetary return, volatility of payout, and hit probability statistics obtained are shown in Table 2[5].

INSERT TABLE 2

A particular slot machine, played using *a* lines with a bet of *b* per line, can be described by a combination of its expected monetary return, volatility of payout, and entertainment value i.e. as a single point in the expected win-volatility-hit probability space $(E[W_{a,b}], \sigma_{W_{a,b}}, \phi_{hit}(W_{a,b}))$. More interestingly, slot machine varieties can be represented by three-dimensional risk-return curves characterising their risk and return (that is, monetary and entertainment return) characteristics. These three-dimensional curves can be analysed directly, but particular interpretations or points-of-view can be obtained by projecting those curves onto the different two-dimensional subspaces that exist. Figure 2 shows all possible two-dimensional projections under assumptions of line independence and without any such assumptions.

INSERT FIGURES 2 (a) to (f)

*Effects of Assuming Line Independence*

We begin by briefly considering the effects of assuming line independence i.e. by comparing the projections in Figure 2 (a), (c) and (e), which assume line independence with those in Figure 2 (b), (d) and (f), which do not. The impact that the assumption of line independence has is clear to see. Figure 2 (a) and (b) show the obvious point that hit probabilities on all four machines are substantially overestimated when lines are assumed to be independent. More interestingly, machines *C* and *D* show a strong discontinuity at $a$ =9 with even more substantial overestimation of hit probabilities being experienced above this point. The reason for this discontinuity is that, for machine *C* and *D*, winning lines 10 and 11 overlap with winning lines 3 and 2 respectively on the first 2 reels, as shown in Figure 3. Any line win on the first $P$ =2 reels (which result in small payouts for machines *C* and *D*) must be won on both lines 10 and 3 (or 11 and 2) or not at all. Importantly, this is publicly available information that appears on real-world slot machines – note that the line configurations for machines *A* and *B*, which do not have the same sorts of overlaps, do not behave in this way. The extent to which line dependence affects hit probability has important implications for the design of enjoyable slot machines. Since line dependence greatly reduces the proportionality of hit probability, and since it is extremely unlikely that players will pick up different degrees of dependence (e.g. between machines *B* and *C*), players may feel less satisfaction playing a machine which has many dependent lines than a machine in which dependencies are kept to a minimum. It is also worth mentioning that under the assumption of line independence, one would incorrectly predict that the hypothetical player possessing the indifference curve in Figure 2(a) would prefer to play machine *A* using 9 lines. In fact, after dropping the independence assumption, it is clear from Figure 2(b) that the preferred option would be to play machine *D* using 9 lines.

INSERT FIGURE 3

Figures 2(c) and (d) show that volatilities are also greatly affected when line independence is assumed, and are increasingly understated as more lines are played. There is very little further decrease in volatility for $a$ >9. In the case of machine *C*, volatility per unit bet at $a$=9 and $a$ =25 is 3.11 and 1.87 respectively under the assumption of line independence, but 4.25 and 4.11 if line independence is not assumed and results are obtained using simulation. Machine *A*, which is the model whose hit probability was most sensitive to the assumption of line independence at $a$ =9, has a relatively more robust volatility – under line independence $\sigma_{W_{9,1}}$ =2.92 and without it $\sigma_{W_{9,1}}$ =2.45. From a practical perspective, the sensitivity of the volatilities is a more immediately applicable result, since casinos often base maintenance operations (and perhaps also financial planning) on confidence intervals obtained under assumptions of line independence. Our results indicate that this practice can lead to severe errors in estimation. In the following section, we therefore restrict attention to those projections which do not assume line independence i.e. to the second column of plots in Figure 2.

*Analysis of Hypothetical Slot Machine Play*

Figure 2(b) project the risk-return curves into the the expected win-hit probability space. The curves obtained for each slot machine are shown holding the line-bet constant at a single-unit bet i.e. $b$ =1, and for illustration purposes we have included the indifference curve of a hypothetical slot player. Labels in the figure refer to the slot machine variety i.e. *A*, *B*, *C*, or *D*, and the number of lines played i.e. 1,5,9,15,21, or 25.One might call the projection a "player's view" in that the trade-offs that are captured in this projection seem to best capture the fundamental difficulty facing a slot machine player i.e. wanting to be frequently entertained but having to pay more (and possibly be ruined more quickly) in order to achieve this. Volatilities, either of the win percentage or the payouts themselves, may also influence some decisions, but are likely to play a much smaller role since (for the slot machine player) they are not as psychologically salient or as easy to detect as either monetary return or hit probability.

Figure 2(b) also allows for a description of the behaviour of those playing in various regions of the expected win-hit probability space, since one can hypothesise and plot certain indifference curves for people playing a particular slot machine and line-and-bet combination. The clearest example is those who play a large number of lines. People playing the maximum 25 lines on machines *C* or *D*, for example, are clearly driven nearly exclusively by the excitement of winning and are willing to pay (and lose) far greater amounts for the privilege of frequent excitement. At the other extreme, players of single-line games are willing to put up with long periods of inactivity in order to minimise the amount that they spend per unit time – and hence guarantee a reasonable period of play. Between these two extremes, players can be described in terms of their entertainment-monetary return profiles depending on which machine they are playing, and how many lines they are playing. Alternatively, given a particular indifference curve, it is possible to infer which slot machine would theoretically be most suitable for that player. Because it is possible to obtain the machine curves from a manufacturer's specifications, it would be possible for a casino to ensure that it purchased a portfolio of machines that was able to satisfy a wide range of possible players' indifference curves. For example, machines *A* and *D* seem together to accommodate a fairly wide range of preferences.

Figure 2(d) shows the projection of the risk-return curves into the two-dimensional space comprising risk (volatility) and entertainment (hit probability), again holding the line-bet constant at a single-unit bet. Note that again the volatility of actual payouts refers to the standard deviation of payouts after adjusting for the amount bet to yield a volatility per unit bet. This projection becomes particularly relevant over longer periods of time whenever varieties of slot machines offer very similar expected win percentages, but differ in volatilities. The casino itself, for example, would typically judge a machine by its expected win percentage rather than the expected payout value, since in the long run a certain amount of money will be spent by patrons (regardless of line-and-bet choices) and a certain percentage of that will be taken by the casino. If slot machines offer very similar expected win percentages, as they typically do, interest shifts to other criteria, specifically the volatility around that expected win percentage and the entertainment provided to the player.

From a casino's perspective, the most desirable machines are those that offer high entertainment with low volatility. Low volatility is preferable because it speeds up convergence to the expected win percentage, whatever that may be, and highly entertaining machines are transparently desirable. The volatility-hit probability space allows casinos to identify those machines that are particularly variable for the level of entertainment that they provide, and those machines whose returns can be expected to be more stable. For example, machine *C*, which was dominated by machine *D* from the player's point of view, might be preferred to machine *D* on the basis that it provides consistently less variable payouts – the volatility per unit bet when playing 25 lines on machine *C* is 4.06, while machine *D*'s volatility for a 25-line game is more than 25% higher at 5.31.

Since volatility per unit bet decreases as the number of lines played increases, a maximum-line betting strategy will always give the lowest volatility per unit bet for that particular machine. There is thus a sense in which, from the casino's perspective, the maximum-line strategy dominates all other strategies on that particular machine (since the maximum-line play also offers the largest hit probability); casinos would certainly be happy if all their patrons played on the maximum number of lines. This being true, it is an interesting observation that the effect of having increasingly dependent lines is to reduce hit probability while simultaneously increasing volatility – exactly the opposite of what is desired from a casino's point of view. This observation would suggest that winning line configurations, which are at the discretion of slot machine manufacturers, be selected so that they are minimally dependent. Certainly, the type of dependence identified in Figure 3 is both unappealing to casinos and avoidable.

A final view is offered by projecting the risk-return curves into the remaining two-dimensional subspace, comprising expected win and volatility. This is done in Figure 2(f), which shows the projection onto the expected win-volatility space using expected monetary win and standard deviation per unit bet respectively. Given the importance of entertainment to slot machine play, this projection is perhaps less useful than the other two already discussed, and is included mostly for completeness. The difficulty of examining slot machines and slot machine behaviour independently of an entertainment dimension is clearly illustrated by the apparent irrationality of the behaviour as it appears in Figure 2(f). Yet this may offer a potential use for this particular projection. Two points clearly emerge from this plot: firstly, the more one plays, the more one is expected to lose – using actual expected monetary win clearly illustrates this point, which is obscured when considering expected win percentages. Secondly, the more one plays, the more one increases the certainty that one is going to lose. Playing more i.e. by increasing the number of lines played, speeds convergence towards the (negative) expected win percentage – this perspective is obscured by considering the volatility of payouts rather than volatility per unit bet. From this perspective, the argument that the slot machine gambler is paying for entertainment value alone is strongly made.

Before concluding this section it is necessary to return to the subject of dominance and mention the effect of changing the amount bet per line. Since increasing $b$ has the effect of increasing expected costs while leaving the hit probability and volatility per unit bet unaffected, playing with any more than the minimum unit bet is equivalent to the selection of a dominated alternative. Prior observations in Dowling et al. (2005) indicate that it is relatively uncommon for players to bet more than the minimum bet, but that it does occur on occasion, usually in conjunction with playing the maximum number of lines, the so-called "maxbet". Again, the utility of money explains this: any winnings should be viewed relative to the existing wealth of the player, so that those players who are already wealthy would tend to play with the higher denomination implied by the "maxbet" in order to get excitement from any successful play. The behaviour can therefore be explained by the need for players with greater wealth to scale up their bets in order than any success has meaningful monetary value – that is, as a rescaling of the problem rather than as the choice of a dominated alternative.

## Discussion and Conclusions

Previous research into gambling behaviour indicates that it appears to persist because the act of gambling is a form of entertainment, and because gamblers consistently make use of a range of cognitive distortions, so that they may explain away or even be unaware of the actual extent to which they are losing money. Behavioural research into the roots of persistent gambling, though well-developed in each of the two traditions of entertainment/excitement and cognitive biases, has tended to focus more on characteristics of the gambler or on qualitative aspects of the slot machine e.g. attractive audiovisual displays, bonus features, rather than on statistical characteristics of the slot machines themselves. In this paper we attempt to address this imbalance by using Monte Carlo simulation as a means of describing various aspects of slot machine play. The measures we used – expected monetary win, volatility of payouts, and hit probability – have the advantage that they are mathematically precise and can be linked to psychological risk and return criteria that people may look to as they decide both whether to gamble or not and how to play i.e. which machine to use and what number of lines to play. For example, people often choose to play multiple lines, which decreases expected monetary return while decreasing volatility (hence speeding convergence to this loss over a fixed period of time), because of the increased action that comes with the increased frequency of winning events. Also, the tendency of gamblers to overvalue their wins and explain away their loses may result in an overestimation of the hit probability and a general upward shifting of the risk-return curves that has the effect of making *all* gambles more attractive.

An empirical investigation of real-world slot machine play systematically varying the three dimensions of monetary return, volatility and hit probability would be an invaluable direction for future research on this topic. Nevertheless, even without such results the simulations provide insights for both understanding slot machine play and aiding casino management. Without taking a judgmental view on the rationality of slot machine play, the results are suggestive of the following:

- Slot machine gambling can be described as ‘paying to play’. In order to increase the probability of winning something on a single play, one has to play more lines, and this leads on average to greater expected losses (or a faster loss of a given amount) while simultaneously accelerating the convergence of the realised win percentage to the expected win percentage.
- Playing many lines does not offer a proportional increase in the probability of winning something. In particular, there is relatively little additional excitement to be gained from playing the maximum number of lines e.g. 25 lines, relative to an intermediate number e.g. 9 lines. This is particularly true when lines are dependent, as they often are. However, expected losses are always proportional to the number of lines played. Playing many lines does, of course, increase the hit percentage to the extent that the lines are independent and hence gives players the feeling that they are winning something an increased proportion of the time. The something can, of course, include bonuses or free spins which further adds to the player’s experience, a fact that is emphasised in Walker (2004).
- One might think of the additional cost of playing the maximum number of lines as a ‘time-saving cost’. That is, it is quicker to play once on 25 lines than to play 5 times on 5 lines, even though additional costs are incurred. Because of the dependence between lines, the most cost-effective method of playing is to play on a single line; in contrast, the largest concentration of entertainment results from playing the maximum number of lines. If a player is consistently losing money quicker than he or she would like for a given entertainment value, a reasonable response is to play fewer lines – the trade-off may be more attractive than intuition would suggest.

And for casino owners and slot machine manufacturers:

- If any multiple-line volatilities are used, for example to monitor slot machine payouts or to plan financially for the future, it is imperative that a simulation approach be used and that line independence is not assumed for reasons of analytical tractability. Incorrectly assuming line independence results in volatility estimates that are too low, confidence intervals that are too narrow and unrealistic certainty about machine performance.
- Line dependence has the effect of reducing hit probability (and hence entertainment for the player) while increasing volatility (and hence uncertainty for the casino). Slot machines should be constructed so that their winning lines are minimally dependent, particularly when those games use a large number of winning lines.

Slot machines are thought-provoking research tools because on the one hand they are mechanical instruments for which precise results can be calculated. On the other hand, many slot machines currently in use are sophisticated enough to allow for an analysis of decision making under risk that includes strategic elements, real computational complexity, and perceptual subtleties such as line dependence that serve to make the decision task less artificial and more like a general real-world decision.

# Notes

[1]Casino revenue in South Africa in 2006 amounted to US$1.4 billion, the next biggest gambling revenue coming from the state lottery (US$314 million) and horserace betting (US$168 million) (National Gambling Statistics 2005/2006, `http://www.ngb.org.za/home.asp?pid=138`). Taxable revenue from casinos in Nevada, USA totalled US$11 billion for the year 2004-2005 (State of Nevada Gaming Revenue Report 2005, `http://www.gaming.nv.gov/documents/pdf/1g 05oct.pdf`). Research in Dowling et al. (2005) indicates that over half of all gambling expenditure is lost on slot machines, and the 2006 National Survey on Gambling and Problem Gambling in South Africa (Collins and Barr, 2006) indicates that average monthly expenditure on slots is $111, far greater than the state lottery ($12) or horserace betting ($75). The casinos in South Africa, in terms of the configuration and type of their slot machines follow very much the American (USA) casino model. That is, the floor is dominated by modern video slot machines manufactured typically by the large USA manufacturers such as IGT, Ballys and Williams. To a lesser extent there are machines from the German manufacturer Atronic and Australian manufacturer Aristocrat.

[2]It is worth noting that the volatility (of payout) for a particular machine is generally defined for the case of 1 line play of denomination equal to 1. This core volatility of a machine is a key factor in determining the playing experience of a machine. For example, if a machine has a potential bonus high payout (occurring clearly with low probability) then the volatility of the machine will be high compared to a machine that has a configuration of a lower bonus payout (probably occurring with higher probability). Thus, in the industry volatility of a machine is often associated with the potential excitement value of a machine and thus often its attractiveness to players. An important point of this paper is to clarify how players, by selecting the number of lines and quantum of bet for some machine with a given core volatility, will have exercised control over the effective volatility of payout for the spin in question; selecting a larger bet simply raises

the volatility proportionately while selecting many lines (and keeping the bet per line the same) raises the volatility less than proportionately and massively increases the hit percentage. Thus while machines may be selected by casinos according to their core volatility and hence perceived excitement potential, the focus here is on the player's own decision process to self-select some number of lines and quantum of bet given this core volatility, in order to maximise their own player experience.

[3]In many instances it is useful to also refer to the expected win per unit bet i.e. $E\,[W_{a,b}\ /\ ab]$, which when expressed as a percentage gives a single summary of a particular machine's propensity to pay out. The casino industry itself makes uses of the rather more obscure measure 100 x $(1+E\,[W_{a,b}\ /\ ab])\,\%$, apparently without irony called the "win percentage". A player putting $1 into a machine with a win percentage of 95% thus 'wins' 95c (for a net loss of 5c). In this paper we generally use the expected win measure $E\,[W_{a,b}]$, on the basis that it is consistent with the general literature on choice between risky lotteries (e.g. Diecidue et al., 2004)

[4]The lines in Figure 1(a) are almost always line 1, 2, and 3 of a real-world slot machine. Lines 2 and 3 in Figure 1(c) are typical of higher-numbered lines. The configuration in Figure 1(b), with lines overlapping completely, is never encountered.

[5]The expected win results given here are all theoretical. Expected wins obtained from the Monte Carlo simulations are guaranteed to converge to these values in the limit, and after 5 000 000 simulations typically only differ by 1–2% from the theoretical expected win value (e.g. for $a$ =25 on Machine $C$, a simulated expected win of –0.565 was obtained). Any differences merely obscure the substantive issues. Also note that the expected win figures given here are the absolute expected returns $E\,[W_{a,b}]$ and have not been adjusted for the increasing amounts bet when more lines are played. The expected win *percentage* 100 $E\,[W_{a,b}\ /\ ab]$ is constant at 2.3% for any number of lines played (or amount bet per line). The volatilities *have* been adjusted for the amount bet, following the earlier discussion on this point

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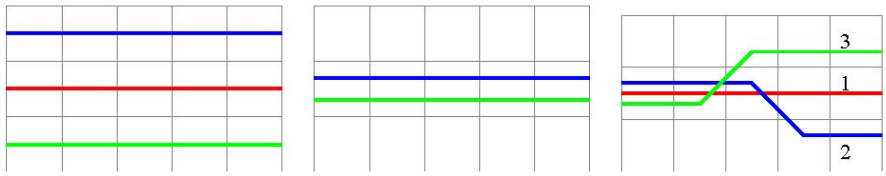



(a) (b) (c)

Figure 1: Illustration of line structure where lines are (a) independent, (b) completely dependent, (c) partially dependent.

| | $a_{\max}$ | $E[W_{1,1}]$ | $\sigma W_{1,1}$ | $\phi_{sc}$ | $\phi_{nsc}$ |
|---|---|---|---|---|---|
| *A* | 9 | -0.023 | 7.365 | 0.050 | 0.105 |
| *B* | 15 | -0.023 | 14.815 | 0.019 | 0.032 |
| *C* | 25 | -0.023 | 9.344 | 0.071 | 0.047 |
| *D* | 25 | -0.023 | 16.422 | 0.075 | 0.071 |

Table 1: Risk and entertainment specifications for 4 hypothetical slot machines

| | $a$ | Machine *A* $E[W_{a,1}]$ | $\frac{\sigma_{W_{a,1}}}{a}$ | $\phi_{hit}(a,1)$ | Machine *B* $E[W_{a,1}]$ | $\frac{\sigma_{W_{a,1}}}{a}$ | $\phi_{hit}(a,1)$ | Machine *C* $E[W_{a,1}]$ | $\frac{\sigma_{W_{a,1}}}{a}$ | $\phi_{hit}(a,1)$ | Machine *D* $E[W_{a,1}]$ | $\frac{\sigma_{W_{a,1}}}{a}$ | $\phi_{hit}(a,1)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Without assuming lines independent (simulated) | 1 | -0.023 | 7.365 | 0.155 | -0.023 | 14.815 | 0.051 | -0.023 | 9.344 | 0.118 | -0.023 | 16.422 | 0.146 |
| | 3 | -0.070 | 4.585 | 0.335 | -0.070 | 8.250 | 0.119 | -0.070 | 5.965 | 0.204 | -0.070 | 9.381 | 0.269 |
| | 5 | -0.117 | 3.678 | 0.389 | -0.117 | 6.506 | 0.179 | -0.117 | 5.014 | 0.274 | -0.117 | 7.793 | 0.377 |
| | 9 | -0.210 | 2.928 | 0.463 | -0.210 | 5.075 | 0.259 | -0.210 | 4.255 | 0.392 | -0.210 | 6.471 | 0.544 |
| | 15 | | | | -0.351 | 4.014 | 0.326 | -0.351 | 4.192 | 0.431 | -0.351 | 5.737 | 0.590 |
| | 21 | | | | | | | -0.491 | 4.057 | 0.466 | -0.491 | 5.499 | 0.629 |
| | 25 | | | | | | | -0.584 | 4.063 | 0.489 | -0.584 | 5.310 | 0.654 |
| Assuming lines independent (theoretical) | 1 | -0.023 | 7.365 | 0.158 | -0.023 | 14.815 | 0.051 | -0.023 | 9.344 | 0.118 | -0.023 | 16.422 | 0.146 |
| | 3 | -0.070 | 4.252 | 0.336 | -0.070 | 8.554 | 0.113 | -0.070 | 5.394 | 0.206 | -0.070 | 9.481 | 0.275 |
| | 5 | -0.117 | 3.294 | 0.479 | -0.117 | 6.626 | 0.171 | -0.117 | 4.179 | 0.286 | -0.117 | 7.344 | 0.386 |
| | 9 | -0.210 | 2.455 | 0.684 | -0.210 | 4.938 | 0.276 | -0.210 | 3.115 | 0.424 | -0.210 | 5.474 | 0.563 |
| | 15 | | | | -0.351 | 3.825 | 0.409 | -0.351 | 2.412 | 0.588 | -0.351 | 4.240 | 0.748 |
| | 21 | | | | | | | -0.491 | 2.039 | 0.710 | -0.491 | 3.584 | 0.865 |
| | 25 | | | | | | | -0.584 | 1.869 | 0.773 | -0.584 | 3.284 | 0.919 |

Table 2: Theoretical and simulated risk and return outcomes for four hypothetical machines

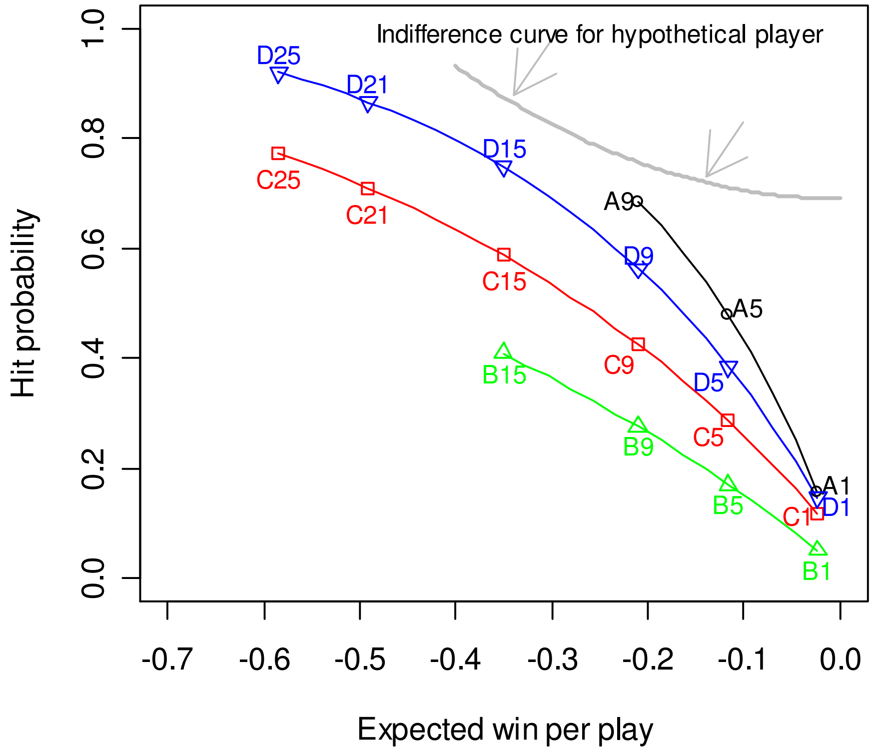



Figure 2 (a)

NOTE: Caption for Figure 2 (a) to (f) follows Figure 2 (f)

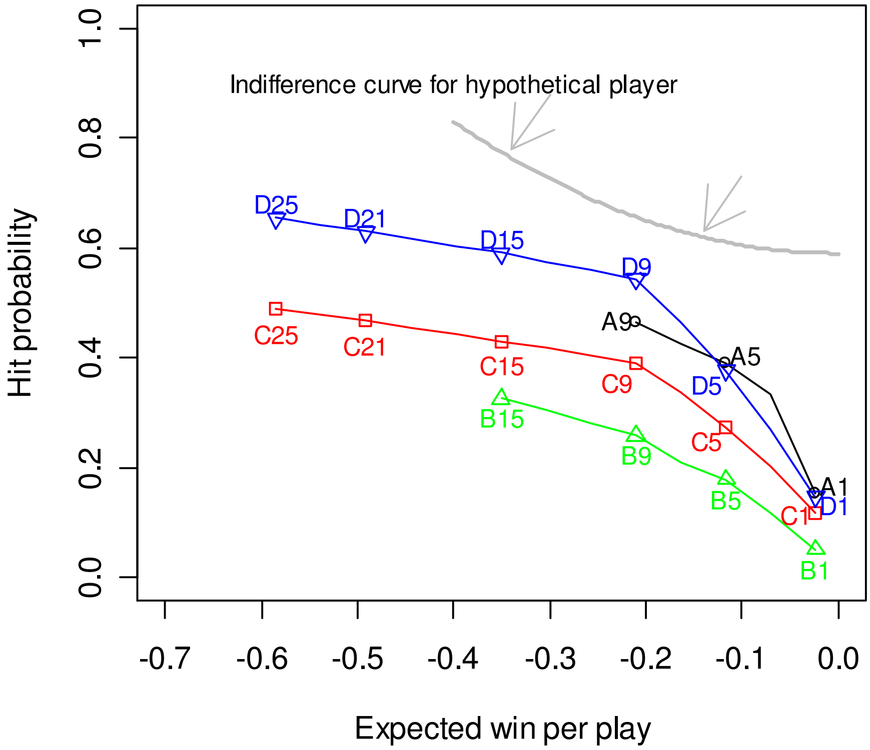

Indifference curve for hypothetical player
Hit probability
Expected win per play
0.0
0.2
0.4
0.6
0.8
1.0
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
D25
D21
D15
D9
D5
D1
C25
C21
C15
C9
C5
C1
B15
B9
B5
B1
A9
A5
A1

Figure 2 (b)

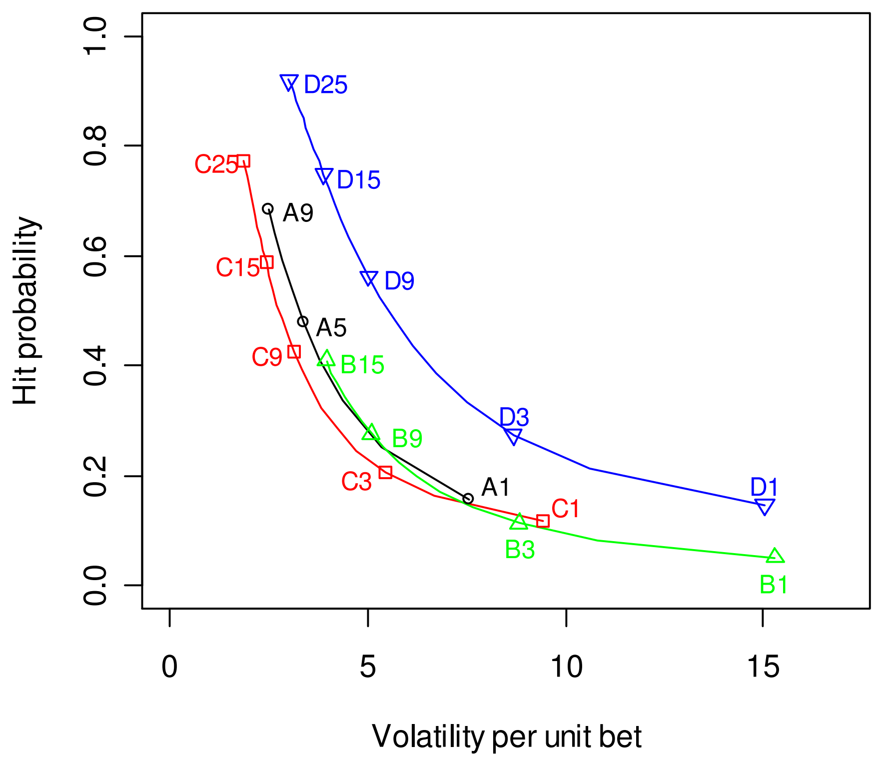
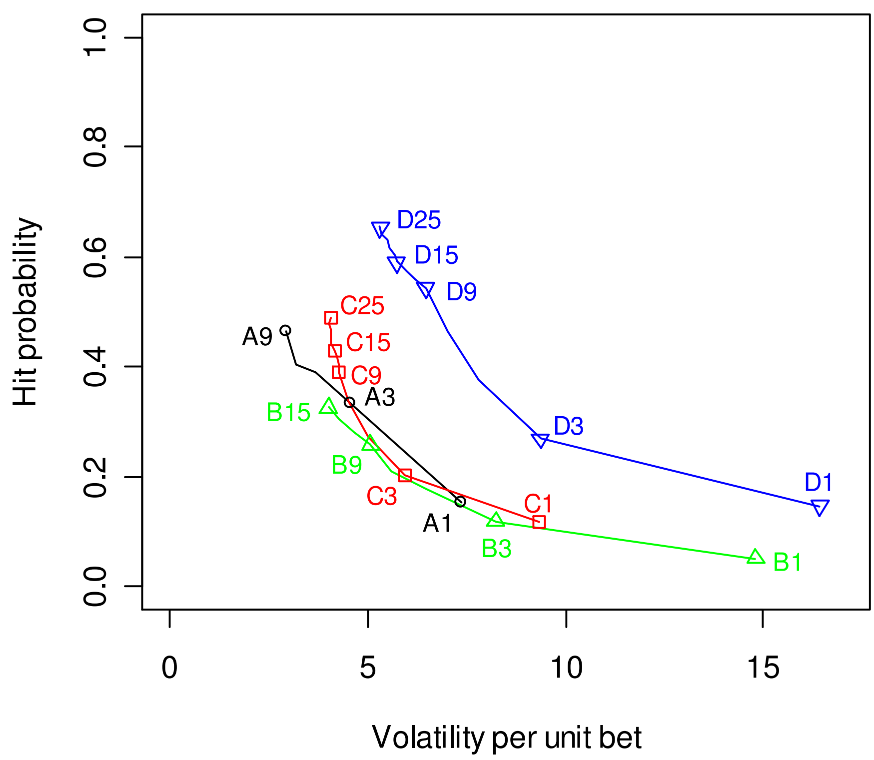
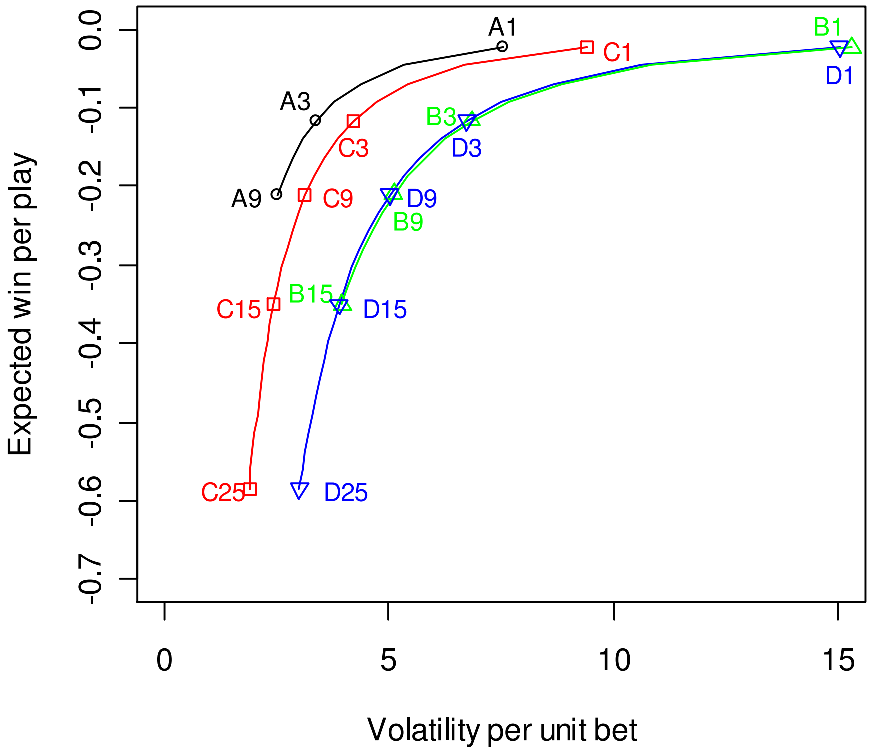

Hit probability
Volatility per unit bet
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
D25
D15
D9
D3
D1
C25
C15
C9
C3
C1
A9
A5
A1
B15
B9
B3
B1

Figure 2 (c)



Hit probability
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
Volatility per unit bet
D25
D15
D9
D3
D1
C25
C15
C9
C3
C1
A9
A3
A1
B15
B9
B3
B1


Figure 2 (d)



A1
A3
A9
C1
C3
C9
C15
C25
B1
B3
B9
B15
D1
D3
D9
D15
D25
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
5
10
15
Expected win per play
Volatility per unit bet


Figure 2 (e)

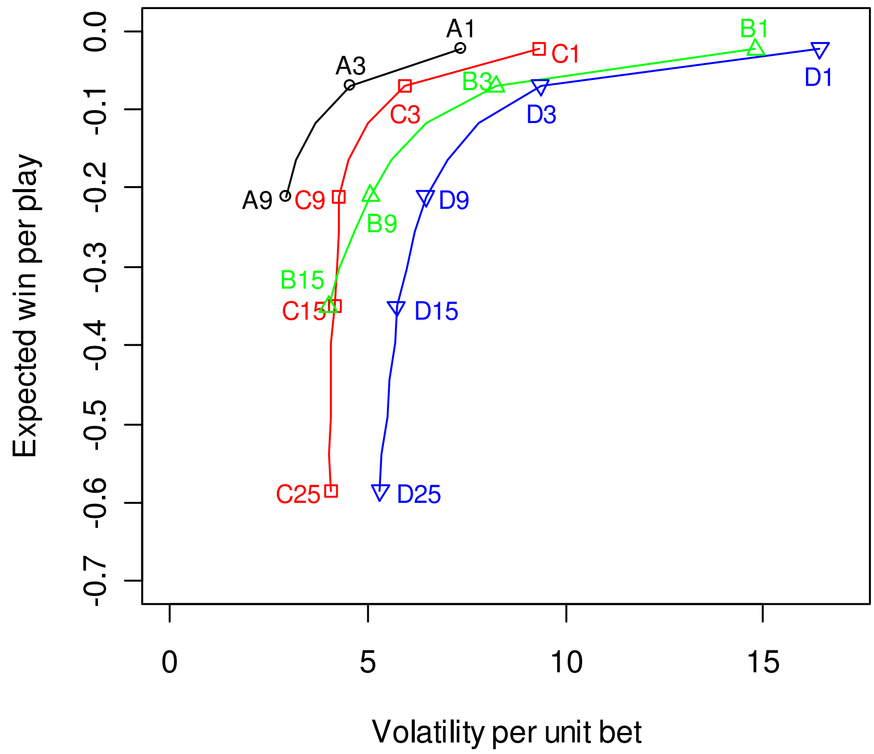



Figure 2 (f)

Figure 2: Projection of four hypothetical slot machines (a) into the expected win-hit probability space, assuming that lines are independent, (b) into the expected win-hit probability space, without the assumption of line independence, (c) into the volatility-hit probability space, assuming that lines are independent, (d) into the volatility-hit probability space, without the assumption of line independence, (e) into the expected win-hit probability space, assuming that lines are independent, (f) into the expected win-hit probability space, without the assumption of line independence

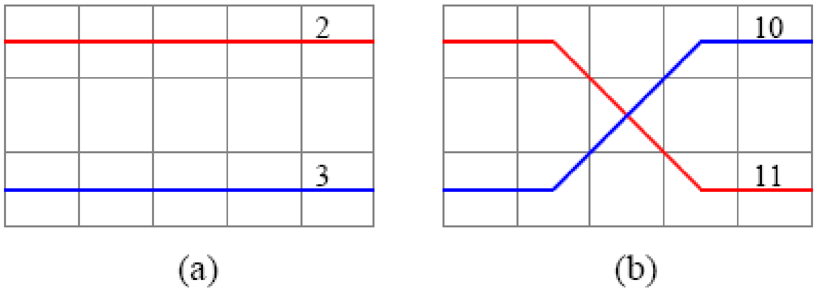



Figure 3: Winning lines for slot machine *C* and *D* (a) lines 2 and 3, (b) lines 10 and 11. Note the dependence on the first two reels