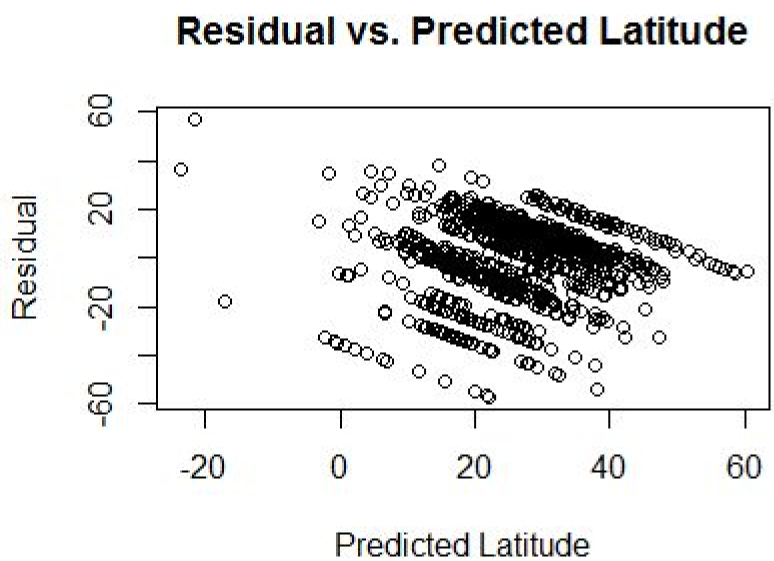
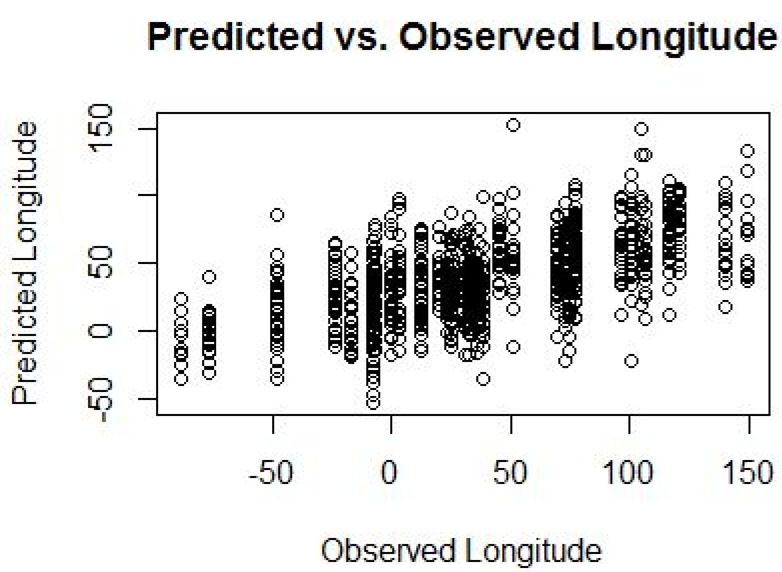
Assignment 6 Report
Harrison Kiang hkiang2, Umberto Ravaioli urjav2, Annlin Sheih sheih2

Q1.

1) Build a straightforward linear regression of latitude (resp. longitude) against features. What is the R-squared? Plot a graph evaluating each regression. (see regression.R)

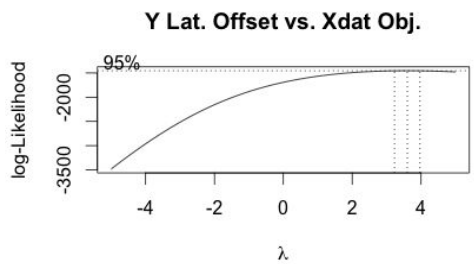
   R-squared: 0.3645767



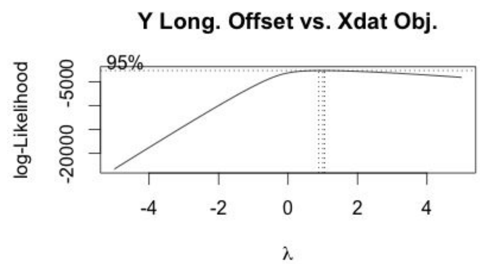


2) Does a Box-Cox transformation improve the regressions? **Notice that the dependent variable has some negative values, which Box-Cox doesn't like. You can deal with this by remembering that these are angles, so you get to choose the origin.** Why do you say so? For the rest of the exercise, use the transformation if it does improve things, otherwise, use the raw data. (see regression.R)

Offset of 90 was chosen to generate the below graphs:



$\lambda_{max\_log\_likelihood} = 3.6$



$\lambda_{max\_log\_likelihood} = 1$

Box-Cox transformation does not improve the regressions. Box-Cox produced values $R^2_{lat} = 0.248$ and $R^2_{long} = 0.365$, which are not higher than the values produced by the previously performed linear regression.

3) Use glmnet to produce… (see q1.R) Note: a 70/30 split was used

Unregularized regression

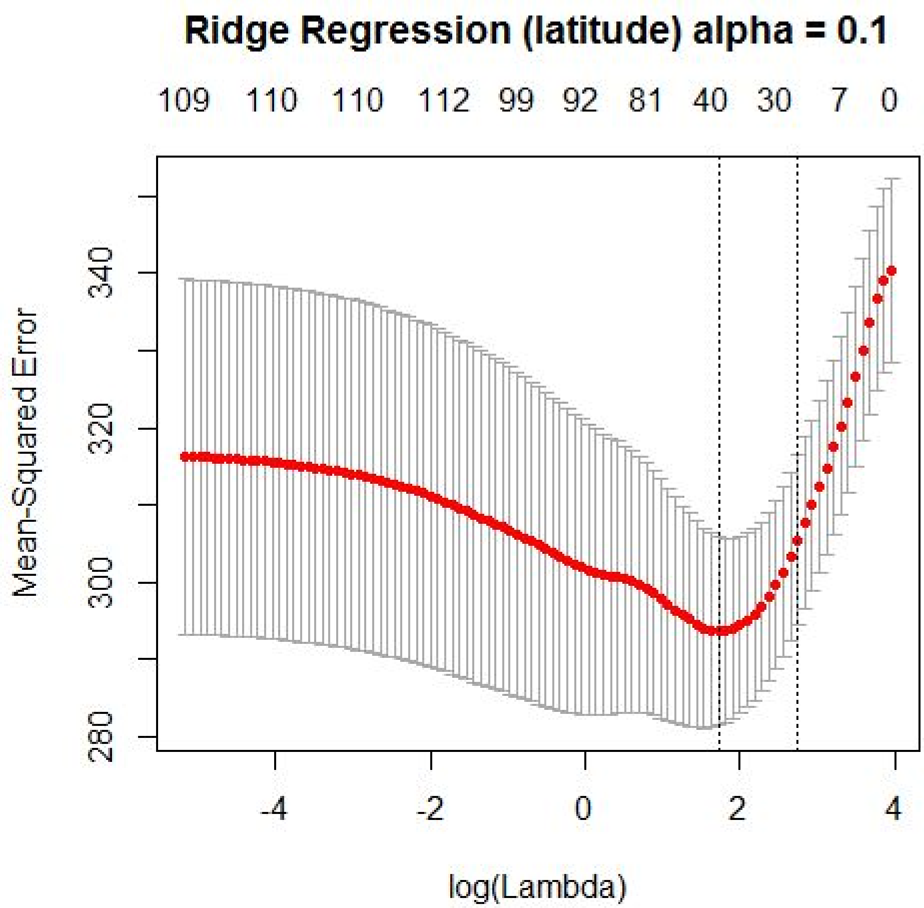
Longitude mean square error: 279.5661

Latitude mean square error: 1978.276

a) A regression regularized by L2 (equivalently, a **ridge** regression). You should estimate the regularization coefficient that produces the minimum error. Is the regularized regression better than the unregularized regression?

Latitude

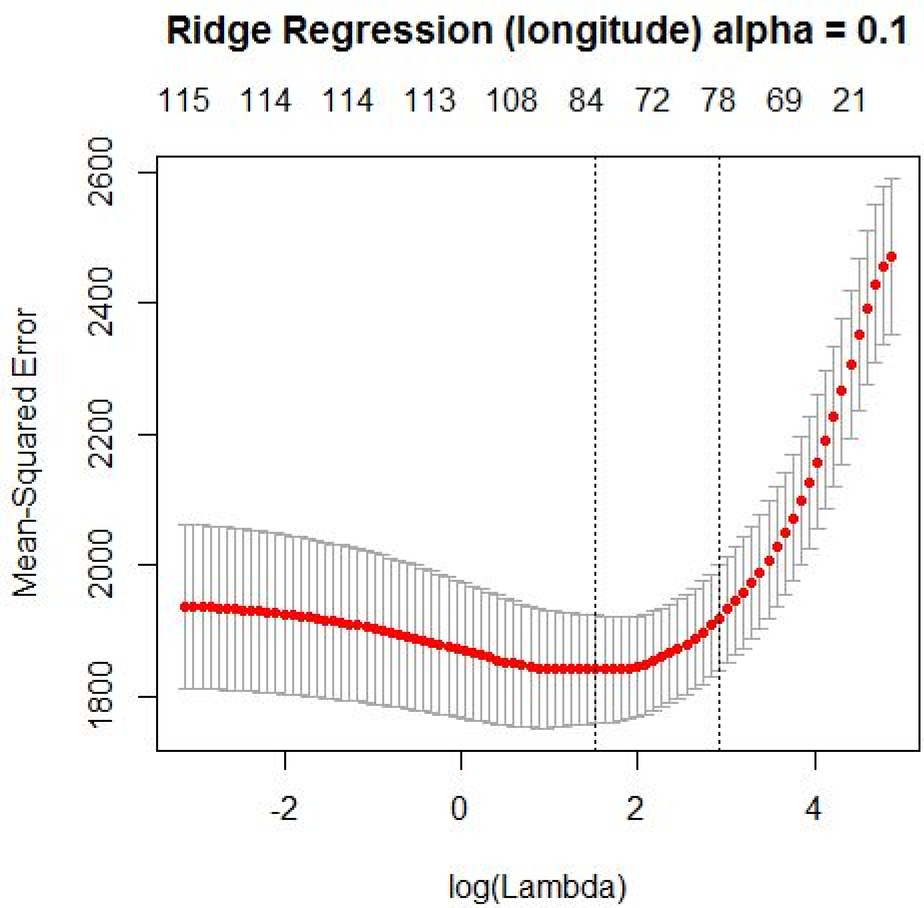
Alpha	Best Regularization Coefficient	Mean Square Error
0	14.911032	265.9078
0.1	5.613908	259.9988
0.2	4.072410	262.7145



There is a small level of variability of error. The number of nonzero components of beta ranges from 20 to 40 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped. Eventually, there are no nonzero constants as log(λ) approaches 4.

Longitude

Alpha	Best Regularization Coefficient	Mean Square Error
0	13.361476	1994.300
0.1	4.583613	1983.270
0.2	4.004997	1994.585



There is a moderate level of variability of error. The number of nonzero components of beta ranges from 72 to 84 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped, with the exception of extremely small log(λ) that range from -4 to -1, and when log(λ) is near the value producing the smallest MSE, resulting in a small local peak of the number of nonzero components.

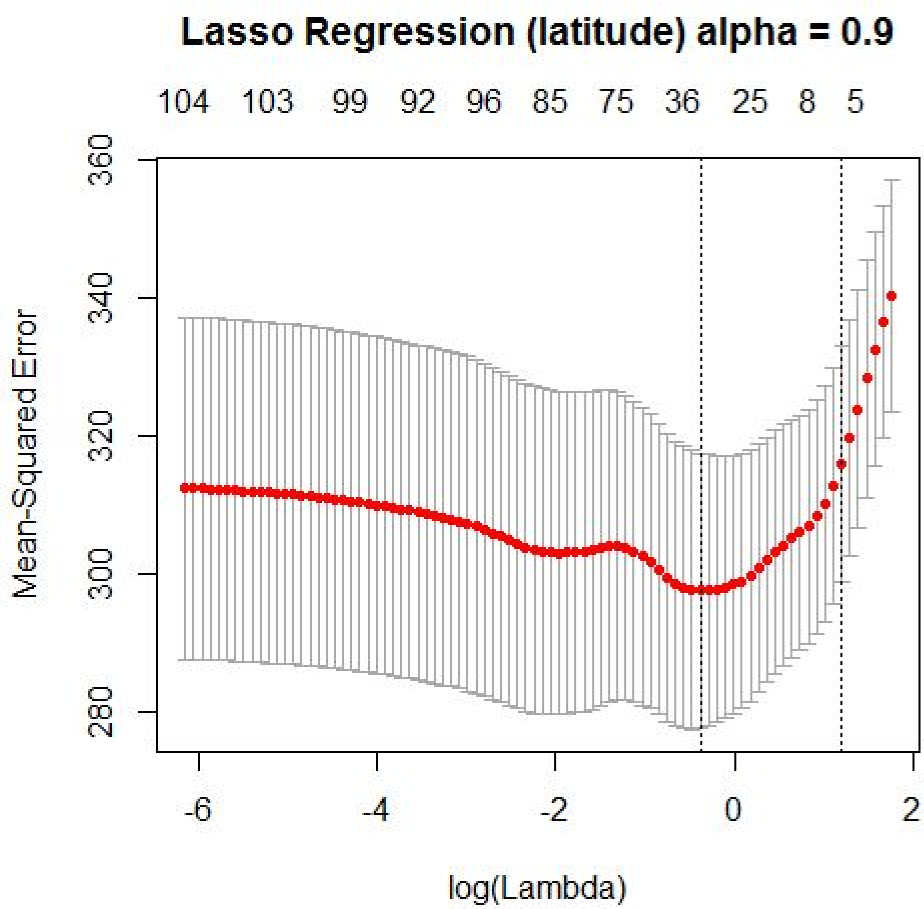
A ridge regression with α = 0 is comparable with the unregularized regression.

b) A regression regularized by L1 (equivalently, a **lasso** regression). You should estimate the regularization coefficient that produces the minimum error. How many variables are used by this regression? Is the regularized regression better than the unregularized regression?

Latitude

Alpha	Best Regularization Coefficient	Mean Square Error
0.8	0.7701571	255.1694

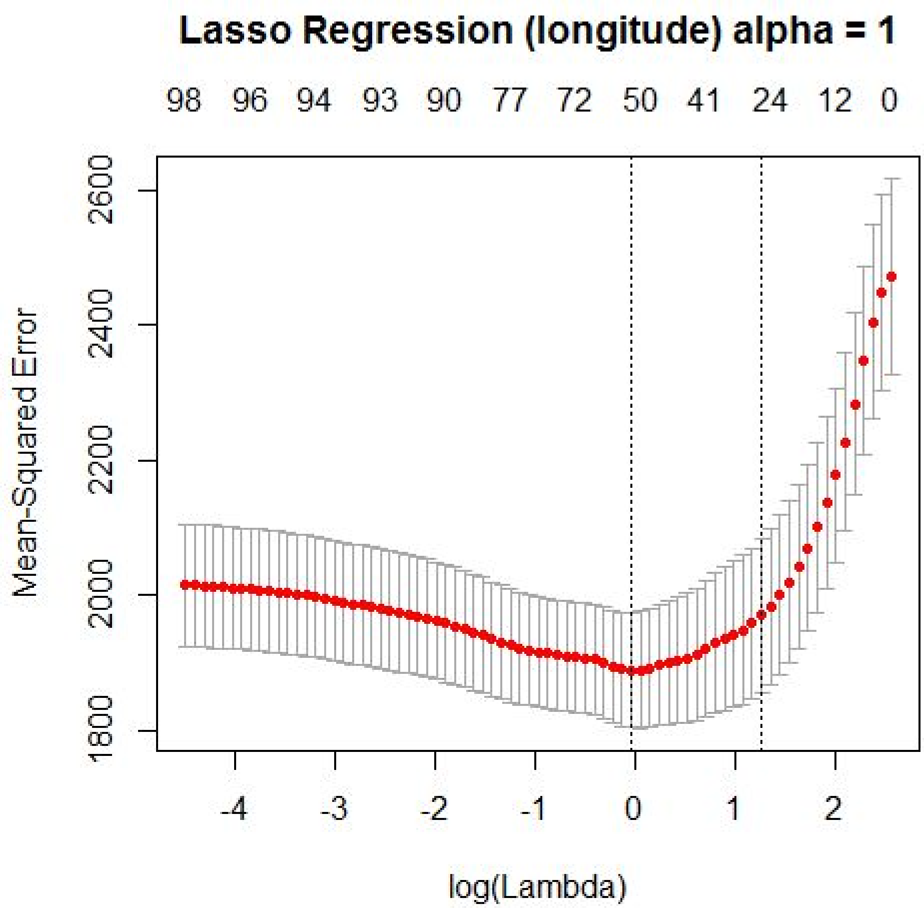
0.9	0.6845841	255.0576
1	0.6761972	255.9621



There is a large level of variability of error. The number of nonzero components of beta ranges from 6 to 35 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped.

Longitude

Alpha	Best Regularization Coefficient	Mean Square Error
0.8	1.2060084	1992.582
0.9	1.1765268	1995.617
1	0.9648067	1991.925



There is a moderate level of variability of error. The number of nonzero components of beta ranges from 25 to 51 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped.

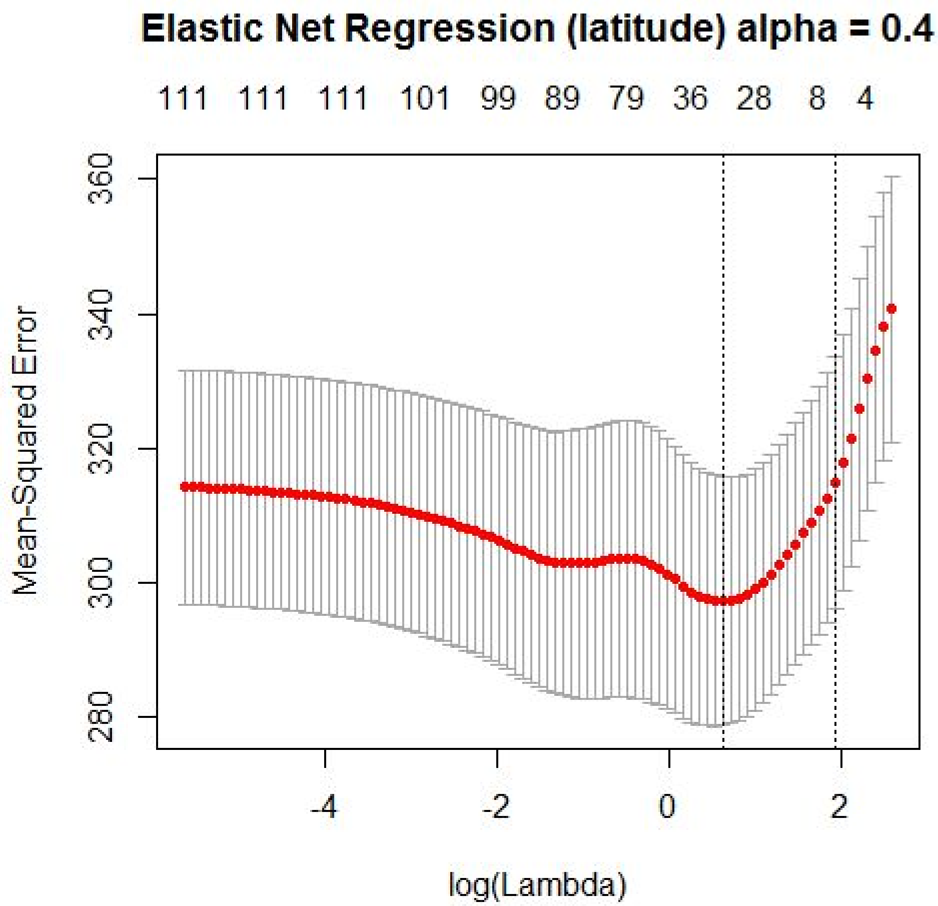
A lasso regression with α = 1 is worse than the unregularized regression.

c) A regression regularized by elastic net (equivalently, a regression regularized by a convex combination of L1 and L2). Try three values of alpha, the weight setting how big L1 and L2 are. You should estimate the regularization coefficient that produces the minimum error. How many variables are used by this regression? Is the regularized regression better than the unregularized regression?

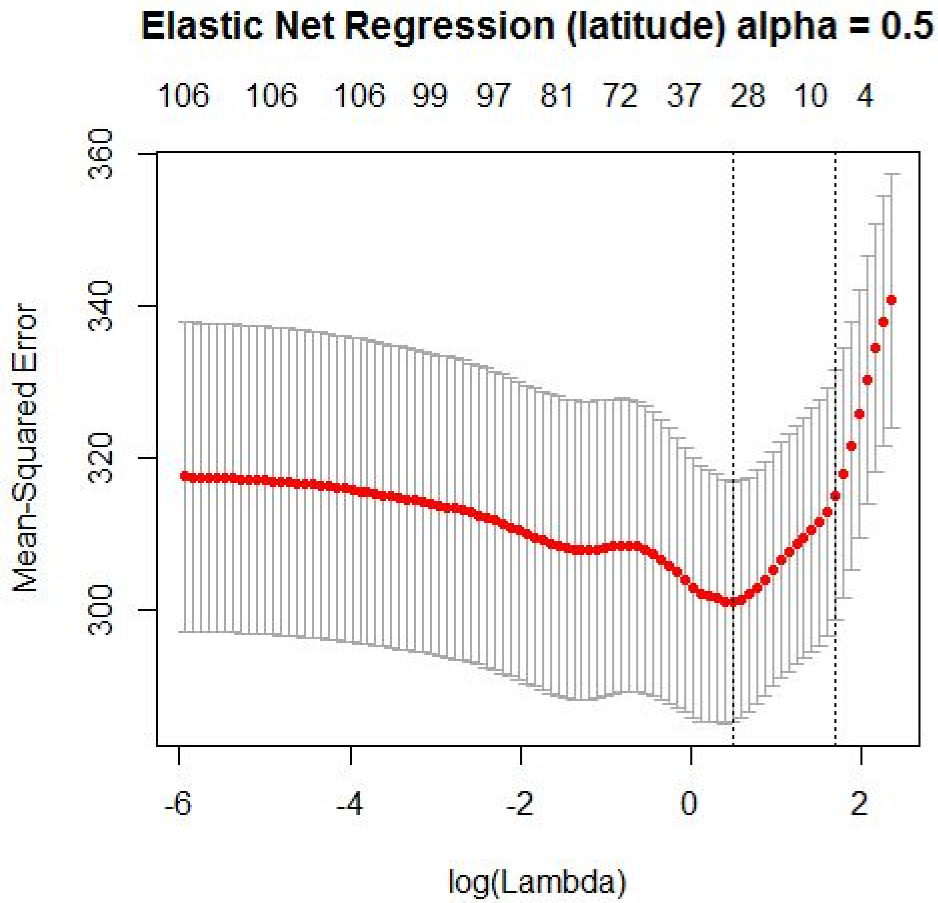
Latitude

Alpha	Best Regularization Coefficient	Mean Square Error
0.4	1.855314	258.6537
0.5	1.628964	259.7221

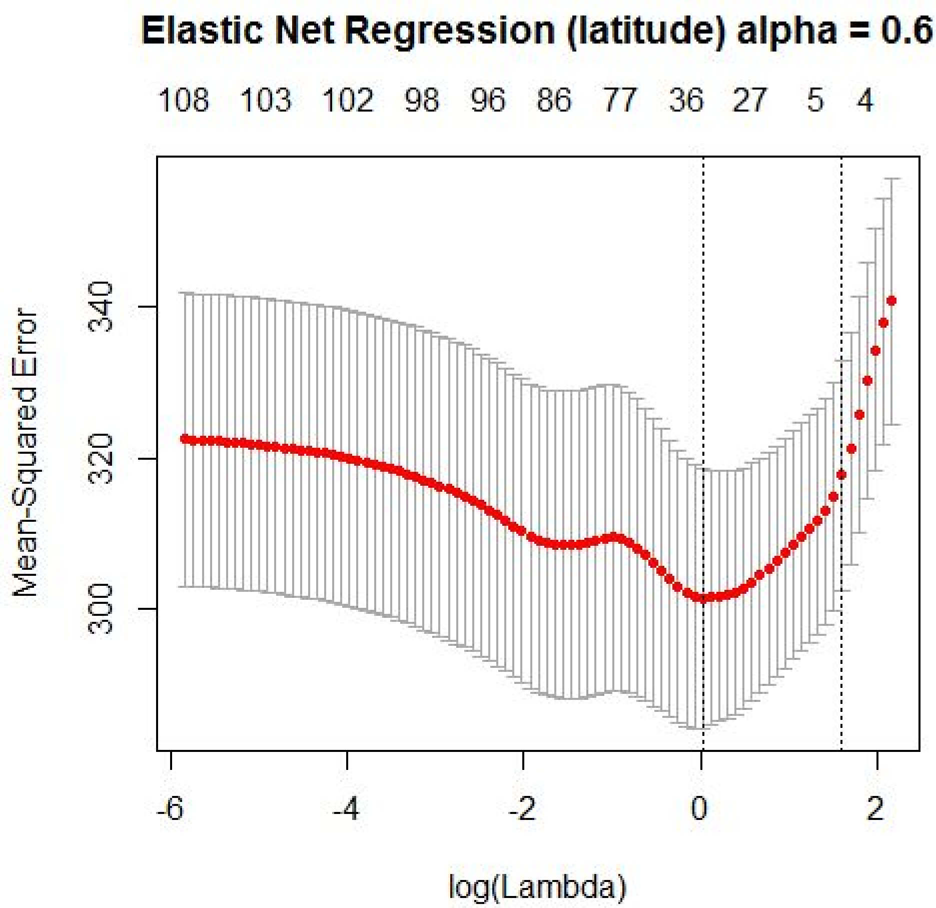
0.6	1.026876	255.5064



There is a moderate level of variability of error. The number of nonzero components of beta ranges from 7 to 35 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped, with the exception of extremely log(λ) that range from -6 to -4.



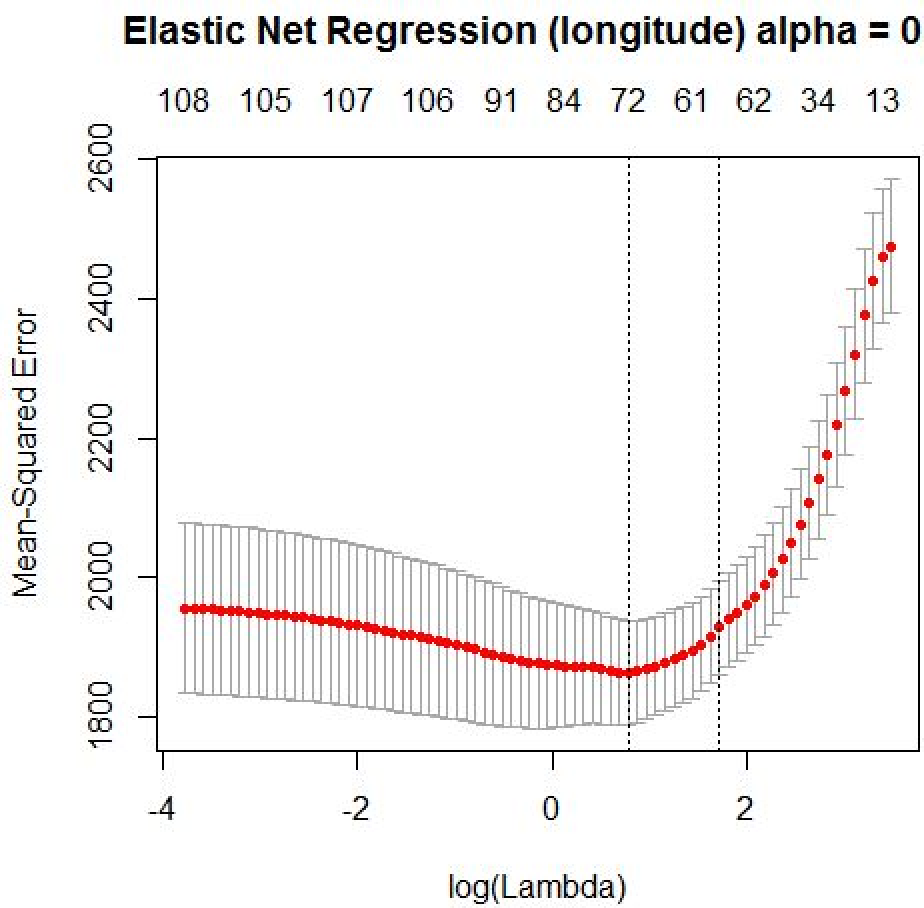
There is a moderate level of variability of error. The number of nonzero components of beta ranges from 8 to 30 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped.



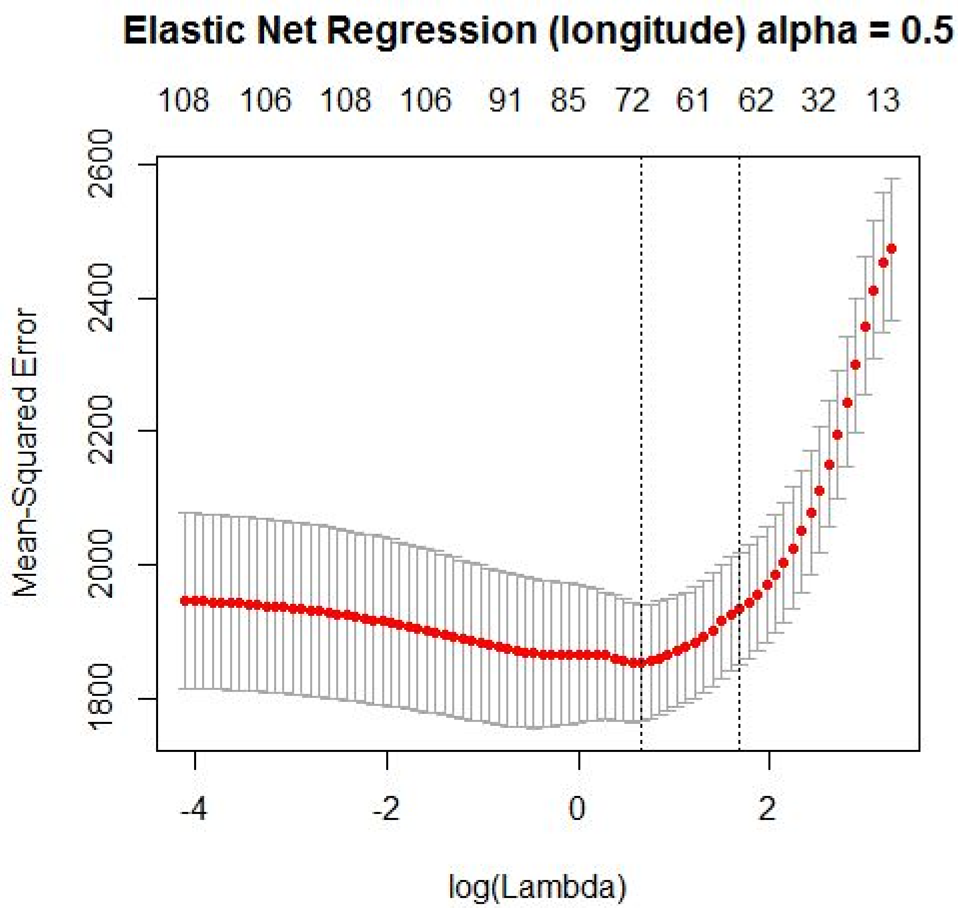
There is a large level of variability of error. The number of nonzero components of beta ranges from 4.5 to 35 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped, with the exception of log(λ) that lie just to the right of the variability of error, where it remains relatively constant, and for log(λ) that range from -6 to -2.

Longitude

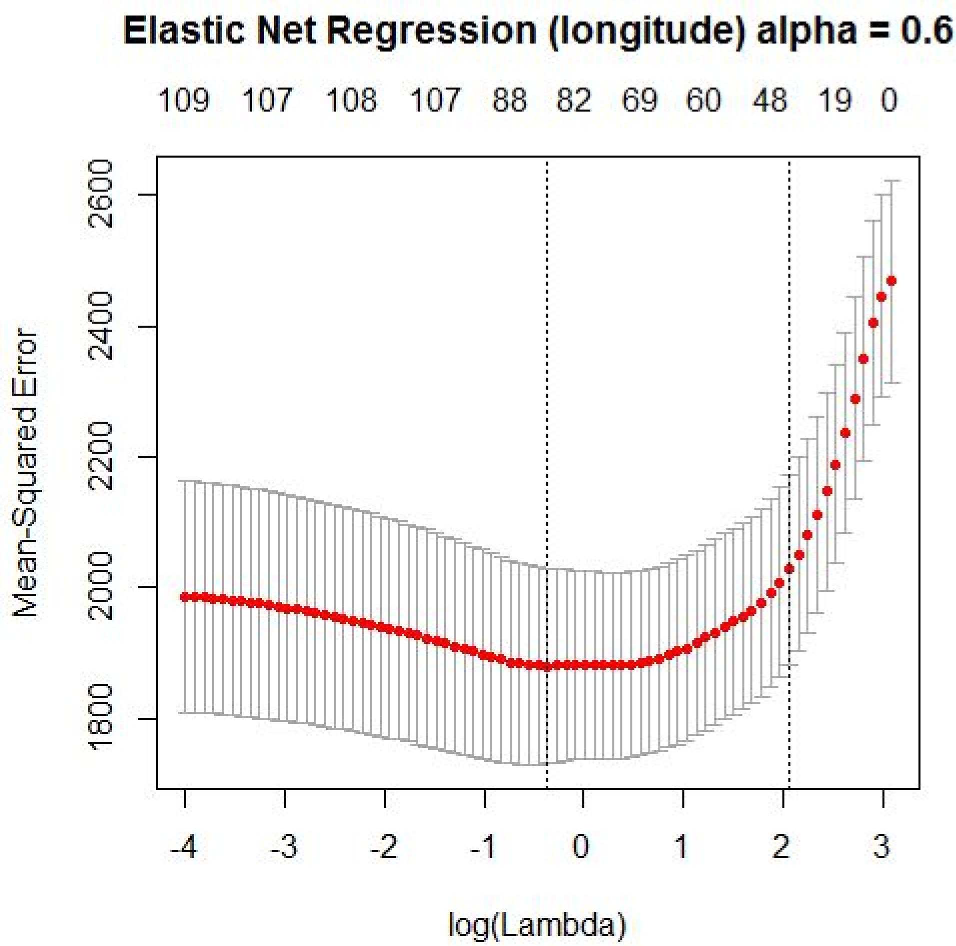
Alpha	Best Regularization Coefficient	Mean Square Error
0.4	1.4369538	2074.148
0.5	0.7219591	2061.794
0.6	0.9579692	2070.515



There is a moderate level of variability of error. The number of nonzero components of beta ranges from 61 to 72 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped, with the exception of log(λ) that lie just to the right of the variability of error, where it remains relatively constant.



There is a moderate level of variability of error. The number of nonzero components of beta ranges from 61 to 72 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped, with the exception of log(λ) that lie just to the right of the variability of error, where it remains relatively constant.



There is a large variability of error. The number of nonzero components of beta ranges from 46 to 85 for regularization constants that produce data within the variability of error. As log(λ) increases, the number of nonzero components fall, ensuring that explanatory variables with small coefficients are dropped.

A regression regularized by elastic net with α = 0.6 is worse than the unregularized regression.

Q2.

The unregularized regression ended up yielding the best accuracy result. This is likely due to the fact that outlier points were not removed, potentially interfering with the ability of the regularization to better model the data.

Lasso regression outperformed ridge regression with this dataset. The elastic net regression did not vary too greatly with different values of alpha. The small differences peaked at an alpha value of 0.7

Without removing outlier points, unregularized regression is the best strategy for this problem.

Accuracy for each regularization scheme (80-20 train-test split) in classification

Unregularized:

- Train: 81.0%
- Test: 81.1%

Lasso:

- Regularization Constant: 0.000668381
- Train: 78.95%
- Test: 79.22%

Ridge:

- Regularization Constant: 0.01473867
- Train: 78.7%
- Test: 78.95%

Elastic Net:

- alpha = 0.1:
  - Regularization Coef: 0.0034849438
  - Train: 78.888%
  - Test: 79.2%
- alpha = 0.2:
  - Regularization Coef: 0.0027745080
  - Train: 78.892%
  - Test: 79.2%
- alpha = 0.3:
  - Regularization Coef: 0.0020300128
  - Train: 78.921%
  - Test: 79.2%
- alpha = 0.4:
  - Regularization Coef: 0.0018338682
  - Train: 78.904
  - Test: 79.2%
- alpha = 0.5:
  - Regularization Coef: 0.0009213781
  - Train: 78.971%
  - Test: 79.233%
- alpha = 0.6:
  - Regularization Coef: 0.0011139683
  - Train: 78.933%

Test: 79.2%

alpha = 0.7:

Regularization Coef: 0.0005463888

Train: 78.996%

Test: 79.233%

alpha = 0.8:

Regularization Coef: 0.0010063340

Train: 78.925%

Test: 79.2%

alpha = 0.9:

Regularization Coef: 0.0008150525

Train: 78.938%

Test: 79.183%