Unit 6. Integer Programming

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Preliminary

This course is strongly based on the monography on Operations Research by Carter, Price and Rabadi [1], and in material obtained from different sources (quoted when needed through the slides).

Learning outcomes

- Getting familiar with the use of integer programming
- Solving integer programming problems

The concept

- Problems in which the feasible set is composed of only integer values.
- The feasible set is neither continuous nor feasible.
- NP-hard problems in general.
- Integer problems that have a network structure are easy to solve using the Simplex method (assignment and matching problems, transportation and transshipment problems, and network flow problems always produce integer results, provided that the problem bounds are integers).
- Rounding can be effective in some problems and clearly not in others:
  - not the same tires than aircrafts!
  - values 0/1 for variable: zero-one or binary integer programming (produce or not produce cars in this factory)
  - mixed integer programming problems

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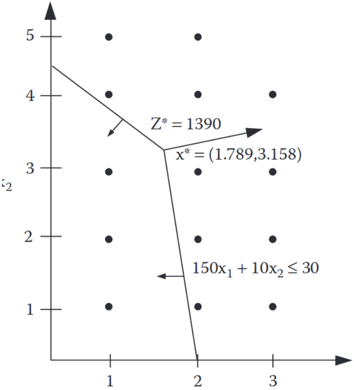
Integer programming

$$
\begin{array}{ll}
\text{maximize} & \displaystyle\sum_{j=1}^{n} c_j x_j \\
\text{subject to} & \begin{array}{rll} \sum_{j=1}^{n} a_{ij} x_j = b_i & i = 1, 2, \ldots, m & \\ x_j > 0 & j = 1, 2, \ldots, n & \\ x_j \quad \text{integer} & \text{for some or all} & j = 1, 2, \ldots, n \end{array}
\end{array}
$$

General integer programming problems



Graphical representation of a typical 2 dimensional integer programming[1].

Integer problem examples

- Capital budgeting: deciding between a collection of investiments
- Warehouse location: in modelling distribution systems, we should decide about tradeoffs between transportation costs and costs for operating distributions centers
- Scheduling: students-faculty-classrooms allocations, vehicle dispatching, etc (see next slide)

Zero-One (0-1) scheduling problem I

Airline crew scheduling problem: The airlines first design a flight schedule composed of a large number of *flight legs* (specific flight on a specific piece of equipment, such as a 747 from New York to Chicago departing at 6:27 a.m.). A flight crew is a complete set of people, including pilots, navigator, and flight attendants who are trained for a specific airplane. A work schedule or rotation is a collection of flight legs that are feasible for a flight crew, and that normally terminate at the point of origin. Variables x_{ij} have value 1 if flight leg i is assigned to crew j. All flight legs should be covered at minimum total cost.

Also called a *set-partitioning problem*

$$\text{minimize} \quad \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to} \quad \begin{aligned} \textstyle\sum_{j=1}^{n} a_{ij} x_j &= 1 \quad i = 1, 2, \ldots, m \\ x_j &= 0, 1 \quad j = 1, 2, \ldots, n \end{aligned}$$

Travelling salesman problem

Exercise 1

Consider the travelling salesman problem. Starting from his home, a salesman wishes to visit each of $(n-1)$ other cities and return home at minimal cost. He must visit each city exactly once and the cost to travel from city i to j is c_{ij}. Let x_{ij} be 1 or 0 depending on the fact that he goes or not from city i to city j

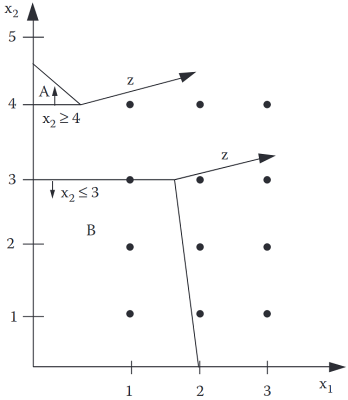
- formulate the optimization problem
- how to avoid disjoint tours?

Solving Integer problems

Simplex is not useful to solve, in general, integer problems. Instead, many other techniques have been proposed:

- Enumeration techniques, including the branch-and-bound procedure;
- cutting plane techniques; and
- group-theoretic techniques,

as well as several composite techniques



Separation into two subproblems in the Branch-and-Bound method[1].

Exercise 2

Using the graphical representation and the branch-and-bound procedure, solve this integer program:

$$
\begin{aligned}
\text{maximize} \quad & z = x_1 + 5x_2 \\
\text{subject to} \quad & -4x_1 + 3x_2 \leq 6 \\
& 3x_1 + 2x_2 \leq 18 \\
& x_1, x_2 \geq 0 \quad \text{and integer}
\end{aligned}
$$

Summary

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Summary

1 References

2 References

References

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[2] David Harel, with Yishai Feldman. Algorithmics: the spirit of computing, 3rd Edition. Addison-Wesley.

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