Unit 3. Sensitivity Analysis

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Preliminary

This course is strongly based on the monography on Operations Research by Carter, Price and Rabadi [**?**], and in material obtained from different sources (quoted when needed through the slides).

Learning outcomes

- Understanding the concept of postoptimality analysis
- Understanding and solving LP problems requiring sensitvity analysis
- Understanding the matrix representation of the Simplex solution

The concept

- After an optimal solution is found, the analyst needs to review the problem parameters and the solution.
- This process is called **postoptimality analysis**:
  - confirming or updating problem parameters (cost and availabilities of activities and resources)
  - if changes need to be introduced in the original parameters, assessing their impact on the optimality of the solution.
- If changes are smallre-optimization may not be needed.
- **Sensitivity analysis** is the study of the effect that types, ranges , and magnitude of changes in problem parameters have in the value of the objective function, *without the need to solving again the new linear problem*.

Two types of parameter modifications

In a LP problem, I can have two situations

$$\begin{aligned} & \max & \mathbf{c}^t\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b}, \forall x_i \geq 0 \end{aligned}$$

I may have interest in knowing what happens with modifications in the parameters c or modifications in the parameters b.

Case 1: sensitivity with respect to c

Exercise 1

A farmer wants to minimize the cost of the food given of two different types of nutrients A and B, provided that she needs a minimum nutrition to be achieved.

	A	B	price
Feed 1	10	3	16
Feed 2	4	5	14
requirements	124	60	

Find how resilient is she to the changes in the price?

Case 2: sensitivity with respect to b

In such cases we will explore the coeficients of the objective function in the dual problem, instead.

Exercise 2

Given the LP problem

$$\begin{aligned} \text{maximize} \quad & z = 16x_1 + 14x_2 \\ & 10x_1 + 4x_2 \geq 124 \\ \text{subject to} \quad & 3x_1 + 5x_2 \geq 60 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Explore the sensitivity of the minimum value of z with respect to the parameters in the RHS of the contraints.

Shadow price is the solution of the dual problem

Solving the dual problem gives a lot of insight into the actual situation.

Exercise 3

In a company that manufactures two types of cans, A and B, the owners want to maximize the production, which dependence on a restricted amount of aluminum, the time the machines can devote to the work and the manpower is given the following table:

	A	B	limit
Aluminium	50	30	2000
Machine time	6	5	300
Labor	3	5	200
price	124	60	

Given an optimal solution of the problem, how the objective function will depend on variations in the different constraints?

Some tools that help interpreting the calculations

Find *here* a code with the solution using ORtools.
The Senstivity analysis tool in Excel is nicely explained *here*.

References


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