Unit 4. Sensitivity Analysis

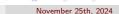
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Preliminary

This course is strongly based on the monography on Operations Research by Carter, Price and Rabadi [1], and in material obtained from different sources (quoted when needed through the slides).

Learning outcomes

- Understanding the concept of postoptimality analysis
- Understanding and solving LP problems requiring sensitivity analysis
- Understanding the matrix representation of the Simplex solution

The concept

- After an optimal solution is found, the analyst needs to review the problem parameters and the solution.
- This process is called postoptimality analysis:
 - confirming or updating problem parameters (cost and availabilities of activities and resources)
 - if changes need to be introduced in the original parameters, assessing their impact on the optimality of the solution.
- If changes are small, re-optimization may not be needed.
- Sensitivity analysis is the study of the effect that types, ranges, and magnitude of changes in problem parameters have in the value of the objective function, without the need to solve again the new linear problem.

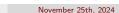
Two types of parameter modifications

In a LP problem,

$$\max \quad \mathbf{c}^t \mathbf{x}$$
 subject to
$$A\mathbf{x} \leq \mathbf{b}, \forall x_i \geq 0$$

one can have two situations: one may have interest in knowing what happens with modifications in the parameters c or modifications in the parameters b.

In addition, one can explore what occurs when adding an extra constraint or adding a new variable.



Case 1: sensitivity with respect to *c*

Exercise 1

A farmer wants to minimize the cost of the food given to her lifestock. Two different types of nutrients A and B are needed by the animals, and she needs a minimum nutrition to be achieved.

	Α	В	price
Feed 1	10	3	16
Feed 2	4	5	14
requirements	124	60	

Find how resilient is she to the changes in the price? What happens with respect to basic vs non-basic variables in a general case?



Case 2: sensitivity with respect to b

In such cases we will explore the coeficients of the objective function in the dual problem, instead.

Exercise 2

Given the LP problem

minimize
$$z = 16x_1 + 14x_2$$

 $10x_1 + 4x_2 \ge 124$
subject to $3x_1 + 5x_2 \ge 60$
 $x_1, x_2 \ge 0$

Explore the sensitivity of the minimum value of z with respect to the parameters in the RHS of the contraints. Hint: consider the dual problem.



Shadow price is the solution of the dual problem

Solving the dual problem gives a lot of insight into the actual situation.

Exercise 3

* In a company that manufactures two types of bikes, A and B, the owners want to maximize the benefits, taking into account that the production depends on a restricted amount of titanium, the time the machines can devote to the work and the manpower, which are all given below:

	Α	В	limit
Titanium	50	30	2000
Machine time	6	5	300
Labor	3	5	200
price	50	60	

Some tools that help interpreting the calculations

Find *here* a code with the solution using ORtools.

The Sensitivity analysis tool in Excel is nicely explained here.

Notice that in a non-linear scenario, what we have identified here as reduced costs are in fact reduced gradient (or actual gradient of the optimal solution) while shadow prices (margin for profit to be obtained by changing 1 unit in each constraint RHS term) correspond to the Lagrange multipliers.

Summary

References





References

[3]

- [1] Michael W. Carter, Camille C. Price, and Ghaith Rabadi. Operations Research, 2nd Edition. CRC Press.
 - David Harel, with Yishai Feldman. Algorithmics: the spirit of computing, 3rd Edition. Addison-Wesley.
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