

# Unit 3. Sensitivity Analysis

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This course is strongly based on the monography on Operations Research by Carter, Price and Rabadi [1], and in material obtained from different sources (quoted when needed through the slides).

# Learning outcomes

- Understanding the concept of postoptimality analysis
- Understanding and solving LP problems requiring sensitivity analysis
- Understanding the matrix representation of the Simplex solution

# The concept

- After an optimal solution is found, the analyst needs to review the problem parameters and the solution.
- This process is called **postoptimality analysis**:
  - confirming or updating problem parameters (cost and availabilities of activities and resources)
  - if changes need to be introduced in the original parameters, assessing their impact on the optimality of the solution.
- If changes are small, re-optimization may not be needed.
- **Sensitivity analysis** is the study of the effect that types, ranges, and magnitude of changes in problem parameters have in the value of the objective function, *without the need to solve again the new linear problem*.

# Two types of parameter modifications

In a LP problem,

$$\begin{array}{ll}\max & \mathbf{c}^t \mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b}, \forall x_i \geq 0\end{array}$$

one can have two situations: one may have interest in knowing what happens with modifications in the parameters  $c$  or modifications in the parameters  $b$ .

In addition, one can explore what occurs when adding an extra constraint or adding a new variable.

# Case 1: sensitivity with respect to $c$

## Exercise 1

A farmer wants to minimize the cost of the food given of two different types of nutrients  $A$  and  $B$ , provided that she needs a minimum nutrition to be achieved.

|              | A   | B  | price |
|--------------|-----|----|-------|
| Feed 1       | 10  | 3  | 16    |
| Feed 2       | 4   | 5  | 14    |
| requirements | 124 | 60 |       |

Find how resilient is she to the changes in the price? What happens with respect to basic vs non-basic variables in a general case?

## Case 2: sensitivity with respect to $b$

In such cases we will explore the coefficients of the objective function in the dual problem, instead.

### Exercise 2

Given the LP problem

$$\begin{array}{ll}\text{maximize} & z = 16x_1 + 14x_2 \\ & 10x_1 + 4x_2 \geq 124 \\ \text{subject to} & 3x_1 + 5x_2 \geq 60 \\ & x_1, x_2 \geq 0\end{array}$$

Explore the sensitivity of the minimum value of  $z$  with respect to the parameters in the RHS of the constraints.

# Shadow price is the solution of the dual problem

Solving the dual problem gives a lot of insight into the actual situation.

## Exercise 3

In a company that manufactures two types of bikes,  $A$  and  $B$ , the owners want to maximize the benefits, taking into account that the production depends on a restricted amount of titanium, the time the machines can devote to the work and the manpower, which are all given below:

|              | A    | B   | limit |
|--------------|------|-----|-------|
| Titanium     | 50   | 30  | 2000  |
| Machine time | 6    | 5   | 300   |
| Labor        | 3    | 5   | 200   |
| price        | 1240 | 600 |       |

Check the link <https://app.wooclap.com/PDHGSS> and answer the questions



# Some tools that help interpreting the calculations

Find *here* a code with the solution using ORtools.

The Sensitivity analysis tool in Excel is nicely explained *here*.

Notice that in a non-linear scenario, what we have identified here as *reduced costs* are in fact *reduced gradient* (or actual gradient of the optimal solution) while *shadow prices* (margin for profit to be obtained by changing 1 unit in each constraint RHS term) correspond to the *Lagrange multipliers*.

# Summary

## 1 References

# References

- [1] Michael W. Carter, Camille C. Price, and Ghaith Rabadi. Operations Research, 2nd Edition. CRC Press.
- [2] David Harel, with Yishai Feldman. Algorithmics: the spirit of computing, 3rd Edition. Addison-Wesley.
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- [5] K.F. Riley, M.P. Hobson, S.J. Bence. Mathematical Methods for Physics and Engineering (2nd Ed). McGraw Hill.
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