Unit 5. Stochastic processes

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Preliminary

This course is strongly based on the monography on Operations Research by Carter, Price and Rabadi [**?**], and in material obtained from different sources (quoted when needed through the slides).

Here is a simple game:

- a player bets on coin tosses, a dollar each time,
- the game ends either when the player has no money left or is up to five dollars.

If the player starts with three dollars, what is the chance that the game takes at least five flips? Twenty-five flips?
At any point, this player has either \$0, or \$1, ..., or \$5. We say that the player is in the state s_0, s_1, ..., or s_5. A game consists of moving from state to state. For instance, a player now in state s_3 has on the next flip a .5 chance of moving to state s_2 and a .5 chance of moving to s_4. The boundary states are a bit different; once in state s_0 or state s_5, the player never leaves.

Let $p_i(n)$ be the probability that the player is in state s_i after n flips. Then, for instance, we have that the probability of being in state s_0 after flip $n+1$ is $p_0(n+1) = p_0(n) + 0.5 \cdot p_1(n)$. This matrix equation sumarizes.

$$\begin{pmatrix} 1 & .5 & 0 & 0 & 0 & 0 \\ 0 & 0 & .5 & 0 & 0 & 0 \\ 0 & .5 & 0 & .5 & 0 & 0 \\ 0 & 0 & .5 & 0 & .5 & 0 \\ 0 & 0 & 0 & .5 & 0 & 0 \\ 0 & 0 & 0 & 0 & .5 & 1 \end{pmatrix} p_0(n)$$

$\mathrm{p}_1(n)$
$\mathrm{p}_2(n)$
$\mathrm{p}_3(n)$
$\mathrm{p}_4(n)$
$\mathrm{p}_5(n) = p_0(n+1)$
$\mathrm{p}_1(n+1)$
$\mathrm{p}_2(n+1)$
$\mathrm{p}_3(n+1)$
$\mathrm{p}_4(n+1)$