Unit 3. Sensitivity Analysis

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Preliminary

This course is strongly based on the monography on Operations Research by Carter, Price and Rabadi [1], and in material obtained from different sources (quoted when needed through the slides).

Learning outcomes

- Understanding the concept of postoptimality analysis
- Understanding and solving LP problems requiring sensitvity analysis
- Understanding the matrix representation of the Simplex solution

The concept

- After an optimal solution is found, the analyst needs to review the problem parameters and the solution.
- This process is called **postoptimality analysis**:
  - confirming or updating problem parameters (cost and availabilities of activities and resources)
  - if changes need to be introduced in the original parameters, assessing their impact on the optimality of the solution.
- If changes are small, re-optimization may not be needed.
- **Sensitivity analysis** is the study of the effect that types, ranges, and magnitude of changes in problem parameters have in the value of the objective function, *without the need to solve again the new linear problem*.

Two types of parameter modifications

In a LP problem,

$$\begin{aligned} \max \quad & \mathbf{c}^t\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b}, \forall x_i \geq 0 \end{aligned}$$

one can have two situations: one may have interest in knowing what happens with modifications in the parameters c or modifications in the parameters b.
In addition, one can explore what occurs when adding an extra constraint or adding a new variable.

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Case 1: sensitivity with respect to c

Exercise 1

A farmer wants to minimize the cost of the food given to her lifestock. Two different types of nutrients A and B are needed by the animals, and she needs a minimum nutrition to be achieved.

	A	B	price
Feed 1	10	3	16
Feed 2	4	5	14
requirements	124	60	

Find how resilient is she to the changes in the price? What happens with respect to basic vs non-basic variables in a general case?

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Case 2: sensitivity with respect to b

In such cases we will explore the coeficients of the objective function in the dual problem, instead.

Exercise 2

Given the LP problem

$$\begin{array}{ll} \text{minimize} & z = 16x_1 + 14x_2 \\ & \begin{array}{rcl} 10x_1 + 4x_2 & \geq & 124 \\ 3x_1 + 5x_2 & \geq & 60 \\ x_1, x_2 & \geq & 0 \end{array} \end{array}$$

subject to

Explore the sensitivity of the minimum value of z with respect to the parameters in the RHS of the contraints. Hint: consider the dual problem.

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Shadow price is the solution of the dual problem

Solving the dual problem gives a lot of insight into the actual situation.

Exercise 3

* In a company that manufactures two types of bikes, A and B, the owners want to maximize the benefits, taking into account that the production depends on a restricted amount of titanium, the time the machines can devote to the work and the manpower, which are all given below:

	A	B	limit
Titanium	50	30	2000
Machine time	6	5	300
Labor	3	5	200
price	50	60	

Check the link https://app.wooclap.com/PDHGSS and answer the questions

Some tools that help interpreting the calculations

Find *here* a code with the solution using ORtools.
The Senstivity analysis tool in Excel is nicely explained *here*.
Notice that in a non-linear scenario, what we have identified here as *reduced costs* are in fact *reduced gradient* (or actual gradient of the optimal solution) while *shadow prices* (margin for profit to be obtained by changing 1 unit in each constraint RHS term) correspond to the *Lagrange multipliers*.

Summary

1. References

References

[1] Michael W. Carter, Camille C. Price, and Ghaith Rabadi. Operations Research, 2nd Edition. CRC Press.

[2] David Harel, with Yishai Feldman. Algorithmics: the spirit of computing, 3rd Edition. Addison-Wesley.

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[4] J. Hefferon. Linear algebra (4th Ed).

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[7] Kenneth J. Beers. Numerical methods for chemical engineering: applications in Matlab. Cambridge University Press.

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