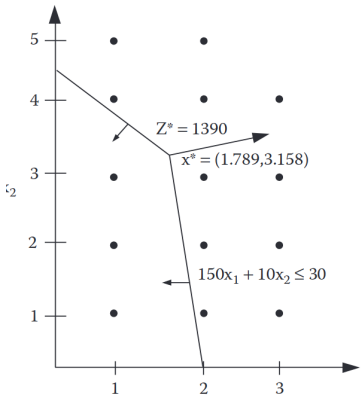
1 Session 15. Integer programming

material: cuttingstock.pdf

- Getting familiar with the use of integer programming
- Solving integer programming problems
- Problems in which the feasible set is composed of only integer values.
- The feasible set is neither continuous nor feasible.
- NP-hard problems in general.
- Integer problems that have a network structure are easy to solve using the Simplex method (assignment and matching problems, transportation and transshipment problems, and network flow problems always produce integer results, provided that the problem bounds are integers).
- Rounding can be effective in some problems and clearly not in others:
  - not the same tires than aircrafts!
  - values 0/1 for variable: zero-one or binary integer programming (produce or not produce cars in this factory)
  - mixed integer programming problems

$$\begin{aligned} \text{maximize} \quad & \sum_{j=1}^{n} c_j x_j \\ & \textstyle\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = 1, 2, \ldots, m \\ \text{subject to} \quad & x_j > 0 \quad j = 1, 2, \ldots, n \\ & x_j \quad \text{integer} \quad \text{for some or all} \quad j = 1, 2, \ldots, n \end{aligned}$$



Graphical representation of a typical 2 dimensional integer programming[1].

- Capital budgeting: deciding between a collection of investiments

- Warehouse location: in modelling distribution systems, we should decide about tradeoffs between transportation costs and costs for operating distributions centers

- Scheduling: students-faculty-classrooms allocations, vehicle dispatching, etc (see next slide)

Airline crew scheduling problem: The airlines first design a flight schedule composed of a large number of *flight legs* (specific flight on a specific piece of equipment, such as a 747 from New York to Chicago departing at 6:27 a.m.). A flight crew is a complete set of people, including pilots, navigator, and flight attendants who are trained for a specific airplane. A work schedule or rotation is a collection of flight legs that are feasible for a flight crew, and that normally terminate at the point of origin. Variables x_{ij} have value 1 if flight leg i is assigned to crew j. All flight legs should be covered at minimum total cost.

Also called a *set-partitioning problem*

$$\begin{aligned} \text{minimize} \quad & \sum_{j=1}^{n} c_j x_j \\ \text{subject to} \quad & \textstyle\sum_{j=1}^{n} a_{ij} x_j = 1 \quad i = 1, 2, \ldots, m \\ & x_j = 0, 1 \quad j = 1, 2, \ldots, n \end{aligned}$$

Exercise 1

Consider the travelling salesman problem. Starting from his home, a salesman wishes to visit each of $(n-1)$ other cities and return home at minimal cost. He must visit each city exactly once and the cost to travel from city i to j is c_{ij}. Let x_{ij} be 1 or 0 depending on the fact that he goes or not from city i to city j

- formulate the optimization problem

- how to avoid disjoint tours?

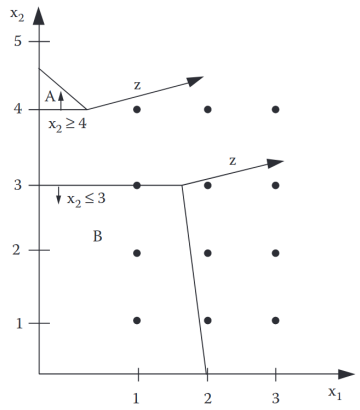
Simplex is not useful to solve, in general, integer problems. Instead, many other techniques have been proposed:

- Enumeration techniques, including the branch-and-bound procedure;

- cutting plane techniques; and

- group-theoretic techniques,

as well as several composite techniques



Separation into two subproblems in the Branch-and-Bound method[1].

Exercise 2

Using the graphical representation and the branch-and-bound procedure, solve this integer program:

$$\begin{aligned} \text{maximize} \quad & z = x_1 + 5x_2 \\ & -4x_1 + 3x_2 \leq 6 \\ \text{subject to} \quad & 3x_1 + 2x_2 \leq 18 \\ & x_1, x_2 \geq 0 \quad \text{and integer} \end{aligned}$$

References

[1] Michael Carter, Camille C Price, and Ghaith Rabadi. *Operations Research. A Practical Introduction. Second Edition.* CRC Press, 2019.