Unit 2: Derivatives
Part 2

Bionic Who

November 20, 2024

1 Problem 1

Consider the function $f(x, y) = x^2y^3 + 2xy$.

(a) Compute the partial derivatives f_x, f_y, f_{xx}, f_{yy}, and f_{xy}.

(b) Find the directional derivative of f at the point $P(1, 2)$ in the direction of the vector $\vec{u} = \langle 3, -4 \rangle$.

(c) Find the equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(1, 2)$.

2 Problem 2

Given the function $g(x, y, z) = x^2yz + e^{xyz}$,

(a) Find the gradient vector ∇g.

(b) Calculate the directional derivative of g at the point $P(1, 0, 1)$ in the direction of the vector $\vec{v} = \langle 2, -1, 2 \rangle$.

(c) Determine the maximum rate of change of g at the point $P(1, 0, 1)$ and the direction in which it occurs.

3 Problem 3

A rectangular box has dimensions x, y, and z. The volume of the box is given by $V(x, y, z) = xyz$.

(a) Find the differential dV of the volume.

(b) If the dimensions of the box are measured with errors of ± 0.1, estimate the maximum possible error in the calculated volume when $x = 2$, $y = 3$, and $z = 4$.

4 Problem 4

The temperature at a point (x, y, z) in space is given by $T(x, y, z) = x^2 + 2y^2 + 3z^2$.

(a) Find the rate of change of temperature at the point $P(1, 2, 3)$ in the direction toward the point $Q(2, 1, 4)$.

(b) In what direction does the temperature increase most rapidly at P? What is the maximum rate of increase?

(c) Find the equation of the tangent plane to the level surface $T(x, y, z) = 22$ at the point $P(1, 2, 3)$.