

# Ancestry Specific Allele Frequency Estimation (ASAFE)

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July 12, 2016

Some commentary is typed in, in red like this.

① Motivation for ASAFE

② Available Data

③ Proposed Approach

④ Data Simulation

⑤ Results

## Genetic Terminology

Human genomes are packaged into pairs of homologous chromosomes. 2 pairs shown below. Rows are SNPs.

- **"SNP"** := Point along a chromosome where genomes differ.  
Often there are two variants or "alleles" at a SNP, labeled 0 and 1.

|                         |   |   |                               |   |   |  |
|-------------------------|---|---|-------------------------------|---|---|--|
| SNP1                    | 0 | 1 | SNP1                          | 0 | 1 | <div>Both pairs: SNP1 0/1 and SNP2 0/1.<br/>Left pair: SNP1 0 1 and SNP2 1 0.<br/>Right pair: SNP1 0 1 and SNP2 0 1.</div> |
| SNP2                    | 1 | 0 | SNP2                          | 0 | 1 |  |
| SNP3                    | 0 | 0 | SNP3                          | 0 | 0 |  |
| SNP4                    | 1 | 1 | SNP4                          | 1 | 1 |  |
| 1 pair of<br>hom. Chr's |   |   | Another pair of<br>hom. Chr's |   |   |  |

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Often there are two variants or "alleles" at a SNP, labeled 0 and 1.
- **"Genotype"** := 2 homologous chromosomes' alleles at a SNP  
Ex: SNP1's genotype is 0/1, or 0|1 or 1|0. / and | denote phase.

|                         |   |   |                               |   |   |  |
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| SNP2                    | 1 | 0 | SNP2                          | 0 | 1 |  |
| SNP3                    | 0 | 0 | SNP3                          | 0 | 0 |  |
| SNP4                    | 1 | 1 | SNP4                          | 1 | 1 |  |
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|                      |   |   |                            |   |   |  |
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- "Unphased genotype" (/): SNP's genotype is NOT ordered with respect to another SNP's genotype
- "Phased genotype" (|): SNP's genotype IS ordered with respect to another SNP's genotype
- Alleles on the same side of | are on the same chromosome, but not necessarily for /

|                      |   |   |                            |   |   |  |
|----------------------|---|---|----------------------------|---|---|--|
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| SNP2                 | 1 | 0 | SNP2                       | 0 | 1 |  |
| SNP3                 | 0 | 0 | SNP3                       | 0 | 0 |  |
| SNP4                 | 1 | 1 | SNP4                       | 1 | 1 |  |
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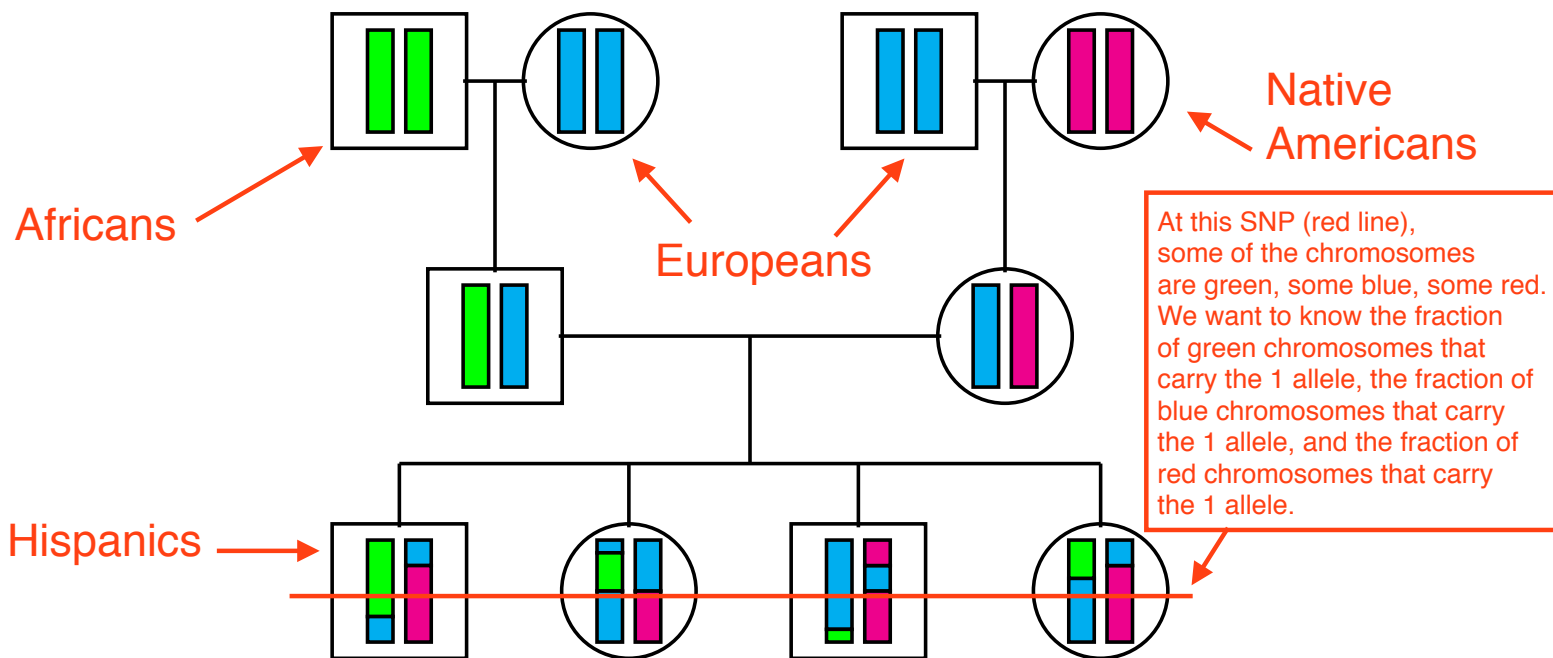
# Hispanic Community Health Study (HCHS)

- Cohort study of 13,000 US Hispanics
- Hispanics are admixed, descended from multiple ancestral populations: Africans, Europeans, and Native Americans
- In this cohort, we test each bi-allelic SNP for association with a trait, say diabetes
- If a SNP is significantly associated with a trait, we want to perform a follow-up study on new individuals to see if we can replicate the association



# The Problem that ASAFE Solves

- For a significant SNP, want **ancestry-specific allele frequencies** :=  $P(\text{Allele 1} \mid \text{African})$ ,  $P(\text{Allele 1} \mid \text{European})$ , and  $P(\text{Allele 1} \mid \text{Native American})$ , i.e. frequencies of allele 1 amongst chromosomes of African, European, or Native American origin at the SNP



- ASAFE := EM algorithm for estimating these frequencies, for a SNP

# How Ancestry Specific Allele Frequencies Relate to HCHS

- These frequencies inform the design of a replication study: If allele 1 were more common amongst the African chromosomes than amongst the other two ancestries' chromosomes, then one would want to recruit a population of predominantly African descent for the replication study

① Motivation for ASAFE

② Available Data

③ Proposed Approach

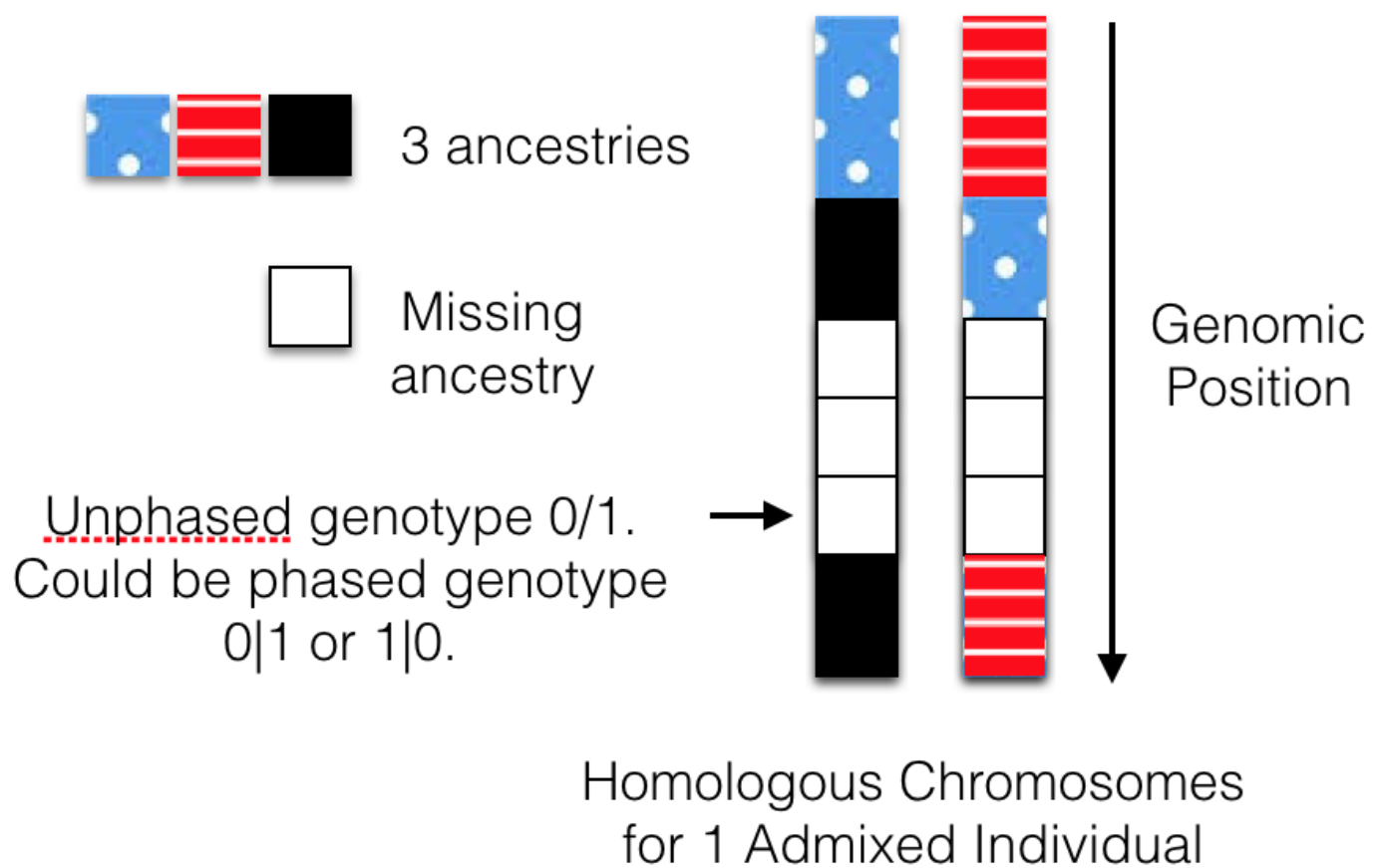
④ Data Simulation

⑤ Results

## Available Data

- At some SNPs, the RFMix program takes phased genotypes as input, and outputs admixed individuals' phased ancestries
- Available data on admixed individuals
  - Phased ancestries, Phased genotypes: Some SNPs
  - No ancestry calls, Unphased genotypes: Other SNPs

## Available Data



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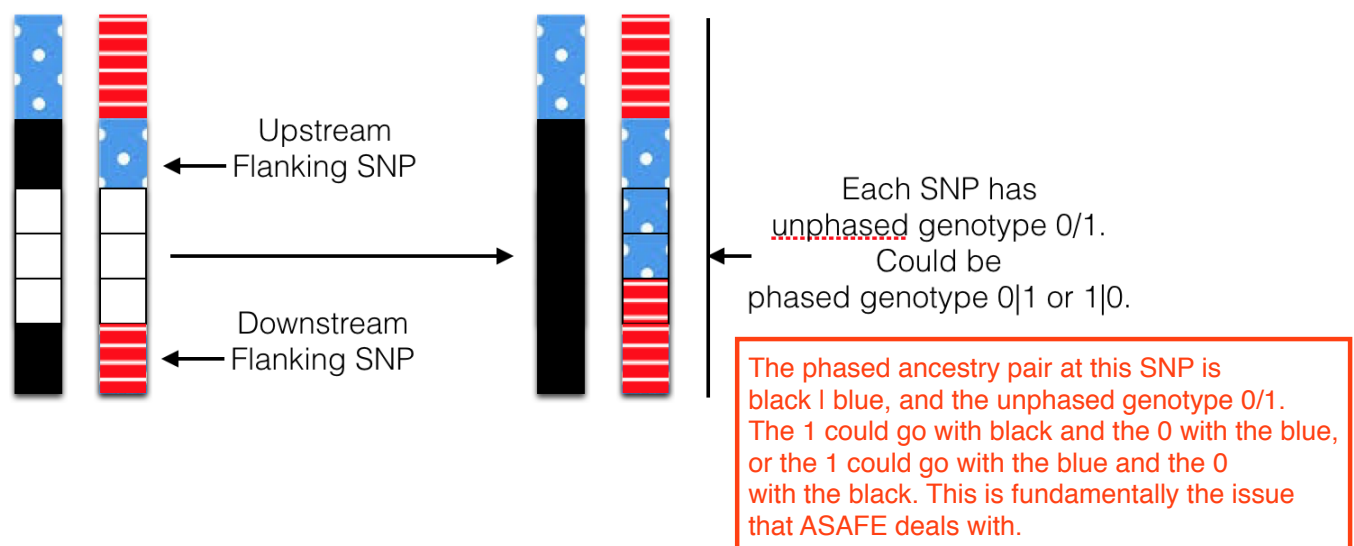
⑤ Results

## Proposed Approach: Fill in Ancestries

Consider a block of SNPs that have been genotyped in the admixed sample, but that do not have ancestries called. For any SNP in this block:

- Call the SNP's ancestry the nearest flanking ancestry

Then we know all SNPs' unphased genotypes and local ancestry pairs.



# Proposed Approach: EM Algorithm to Deal with Unknown Phase of Genotype Relative to Ancestry Pair (ASAFE)

You can view this 2-vector as a new kind of allele, and a  $(g,a)/(g,a)$  genotype as a new kind of genotype.

Consider an ancestry-specific allele (allele, ancestry) =  $(g,a)$ :

- Allele  $g = 0$  or  $1$
- Ancestry  $a = \text{African (A), European (E), Native American (N)}$

There are 6 possible  $(g,a)$  alleles, so 21 values for unordered  $(g,a)/(g,a)$  genotype. We call these values unordered  $(g,a)$ -genotype categories.

**1 complete observation** = The  $(g,a)$ -genotype category that **an individual** belongs to at a SNP.



# Complete, Unobserved Data Categories

Entry  $C_i$  is the name of the  $i$ -th complete, unobserved category.

**Table:** Complete Data Unordered (g,a)-genotype Categories.

| $(g,a) \backslash (g',a')$ | (0, A)   | (0, E)   | (0,N)    | (1, A)   | (1, E)   | (1, N)   |
|----------------------------|----------|----------|----------|----------|----------|----------|
| (0, A)                     | $C_1$    |          |          |          |          |          |
| (0, E)                     | $C_2$    | $C_4$    |          |          |          |          |
| (0, N)                     | $C_3$    | $C_5$    | $C_6$    |          |          |          |
| (1, A)                     | $C_7$    | $C_9$    | $C_{11}$ | $C_{16}$ |          |          |
| (1, E)                     | $C_8$    | $C_{12}$ | $C_{14}$ | $C_{17}$ | $C_{19}$ |          |
| (1, N)                     | $C_{10}$ | $C_{13}$ | $C_{15}$ | $C_{18}$ | $C_{20}$ | $C_{21}$ |

If an individual is in category C9, that means the individual has (g,a) alleles (1, A) and (0,E).

Hardy-Weinberg Equilibrium assumed in the admixed population to get probability  $p_j$  of an individual falling into the  $j$ -th complete data category:

The (g,a) alleles combine independently

$$p_j = \begin{cases} p_{ga}p_{g'a'}, & \text{if } (g, a) = (g', a') \\ 2p_{ga}p_{g'a'}, & \text{otherwise} \end{cases}$$

# Incomplete, Observed Data Categories

Entry  $O_i$  is the name of the  $i$ -th incomplete, observed category. Colored entries are observed data categories that map to multiple complete data categories.

**Table:** Incomplete, Observed Data Categories.

| $(g,a) \backslash (g',a')$ | (0, A) | (0, E)   | (0,N)    | (1, A)   | (1, E)   | (1, N)   |
|----------------------------|--------|----------|----------|----------|----------|----------|
| (0, A)                     | $O_1$  |          |          |          |          |          |
| (0, E)                     | $O_2$  | $O_4$    |          |          |          |          |
| (0, N)                     | $O_3$  | $O_5$    | $O_6$    |          |          |          |
| (1, A)                     | $O_7$  | $O_8$    | $O_9$    | $O_{13}$ |          |          |
| (1, E)                     | $O_8$  | $O_{10}$ | $O_{11}$ | $O_{14}$ | $O_{16}$ |          |
| (1, N)                     | $O_9$  | $O_{11}$ | $O_{12}$ | $O_{15}$ | $O_{17}$ | $O_{18}$ |

An individual in observed category  $O_8$  has phased ancestry A/E and unphased genotype 0/1. We don't know if the 1 goes with the A or with the E, so two complete categories correspond to this one observed category.

Overlaying complete and observed categories gives their correspondence. This correspondence allows us to express the probability  $p'_{j'}$  of an individual being in observed data category  $j', j' \in \{1, \dots, 18\}$  in terms of complete data category probabilities  $p_j, j \in \{1, \dots, 21\}$ .

# Outline Approach to Estimating Ancestry-Specific Allele Frequencies

Because of the connection

- Between  $p'_j$  and  $p_j$ , and
- Between  $p_j$  and (g,a)-allele probabilities

$$\vec{p} = [p_{ga} : g \in \{0, 1\}, a \in \{A, E, N\}],$$

maximizing the observed data log likelihood (e.g. via EM algorithm ASAFE)

$$\log(P(\vec{o} = [o_1, \dots, o_n] | \vec{p}' = [p'_1, \dots, p'_{18}])) = \sum_{j'=1}^{18} m'_{j'} \log(p'_{j'})$$

where  $o_i$  = Observed category of the i-th individual, and  $m'_{j'}$  = Number of individuals in observed category  $j'$

gives us a maximum likelihood estimate (MLE)

$\hat{\vec{p}} = [\hat{p}_{0A}, \hat{p}_{0E}, \hat{p}_{0N}, \hat{p}_{1A}, \hat{p}_{1E}, \hat{p}_{1N}]$  of  $\vec{p}$ , from which we obtain ancestry-specific allele frequency estimates:

$$\hat{p}_{1|a} = \hat{p}_{1a} / \hat{p}_a = \hat{p}_{1a} / (\hat{p}_{1a} + \hat{p}_{0a}), a \in \{A, E, N\} \leftarrow \text{THE GOAL!}$$

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## Simulated Genetic Data

- Used MaCS [Chen et al. (2009)] to simulate Hispanic individuals' sequence data
- For each of the 56,003 SNPs in the sequence data, ran ASAFE with inputs: Unphased admixed genotypes (ignoring known phase) and phased admixed ancestries
- Got ancestry-specific allele 1 frequencies for each ancestry (African, European, Native American), at each SNP
- For each SNP, calculated  
error = Estimated  $p_{1|a}$  - True  $p_{1|a}$ ,  $a \in \{A, E, N\}$

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## Low Error on Simulated Data

Mean and SD of errors  $\{\hat{p}_{1|a} - p_{1|a} : a \in \{A, E, N\}\}$ , grouped by:

- True allele frequency bin that  $p_{1|a}$  falls into: Columns
- Ancestry  $a \in \{A, E, N\}$ : Rows

How to interpret this cell: Say 10,000 of our 56,003 SNPs have  $P(1 | \text{African})$  in (0-0.2]. These 10,000 SNPs have 10,000 associated errors. The mean of those 10,000 errors is -0.0011. Their SD is 0.0065.

|          |           | True Allele 1 Frequency Bins |           |           |           |         |
|----------|-----------|------------------------------|-----------|-----------|-----------|---------|
| Ancestry | Statistic | (0-0.2]                      | (0.2-0.4] | (0.4-0.6] | (0.6-0.8] | (0.8-1] |
| African  | Mean      | -0.0011                      | -0.0003   | -0.0004   | 0.0004    | -0.0004 |
| African  | SD        | 0.0065                       | 0.0185    | 0.0233    | 0.0186    | 0.0118  |
| European | Mean      | -0.0015                      | -0.0004   | -0.0007   | -0.0010   | <0.0001 |
| European | SD        | 0.0077                       | 0.0209    | 0.0249    | 0.0220    | 0.0122  |
| Nat. Am. | Mean      | -0.0004                      | -0.0017   | 0.0021    | 0.0048    | 0.0007  |
| Nat. Am. | SD        | 0.0083                       | 0.0235    | 0.0238    | 0.0257    | 0.0118  |

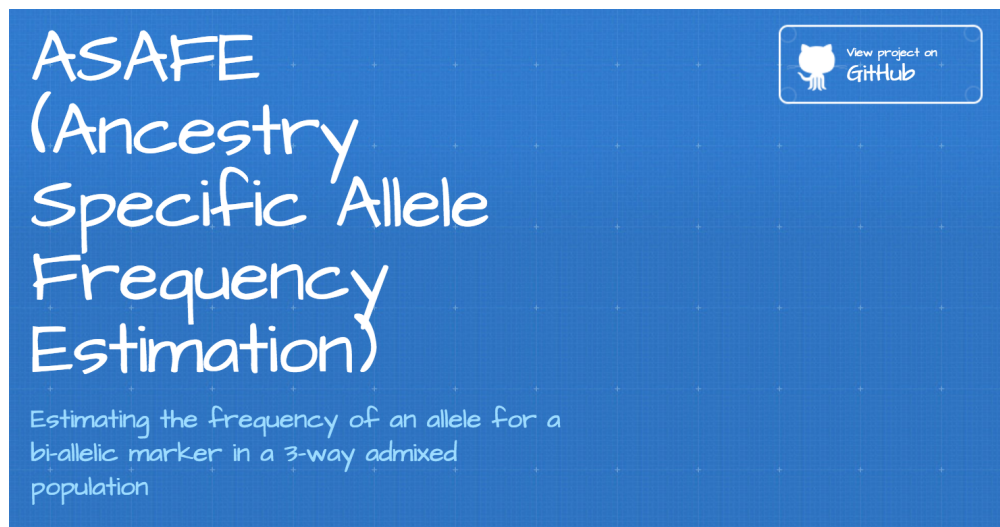
Regardless of true ancestry-specific allele frequency  $p_{1|a}$  bin, errors are low:  
Largest  $|\text{Mean}| = 0.005$ . Largest SD = 0.03.

More Results in paper supplement.

## More Info: Paper and Code

- Qian S. Zhang, Brian L. Browning, and Sharon R. Browning. Asafe: ancestry-specific allele frequency estimation. *Bioinformatics*, 32(14):2227-2229, 2016.
- Package "ASAFE" on Bioconductor 

Oops this is wrong! It's been accepted by Bioconductor, but isn't on there as of 7/12/2016.
- Code to reproduce analysis at <http://biostatqian.github.io/ASAFE/>





## ⑥ Appendix: Extra Slides

## Available Data

- Unphased bi-allelic SNP genotypes from three reference panels proxying ancestral Africans, Europeans, and Native Americans. Unphased admixed sample genotypes.
  - Phase all SNP genotypes
  - Program RFMix [Maples et al. (2013)] takes phased SNP genotypes, internally re-phases admixed genotypes, and outputs ancestry calls for each admixed person's chromosomes only at SNPs typed in all 3 reference panels
- Using a program called RFMix, we obtain admixed individuals' phased ancestries at some SNPs
- Available data on admixed individuals
  - Phased ancestries, Unphased genotypes: For SNPs that RFMix did not output re-phased genotypes for
  - No ancestry calls, Unphased genotypes: For SNPs that RFMix could NOT make ancestry calls for (e.g. not typed in a reference panel)

You get phased ancestries and phased genotypes at the SNPs that RFMix does output re-phased genotypes for. It depends on the options you use with RFMix.

## Inadequate Alternative Approaches

Goal: Estimate ancestry-specific allele frequencies, for each marker typed in the admixed sample, with available data.

Approaches:

- (1) Find the allele 1 frequency in each reference panel
- (2) Find allele 1 frequencies in populations sequenced by the 1000 Genomes project [Via García et al. (2012)]
- (3) ADMIXTURE, STRUCTURE

Weaknesses of Approaches:

- (1), (2), (3): If a marker is not typed in a reference panel or sequenced by the 1000 Genomes project, these approaches cannot be taken
- (3) assume linkage equilibrium (independence amongst SNPs), e.g. do not make use of linkage disequilibrium from local ancestry calls

# ASAFE EM Algorithm

We approximate the MLE  $\hat{\vec{p}}$  using an EM algorithm:

(0) Start with an initial estimate of

$$\vec{p} = \vec{p}_0 = [p_{0,ga}, g \in \{0, 1\}, a \in \{A, E, N\}] = [1/6, 1/6, 1/6, 1/6, 1/6, 1/6]$$

Iterate E-M steps (1 E-M = 1 iteration) until  $\|\vec{p}_{k+1} - \vec{p}_k\|_2 < \epsilon = 10^{-8}$ , where  $\vec{p}_{k+1}$  is the latest estimate of  $\vec{p}$  and  $\vec{p}_k$  is the 2nd to latest estimate.

(1) E-Step : Evaluate

$$E_{\vec{c}}(\log(P(\vec{c}|\vec{p}))|\vec{o}, \vec{p}_k) = E_{\vec{c}}(\sum_{j=1}^{21}[m_j \log(p_j)]|\vec{o}, \vec{p}_k) = \sum_{j=1}^{21}[E_{\vec{c}}(m_j|\vec{p}, \vec{p}_k) \log(p_j)]$$

(2) M-Step: Set

$$\vec{p}_{k+1} = \operatorname{argmax}_{\vec{p}} E_{\vec{c}}(\log(P(\vec{c}|\vec{p}))|\vec{o}, \vec{p}_k)$$

## E-Step in More Detail

Intuition behind  $m_{k,j}$  equation: You iterate over the individuals in observed category  $j'$ . Each individual contributes a fractional count towards the expected number of individuals in complete category  $j$ . This fractional count is the probability and individual in observed category  $j'$  is in complete category  $j$ .

On the  $k$ -th iteration of the algorithm, let the expected value of the number  $m_j$  of individuals in complete category  $j$  be denoted

$$m_{k,j} = E_{\vec{c}}(m_j | \vec{o}, \vec{p}_k) = \sum_{i=1}^{m'_{j'}} \frac{P(c_i=j | \vec{p}_k)}{P(c_i=\text{Any } j \text{ that is consistent with } o_i=j' | \vec{p}_k)}$$

Fractional count

where

- $\vec{c} = [c_1, \dots, c_n]$  are complete categories for  $n$  admixed individuals
- $\vec{o} = [o_1, \dots, o_n]$  are observed categories for  $n$  admixed individuals
- $\vec{p}_k = [p_{k,0A}, p_{k,0E}, p_{k,0N}, p_{k,1A}, p_{k,1E}, p_{k,1N}]$  is the  $k$ -th estimate for the  $\vec{p}$  that maximizes the observed data log likelihood
- $m'_{j'}$  is the number of individuals in observed category  $j'$  that is consistent with complete category  $j$

## M-Step in More Detail

$\hat{\vec{p}}_{k+1} = [\hat{p}_{k+1,0A}, \hat{p}_{k+1,0E}, \hat{p}_{k+1,0N}, \hat{p}_{k+1,1A}, \hat{p}_{k+1,1E}, \hat{p}_{k+1,1N}]$ , where

Intuition behind this equation: Consider the definition of complete categories on Slide 14. Imagine you could observe the complete categories. Take the fraction of  $2n$  chromosomes that carry (0,A), and that is the expression on the right-hand side of the equation.

$$\hat{p}_{k+1,0A} = \frac{2m_{k,1} + m_{k,2} + m_{k,3} + m_{k,7} + m_{k,8} + m_{k,10}}{2n}$$

$$\hat{p}_{k+1,0E} = \frac{m_{k,2} + 2m_{k,4} + m_{k,5} + m_{k,9} + m_{k,12} + m_{k,13}}{2n}$$

$$\hat{p}_{k+1,0N} = \frac{m_{k,3} + m_{k,5} + 2m_{k,6} + m_{k,11} + m_{k,14} + m_{k,15}}{2n}$$

$$\hat{p}_{k+1,1A} = \frac{m_{k,7} + m_{k,9} + m_{k,11} + 2m_{k,16} + m_{k,17} + m_{k,18}}{2n}$$

$$\hat{p}_{k+1,1E} = \frac{m_{k,8} + m_{k,12} + m_{k,14} + m_{k,17} + 2m_{k,19} + m_{k,20}}{2n}$$

$$\hat{p}_{k+1,1N} = \frac{m_{k,10} + m_{k,13} + m_{k,15} + m_{k,18} + m_{k,20} + 2m_{k,21}}{2n}$$

where  $n$  = Number of individuals.

- Chen, G. K., Marjoram, P., and Wall, J. D. (2009). Fast and flexible simulation of dna sequence data. *Genome research*, 19(1):136–142.
- Maples, B. K., Gravel, S., Kenny, E. E., and Bustamante, C. D. (2013). Rfmix: a discriminative modeling approach for rapid and robust local-ancestry inference. *The American Journal of Human Genetics*, 93(2):278–288.
- Via García, M., Consortium, . G. P., et al. (2012). An integrated map of genetic variation from 1,092 human genomes. *Nature*, 2012, vol. 491, p. 56-65.