Consider a dataset $\mathcal{D} = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_N, y_N)\}$ of N independent and identically distributed observations where each $\mathbf{x}_n$ is a p-dimensional real vector. Assume the random variable Y_n is distributed Laplacian with a mean $\boldsymbol{\beta}^T\mathbf{x}_n$ and known scale parameter $b > 0$. In other words, $Y_n|\mathbf{x}_n, \boldsymbol{\beta} \sim \mathcal{L}(\boldsymbol{\beta}^T\mathbf{x}_n, b)$. Define the a priori distribution of the parameters $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_p)$ {\em multivariate} Gaussian with parameters mean $\mathbf{0}$ and variance $\sigma^2\mathbf{I}$.

$Pr(y_n|\mathbf{x}_n, \boldsymbol{\beta}) = \frac{1}{2b}\exp\left[-\frac{|y_n - \mathbf{x}_n^T\boldsymbol{\beta}|}{b}\right]$

$P(\boldsymbol{\beta}) = \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
params = {'legend.fontsize': 'x-large','figure.figsize': (12, 5),
         'axes.labelsize': 'x-large','axes.titlesize':'x-large',
         'xtick.labelsize':'x-large','ytick.labelsize':'x-large'}
plt.rcParams.update(params)

import cvxpy as cp
dim = 3
numrepetitions = 100
betaTRUE = np.random.multivariate_normal(np.zeros(dim),np.eye(dim))
```

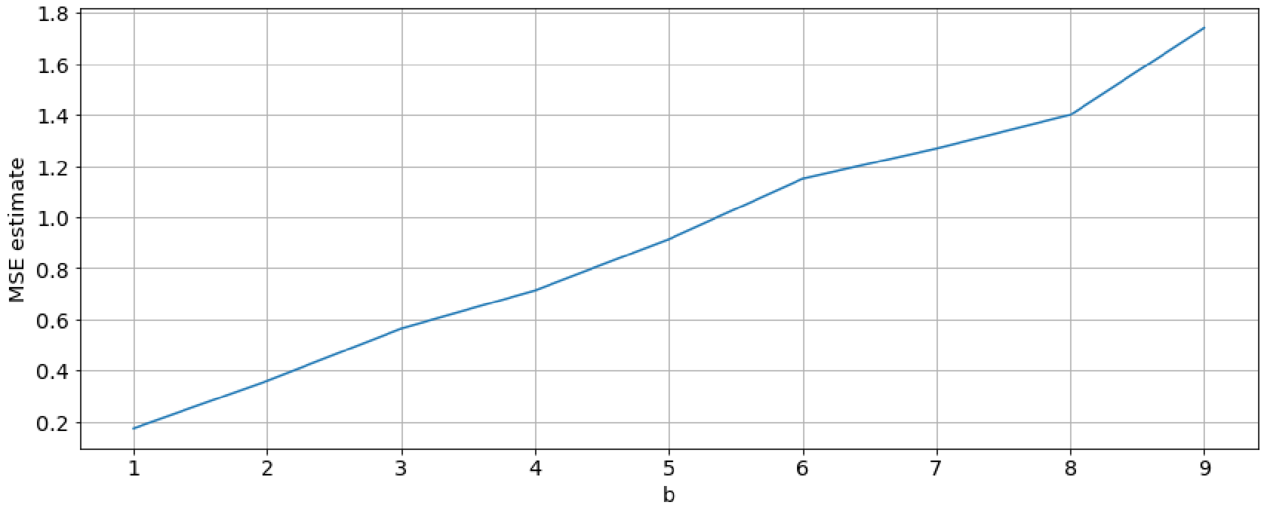
What happens to $\mathrm{E}(||\beta - \hat{\beta}||_2)$ as b increases and N remains constant? Or, equivalently, as b decreases and N remains constant?

In [82]:

```
bTRUE = np.arange(1,10)
N = 100

error = []
for b in bTRUE:
    error2 = []
    for nn in range(numrepetitions):
        X_train = 
np.random.multivariate_normal(np.zeros(dim),np.eye(dim),N)
        Y_train = np.asarray([np.random.laplace(np.dot(xx,betaTRUE),b) for 
xx in X_train])
        problem = cp.Problem(cp.Minimize(cp.norm1(X_train @ betaHAT - 
Y_train)))
        problem.solve()
        error2.append(np.linalg.norm(betaHAT.value-betaTRUE))
    error.append(np.mean(error2))
```

```python
fig, ax1 = plt.subplots()
plt.plot(bTRUE,error)
plt.grid()
plt.xlabel("b")
plt.ylabel("MSE estimate")
fig.tight_layout()
```



What happens to $\mathrm{E}(\|\beta - \hat{\beta}\|_2)$ as N increases and b remains constant? Or equivalently, as N decreases and b remains constant?

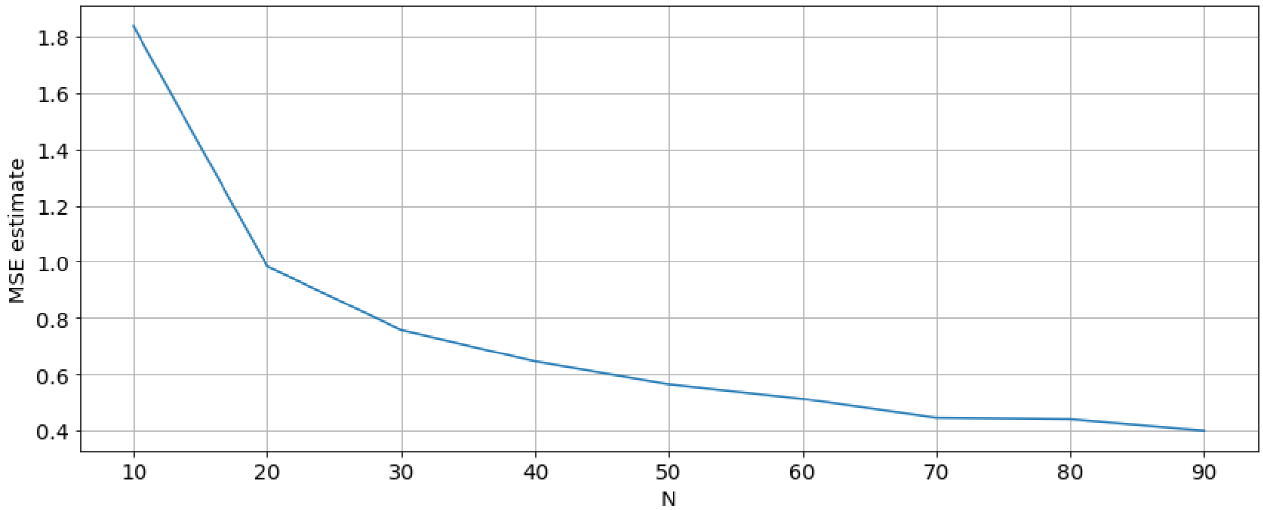
In [84]:

```python
bTRUE = 2
Nvals = np.arange(10,100,10)

error = []
for N in Nvals:
    error2 = []
    for nn in range(numrepetitions):
        X_train =
np.random.multivariate_normal(np.zeros(dim),np.eye(dim),N)
        Y_train = np.asarray([np.random.laplace(np.dot(xx,betaTRUE),bTRUE)
for xx in X_train])
        problem = cp.Problem(cp.Minimize(cp.norm1(X_train @ betaHAT -
Y_train)))
        problem.solve()
        error2.append(np.linalg.norm(betaHAT.value-betaTRUE))
    error.append(np.mean(error2))

fig, ax1 = plt.subplots()
plt.plot(Nvals,error)
plt.grid()
```

```python
plt.xlabel("N")
plt.ylabel("MSE estimate")
fig.tight_layout()
```



In [ ]:

```python

```