Note - Kindly check corresponding R & Excel files for final solutions.

QM-1b ASSIGNMENT 3 GROUP-14

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Q1

a. SRSWOR $N = 150$, $n = 30$

- Picked sample of size 30 using R (Q1.R) (SRSWOR)
- Computed required values in Excel

- Formulas used:- for each characteristic

$\hat{\mu}$ = sample mean = $\bar{y} = \frac{\sum y_i}{n}$

$\hat{SE}(\bar{y}) = \sqrt{1 - \frac{n}{N}} \frac{s}{\sqrt{n}}$; where s = std dev of sample

b. Stratas of 10 each since $N_i = 50$, $N = 150$

$$n_i = \frac{N_i}{N} \times n = \frac{50}{150} \times 30 = 10$$

- Made samples using R (Q1.R)
- Computed required values in Excel.

$$\hat{\mu} = \bar{y} = \sum_{i=1}^{3} \frac{N_i}{N} * \bar{y}_i \quad (3 \text{ stratas})$$

↳ sample mean of each strata

$$\hat{SE}(\bar{y}) = \sqrt{\sum_{i=1}^{3} \left(\frac{N_i}{N}\right)^2 \left(1 - \frac{n_i}{N_i}\right) \frac{S_i^2}{n_i}}$$; $S_i \rightarrow$ Std dev for samples of i^{th} strata.

c. Used Optimal allocation

$n_{setosa} = 7$

$n_{virginica} = 13$

$n_{versicolor} = 10$

- Made samples using R (Q1.R)
- Computed Required Values in Excel.

Q4. $\mu = 60, \sigma = 15$

$n = 50$

X_i – weight of each individual

$S_{50} = X_1 + X_2 + \ldots + X_{50}$

$P(S_{50} > S') = 0.001$

$P\left(\frac{S_{50}}{50} > \frac{S'}{50}\right) = 0.001$

$P\left(\bar{X}_{50} > \frac{S'}{50}\right) = 0.001 \Rightarrow 1 - P\left(\bar{X}_{50} \leq \frac{S'}{50}\right) = 0.001$

From CLT, $\bar{X}_{50} \sim N\left(60, 15/\sqrt{50}\right)$

$\Rightarrow \frac{S'}{50} = \text{NORM.INV}\left(1-0.001, 60, 15/\sqrt{50}\right) = 66.5537$

$\Rightarrow S' = 50 * 66.5537 = 3327.769$

Q5.

$n = 100$

X_i	1	2	3	4	5
$P(X_i)$	0.3	0.4	0.05	0.2	0.05

X_i – no of candies bought by i^{th} customer (i^{th} Customer)

$E[X_i] = 0.3\times1 + 0.4\times2 + 0.05\times3 + 0.2\times4 + 5\times0.05$

$E[X_i] = 2.3$

$V(X_i) = E[X_i^2] - E[X_i]^2 = 6.8 - (2.3)^2 = 1.51$

$S_{100} = X_1 + \ldots + X_{100}$

$P(S_{100} \leq S') = 0.9$

$P\left(\frac{S_{100}}{100} \leq \frac{S'}{100}\right) = 0.9$

$$P\left(\bar{X}_{100} \leq \frac{S'}{100}\right) = 0.9$$

By CLT

$$\bar{X}_{100} \sim ND\left(2.3, \sqrt{(1.51/100)}\right)$$

$$\therefore \quad \frac{S'}{100} = NORM.INV\left(0.9, 2.3, \sqrt{\frac{1.51}{100}}\right)$$

$$\Rightarrow \quad S' = 245.748$$

$$\Rightarrow \text{246 candies should be stocked.}$$

Q3.

a) Done using R (Q3.R) & pasted in Excel
b) Done in Excel

Q2. a), b) Sampling done using R

b) All parts solved using Excel (sheets attached)

a, b, c → Mean $= \hat{\mu} = \bar{y} = \sum_i \frac{N_i}{N} \times \bar{y}_i$ ← sample mean of each strata

$$\hat{SE}(\bar{y}) = \sqrt{\sum_i \left(\frac{N_i}{N}\right)^2 \left(1 - \frac{n_i}{N_i}\right) \frac{S_i^2}{n_i}}$$

S_i → std dev of samples for ith strata

d, e, f, g → ~~Sample~~ Population mean same as a, b, c

but $\bar{y}_i = (k \sum y_j)/N_i$

Koops method $\hat{SE}(\bar{y}) = 0.5 * |\bar{y}_{odds} - \bar{y}_{evens}|$

For population estimates,

$$\hat{SE}(\bar{Y}) = \sum_{i=1}^{6} \frac{N_i}{N} * \hat{SE}(\bar{y}_i)$$

↓ for all strata