№3.1

№3.1

2. $n^{\frac{7}{2}} + 7n^3 \log n + n^2 = f(n)$

1.) Let us find a supremum of the polinomial of $f(n)$.

1.1) $n^{\frac{7}{2}} < 7n^3 \log n$
1.2) $7n^3 \log n > n^2$
$\Big\}$ $7n^3 \log n$ is a supremum of $f(n)$

2.) For a given supremum let us prove that it is an upper bound, i.e. $O(n)$.

$c_1 7n^3 \log n > 7n^3 \log n + n^{\frac{7}{2}} + n^2$

$c n^3 \log n > 7n^3 \log n + n^{\frac{7}{2}} + n^2$, $c = 7c_1$.

Let c be equal 9, then

$n \to \infty$: $\cancel{9n^3 \log n > 7n^3 \log n + n^{\frac{7}{2}} + n^2}$

$\cancel{n^3 \log n >}$

$$\frac{7n^3 \log n + n^{\frac{7}{2}} + n^2}{7n^3 \log n} \approx 1 + 0 + 0, \text{ hence}$$

$f(n)$ and $g(n) = n^3 \log n$ grow assymptotically equally.

For $c = 9$:

$7n^3 \log n + n^{\frac{7}{2}} + n^2 < 9n^3 \log n$

$\begin{cases} n^{\frac{7}{2}} < n^3 \log n \\ n^2 < n^3 \log n \end{cases}$ satisfies condition above, then

$\begin{cases} \frac{1}{n} < \log n \\ 1 < n \log n \end{cases}$ holds for $n \geq 9$, since $g(n) = n^3 \log n$ grows faster as a supremum.

$\forall n \geq 9$, $c = 9 \Rightarrow f(n) < g(n) \Rightarrow f(n) = O(g(n))$

№ 3.1

DSA Assignment I Theoretical Part

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№ 3.1

1.) $n^2 + 10n\log(n) + 50n + 100 = f(n)$

~~$c = 3, \forall n: cA$~~

$c = 3, \forall n > 100: cn^2 \geq f(n) \geq 0 \Rightarrow f(n) = O(n^2)$

Answer: $f(n) = O(n^2)$

Proof:

I. Let us find a supremum of the polinomial of $f(n)$:

$$\lim_{n\to\infty} \frac{n^2}{10n\lg n} = \lim_{n\to\infty} \frac{n}{10\lg n} = \infty \Rightarrow n^2 > 10n\lg(n)$$

$$\lim_{n\to\infty} \frac{50n}{n^2} = \lim_{n\to\infty} \frac{50}{n} = 0 \Rightarrow n^2 > 50n$$

$$\lim_{n\to\infty} \frac{100}{n} = 0 \Rightarrow n^2 > 100$$

II. Let us proof that $g(n) \mid f(n) = O(g(n))$ equals n^2:

$$cg(n) = cn^2 \geq n^2 + 10n\log n + 50n + 100 = f(n)$$

$$cn \geq n + 10\log n + 50 + \frac{100}{n}$$

$$(c-1) \geq 1 + \frac{10\log n}{n} + \frac{50}{n} + \frac{100}{n^2}$$

Assume: $n = 100$, then

$$(c-1) \geq 1 + \frac{\log 100}{10} + \frac{50}{100} + \frac{1}{100}$$

$$c-1 \geq \frac{\log 128}{10} + 0.5 + 0.01 \geq \frac{\log 100}{100} + 0.5 + 0.01$$

$$c \geq 1.7 + 0.51$$

$c \geq 2.31$, ~~Hence~~ Since $g(n)$ is ~~not~~ monotoneously not less, than $f(n)$, $cg(n) \geq f(n)$. Q.E.D.

№ 3.3

$T(n) = \sqrt{k} \cdot T\left(\frac{n}{k^2}\right) + c \cdot \sqrt[4]{n}$ $\quad T(1) = 0$

$$\boxed{T(n) = a\, T\left(\frac{n}{b}\right) + f(n)}$$

$a = \sqrt{k}$

$b = k^2$

$f(n) = c \cdot \sqrt[4]{n} = c \cdot n^{\frac{1}{4}}$

$\log_b(a) = \log_{k^2}(\sqrt{k}) = \frac{1}{4} = \log_n\left(\frac{f(n)}{c}\right) \Rightarrow$

$\Rightarrow n^{\log_b(a)} = n^{\log_n\left(\frac{f(n)}{c}\right)} \Rightarrow n^{\log_b(a)} = \frac{f(n)}{c} \Rightarrow$

$\Rightarrow c \cdot n^{\log_b(a)} = f(n) \Rightarrow f(n) = \Theta(n^{\log_b(a)})$. Second case of the Master Theorem, hence $T(n) = \Theta(n^{\log_b(a)} \log(n))$.

$T(n) = \Theta(n^{\log_b a} \log(n)) = \Theta(n^{\frac{1}{4}} \log(n)) = \boxed{\Theta(\sqrt[4]{n} \log(n))}$

Answer: $T(n) = \Theta(\sqrt[4]{n} \log(n))$.

№ 3.2

```
void calculateComplexity(int n) { //[Σ_{i=0}^{n-1}((Σ_{j=0}^{i-1} c_1)+1)]·c_2·((log_3 n)+1)
  for(int i=n; i>0; i--)                    //(n+1)
    for(int j=0; j<i; j++)                  // Σ_{i=0}^{n-1}((Σ_{j=0}^{i} c_1)+1)
      checkpoint(n);   //[Σ_{i=0}^{n-1}((Σ_{j=0}^{i-1} c_1)+1)]·α(n)

  void checkpoint(int number) { //c_2·((log_3 number)+1) = α(number)
    for (int i=1; i<number; i=i*3)     //(log_3 number)+1
      System.out.println("DSA Homework"+i); //c_2·((log_3 number)+1)

  }
}
```

$f(n) = \left[\sum_{i=0}^{n}\left(\left(\sum_{j=0}^{i} c_1\right)+1\right)\right]\cdot c_2\cdot\left((\log_3 n)+1\right)$; c_1 and c_2 are some constants

$f(n) = \left(\frac{n(n+1)}{2}\cdot c_1 + n\right)(c_2 + c_2\log_3 n)$

Supremum of the polinomial form of $f(n)$ ~~of~~ is

$sup(g_1(n)) = \frac{c_2 c_1}{2}\cdot n(n+1)\log_3 n \approx \frac{c_2 c_1}{2}\cdot n^2\log_3 n = g(n)\cdot\frac{c_1 c_2}{2}$

$g(n)$ ~~grows~~ grows faster than any of the other terms of polinomial form of $f(n)$ by definition of supremum.

Then: $\lim_{n\to\infty}\frac{f(n)}{g(n)} = 1$, means, by multiplying $g(n)$ by different constant ~~a~~ we can make it eithe less equal of bigger than $f(n)$ after certain n_0 as equivalent functions. Therefore $f(n) = \Theta(g(n)) = O(g(n))$.

Answer: $f(n) = \underline{O(n^2\log_3 n)}$.

№ 3.1

3. $6^{n+1} + 6(n+1)! + 24n^{42} = f(n) \le g(n)$

$6(n+1)!$ – supremum

$f(n) = O((n+1)!)$, Proof:

$$\left\{\begin{array}{l} \left\{\begin{array}{l} 6^{n+1} < (n+1)! \quad \#1 \\ 24n^{42} < (n+1)! \cdot 24 \quad \#2 \end{array}\right. \\ 6(n+1)! = 6(n+1)! \text{ By construction} \end{array}\right.$$

For $c = 31$ there're natural numbers n such that:

$c\,g(n) = c(n+1)! > f(n)$ (use a Graham's number to calculate)

and hence $f(n) = O(g(n))$

#1

Let us consider a relation $\frac{6^n}{(n)!} = a_n$ for finite n.

Then $a_{n+1} = \frac{6^{n+1}}{(n+1)!} = a_n \cdot \frac{6}{n+1}$, considering

finite $n > 6$ we get $a_{n+1} < a_n$.

Then we can decompose $\frac{6^k}{(k)!}$ to $\prod_{k=1}^{\infty} \frac{6}{k}$, that

converges to 0, hence $(n+1)! > 6^{n+1}$

#2

Let us consider a ~~relation~~ $\frac{n^{42}}{(n+1)}$ limit

$a = \lim_{n\to\infty} \frac{n^{42}}{(n+1)!}$. After using ~~Lhost~~ L'Hospital's

Rule 42 times, $a \approx \lim_{n\to\infty} \frac{42!}{c_1 n^{n-42} + c_2 n^{n-43} \dots 1}$, hence

$(n+1)! > n^{42}$