

# Notation

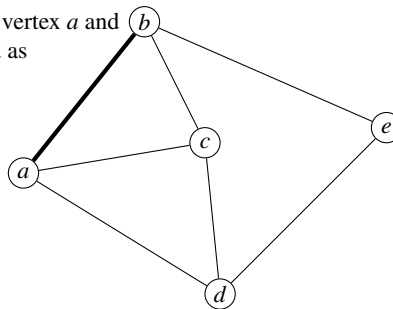
## Strings, arrays, sets

$i, j, k, n$	Integer numbers.
$[i..j],  [i..j] $	The set of integers $\{i, i + 1, \dots, j - 1, j\}$ and its cardinality $j - i + 1$ .
$\mathbb{I}(x, y), \mathbb{I}(x)$	A function that returns the set of integers $[i..j]$ associated with the pair of objects $(x, y)$ . We use $\mathbb{I}(x)$ when $y$ is clear from the context.
$\tilde{x}, \tilde{x}, \vec{x},  \tilde{x} $	Assume that $\mathbb{I}(x, y) = [i..j]$ , where both function $\mathbb{I}$ and object $y$ are clear from the context. Then, $\tilde{x} = [i..j]$ , $\tilde{x} = i$ , $\vec{x} = j$ , and $ \tilde{x}  =  [i..j] $ .
$\Sigma = [1..\sigma]$	Alphabet of size $\sigma$ . All integers in $\Sigma$ are assumed to be used.
$\Sigma \subseteq [1..u]$	An ordered alphabet, in which not all integers in $[1..u]$ are necessarily used. We denote its size $ \Sigma $ with $\sigma$ .
$a, b, c, d$	<i>Characters</i> , that is, integers in some alphabet $\Sigma$ . We also call them <i>symbols</i> .
$T = t_1 t_2 \dots t_n$	A <i>string</i> , that is, a concatenation of characters in some alphabet $\Sigma$ , with character $t_i$ at position $i$ . We use the term <i>sequence</i> in a biological context.
$T \cdot S, t \cdot s$	Concatenation of strings $T$ and $S$ or multiplication of integers $t$ and $s$ , with the operation type being clear from the context. We sometimes omit $\cdot$ if the operands of the concatenation/multiplication are simple.
$T = \text{ACGATAGCTA}$	A <i>string</i> , with characters given explicitly and represented as letters of the English alphabet.
$\underline{T}$	The <i>reverse</i> of a string $T$ , i.e. string $T$ read from right to left.
$\overline{T}$	The <i>reverse complement</i> of a DNA sequence $T$ , that is, string $T$ read from right to left, replacing A with T and C with G, and vice versa.
$T_{i..j}$	The <i>substring</i> $t_i t_{i+1} \dots t_{j-1} t_j$ of string $T$ induced by the indexes in $[i..j]$ .
$T[i..j]$	Equivalent to $T_{i..j}$ , used for clarity when $i$ or $j$ are formulas rather than variables.

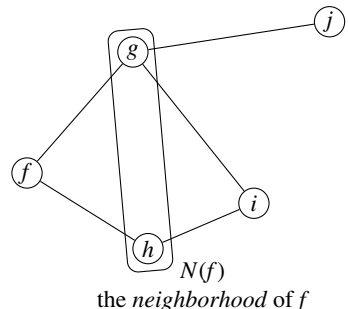
<i>subsequence</i>	A string $t_{i_1}t_{i_2}\cdots t_{i_k}$ obtained by selecting a set of positions $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ and by reading the characters of a string $T = t_1t_2\cdots t_n$ at those positions. In a biological context, subsequence is used as a synonym of substring.
$\mathcal{S} = \{T^1, T^2, \dots, T^n\}$	Set of strings, with $T^i$ denoting the $i$ th string.
$\Sigma^*, \Sigma^+, \Sigma^n$	The set of all strings over alphabet $\Sigma$ , the set of all non-empty strings over alphabet $\Sigma$ , and the set of strings of length $n$ over alphabet $\Sigma$ , respectively. We use shorthand $A = a^n$ for $A \in \{a\}^n$ , that is, for a string consisting of $n$ occurrences of $a$ .
$\delta(i..j, c)$	A function that maps an interval $[i..j]$ and a character $c \in \Sigma$ onto exactly one interval $[i'..j']$ .
$\dots, \#_2, \#_1, \#_0$	Shorthands for non-positive integers, with $\#_0 = 0$ and $\#_i = -i$ .
$\#$	Shorthand for $\#_0$ .
$\$1, \$2, \dots$	Shorthands for positive integers greater than $\sigma$ , with $\$_i = \sigma + i$ .
$\$$	Shorthand for $\$_1$ .
$A[1..n]$	Array $A$ of integers, indexed from 1 to $n$ .
$A[i..j], A[\vec{x}]$	The <i>subarray</i> of array $A$ induced by the indexes in $[i..j]$ and in $\vec{x}$ , respectively.
$X = (p, s, v)$	<i>Triplet</i> of integers, with primary key $X.p$ , secondary key $X.s$ , and value $X.v$ .
$D[1..m, 1..n]$	An <i>array/matrix</i> with $m$ rows and $n$ columns.
$D_{i_1..j_1, i_2..j_2}$	<i>Subarray</i> of $D$ .
$D[i_1..j_1, i_2..j_2]$	Same as above.
$d_{i,j} = D[i,j]$	An element of the array $D$ .

## Undirected graphs

an *edge* between vertex  $a$  and vertex  $b$ , denoted as  $(a, b)$  or as  $(b, a)$



one *connected component* of  $G$

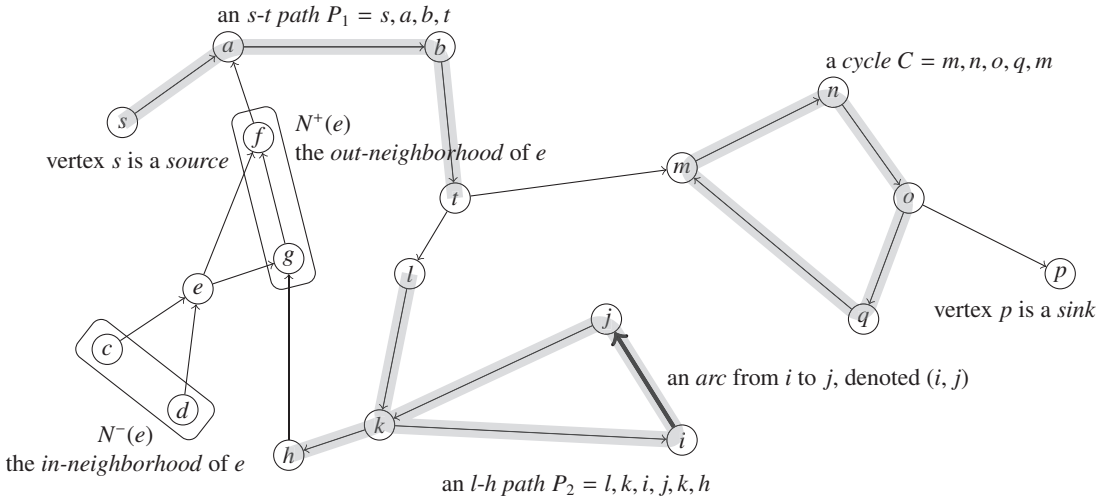


another *connected component* of  $G$

**Figure 2** An undirected graph  $G = (V, E)$ , with vertex set  $V$  and edge set  $E$ .

$V(G)$	Set $V$ of <i>vertices</i> of a graph $G = (V, E)$ .
$E(G)$	Set $E$ of <i>edges</i> of an undirected graph $G = (V, E)$ .
$(x, y) \in E(G)$	An edge of an undirected graph $G$ ; the same as $(y, x)$ .
$N(x)$	The <i>neighborhood</i> of $x$ in $G$ , namely the set $\{y \mid (x, y) \in E(G)\}$ .

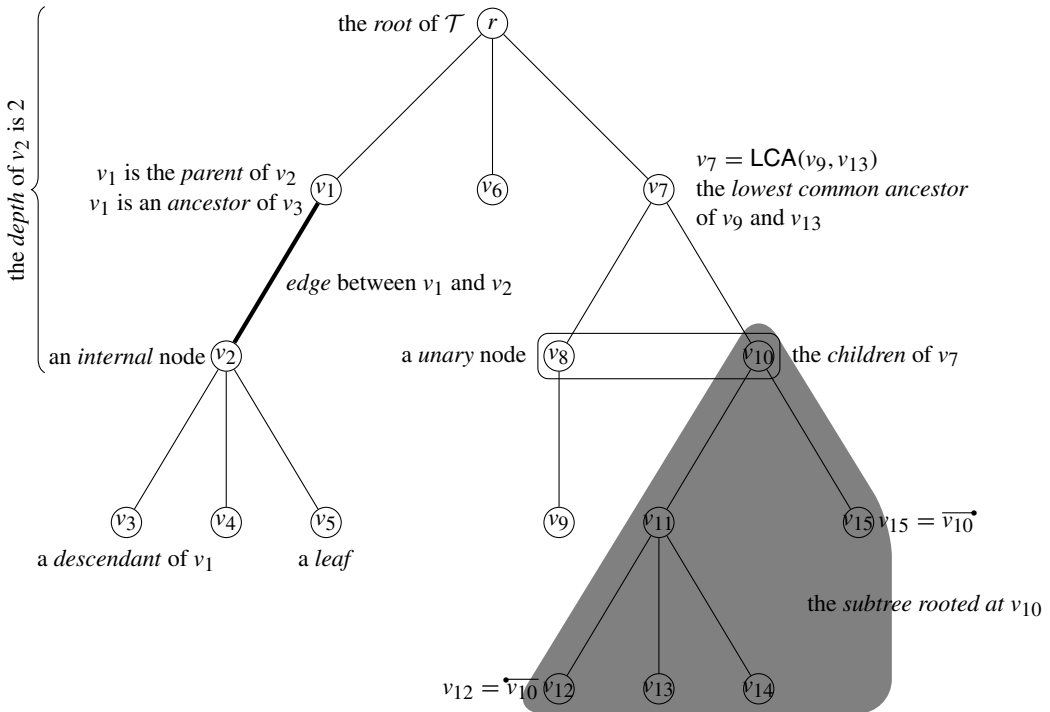
## Directed graphs



**Figure 3** A directed graph  $G = (V, E)$ , with vertex set  $V$  and arc set  $E$ .

$(x, y) \in E(G)$	An arc of a directed graph $G$ ; the arc $(x, y)$ is different from $(y, x)$ .
$N^-(x)$	The in-neighborhood of $x$ in $G$ , namely the set $\{y \mid (y, x) \in E(G)\}$ .
source	A vertex $v$ is a source if $N^-(v) = \emptyset$ .
$N^+(x)$	The out-neighborhood of $x$ in $G$ , namely the set $\{y \mid (x, y) \in E(G)\}$ .
sink	A vertex $v$ is a sink if $N^+(v) = \emptyset$ .
$P = v_1, \dots, v_k$	A path in $G$ , namely a sequence of vertices of $G$ connected by arcs with the same orientation, from $v_1$ to $v_k$ ; depending on the context, we allow or not $P$ to have repeated vertices.
$s$ - $t$ path	Path from vertex $s$ to vertex $t$ .
$C = v_1, \dots, v_k, v_1$	A cycle in $G$ , namely a path in $G$ in which the first and last vertex coincide; depending on the context, we allow or do not allow $C$ to have other repeated vertices than its first and last elements.
$(x, y) \in S$	Arc $(x, y) \in E(G)$ appears on $S$ , where $S$ is a path or cycle of $G$ .

## Trees



**Figure 4** A tree  $\mathcal{T} = (V, E)$ , with node set  $V$  and edge set  $E$ . Unless stated otherwise, we assume all trees to be *ordered*, that is, we assume that there is a total order on the children of every node.

$v_2$ is a child of $v_1$	If there is an edge between $v_1$ and $v_2$ and $v_1$ appears on the path from the root to $v_2$ .
$v_1$ is the parent of $v_2$	If $v_2$ is the child of $v_1$ .
degree of a node $v$	The number of children of $v$ .
leaf	A node with degree 0.
internal node	A node with degree at least 1.
unary node	A node with degree 1.
depth of $v$	The number of edges of the path from the root to $v$ .
subtree rooted at $v$	The subtree of $\mathcal{T}$ having root $v$ and consisting of all nodes reachable through a path starting at $v$ made up only of nodes of depth at least the depth of $v$ .
$v_2$ is descendant $v_1$	If $v_2 \neq v_1$ belongs to the subtree rooted at $v_1$ .
$v_1$ is ancestor $v_2$	If $v_2 \neq v_1$ belongs to the subtree rooted at $v_1$ .
$\text{LCA}(v_1, v_2)$	The <i>lowest common ancestor</i> of $v_1$ and $v_2$ , that is, the deepest node which is an ancestor of both $v_1$ and $v_2$ .
$\vec{v}$	The left-most leaf of the subtree rooted at a node $v$ .
$\overrightarrow{v}$	The right-most leaf of the subtree rooted at a node $v$ .

