

Chapter 0

Quick Tricks: Easy (and Impressive) Calculations In the pages that follow, you will learn to do math in your head faster than you ever thought possible. Consider the problem: 32×11 To solve this problem, simply add the digits, $3 + 2 = 5$, put the 5 between the 3 and the 2, and there is your answer:

Benj_0307338401_4p_c00_r1.r.qxd 5/4/06 1:46 PM Page 2 Secrets of Mental Math 2 35×2 What could be easier? Now you try: 53×11 Since $5 + 3 = 8$, your answer is simply 583 One more. As before, the 3 goes in between the numbers, but the 1 needs to be added to the 8 to get the correct answer: 935 Think of the problem this way: $1\ 835 \times 935$ Benj_0307338401_4p_c00_r1.r.qxd 5/4/06 1:46 PM Page 3 Quick Tricks: Easy (and Impressive) Calculations 3

Here is another example. Since $5 \times 7 = 35$, the answer is 1 527 627 Okay, now it's your turn. Now, I know from experience that if you tell a friend or teacher that you can multiply, in your head, any two-digit number by eleven, it won't be long before they ask you to do 99×11 . Since $9 + 9 = 18$, the answer is: 1 989 1089 Okay, take a moment to practice your new skill a few times, then start showing off. At this point, you probably have a few questions, such as: Can we use this method for multiplying three-digit numbers (or larger) by eleven? Benj_0307338401_4p_c00_r1.r.qxd 5/4/06 1:46 PM Page 4 Secrets of Mental Math 4

Absolutely. For instance, for the problem 314×11 , the answer still begins with 3 and ends with 4. Since $3 + 1 = 4$, and $1 + 4 = 5$, the answer is 3454. In Chapters 2, 3, 6, and 8, you will learn methods for multiplying together just about any two numbers. For example, the square of 7 is $7 \times 7 = 49$. Benj_0307338401_4p_c00_r1.r.qxd 5/4/06 1:46 PM Page 5 Quick Tricks: Easy (and Impressive) Calculations 5

For example, to square the number 35, we simply multiply the first digit (3) by the next higher digit (4), then attach 25. Since $3 \times 4 = 12$, the answer is 1225. Therefore, $35 \times 35 = 1225$. Since $8 \times 9 = 72$, we immediately get $85 \times 85 = 7225$. $85 \times 85 = 7225$ $8 \times 9 = 72$ $5 \times 5 = 25$ Answer: 7225

We can use a similar trick when multiplying two-digit numbers with the same first digit, and second digits that sum to 10. For example, let's try 83×87 . (Both numbers begin with 8, and the last digits sum to $3 + 7 = 10$.) Since $8 \times 9 = 72$, and $3 \times 7 = 21$, the answer is 7221. $83 \times 87 = 7221$ $8 \times 9 = 72$ $3 \times 7 = 21$ Answer: 7221 Benj_0307338401_4p_c00_r1.r.qxd 5/4/06 1:46 PM Page 6 Secrets of Mental Math 6

Similarly, $84 \times 86 = 7224$. Thus, we can use this method to instantly determine that $31 \times 39 = 1209$ $32 \times 38 = 1216$ $33 \times 37 = 1221$ $34 \times 36 = 1224$ $35 \times 35 = 1225$ You may ask, What if the last digits do not sum to ten? (For 22×23 , you would do 20×25 plus 2×3 , to get $500 + 6 = 506$, but I'm getting ahead of myself!) Benj_0307338401_4p_c00_r1.r.qxd 5/4/06 1:46 PM Page 7 Quick Tricks: Easy (and Impressive) Calculations 7

Consider the subtraction problem $1241 - 587$ Most people would not like to do this problem in their head (or even on paper! Since $1200 - 600 = 600$, we have that $1241 - 600 = 641$ But we have subtracted 13 too much. Thus, our painful-looking subtraction problem becomes the easy addition problem $641 + 13 = 654$ which is not too hard to calculate in your head (especially from left to right). Thus, $1241 - 587 = 654$. 9 5 14 19 33 52 85 137 222 359 935 Benj_0307338401_4p_c00_r1.r.qxd 5/4/06 1:46 PM Page 8 Secrets of Mental Math 8

Although I won't reveal the magical secret right now, here is a hint. Furthermore, you will be able to quickly give the quotient of the last two numbers: $359 \div 222 = 1.61$ (first three digits) We will have much more to say about division (including decimals and fractions) in Chapter 4. Adding these numbers together gives us \$6.30, which is exactly 15% of the bill. You can use this to figure

Benj_0307338401_4p_c00_r1.r.qxd 5/4/06 1:46 PM Page 9 Quick Tricks: Easy (and Impressive) Calculations 9 out birth dates, historical dates, future appointments, and so on. Monday Tuesday Wednesday Thursday Friday Saturday Sunday 1 2 3 4 5 6 7 or 0 For instance, let's determine the day of the week of January 1, 2030. To figure out the day of the week, subtract the biggest multiple of 7 (0, 7, 14, 21, 28, 35, 42, 49, . In this case, $37 - 35 = 2$, and so January 1, 2030, will occur on 2's day, namely Tuesday: Bill: 30 Tip: 37 subtract 7s: 33 35 2 Tuesday How about January 1,

Chapter 1

The following exercises are good practice, however, because the two-digit addition skills that you acquire here will be needed for larger addition problems, as well as virtually all

Benj_0307338401_4p_c01_r1.r.qxd 5/4/06 1:45 PM Page 13 A Little Give and Take: Mental Addition and Subtraction 13 multiplication problems in later chapters. For example: $47 + 32$?? ($30 + 2$) To solve $47 + 32$, first add 30, then add 2. After adding 30, you have the simpler problem $77 + 2$, which equals 79. We illustrate this as follows: $47 + 32$ $77 + 2$ 79 ?? (first add 30) (then add 2) The above diagram is simply a way of representing the mental processes involved in arriving at an answer using our method. Now let's try a calculation that requires you to carry a number: $67 + 28$?? ($20 + 8$) Adding from left to right, you can simplify the problem by adding $67 + 20 = 87$; then $87 + 8 = 95$.

Benj_0307338401_4p_c01_r1.r.qxd 5/4/06 1:45 PM Page 14 Secrets of Mental Math 14 $67 + 28$ $87 + 8$ 95 ?? (first add 20) (then add 8) Now try one on your own, mentally calculating from left to right, and then check below to see how we did it: $84 + 57$?? ($50 + 7$) How was that? $84 + 57$ $134 + 7$ 141 ?? (first add 50) (then add 7) If carrying numbers trips you up a bit, don't worry about it. Try another problem for practice, again computing it in your mind first, then checking how we did it: $68 + 45$?? ($40 + 5$) You should have added $68 + 40 = 108$, and then $108 + 5 = 113$, the final answer.

Benj_0307338401_4p_c01_r1.r.qxd 5/4/06 1:45 PM Page 15 A Little Give and Take: Mental Addition and Subtraction 15 EXERCISE:TWO-DIGIT ADDITION $23 + 16$?? $64 + 43$?? $95 + 32$?? $34 + 26$?? $89 + 78$?? 1. Let's try the following: $538 + 327$?? ($300 + 20 + 7$) Starting with 538, we add 300, then add 20, then add 7. After adding 300 ($538 + 300 = 838$), the problem becomes $838 + 27$. After adding 20 ($838 + 20 = 858$), the problem simplifies to $858 + 7 = 865$. This thought process can be diagrammed as follows: $538 + 327$ $838 + 27$ $858 + 7$ 865 ?? $300 + 20 + 7$ All mental addition problems can be done by this method. Notice that $538 + 327$ requires you to hold on to six digits in your head, whereas $838 + 27$ and Benj_0307338401_4p_c01_r1.r.qxd 5/4/06 1:46 PM Page 16 Secrets of Mental Math 16 $858 + 7$ require only five and four digits, respectively. Try the following addition problem in your mind before looking to see how we did it: $623 + 159$?? ($100 + 50 + 9$) Did you reduce and simplify the problem by adding left to right? After adding the hundreds ($623 + 100 = 723$), you were left with $723 + 59$. Next you should have added the tens ($723 + 50 = 773$), simplifying the problem to $773 + 9$, which you then summed to get 782. Diagrammed, the problem looks like this: $623 + 159$ $723 + 59$ $773 + 9$ 782 ?? $100 + 50 + 9$ When I do these problems mentally, I do not try to see the numbers in my mind! I try to hear them. I hear the problem $623 + 159$ as six hundred twenty-three plus one hundred fifty-nine; by emphasizing the word hundred to myself, I know where to begin adding. Three-digit addition problems really do not get much harder than the following: $858 + 634$??

Benj_0307338401_4p_c01_r1.r.qxd 5/4/06 1:46 PM Page 17 A Little Give and Take: Mental Addition and Subtraction 17 Now look to see how we did it: $858 + 634$ $1458 + 34$ $1488 + 4$ 1492 ?? $600 + 30 + 4$ At each step I hear (not see) a new addition problem. Let's try another one for practice: $759 + 496$?? ($400 + 90 + 6$) Do it in your mind first, then check our computation below: $759 + 496$ $1159 + 96$ $1249 + 6$ 1255 ?? $400 + 90 + 6$ This addition problem is a little more difficult than the last one since it requires you to carry numbers in all three steps. I am sure you will agree that it is a Benj_0307338401_4p_c01_r1.r.qxd 5/4/06 1:46 PM Page 18 Secrets of Mental Math 18 lot easier to add 500 to 759 than it is to add 496, so try adding 500 and then subtracting the difference: $759 + 496$?? ($500 + 4$) $759 + 496$ $1259 + 4$ 1255 ?? (first add 500) (then subtract 4) So far, you have consistently broken up the second number in any problem to add to the first. If the second number happens to be a lot simpler than the first, I sometimes switch them around, as in the following example: $207 + 528$?? $207 + 528$ $528 + 207$ $728 + 7$ 735 ?? (switch) $200 + 7$ Let's finish up by adding three-digit to four-digit numbers. We begin with an easy one: $2700 + 567$???