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Assignment 4
Optimization Methods – SP24

May 21, 2024
Deadline: June 3, 2024

Please refer to the Assignment rules document.

Exercise 1(20/100)

Consider the quadratic function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined as:

$$f(\mathbf{x}) = 7x^2 + 4xy + y^2 \tag{1}$$

where $\mathbf{x} = (x,\ y)^T$.

1. Write this function in canonical form, i.e. $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} - \mathbf{b}^T\mathbf{x} + c$, where $\mathbf{A}$ is a symmetric matrix.
2. Describe briefly how the Conjugate Gradient (CG) Method works and discuss whether it is suitable to minimize f from equation (1). Explain your reasoning in detail **(max. 30 lines)**.

Exercise 3 (20/100)

Consider the following constrained minimization problem for $\mathbf{x} = (x, y, z)^T$

$$\begin{aligned} \min_{\mathbf{x}} f(\mathbf{x}) &:= -3x^2 + y^2 + 2z^2 + 2(x + y + z) \\ \text{subject to} \quad c(\mathbf{x}) &= x^2 + y^2 + z^2 - 1 = 0 \end{aligned} \tag{2}$$

Write down the Lagrangian function and derive the KKT conditions for (2).

Exercise 3 (60/100)

1. Read the chapter on Simplex method, in particular the section 13.3 The Simplex Method, in Numerical Optimization, Nocedal and Wright. Explain how the method works, with a particular attention to the search direction.
2. Consider the following constrained minimization problem, $\mathbf{x} = (x_1,\ x_2)^T$:

$$\min_{\mathbf{x}} f(\mathbf{x}) := 4x_1 + 3x_2 \tag{3}$$

subject to:

$$\begin{aligned} 6 - 2x_1 - 3x_2 &\geq 0 \\ 3 + 3x_1 - 2x_2 &\geq 0 \\ 5 - 2x_2 &\geq 0 \\ 4 - 2x_1 - x_2 &\geq 0 \\ x_2 &\geq 0 \\ x_1 &\geq 0 \end{aligned}$$

   (a) Sketch the feasible region for this problem.
   (b) Which are the basic feasible points of the problem (3)? Compute them by hand using the geometrical interpretation and find the optimal point $\mathbf{x}^*$ that minimizes f subject to the constraints.
   (c) Prove that the first order necessary conditions holds for the optimal point.