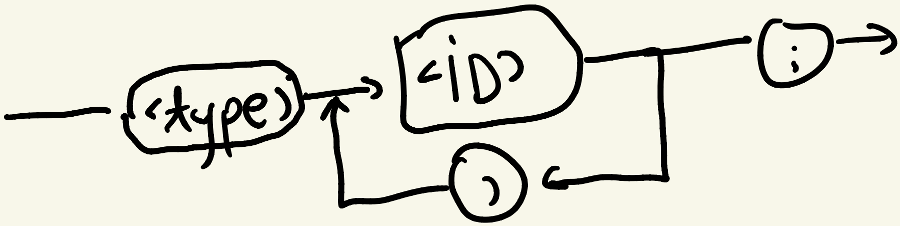
Seminar 1

1.1. (,,) + $ FF

1.2.



2.

<exp> ::= <p-des> <ids> <p-inchis>

<p-des> ::= (

<p-inc> ::=)

<ids> ::= <ID> | <ID><v><ids>

<v> ::= ,

EBNF:

exp = "(" ids ")"

ids = val | val "," ids

2) BNF: <exp> ::= + <numar> | - <numar> | <numar>

<numar> ::= <normal> | $ <hexa>

<normal> ::= <cifra> | <normal> <cifra>

<hexa> ::= <cifra_hex> | <hexa> <cifra_hex>

EBNF

exp = "+" nr | "-" nr | nr

nr = dec | "\$" hex

dec = cifra { dec }

hex = c-hex { hex }

cifra = 0|1|...|9

c-hex = 0|1|...|F

3.2. id = f, a1, a2, a3

const = 7, 11, 3, 5

op = :=, +

cuv = cheie

3) a) t: ID if then ; begin end + =

net: intre <>

```
begin
    a bb = aaa
end
```

```
begin
    if (aaa + ab) then
        aa = abb
end
```

Seminar 2

$L_A = \{1w \mid w \in \{0,1\}^*\}$

$L_B = -L_A \cup +L_A \cup L_A$

$L_C = \{z, x \mid z, x \in L_A\}$

2/ 1) $G = (N, \Sigma, S, P)$

$N = \{A, B\}$

$\Sigma = \{a, b\}$

$S = A$

P: $A \to aB$

$A \to B$

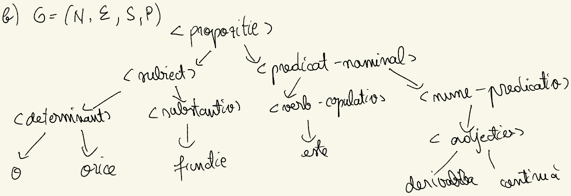
$B \to b$

$A \to aB \to ab$

$A \to B \to b$

$L(G) = \{ab, b\}$

b) $G = (N, \Sigma, S, P)$



$L(G) = \{$ o functie este derivabila, ... $\}$

2. a) ab, ac

$G = (N, \Sigma, S, P)$

$N = A, B$

$\Sigma = a, b, c$

$S = A$

P: $A \to aB$

$B \to b$

$B \to c$

BNF:

<exp> ::= a <lit>

<lit> ::= b | c

4/ $L = \{ab\} \mid \{x, ab, axb\}$

$L = \{a^{2n} bc \mid n \in \mathbb{N}\}$

2. $S \to B$

$S \to AB$

$B \to bc$

$A \to aa$

$A \to aaA$

3. $L = \{a^{2n+1} \mid n \in \mathbb{N}\}$

$S \to aA$

$A \to aa$

$A \to aaA$

Seminar 3

1.

δ	a	b	
p	q	p	0
q	p	p	1

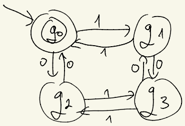
AFD

AFN

δ	0	1	
q_0	q_1	q_2	0
q_1	q_1, q_2	q_1	0
q_2	q_2	q_1, q_2	1

2.

	0	1	
q_0	q_2	q_1	1
q_1	q_3	q_0	0
q_2	q_0	q_3	0
q_3	q_1	q_2	0



$(q_0, 1010) \vdash$

$(q_1, 010) \vdash$

$(q_3, 10) \vdash$

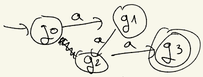
$(q_2, 0) \vdash$

(q_0, ε) acc de aut

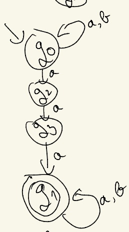
$= q_0 \in F$

$\Rightarrow 1010 \in L(A)$

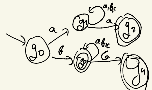
3/ a) $L = \{aaa\}$, $\Sigma = \{a\}$



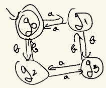
b)



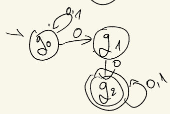
d)



g)



c)



Seminar 4

1/ a) $L=\{a\}$

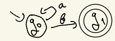
$S \to a$

G. regulară

b) $L=\{a^n \mid n\in\mathbb{N}\}$

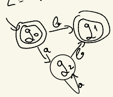
$S \to \varepsilon$
$S \to aA$
$S \to$
$A \to aA$
$A \to a$

c) $L=\{a^n b \mid n\in\mathbb{N}\}$



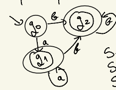
$S \to b$
$S \to aS$

d) $L=\{\varepsilon\}\cup\{a^n b \mid n\in\mathbb{N}\}$



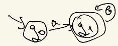
$S \to \varepsilon$
$S \to b$
$S \to aA$
$A \to b$
$A \to aA$

e) $L=\{a^m b^n \mid m,n\in\mathbb{N},\ m+n\}$



$S \to a$
$S \to b$
$S \to aS$
$S \to bB$
$B \to b$
$B \to bB$

f) $L=\{ab^n \mid n\in\mathbb{N}\}$



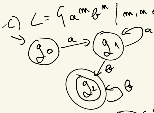
$S \to a$
$S \to aA$
$A \to bA$
$A \to b$

3/ $L=\{a^{3n} \mid n\in\mathbb{N}^*\}$

$S \to aA$
$A \to aB$
$B \to a$
$B \to aS$

b) $S \to \varepsilon$
$S \to aA$
$A \to aB$
$B \to a$
$B \to aC$
$C \to aA$

c) $L=\{a^m b^n \mid m,n\in\mathbb{N}^*\}$

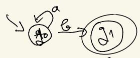


$S \to aA$
$A \to b$
$A \to aA$
$A \to bB$
$B \to b$
$B \to bB$

d)

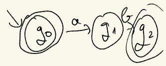
$S \to 0$
$S \to 1$
$S \to 1A$
$A \to 0$
$A \to 1$
$A \to 0A$
$A \to 1A$

4| $A \to aA$
$A \to b$



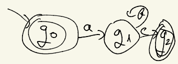
$L=\{a^n b \mid n\in\mathbb{N}\}$

b) $S \to \varepsilon$
$S \to aA$
$A \to b$



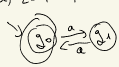
$L=\{ab\}\cup\emptyset$

c) $S \to \varepsilon$
$S \to aA$
$A \to bA$
$A \to c$

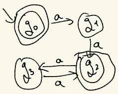


$L=\{ab^n c \mid n\in\mathbb{N}^*\}\cup\emptyset$

6) a) $L=\{a^{2n} \mid n\in\mathbb{N}\}$

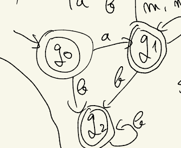


$S \to \varepsilon$
$S \to aA$
$A \to a$
$A \to aS$



$S \to \varepsilon$
$S \to aA$
$A \to a$
$A \to aB$
$B \to aC$
$C \to a$
$C \to aB$

b) $L=\{a^m b^n \mid m,n\in\mathbb{N}\}$



$S \to \varepsilon$
$S \to a$
$S \to b$
$S \to aA$
$S \to bB$
$A \to a$
$A \to aA$
$A \to b$
$A \to bB$
$B \to b$
$B \to bB$

Lema de pompare

L = lb. regular

$\exists p\in\mathbb{N}^*$ (unic) a.î. $\forall w\in L$ cu prop că $|w|\geq p$,

$\exists$ o descompunere $w=xyz$ unde $0<|y|\leq p$

cu prop că $xy^iz\in L$ $\forall i\in\mathbb{N}$

L regular $\Rightarrow$ are loc lema de pompare

Seminar 5

1.1.

1. a. $L=\{a^n b^{2n} \mid n\in\mathbb{N}^*\}$

$\forall p\in\mathbb{N}^*,\ \exists w\in L,\ w=a^p b^{2p}$

$|w|=3p\geq p$

$\forall\ w=xyz$

I
$x=a^m$ $\quad 0\leq m\leq p-n$
$y=a^n$ $\quad 0<n\leq p$
$z=a^{p-n-m}b^{2p}$

(cat I $x=aa\ldots a$, $y=aa\ldots a$, $z=$ rest

cat 2 $x=aa\ldots a$, $y=$ rest, $z=b\ldots b$

cat 3 $x=$ rest, $y=b\ldots b$, $z=b\ldots b$

$xy^iz\in L\ \ \forall i\in\mathbb{N}$

pt $i=2 \Rightarrow a^m a^{2n} a^{p-n-m} b^{2p}$

$a^{p+n}b^{2p}\notin L$ pt. $n>0$ (1)

II
$x=a^m$ $\quad 0\leq m<p$
$y=a^{p-m}b^j$ $\quad 0<j\leq p$ & $m<p$
$z=b^{2p-j}$

pt. $i=2$ $\quad a^m(a^{p-m}b^j)^2 b^{2p-j}=$

$=a^m a^{p-m}b^j a^{p-m}b^j b^{2p-j}=$

$=a^p b^j a^{p-m} b^{2p}\notin L$ $\left(\begin{matrix}p-m>0\\ j>0\end{matrix}\right)$ (2)

III $x = a^m \quad 0 \le m < p$
$y = a^l \quad 0 < l \le p$
$z = a^{p-l-m} b^{2p} \quad 0 \le m < p-l$

pt. $i = 2$
$a^m a^{2l} a^{p-l-m} b^{2p} = a^{p-l} b^{2p} \notin L$ (3)
$(l > 0)$

(1)(2)(3) $\Rightarrow$ lema de pompare nu are loc $\Rightarrow L$ nu e regular

b) $L = \{a^k \mid k - prim\}$
$\forall p \in \mathbb{N}^* \; \exists w \in L, w = a^k, |w| = k \ge p$
k prim
$\forall w = xyz, \quad 0 < |y| \le p$
$x = a^m \quad 0 \le m < p$
$y = a^n \quad 0 < n \le p$
$z = a^{k-m-n} \quad n \le k - m$
$m \le k$

$i = k+1$
$a^m a^{n(k+1)} a^{k-m-n} =$
$a^{nk+n} a^{k-n} =$
$a^{nk} a^k = a^{k(n+1)} \notin L$
$n > 0$

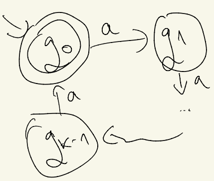
$\Rightarrow$ Lema de pompare nu are loc

c) $L = \{a^{n^2} \mid n \in \mathbb{N}^*\}$
$\forall p \in \mathbb{N}^*, \exists w = a^{p^2} \in L, |w| = p^2 \le p$
$\forall w = xyz$
$x = a^m \quad 0 < m < p^2 - n$
$y = a^n \quad 0 < n \le p$
$z = a^{p^2-m-n}$
pt $i = 2 \Rightarrow a^m a^{2n} a^{p^2-m-n}$
$= a^{p^2+n} \Rightarrow xy^iz \notin L$
$\Rightarrow$ nu are loc lema de pompare

d) $L = \{a^{2^n} \mid n \in \mathbb{N}^*\}$
$\forall p \in \mathbb{N}^*, \exists w = a^{2^p} \in L, |w| = 2^p \le p$
$\forall w = xyz \quad 0 < |y| \le p$
$x = a^m \quad 0 \le m < 2^p - n$
$y = a^n \quad 0 < n \le p$
$z = a^{2^p - n - m}$

$i = 2 \Rightarrow xy^iz = a^m a^{2n} a^{2^p-n-m}$
$= a^{2^p+n} \notin L$
$0 < n \le p$
$\Rightarrow$ nu are loc lema de pompare

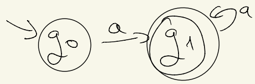
e) k - nr nat fixat
$L = \{a^{kn} \mid n \in \mathbb{N}\}$



2. $L = \{a^k \mid k - neprim\}$
1.b nu e regular
$L = \overline{L_{1b}}$
$\Rightarrow L$ - nu e regular

b) $p = 2$, $w = aaaa \Rightarrow |w| = 4 \ge 2$
$w = xyz$
$x = a$
$y = aa$
$z = a$
$xy^iz = a(aa)^i a = (aa)^{i+1}$ pt. $\forall i \in \mathbb{N}^*$ $\Rightarrow xy^iz \in L$

1.2. $L = \{a^n \mid n \in \mathbb{N}^*\}$



$S \to a$
$S \to aS$

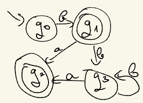
$S \to a$
$S \to Sa$

b) $L = \{a^n b^n \mid n \in \mathbb{N}^*\}$
$S \to aSb$
$S \to ab$

1.3
a) $S \to abS$
$S \to ab$

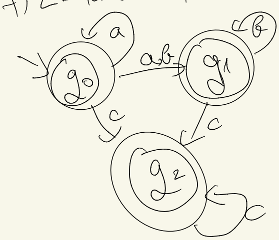
$S \to aA$
$A \to b$
$A \to bS$

b) $S \to Sa$
$S \to b$



$S \to b$
$S \to bA$
$A \to a$
$A \to bB$
$B \to a$
$B \to bB$

f) $L = \{a^n b^m c^p \mid m, n, p \in \mathbb{N}\}$



Seminar 6

1.a) 0111 0111 $(1^*01)^*(11+0)^*$

(01)(1101)(11)

b. 11100 111 $(1^*0)^* + (0^*11)$

nu e matching

c. 111 00 11 $(1^*0)^* + (0^*11)$

nu e matching

d. 11100 11 $(1^*0)^*(0^*11)$

match

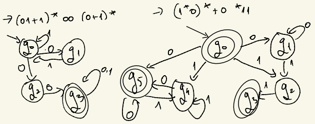
e. 0111 00 101 $01^* 01^* (11^*0)^*$

nu e matching

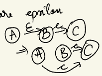
f. 10000 11 $(10^* + 11)^* (0+1)^*$

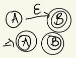
match

2. $\to (01+1)^* 00 (0+1)^*$ $\to (1^*0)^* + 0^*11$



eliminare epsilon

1. 

2. 

3. 

-cu sistem

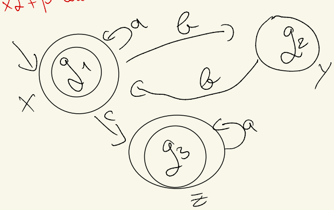
calculăm pt. toate var. corespunzătoare stărilor finale!

$$\begin{cases} X = Xa + Yb + \varepsilon \\ Y = Xb \end{cases}$$

$\Rightarrow X = Xa + Xbb + \varepsilon$

$X = X\underbrace{(a+bb)}_{\alpha} + \underbrace{\varepsilon}_{\beta} \Rightarrow X = \varepsilon(a+bb)^* = (a+bb)^*$

REGULĂ: $X = X\alpha + \beta$ are sol. $X = \beta\alpha^*$



$$\begin{cases} X = Xa + Yb + \varepsilon \\ Y = Xb \\ Z = Xc + Za \end{cases} \Rightarrow \begin{matrix} X = Xa + Xbb + \varepsilon \\ Z = Xc + Za \end{matrix}$$

$\Rightarrow X = X\underbrace{(a+bb)}_{\alpha} + \underbrace{\varepsilon}_{\beta}$

$\Rightarrow X = \varepsilon(a+bb)^* = (a+bb)^*$

$Z = (a+bb)^* c + Za$

$Z = (a+bb)^* c a^*$

2 stări finale $\Rightarrow (a+bb)^* + (a+bb)^* c a^*$

exp regulară $= (a+bb)^*(\varepsilon + ca^*)$

Se poate și cu mulțimi

Seminar 7

1. $M = \{ a^{2n} \mid n \in \mathbb{N} \}$

$\begin{cases} X = Ya + \varepsilon \\ Y = Xa \end{cases} \Rightarrow X = X\underbrace{aa}_{\alpha} + \underbrace{\varepsilon}_{\beta}$

$\Rightarrow X = \varepsilon(aa)^* \Rightarrow$ exp. reg $= (aa)^*$

g. reg: $S \to \varepsilon$
$S \to aA$
$A \to a$
tip 3 $A \to aB$
$B \to aA$

g. ind. c.: $S \to \varepsilon$
$S \to aSa$

tip 2

tip 1
gr. dep. context:
$S \to \varepsilon$
$S \to aA$
$aA \to aaaA$
$A \to a$

2. 1. $L = \{ ww \mid w \in \{a,b\}^* \}$

$S \to \varepsilon$
$S \to AA$
$A \to a$
$A \to b$
$A \to aA$
$A \to bA$

2. $L = \{ wxw \mid w \in \{a,b\}^*, x \in \{a,b\}^* \}$

$S \to \varepsilon$
$S \to ABA$
$A \to a$
$A \to b$
$A \to aA$
$A \to bA$
$B \to a$
$B \to b$
$B \to aB$
$B \to bB$

3. $L = \{ a^n b^n c^n \mid n \in \mathbb{N}^* \}$

$S \to abc$
$S \to aSAC$
$cA \to Ac$
$bA \to bb$

4. $L = \{ a^n b^n c^n d^n \mid n \in \mathbb{N}^* \}$

$S \to abcd$
$S \to aSBCd$
$dC \to Cd$
$cC \to cc$
$cdB \to Bcd$
$bB \to bb$

a abcd Bcd

a abB cdcd
Cdd
aa bbccdd

5. $L = \{ a^{2^n} \mid n \in \mathbb{N} \}$

$S \to LaR$
$L \to \varepsilon$
$R \to \varepsilon$
$L \to LD$
$Da \to aaD$
$DR \to R$

6. $L = \{ w \mid w \in \{a,b,c\}^*, nr_a(w) = nr_b(w) = nr_c(w) \}$

$S \to \varepsilon$
$S \to abc$
$S \to acb$
$S \to bac$
$S \to bca$
$S \to cab$
$S \to cba$

$S \to SaSbScS$
$S \to SaScSbS$
$S \to SbSaScS$
$S \to SbScSaS$
$S \to ScSaSbS$
$S \to ScSbSaS$

7. $L = \{ a^n b^n c^m d^m \mid n, m \in \mathbb{N} \}$

$S \to \varepsilon$
$S \to AB$
$A \to \varepsilon$
$A \to aAb$
$B \to \varepsilon$
$B \to cBd$

8. $L = \{ a^n b^m c^m d^n \mid n, m \in \mathbb{N} \}$

$S \to \varepsilon$
$S \to aABDd$
$A \to \varepsilon$
$A \to aA$
$D \to \varepsilon$
$D \to dD$
$B \to bBc$
$B \to \varepsilon$

9. $L = \{ a^n b^m c^k \mid n, m, k \in \mathbb{N}, (n=m) \mid (m=k) \}$

I $n = m$
$a^n b^n c^k$
$S \to \varepsilon$
$S \to AC$
$A \to aAb$
$A \to \varepsilon$
$C \to \varepsilon$
$C \to cC$

II $m = k$
$a^n b^k c^k$
$S \to \varepsilon$
$S \to AC$
$A \to aA$
$A \to \varepsilon$
$C \to bCc$
$C \to \varepsilon$

10. $L = \{ a^n b^m c^k \mid n, m, k \in \mathbb{N}, m+n=k \}$

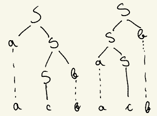
$(a^n b^m c^{n+m})$

$S \to \varepsilon$
$S \to aSBc$
$B \to bBc$
$B \to \varepsilon$

11. $L = \{ w \in \{a,b\}^* \mid w$ începe și se termină cu aceeași literă $\}$

$S \to \varepsilon$
$S \to aMa$
$S \to bMb$
$M \to a$
$M \to b$
$M \to aM$
$M \to bM$

2\. 1. a) S → aS | Sb | c



→ gr. ambiguă acb

gr. neambiguă:

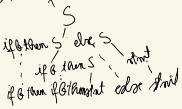
S → aS
S → A
A → Ab
S → c
A → c

3) G = ({E, T, F}, {a, +, *, (,)}, P, E)

P:
E → E+T
E → T
T → T*F
T → F
F → (E)
F → a

b) S → if b then S else S | if b then S | stmt

if b then if b then stmt else stmt



S → if b then A else S
S → stmt
A → if b then A
A → stmt

Convertire CFG → CNF

P: S → ASA | aB, A → B | S, B → b | ε

1) S' → S, S → ASA | aB, A → B | S, B → b | ε

2) Null prod: B → ε
S' → S, S → ASA | aB | a, A → B | S | ε, B → b
A → ε: S' → S, S → ASA | aB | a | SA | AS | S, A → B | S, B → b

3) Unit prod.
S → S: S' → S, S → ASA | aB | a | SA | AS, A → B | S, B → b
S' → S: S' → ASA | aB | a | SA | AS
S → ASA | aB | a | SA | AS
A → B | S
B → b

A → B: S' → ASA | aB | a | SA | AS
S → ASA | aB | a | SA | AS
A → b | S
B → b

A → S: S' → ASA | aB | a | SA | AS
S → ASA | aB | a | SA | AS
A → b | ASA | aB | a | SA | AS
B → b

4) More than 2 vars on RHS
S' → AX | aB | a | SA | AS
S → AX | aB | a | SA | AS
A → b | AX | aB | a | SA | AS
B → b
X → SA

5) S' → aB, S → aB, A → aB
S' → AX | YB | a | SA | AS
S → AX | YB | a | SA | AS
A → b | AX | YB | a | SA | AS
B → b
X → SA
Y → a

GNF - Greibach Normal Form

CFG → GNF

CNF →
S → CA | BB
B → b | SB
C → b
A → a

Replace:
S with A_1
C with A_2
A with A_3
B with A_4
correct order

$A_1 \to A_2A_3 \mid A_4A_4$
$A_4 \to b \mid A_1A_4$! 4>1
$A_2 \to b$
$A_3 \to a$

$A_4 \to b \mid \underline{A_1}A_4$

$A_4 \to b \mid \underline{A_2}A_3A_4 \mid A_4A_4A_4$
$A_4 \to b \mid bA_3A_4 \mid A_4A_4A_4$ left recursion

remove left recursion

$Z \to A_4A_4Z \mid A_4A_4$
$A_4 \to b \mid bA_3A_4 \mid bZ \mid bA_3A_4Z$

$A_1 \to bA_3 \mid bA_4 \mid bA_3A_4A_4 \mid bZA_4 \mid bA_3A_4ZA_4$
$A_4 \to b \mid bA_3A_4 \mid bZ \mid bA_3A_4Z$
$Z \to bA_4 \mid bA_3A_4A_4 \mid bZA_4 \mid bA_3A_4ZA_4 \mid bA_4Z \mid bA_3A_4A_4Z \mid bZA_4Z \mid bA_3A_4ZA_4Z$
$A_2 \to b$, $A_3 \to a$

GNF

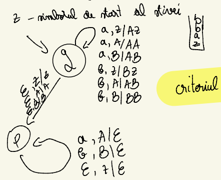
Seminar 8

Automate Push-Down (APD)

1. e) $L = \{ w w^{tilda} \mid w \in \{a, b\}^*, w^{tilda}$ este inv lui $w \}$

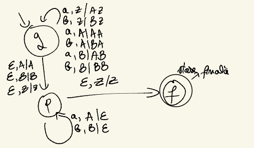
ex: abb bba $\in L$

z – simbolul de start al stivei



criteriul stivei goale

CRITERIUL STĂRII FINALE



stări	stiva	a	b	ε	
q	Z	(q, AZ)	(q, BZ)	(p, Z)	0
	A	(q, AA)	(q, BA)	(p, A)	
	B	(q, AB)	(q, BB)	(p, B)	
p	Z	—	—	(p, ε)	0
	A	(p, ε)	—	—	
	B	—	(p, ε)	—	
f	Z	—	—	—	1
	A	—	—	—	
	B	—	—	—	

în stare finală nu trb stiva goală!

stări	stiva	a	b	ε
q	Z	(q, AZ)	(q, BZ)	(p, Z)
	A	(q, AA)	(q, BA)	(p, A)
	B	(q, AB)	(q, BB)	(p, B)
p	Z	—	—	(p, ε)
	A	(p, ε)	—	—
	B	—	(p, ε)	—

mijloc

$(q, abbbba, Z) \vdash (q, bbbba, AZ) \vdash (q, bbba, BAZ) \vdash (q, \varepsilon bba, BBAZ)$

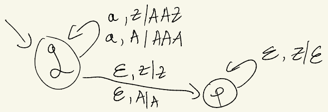
$\vdash (p, bba, BBAZ) \vdash (p, ba, BAZ) \vdash (p, a, AZ) \vdash (p, \varepsilon, Z) \vdash (p, \varepsilon, \varepsilon)$

⇓

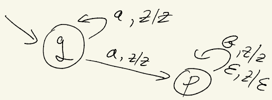
stiva goală + banda s-a golit

$\Rightarrow abb\,bba \in L_\varepsilon(M)$

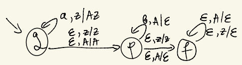
a) $L = \{ a^n b^{2n} \mid n \geq 0 \}$



b) $L = \{ a^n b^m \mid m, n >= 0 \}$

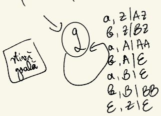


c) $L = \{ a^n b^m \mid n \geq m \geq 0 \}$



f) $L = \{ w \mid w \in \{a, b\}^*, nr_a(w) = nr_b(w) \}$

I f)



g.i.c.

$S \to \varepsilon$

$S \to aSb$

$S \to bSa$

$S \to SS$

⇓

$\delta(q, \varepsilon, S) = (q, \varepsilon)$

$\delta(q, \varepsilon, S) = (q, aSb)$

$\delta(q, \varepsilon, S) = (q, bSa)$

$\delta(q, \varepsilon, S) = (q, SS)$

stări	stiva	a	b	ε
q	S	—	—	(q, ε) | (q, aSB) | (q, bSa) | (q, SS)
	a	(q, ε)	—	
	b	—	(q, ε)	—

Seminar 9

Analizator descendent cu reveniri

$(S, i, [\,,]\,)$

S - stare
- g - stare normală
- r - stare revenire
- t - stare terminare
- e - stare eroare

$(g, i, \alpha, \boxed{A}\beta) \overset{\text{expandare}}{\vdash} (g, i, \alpha A_1, \gamma_1\beta)$ — A: net.; $A_1 \to \gamma_1$ producție

$(g, i, \alpha, a_j\beta) \underset{a_j = a_i}{\overset{\text{avans}}{\vdash}} (g, i+1, \alpha a_j, \beta)$

$(g, i, \alpha, a_j\beta) \underset{a_j \neq a_i}{\overset{\text{insucces de moment}}{\vdash}} (r, i, \alpha, a_j\beta)$

$(g, n+1, \alpha, \varepsilon) \overset{\text{succes}}{\vdash} (t, n+1, \alpha, \varepsilon)$

$(r, i, \alpha a, \beta) \overset{\text{revenire}}{\vdash} (r, i-1, \alpha, a\beta)$

$(r, i, \alpha A_i, \gamma_i\beta) \underset{\exists A_{i+1} \to \gamma_{i+1}}{\overset{\text{altă încercare}}{\vdash}} (g, i, \alpha A_{i+1}, \gamma_{i+1}\beta)$

$\underset{i=1,\ A_1 = S}{\vdash} (e, 1, \varepsilon, S)$

$\underset{\nexists A_{i+1}}{\vdash} (r, 1, \alpha, A\beta)$

Nu funcționează dacă gr. e recursivă la stânga!

1/ $S \to aSbS$ (S_1)
$S \to aS$ (S_2)
$S \to c$ (S_3)

$acbc \in L(G)$?

a) $(g, 1, \varepsilon, S) \overset{\text{expandare}}{\vdash} (g, 1, S_1, aSbS) \overset{\text{avans}}{\vdash} (g, 2, S_1a, SbS)$
$\overset{\text{expandare}}{\vdash} (g, 2, S_1aS_1, aSbSbS) \overset{\text{insucces de moment}}{\vdash} (r, 2, S_1aS_1, aSbSbS)$
$\underset{(I)}{\overset{\text{altă încercare}}{\vdash}} (g, 2, S_1aS_2, aSbS) \overset{\text{insucces de moment}}{\vdash} (r, 2, S_1aS_2, aSbS)$
$\underset{(I)}{\overset{\text{altă încercare}}{\vdash}} (g, 2, S_1aS_3, cbS) \overset{\text{avans}}{\vdash} (g, 3, S_1aS_3c, bS)$
$\overset{\text{avans}}{\vdash} (g, 4, S_1aS_3cb, S) \overset{\text{expandare}}{\vdash} (g, 4, S_1aS_3cbS_1, aSbS)$
$\overset{\text{insucces de moment}}{\vdash} (r, 4, S_1aS_3cbS_1, aSbS) \overset{\text{altă încercare (I)}}{\vdash} (g, 4, S_1aS_3cbS_2, aS)$
$\overset{\text{insucces de moment}}{\vdash} (r, 4, S_1aS_3cbS_2, aS) \overset{\text{altă încercare (I)}}{\vdash} (g, 4, S_1aS_3cbS_3, c)$
$\overset{\text{avans}}{\vdash} (g, 5, S_1aS_3cbS_3, \varepsilon) \overset{\text{succes}}{\vdash} (t, 5, S_1aS_3cbS_3, \varepsilon)$

$\Rightarrow acbc \in L(G)$ și S_1, S_3, S_3 șir de prod.

b) $cb \in L(G)$?

$(g, 1, \varepsilon, S) \overset{\text{expandare}}{\vdash} (g, 1, S_1, aSbS) \overset{\text{insucces de moment}}{\vdash} (r, 1, S_1, aSbS)$
$\underset{(I)}{\overset{\text{altă încercare}}{\vdash}} (g, 1, S_2, aS) \overset{\text{insucces de moment}}{\vdash} (r, 1, S_2, aS)$
$\underset{(I)}{\overset{\text{altă încercare}}{\vdash}} (g, 1, S_3, c) \overset{\text{avans}}{\vdash} (g, 2, S_3c, \varepsilon) \overset{\text{insucces de moment}}{\vdash} (r, 2, S_3C, \varepsilon)$
$\overset{\text{revenire}}{\vdash} (r, 1, S3, c) \underset{(II)}{\overset{\text{altă încercare}}{\vdash}} (e, 1, \varepsilon, S) \Rightarrow cb \notin L(G)$

2. $S \to +SS$ (S_1)
$S \to -SS$ (S_2)
$S \to a$ (S_3)

$+a-aa$

$(g, 1, \varepsilon, S) \overset{\text{expandare}}{\vdash} (g, 1, S_1, +SS) \overset{\text{avans}}{\vdash} (g, 2, S_1+, SS)$
$\overset{\text{expandare}}{\vdash} (g, 2, S_1+S_1, +SSS) \overset{\text{insucces de moment}}{\vdash} (r, 2, S_1+S_1, +SSS)$
$\underset{(I)}{\overset{\text{altă încercare}}{\vdash}} (g, 2, S_1+S_2, -SSS) \overset{\text{insucces de moment}}{\vdash} (r, 2, S_1+S_2, -SSS)$
$\underset{(I)}{\overset{\text{altă încercare}}{\vdash}} (g, 2, S_1+S_3, aS) \overset{\text{avans}}{\vdash} (g, 3, S_1+S_3a, S)$
$\overset{\text{expandare}}{\vdash} (g, 3, S_1+S_3aS_1, +SS) \overset{\text{insucces de mom.}}{\vdash} (r, 3, S_1+S_3aS_1, +SS)$
$\overset{\text{altă încercare (I)}}{\vdash} (g, 3, S_1+S_3aS_2, -SS) \overset{\text{avans}}{\vdash} (g, 4, S_1+S_3aS_2-, SS)$
$\overset{\text{expandare}}{\vdash} (g, 4, S_1+S_3aS_2-S_1, +SSS) \overset{\text{i.m.}}{\vdash} (r, 4, S_1+S_3aS_2-S_1, +SSS)$
$\underset{(I)}{\overset{\text{a.î.}}{\vdash}} (g, 4, S_1+S_3aS_2-S_2, -SSS) \overset{\text{i.m.}}{\vdash} (r, 4, S_1+S_3aS_2-S_2, -SSS)$
$\underset{I}{\overset{\text{a.î.}}{\vdash}} (g, 4, S_1+S_3aS_2-S_3, aS) \overset{\text{avans}}{\vdash} (g, 5, S_1+S_3aS_2-S_3a, S)$
$\overset{\text{exp.}}{\vdash} \ldots\ldots (g, 5, S_1+S_3aS_2-S_3aS_3, a) \overset{\text{avans}}{\vdash} (g, 6, S_1+S_3aS_2-S_3aS_3a, \varepsilon) \overset{\text{succes}}{\vdash}$

$S_1\, S_3\, S_2\, S_3\, S_3$

Seminar 10/11

1. FIRST și FOLLOW

$S \to abA$
$S \to \varepsilon$
$A \to Saa$
$A \to b$

	FIRST	FOLLOW
S	(a, ε)	(a, $)
A	(b, a, ε)	(a, $)

2. $S \to aSb$ (1) verif. dacă este LL(1)
$S \to bSa$ (2)
$S \to SS$ (3)
$S \to \varepsilon$ (4)

	FIRST	FOLLOW
S	(a, b, ε)	$, b, a

	a	b	$
S	(aSb, 1) (ε, 4) (SS, 3)	(SS, 3) (ε, 4) (bSa, 2)	(SS, 3) (ε, 4)
a	pop	err	err
b	err	pop	err
$	err	err	acc

→ nu este LL(1)

FACT. LA STÂNGA

I $A \to A\alpha \mid \beta$
$A \to \beta A'$
$A' \to \alpha A' \mid \varepsilon$

II $A \to \alpha\beta \mid \alpha\gamma \mid \alpha$
$A \to \alpha A'$
$A' \to \beta \mid \gamma \mid \varepsilon$

Ex: $A \to AC \mid Aad \mid bd \mid c$
$A \to bdA' \mid cA'$
$A' \to CA' \mid adA' \mid \varepsilon$

3 1. S → if c then S endif
S → if c then S else S endif
S → stmt

if c then : a
else : b
endif : c
stmt : i

⇒ $S \to aSc$ (1)
$S \to aSbSc$ (2)
$S \to i$ (3)

	FIRST	FOLLOW
S	a, i	$, c, b

	a	b	c	i	$
S	aSc,1 aSbSc,2			i,3	
a					
b					
c					
i					
$					

nu e LL(1)

fact. la stânga: $S \to aSA$ (1)
$A \to c$ (2)
$A \to bSc$ (3)
$S \to i$ (4)

	FIRST	FOLLOW
A	b, c	$, c, b
S	i, a	$, c, b

	a	b	c	i	$
A		bSc,3	c,2		
S	aSA,1			i,4	
a	pop				
b		pop			
c			pop		
i				pop	
$					acc

∈ LL(1)

aaibicc ∈ LL

(aaibicc$, S$, ε) $\vdash_{push 1}$ (aaibicc$, aSA$, 1)
$\vdash_{pop}$ (aibicc$, SA$, 1) $\vdash_{push 1}$ (aibicc$, aSAA$, 11)
$\vdash_{pop}$ (ibicc$, SAA$, 11) $\vdash_{push 4}$ (ibicc$, iAA$, 114)
$\vdash_{pop}$ (bicc$, AA$, 114) $\vdash_{push 3}$ (bicc$, bScA$, 1143)
$\vdash_{pop}$ (icc$, ScA$, 1143) $\vdash_{push 4}$ (icc$, icA$, 11434)
$\vdash_{pop}$ (cc$, cA$, 14434) $\vdash_{pop}$ (c$, A$, 14434)
$\vdash_{push 2}$ (c$, c$, 144342) $\vdash_{pop}$ ($, $, 144342) $\vdash_{accept}$

2. $S \to aS$ (1)
$S \to aSbS$ (2)
$S \to i$ (3)

a)

	FIRST	FOLLOW
S	a, i	$, b

	a
S	aS,1 aSbS,2

⇒ nu e LL(1)

b) Fact. la stânga:
$S \to aSS'$ (1)
$S' \to bS$ (2)
$S' \to \varepsilon$ (3)
$S \to i$ (4)

	FIRST	FOLLOW
S	i, a	$, b
S'	b, ε	$, b

	a	b	i	$
S	aSS',1		i,4	
S'		bS,2		ε,3

?

$E \to T + F \mid T$
$T \to T * F \mid F$
$F \to (E) \mid a$
⇓
FACT. LA STÂNGA:
$E \to TA$ (1)
$A \to +E$ (2)
$A \to \varepsilon$ (3)
$T \to FB$ (4)
$B \to *FB$ (5)
$B \to \varepsilon$ (6)
$F \to (E)$ (7)
$F \to a$ (8)

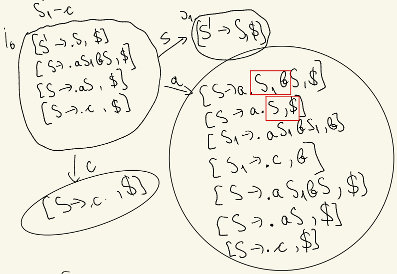
	FIRST	FOLLOW
E	(, a	$,)
A	+, ε	$,)
T	(, a	+, $,)
B	*, ε	+, $,)
F	(, a	*, +, $,)

	+	*	(	)	$	a
E			TA,1	TA,1		TA,1
A	+E,2			+E,2	ε,3	
T			FB,4	FB,4		FB,4
B	ε,6	*FB,5		*FB,5	ε,6	
F		(E),7		(E),7		a,8

a+a ∈ L(G)?
(a+a$, E$, ε) $\vdash_{push 1}$ (a+a$, TA$, 1)
$\vdash_{push 4}$ (a+a$, FBA$, 14) $\vdash_{push 8}$ (a+a$, aBA$, 148)
$\vdash_{pop}$ (+a$, BA$, 148) $\vdash_{push 6}$ (+a$, A$, 1486) $\vdash_{push 2}$
$\vdash$ (+a$, +E$, 14862) $\vdash_{pop}$ (a$, E$, 14862)
$\vdash_{push 1}$ (a$, TA$, 148621) $\vdash_{push 4}$ (a$, FBA$, 1486214)
$\vdash_{push 8}$ (a$, aBA$, 14862148) $\vdash_{pop}$ ($, BA$, 14862148)
$\vdash_{push 6}$ $\vdash_{push 3}$ $\vdash_{acc.}$

LR(1)
S → aS₁bS
S → aS
S → c
S₁ → aS₁bS₁
S₁ → c
S' → S

	FIRST₁
S'	a, c
S	a, c
S₁	a, c



E → TX
X → +E
X → ε
T → T*F
T → F
F → (E)
F → a

	FIRST	FOLLOW
E	(, a	$,)
X	+, ε	$,)
T	(, a	+, $, *,)
F	(, a	+, $, *,)

	(	)	a	+	*	$
E	TX, 1		TX, 1			
X		ε, 3		E		ε, 3
T						
F						

LR(1)
S' → S
S → aS
S → bS
S → c

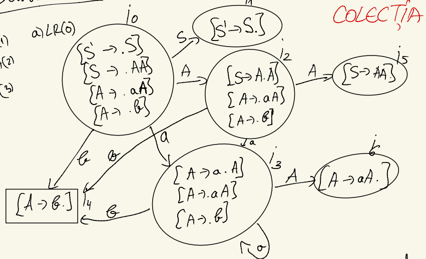
	first₁
S'	a, b, c
S	a, b, c

Seminar 11

S' → S (0)
S → AA (1)
A → aA (2)
A → b (3)

a) LR(0)

COLECȚIA CANONICĂ LR(0)



tabel de analiză LR(0)

	Acțiune	S	A	a	b
i0	shift	i1	i2	i3	i4
i1	accept				
i2	shift		i5	i3	i4
i3	shift		i6	i3	i4
i4	reduce(3)				
i5	reduce(1)				
i6	reduce(2)				

nu este conflict
⇒ este de tip LR(0)

abab

($0, abab$, ε) ⊢shift ($0a3, bab$, ε) ⊢shift ($0a3b4, ab$, ε) ⊢reduce(3) ($0a3A6, ab$, 3) ⊢reducere(2) ($0A2, ab$, 23) ⊢shift ($0A2a3, b$, 23) ⊢shift ($0A2a3b4, $, 23) ⊢reduce(3) ($0A2a3A6, $, 323) ⊢reduce(2) ($0A2A5, $, 2323) ⊢reduce(1) ($0S1, $, 12323) ⊢acc

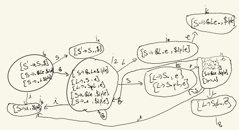
⇒ LR(0) ⇒ LR(1) ⇒ LALR

continuare 6

	S	L	b	e	i	p	$
i0	s1		s2		s3		
i1							acc
i2	s5	s4	s2		s3		
i3				r3		r3	r3
i4				s6			
i5				r4		s7	
i6				r2		r2	r2
i7	s5	s8	s2		s3		
i8				r5			

bipie

($o, bipie$, ε) ⊢shift ($ob2, ipie$, ε) ⊢shift ($ob2i3, pie$, ε) ⊢reduce(3) ($ob2S5, pie$, 3) ⊢shift ($ob2S5p7, ie$, 3) ⊢shift ($ob2S5p7i3, e$, 3) ⊢reduce(3) ($ob2S5p7S5, e$, 33) ⊢reduce(4) ($ob2S5p7L8, e$, 433) ⊢reduce(5) ($ob2L4, e$, 5433) ⊢shift ($ob2L4e6, $, 5433) ⊢reduce(2) ($oS1, $, 25433) ⊢acc.



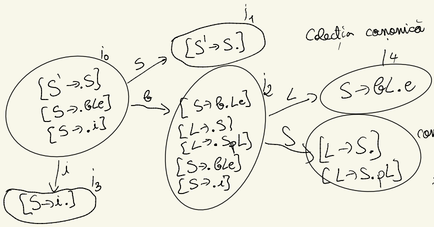
6/ S → begin SL end
S → stmt
SL → S
SL → S semicolon SL

S → bLe
S → i
L → S
L → SpL

înlocuim ⇒

S' → S (1)
S → bLe (2)
S → i (3)
L → S (4)
L → SpL (5)

Colecția canonică

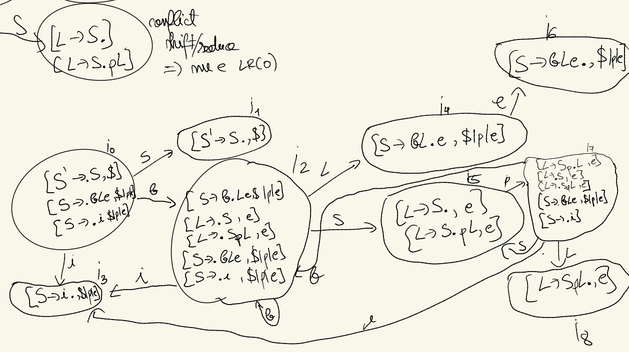


conflict shift/reduce
⇒ nu e LR(0)

* SLR

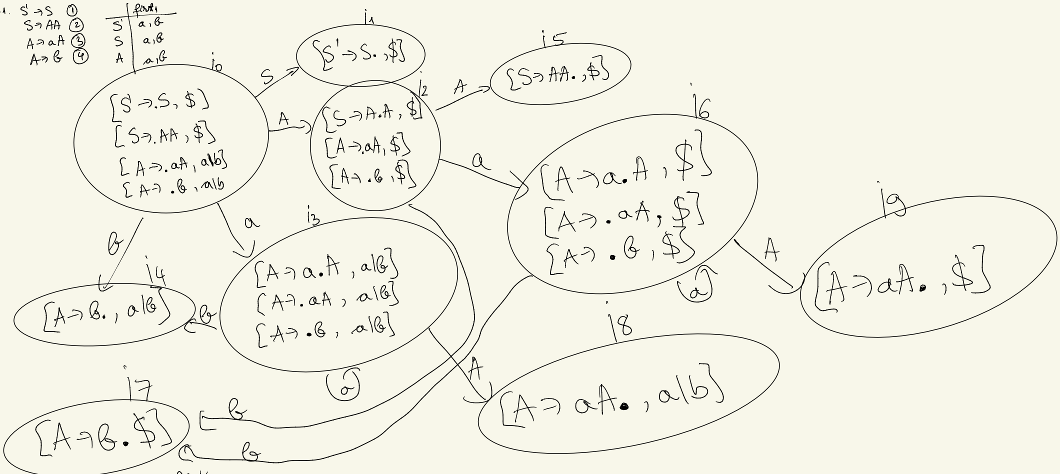
S' → S (1)
S → bLe (2)
S → i (3)
L → S (4)
L → SpL (5)

	FIRST	FOLLOW
S'	b, i	$
S	b, i	$, p, e
L	b, i	e



4.1. $S' \to S$ (1)
$S \to AA$ (2)
$A \to aA$ (3)
$A \to b$ (4)

	$first_1$
S'	a, b
S	a, b
A	a, b



Tabel de analiză

	S	A	a	b	$
i_0	s1	s2	s3	s4	
i_1					acc
i_2		s5	s6	s7	
i_3		s8	s3	s4	
i_4			r4	r4	
i_5					r2
i_6		s9	s6	s7	
i_7					r4
i_8			r3	r3	
i_9					r3

$abab \in L(G)$

$(\$0, abab\$, \varepsilon) \xvdash{shift3} (\$0a3, bab\$, \varepsilon) \xvdash{shift4} (\$0a3b4, ab\$, \varepsilon) \xvdash{r(4)} (\$0a3A8, ab\$, 4) \xvdash{r(3)} (\$0A2, ab\$, 34) \xvdash{s6} (\$0A2a6, b\$, 34) \xvdash{s7} (\$0A2a6b7, \$, 34) \xvdash{r(4)} (\$0A2a6A9, \$, 434) \xvdash{r(3)} (\$0A2A5, \$, 3434) \xvdash{r(2)} (\$0S1, \$, 23434) \xvdash{acc}$

Subiectul 1 Bilet nr. 6/2023

1. a) $a+(aaa)^* + (aa)^* = a^*$ $\boxed{\text{Fals}}$ aaaaa

b) $(aa^*)^* = a^*$ (A)

c) $(ab)^* = a^* b^*$ (F) abab

d) $(a^* + b^*)^* = (a+b)^*$ (A)

2?

3. $A \to \alpha B \beta$

A, B - net.

α, β - secv. net + term / ε

$FOLLOW_1(B) = FIRST_1(\beta) + FOLLOW_1(A)$

$FIRST_1(\beta) \subseteq FOLLOW_1(B)$

$FOLLOW_1(A) \subseteq FOLLOW_1(B)$

4. $L = \{a^n b^{2n} / n \in \mathbb{N}^*\}$

up. că e regular $\Rightarrow \forall p \in \mathbb{N}^*$ a.î. $\exists w = a^p b^{2p} \in L$, $|a^p b^{2p}| = 3p \geq p$

$\forall w = xyz$

$\exists$ $x = a^l$ $0 < l < p$

$y = a^{p-l} b^{\cancel{2p-k}}$

$z = b^k$ $0 < k < 2p$

pt. $i=2$ $xy^i z = a^l a^{p-l} b^{2p-k} a^{p-l} b^{2p-k} b^k$

$= a^p b^{2p-k} a^{p-l} b^{2p} \notin L$

5. $S' \to S$ (1)

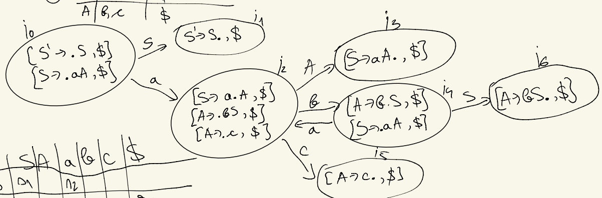
$S \to aA$ (2)

$A \to bS$ (3)

$A \to c$ (4)

	FIRST	FOLLOW
S'	a	\$
S	a	\$
A	b, c	\$

LR(1):



	S	A	a	b	c	\$
i_0	s1		s2			
i_1						r1
i_2		s3		s4	s5	
i_3						r2
i_4	s6		s2			
i_5						r4
i_6						r3

ab abc $\in L(G)$

$(\$0, ababc\$, \varepsilon) \vdash^{s2} (\$0a2, babc\$) \vdash^{s4} (\$0a2b4, abc\$, \varepsilon)$

$\vdash^{s4} (\$0a2b4a2b4, c\$, \varepsilon)$ $(\$0a2b4a2, bc\$, \varepsilon)$

$\vdash^{s2}$

$\vdash^{err.} \Rightarrow \notin L(G)$

G. reg.

Sub. 2 Bilet 5/2023

1. $(a^* + a^*)^* = a^*$

$(a^*)^* = a^*$ A

$a^* a^* = a^*$