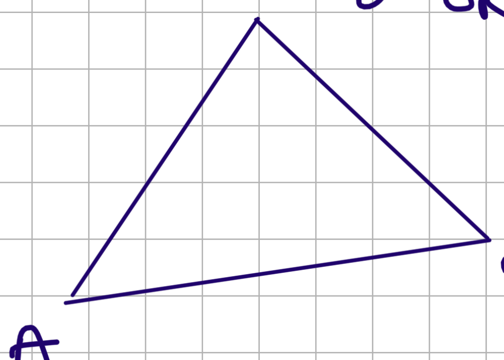
Seminar 8

02.05.2023

1. $K = (O, \vec{i}, \vec{j})$

$K' = (O', \vec{i}', \vec{j}')$



$C = \begin{bmatrix} 3 \\ 1 \end{bmatrix}_K$, $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}_K$, $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_K$

$$[O']_K = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$[\vec{i}']_K = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$[\vec{j}']_K = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$[P]_{K'} = \underline{M_{KK'}}\left([P]_K - [O']_K\right)$$

$$M_{KK'} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \left(-\frac{1}{5}\right) = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$M_{KK'}^{-1}$ det

$$[C]_K = M_{KK'} \cdot \left([C]_{K'} - [O]_{K'}\right)$$

$$\parallel$$

$$M_{K'K}\left([C]_K - [O']_K\right)$$

$$= M_{KK'}\left(M_{K'K}\left([C]_{K'} - [O']_K\right) - [O]_{K'}\right)$$

$$= \mathcal{M}_{KK'} \cdot \mathcal{M}_{K'K}([C]_K - [O']_K) - \mathcal{M}_{KK'}[O]_{K'}\} = (\overrightarrow{O'O})_{K'}$$

$$= [C]_K - \cancel{[O']_K} + \cancel{[O']_K}$$

$$[\overrightarrow{O'O}]_K = -[\overrightarrow{OO'}]_K = -[O']_K$$

~~Def~~ Par. + Cart. eq:

$$\mathcal{M}_{KK'} = \frac{1}{5}\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$$

im K:

$$\overrightarrow{AC} = (2,-1)$$

$$AC: \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + t\underbrace{\begin{bmatrix} 2 \\ -1 \end{bmatrix}}_{v}$$

cart. eq:

$$\frac{x - x_A}{v_x} = \frac{y - y_A}{v_y}$$

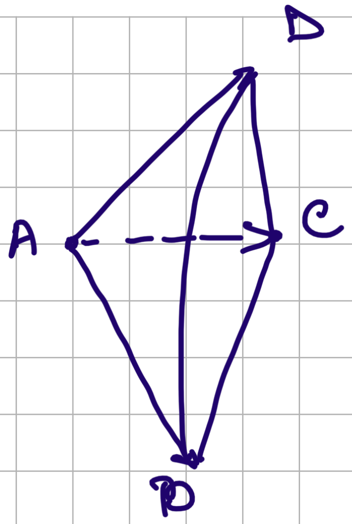
im K':

$$\mathcal{M}_{KK'}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} -5 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$AC: \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

3.



$K_A = (A, \vec{AB}, \vec{AC}, \vec{AD})$

$K_A' = (A, \vec{AB}, \vec{AB}, \vec{AC})$

$K_B = (B, \vec{BA}, \vec{BC}, \vec{BD})$

a) Det. coord of A, B, C, D w.r.t. K_A, K_A', K_B

b) Det. the base change matrix from

$K_A \to K_A'$ and $K_B \to K_A$

Sol:

in K_A: $A(0,0,0)$
$B(1,0,0)$
$C(1,1,0)$
$D(0,0,1)$

in K_B: $B(0,0,0)$
$A(1,0,0)$
$C(0,1,0)$
$D(0,0,1)$

in K_A': $A(0,0,0)$
$B(1,0,0)$
$C(0,0,1)$
$D(0,1,0)$

$K_A \to K_A'$

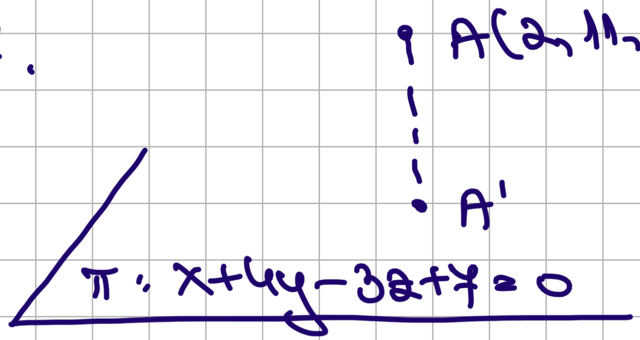
$$M_{K_A' K_A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\overrightarrow{BA} = -\overrightarrow{AB}$

$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

$$M_{K_B K_A} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7.



Det $Pr_{\pi}^{\perp} A = ?$

Sol :

$$[Pr_{\pi}^{\perp} A] = [Pr_{\pi,n} A] = \Big(I_3 - \underbrace{\frac{n \otimes a}{n \cdot a}}_{\frac{n \cdot a^t}{n^t \cdot a}}\Big)[A] - \frac{a_{n+1}}{n \cdot a} \cdot n$$

$$\pi : l_p : \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$$

$$p' = l_p \cap \pi : (x_0 + t) + 4(y_0 + 4t) - 3(z_0 - 3t) + 7 = 0$$

$$t = -\frac{x_0 + 4y_0 - 3z_0 + 7}{1 + 16 + 9} \quad \ldots$$

9. $\ell : \begin{cases} 2x - y - 1 = 0 \\ x + y - z + 1 = 0 \end{cases}$

$\pi : x + 2y - 3$