Algebra theory: good milion : (+, 9) si pois (1): (. +, 8): too o 8.

·  $\pi = (A, A, R)$  on A is called:

(1) neflexive (n) : VXEA XXX = XXX

(2) transitive (t): x,4,2 c A XRY and YRZ => XRZ

(3) 54 m metric (s.): x,4 Ex xx4 =>4xx

r equivalence relation if n has: (n),(t),(s) comes : the op "." comes

"ITEE(A) AIREP(A)

 $\pi = (A_i)_{i \in I} \in P(A) . \pi_{\pi} \in E(A)$ 

iii) F: E(A) -> P(A) be det loy: F(n)=A/n il neE(A) aporto (ie) (ie).

=> 7 a big. whose inverse 15 G:P(A) → E(A):G(T) = TT UTEP(A) isom orginism. A/r- portition caresponding to r.

C2:

· Let "." be an operation on A: (x) = ((x)): '1=(1) = ngromomail 18 4.

(1) Associative law: (x·y). = x·(y.z) +x,y, = 6A

(2) Commutative law: x.y=4.x +x,46A

(3) Identity law: FeEA n.t tabA a.e=e.a=a (e-identity element)

(4) Inverse law: tack fa' s.t a.a' = a'. a=e

· "." op om A:

(i) If Ie => it is unique

(ii) If Ix'=> it is unique

· ntable subset: 9: A \* A -> A B C A , B stable nubset if: 4 x, y & B & (x, y) & B

· (A,·) called: (1) semigroup if it is asoc. law holds

(2) momaid if: semigroup with idel

(3) group if it is a monaid txeA is inversable.

(4) obelian group if it is comutative group

toodoo Happy Him: 41 quarodoo H aght.

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· R a set: (R,+,·): (1) ring if (R,+) is abelian group, (R,·) semigroup
                       and distributive law holds:
                             X: (4+2) = x.4+ x.2 + x,4,2eR
                             (4+5). X = 4x+2xx Axinises 1) 2 (1)
                    (2) unitory pring my enx ADE, P(x: (3) sufferent (3)
                    (3) division ring: (R,+) abel gr (R*, ·) gr. dist. law
                   (4) field: cam watere division ring lociops
                  (R;+,·) camet if the op "." comet
· HCG H subgroup it: (1) H stable subset
                                                'UNEELA) ALREPIA)
(G,·) -gr
                      (2) (H,·) is a group quange p si (·, H) (2)
· (G, ·) (G", ·) groups (f: G= G' f group homomorph:
     f(x.y)=f(x).f(y) +xyeg 3 - 4)9:0 21 soon solv . id a 3 5
 f - hij => isomorphism.
                                  A/n- portition corresponding to n.
• f gr homomorph => f(i)=i'; (f(x))^{-1}=f(x^{-1}); f no notating no ii' od.
                             (1) Associative law: (x.4) = = x. (4.2) + x. 4 = 6A
                                    (4) Commutative laws x-y=y-x +x, yen
          (3) Identity law: 3864 at 6 as a caca (e-identity element)
                             (4) hourse law: Hack to st and adiane
                                                          : A NO 40 "." .
                                                  Supino - 14: (= SE $1 11)
                                                  signorest G x E D (10)
         Clary of the Artak Dea , bedie of the Brayes the trayes
                              There we soon with fi granginess (it is below ( ).
                            is, anomorphis : it is biomorphism is. of
              3) group if it is a monoid treat is interpolate.
                     quare stitutions of it if quare relieve (r)
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· A vector space over K is an abelian group (V,+)

$$: K \times V \rightarrow V$$
,  $(k, v) \mapsto kv$ 

satis fying :

(1) 
$$k \cdot (v_1 + v_2) = k \cdot v_1 + k \cdot v_2$$

(1) 
$$k \cdot (v_1 + v_2) = k \cdot v_1 + k \cdot v_2$$

(2)  $(k_1 + k_2) \cdot v = k_1 \cdot v + k_2 \cdot v$ 

(3) 
$$(k_1 \cdot k_2) \cdot v = k_1 \cdot (k_2 \cdot v)$$

(4) out a morphism of big and U=V' · Let V v.s over K SCV, 5 wb space of Vif: 10=1014 | Nov f-frey + No-0.1.

C 4:

· V v.s XCV:

we call it the subspace generated by X

· V v.s over K

finitely generated if  $\exists u_1...u_n \in V(n \in N)$  s.b.  $V = \langle v_1...v_n \rangle$ 

· V v.s over K

k, v, + ... + kn um 11 most c= ( 195 4 ( ) + h = ( 19 ) ( ) ) is called a linear combination of the vectors u ... un

· V v.s over k x1...×MEU

V t.s over 
$$k$$
  $x_1...x_m \in U$   $(x_1...x_m) = \{k_1x_1+...+k_mx_m \mid k_i \in K\}$ 

· VV.S over K SITSU

Vario 12 Telostel

(No.0) (0) } = 1 mil

(2) p+(2) = (1) (p+2)

fund to lefel of the form

· V and V' ess over K

· KontsV

· V oxs K S,T < V >> S+T < V · let v, v' vs over the field Kano moilede me i i romo sauga notare h. f: U->V' is: (1) (K-) linear map (homomorphism or linear trans.) if:  $f(k_1v_1+k_2v_2)=k_1f(v_1) \iff \begin{cases} f(v_1+v_2)=f(v_1)+f(v_2) & \forall v_1,v_2 \in V_1 \\ f(kv)=k_1f(v_1) & \forall k \in K \forall v \in V \end{cases}$ (3. (4. 62) · v= (4. 4) (6) (2) isomorphism it bij (3) endomorphism if K lm => V=V' (4) automorphism if big and b=v' · Fet A 22.2 avoit K 26A 2 220 Ebors of Att. Q72 (1) · f: U->V': Kerf={veV|f(v)=o'} (1) 4 21 , 25 62 14 21 425 62 1mf = 3 f(v) | v = 43 (3) A Well Abez P. 262 ·N) · Kerf < V · V B.S XEV: imf < v' <x>= 0 fs < v | x = < x > · f: U -> U' => Ken f= 207 (=) f inj we call it the subspace generated by X · Vand V' v.s over K · V 3.5 OHON K Homk (v,v'): Afige Hom K(v,v') AFEK: fig, & fe Homk (v,v') (f+g)(v) = f(v)+g(v) (k!)(v) = k!(v)  $\forall v \in V$  = 0 + 0 (kf)(v) = kf(v)· V v.s over K => End K(V) v.s over K (x1..x4) = { B. Y. + - - + R. X | R. EKY {Tee; 101|040} = T+L To be for between is To be for The gill <TUZ>= T+2.

- · V v.s over K V. . . vm EV
- (1) liniarly independent UR; EK

- · V v.s over k . B=(v1 ·· vn) evm is a basin of Vif:
  - (1) B linionly independent in V. I become it is sman and (im) mile.
  - (2) B is a syst of generators for V, <B>=V=V=(Soot) mib = Vmib. 4 or. 5 has a base dim Stdim To Stone (SAT) +dim (STT)
- · V vs over K B=(u1...um) basis if 4 ve V can be uniquely voriften as a linear cambination:

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complement of Siny

- · B=(U1...un) basin of V and vev Then K1...Knek from the timear v= le, v,+.. know of vectors B ore the coordinates of v in B **C** 6
- · Any 2 horses of v.s have the name me of el
- · U v.s over K. The m of el is called the dim V or dim V
- · The max mr of linearly independent vector in V is dim V = n
- · The min or of generators for Vin dim V=n
- · × lim indep in V => × is a syst of generators for V
- · I linearly independent list of vectors in v.s can be completed to a
- · V v.s over k S S V:
  - (1) I havis of S is part of a bosis of U
  - (2) dim 5 ≤ dim V
  - (3) dim S = dim V (=) S=V

- . Sunspace 5 of V s.t V=S # 5 is called a · V v.s over K SSV camplement of sinv Value of a state of V.
- V ~ V' (=) dim V = dim V'
- dim V=n is isomorphic to the can anical vector space Km over K
- · dim(kerf) the mullity off and is denoted by null(f).
- · dirm(Imf) the nank of f denoted by nank(f)
- · dim V= dim (Kerf) + dim (Imf)
- dim S+dim T= dim (SAT) +dim (S+T)
- · V=SBT: dim V=dim S+dimT (2004 5: circol (m. 1)=0 21 1000 CELV.

· Asy 2 houses of v.s have the source me of el

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V= 10, V t .. hwon

· V on over 15. The not of el in colled the dim v or dim v

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ore al, of the sound of vectors is one the coordinates of or in B

· The max our of linearly independent victor in Vis Wilm V = or N= V mode air ref austrones for an aim sor.

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