

Computational logic

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Project

Group 911

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Subject 1

Course Theory  
Group: 911

## Homework - teams Student 1

### Subject 1 - Student 1.

$$b_1 = 16$$

$$x = 1257A9(16)$$

$$y = 35076(16)$$

The addition of the two numbers is performed starting with the units digit (with index 0), from right to left, the process being repetitive with a number of  $\max(m, n) + 1$  iteration ( $m, n$  - number of digits - 1).

At each iterations, the digits (after they have been converted to decimal) from the homologous positions and the carry digit (0 or 1) from the previous iteration are added.

The sum provides 2 digits:

- the transport figure used in the next iteration is the quotient of dividing the sum by base.
- the positionally corresponding figure in number C is the remainder of dividing the sum by base

$$1257A9(16) + 35076(16) = ?(16)$$

$$m=6$$

$$n=5$$

$$\begin{array}{r} 000^{\wedge}00 \\ 1257A9(16) + \\ 035076(16) \\ \hline 15A897(16) \end{array}$$

$$i=0 \quad x_0=0$$

$$a_{(6)} + b_{(4)} + c_{(0)} = 9 + 6 + 0 = 15(16)$$

$$x_1 = \lfloor 15/16 \rfloor = 0$$

$$c'_0 = 15 - 0 \cdot 16 = 15 \Rightarrow c_0 = 7(16)$$

$$i=1 : x_1 = 0$$

~~$$A(16) + F(16) = 10 + 15 + 0_{(16)} = 25$$~~

$$A(16) + F(16) + Q(16) = 10 + 15 + 0 = 25$$

$$x_2 = \lfloor 25/16 \rfloor = 1, \quad c'_q = 25 - 1 \cdot 16 = 9$$

$$c_1 = 9_{(16)}$$

$$i=2 : x_2 = 1.$$

$$F(16) + 0_{(16)} + 1_{(16)} = 7 + 0 + 1 = 8.$$

$$x_3 = \lfloor 8/16 \rfloor = 0, \quad c'_x = 8 - 0 \cdot 16 = 8.$$

$$c_2 = 8_{(16)}$$

$$i=3 : x_3 = 0.$$

$$5_{(16)} + 5_{(16)} + 0_{(16)} = 5 + 5 + 0 = 10.$$

$$x_4 = \lfloor 10/16 \rfloor = 0, \quad c'_3 = 10 - 0 \cdot 16 = 10$$

$$c_3 = 10_{(16)}$$

$$i=4 : x_4 = 0.$$

$$2_{(16)} + 3_{(16)} + 0_{(16)} = 2 + 3 + 0 = 5$$

$$x_5 = \lfloor 5/16 \rfloor = 0, \quad c'_4 = 5 - 0 \cdot 16 = 5$$

$$c_4 = 5_{(16)}$$

$$i=5 : x_5 = 0.$$

$$1_{(16)} + 0_{(16)} + 0_{(16)} = 1 + 0 + 0 = 1.$$

$$x_6 = \lfloor 1/16 \rfloor = 0, \quad c'_5 = 1 - 0 \cdot 16 = 1$$

$$c_5 = 1_{(16)}$$

$$\Rightarrow \boxed{D = 15A89F(16)}$$

$$b_2 = 8$$

$$a = 135246_8$$

$$L = 5_8$$

The multiplication is performed starting with the units digit (with index 0), from right to left, the process being repetitive with a number of  $n+1$  iterations.

At each iteration, the current digit  $a_i$  is multiplied by  $b$ , after they have been converted to decimal, and the carry digit from the previous iteration is added.

The calculated value provides 2 digits:

- the transport figure used in the next iteration is the quotient of dividing the calculated value by  $b_{10}$ ;
- the positionally corresponding figure in number  $C$  is the remainder of the division of the calculated value by  $b_{10}$ .

$$135246_8 \cdot 5_8 = ?_8$$

$$m=6.$$

$$\begin{array}{r} 135246_8 \\ \times 5_8 \\ \hline \end{array}$$

$$722476_8$$

$$i=0 : x_0=0.$$

$$(6_8) \cdot 5_8 + 0_8 = 6 \cdot 5 + 0 = 30.$$

$$x_1 = \lfloor 30/8 \rfloor = 3, C'_0 = 30 - 3 \cdot 8 = 30 - 24 = 6$$

$$C_0 = 6_8$$

$$i=1 : x_1=3$$

$$4_8 \cdot 5_8 + 3_8 = 4 \cdot 5 + 3 = 23$$

$$x_2 = \lfloor 23/8 \rfloor = 2, C'_1 = 23 - 2 \cdot 8 = 23 - 16 = 7$$

$$C_1 = 7_8$$

$$i=2 : x_2=2$$

$$2_8 \cdot 5_8 + 2_8 = 2 \cdot 5 + 2 = 12 \Rightarrow x_3 = \lfloor 12/8 \rfloor = 1$$

$$C'_2 = 12 - 8 \cdot 1 = 12 - 8 = 4$$

$$C_2 = 4_8$$

$$i=3 : t_3 = 1.$$

$$5(0) + 5(0) + 1(0) = 5 \cdot 5 + 1 = 26.$$

$$t_4 = \lfloor 26/8 \rfloor = 3 \quad C'_3 = 26 - 8 \cdot 3 = 26 - 24 = 2$$

$$C_3 = 2(0)$$

$$i=4 : t_4 = 3$$

$$2(0) + 5(0) + 3(0) = 3 \cdot 5 + 3 = 18.$$

$$t_5 = \lfloor 18/8 \rfloor = 2 \quad C'_4 = 18 - 8 \cdot 2 = 18 - 16 = 2.$$

$$C_4 = 2(0)$$

$$i=5 : t_5 = 2$$

$$1(0) + 5(0) + 2(0) = 1 \cdot 5 + 2 = 7$$

$$t_6 = \lfloor 7/8 \rfloor = 0 \quad C'_5 = 7 - 8 \cdot 0 = 7.$$

$$C_5 = 7(0)$$

$$\Rightarrow \boxed{n = 722 \ 476(0)}$$

Subject 1  
Student 2

$$b_1 = 16 \quad S = 15A89F$$

$$b_2 = 8 \quad Z = 135246$$

$$X = 1257A9 \quad f = 5$$

$$y = 350F6 \quad p = 422476$$

$$S(b_1) - y(b_1) = ?(b_1)$$

~~15A89F~~

$$15A89F_{(16)} -$$

$$350$$

$$350F6_{(16)}$$

$$1257A9_{(16)} = X$$

$$P(b_2) : f(b_2) = ?(b_2)$$

$$422476_{(8)} : 5_{(8)} = 135246 = Z$$

$$4_{(8)} = 4 : 5 = 0r2$$

$$22_{(8)} = 2 \cdot 8 + 2 = 18 : 5 = 3r3$$

$$32_{(8)} = 3 \cdot 8 + 2 = 26 : 5 = 5r1$$

$$14_{(8)} = 1 \cdot 8 + 4 = 12 : 5 = 2r2$$

$$27_{(8)} = 2 \cdot 8 + 7 = 23 : 5 = 4r3$$

$$36_{(8)} = 3 \cdot 8 + 6 = 30 : 5 = 6r0$$

$$\begin{array}{r} P(b_2) : f(b_2) = ?(b_2) \\ 422476_{(8)} : 5_{(8)} = \\ 4_{(8)} = 4, 2 : 5 = 1r2 \\ 22_{(8)} : 5_{(8)} \\ 22 \\ 22 \end{array}$$



## Subject 2

Subject 2  
Student 2

$$b=6, h=16, \lambda=52121, 402$$

$$52121, 402_{(16)} = 5 \cdot 6^4 + 2 \cdot 6^3 + 1 \cdot 6^2 + 2 \cdot 6^1 + 1 \cdot 6^0 + 4 \cdot 6^{-1} + 0 \cdot 6^{-2} + 2 \cdot 6^{-3}$$

$$= 5 \cdot 6^4 + 2 \cdot 6^3 + 1 \cdot 6^2 + 2 \cdot 6^1 + 1 \cdot 6^0 + 4 \cdot 6^{-1} + 0 \cdot 6^{-2} + 2 \cdot 6^{-3} = 1B31, tCF$$

$$a) 5 \cdot 6^4 = 5_{(16)} \cdot 510_{(16)} = 1950_{(16)}$$

$$6^4 = 36$$

$$36$$

$$216$$

$$108$$

$$1296$$

$$128$$

$$16$$

$$16$$

$$0$$

$$16$$

$$16$$

$$16$$

$$16$$

$$16$$

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$$b) 2 \cdot 6^3 = 2_{(16)} \cdot 216_{(16)} = 1B0$$

$$6^3 = 216$$

$$16$$

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$$c) 1_{(16)} \cdot 6^2_{(16)} = 36_{(16)} = 24$$

$$d) 2_{(16)} \cdot 6^1_{(16)} = 12_{(16)} = C$$

$$e) 1 \cdot 6^0_{(16)} = 1_{(16)}$$

$$f) 4 \cdot 6^{-1}_{(16)} = 0, AAA$$

$$4$$

$$4$$

$$4$$

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$$4$$

$$4$$

$$4$$

$$4$$

$$4$$

$$4$$

$$1950 +$$

$$510$$

$$1B0$$

$$24$$

$$C$$

$$1$$

$$1B31 + 0, tCF = 1B31, tCF$$

$$1B31$$

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$$1B31$$

$$g) 0 \cdot 6^{-2}_{(16)} = 0_{(16)}$$

$$h) 2 \cdot 6^{-3}_{(16)} =$$

$$2,000$$

$$20$$

$$20$$

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$$20$$

$$20_{(16)} = 2 \cdot 16 + 8 = 32 + 8 = 40$$

$$20_{(16)} = 2 \cdot 16 + 8 = 32 + 8 = 40$$

$$55_{(16)} = 5 \cdot 16 + 5 = 80 + 5 = 85$$

$$15_{(16)} = 1 \cdot 16 + 5 = 16 + 5 = 21$$

$$0,0E3 \mid 6 \quad E_{(16)} = 14 \cdot 16 = 224$$

$$23_{(16)} = 2 \cdot 16 + 7 = 32 + 7 = 39$$

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exam Nov 03  
Group 511

## Subject 2 - Student 1

$$b = 6$$

$$h = 16$$

$$y(h) = 1031, A05$$

$16 > 6 \Rightarrow$  we will use the method of successive divisions and multiplications.

The calculations will be in base 16

a) Integer part: 1031

$$\begin{array}{r} 1031_{16} \mid 6_{16} \\ \hline 168_{16} \\ \hline 33 \\ \hline 1 \\ \hline 31 \\ \hline 1 \\ \hline \textcircled{1} \end{array}$$

$$it_1: 1031_{16} = 16 \times 64 + 11 = 27$$

$$27 : 6 = 4, 13$$

$$it_2: 33 = 3 \times 16 + 3 = 48 + 3 = 51$$

$$51 : 6 = 8, 13$$

$$it_3: 31 = 3 \times 16 + 1 = 49$$

$$49 : 6 = 8, 11$$

$$\begin{array}{r} 488_{16} \mid 6_{16} \\ \hline 80_{16} \\ \hline 08 \\ \hline \textcircled{2} \end{array}$$

$$it_1: 48 = 4 \times 16 + 8 = 72$$

$$72 : 6 = 12, 10$$

$$it_2: 8 = 0 \times 16 + 8 = 8$$

$$8 : 6 = 1, 2$$

$$\begin{array}{r} 01_{16} \mid 6_{16} \\ \hline 01 \\ \hline \textcircled{1} \end{array}$$

$$it_1: C = 0 \times 16 + 12 = 12$$

$$12 : 6 = 2, 10$$

$$it_2: 1 = 0 \times 16 + 1 = 1$$

$$1 : 6 = 0, 11$$



$$\begin{array}{r|l} 20_{(10)} & 6_{(16)} \\ \hline 1 & 5_{(16)} \end{array}$$

②

$$\text{It 1: } 20 = 2 \cdot 16 + 0 = 32$$

$$32 : 6 = 5,12$$

$$\begin{array}{r|l} 5_{(16)} & 6_{(16)} \\ \hline 1 & 0_{(16)} \end{array}$$

⑤

$$\text{It 1: } 5 = 0 \cdot 16 + 5 = 5$$

$$5 : 6 = 0,15$$

We take the remainders from last to first

$$\Rightarrow 1531_{(16)} = 52121_{(6)}$$

2) Fractional part: 0,105

$$\begin{array}{r} 0,105_{(10)} \\ \cdot 6_{(16)} \\ \hline 0,075_{(16)} \end{array}$$

④,075<sub>(16)</sub>

$$\text{It 1: } 0_{(16)} + 6_{(16)} = 0 + 5 \cdot 6 = 30$$

$$30 : 16 = 1,14$$

$$\text{It 2: } 1_{(16)} + 0_{(16)} \cdot 6_{(16)} = 1 + 13 \cdot 6 = 79$$

$$79 : 16 = 4,115$$

$$\text{It 3: } 4_{(16)} + 1_{(16)} \cdot 6_{(16)} = 4 + 10 \cdot 6 = 64$$

$$64 : 16 = 4,10$$

$$\text{It 4: } 4 + 0 \cdot 6 = 4$$

$$4 : 16 = 0,14$$

$$\begin{array}{r} 0,075_{(16)} \\ \cdot 6_{(16)} \\ \hline 0,045_{(16)} \end{array}$$

①,045<sub>(16)</sub>

$$\text{It 1: } 0_{(16)} + 4_{(16)} \cdot 6_{(16)} = 0 + 14 \cdot 6 = 84 \quad 84 : 16 = 5,14$$

$$\text{It 2: } 5_{(16)} + 0_{(16)} \cdot 6_{(16)} = 5 + 15 \cdot 6 = 95$$

$$95 : 16 = 5,15$$

$$\text{It 3: } 5_{(16)} + 0_{(16)} \cdot 6_{(16)} = 5 \quad 5 : 16 = 0,15$$

$$\text{It 4: } 0_{(16)} + 0_{(16)} \cdot 6_{(16)} = 0 \quad 0 : 16 = 0,10$$

$$\begin{array}{r} 0, \overset{2}{5} \overset{3}{7} \overset{1}{4}_{(6)} \\ \hline \end{array}$$

$$\textcircled{2}, 3, 5, 8$$

$$\text{it}_1: 0_{(6)} + 4_{(6)} \cdot 6_{(6)} = 0 + 4 \cdot 6 = 24 \quad 24 : 16 = 1, 8$$

$$\text{it}_2: 1_{(6)} + 7_{(6)} \cdot 6_{(6)} = 1 + 7 \cdot 6 = 91 \quad 91 : 16 = 5, 11$$

$$\text{it}_3: 5_{(6)} + 5_{(6)} \cdot 6_{(6)} = 5 + 5 \cdot 6 = 35 \quad 35 : 16 = 2, 13$$

$$\text{it}_4: 2_{(6)} + 0_{(6)} \cdot 6_{(6)} = 2 \quad 2 : 16 = 0, 12$$

We take the integer part of which result from first to last:  
 $0, \text{ABS}_{(6)} = 0, 402_{(6)}$

$$\Rightarrow R/s: x_{(6)} = 52121, 402_{(6)}$$

## Subject 3

Birou Roes

Group: 511

### Subject 3 - Student 1

- $x = 84230,12$
- $SP = 32$  bits, mantissa  $> 1$
- $SP$  - single precision  $\rightarrow m = 32$  bits
  - $e$  on 8 bits
  - $q = 127$
  - $m$  on 23 bits

Floating point representation of real numbers

- used to represent very large and very small numbers with a high precision
- if there is an overflow, then the least significant digits are lost.

- Conversion of the integer part: successive divisions by 8

$$\begin{array}{r} 84230 \div 8 \\ \hline 10528 \\ \hline 04 \\ \hline 42 \\ \hline 23 \\ \hline 70 \\ \hline 64 \\ \hline 6 \end{array}$$

$$\begin{array}{ll} \text{it}_1 & 8:8 = 1, \text{r } 0 \\ \text{it}_2 & 4:8 = 0, \text{r } 4 \\ \text{it}_3 & 42:8 = 5, \text{r } 2 \\ \text{it}_4 & 23:8 = 2, \text{r } 7 \\ \text{it}_5 & 70:8 = 8, \text{r } 6 \end{array}$$

$$\begin{array}{r}
 10528 \overline{)8} \\
 \underline{8} \phantom{0000} \\
 25 \phantom{0000} \\
 \underline{24} \phantom{0000} \\
 12 \phantom{0000} \\
 \underline{8} \phantom{0000} \\
 48 \phantom{0000} \\
 \underline{48} \phantom{0000} \\
 0
 \end{array}$$

it 1  $10:8 = 1, \wedge 2$   
 it 2  $25:8 = 3, \wedge 1$   
 it 3  $12:8 = 1, \wedge 4$   
 it 4  $48:8 = 6, \wedge 0$

$$\begin{array}{r}
 1316 \overline{)8} \\
 \underline{8} \phantom{0000} \\
 51 \phantom{0000} \\
 \underline{48} \phantom{0000} \\
 36 \phantom{0000} \\
 \underline{32} \phantom{0000} \\
 4
 \end{array}$$

it 1  $13:8 = 1, \wedge 5$   
 it 2  $51:8 = 6, \wedge 3$   
 it 3  $36:8 = 4, \wedge 4$

$$\begin{array}{r}
 164 \overline{)8} \\
 \underline{16} \phantom{0000} \\
 04 \phantom{0000} \\
 \underline{0} \phantom{0000} \\
 4
 \end{array}$$

it 1  $16:8 = 2, \wedge 0$   
 it 2  $04:8 = 0, \wedge 4$

$$\begin{array}{r}
 20 \overline{)8} \\
 \underline{16} \phantom{0000} \\
 4
 \end{array}$$

it 1  $20:8 = 2, \wedge 4$

$$\begin{array}{r}
 2 \overline{)8} \\
 \underline{0} \phantom{0000} \\
 2
 \end{array}$$

it 1  $2:8 = 0, \wedge 2$

Now we take the remainders from the last one

$$\Rightarrow 87230_{(10)} = 24440_{(8)}$$

Furthermore we will use rapid conversions.

$$24440_{(8)} = 010 \ 100 \ 100 \ 100 \ 000 \ 110_{(2)} \text{ (rapid conversion)}$$

- Conversion of the fractional part: successive multiplication by 8

$$\begin{array}{r} 0,12_8 \\ \times 8 \\ \hline 0,96_8 \end{array} \quad \begin{array}{r} 0,96_8 \\ \times 8 \\ \hline (7),68_8 \end{array} \quad \begin{array}{r} 0,68_8 \\ \times 8 \\ \hline (6),44_8 \end{array}$$

So we have  $0,12 = 0,075_{(8)} = 0,000\ 111\ 101_{(2)}$  (fixed conversion)

From the number above we will only take 7 digits from fractional part, as we have 16 bits in total part, so  $23 - 16 = 7$

- $X = 842\ 30,11_{(10)} = 10\ 100\ 100\ 100\ 000\ 110,000\ 111\ 11_{(2)}$

$$X = 1, \underbrace{0100\ 100\ 100\ 000\ 110\ 000\ 111\ 11}_{\text{mantissa}}_{(2)} \times 2^{16-2}$$

↓  
hidden bit

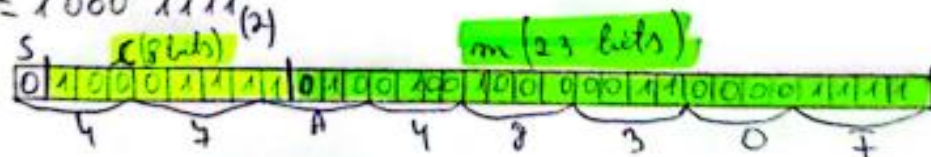
- $C = 8 + 127$

$$= 16 + 127$$

$$= 143$$

$$= 2^7 + 2^3 + 2^2 + 2^1 + 2^0$$

$$= 1\ 000\ 1111_{(2)}$$



$$\Rightarrow 47A48307_{(16)}$$

Subject 3. Option 4  
Student 2

$$M_{16} = 47A4830F_{(16)}$$

$$47A4830F = 1000111101001001000001100001111$$

$$MEM. REPR: \begin{array}{c} 5 \quad c(8 \text{ bits}) \quad m(23 \text{ bits}) \\ \boxed{0 \mid 1000 \ 1111 \mid 0100 \ 1001 \ 0000 \ 0110 \ 000 \ 111} \end{array}$$

$$\text{Exp: } 1000 \overset{\swarrow}{1111} = 143_{(10)} - 127 = 16$$

$$\text{Denormalization: } 1, 0100 \ 1001 \ 0000 \ 0110 \ 000 \ 111 : 2^{16} =$$

$$= 1010010010000110,000111 = 2^{16} + 2^{14} + 2^{11} + 2^8 + 2^2 + 2^1 = 65,536 + 16,384 + 2048 + 256 + 4 + 2 =$$

~~==~~

$$= 1010010010000110,000111 = 2^{16} + 2^{14} + 2^{11} + 2^8 + 2^2 + 2^1 + 2^{-4} + 2^{-5} + 2^{-6} =$$

$$= 65,536 + 16,384 + 2048 + 256 + 4 + 2 + 0,06 + 0,03 + 0,01 = 84230,10 \simeq 84230,12$$