QUEEN'S UNIVERSITY BELFAST

PHY2005

Any calculator, except one with pre-programmable memory, may be used in this examination.

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2005
Atomic and Nuclear Physics

Duration: 3 hours plus additional 1 hour for upload of work

Tuesday 3rd of August 2021
09:30 AM – 1:30 PM

Examiners: Prof S Matthews, Prof F. Peters
and the internal examiners
Dr S Sim (s.sim@qub.ac.uk)

Answer ALL questions in Section A for 4 marks each.
Answer ONE question from Section B for 30 marks.
Answer ONE question from Section C for 30 marks.

If you have any problems or queries, contact the School Office at mpts@qub.ac.uk or 028 9097 1907, and the module coordinator s.sim@qub.ac.uk

QUEEN'S UNIVERSITY BELFAST
SCHOOL OF MATHEMATICS AND PHYSICS

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \text{ms}^{-1}$
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$ $\approx 1.26 \times 10^{-6}\ \text{Hm}^{-1}$
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{Fm}^{-1}$
Elementary charge	$e = 1.60 \times 10^{-19}\ \text{C}$
Electron charge	$= -1.60 \times 10^{-19}\ \text{C}$
Planck Constant	$h = 6.63 \times 10^{-34}\ \text{Js}$
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}\ \text{Js}$
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}\ \text{m}^{-1}$
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}\ \text{kg}$ $1u = 931.5\ \text{MeV}$
1 electron volt (eV)	$= 1.60 \times 10^{-19}\ \text{J}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ \text{kg}$
Mass of proton (three sig. fig.)	$m_p = 1.67 \times 10^{-27}\ \text{kg}$
Mass of neutron (three sig. fig.)	$m_n = 1.67 \times 10^{-27}\ \text{kg}$
Molar gas constant	$R = 8.31\ \text{JK}^{-1}\text{mol}^{-1}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\ \text{JK}^{-1}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11}\ \text{Nm}^{2}\text{kg}^{-2}$
Acceleration of free fall on the Earth's surface	$g = 9.81\ \text{ms}^{-2}$
Nuclear magneton	$\mu_N = 5.05 \times 10^{-27}$ J/T

DATA FOR NUCLEAR MASSES

$m(e^-) = m(e^+) = 0.511\ \mathrm{MeV}/c^2$

$m(^1H) = 1.007825\ u = 938.28\ \mathrm{MeV}/c^2$

$m(n) = 1.008665\ u = 939.57\ \mathrm{MeV}/c^2$

$m(^3He) = 3.016030\ u$

$m(^4He) = 4.002603\ u$

$m(^{96}Sr) = 95.921750\ u$

$m(^{140}Xe) = 139.921647\ u$

$m(^{226}Ra) = 226.025402\ u$

$m(^{230}Th) = 230.033134\ u$

$m(^{238}U) = 238.050788\ u$

SECTION A

Answer <u>ALL</u> 10 questions in this section
Full explanations of your answers are required to attain full marks

1 Discuss the extent to which the Bohr model accounts for the observed properties of atomic spectra. Your answer should include comments on both successes and shortcomings of the model. **[4]**

2 Consider a multi-electron atom in which the total orbital angular momentum quantum number is $L = 3$ and the total spin quantum number is $S = 3$. Give all the allowed values for the total angular momentum quantum number J. How many distinct angular momentum states exist for this combination of L and S ? **[4]**

3 The time-independent Schrödinger equation for an atom with n electrons can be written as

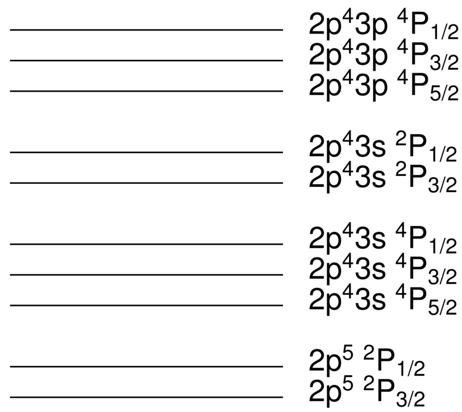
$$\sum_{i=1}^{n}\left[-\frac{\hbar^2}{2m_e}\nabla_i^2\psi - \frac{Ze^2}{4\pi\varepsilon_0|\boldsymbol{r_i}|}\psi + \sum_{j=i+1}^{n}\frac{e^2}{4\pi\varepsilon_0|\boldsymbol{r_i}-\boldsymbol{r_j}|}\psi\right] = E\psi$$

where $\boldsymbol{r_i}$ is the position vector of electron i, ∇_i^2 is the Laplacian operator with respect to the coordinates of particle i, and all other symbols have their usual meanings. State the physical meaning of each of the terms in this equation. Explain why finding wavefunctions (ψ) that are solutions to this equation is much more difficult than for the Schrödinger equation of single-electron systems. **[4]**

4 Apply the Aufbau principle to determine the ground state electron configuration of neutral zirconium, which has atomic number $Z = 40$. Show all your working. **[4]**

continued …

5 The energy level diagram below shows the first 10 energy levels of neutral fluorine ($Z = 9$). Note that in all cases the filled inner shells (1s and 2s) are omitted from the electron configurations given in the figure. Make a copy of this diagram and indicate all permitted transitions between these states. **[4]**



6 The measured binding energy per nucleon of a nucleus X is 8.580 MeV. Its atomic mass number is A = 102, and it contains N = 56 neutrons. Calculate:

(a) The atomic mass of X in atomic mass units using a precision of six significant figures. **[3]**

(b) The average nuclear radius, given the proportionality constant $R_0 \approx 1.2$ fm. **[1]**

Clearly show all working for your answers.

7 Calculate the rest mass in MeV/c^2 for π^+ (Pion), K^+ (Kaon) and D^+ (D-meson) particles, assuming that their nuclear interaction range is 1.433 fm, 0.405 fm, 0.107 fm, respectively, and compare the calculated values with the electron and proton masses.
Clearly show all working for your calculations. **[4]**

continued …

SECTION A

8 Recent scattering experiments of energetic α-particles fusing into ^{12}C nuclei provide evidence for the existence of the 2^+ excited rotational state in ^{12}C. This is known as the "Hoyle state" and consists of a nucleus (^{12}C) in which three α -particles coexist in a form of cluster state. Assuming a moment of inertia of the nucleus of 2.1×10^{-56} kg m^2, calculate:

(a) the energy of the rotating nucleus in MeV **[2]**

(b) its magnetic dipole moment. **[2]**

9 An extravagant scientist announces the end of the world due to the following spontaneous decay reaction:

$$p \rightarrow n + e^+ + \nu$$

(a) Identify the type of nuclear decay and calculate the energy exchanged in this reaction. **[2]**

(b) Explain why the prediction of the scientist is wrong. **[2]**

10 Consider the following nuclear reaction:

$$n + {}^{238}U \rightarrow {}^{96}Sr + {}^{140}Xe + 3n$$

(a) Identify the type of reaction and calculate the energy released. **[2]**

(b) Demonstrate that the total number of nucleons and total charge are conserved. **[2]**

[^{238}U has neutron number N = 146; ^{96}Sr has N = 58; ^{140}Xe has N = 86.]

[END OF SECTION A]

continued …

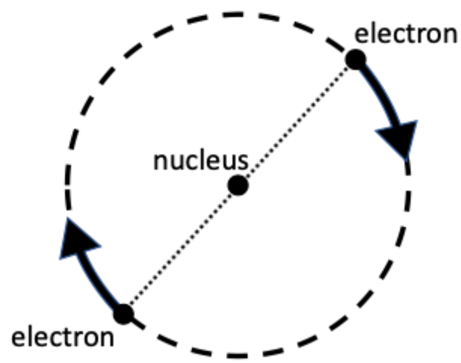
SECTION B

Answer ONE question from this section

11(a) Use the Bohr model to estimate the wavelength of the transition between states with principal quantum number $n = 5$ and $n = 3$ for seven-times ionized oxygen (O^{7+}). Oxygen has atomic number $Z = 8$. Give your answer to three significant figures. **[4]**

(b) Discuss what is meant by the term *effective nuclear charge*, Z_{eff}, when used in the context of electron orbitals for multi-electron atoms. Your answer should include comments on the principles of using a *central field approximation* for the potential energy, and how this potential is conveniently expressed in terms of Z_{eff}. **[6]**

(c) Consider the semi-classical model for a two-electron atom illustrated in the diagram below. This model attempts to extend the Bohr model to a two-electron system by assuming both electrons follow a circular orbit around the nucleus while remaining diametrically opposite to each other.



(i) Carefully explain why, for the ground state of helium, this semi-classical model predicts that both electrons would experience an effective nuclear charge $Z_{\mathrm{eff}} = \frac{7}{4}$. **[7]**

(ii) Assuming that each electron's angular momentum is quantized in the same manner as the usual Bohr model for single-electron atoms, calculate the total binding energy of the ground state of helium according to this semi classical model. **[6]**

[Question 11 continues on next page]

[Question 11 continued]

(iii) The experimentally measured total binding energy for the ground state of helium is around 79 eV. Comment on the comparison of your result from part (c)(ii) to this value and explain why the semi-classical model does not predict this value to very high accuracy. **[3]**

(iv) If this model were extended to calculate the total binding energy of the ground state of singly-ionized lithium (Li^+), explain whether the fractional error in comparison to experiment would be better or worse than for the ground state of helium. **[4]**

[Helium has atomic number $Z = 2$. Lithium has atomic number $Z = 3$.]

continued …

SECTION B

12(a) The energy associated with the interaction of a magnetic dipole moment $\boldsymbol{\mu}$ in a magnetic field $\boldsymbol{B}$ can be written as

$$\Delta E = -\,\boldsymbol{\mu} \cdot \boldsymbol{B}$$

Explain how this relationship can be used to understand

(i) fine-structure in spectral lines due to the spin-orbit interaction;

(ii) splitting of spectral lines via the Zeeman effect.

Your answer does not need to include detailed calculations but should explain what these phenomena are and the nature of the magnetic dipole moment and magnetic field involved.

[12]

(b) Consider the electron configuration $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d\,4d$. Apply Hund's rules to determine the angular momentum quantum numbers (L, S and J) of the lowest energy term associated with this configuration. Give you answer in spectroscopic notation. **[6]**

(c) The spin-orbit energy of interaction is often written in the form

$$\Delta E = \frac{2K}{\hbar^2} \boldsymbol{S} \cdot \boldsymbol{L}$$

Where $\boldsymbol{S}$ and $\boldsymbol{L}$ are the total spin and orbital angular momentum vectors, respectively and the coefficient K has dimensions of energy.

(i) The three lowest energy levels of neutral oxygen all have the electron configuration $1s^2\,2s^2\,2p^4$, total spin quantum number $S = 1$ and total orbital angular momentum number $L = 1$. The energies of these three states are approximately 0 eV, 0.020 eV and 0.029 eV. Assuming that the different energies of these three states are due to the spin-orbit interaction, determine the value of K (including its sign) for this case.

[8]

(ii) The two lowest energy state of neutral boron are also separated in energy by the spin-orbit interaction. In this case the energies are 0 eV and 0.0019 eV. Determine K, making clear why the sign is opposite to that obtained in (c)(i). **[4]**

[Oxygen has atomic number $Z = 8$. Boron has atomic number $Z = 5$.]

[END OF SECTION B]

continued …

SECTION C

Answer ONE question from this section

13(a) $^{230}_{90}\mathrm{Th}$ is one of the seven naturally occurring isotopes of thorium which undergoes α decay.

(i) Identify the nucleus produced after α decay, including its atomic mass number, atomic number, and number of neutrons. **[1]**

(ii) Calculate the Q-value of the reaction to an accuracy of four significant figures (use the nuclear masses provided in the nuclear mass table). **[3]**

(iii) Estimate the Q-value of the reaction assuming a kinetic energy of ~5 MeV for the produced α particles, which was measured using an ion spectrometer with an experimental resolution of 1 MeV. Compare the two estimated values and comment on the potential discrepancy. Assume the initial $^{230}\mathrm{Th}$ nucleus to be at rest. **[4]**

(iv) Under the same assumption ($^{230}\mathrm{Th}$ at rest), calculate an accurate value (to four significant figures) of the kinetic energy of the α particle and the nucleus produced after the decay. **[4]**

(v) Assuming the initial nucleus to have a kinetic energy of 3.0 MeV, and the produced nucleus to be at rest, calculate the kinetic energy of the α particle. **[3]**

(b) Using your own words, briefly explain:

(i) Basic principles of the interaction of radiation (ions, electrons, and photons) with matter, also including a graph which shows how the flux of transmitted radiation (ions, electrons, and photons) depends on the thickness of a given material. **[9]**

(ii) Basic working principles of different gas-filled counters (ionization chamber, proportional counters, Geiger counters), also including a graph that shows how the signal amplitude depends on the voltage applied to the given detector. **[6]**

continued …

SECTION C

14(a) The following fusion reaction:

$$^{2}H + ^{2}H \rightarrow ^{3}He + n$$

is known as D-D fusion and releases a net energy of 3.3 MeV. If a mixture containing $8x10^{24}$ deuterium nuclei undergoes nuclear fusion, and the conversion efficiency into electricity is 50%, calculate:

(i) The total energy released before conversion into electricity and the total energy transferred to the electrical network. **[4]**

(ii) The average electrical power if all the initial mixture undergoes nuclear fusion in one day of continuous operation. **[2]**

(iii) The energy carried by the ^{3}He nucleus produced in a single reaction. **[5]**

(iv) The power output if only the kinetic energy of the ^{3}He nuclei would be converted into electricity with 100% efficiency in one day of continuous operation. **[2]**

(v) The amplitude of the Coulomb potential barrier for D-D fusion, given the proportionality constant $R_0 \approx 1.2$ fm. **[2]**

(b) Using your own words, briefly explain basic principles of:

(i) Rutherford backscattering analysis and C-14 dating techniques. **[9]**

(ii) Thermonuclear fusion in the Sun, including differences between p-p and CNO cycles. **[6]**

END OF EXAMINATION