QUEEN'S UNIVERSITY BELFAST

PHY2005

Any calculator, except one with pre-programmable memory, may be used in this examination.

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2005
Atomic and Nuclear Physics

Duration: 3 hours plus additional 1 hour for upload of work

Thursday 20th of May 2021
09:30 AM – 1:30 PM

Examiners: Prof S Matthews, Prof F. Peters
and the internal examiners
Dr S Sim (s.sim@qub.ac.uk)

Answer ALL questions in Section A for 4 marks each.
Answer ONE question from Section B for 30 marks.
Answer ONE question from Section C for 30 marks.

If you have any problems or queries, contact the School Office at mpts@qub.ac.uk or 028 9097 1907, and the module coordinator s.sim@qub.ac.uk

QUEEN'S UNIVERSITY BELFAST

SCHOOL OF MATHEMATICS AND PHYSICS

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}$ ms^{-1}
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1} $\approx 1.26 \times 10^{-6}$ Hm^{-1}
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}
Elementary charge	$e = 1.60 \times 10^{-19}$ C
Electron charge	$= -1.60 \times 10^{-19}$ C
Planck Constant	$h = 6.63 \times 10^{-34}$ Js
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}$ Js
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}$ m^{-1}
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}$ kg $1u = 931.5$ MeV
1 electron volt (eV)	$= 1.60 \times 10^{-19}$ J
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton (three sig. fig.)	$m_p = 1.67 \times 10^{-27}$ kg
Mass of neutron (three sig. fig.)	$m_n = 1.67 \times 10^{-27}$ kg
Molar gas constant	$R = 8.31$ JK^{-1}mol^{-1}
Boltzmann constant	$k = 1.38 \times 10^{-23}$ JK^{-1}
Avogadro constant	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}
Acceleration of free fall on the Earth's surface	$g = 9.81$ ms^{-2}

DATA FOR NUCLEAR MASSES

$m(n) = 1.008665$ u

$m(^{1}H) = 1.007825$ u

$m(^{2}H) = 2.014102$ u

$m(^{3}H) = 3.016049$ u

$m(^{3}He) = 3.016030$ u

$m(^{4}He) = 4.002603$ u

$m(^{15}N) = 15.000109$ u

$m(^{16}O) = 15.994915$ u

$m(^{238}U) = 238.050788$ u

$m(^{234}Th) = 234.043601$ u

SECTION A

Answer ALL 10 questions in this section
Full explanations of your answers are required to attain full marks

1 Estimate the photon frequency for the $n = 4$ to $n = 3$ transition of singly-ionized helium. **[4]**

2 The total angular momentum of a particular atomic state is characterized by the quantum numbers $J = 3$ and $M_J = -1$. Give the physical meaning of these two quantum numbers and draw an appropriate sketch to illustrate the total angular momentum vector. **[4]**

3 Explain why the ground-state term of helium is $^1S_0$. **[4]**

4 In the Hartree theory for multi-electron atoms, electrons are assumed to experience an effective potential energy, $U(r)$, which depends on their radial coordinate, r. Give expressions for $U(r)$ that would be appropriate for the ground configuration of twice ionized calcium (i.e. Ca^{2+}) for the following two limits:

(a) r is very small ($r \to 0$)

(b) r is large ($r \to \infty$). **[4]**

[Calcium has atomic number $Z = 20$.]

5 For each of the following transitions, indicate if the transition is allowed. If not allowed briefly explain why.

(a) 5g $^2G_{9/2} \to$ 5f $^2F_{7/2}$

(b) $1s^2 2p3d$ $^1P_1 \to 1s^2 2s2p$ $^1P_1$

(c) $1s^2 2s^2 2p^3 3d$ $^5D_3 \to 1s^2 2s^2 2p^4$ $^3P_2$ **[4]**

6 Estimate the energy (in MeV) necessary to separate a proton from ^{16}O. Clearly show all working for your answer. **[4]**

continued …

SECTION A

7 Consider the α-decay $^{238}U \rightarrow {}^{234}Th$. Calculate:

(a) The energy (in MeV) released in a single decay. **[2]**

(b) The fraction of the mass of a single ^{238}U nucleus converted to kinetic energy in the decay. **[2]**

Show your working for both calculations.

8 A 2 kg mixture of deuterium and tritium (50:50) undergoes nuclear fusion producing helium (Q-value = 2.8×10^{-12} J). Calculate the total energy released, and the average power output if all the initial mixture is used during a year of continuous and stable operation. **[4]**

9 A smoke alarm prototype is tested and shows a detector sensitivity (smallest current detectable) for α radiation of approximately 2 μA. Calculate the activity in Curie corresponding to a 2 μA current of α-particles, and explain why the result is unreasonable for a smoke alarm for home use.
[The activity of a typical Am-241 α-based smoke detector is ~ 1 μCi.] **[4]**

10 A scientist discovers a particle and measures its charge (zero) and mass (2.03 u). They conclude that the particle is made of two neutrons bound together. Calculate the binding energy of such particle and explain why the conclusion of the scientist is unreasonable. **[4]**

[END OF SECTION A]

continued …

SECTION B

Answer ONE question from this section

11(a) Briefly review the main successes and limitations of the Bohr model in accounting for atomic spectra. **[4]**

(b) In the Bohr model, the radius of orbit for an electron with principal quantum number n is

$$r_n = a_0 \frac{n^2}{Z} \ ,$$

where Z is the nuclear charge and $a_0 \approx 5.3 \times 10^{-11}$ m is the Bohr radius.

(i) Combine this expression for the radius with formulae for the Bohr-model orbital angular momentum to derive an expression for the orbital speed. Calculate this speed for $n = 2$ in hydrogen, giving the answer as a fraction of the speed of light. **[7]**

(ii) **Briefly** comment on the significance of your answer for the accuracy of the Bohr and Schrödinger models of single-electron atoms. **[2]**

(c) (i) Use the Aufbau principle to deduce the ground configuration of the potassium atom. **[4]**

(ii) The energy associated with the occupied 4s orbital in the ground configuration of potassium is -4.34 eV. Estimate the effective nuclear charge for this orbital and comment on the result with reference to the concept of *electron screening* in multi-electron atoms. **[5]**

(d) Consider each of the following electron configurations. For each case, discuss whether the binding energy of the 4d orbital will be larger or smaller than that of the 4s orbital considered in part (c). Include in your answer a list ranked by binding energy (from most tightly bound to least tightly bound) and justify your ranking.

(i) The $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4d$ configuration of potassium.

(ii) The $1s^2\,2s^2\,2p^6\,4d$ configuration of sodium.

(iii) The $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4d$ configuration of doubly-ionized scandium (Sc^{2+}). **[8]**

[Potassium, sodium and scandium have atomic numbers $Z = 19$, 11 and 21, respectively.]

continued …

SECTION B

12(a) Consider the electron configuration $1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p\, 4p$

(i) List all the allowed spectroscopic terms associated with this configuration. **[6]**

(ii) How many distinct quantum states does this configuration have with $M_J = -1$? **[3]**

(iii) In total, how many distinct quantum states does this configuration have? **[4]**

(b) Explain whether the configuration $1s^2\, 2s^2\, 2p^2$ has more, fewer, or the same number of quantum states as the configuration in part (a). **[3]**

(c) The spin-orbit interaction splits the $1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p\, 4s$ ^3P term of neutral silicon into three energy levels. To attain full marks you must fully explain your rationale for each of the sub-questions below.

(i) Give the J-values for the three energy levels. **[2]**

(ii) Will any of these three states have permitted transitions to states of the ground configuration of neutral silicon? **[2]**

(iii) The first of the three energy levels lies 4.920 eV above the ground state of neutral silicon. The third of the three (i.e., the highest energy) lies 4.954 eV above the ground state. Deduce the energy of the second energy level. **[7]**

(iv) Make a **very rough** estimate of the strength of an externally applied magnetic field that would be required to induce energy differences between states with different M_J values that are comparable to the spin-orbit energy splitting discussed in part (c)(iii). **[3]**

[Silicon has atomic number $Z = 14$.

The Bohr magneton is given by $\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$]

[END OF SECTION B]

continued …

SECTION C

Answer <u>ONE</u> question from this section

13(a) The α decay of $^{151}_{63}\mathrm{Eu}$ to the $^{147}_{61}\mathrm{Pm}$ ground state can be measured experimentally. Given that a total of 55 events are detected with an efficiency of 98.9% in a certain time, and that each event releases an energy of 1.95 MeV, calculate:

(i) the corresponding α-particle kinetic energy and the recoil energy of the daughter nucleus. State if the recoiling daughter nucleus is radioactive and provide a qualitative explanation for your answer; **[5]**

(ii) the number of decays of $^{151}_{63}\mathrm{Eu}$ that occurred in the sample, and the *half-life* of $^{151}_{63}\mathrm{Eu}$, given that the characteristic decay *lifetime* (required time for undecayed nuclei to decrease by a factor of *e*) is $T = 6.7 \times 10^{18}$ years. State the main assumption(s) you make in this estimation; **[4]**

(iii) Explain the main concepts of the quantum mechanical theory used by Gamow and Gurney to describe α emission. **[6]**

(b) Using your own words, briefly explain the basic principles of nuclear fission and nuclear fusion. Structure your answer as follows:

(i) Explain the physics of these processes and why they occur. **[5]**

(ii) Give an example of a fission reaction, and an example of a fusion reaction. In both cases comment on the stability of the products. **[5]**

(iii) Highlight fundamental differences between nuclear fission and fusion, including their current (or potential) use for energy production. **[5]**

continued …

SECTION C

14(a) Consider the following fusion reactions:

$$(1) \quad \mathrm{n} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + \gamma$$

$$(2) \quad \mathrm{n} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + \gamma$$

(i) Confirm that both reactions release energy (explain the reason) and calculate the energy of each fusion reaction. State which product nuclide is most tightly bound and explain why; **[4]**

(ii) Explain why it is difficult to capture and utilize the energy released, and briefly describe the main processes behind the interaction of electromagnetic radiation (high energy photons) with matter. **[6]**

(iii) Explain the general effect and role of the Coulomb barrier that is relevant to two particles (*a* and *X*) involved in nuclear fusion. **[5]**

(b) Using your own words, briefly explain the basic principles of α and β decays. Structure your answer as follows:

(i) Explain the physics of these processes and why they occur. **[5]**

(ii) Give an example of both an α and a β decay reaction and explain how their respective Q-values can be determined experimentally. **[5]**

(iii) Highlight fundamental differences between α and β decays, including energy distribution of the respective decay products. **[5]**

END OF EXAMINATION