Part 2: Optics

Dr. Fumin Huang

f.huang@qub.ac.uk

Course Contents

- Polarisation of light
  - Types of polarisation
  - Linear and circular polarisation conditions
  - Stokes polarisation parameters
    Jones Vector
  - Faraday effect
- Light reflection and refraction at a planar interface
  - Fresnel formulae
  - Brewster angle, critical angle
  - Total internal reflection
  - Phase changes during reflection
- Principles of lasers
  - Einstein coefficients
  - Population inversion
  - Three energy level and four energy level laser systems

Suggested Textbooks

1. Polarized light, 3rd Edition, Dennis H Goldstein.
2. Principles of Optics, Max Born & Emil Wolf
3. Lasers : theory and applications, K. Thyagarajan and A.K. Ghatak

Please submit your attendance at:

https://go.qub.ac.uk/mpattendance

Or Scan the QR code below with a mobile device by:
Open the camera app on your device
Point the camera towards the QR code
Login with your QUB credentials

If attending On Campus, please specify a location and room number

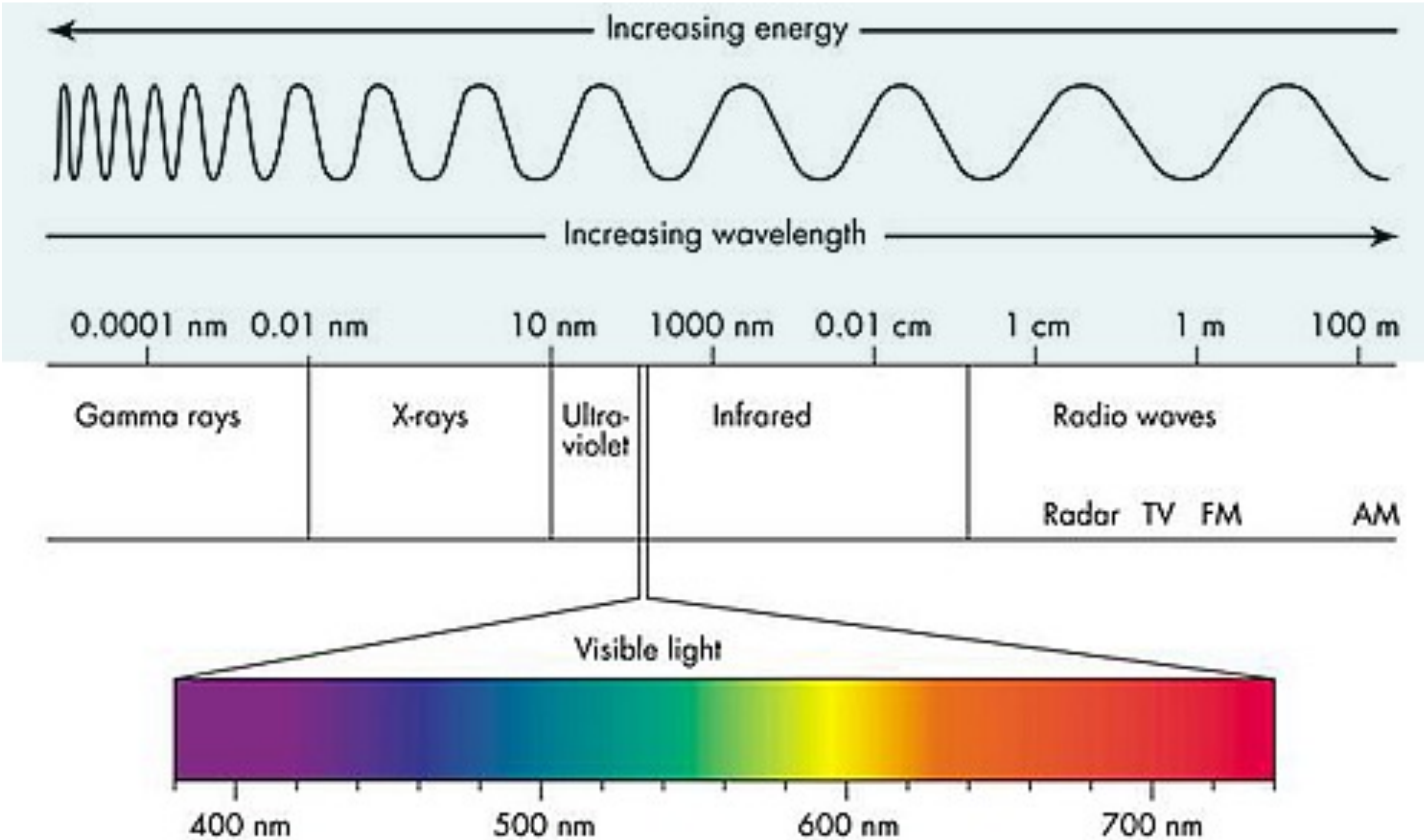
Part 2: Optics

Topic 1
Polarisation of Light

Electromagnetic Spectrum

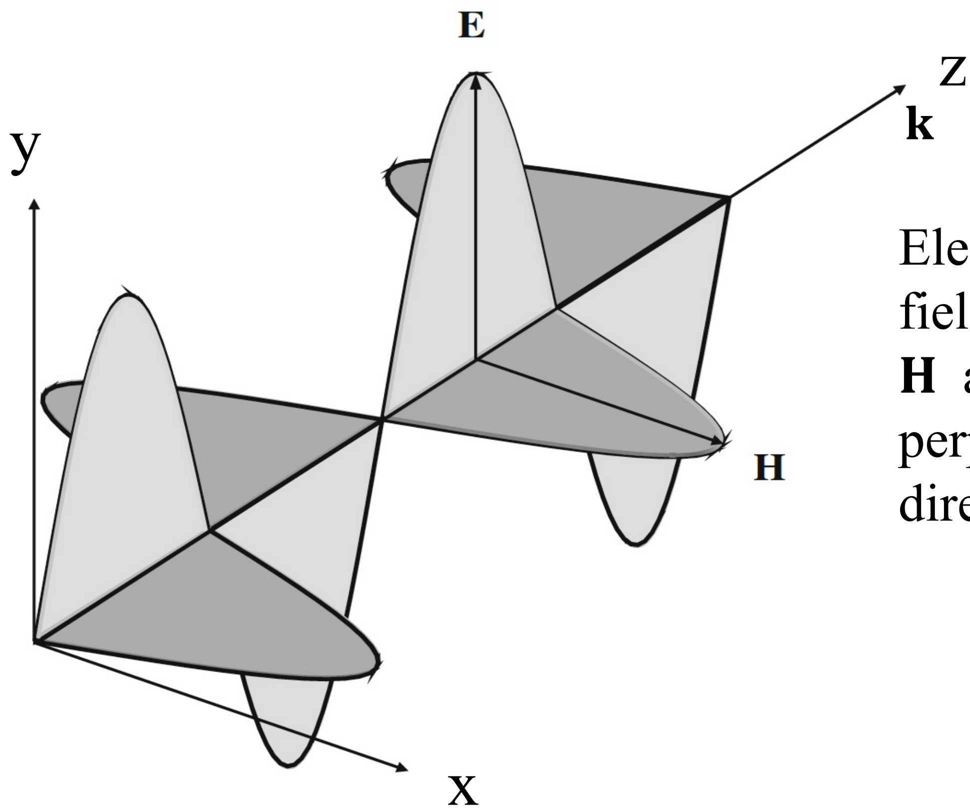
Light is an electromagnetic wave (λ~ 400-700 nm).

$$\text{Energy } E = h\upsilon = \frac{h}{T} = \frac{hc}{\lambda}$$



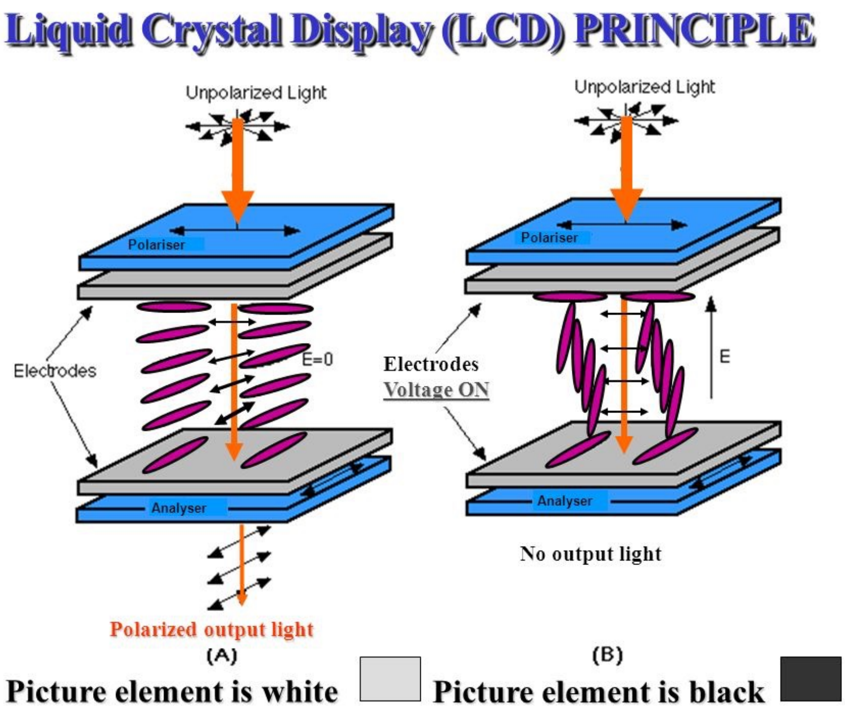
What is Polarisation?

<u>Polarisation</u>: a fundamental property of EM field, specifying the oscillating orientation of the **E-field** in space and how it evolves with time.



Electromagnetic wave is a **transverse** field: electric field **E**, magnetic field **H** are both oscillating in a plane perpendicular to the wave propagation direction (**k**).

Why Need to Know Polarisation?

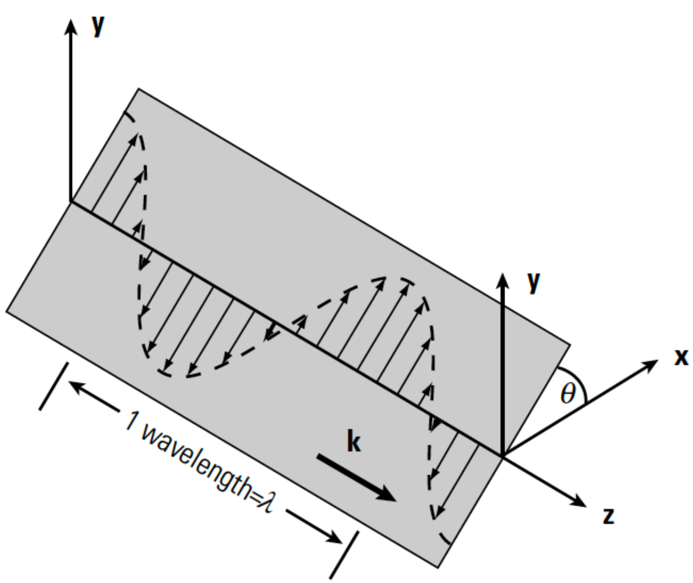


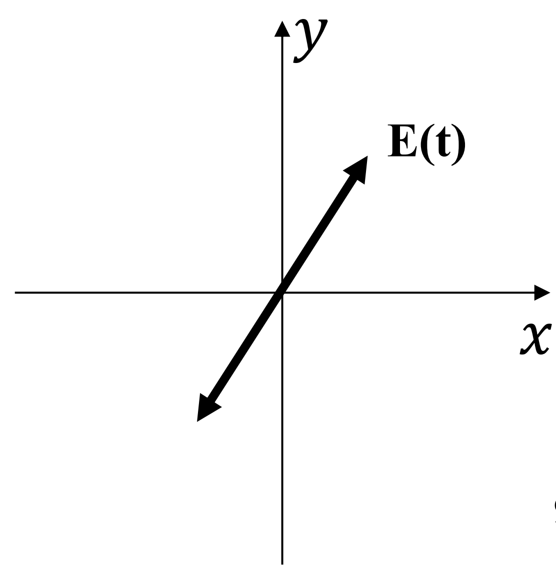
Types of Polarisation

Based on the trajectory of the tips of E-field, polarisation can be classified as:

1. Linear polarisation
2. Circular polarisation
   - Right-handed circular polarisation
   - Left-handed circular polarisation
3. Elliptic polarisation

<u>Linear polarisation</u>: E-field oscillates along a *fixed* orientation, which remains unchanged in *time* (could vary in space).





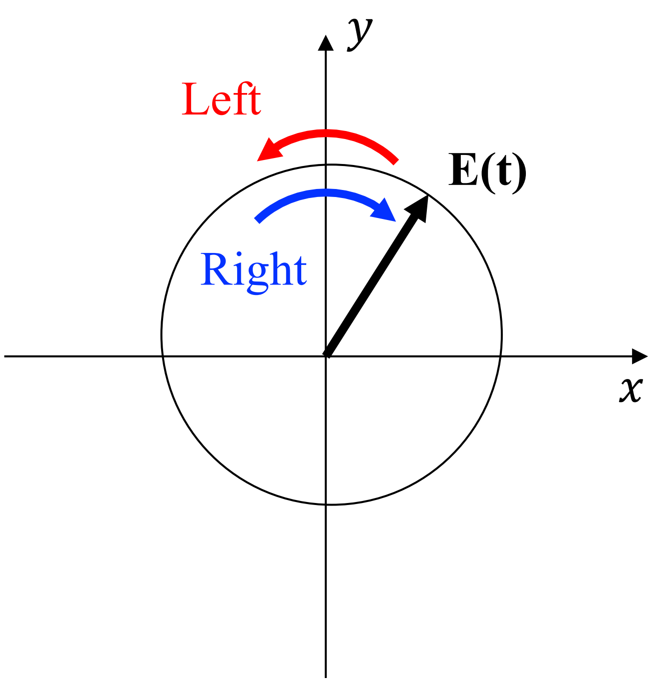
Circular Polarisation

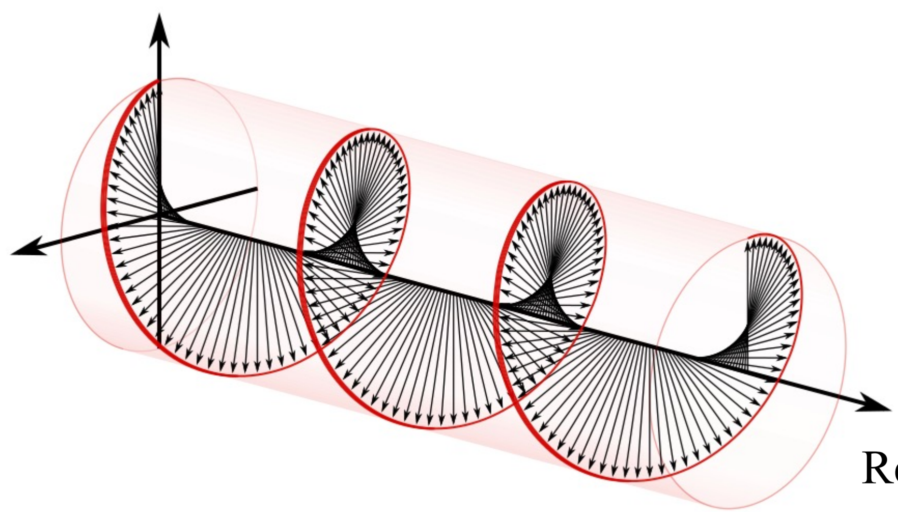
<u>Circular polarisation</u>:

- The *orientation* of E-field rotates circularly with *time*.
- The *magnitude* of the field remains constant.

Right/Left-handed circular light: E-vector rotates clockwise/anticlockwise with time (viewed by the source).

From the Receiver's view, the tip of the E-vector of right-handed circular polarisation forms a shape like a right-handed screw.

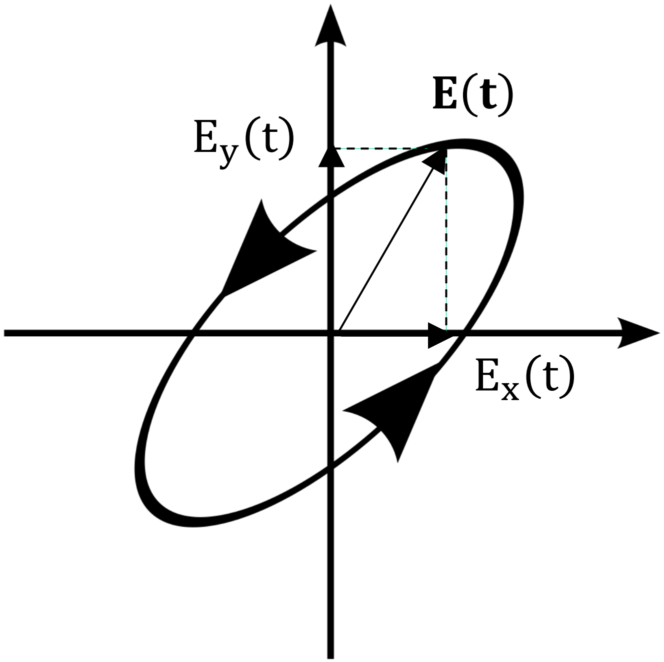




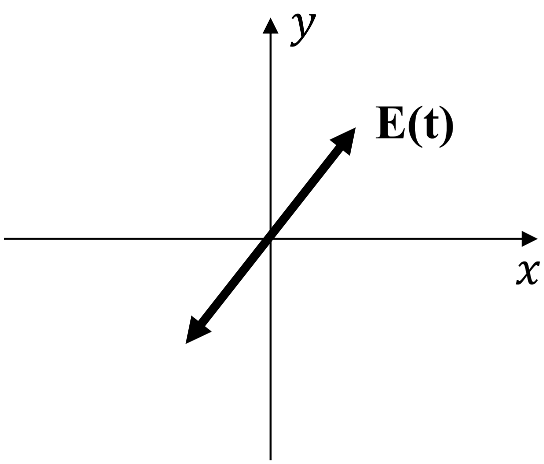
Right-handed circular polarisation

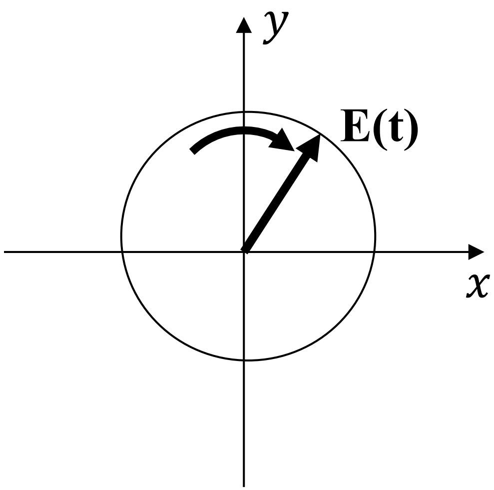
Elliptic Polarisation

<u>Elliptic polarisation</u>: in the general case, the tip of E-field moves along an ellipse.

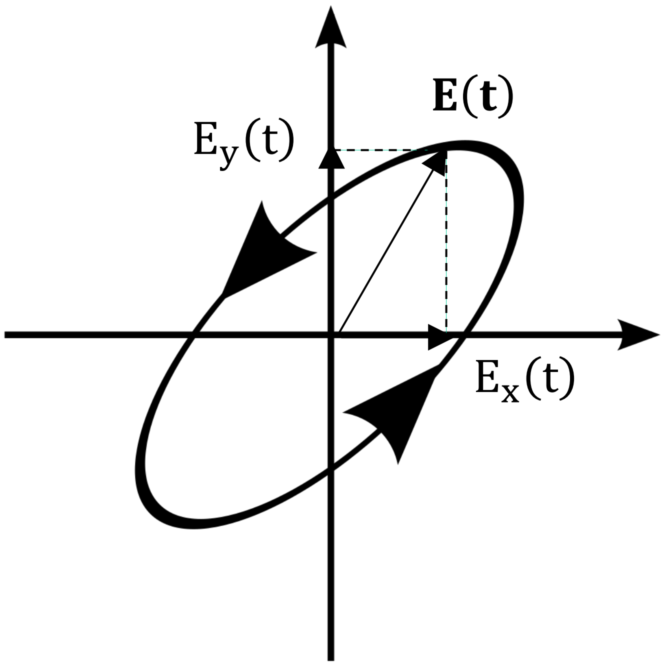


Linear polarisation and circular polarisation are the special cases of elliptic polarisation (as line and circle are special shapes of ellipse)





Polarisation Ellipse



$$\mathbf{E}(\mathbf{t}) = E_x(t)\hat{\mathbf{x}} + E_y(t)\hat{\mathbf{y}}$$

$$E_x(t) = E_{0x}\cos(\omega t - kz + \delta_x)$$

$$E_y(t) = \underbrace{E_{0y}}_{\text{magnitude}}\cos\left(\omega t - kz + \underbrace{\delta_y}_{\text{phase}}\right)$$

$\delta = \delta_y - \delta_x$ Phase difference

Ellipse Equation

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos\delta = \sin^2(\delta)$$

Linear Polarisation

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos\delta = \sin^2(\delta)$$

<u>What are the conditions of linear polarisation?</u>

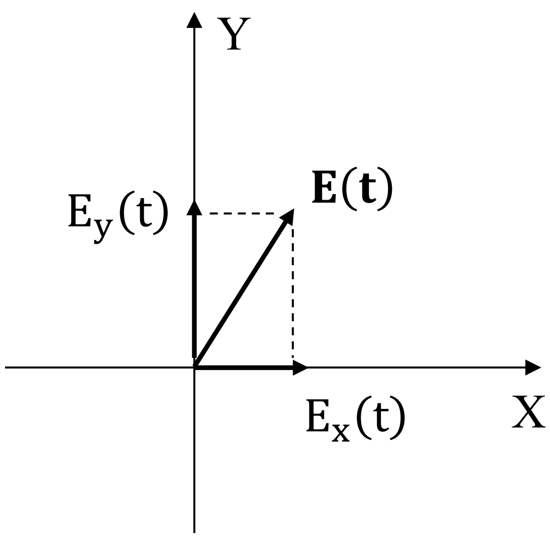
1. Ordinary cases: $E_{0y} = 0$, $\mathbf{E(t)} = E_x(t)\hat{\mathbf{y}}$, Linear horizontal polarisation

   or $E_{0x} = 0$, $\mathbf{E(t)} = E_y(t)\hat{\mathbf{y}}$, Linear vertical polarisation

2. $\frac{E_y}{E_x} = \text{const.}$ This can be achieved when $\delta = 0$ or π

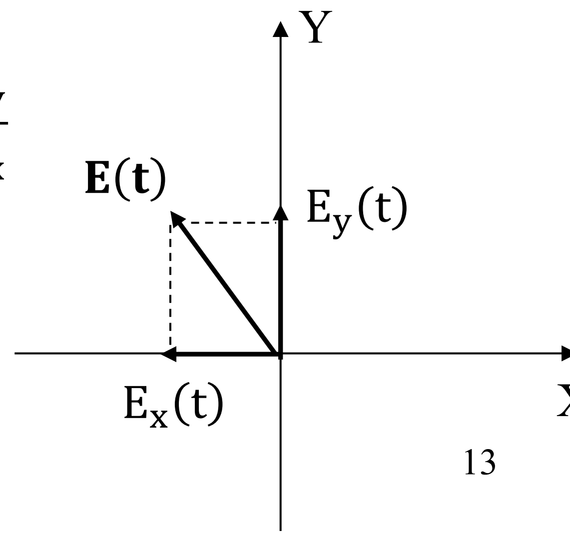
(a) $\delta = 0$

$$\frac{E_y}{E_x} = \frac{E_{0y}}{E_{0x}}$$



(b) $\delta = \pi$

$$\frac{E_y}{E_x} = -\frac{E_{0y}}{E_{0x}}$$



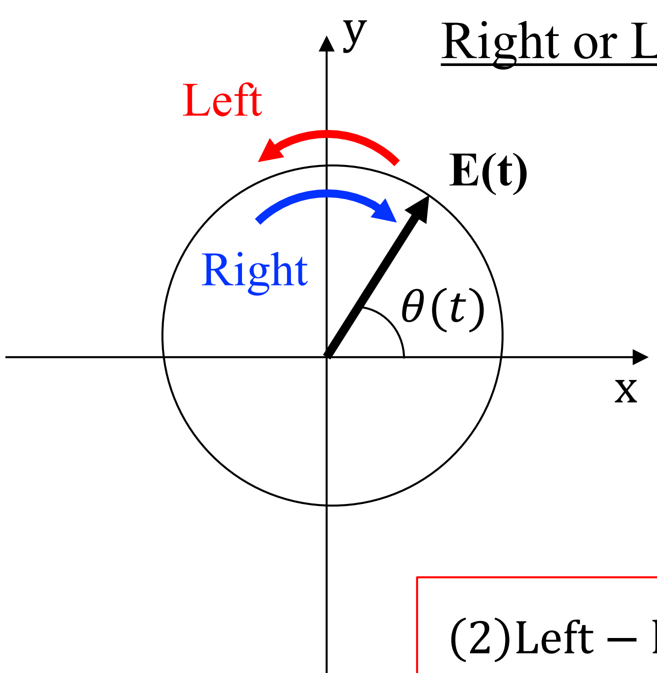
Circular Polarisation

Conditions for circular polarisation:

$$\delta = \pm\frac{\pi}{2} \quad \text{AND} \quad E_{0x} = E_{0y} = E_0$$

$$\frac{E_x^2}{E_0^2} + \frac{E_y^2}{E_0^2} = 1 \qquad \text{circle}$$

Right or Left?



(1) Case: $\delta = \delta_y - \delta_x = +\frac{\pi}{2}$

$$E_y(t) = E_0 \cos\left(\omega t - kz + \delta_x + \frac{\pi}{2}\right)$$
$$= E_0 \sin\left[\frac{\pi}{2} - \left(\omega t - kz + \delta_x + \frac{\pi}{2}\right)\right]$$
$$= E_0 \sin(kz - \delta_x - \omega t)$$

$$E_x(t) = E_0 \cos(\omega t - kz + \delta_x)$$
$$= E_0 \cos(kz - \delta_x - \omega t)$$

$$\boxed{\theta(t) = kz - \delta_x - \omega t}$$

$$E_x(t) = E_0 \cos\theta(t)$$

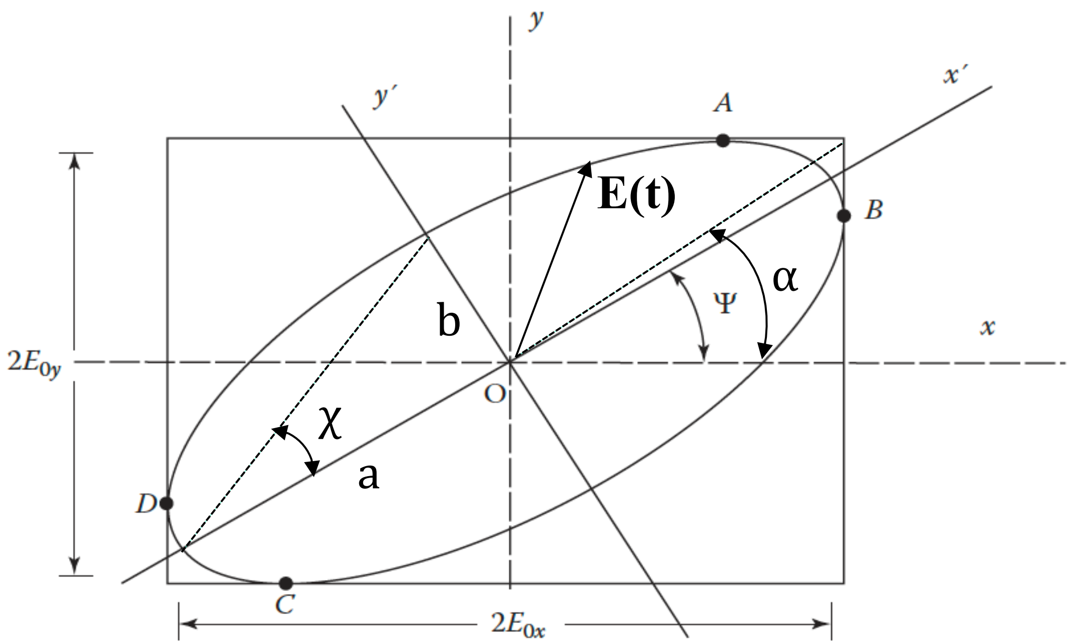
$$E_y(t) = E_0 \sin\theta(t)$$

$\theta(t)$ decreases with time, *clockwise rotation*:
right-handed circular polarisation

$$\boxed{(2)\text{Left} - \text{handed circular:}\ \delta = \delta_y - \delta_x = -\frac{\pi}{2}}$$

Polarisation Ellipse

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos\delta = \sin^2(\delta)$$



The ellipse can be described by the orientation Ψ and ellipticity χ angles.

$$0 \le \Psi < \pi, -\frac{\pi}{4} \le \chi \le \frac{\pi}{4}$$

$$\tan(2\Psi) = \tan(2\alpha)\cos\delta$$

$$\sin(2\chi) = \sin(2\alpha)\sin\delta$$

$$a^2 + b^2 = E_{0x}^2 + E_{0y}^2$$

$$\alpha = \tan^{-1}\left(\frac{E_{0y}}{E_{0x}}\right)$$

Ellipticity angle χ: $\tan\chi = \pm\frac{b}{a}$

- Line: $\chi=0$

(1) $\alpha = 0\ (E_{0y} = 0), \frac{\pi}{2}\ (E_{0x} = 0)$

or

(2) $\delta = 0, \pi$

- Circle: $\chi = \pm\frac{\pi}{4}$

(1) $\alpha = \frac{\pi}{4}\ (E_{0x} = E_{0y})$ **and**

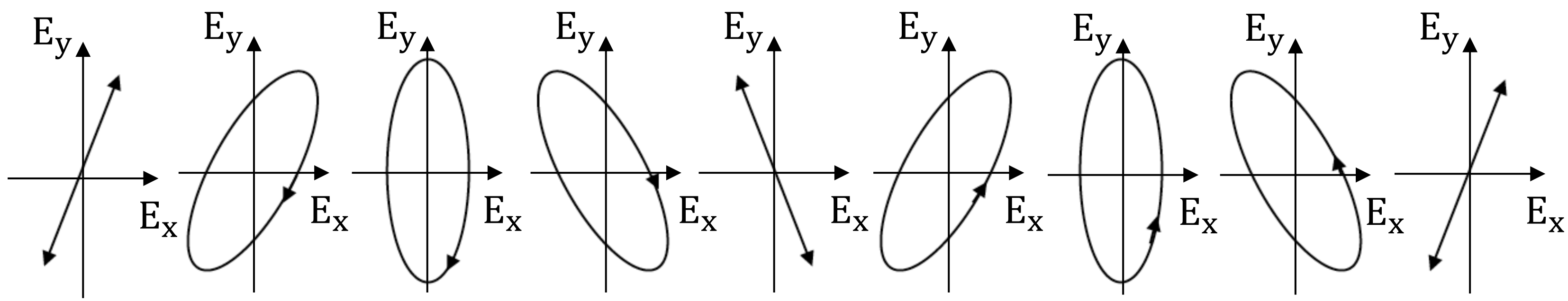
(2) $\delta = \pm\frac{\pi}{2}$

Polarisation Ellipse

- Linear polarisation
  1) $E_{0x} = 0$ or $E_{0y} = 0$, **or**
  2) $\delta = 0$ or π

- Circular polarisation
  1) $E_{0x} = E_{0y}$, **and**
  2) $\delta = \frac{\pi}{2}$ (right), or $\delta = -\frac{\pi}{2}$ (left),

- Other conditions: elliptic polarisation



$\delta = 0$ $\delta = \pi/4$ $\delta = \pi/2$ $\delta = 3\pi/4$ $\delta = \pi$ $\delta = 5\pi / 4$ $\delta = 3\pi/2$ $\delta = 7\pi / 4$ $\delta = 2\pi$

Examples of Questions

1. $E_x(t) = A\cos(kz - \omega t)$, $E_y(t) = A\sin(\omega t - kz + \frac{\pi}{4} + \delta)$. Describe the conditions of δ $(0 \leq \delta < 2\pi)$ for linear, right- and left-circular polarisation, respectively.

2. Determine the polarisation states of the following cases $(A > 0)$

(a) $E_x(t) = A\cos(\omega t - kz)$, $E_y(t) = 2A\cos(\omega t - kz)$

(b) $E_x(t) = A\sin(\omega t - kz)$, $E_y(t) = A\sin(\omega t - kz + \frac{\pi}{2})$

(c) $E_x(t) = A\cos(\omega t - kz)$, $E_y(t) = A\sin(\omega t - kz)$

(d) $E_x(t) = A\sin(\omega t - kz)$, $E_y(t) = A\cos(\omega t - kz)$

(e) $E_x(t) = A\cos(\omega t - kz)$, $E_y(t) = 2A\sin(\omega t - kz)$

Stokes Polarisation Parameters

How to measure and characterize the polarisation state of light?

In 1852, Lord George Stokes discovered that any polarisation state of light can be completely described by four ***measurable*** real quantities (S_0, S_1, S_2, S_3), which are called Stokes polarisation parameters.

$$S_0 = \langle E_x(t)E_x^*(t)\rangle + \langle E_y(t)E_y^*(t)\rangle$$

$$S_1 = \langle E_x(t)E_x^*(t)\rangle - \langle E_y(t)E_y^*(t)\rangle$$

$$S_2 = \langle E_x(t)E_y^*(t)\rangle + \langle E_y(t)E_x^*(t)\rangle$$

$$S_3 = i\left(\langle E_x(t)E_y^*(t)\rangle - \langle E_y(t)E_x^*(t)\rangle\right)$$

$\langle\quad\rangle$ means time average, $\langle E_i(t)E_j^*(t)\rangle = \frac{1}{T}\int_0^T E_i(t)E_j^*(t)dt$

T: Period of EM field; *: complex conjugate

Completely Polarised Light

For completely polarized light, the y- and x- component electric fields have a constant phase difference $\delta = \delta_y - \delta_x$, Stokes parameters are given by

$$S_0 = \langle E_x(t)E_x^*(t)\rangle + \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 + E_{0y}^2$$

$$S_1 = \langle E_x(t)E_x^*(t)\rangle - \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 - E_{0y}^2$$

$$S_2 = \left\langle E_x(t)E_y^*(t)\right\rangle + \left\langle E_y(t)E_x^*(t)\right\rangle = 2E_{0x}E_{0y}\cos\delta$$

$$S_3 = i\left(\left\langle E_x(t)E_y^*(t)\right\rangle - \left\langle E_y(t)E_x^*(t)\right\rangle\right) = 2E_{0x}E_{0y}\sin\delta$$

$$S_0^2 = S_1^2 + S_2^2 + S_3^2$$

Knowing the Stokes parameters, E_{0x}, E_{0y} and δ can be worked out, therefore the polarization state of light can be completely determined.

Generally, we can write $E_x(t)$ and $E_y(t)$ in complex term

$$E_x(t) = E_{0x}e^{i(\omega t - kz + \delta_x)}$$

$$E_y(t) = E_{0y}e^{i(\omega t - kz + \delta_y)} = E_{0y}e^{i(\omega t - kz + \delta_x + \delta)}$$

$$\langle E_x(t)E_x^*(t)\rangle = \frac{1}{T}\int_0^T E_x(t)E_x^*(t)dt = \frac{1}{T}\int_0^T E_{0x}e^{i(\omega t - kz + \delta_x)}E_{0x}e^{-i(\omega t - kz + \delta_x)}dt$$
$$= \frac{1}{T}\int_0^T E_{0x}^2 dt = E_{0x}^2$$

$$\left\langle E_y(t)E_y^*(t)\right\rangle = E_{0y}^2$$

$$S_0 = \langle E_x(t)E_x^*(t)\rangle + \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 + E_{0y}^2$$ Intensity

$$S_1 = \langle E_x(t)E_x^*(t)\rangle - \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 - E_{0y}^2$$

$$E_x(t) = E_{0x}e^{i(\omega t - kz + \delta_x)},\ E_y(t) = E_{0y}e^{i(\omega t - kz + \delta_x + \delta)}$$

$$S_2 = \langle E_x(t)E_y^*(t)\rangle + \langle E_y(t)E_x^*(t)\rangle$$

$$\langle E_x(t)E_y^*(t)\rangle = \frac{1}{T}\int_0^T E_{0x}e^{i(\omega t - kz + \delta_x)}E_{0y}e^{-i(\omega t - kz + \delta_x + \delta)}dt$$

$$= \frac{E_{0x}E_{0y}}{T}\int_0^T e^{-i\delta}dt = E_{0x}E_{0y}e^{-i\delta}$$

$$\langle E_y(t)E_x^*(t)\rangle = \frac{1}{T}\int_0^T E_{0y}e^{i(\omega t - kz + \delta_x + \delta)}E_{0x}e^{-i(\omega t - kz + \delta_x)}dt = E_{0x}E_{0y}e^{i\delta}$$

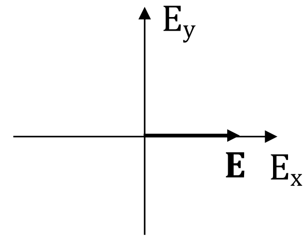
$$S_2 = \langle E_x(t)E_y^*(t)\rangle + \langle E_y(t)E_x^*(t)\rangle = E_{0x}E_{0y}\left(e^{-i\delta} + e^{i\delta}\right) = 2E_{0x}E_{0y}\cos\delta$$

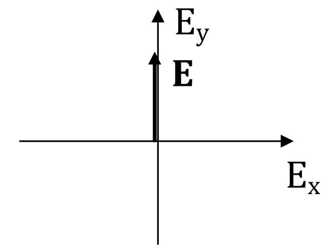
Similarly, $S_3 = i\left(\langle E_x(t)E_y^*(t)\rangle - \langle E_y(t)E_x^*(t)\rangle\right) = 2E_{0x}E_{0y}\sin\delta$

Stokes Parameters

1. Linear horizontal polarisation (LHP): $E_{0y} = 0$

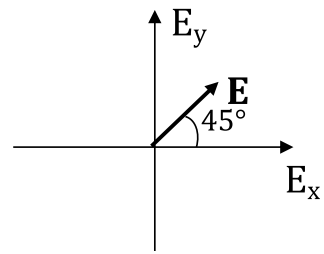
$$S_0 = S_1 = E_{0x}^2,\ S_2 = S_3 = 0$$

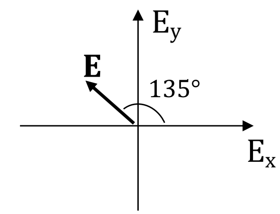




2. Linear vertical polarisation (LVP): $E_{0x} = 0$

$$S_0 = S_1 = E_{0y}^2,\ S_2 = S_3 = 0$$



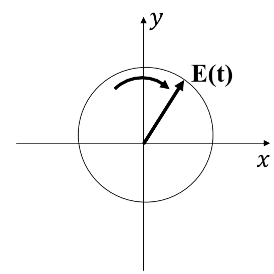


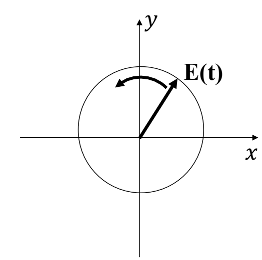
3. Linear +45° polarisation: $E_{0x} = E_{0y} = E_0, \delta=0$

$$S_0 = 2E_0^2, S_1 = 0,\ S_2 = 2E_0^2,\ S_3 = 0$$

4. Linear -45° polarisation: $E_{0x} = E_{0y} = E_0, \delta=\pi$

$$S_0 = 2E_0^2, S_1 = 0,\ S_2 = -2E_0^2,\ S_3 = 0$$





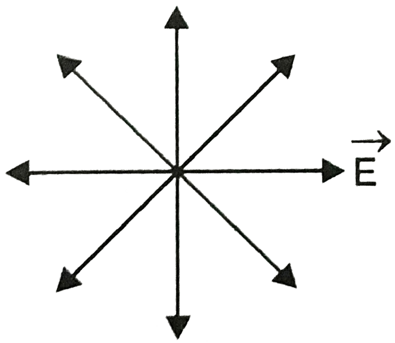
5. Right-handed circular polarisation (RCP): $E_{0x} = E_{0y} = E_0, \delta = \frac{\pi}{2}$

$$S_0 = 2E_0^2, S_1 = 0,\ S_2 = 0,\ S_3 = 2E_0^2$$

6. Left-handed circular polarisation (LCP): $E_{0x} = E_{0y} = E_0, \delta = -\frac{\pi}{2}$

$$S_0 = 2E_0^2, S_1 = 0,\ S_2 = 0,\ S_3 = -2E_0^2$$

Unpolarised Light



$$E_x(t) = E_0 e^{i[\omega t - kz + \delta_x(t)]}$$

$$E_y(t) = E_0 e^{i[\omega t - kz + \delta_y(t)]}$$

Random phases
$\delta_x(t)$ and $\delta_y(t)$ are unrelated

$$\langle E_x(t)E_x^*(t)\rangle = E_0^2 \qquad \left\langle E_y(t)E_y^*(t)\right\rangle = E_0^2$$

$$S_0 = \langle E_x(t)E_x^*(t)\rangle + \left\langle E_y(t)E_y^*(t)\right\rangle = 2E_0^2$$

$$S_1 = \langle E_x(t)E_x^*(t)\rangle - \left\langle E_y(t)E_y^*(t)\right\rangle = 0$$

As $\delta(t)$ is a random function of t, so

$$\int_0^T e^{-i\delta(t)}dt = 0$$

$$\left\langle E_x(t)E_y^*(t)\right\rangle = \frac{1}{T}\int_0^T E_0 e^{i(\omega t - kz + \delta_x(t))} E_0 e^{-i(\omega t - kz + \delta_y(t))} dt = \frac{E_0^2}{T}\int_0^T e^{-i\delta(t)}dt = 0$$

Similarly, $\left\langle E_y(t)E_x^*(t)\right\rangle = 0$, therefore $S_2 = S_3 = 0$

Polarised and Unpolarized Light

$S_0 = E_{0x}^2 + E_{0y}^2 = |E|^2$ represents the total intensity of light

1) For completely polarized light: $S_0^2 = S_1^2 + S_2^2 + S_3^2$

2) For unpolarized light: $S_0 > 0, S_1 = S_2 = S_3 = 0$

In general: $S_0^2 \geq S_1^2 + S_2^2 + S_3^2$

'=' condition is matched for completely polarized light

Intensity of completely polarized light: $I_p = \sqrt{S_1^2 + S_2^2 + S_3^2}$

Intensity of unpolarized light: $I_u = S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2}$

$$I_{tot} = S_0 = I_p + I_u$$

Stokes parameters can be used to separate the unpolarized and polarized parts of light

Degree of Polarisation

Degree of polarisation: the ratio of polarized light to the total intensity

$$P = \frac{I_p}{I_{tot}} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \quad (0 \le P \le 1)$$

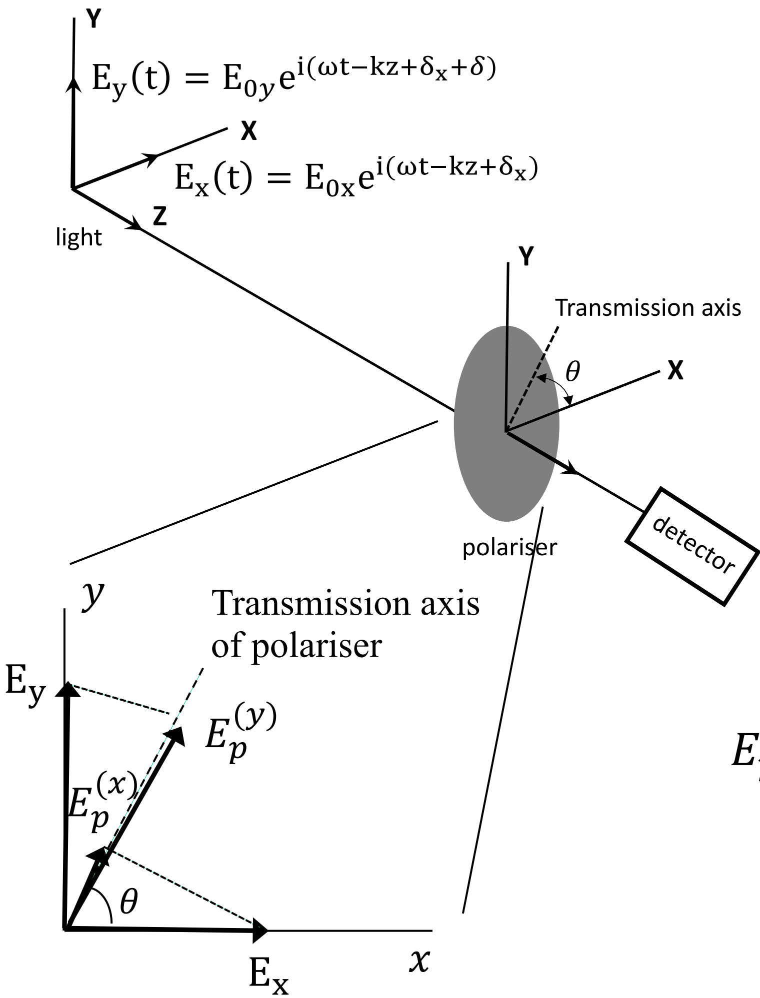
$P = 1$ Completely polarised light

$P = 0$ Unpolarised light

$0 < P < 1$ Partially polarised light

Measurement of Stokes Parameters

S_0, S_1 and S_2 can be measured with a polarizer and a detector.



Assuming the polarizer has an angle θ with regard to the x-axis.

The transmitted field from $E_x(t)$ is

$$E_p^{(x)} = E_x(t)\cos\theta$$

The transmitted field from $E_y(t)$ is

$$E_p^{(y)} = E_y(t)\sin\theta$$

The total transmitted field is

$$E_p(\theta) = E_p^{(x)} + E_p^{(y)} = E_x(t)\cos\theta + E_y(t)\sin\theta$$

The intensity measured by detector is

$$I_D(\theta) = \langle E_p(\theta)E_p^*(\theta)\rangle$$

Measurement of Stokes Parameters

The measured intensity is $I_D(\theta) =< E_p(\theta)E_p^*(\theta) >$

$$E_p(\theta) = E_p^{(x)} + E_p^{(y)} = \mathrm{E_x(t)} \cos\theta + \mathrm{E_y(t)} \sin\theta$$

$$I_D(\theta) = \left\langle (\mathrm{E_x} \cos\theta + \mathrm{E_y} \sin\theta)(E_x^* \cos\theta + E_y^* \sin\theta) \right\rangle$$

$$= \left\langle \mathrm{E_x} E_x^* \cos^2\theta + \mathrm{E_x} E_y^* \sin\theta\cos\theta + \mathrm{E_y} E_x^* \sin\theta\cos\theta + \mathrm{E_y} E_y^* \sin^2\theta \right\rangle \qquad \text{(eq.1)}$$

(1) $\theta = 0°, \quad I_D(0°) = \langle \mathrm{E_x} E_x^* \rangle = \mathrm{E_{0x}^2}$

$\mathrm{S_0 = I_D(0°) + I_D(90°)}$

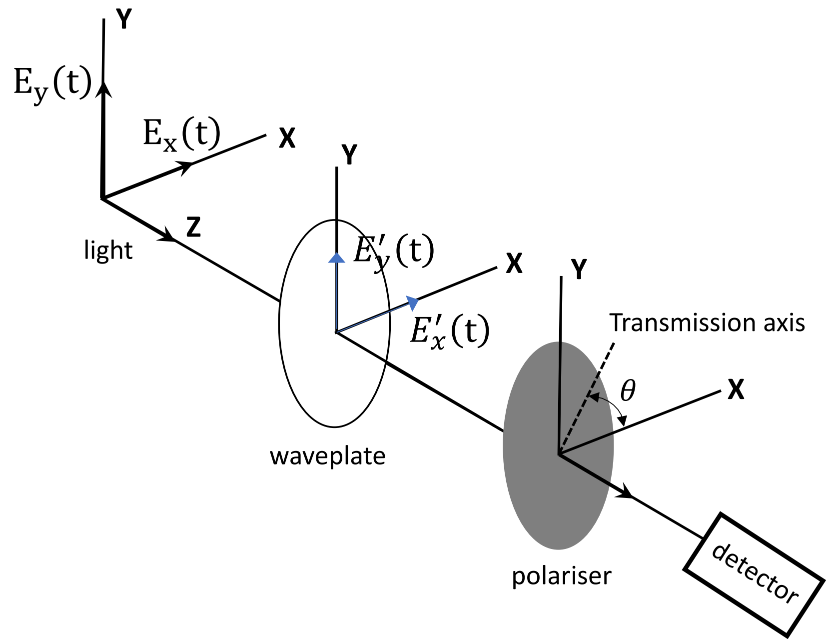
(2) $\theta = 90°, I_D(90°) = \left\langle \mathrm{E_y} E_y^* \right\rangle = \mathrm{E}_{0y}^2$

$\mathrm{S_1 = I_D(0°) - I_D(90°)}$

(3) $\theta = 45°, I_D(45°) = \frac{1}{2}\left\langle \mathrm{E_x} E_x^* + \mathrm{E_x} E_y^* + \mathrm{E_y} E_x^* + \mathrm{E_y} E_y^* \right\rangle = \frac{1}{2}(\mathrm{S_0 + S_2})$

$$\mathrm{S_2 = 2I_D(45°) - S_0 = 2I_D(45°) - I_D(0°) - I_D(90°)}$$

Measurement of S_3



S_3 can be measured with the combination of a waveplate and a polarizer.
A waveplate introduces a phase difference of ϕ between the x- and y-components of E-field, i.e., after the waveplate,

$$E'_x(\mathrm{t}) = \mathrm{E_x}(\mathrm{t})e^{i\phi},\ E'_y(\mathrm{t}) = \mathrm{E_y}(\mathrm{t})$$

The intensity measured by the detector is (this can be obtained by replacing the $\mathrm{E_x}$ and $\mathrm{E_y}$ in Eq.1 in slide 27 with $E'_x = \mathrm{E_x}e^{i\phi}$ and $E'_y = \mathrm{E_y}$)

$$I_D(\theta,\phi) = \left\langle E'_x E'^*_x \cos^2\theta + E'_x E'^*_y \sin\theta\cos\theta + E'_y E'^*_x \sin\theta\cos\theta + E'_y E'^*_y \sin^2\theta \right\rangle$$

$$= \langle E'_x E'^*_x \rangle \cos^2\theta + \left\langle E'_x E'^*_y + E'_y E'^*_x \right\rangle \sin\theta\cos\theta + \left\langle E'_y E'^*_y \right\rangle \sin^2\theta$$

Measurement of S_3

$$I_D(\theta,\phi) = \langle E_x' E_x'^* \rangle \cos^2\theta + \left\langle E_x' E_y'^* + E_y' E_x'^* \right\rangle \sin\theta\cos\theta + \left\langle E_y' E_y'^* \right\rangle \sin^2\theta$$

$$\langle E_x' E_x'^* \rangle = \mathrm{E}_{0\mathrm{x}}^2 \qquad \left\langle E_y' E_y'^* \right\rangle = \mathrm{E}_{0y}^2$$

$$\left\langle E_x' E_y'^* + E_y' E_x'^* \right\rangle = \left\langle \mathrm{E_x} e^{i\phi} E_y^* + \mathrm{E_y} E_x^* e^{-i\phi} \right\rangle$$

$$e^{i\phi} = \cos\phi + i\sin\phi \qquad e^{-i\phi} = \cos\phi - i\sin\phi$$

$$\left\langle E_x' E_y'^* + E_y' E_x'^* \right\rangle = \left\langle \mathrm{E_x} E_y^* (\cos\phi + i\sin\phi) + \mathrm{E_y} E_x^* (\cos\phi - i\sin\phi) \right\rangle$$

$$= \left\langle \mathrm{E_x} E_y^* + \mathrm{E_y} E_x^* \right\rangle \cos\phi + i\left\langle \mathrm{E_x} E_y^* - \mathrm{E_y} E_x^* \right\rangle \sin\phi$$

$$= S_2 \cos\phi + S_3 \sin\phi$$

$$I_D(\theta,\phi) = \mathrm{E}_{0\mathrm{x}}^2 \cos^2\theta + S_2\cos\phi\sin\theta\cos\theta + S_3\sin\phi\sin\theta\cos\theta + \mathrm{E}_{0y}^2\sin^2\theta$$

$$I_D(45°, 90°) = \frac{1}{2}(\mathrm{E}_{0\mathrm{x}}^2 + S_3 + \mathrm{E}_{0y}^2) = \frac{1}{2}(S_0 + S_3)$$

$$S_3 = 2I_D(45°, 90°) - S_0$$

Determine the polarisation states from Stokes parameters

1. Intensity of unpolarized light: $I_u = S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2}$

   Intensity of polarized light: $I_p = \sqrt{S_1^2 + S_2^2 + S_3^2}$

   Degree of polarisation: $P = \dfrac{I_p}{I_{tot}} = \dfrac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$

2. For the completely polarised component

$$I_p = \sqrt{S_1^2 + S_2^2 + S_3^2} = E_{0x}^2 + E_{0y}^2$$

$$S_1 = E_{0x}^2 - E_{0y}^2$$

$$S_2 = 2E_{0x}E_{0y}\cos\delta$$

$$S_3 = 2E_{0x}E_{0y}\sin\delta$$

$$E_{0x} = \sqrt{\frac{\sqrt{S_1^2 + S_2^2 + S_3^2} + S_1}{2}}$$

$$E_{0y} = \sqrt{\frac{\sqrt{S_1^2 + S_2^2 + S_3^2} - S_1}{2}}$$

$$\tan\delta = \frac{S_3}{S_2}$$

E_{0x}, E_{0y} and δ can be completely determined, hence the polarisation ellipse.

Waveplate

A e
a
re

- Cry
refr
orie
fast
call
(n_e)

<u>Pha</u>

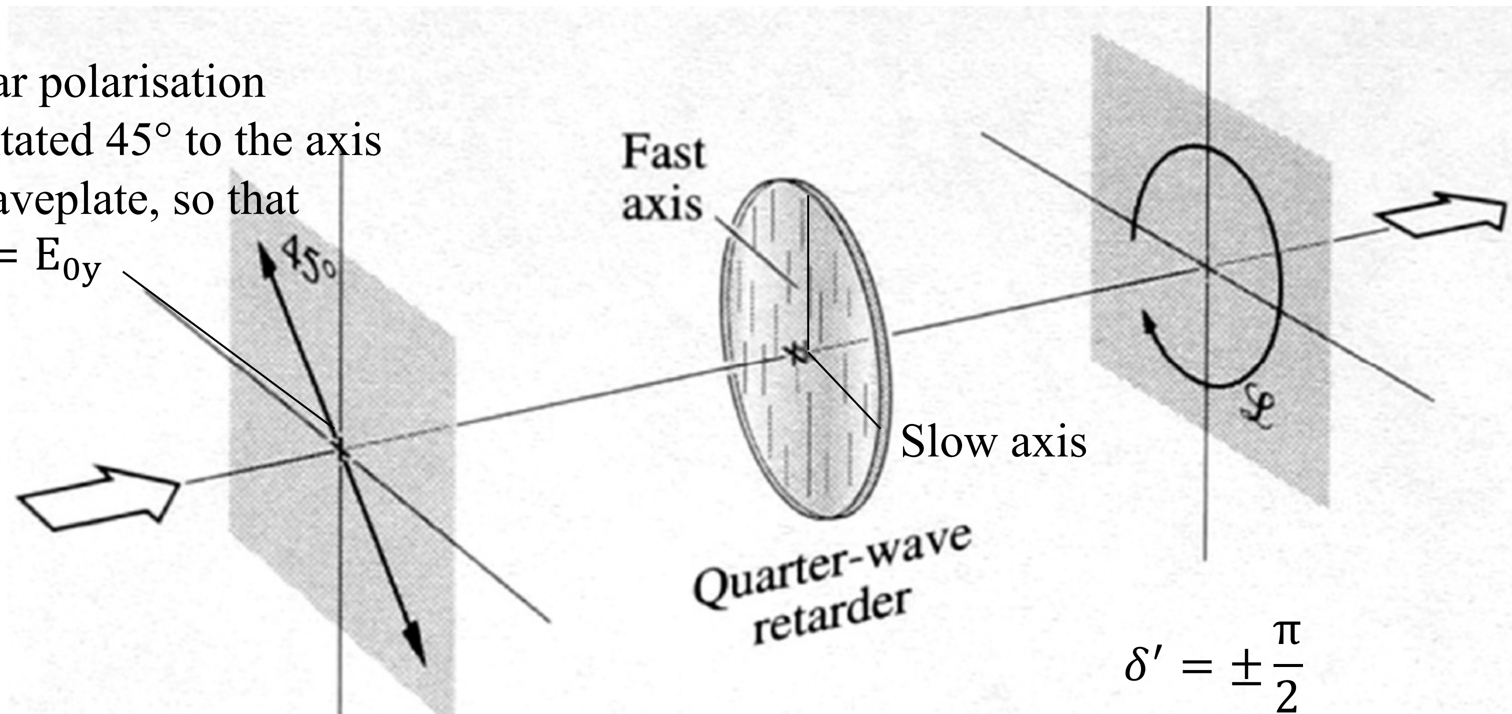
Qu

Ha

Polarisation Transformation

Linear polarization can be transformed to circular polarization by passing through a quarter waveplate which introduces a $\pm\frac{\pi}{2}$ phase change between the x- and y-component fields. Vice versa, circular polarization light can be converted to linear polarization by passing through a quarter waveplate.

Linear polarisation orientated 45° to the axis of waveplate, so that $E_{0x} = E_{0y}$



$\delta = 0, \pi$

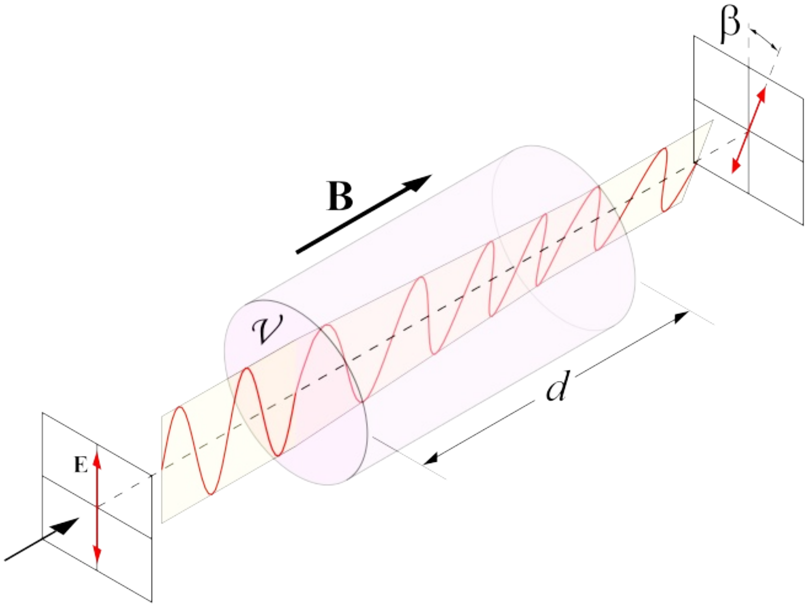
$\Delta\delta = \pm\frac{\pi}{2}$

$\delta' = \pm\frac{\pi}{2}$

$E_{0x} = E_{0y}$

The Faraday Effect

- The **Faraday effect** or **Faraday rotation** is a magneto-optical phenomenon discovered by Faraday in 1845.
- A linearly-polarised plane wave will rotate its plane-of-polarisation after passing through a piece of material, when a magnetic field along the light propagation direction is applied .
- The rotation is **linearly proportional** to the magnetic field and the thickness of the material.



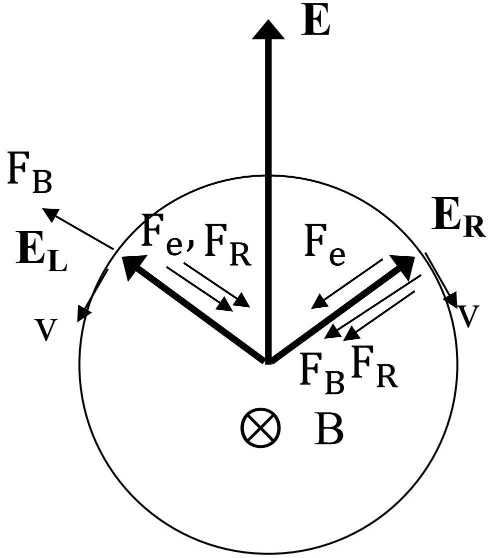
<u>Rotated polarisation angle</u>

$$\theta = VBd$$

V: Verdet constant

Magneto-Optic Effect

A linear polarisation can be decomposed into equal proportion of right- and left-circular polarisation



Assuming the forces pointing towards the circle centre is positive

Forces experienced by electrons

(1) E-field force $F_e = -e(-E) = eE$

(2) Restoring force $F_R = kr = m\omega_0^2 r$

(3) Lorentz force $\mathbf{F_B} = -e\mathbf{v}\times\mathbf{B}$

$$F_B^{R,L} = \pm evB = \pm eB\omega r$$

$$eE_{R,L} \pm e\omega Br + m\omega_0^2 r = m\omega^2 r$$

$$r_{R,L} = \frac{eE_{R,L}/m}{\omega^2 - \omega_0^2 \mp e\omega B/m}$$

With applied magnetic field, electrons move on circles of different radius for right and left circular polarisations

Circular Birefringence

Total induced dipole moment per unit volume

Number of electrons per unit volume

$$P = Ner = \frac{Ne^2 E_{R,L}/m}{\omega^2 - \omega_0^2 \mp e\omega B/m}$$

Relative permittivity

$$\varepsilon_r^{R,L} = 1 + \frac{P}{\varepsilon_0 E_{R,L}} \quad = 1 + \frac{\frac{Ne^2}{m\varepsilon_0}}{\omega^2 - \omega_0^2 \mp e\omega B/m}$$

$$= 1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 \mp e\omega B/m}$$

$$\omega_p = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$ Plasma frequency

Refractive index

$$n = \sqrt{\varepsilon_r}$$

$$n_R = \sqrt{1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 - e\omega B/m}}$$

$$n_L = \sqrt{1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 + e\omega B/m}}$$

$$n_R \neq n_L$$

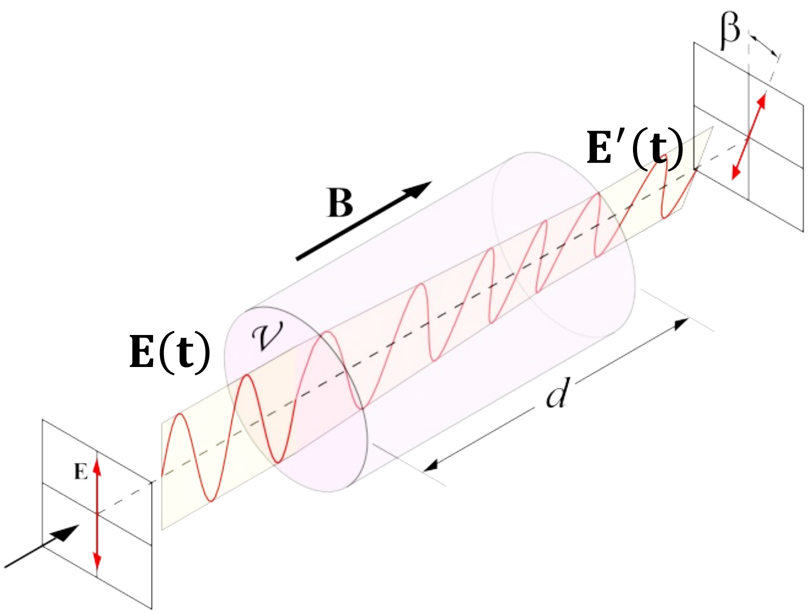
The refractive indices of right and left-circular polarisation are different, which is called **circular birefringence**.

Polarisation Rotation

$$\mathbf{E}_{\mathbf{R}}'(\mathbf{t}) = \frac{1}{2}\mathrm{E}_0\left\{\cos(\omega t - k_R d)\,\hat{\mathbf{x}} + \cos\left(\omega t - k_R d + \frac{\pi}{2}\right)\hat{\mathbf{y}}\right\}$$

$$= \frac{1}{2}\mathrm{E}_0\left\{\cos(\omega t - n_R k_0 d)\,\hat{\mathbf{x}} + \cos\left(\omega t - n_R k_0 d + \frac{\pi}{2}\right)\hat{\mathbf{y}}\right\}$$

$$\mathbf{E}_{\mathbf{L}}'(\mathbf{t}) = \frac{1}{2}\mathrm{E}_0\left\{\cos(\omega t - n_L k_0 d)\,\hat{\mathbf{x}} + \cos\left(\omega t - n_L k_0 d - \frac{\pi}{2}\right)\hat{\mathbf{y}}\right\}$$



$$\mathbf{E}'(\mathbf{t}) = \mathbf{E}_{\mathbf{R}}'(\mathbf{t}) + \mathbf{E}_{\mathbf{L}}'(\mathbf{t})$$

$$= \mathrm{E}_0\cos\left(\omega t - \frac{k_R + k_L}{2}d\right)\left\{\cos\left(\frac{n_R - n_L}{2}k_0 d\right)\hat{\mathbf{x}} + \sin\left(\frac{n_R - n_L}{2}k_0 d\right)\hat{\mathbf{y}}\right\}$$

$$\theta = \frac{n_R - n_L}{2}k_0 d = VBd$$

$$V = \frac{n_R - n_L}{2B}k_0 = \frac{\pi(n_R - n_L)}{B\lambda}$$

Verdet Constant

$$V = \frac{n_R - n_L}{2B} k_0$$

$$n_{R,L}(\omega) = \sqrt{1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 \mp e\omega B/m}}$$

$$\approx \sqrt{1 + \frac{\omega_p^2}{\left(\omega \mp \frac{eB}{2m}\right)^2 - \omega_0^2}} = n(\omega \mp \Delta\omega)$$

$$\Delta\omega = \frac{eB}{2m} \qquad n(\omega) = \sqrt{1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2}}$$

$$n(\omega \mp \Delta\omega) \approx n(\omega) \mp \Delta\omega \frac{dn}{d\omega}$$

$$n_R - n_L = -2\Delta\omega \frac{dn}{d\omega} = -\frac{eB}{m}\frac{dn}{d\omega}$$

$$V = \frac{\pi(n_R - n_L)}{B\lambda} = -\frac{e\pi}{m\lambda}\frac{dn}{d\omega}$$

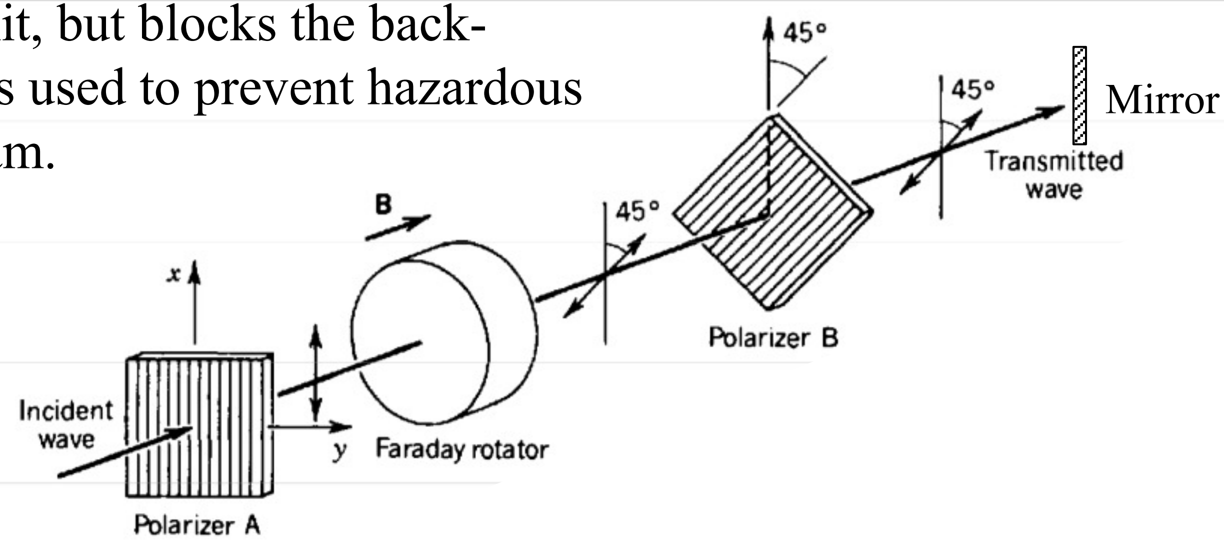
$$\omega = \frac{2\pi c}{\lambda}$$

$$\frac{dn}{d\omega} = \frac{\frac{dn}{d\lambda}}{\frac{d\omega}{d\lambda}} = -\frac{\lambda^2}{2\pi c}\frac{dn}{d\lambda}$$

$$V = \frac{e\lambda}{2mc}\frac{dn}{d\lambda}$$

Optical Isolator

Optical isolator is an optical element which allows light transmit, but blocks the back-reflection light. It is used to prevent hazardous back-reflection beam.



- Two polarisers are orientated 45° to each other. A magnetic field is applied to rotate the polarisation of linearly-polarized light by 45° so that light can completely transmit through the second polariser.
- When the reflected light passes through the Faraday rotator, its polarisation is rotated by another 45° so that it is 90° to the axis of the first polariser, hence is blocked.