Lecture 1:
introduction

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Course content

- Fundamental laws of electromagnetisms and their in-depth mathematical structure

- Electrostatics and magnetostatics and time-varying fields.

- Maxwell's equations and their applications

- Electromagnetic waves and their general properties in vacuum and in a medium

- Elements on non-linear optics.

- Temporally and spatially coherent light: lasers

Why electromagnetism?

The theory of electromagnetism represents one of the major successes in physics, since, by using only a limited set of equations, it explains:

EVERYTHING

(*except gravity and nuclear forces*)

It is by far the strongest force acting on our spatial and temporal typical scales.

Seemingly disparate phenomena such as friction, sight, lightning, and magnetism are effectively aspects of the same fundamental force.

A couple of words on Mathematics...

We will focus our attention on **classical electromagnetism**

- We will neglect quantum effects

- We will not dwell on relativistic effects

We will assume that forces are represented by **fields**, which, in turn, can be obtained from **potentials** (either scalar or vectorial).

DEF A **field** is a physical quantity that is defined, continuously, everywhere in space.

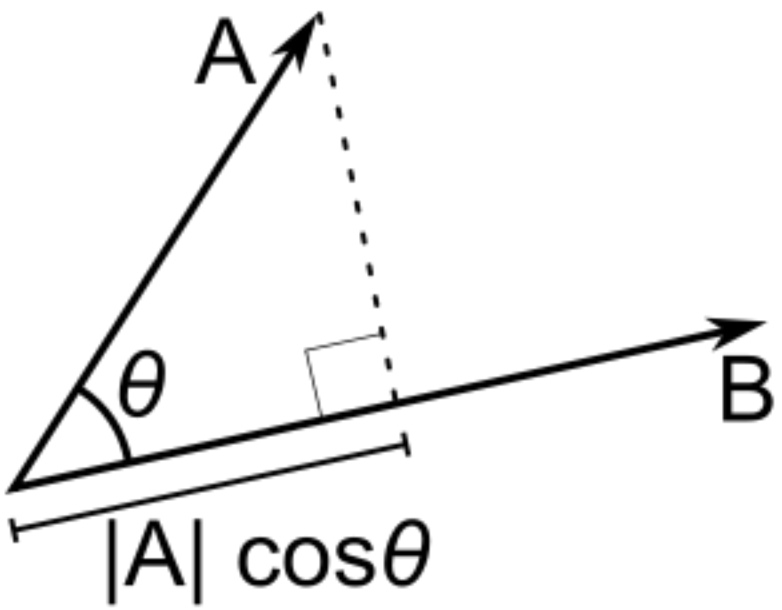
DEF A **conservative field** is a field that can be obtained as the gradient of a scalar function (the potential).

REMINDER: Scalar Product

An operation that, given two vectors, returns a **scalar.**

Operation that is defined in any dimensions.

$$\underline{A} \,.\, \underline{B} = \text{scalar} = A_x B_x + A_y B_y + A_z B_z == |A||B| \mathrm{Cos}\theta$$



REMINDER: Vector Product

An operation that, given two vectors, returns another vector perpendicular to the two.

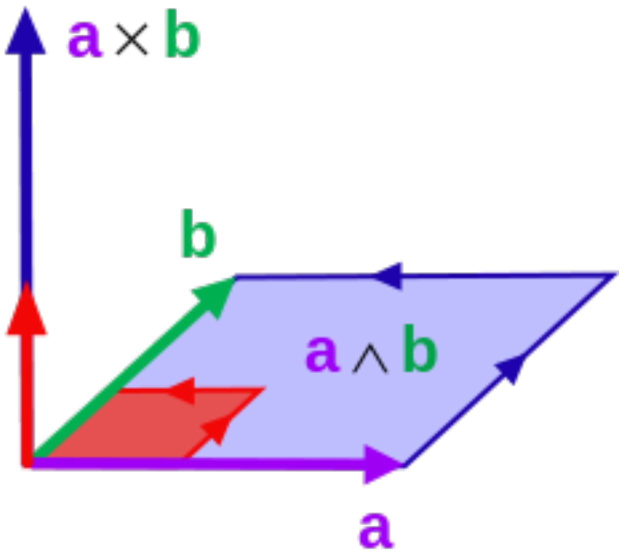
Operation that is **only** defined in three-dimensional spaces.

$$\underline{A} \times \underline{B} = \underline{C}$$

$$C_x = A_y B_z - A_z B_y$$

$$C_y = A_z B_x - A_x B_z$$

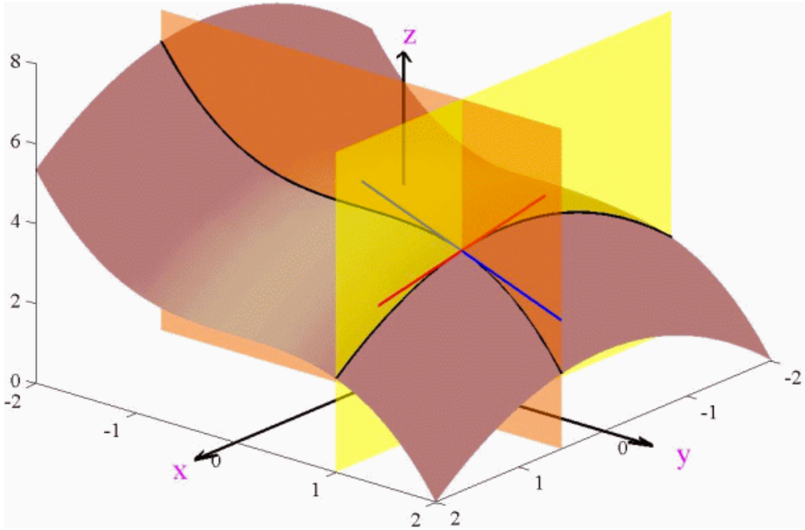
$$C_z = A_x B_y - A_y B_x$$



$$\underline{A} \times \underline{A} = 0 \qquad \underline{A} \cdot \left(\underline{A} \times \underline{B}\right) = 0$$

REMINDER: vectorial differentiation

If a scalar quantity varies in space, it might do so differently depending on the chosen direction. (see this video)



To take this into account, it is useful to define a pseudo-vector, whose components are partial derivatives (called the **del** operator):

$$\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

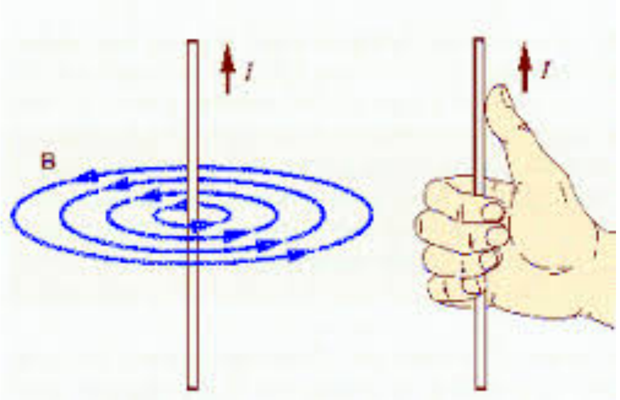
The "variation" of a scalar quantity (V) in space is then defined as its **gradient**: $\nabla V(x,y,z) \equiv \hat{x}\frac{\partial V(x,y,z)}{\partial x} + \hat{y}\frac{\partial V(x,y,z)}{\partial y} + \hat{z}\frac{\partial V(x,y,z)}{\partial z}$

REMINDER: vectorial differentiation

It is possible that a vector might vary in space. In this case, it is useful to define the curl product as the vector product between the del operator and the vector itself.

$$\nabla \times \mathrm{A} \equiv \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

In practice, it represents, how a vector “rotates” in space. A typical example of it is the magnetic field generated by a straight current:



The current tells us what is the axis around which the field rotates:

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

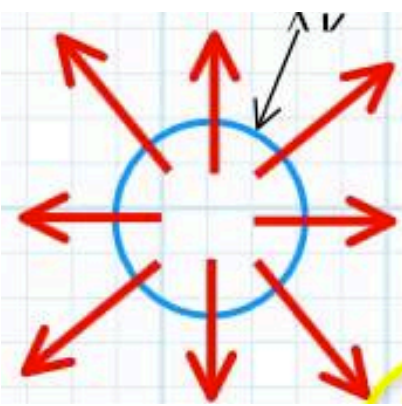
REMINDER: vectorial differentiation

Another useful operation is the **divergence** of a vector, i.e., the scalar product between the del operator and a vector.

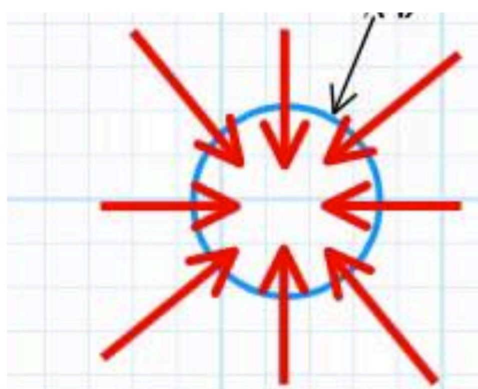
$$\nabla \cdot \vec{A} \equiv \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \text{(this is now a scalar!)}$$

Intuitively, the divergence tells us how a vector escapes an enclosed surface. A positive divergence implies vectors pointing outward, whereas a negative divergence implies vectors pointing inward.

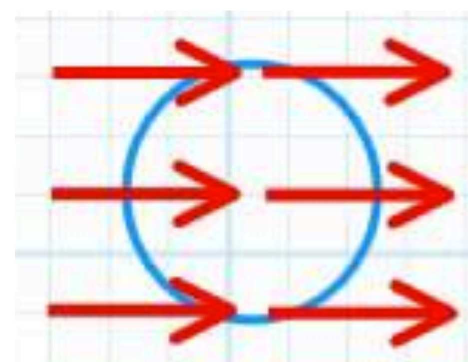
div A > 0



div A < 0



div A = 0



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The final goal of this module

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} \longrightarrow$$ Source of magnetic fields

$$\nabla \cdot \underline{D} = \rho \longrightarrow$$ Source of electrostatic fields

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \longrightarrow$$ Induction (relation between fields)

$$\nabla \cdot \underline{B} = 0 \longrightarrow$$ Topology of magnetic fields

$$\underline{D} = \varepsilon \underline{E} = \varepsilon_0 \underline{E} + \underline{P}$$

$$\underline{B} = \mu \underline{H} = \mu_0 \underline{H} + \mu_0 \underline{M}$$

$\longrightarrow$ Constitutive relations

(macroscopic properties of media)

$$\frac{d\underline{p}}{dt} = q\left(\underline{E} + \underline{v} \times \underline{B}\right) \longrightarrow$$ Lorentz force

(how charges react to fields)

Homework

Demonstrate that:

$$\nabla\left(2x+3y^2-\sin z\right)=\left(2,6y,-\cos z\right)$$

$$\nabla\cdot\left(xy,y^2z,\sin z\right)=y+2yz+\cos z$$

$$\nabla\times\left(xy,y^2z,\sin z\right)=\left(-y^2,0,-x\right)\qquad \textit{prove that}\ \nabla\cdot\left(\nabla\times\underline{A}\right)=0$$