Lecture 2:
Fundamental principles of electrostatics

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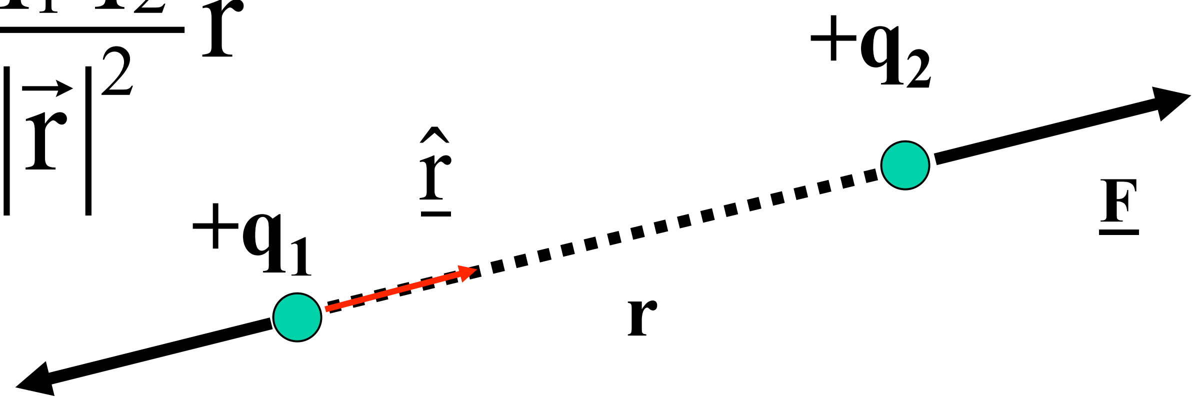
The Coulomb force

- It is an **empirical** fact that two charged objects at rest experience a mutual force of the kind:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$



- Where: F is the experienced force (in Newton)
  q_1 and q_2 are the charges (in Coulomb)
  r is the distance between the two charges
  ε_0 is a constant (8.85×10^{-12} Fm^{-1})

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Strength and range of the electrostatic force

- How "strong" is the electromagnetic force?

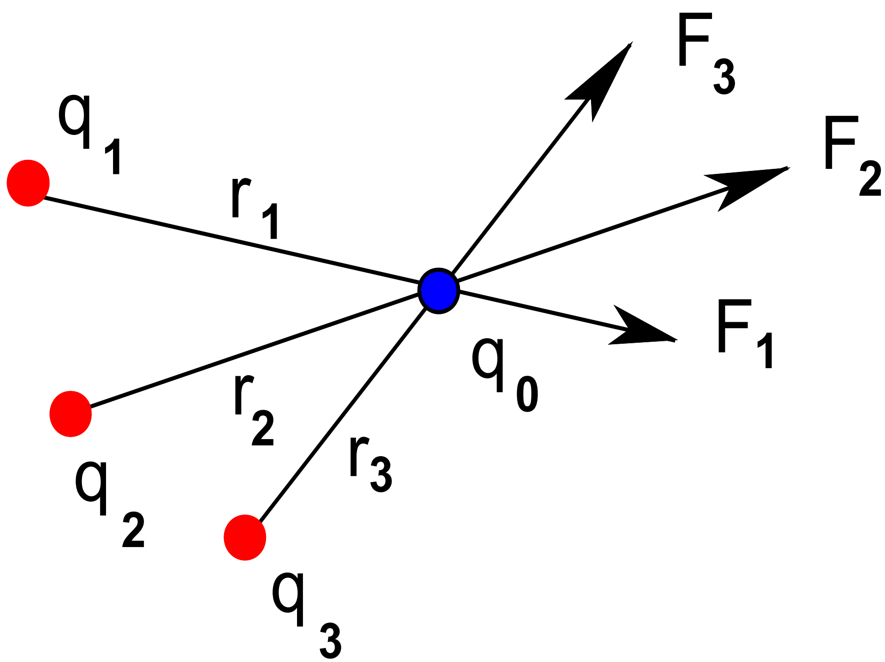
Interaction	Current theory	Field particle	Charge	Particle mass	Strength	Range
Strong	Quantum Chromodynamics	gluons	colour	0	10^2	10^{-15} m
Weak	Electroweak theory	$W^{\pm}$, Z^0	/	80.4, 91.2 GeV/c^2	10^{-11}	10^{-18} m
Electromagnetic	Quantum Electrodynamics	photons	el. charge	0	1	∞
Gravitation	General Relativity	(graviton)	mass	(0)	10^{-36}	∞

- If we exclude the strong force (effective only over extremely short distances), the electromagnetic force is the strongest force in Nature.

- It has infinite range and it is 36 orders of magnitude stronger than gravity!

The electrostatic force is linear

- If we have more than one charge involved, the resulting force is simply the sum of the forces exerted by every single charge:



$$\underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_0 q_i}{r_i^2} \hat{\underline{r}}_i$$

- This is sometimes called the **superposition principle**

The concept of electric field

- The Coulomb force depends on both the external charge and the one experiencing it. It thus appears as if a charge placed in space creates a "disturbance" that is felt by any charge nearby.

- This observation justifies the definition of **electric field:**

$$\underline{\mathbf{E}} = \left(\frac{\underline{\mathbf{F}}}{\mathbf{q}} \right)_{\mathbf{as\ q \to 0}}$$

it is, technically, the force felt by a test particle of infinitesimally small charge.

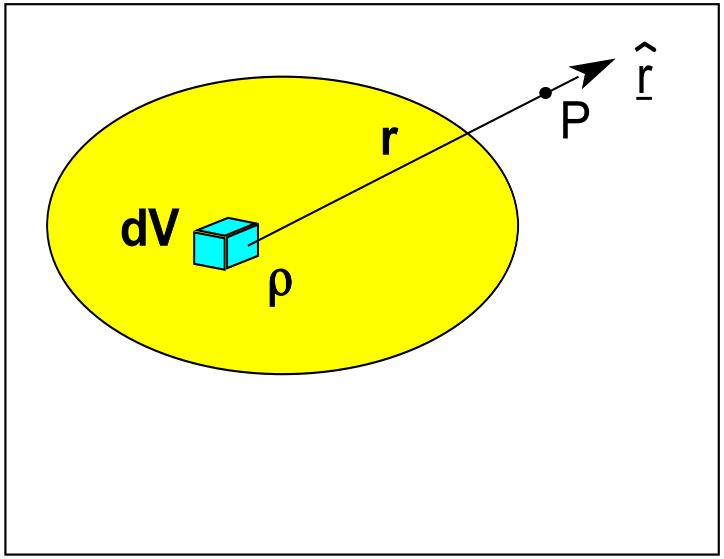
- The electric field generated by a point charge at rest (q) is then:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{|\vec{r}\,|^2} \hat{r}$$

Charge distribution

- The superposition principle allows to extract a simple formula to calculate the electric field generated by a **distribution of charges**

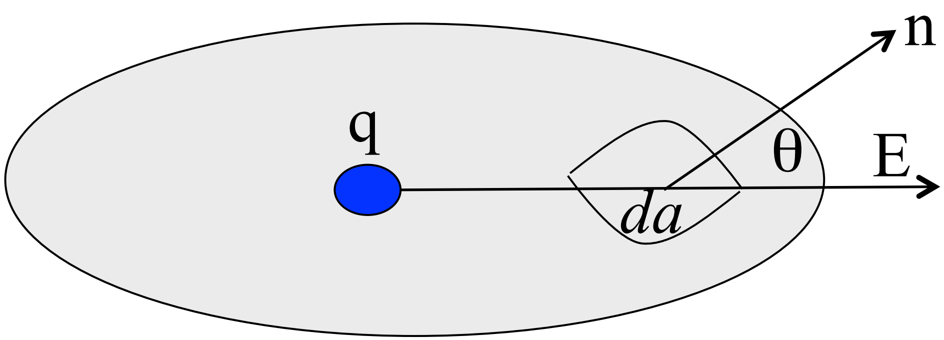
- Volumetric charge density: $\rho = Q / Volume \quad \rightarrow \quad \rho = dQ / dV$
- Areal charge density: $\sigma = Q / Area \quad \rightarrow \quad \sigma = dQ / dA$
- Linear charge density: $\lambda = Q / Length \quad \rightarrow \quad \lambda = dQ / dL$



$$\underline{E}_p = \frac{1}{4\pi\varepsilon_0} \int_{\text{all space}} \frac{\rho dV}{r^2} \hat{r}$$

Gauss' law

- Is there a simpler way to express the electrostatic field?
  Let us assume a charge enclosed in a surface.



a is an infinitesimal portion of the surface
E is the electric field generated by q
n is the vector normal to *a*

Normal component of E times *da* = $\vec{E}\cdot\vec{n}\ da = \dfrac{q}{4\pi\varepsilon_0}\dfrac{\cos\theta}{r^2}da$

However, $\cos\theta\ da = r^2 d\Omega$ ($d\Omega$ is the solid angle under the surface *da*)

If we take that into account and integrate over the whole surface we get:

$$\oint_S \vec{E}\cdot\vec{n}\ da = \begin{cases} q/\varepsilon_0 & \text{if q lies inside S} \\ 0 & \text{if q lies outside S} \end{cases} \rightarrow \oint_S \vec{E}\cdot\vec{n}\ da = \frac{1}{\varepsilon_0}\int_V \rho\, dV$$

Gauss' law in differential form

- Let's recall the divergence theorem: $\oint_S \vec{A}\cdot\vec{n}\ da = \int_V \nabla\cdot\vec{A}\,dV$
- This theorem is **general,** valid for any A, S, and V.
- We can then write the previous equation as:

$$\int_V \nabla\cdot\vec{E}\ dV = \frac{1}{\varepsilon_0}\int_V \rho\ dV$$

- The two integrals must be equal, regardless of the choice of V, implying that the two integrands are exactly the same:

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$ **First Maxwell's equation**

Homework

Charges $+q$, $+2q$ and $-q$ are placed at three corners of a square of side a as shown. What is the magnitude and direction of the electric field at the forth?