

Section A, B and C Answer Books.

Any calculator, except one with pre-programmable memory, may be used in this examination.

Level 2 Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2003 Astrophysics I

Friday, 9th August 2019 9:30 AM - 11:30 AM

Examiners: Prof P Browning
Dr P van der Burgt
and the Internal Examiners

Answer ALL FOUR QUESTIONS in Section A for 10 marks each.

Answer ONE QUESTION in Section B for 30 marks.

Answer ONE QUESTION in Section C for 30 marks.

Use a separate answer book for each Section You have TWO HOURS to complete this paper.

THE QUEEN'S UNIVERSITY OF BELFAST School of Mathematics & Physics

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^8 \text{ ms}^{-1}$
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Permeability of a vacuum
$$\mu_0 = 4\pi \times 10^{-7}~\mathrm{Hm^{-1}}$$

$$\approx 1.26 \times 10^{-6}~\mathrm{Hm^{-1}}$$

Permittivity of a vacuum
$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$$

Elementary charge
$$e = 1.60 \times 10^{-19} \text{ C}$$

Electron charge
$$= -1.60 \times 10^{-19} \text{ C}$$

Planck Constant
$$h = 6.63 \times 10^{-34} \text{ Js}$$

Reduced Planck Constant
$$\hbar = 1.05 \times 10^{-34} \text{ Js}$$

Rydberg Constant for hydrogen
$$R_{\infty} = 1.097 \times 10^7 \text{ m}^{-1}$$

Unified atomic mass unit
$$1u = 1.66 \times 10^{-27} \text{ kg}$$

$$1u = 931 \text{ MeV}$$

1 electron volt (eV)
$$= 1.60 \times 10^{-19} \text{ J}$$

Mass of electron
$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

Mass of proton
$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

Mass of neutron
$$m_n = 1.67 \times 10^{-27} \text{ kg}$$

Molar gas constant
$$R = 8.31 \text{ JK}^{-1} \text{ mol}^{-1}$$

Boltzmann constant
$$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Avogadro constant
$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

Gravitational constant
$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$$

Acceleration of free fall on the Earth's surface $g = 9.81 \text{ ms}^{-2}$

Level 2 Astronomy Data Sheet for PHY2003

Astronomical Constants

Mass of Sun $M_{Sun}=1.99\times 10^{30} {
m kg}$ Radius of Sun $R_{Sun}=6.96\times 10^8 {
m m}$ Luminosity of Sun $L_{Sun}=3.90\times 10^{26} {
m W}$

Effective temperature of Sun $T_{Sun} = 5770 \text{K}$

Mass of Earth $M_{\oplus} = 5.98 \times 10^{24} \mathrm{kg}$ Radius of Earth $R_{\oplus} = 6.38 \times 10^{6} \mathrm{m}$ Astronomical Unit $1 \mathrm{AU} = 1.496 \times 10^{11} \mathrm{m}$ Parsec $1 \mathrm{pc} = 3.08 \times 10^{16} \mathrm{m}$ Megaparsec $1 \mathrm{Mpc} = 10^{6} \mathrm{pc}$

Hubble Constant $H_0 = 71 \text{ km/sec/Mpc}$

Solar System Data

Planet	a (AU)	е	i (°)	M/M _{Earth}	R/R _{Earth}	Albedo
Mercury	0.387	0.206	7.00	0.056	0.38	0.06
Venus	0.723	0.007	3.39	0.815	0.95	0.76
Earth	1.000	0.017	0.00	1.000	1.00	0.40
Mars	1.524	0.093	1.85	0.107	0.53	0.16
Jupiter	5.203	0.048	1.31	318	11.19	0.51
Saturn	9.540	0.056	2.49	95	9.41	0.50
Uranus	19.18	0.047	0.77	14.5	4.01	0.66
Neptune	30.06	0.009	1.77	17.2	3.89	0.62
Dwarf	a (AU)	е	i (°)	M/M _{Earth}	R/R _{Earth}	Albedo
Planet			, ,			
Ceres	2.77	0.080	10.59	1.6x10 ⁻⁴	0.15	0.37
Pluto	39.44	0.249	17.15	2.0x10 ⁻³	0.18	0.50
Huamea	43.22	0.191	28.19	6.6x10 ⁻⁴	~0.11	0.80
Makemake	45.56	0.158	28.98	7.4x10 ⁻⁴	0.11	0.81
Eris	67.67	0.442	44.19	2.8x10 ⁻³	0.18	0.96
Satellite	a (km)	е	i (°)	M/MEarth	R/R _{Earth}	Albedo
Moon	384,400	0.055	5.15	0.012	0.27	0.08

SECTION A

Use a section A answer book Answer all questions in this section (All questions are worth 10 marks)

- 1 Describe two observational measurements that allow us to measure physical properties and processes within the Sun *i.e.* at radii less than the photospheric radius.
- Show that for an airless body a distance R_h from the Sun, the maximum surface temperature is given by

$$T = (1 - A)^{1/4} \frac{393}{\sqrt{R_h}}$$

where A is the albedo of the body.

- 3 Describe the evolution of a 1 solar mass star from the point it arrives onto the mainsequence until its death. Include in your description a sketch of its evolutionary path on a Hertzsprung-Russell diagram.
- The velocities of individual galaxies do not obey Hubble's law exactly. In particular, galaxies have 'peculiar velocities' (resulting from gravitational interactions with neighbouring galaxies) in addition to 'cosmological velocities'. Suppose that the typical 'peculiar velocity' is 500 km s⁻¹. At what distance would a galaxy have to be if you wanted to determine the Hubble constant to an accuracy of 10%? Clearly outline your working and any assumptions you have made.

SECTION B

Use a Section B answer book Answer ONE question from this section

- 5 (a) Radio telescopes can operate at millimetre (mm) wavelengths. Using simple arguments based on diffraction theory, derive an expression for the angular resolution of a telescope with a circular aperture. Calculate the angular resolution for a 12-m diameter dish of the ALMA radio telescope observing at a wavelength of 1 mm, and comment on whether this is good or poor relative to the largest optical telescopes currently operating. [10]
 - (b) ALMA is an array of 66 radio telescopes, each of 12-m diameter, which are linked to form a large radio interferometer. Show that the angular resolution θ is related to the number of wavelengths n separating the telescopes by

$$\theta = \frac{1}{n}$$

Thereby calculate the relative improvement in resolution of the telescope array over using a single ALMA telescope. [10]

- (c) ALMA has been used to observe the nearby star HL Tau at a distance of 140 pc, to search for planets forming around this star.
 - (i) ALMA has detected a disk forming planets around HL Tau, with a total measured flux at Earth of 10⁻¹⁶ Joules/sec/m². What is the luminosity of the disk?
 [3]
 - (ii) The disk was resolved with an apparent diameter of 0.70 arcseconds.What is its true physical diameter in AU? [3]
 - (iii) If the ALMA array spans 16km and is operated at a wavelength 1 mm, what is the size of the smallest resolvable feature in the HL Tau planet-forming disk?

 [4]

SECTION B

6 (a) Consider two bodies of mass m_1 and m_2 in orbit about the centre of mass. By balancing the centripetal and gravitational forces in the system, derive Kepler's third law of planetary motion relating the orbital period P to the mutual semi-major axis a

$$P^2 = \frac{4\pi a^3}{G(m_1 + m_2)}$$

[10]

(b) For a body in a circular orbit about a much more massive object, show that the orbital velocity v_{orb} and the escape velocity v_{esc} are related by

$$v_{esc} = \sqrt{2}v_{orb}$$

[8]

- **(c)** An interstellar spacecraft is launched from Earth.
 - (i) The spacecraft is initially placed into an orbit 400 km above the Earth's surface. Calculate the orbital period and velocity of the spacecraft in this initial orbit.
 [3]
 - (ii) The spacecraft engine is fired again to place it on an escape trajectory from the Solar system. It is possible to point the spacecraft directly away from the Sun, or in the same direction as the Earth's orbital motion. Explain which trajectory would be easier to use, and calculate the necessary velocity of the spacecraft in both these cases. Assume the initial spacecraft orbit was in the same direction and plane as the Earth's orbit around the Sun.
 - (iii) When the New Horizons spacecraft crossed the orbit of Pluto at 32.9 AU, it was travelling at 14 km/sec. Will New Horizons escape the Solar System and also become an interstellar spacecraft? [2]

SECTION C

Use a Section C answer book Answer ONE question from this section

- (a) Explain the observational technique of photometry using the UBV system for illustration. Pay particular attention to the magnitude scale for expressing observed brightnesses and colours.
 [8]
 - (b) Explain what is meant by the terms absolute magnitude and distance modulus. Hence derive the relationship

$$(m_V - M_V) = 5\log d - 5$$

where *d* is the distance to an object in parsecs.

[6]

- (c) What effect does interstellar dust have on the distance modulus? Derive the modified version of the distance modulus in the presence of dust. [3]
- (d) Initial main-sequence fitting of the open cluster M7, ignoring the effects of the interstellar medium, results in an apparent distance to the cluster of 282 pc. A study shows that stars in the cluster matching the spectral type of Vega have a colour index (B-V) = +0.11.
 - (i) What is the true distance to the cluster? [7]
 - (ii) What is the mean extinction per kpc in this direction at visual wavelengths?[1]
- (e) Briefly describe the key differences between population I and population II stars.

 Explain how the interstellar medium (ISM) accounts for some of their differences, and why the ISM is crucial for the formation of life.

 [5]

[For Vega, $M_V = 0.582$]

SECTION C

- 8 (a) Estimate the main-sequence lifetime of a 10 M_{Sun} star if the exponent of the mass-luminosity relationship is $\alpha = 3.5$. Would you expect this star to ultimately form a white dwarf [explain your reasoning]? [4]
 - (b) Sketch and outline the 'lighthouse' model for pulsars, and thus explain why not all neutron stars are seen as pulsars. [5]
 - (c) Show that the density, ρ , of a pulsar (in kg m⁻³) can be related to its rotation period, P, by

$$P = \frac{3.8 \times 10^5}{\rho^{1/2}}$$
 seconds.

Clearly state any assumptions you have made.

- [8]
- (d) Write down reasonable values bounding the typical range of pulsar rotation periods that are observed. From this, and the equation in part c), calculate a range of densities for the constituent matter making up pulsars (in kg m⁻³). What stops the pulsar from further gravitational collapse? [4]
- (e) Sketch a diagram showing the typical form of the rotation curve of a spiral galaxy, ensuring that the axes are labelled (absolute values on the axes are not required). Contrast this with the theoretically expected form (ensure both sketches are clearly identified). Comment on the nature of the observed rotation curves, and the main inference that has been drawn from such observations. [9]

END OF EXAMINATION