QUEEN'S UNIVERSITY BELFAST

PHY2004
Exam Time Table
Code PHY2004

Answer Books A, B and C.

Any calculator, except one with pre-programmable memory, may be used in this examination.

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2004
Electricity, Magnetism and Optics

Tuesday, 13th August 2019 9:30 AM - 12:30 PM

Examiners: Professor P Browning
Dr P van der Burgt
and the Internal Examiners

Answer ALL TEN questions in Section A for 4 marks each.
Answer TWO questions in Section B for 20 marks each.
Answer ONE question in Section C for 20 marks.

Use a separate Answer Book for each Section.
You have THREE hours to complete this paper.

Mathematical identities and physical constants

In answering the questions on this paper you may make use of any of the following:

Divergence theorem $\int_V \nabla \cdot \underline{E}\, dV = \oint_S \underline{E} \cdot d\underline{S}$

Stoke's Theorem $\int_S \nabla \times \underline{E} \cdot d\underline{S} = \oint \underline{E} \cdot d\underline{\ell}$

Identities

$$\nabla \times \nabla(Scalar) = \underline{0}$$

$$\nabla \cdot \nabla \times (Vector) = 0$$

$$\nabla \times (\nabla \times \underline{E}) = \nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E}$$

$$\nabla(y\,\underline{E}) = y\,\nabla \cdot \underline{E} + \underline{E} \cdot \nabla y$$

$$\nabla \cdot (\underline{E} \times \underline{H}) = \underline{H} \cdot (\nabla \times \underline{E}) - \underline{E} \cdot (\nabla \times \underline{H})$$

Permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12} Fm^{-1}$

Permeability of free space $\mu_0 = 4\pi \times 10^{-7} Hm^{-1}$

Material Equations $\underline{J} = \sigma \underline{E} \quad \underline{B} = \mu \underline{H} \quad \underline{D} = \varepsilon \underline{E}$

Poynting Vector $\underline{S} = \underline{E} \times \underline{H}$

Trigonometric identities

$$cos(A \pm B) = cos\,A\,cos\,B \mp sin\,A\,sin\,B$$

$$sin(A \pm B) = sin\,A\,cos\,B \pm sin\,B\,cos\,A$$

$$cos\,A + cos\,B = 2\,cos\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right)$$

$$sin\,A - sin\,B = 2\,cos\left(\frac{A+B}{2}\right)sin\left(\frac{A-B}{2}\right)$$

continued …

SECTION A

Use a Section A answer book

Answer ALL TEN questions in this section

Full explanations of your answers are required to attain full marks

1 Figure 1 shows charges in vacuum in the vicinity of a cubic Gaussian surface of volume V. For each case state the total flux of the electric field passing through the surface. **[4]**

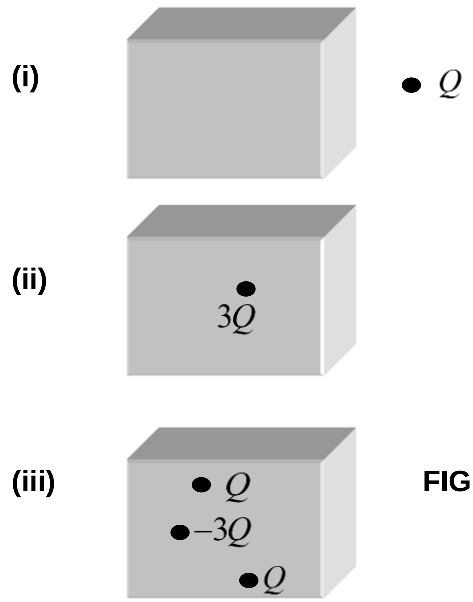


FIGURE 1

2 Show that Maxwell's equations forbid the existence of magnetic monopoles. **[4]**

3 Four point-like charges (q = 5 nC, h = 1 cm, and d = 5 cm) are in vacuum and aligned along the x-axis as in Figure 2.

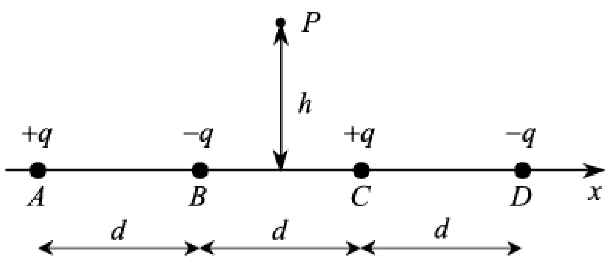


FIGURE 2

Calculate the electrostatic potential and the modulus of the electric field generated by the four charges at the point P. **[4]**

continued …

SECTION A

4 Figure 3 shows a short section of a long cylindrical conductor of permittivity μ and radius R carrying a steady current I. The cylinder is surrounded by vacuum.

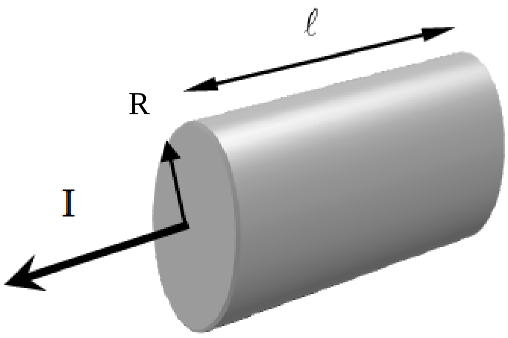


FIGURE 3

Calculate the magnetic field $\boldsymbol{B}$ generated by the current inside and outside of the conductor. Is the magnetic field continuous at the surface of the cylinder? **[4]**

5 Derive the wave equation for an electric field propagating in a pure vacuum in the absence of net charges or currents. **[4]**

6 Draw the Fourier transform pattern of the rectangular aperture shown in Figure 4 and justify your answer. **[4]**

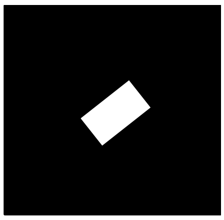


FIGURE 4

7 A fictitious metal has a plasma frequency of ω_p ($\hbar\omega_p = 10$ eV), calculate the free electron density of the material. **[4]**

8 A fictitious material has a complex refractive index of $n = 1 + i$. A plane wave of $\lambda = 500$ nm is incident normal to the surface of a thin slab of this material, 1 mm thick. Calculate the percentages of the reflected and transmitted light, respectively. **[4]**

continued …

SECTION A

9 Describe the phenomena of linear birefringence and circular birefringence. **[4]**

10 The electric field of a plane wave has the following components: $E_x = 0.5\cos(kz - \omega t + \frac{\pi}{2})$ and $E_y = E_{0y}\cos(kz - \omega t + \delta)$, $E_{0y} > 0$. Describe the conditions of E_{0y} and δ for the plane wave to be linearly-polarised and left-circularly polarised, respectively. **[4]**

continued …

SECTION B

Use a Section B answer book

Answer TWO questions from this Section

11 **(a)** Two small Styrofoam balls of mass 3×10^{-3} kg are suspended from the same point by threads 5 cm long. Equal positive charges are placed on the balls. What must the magnitudes of these charges be if the balls are to remain in equilibrium at angles of 30° from the vertical? **[6]**

(b) A conducting sphere of radius R is filled with a dielectric material of relative permittivity ε_r. A total charge of Q is homogeneously distributed over the dielectric material and the total charge on the conducting sphere is $-Q$. The sphere is surrounded by air.

(i) Show that the charge density inside the dielectric material $(r < R)$ is **[1]**

$$r = \frac{3Q}{4\pi R^3}$$

(ii) Calculate D, E and P inside the dielectric material $(r < R)$. **[4]**

(iii) Similarly, derive expressions for D, E and P in air $(r > R)$. **[2]**

(iv) Assuming the total charge on the conducting sphere is doubled to $-2Q$. Calculate D, E and P in air $(r > R)$. **[3]**

(v) Calculate the bound *volume* charge density inside the dielectric. **[4]**

[*You may assume, that in spherical polar co-ordinates* (r, θ, ϕ)

$$\nabla \cdot \underline{P} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P_r) + \frac{1}{r sin\theta}\frac{\partial}{\partial \theta}(P_\theta sin\theta) + \frac{1}{r sin\theta}\frac{\partial P_\phi}{\partial \phi}) \text{]}$$

continued …

SECTION B

12 Two long concentric conducting cylinders of radii a and $b (a < b)$ carry equal and opposite charge per unit length λ (positive on the inner conductor). The space between the two conductors is filled with a dielectric of relative permittivity ε_r (Figure 5).

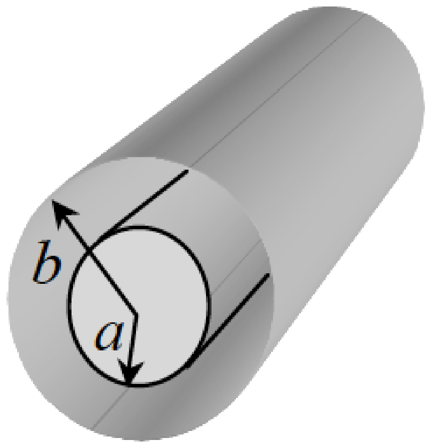


FIGURE 5

(a) Sketch the distribution of charges on the conductors and the dielectric. **[2]**

(b) Sketch the direction of the resulting electric field. **[2]**

(c) Calculate the amplitude of the electric field $\boldsymbol{E}$, polarisation $\boldsymbol{P}$, and electric displacement field $\boldsymbol{D}$ outside of the outer conductor $(r > b)$. **[2]**

(d) Calculate the amplitude of the electric field $\boldsymbol{E}$, polarisation $\boldsymbol{P}$, and electric displacement field $\boldsymbol{D}$ in the dielectric $(b > r > a)$. **[6]**

(e) Calculate the amplitude of the electric field $\boldsymbol{E}$, polarisation $\boldsymbol{P}$, and electric displacement field $\boldsymbol{D}$ inside the inner conductor $(r < a)$. **[2]**

(f) Calculate the potential difference between the two conductors. **[3]**

(g) Calculate the capacitance per unit length between the two conductors. **[3]**

continued …

SECTION B

13 A parallel plate capacitor of capacitance C as shown in Figure 6 holds energy $W = \frac{1}{2}C{V_0}^2$, where V is the initial potential across its circular plates. The capacitor is in vacuum and it is now slowly and completely discharged through a resistor R.

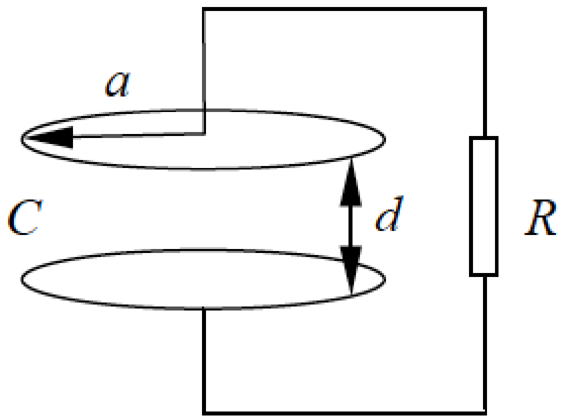


FIGURE 6

(a) Show that the potential, as it discharges across the RC circuit, can be expressed as:

$$\boldsymbol{V(t) = V_0 e^{-t/RC}}$$ **[4]**

(b) Using the expression in (a), calculate the electric field E at the edge of the plates. **[2]**

(c) Calculate the magnetic field H at the edge of the plates using the generalised Ampere's law. **[4]**

(d) Show the direction of these fields on a diagram similar to the one in Figure 6. **[2]**

(e) Show on the same diagram the direction of the Poynting vector. **[1]**

(f) Calculate the amplitude of the Poynting vector. **[2]**

(g) By using the obtained expression for the Poynting vector, show that the total energy radiating out of the gap between the plates is equal to W (remember that for a plane capacitor, the capacitance $C = \varepsilon_0 A/d^2$, where A is the area of each plate). **[5]**

continued …

SECTION B

14 **(a)** The resolution of conventional optical microscopes is limited, due to diffraction effects, to about half the wavelength of light. Scanning near-field optical microscopes (SNOM) are able to beat the diffraction limit, achieving optical resolutions much smaller than the diffraction limit.

(i) Describe the image of a point source imaged by a conventional optical microscope and explain the physics for that. **[3]**

(ii) Explain why SNOM is able to beat the diffraction limit. **[2]**

(iii) Sketch the diagram of the SNOM setup proposed by the Irish scientist Synge and explain its working principles. Name the factors that will determine the resolution of SNOM. **[5]**

(b) Laser is tremendously useful in modern society, with applications in a wide variety of areas.

(i) Name at least three key properties of Laser that are normally not achievable from conventional incandescent light sources. **[3]**

(ii) Describe the process of stimulated light emission. Sketch a diagram to illustrate your answer. **[3]**

(iii) Explain what is the population inversion and why it is crucial to realise lasing. **[4]**

continued …

SECTION C

Use a Section B answer book

Answer ONE question from this Section

15 The complex relative permittivity of a fictitious material is given by $\varepsilon_r(\omega) = 12 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\gamma\omega}$, where $\omega_p = 10\ \mathrm{eV}$, $\omega_0 = 5\ \mathrm{eV}$, $\gamma = 0.4\ \mathrm{eV}$.

(a) Draw the graphs of the real and imaginary parts of the permittivity within the frequency range $4 \le \omega \le 6\ \mathrm{eV}$, respectively. Label the graphs. **[5]**

(b) Calculate the real and imaginary parts of the refractive index at the resonant frequency $\omega_0 = 5\ \mathrm{eV}$. **[4]**

(c) Describe what is anomalous dispersion. Specify the frequency range of anomalous dispersion for the material. **[4]**

(d) Comment on whether this material would be a good transparent conductor, justifying your answer. **[3]**

(e) Explain the difference between the phase velocity and group velocity. Is it possible for this material to have phase velocity and group velocity to be larger than the speed of light, respectively? Justify your answer. **[4]**

continued …

SECTION C

16 Light of a TM-polarised plane wave is incident from a medium 1 of refractive index of $n_1 = 1.5$ to a medium 2 of refractive index of $n_2 = 1.3$.

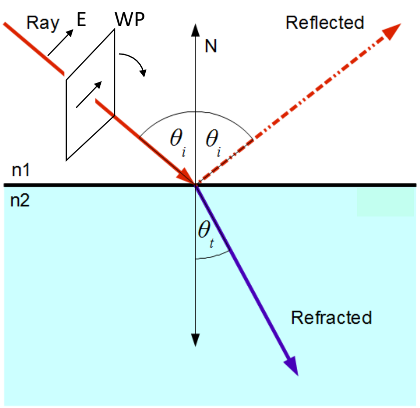
(a) Calculate the Brewster angle for the plane wave **[2]**

(b) Calculate the critical angle for the plane wave **[2]**

(c) Draw the intensity reflectance graphs of the plane wave as a function of the incidence angle θ, for θ increasing from 0° to 90°. Label the axes of the graph. **[5]**

(d) For light incident at 45°, describe the polarisation state of the reflected light and provide rationales to your answer. **[3]**

(e) A half-waveplate (WP) is inserted into the path of the above beam incident at 45°, as shown in the following schematic. Initially the principal axis of the waveplate is parallel to the E-field of the incident light, it then is rotated clockwise around the incident beam. Describe how the intensity and polarisation state of the reflected light changes during the rotation and provide rationales to your answers. **[5]**



(f) If light is incident at the Brewster angle, what would be the answer to the above question? **[3]**

END OF THE EXAMINATION PAPER