PHY2003 ASTROPHYSICS I

Lecture 9. The Planets

What is a planet?

Many objects orbit our Sun of various sizes and compositions. The first official decision on the definition of a planet only occurred in August 2006! A planet is a body in the Solar system that

(a) is in orbit about the Sun;

(b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a shape under hydrostatic equilibrium;

(c) has effectively cleared the neighbourhood around its orbit.

Part (c) implies that the object must be significantly massive than anything else at a similar distance from the Sun.

Part (b) is says a planet must be round, so how large is that? The stress S on a solid body is the force/area (pressure!), and the strain is the fraction change in length under that stress $e = \Delta L/L$. They are related by Young's modulus:

$$Y = S/e$$

We can define a maximum stress beyond which no more change in size occurs and the material breaks or cracks, called the *tensile strength* T. This normally occurs when the change is size is $\simeq 1$ %.

$$S(max) = T = Ye(max) = Y\frac{\Delta L}{L}(max) \simeq 0.01Y$$

So an irregular solid body will deform under self-gravity to hydrostatic equilibrium when the internal pressure overcomes the tensile strength of the material. Using the average internal pressure, this will happen when

$$T \simeq 0.01Y \simeq \bar{P} \simeq -\frac{U_g}{3V} \simeq \frac{GM^2}{R}\frac{1}{3V}$$

Substituting $M^2 = \rho^2 V^2$,

$$0.01Y \simeq \frac{G\rho^2 V^2}{R}\frac{1}{3V} \simeq \frac{1}{3}\frac{G\rho^2}{R}\frac{4}{3}\pi R^3$$

$$R^2 \simeq \frac{0.09}{4\pi}\frac{Y}{G\rho^2}$$

$$\boxed{R \simeq \frac{1}{2\rho}\sqrt{\frac{0.09Y}{\pi G}}}$$

Example: Rock has a Young's modulus of $\simeq 2 \times 10^{10}$ Pa and a density of $\simeq 3000\text{kg/m}^3$. What is the approximate minimum size for a rocky planet?

If a body fulfills (a) and (b) but <u>not</u> (c) and is <u>not</u> a satellite of another body it is termed a "dwarf planet". Rule (c) for massive planets like Jupiter can be assumed to be true, for smaller bodies has to be tested by observation.

Under this definition, the Solar system has EIGHT planets (Mercury to Neptune).

Currently the Solar system also has FIVE dwarf planets, Ceres (discovered 1801), Pluto (discovered 1930), Haumea (discovered 2004), Makemake (discovered 2005) and Eris (discovered 2005).

Temperatures of planetary bodies

Objects are found orbiting the Sun from inside Mercury ($a = 0.39$ au) to beyond Neptune ($a = 30.1$ au). For most planetary systems, the main source of heat will be electromagnetic radiation from the host star. In the case of our Solar System, the main source of electromagnetic radiation is the Sun.

A black body radiates as E = $\sigma T^4 \ \frac{\mathrm{J}}{\mathrm{s \cdot m^2}}$. Thus, the energy per second per unit area emitted (P_*) by the parent star with temperature T_* is:

$$P_* = \sigma T_*^4 \ \frac{\mathrm{J}}{\mathrm{s \cdot m^2}}$$

Assuming the star has a radius of R_*, the total luminosity of parent star (L_*) is:

$$L_* = 4\pi R_*^2 P_* = 4\pi R_*^2 \sigma T_*^4 \ \frac{\mathrm{J}}{\mathrm{s}}$$

If we have a planet (or planetesimal) located at Flux passing through unit area at distance R_h is

$$F_* = \frac{L_*}{4\pi R_h^2} = \frac{R_*^2 \sigma T_*^4}{R_h^2}$$

The albedo A is the fraction of light reflected by a body. Therefore, the fraction of the star's flux absorbed by the planetary body available to re-radiate as thermal energy is $(1 - A)$.

The total power absorbed (P_A) by planet (planetesimal) can be written as:

$$P_A = (1 - A) \times \mathrm{cross} - \mathrm{section} \times F_*$$

Not all of the planetary body will be facing the Sun. The cross-section is the area of the part illuminated by the Sun. The cross section fo the planet (planetesimal) if round will be a circle. Therefore:

$$P_A = (1 - A)\pi R_p^2 \frac{R_\odot^2 \sigma T_\odot^4}{R_h^2}$$

In thermal equilibrium, this power emitted by the surface of the planet (P_E)must be radiated away over the atmosphere/surface of the planet at equilibrium temperature T_p. We assume that the planet's atmosphere/surface radiate the thermal energy in all directions uniformly. Thus we can use the Stefan-Boltzman's law to estimate P_E, like we did for the power emitted by the star.

$$P_E = 4\pi R_p^2 \sigma T_p^4$$

The power radiated by the planet (planetesimal) will be equal to the power absorbed by the planet.

$$P_A = P_E$$

Substituting in the two expressions we have for P_E and P_A:

$$4\pi R_p^2 \sigma T_p^4 = (1 - A)\pi R_p^2 \frac{R_*^2 \sigma T_*^4}{R_h^2}$$

Note that R_p cancels out, thus the equilibrium temperature (T_p) of a planet (planetesimal) is independent of radius.

$$4T_p^4 = (1 - A)\frac{R_*^2 T_*^4}{R_h^2}$$

Thus, for For the above equation R_h and R_* in meters,

$$\boxed{T_p = (1 - A)^{1/4} \left(\frac{R_*}{2R_h}\right)^{1/2} T_*}$$

For R_h in au and for the Solar System's planets and planetesimals:

$$\boxed{T_p = (1 - A)^{1/4} \frac{279}{\sqrt{R_h}}\ \mathrm{K}}$$

The Earth has $A \simeq 0.3 \Rightarrow T_\oplus \simeq 255$K

The difference is caused by some atmospheric gases being very efficient at trapping thermal radiation - the "greenhouse" effect.

For Venus, the difference is even larger, due to the atmosphere being 96% CO_2.

What is the temperature of a typical cold classical Kuiper belt object out at 40 au with a Bond albedo of 20%?