Lecture 9:

Magnetic properties of materials

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Electric properties of materials

We have seen that materials react to an external electric fields in different ways: **conductors and dielectrics**

CONDUCTORS

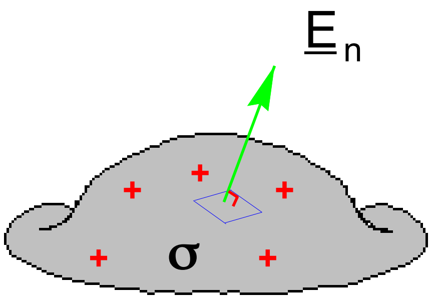
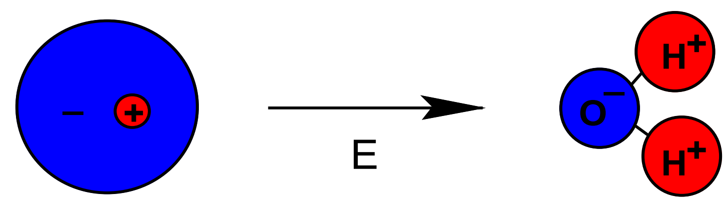


Fig 31

- Free charges on the surface
- no field inside
- field always perpendicular to the surface

DIELECTRICS



induced dipole

orientation of dipole

$$\underline{p} = Ze\underline{\ell}$$

$$\underline{D} = \varepsilon\underline{E} = \varepsilon_0\underline{E} + \underline{P}$$

Can we have a similar behaviour for magnetic fields?

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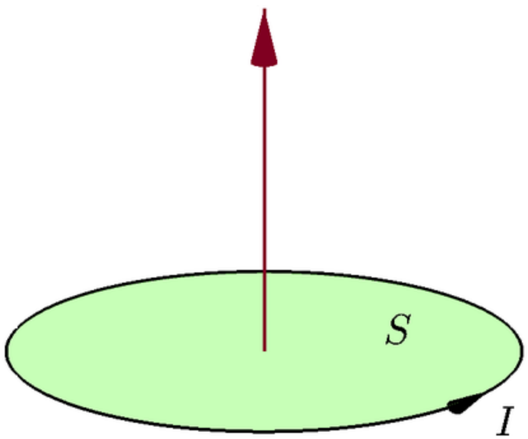
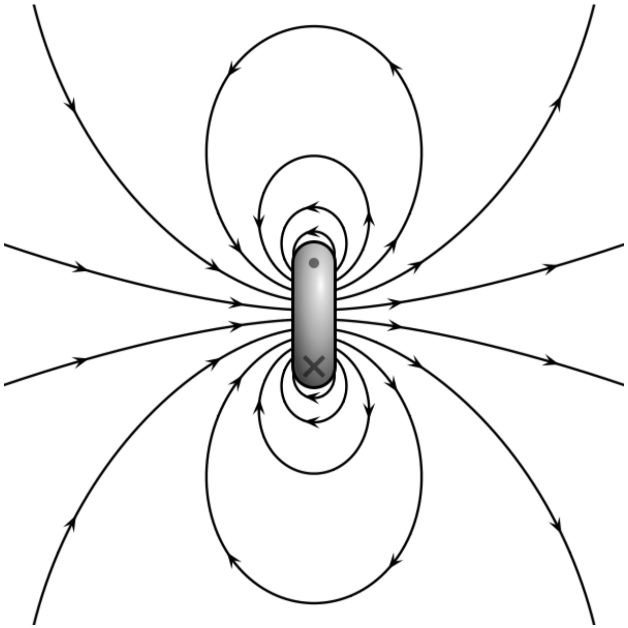
The magnetic moment

Molecules within the material will react to an external magnetic field by setting a magnetic moment (compare with the electric dipole in electrostatics)





This is equivalent to having a current circulating within the molecule.

The magnetic moment is defined by the current times the area over which it circulates $\boxed{m = SI}$. Each molecule will have its own moment and the density of magnetic moment will be the product between m and the density of molecules: $\boxed{M = nm}$

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Magnetic properties of materials

M plays the same role as P in electrostatics. It tells you how the material reacts to the external field.

The total field will then be made of two contributions: the external field (H) and the field induced in the material (M):

$$\underline{\mathrm{B}} = \mu_0\left(\underline{\mathrm{H}} + \underline{\mathrm{M}}\right)$$

Similar to the electrostatic case, we can define a relative permittivity of the material (μ_r) and the magnetic susceptibility ($\chi_m = \mu_r - 1$).

$$\underline{\mathrm{M}} = \chi_m \underline{\mathrm{H}}$$

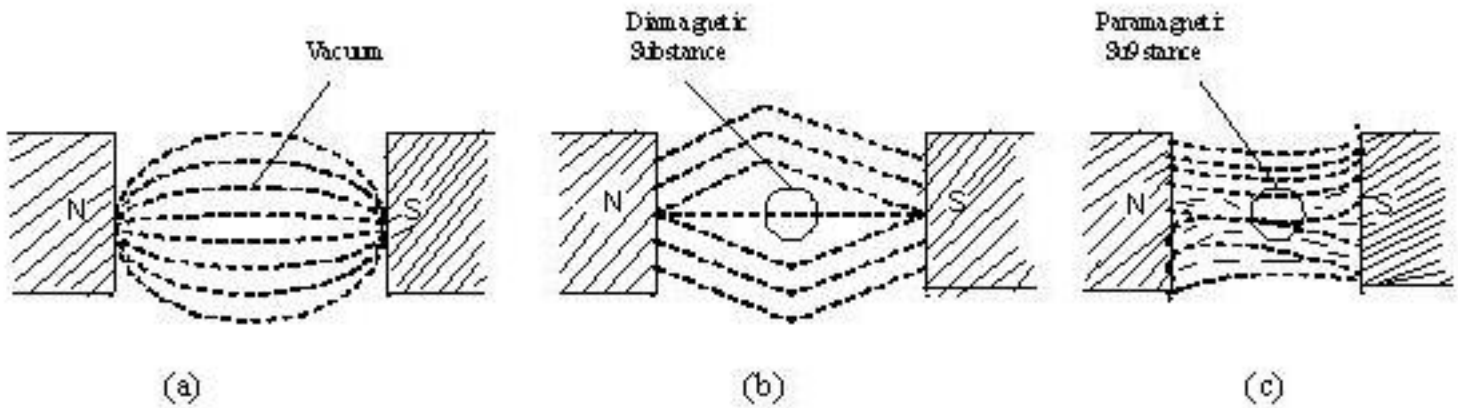
$$\underline{\mathrm{B}} = \mu_0\left(1 + \chi_m\right)\underline{\mathrm{H}}$$

Magnetic properties of materials

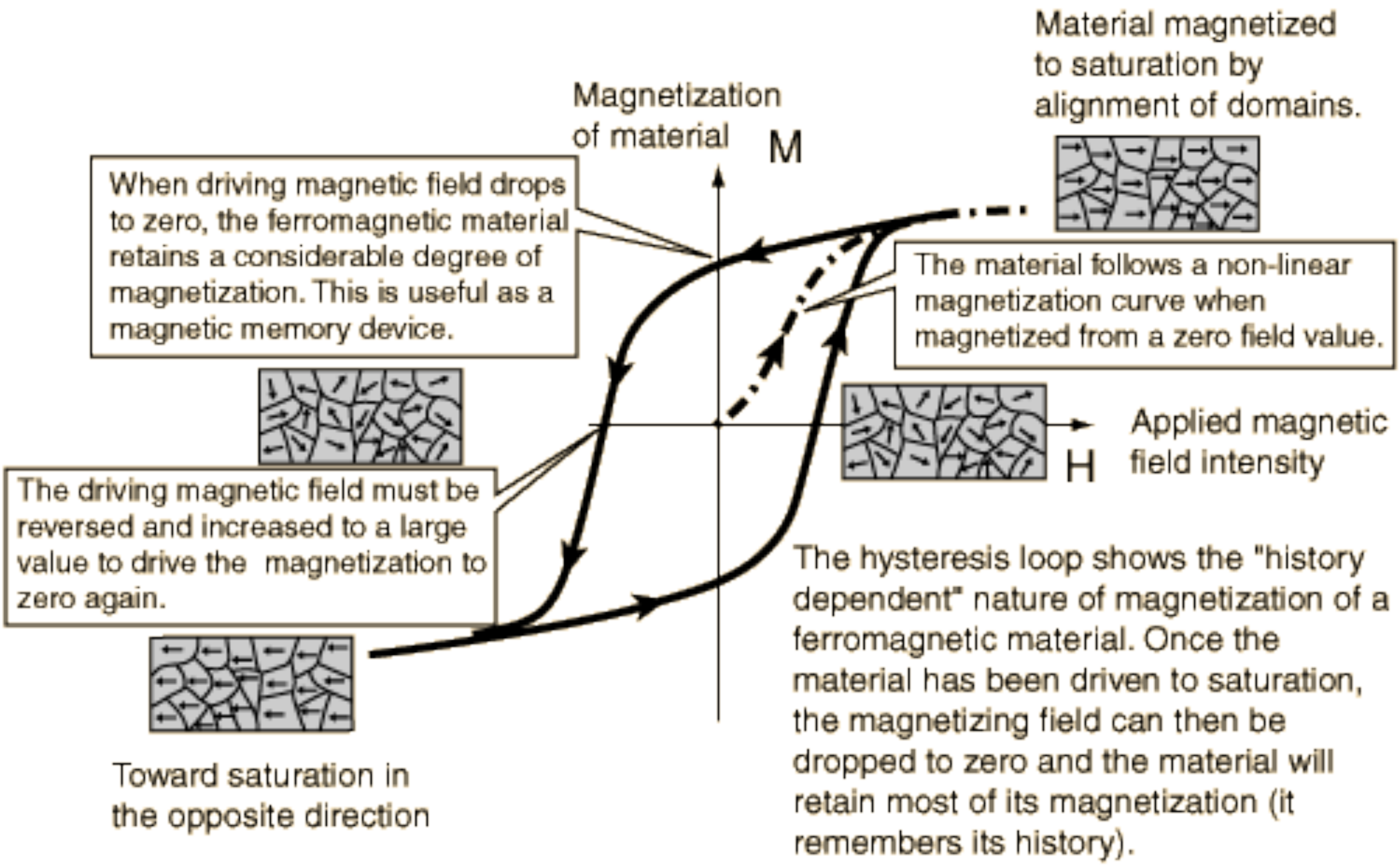
However, magnetic materials do not always act the same way (remember in the electrostatic case $\varepsilon_r > 1$).

$\chi_m > 0$	paramagnetic material
$\chi_m < 0$	diamagnetic material

A paramagnetic material responds by enhancing the field in the medium whereas a diamagentic material counteracts it.



Ferromagnetic materials



Field lines

We have seen that the field B has continuous lines that always go in loops. However, M is clearly discontinuous (zero in vacuum, non-zero in the material). Hence, also H must be discontinuous

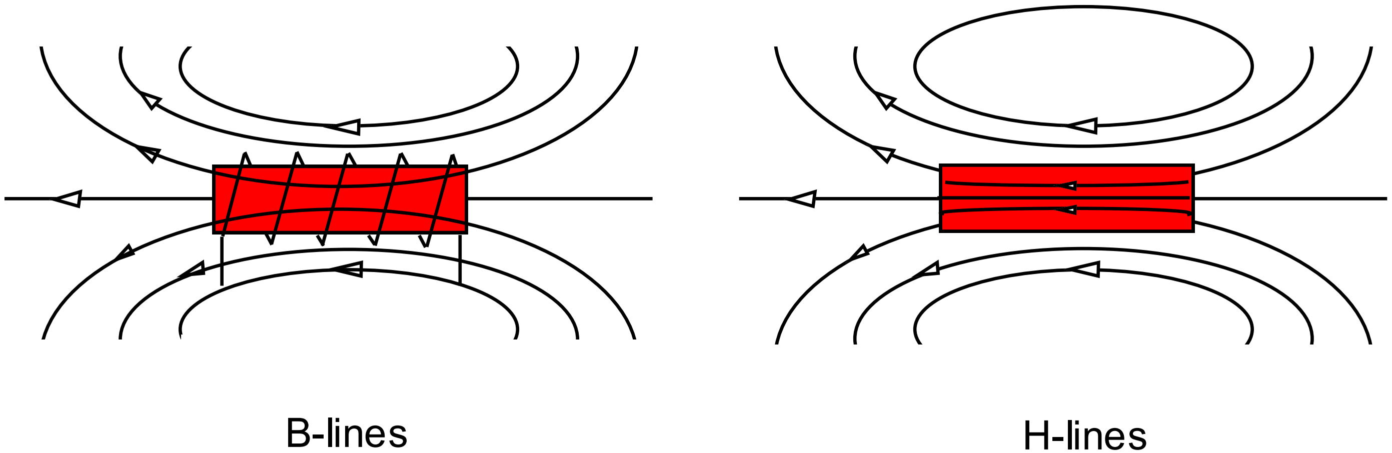
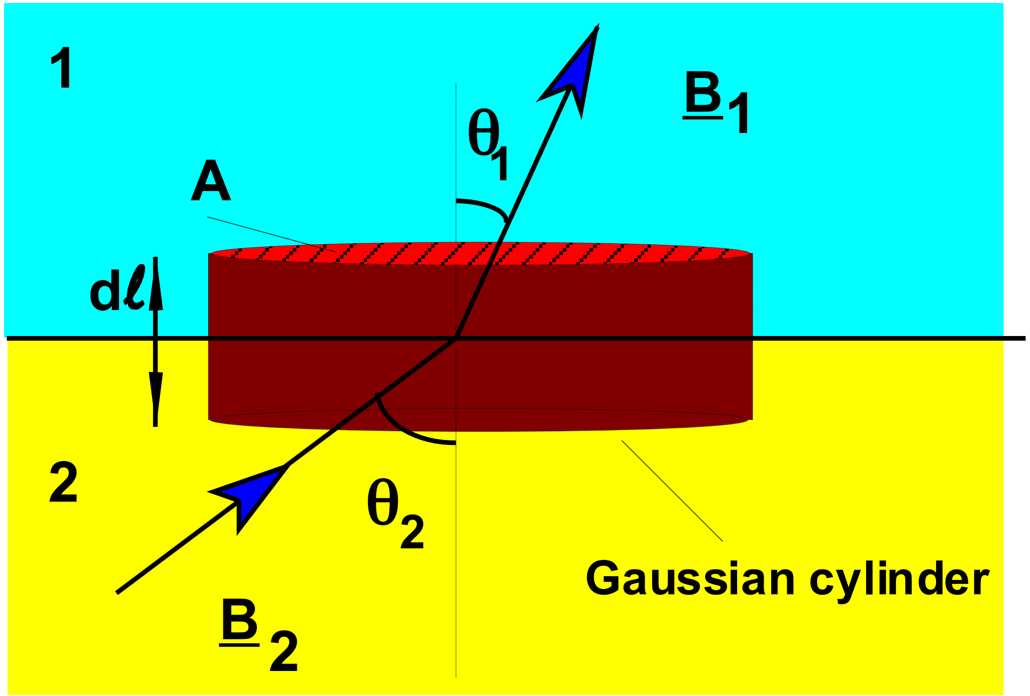


Fig 66 Solenoid with a paramagnetic rod

Boundary conditions for B

What are the boundary conditions for B (or, how does B change whenever it passes through a different material?)



Going from material 2 to 1, the magnetic field must change direction but how?

$$\int_B \underline{B} \cdot d\underline{S} = \int_V \nabla \cdot \underline{B}\, dV = 0 \quad (\nabla \cdot \underline{B} = 0)$$

$$\downarrow$$

$$\underline{B}_1 \cdot \underline{A} = \underline{B}_2 \cdot \underline{A}$$

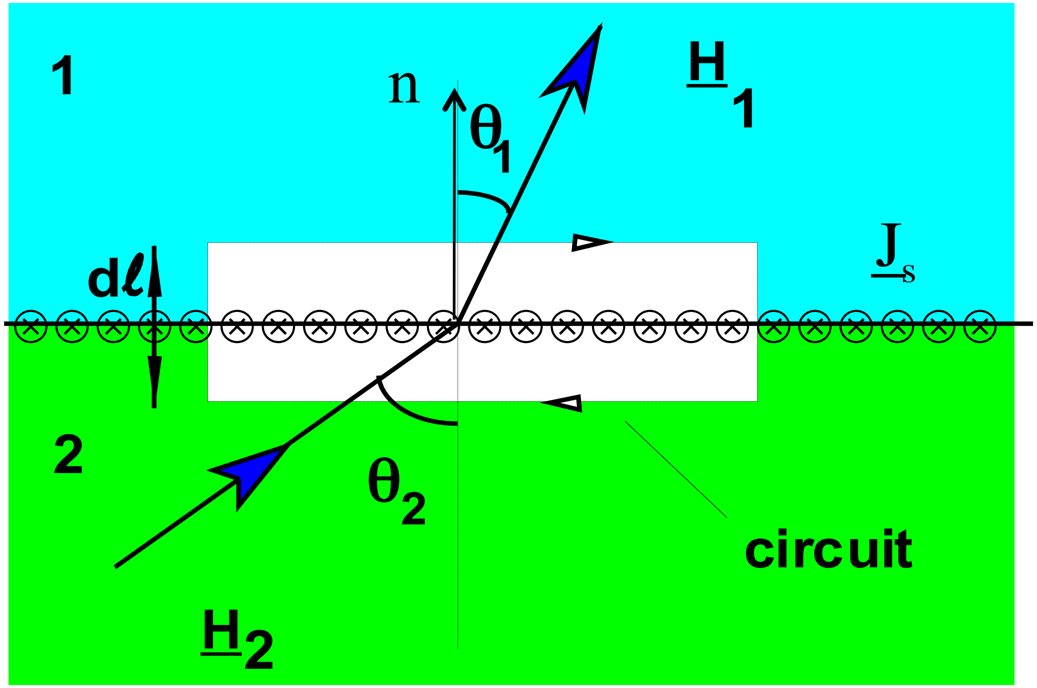
$$\downarrow$$

$$\therefore B_{1n} = B_{2n}$$

The normal component of the field B is always continuous.

Boundary conditions for H

What are the boundary conditions for H (or, how does H change whenever it passes through a different material?)



$$\oint \underline{H} \cdot \underline{d\ell} = I$$

$$H_{1t} - H_{2t} = J_s$$

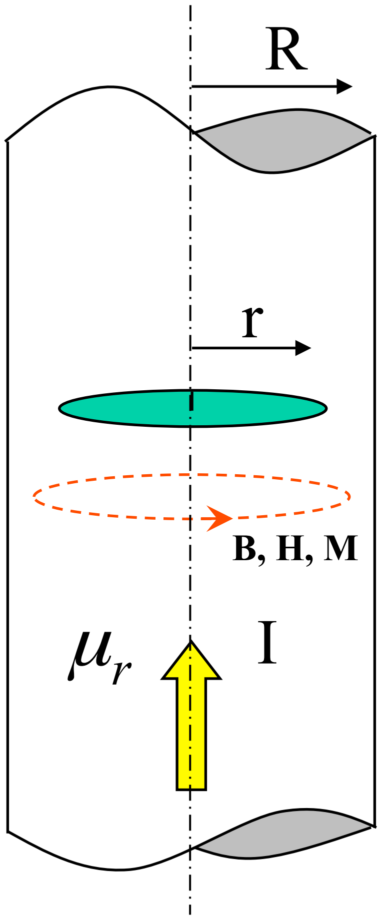
$$H_1 Sin\,\theta_1 - H_2 Sin\,\theta_2 = J_s$$

$$\boxed{\underline{n} \times (\underline{H}_1 - \underline{H}_2) = \underline{J}}$$

The tangential component of H is continuous as long as there are no surface currents

Example

A wire made from a conducting magnetic material of relative permeability μ_r, carries a current I, uniformly distributed across its circular cross section of radius R. Determine expressions for B,H and M inside the wire.



$$\oint \vec{H} \cdot d\vec{\ell} = I$$

$$H.2\pi r = I\frac{\pi r^2}{\pi R^2} \longrightarrow \therefore H = I\frac{r}{2\pi R^2}$$

$$\therefore B = \mu_0 \mu_r H = \mu_0 \mu_r I \frac{r}{2\pi R^2}$$

$$M = \frac{B}{\mu_0} - H \quad \therefore M = I\frac{r}{2\pi R^2}(\mu_r - 1)$$

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