PHY2001 Assignment 1

Deadline: Monday 18th October 2021 10pm

- Assignments to be submitted electronically to Canvas(via appropriate "Assignment" page).
- Please upload a single pdf file, and make sure the scan is readable.
- Show your working and explain your reasoning in all cases.
- The assignment will be marked out of 50.

1. In an experiment, an alpha particle is determined to be within 2.0 nm of a particular point.

(i) Estimate the minimum uncertainty of the alpha particle's momentum.
(ii) Determine an expression for the velocity uncertainty in terms of the position uncertainty.
(iii) What is the corresponding velocity uncertainty?

[5]

2. A quantum particle has an eignenfucntion

$$\psi(x) = \sqrt{\frac{2}{a}} e^{-x/a} \qquad \text{for } x \geq 0$$
$$\psi(x) = 0 \qquad \text{for } x < 0$$

(a) Assuming a is positive, find and sketch the probability density. Label the value of $P(0)$.
(b) What is the probability of finding the particle at any point where $x < 0$?
(c) Show that ψ is normalised.
(d) What is the probability of finding the particle between $x = 0$ and $x = a$?

[8]

3. Consider the quantum energy levels for the following potentials;

- Simple Harmonic
- Infinite Square
- Finite Square
- Coulombic

(a) Explain why the separation of the quantum levels in one of these wells decreases with increasing quantum number.
(b) For the simple harmonic and Infinite square potentials each, sketch the potential energy function and add to each plot, the energies and eigenfunctions corresponding to the $n = 1$ and $n = 2$ quantum numbers (for that system).

[10]

4. The spectral Irradiance of a blackbody is;

$$I(\lambda, T) = \frac{1}{\lambda^5} \frac{2\pi hc^2}{e^{hc/\lambda k_B T} - 1}$$

(i) On the same figure, sketch the $I(\lambda, T)$, in units of $W\, m^{-2}\, nm^{-1}$, against wavelength produced by two blackbodies, one at 4000 K and the other at 5500 K. Label the approximate peak values of $I(\lambda, T)$.

(ii) Describe, in words, what is meant by *spectral irradiance* as it applies to a blackbody.

(iii) Show that;

$$\int_0^\infty I(\lambda, T)\, d\lambda = cT^4$$

where c is a constant independent of T. Clearly show your working. Hint: you will likely need the following integral;

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}$$

(iv) Determine the value of c. What is this constant normally known as? **[15]**

5.(a) For a particle of mass m trapped in an infinite 1-D square potential well in the region $-a/2 \leq x \leq a/2$, the odd and even parity eigenfunctions of the TISE are;

$$\psi_n(x) = A\cos\left(\frac{n\pi}{a}x\right) \quad n = 1, 3, 5...$$
$$and$$
$$\psi_n(x) = B\sin\left(\frac{n\pi}{a}x\right) \quad n = 2, 4, 6...$$

(i) For the eigenfunction corresponding to n=3 determine the value for the normalisation constant. Hint:$\int cos^2(kx)dx = x/2 + sin(2kx)/4k + C$.

(ii) Determine $< x >$ for the eigenstate in part (i).

(b) The potential energy function for a *quantum bouncing ball* of mass *m* is:

$$V(h) = mgh \quad \text{for } h > 0$$
$$V(h) = \infty \quad \text{for } h \leq 0$$

Plot $V(h)$ as a function of h and sketch the wave function associated with the n=3 state.

[12]

End of Examination