PHY20003 ASTROPHYSICS

Lecture 18 Fundamental Stellar Properties

18.1 Binary Stars and Stellar Masses

When we observe stars near the Sun, we find that only a relatively small fraction of stars, around 30%, are single stars.

A system with two gravitationally bound stars is known as a binary system, or binary star.

We can distinguish three main types of binary stars (although note that it is possible for the binary star to be a member of more than one class)

18.1.1 Visual Binary Stars

Gravitationally bound stellar systems that can be directly resolved by imaging are known as visual binary stars (these should not be confused with double stars, which appear close to each other in the sky, but do not need to be gravitationally bound).

Consider a visual binary system that is close enough to the Earth for us to directly measure its parallax, $\pi\ ('')$, as well as the apparent orbital semi-major axis $a('')$ and the orbital period, P.

The true semi-major axis is

$$a(\mathrm{AU}) = \frac{\mathrm{a}('')}{\pi('')} \tag{18.1}$$

We can write the general form of Kepler's 3rd law as:

$$(M_1 + M_2)_{M_\odot} P^2(\mathrm{yr}) = a^3(\mathrm{AU}) \tag{18.2}$$

This will then give us the sum of the stellar masses.

To get the individual masses, we need to find the distance of each star from the centre of mass of the system, where

$$r_1 + r_2 = a \tag{18.3}$$

As the system is balanced

$$M_1 r_1 = M_2 r_2 \tag{18.4}$$

Therefore

$$\frac{M_1}{M_2} = \frac{r_2}{r_1} \tag{18.5}$$

And we can use this to determine the mass of the individual stars.

18.1.2 Spectroscopic Binary Stars

Gravitationally bound systems that are found due to the relative orbital velocities of the two stars are known as spectroscopic binaries.

Consider a spectroscopic binary composed of stars of mass M_1 and M_2 at distance r_1 and r_2 from their centre of gravity.

Now consider the case where only one of these stars is directly observed (for example, it may completely outshine its companion). For this star, we can measure the orbital velocity V and period P (from the regular Doppler shifting).

If the orbit is circular, then the circumference of the orbit is

$$r_1 = \frac{PV}{2\pi} \tag{18.6}$$

But as the system is inclined at an (unknown) angle i relative to the plane of the sky, what is observed is

$$r_1' = \frac{PV \sin i}{2\pi} = r_1 \sin i \tag{18.7}$$

From Kepler's 3rd law

$$\frac{GP^2}{4\pi^2}(M_1 + M_2) = (r_1 + r_2)^3$$

$$= r_1^3 \left(1 + \frac{r_2}{r_1}\right)^3$$
$$= r_1^3 \left(1 + \frac{M_1}{M_2}\right)^3$$

Thus

$$\frac{GP^2}{4\pi^2}(M_1 + M_2) = \frac{r_1^3}{M_2^3}(M_1 + M_2)^3 \tag{18.8}$$

And substituting in equation 18.7 and putting the observables together results in an observable quantity called the mass function of the system:

$$f(M_1, M_2) = \frac{4\pi^2}{G}\frac{r_1'^3}{P^2} = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} \tag{18.9}$$

As M_1 can be estimated from the spectral type of the host star, it is possible to place a *lower limit* on M_2.

18.1.3 Eclipsing Binary Stars

When the inclination of the binary system is (close to) $90°$, the stars will eclipse each other. Such stars are known as eclipsing binary stars.

Each orbital period will give two eclipses, the deeper *primary* eclipse and the shallower *secondary* eclipse.

Generally, the stars will be of different radii, but, of course, the duration of both eclipses will be of equal duration.

From the Doppler shifts it is possible to measure the relative velocity of the stars, v, and using that, we can derive the radius of the secondary star, R_s, (assuming a circular orbit)

$$2R_s = v(t_2 - t_1) = v(t_4 - t_3) \tag{18.10}$$

Where t_1 is the time of first contact (when the primary eclipse starts), t_2 is the first time at which the entire secondary star is in front of the primary, t_3 is the last time the entire secondary star is in front of the primary, and t_4 is the time of last contact (marking the end of the primary eclipse).

The radius of the larger star, R_L, can be determined from

$$2(R_s + R_L) = v(t_4 - t_1) \tag{18.11}$$

And we find

$$R_L = \frac{v}{2}(t_c - t_b) + R_s \tag{18.12}$$

The depth of the eclipses gives the relative luminosity of the two stars.

During both eclipses, the same area of stellar surface, πR_2^2, is hidden.

If the stars have effective temperatures T_1 and T_2, we can use the Stefan-Boltzmann Law to determine the luminosities of the areas hidden during the eclipse

$$\frac{L_1}{L_2} = \frac{\sigma\pi R_2^2 T_1^4}{\sigma\pi R_2^2 T_2^4} = \frac{T_1^4}{T_2^4} \tag{18.13}$$

Therefore, the primary eclipse occurs when the hotter star passes behind the cooler star.

18.2 The Mass-Luminosity Relation

It has been proposed that the luminosity and masses of normal stars would be correlated in the form of a powerlaw with powerlaw index α:

$$L \propto M^\alpha \tag{18.14}$$

Expressing the luminosities and masses relative to our Sun we get

$$\frac{L_*}{L_\odot} = \left(\frac{M_*}{M_\odot}\right)^\alpha \tag{18.15}$$

Modern data shows that α varies depending on the absolute mass of the star (between ~2 and ~ 4), but in general $\alpha \sim 3.5$ is considered typical for a main sequence star.

Note, as the exponent is positive, this means that stars ranging from 0.1 $M_\odot$ to 50 $M_\odot$ have luminosities ranging from $10^{-3}\ L_\odot$ to $10^{6}\ L_\odot$.

The rate at which nuclear fuel is burned is proportional to the luminosity, L. The amount of fuel is proportional to the mass, M. Therefore, stars have a finite lifetime that will depend on their mass:

$$\text{time on the main sequence} = \frac{\text{amount of fuel available}}{\text{rate of burning}}$$

and the lifetime on the main sequence t_{ms} is found to be

$$t_{ms} \propto \frac{M}{L} \propto M^{1-\alpha} \tag{18.16}$$

This result tells us that the more massive and luminous stars spend *less* time on the main sequence.