QUEEN'S UNIVERSITY BELFAST

Any calculator, except one with pre-programmable memory, may be used in this examination.

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2004
Electricity, Magnetism and Optics

Duration: 3 hours plus additional 1 hour for upload of work

Monday 9th of August 2021
09:30 AM – 1:30 PM

Examiners: Prof S Matthews, Prof F. Peters
and the internal examiners
Dr S Sim (s.sim@qub.ac.uk)

Answer ALL questions in Section A for 4 marks each.
Answer TWO questions from Section B for 20 marks each.
Answer ONE question from Section C for 20 marks.

If you have any problems or queries, contact the School Office at mpts@qub.ac.uk or 028 9097 1907, and the module coordinator g.sarri@qub.ac.uk

THE QUEEN'S UNIVERSITY OF BELFAST
DEPARTMENT OF PHYSICS AND ASTRONOMY

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \mathrm{ms^{-1}}$
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$ $\approx 1.26 \times 10^{-6}\ \mathrm{Hm^{-1}}$
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{Fm^{-1}}$
Elementary charge	$e = 1.60 \times 10^{-19}\ \mathrm{C}$
Electron charge	$= -1.60 \times 10^{-19}\ \mathrm{C}$
Planck Constant	$h = 6.63 \times 10^{-34}\ \mathrm{Js}$
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}\ \mathrm{Js}$
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}\ \mathrm{m^{-1}}$
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}\ \mathrm{kg}$ $1u = 931\ \mathrm{MeV}$
1 electron volt (eV)	$= 1.60 \times 10^{-19}\ \mathrm{J}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$
Mass of proton	$m_p = 1.67 \times 10^{-27}\ \mathrm{kg}$
Mass of neutron	$m_n = 1.67 \times 10^{-27}\ \mathrm{kg}$
Molar gas constant	$R = 8.31\ \mathrm{JK^{-1}mol^{-1}}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\ \mathrm{Nm^{2}kg^{-2}}$
Acceleration of free fall on the Earth's surface	$g = 9.81\ \mathrm{ms^{-2}}$

MATHEMATICAL IDENTITIES

In answering the questions on this paper you may make use of any of the following:

Divergence theorem $\int_V \nabla \cdot \underline{E}\, dV = \oint_S \underline{E} \cdot d\underline{S}$

Stoke's Theorem $\int_S \nabla \times \underline{E} \cdot d\underline{S} = \oint \underline{E} \cdot d\underline{\ell}$

Identities

$$\nabla \times \nabla \left(Scalar \right) = \underline{0}$$

$$\nabla \cdot \nabla \times \left(Vector \right) = 0$$

$$\nabla \times \left(\nabla \times \underline{E} \right) = \nabla \left(\nabla \cdot \underline{E} \right) - \nabla^2 \underline{E}$$

$$\nabla \left(\psi\, \underline{E} \right) = \psi\, \nabla \cdot \underline{E} + \underline{E} \cdot \nabla \psi$$

$$\nabla \cdot \left(\underline{E} \times \underline{H} \right) = \underline{H} \cdot \left(\nabla \times \underline{E} \right) - \underline{E} \cdot \left(\nabla \times \underline{H} \right)$$

Material Equations $\underline{J} = \sigma \underline{E} \quad \underline{B} = \mu \underline{H} \quad \underline{D} = \varepsilon \underline{E}$

Poynting Vector $\underline{S} = \underline{E} \times \underline{H}$

Trigonometric identities

$$cos(A \pm B) = cos\, A\, cos\, B \mp sin\, A\, sin\, B$$

$$sin(A \pm B) = sin\, A\, cos\, B \pm sin\, B\, cos\, A$$

$$cos\, A + cos\, B = 2\, cos\left(\frac{A+B}{2} \right) cos\left(\frac{A-B}{2} \right)$$

$$sin\, A - sin\, B = 2\, cos\left(\frac{A+B}{2} \right) sin\left(\frac{A-B}{2} \right)$$

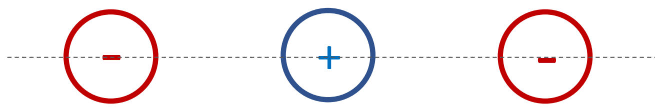
SECTION A

Use a section A answer book

Answer ALL 10 questions in this section

Full explanations of your answers are required to attain full marks

1. Consider two point-like negative charges equidistant from a central positive one (Fig. 1). All charges have the same amplitude. Draw the corresponding electric field lines. How would the field lines change if the central positive charge is made significantly stronger than the other two?



[4]

Figure 1

2. Figure 2 shows charges in vacuum in the vicinity of a cubic surface. For each case, state the total flux of the electric field passing through the surface. **[4]**

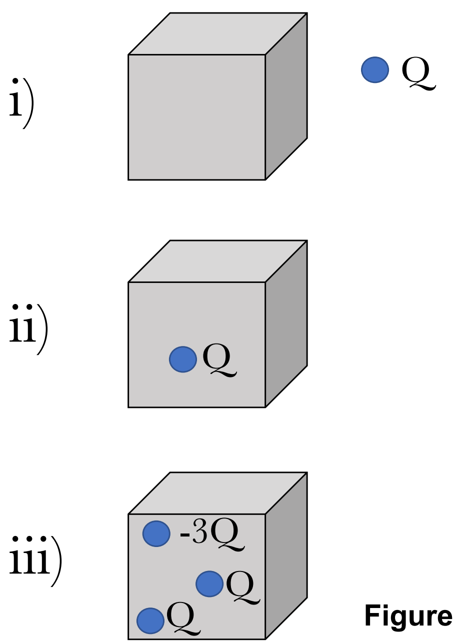


Figure 2

3. Define an electric dipole and show that the electric field it generates tends to zero when its size goes to zero. **[4]**

SECTION A

4. Show that the two vector potentials $\vec{A} = \overrightarrow{A_0} \ln r$ and $\overrightarrow{A_1} = \overrightarrow{A_0} (\ln r + \nabla r^2)$, where r is the radial coordinate and $\overrightarrow{A_0}$ is the potential amplitude, are associated with the same magnetic field. **[4]**

5. In magnetostatics, Ampère's circuital law in vacuum may be written as:

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad (1)$$

How would this equation be modified in a material and why? Show that the integral form of equation (1) is: $\oint \underline{B} \cdot d\underline{\ell} = \mu_0 \int \underline{J} \cdot d\underline{S}$ **[4]**

6. Two small Styrofoam balls of mass 3x10^{-3} kg are suspended from the same point by threads 5 cm long. Equal positive charges are placed on the balls. What must the magnitudes of these charges be if the balls are to remain in equilibrium at angles of 30^{o} from the vertical? **[4]**

7. Consider the circuit in Figure 3:

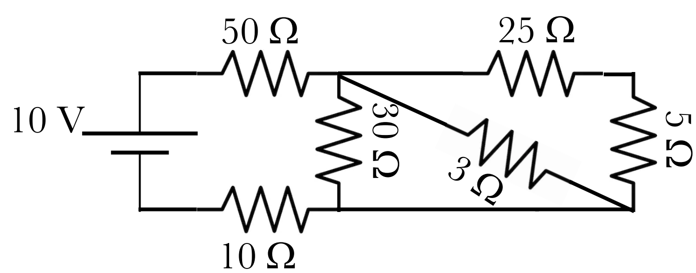


Fig. 3

What is the equivalent resistance of the whole circuit? What is the power dissipated? **[4]**

8. The electric field components of a plane wave are given by $E_x(t) = A\sin(kz - \omega t)$, $E_y(t) = A\cos(kz - \omega t + \delta)$. Describe for which δ (between 0 and 2π) the plane wave will be: (a) linearly polarized, (b) right-handed circularly polarized, and (c) left-handed circularly polarised. **[4]**

SECTION A

9. Use the Jones Vector expression to decompose a linear +45° polarisation into components of right-handed and left-handed circular polarisations. **[4]**

10. Describe the phenomenon of Faraday Effect and explain the physics that underpins the role of magnetic fields in the Faraday Effect. **[4]**

[END OF SECTION A]

SECTION B

Use a Section B answer book

Answer TWO questions from this section

Full explanations of your answers are required to attain full marks

11. Two hollow and thin spherical sheets of conducting material, of radii R_1 and R_2, respectively, are concentric to each other and carry equal and opposite charge Q (positive on the inner conductor). The space between the two conductors is filled with a dielectric of relative permittivity ε_r (volume marked with dashed lines in the figure 4).

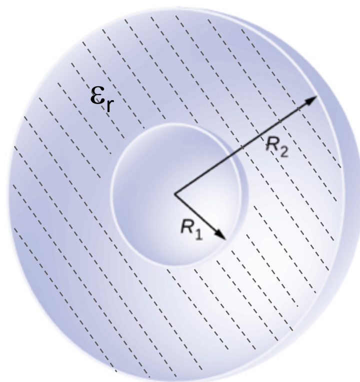


Fig. 4

(a) Sketch the distribution of charges on the conductors and the dielectric. **[2]**

(b) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric induction field ***D*** outside of the outer conductor ($r > R_2$). **[2]**

(c) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric induction field ***D*** in the dielectric ($R_2 > r > R_1$). **[5]**

(d) Calculate the volumetric and surface distribution of bound charges in the dielectric. **[4]**
Hint: you might need to use the divergence in spherical co-ordinates:

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

(e) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric displacement field ***D*** inside the inner conductor ($r < R_1$). **[2]**

(f) Calculate the potential difference between the two conductors and the capacitance of the whole system. **[5]**

SECTION B

12. Two long concentric and hollow conducting cylinders of radii *a* and *b* (*a*<*b*) carry equal and opposite current I. Each cylinder has a negligible thickness L<<a. The space between the two conductors is filled with a diamagnetic material of relative permeability μ_r (Fig. 5).

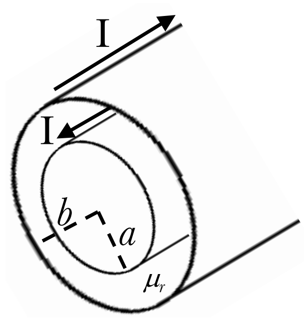


Figure 5

(a) Show that, in the approximation of L<<a,b , the areas traversed by the inner and outer currents are given, respectively, by $A_1 = 2\pi aL$ and $A_2 = 2\pi bL$. Calculate the corresponding current densities. **[3]**

(b) Draw the direction of the magnetic field generated by the inner current. **[1]**

(c) Calculate the amplitude of the ***B***, ***H***, and ***M*** fields for distances r < a. **[2]**

(d) Calculate the amplitude of the ***B***, ***H***, and ***M*** fields between the two conductors ($b > r > a$). **[5]**

(e) Calculate the amplitude of the ***B***, ***H***, and ***M*** fields outside of the whole system (r>b). **[2]**

(f) Discuss how the fields between the two conductors will change if the diamagnetic material is removed. **[2]**

(g) Discuss if wrapping the whole system in a paramagnetic material would change the magnetic field distribution generated by the system **[2]**

(h) In what direction should an external observer with the right velocity move in order not to observe any magnetic field between the two conductors? What type of field would the observer see for $r > b$? **[3]**

SECTION B

13. Consider a disc of conducting material of radius R, with a total charge Q deposited on it. The disc rotates around its axis with angular frequency ω, as in Fig. 6.

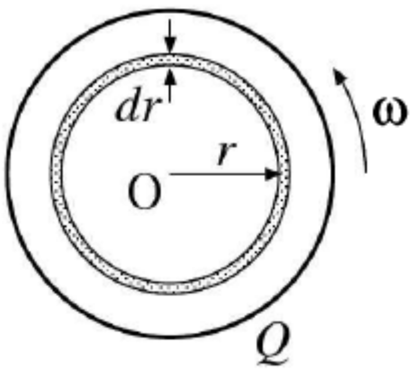


Figure 6

(a) Calculate the areal charge density on the disc. **[1]**

(b) Demonstrate that the infinitesimal area of a ring of width dr at a distance r from the centre is given by $dA=2\pi r\, dr$. **[2]**

(c) Show that the charge contained in the thin ring is: $dq = Q\, 2r\, dr / R^2$. **[2]**

(d) Show that the infinitesimal current generated by each of these thin circular rings once in motion with an angular frequency ω is $dI = Q\omega r / (\pi R^2)\, dr$ **[4]**

(e) Using Bio-Savart law ($d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$), show that the amplitude of the magnetic field generated by a ring of radius r is, in its center, given by $B = \frac{\mu_0 I}{2r}$ **[5]**

(f) Show that the magnetic field generated by each of these thin rings at the centre of the disc is then: $dB = \mu_0\omega Q / (2\pi R^2) dr$ **[2]**

(g) Calculate the total magnetic field generated at the centre of the disc. What is the direction of the field? **[4]**

SECTION B

14. A parallel plate capacitor of capacitance *C* as shown in Figure 7 holds energy $W = \frac{1}{2}CV_0{}^2$, where *V* is the initial potential across its circular plates. The capacitor is in vacuum and it is now slowly and completely discharged through a resistor *R*.

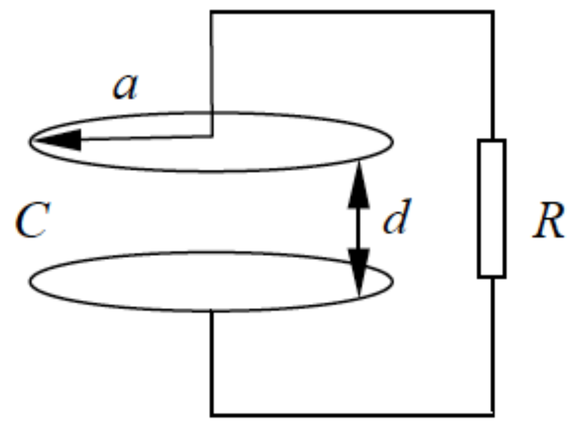


Figure 7

(a) Show that the potential, as the system discharges, can be expressed as:

$V(t) = V_0 e^{-t/_{RC}}$ **[4]**

(b) Using the expression in (a) and neglecting fringe field effects, calculate the electric field **E** at the edge of the plates. **[2]**

(c) Neglecting fringe field effects, calculate the magnetic field **H** at the edge of the plates using the generalised Ampere's law. **[4]**

(d) Show the direction of these fields on a diagram similar to the one in Figure 7. **[2]**

(e) Show on the same diagram the direction of the Poynting vector. **[1]**

(f) Calculate the amplitude of the Poynting vector. **[2]**

(g) By using the obtained expression for the Poynting vector, show that the total energy radiating out of the gap between the plates is equal to *W*. **[5]**

[END OF SECTION B]

SECTION C

Use a Section C answer book

Answer <u>ONE</u> question from this section

Full explanations of your answers are required to attain full marks

15 The reflectivity of a plane wave at the planar interface of two homogeneous media is given by $r_p = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$ and $r_s = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$ for p- and s-polarised light, respectively. A beam of unpolarised light of intensity I_0 is incident from a medium of refractive index $n_1 = 2.0$ to a medium of refractive index $n_2 = 1.6$.

(a) Calculate the Brewster angle and the critical angle, respectively. **[4]**

(b) For a beam of light incident at an angle of 20°, calculate r_p, r_s and the corresponding phase changes. **[5]**

(c) For the case described in (b), calculate the Stokes parameters and the degree of polarisation of the reflected beam. **[7]**

(d) For the case described in (b), calculate the intensities of the s- and p-polarisation components in the transmitted beam, respectively. **[4]**

SECTION C

16(a) Consider a two-level system as shown in Fig. 8 with $E_1 = -13.6$ eV and $E_2 = -11.4$ eV. Assume the Einstein coefficient $A_{21} \approx 6\times10^{8}\ \mathrm{s}^{-1}$.

(i) Calculate the spontaneous emission lifetime due to natural broadening. **[2]**

(ii) Assuming the refractive index of lasing medium is $n = 1.5$, calculate the Einstein coefficient B_{21}. **[3]**

(iii) Calculate the ratio between the spontaneous emission rate and the stimulated emission rate at room temperature. **[2]**

(iv) At what temperature will the stimulated emission rate equal the spontaneous emission rate? **[2]**

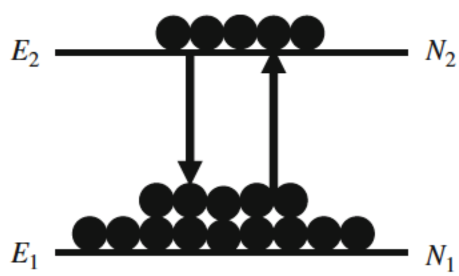


Figure 8

(b) **(i)** Describe what is population inversion and explain why it cannot be achieved in a 2-level system. **[2]**

(ii) Describe how population inversion is achieved in 3-level and 4-level lasing systems, respectively. Sketch diagrams to illustrate your answers. **[7]**

(iii) What is the main advantage of 4-level system compared to 3-level system? **[2]**

END OF EXAMINATION