**RELATIVITY** **Prof Ian Williams**

## SUMMARY OF FORMULAE

Lorentz Factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

Lorentz Transformation:

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{v}{c^2}x)$$

Lorentz-Fitzgerald Contraction:

$$L' = {}^{L_0}/_{\gamma}$$

Time Dilation:

$$\Delta t' = t_2' - t_1' = \gamma(t_2 - t_1)$$

Relativistic Mass:

$$m = \gamma(u)m_0$$

3-D Velocity Transformations:

$$u_x' = \frac{u_x - v}{(1 - \frac{v}{c^2}u_x)}$$

$$u_y' = \frac{u_y}{\gamma(1 - \frac{v}{c^2}u_x)}$$

$$u_z' = \frac{u_z}{\gamma(1 - \frac{v}{c^2}u_x)}$$

Generalised Lorentz Transformation Matrix:

$$\begin{pmatrix} \gamma & 0 & 0 & i\frac{\gamma v}{c} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\frac{\gamma v}{c} & 0 & 0 & \gamma \end{pmatrix}$$

4-Vectors Transform According to:

$$A_\mu{}' = a_{\mu\nu} A_\nu$$

Einstein's Summation Rule:

$$A'_\mu = \sum_{\nu=1}^{4} a_{\mu\nu} A_\nu = a_{\mu 1} A_1 + a_{\mu 2} A_2 + a_{\mu 3} A_3 + a_{\mu 4} A_4$$

4-Displacement:

$$X_\mu = \boldsymbol{X} = (X_1, X_2, X_3, X_4) = (x, y, z, ict) = (\boldsymbol{r}, ict)$$

4-Velocity:

$$U_\mu = \boldsymbol{U} = \gamma(u)(u_x, u_y, u_z, ic) = \gamma(u)(\boldsymbol{u}, ic)$$

4-Momentum:

$$U_\mu = \boldsymbol{P} = (p_x, p_y, p_z, \frac{iE}{c}) = (\boldsymbol{p}, \frac{iE}{c})$$

4-Acceleration:

$$\boldsymbol{A} = \gamma(u)\left(\gamma(u)\boldsymbol{a} + \frac{d\gamma(u)}{dt}\boldsymbol{u}, ic\frac{d\gamma(u)}{dt}\right)$$

4-Force:

$$\boldsymbol{F} = \gamma(u)\left(\boldsymbol{f}, \frac{i}{c}\frac{dE}{dt}\right)$$

4-Frequency:

$$\boldsymbol{N} = f(\frac{c}{\omega}\widehat{\boldsymbol{n}}, i)$$

Useful 4-Vector Relationships:

de Broglie Equation:

$$c\boldsymbol{P} = h\boldsymbol{N}$$

Invariance of Inner Products:

$$\boldsymbol{U}.\boldsymbol{U} = -c^2$$

$$\boldsymbol{U}.\boldsymbol{V} = -\gamma(u_R)c^2$$

$$\boldsymbol{P}.\boldsymbol{P} = -{m_0}^2c^2$$

$$\boldsymbol{A}.\boldsymbol{A} = {a_0}^2$$

$$\boldsymbol{U}.\boldsymbol{A} = 0$$

For 2 Photons:

$$\boldsymbol{P_A}.\boldsymbol{P_B} = -\frac{h^2}{c^2}f_A f_B(1 - cos\theta)$$

For a photon and a particle:

$$\boldsymbol{P}.\boldsymbol{Q} = -hm_0f'$$

Doppler Shift Formula:

$$f' = \gamma f(1 - \frac{v}{\omega}cos\theta)$$

Wave Aberration Formula:

$$tan\theta' = \frac{sin\theta}{\gamma(cos\theta - \frac{v\omega}{c^2})}$$

Compton Scattering Formula:

$$\lambda' - \lambda = \frac{h}{cm_e}(1 - cos\theta)$$

Vector and Scalar Potentials:

$$\boldsymbol{\mathcal{B}} = \boldsymbol{\nabla} \times \boldsymbol{\mathcal{A}}$$

$$\boldsymbol{\mathcal{E}} = -\boldsymbol{\nabla}\phi - \frac{\partial \boldsymbol{\mathcal{A}}}{\partial t}$$

4-D Del Operator:

$$\boxdot = \left(\frac{\partial}{\partial X}, \frac{\partial}{\partial Y}, \frac{\partial}{\partial Z}, -\frac{i}{c}\frac{\partial}{\partial t}\right) = \left(\boldsymbol{\nabla}, -\frac{i}{c}\frac{\partial}{\partial t}\right)$$

d'Alembertian:

$$\boxdot^{\mathbf{2}} = \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) = \left(\boldsymbol{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)$$

4-Potential:

$$\boldsymbol{A} = \left(A_X, A_Y, A_Z, \frac{i}{c}\phi\right)$$

4-Current Density:

$$\boldsymbol{J} = (J_X, J_Y, J_Z, ic\rho)$$

Maxwell's Equations in Terms of $\boldsymbol{A}$ and $\boldsymbol{J}$:

$$\boxdot . \boldsymbol{A} = 0$$

$$\boxdot^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}$$

Continuity Equation:

$$\boxdot . \boldsymbol{J} = 0$$

Electromagnetic Field Tensor:

$$\Gamma_{\mu\nu} = \begin{pmatrix} 0 & \mathcal{B}_Z & -\mathcal{B}_Y & -\frac{i}{c}\,\mathcal{E}_X \\ -\mathcal{B}_Z & 0 & \mathcal{B}_X & -\frac{i}{c}\,\mathcal{E}_Y \\ \mathcal{B}_Y & -\mathcal{B}_X & 0 & -\frac{i}{c}\,\mathcal{E}_Z \\ \frac{i}{c}\,\mathcal{E}_X & \frac{i}{c}\,\mathcal{E}_Y & \frac{i}{c}\,\mathcal{E}_Z & 0 \end{pmatrix}$$

Transformation Property:

$$\Gamma'_{\mu\nu} = a_{\mu\alpha}\, a_{\nu\beta}\, \Gamma_{\alpha\beta}$$

Maxwell's Equations in Terms of $\Gamma_{\mu\nu}$:

$$\frac{\partial \Gamma_{\mu\nu}}{\partial X_\lambda} + \frac{\partial \Gamma_{\nu\lambda}}{\partial X_\mu} + \frac{\partial \Gamma_{\lambda\mu}}{\partial X_\nu} = 0$$

$$\frac{\partial \Gamma_{\mu\nu}}{\partial X_\nu} = \mu_0\, J_\mu$$

Transformation Properties of $\boldsymbol{\mathcal{E}}$ and $\boldsymbol{\mathcal{B}}$:

$$\mathcal{E}'_X = \mathcal{E}_X$$

$$\mathcal{E}'_Y = \gamma\,(\mathcal{E}_Y - v\,\mathcal{B}_z)$$

$$\mathcal{E}'_Z = \gamma\,(\mathcal{E}_Z + v\,\mathcal{B}_Y)$$

$$\mathcal{B}'_X = \mathcal{B}_X$$

$$\mathcal{B}'_Y = \gamma\,(\mathcal{B}_Y + \frac{v}{c^2}\,\mathcal{E}_z)$$

$$\mathcal{B}'_Z = \gamma\,(\mathcal{B}_Z - \frac{v}{c^2}\,\mathcal{E}_Y)$$