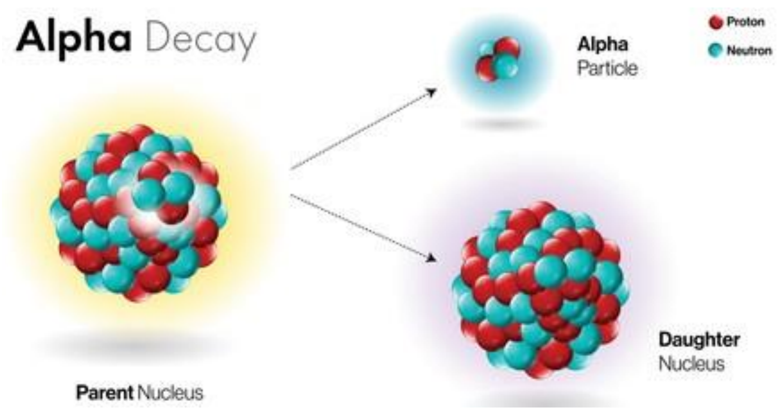
Nuclear and Radiation Physics (PHY2005)

Lecture 7

D. Margarone

2021-2022



QUEEN'S UNIVERSITY BELFAST

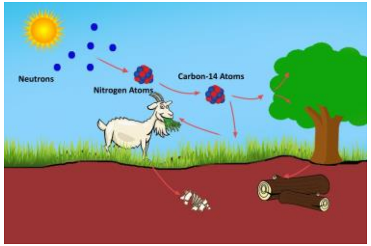
Recap & Learning Goals

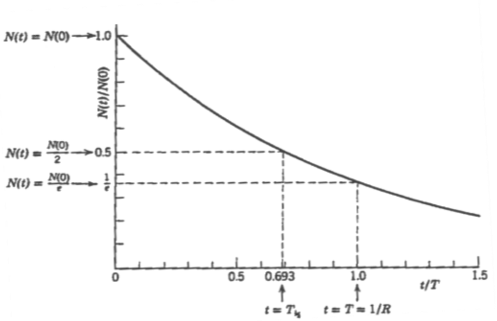
Summary of Lecture 6 *(Chap.4)*

- Nuclear Decay
  - ✓ Decay law
  - ✓ Application: C-14 dating

$$N(t) = N(0)e^{-Rt}$$

$$T_{1/2} = (\ln 2)T = 0.693T$$

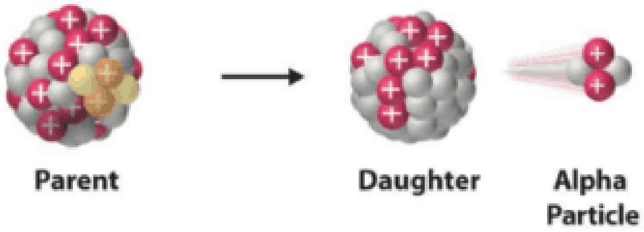




Learning goals of Lecture 7 *(Chap.4)*

- Knowing the theory of *alpha decay*
- Understanding physical reasoning behind *alpha decay*

QUEEN'S UNIVERSITY BELFAST



$$^{A}_{Z}X \rightarrow \, ^{A-4}_{Z-2}Y + \, ^{4}_{2}He$$

Alpha Decay *(general properties)*

- α-particles (^{4}He) → least penetrating radiation *(naturally occurring)*
- mass measured in 1903 *(Rutherford from decay of Rn)*
- typical natural decay in <u>heavy nuclei</u>
- why α-emission over other decay modes?

QUEEN'S UNIVERSITY BELFAST

4. Nuclear Decays
4.2 Alpha Decay II

Alpha Decay *(general properties) cont.*

- Coulomb repulsion ($\sim Z^2$) has a faster increase rate than nuclear binding energy ($\sim A$)
- α decay is <u>spontaneous</u> → kinetic energy emission out of a decrease in mass *(energetically possible only for α-particles)*
- α-emitter → not too small decay rate *($T_{1/2} < 10^{16}$ y)*

Emitted Particle	Energy Release (MeV)	Emitted Particle	Energy Release (MeV)
n	−7.26	^{4}He	+5.41
^{1}H	−6.12	^{5}He	−2.59
^{2}H	−10.70	^{6}He	−6.19
^{3}H	−10.24	^{6}Li	−3.79
^{3}He	−9.92	^{7}Li	−1.94

[a] Computed from known masses.

Alpha Decay Equations

- decay process → conservation of energy and momentum:

$$\boxed{{}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}X'_{N-2} + \alpha}$$

- spontaneous decay → **Q-value** > 0

$$\boxed{Q = (m_X - m_{X'} - m_\alpha)c^2}$$

- α-particle → ~ 98% of Q X' (heavy nucleus) → ~ 2%

$$\boxed{T_\alpha = Q(1 - \frac{4}{A})}$$

- Tα by spectrometer → Q measurement
  (useful for short-lived nuclei mass estimation)

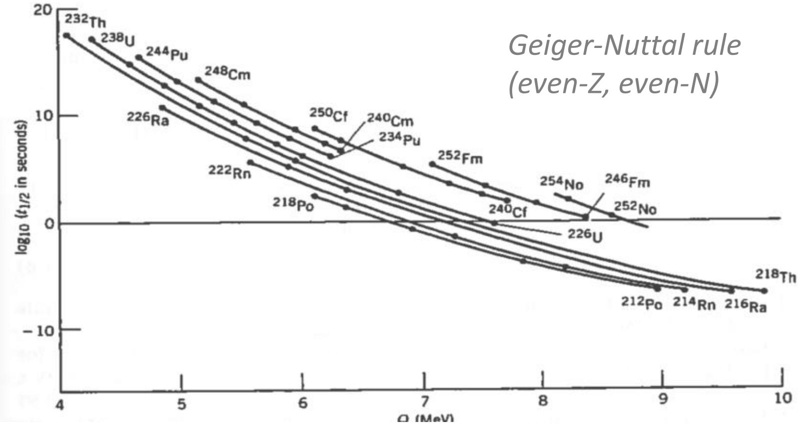
Alpha Decay *(peculiarities)*

- Geiger-Nuttal rule: α emitters with high Q-values → short half-life

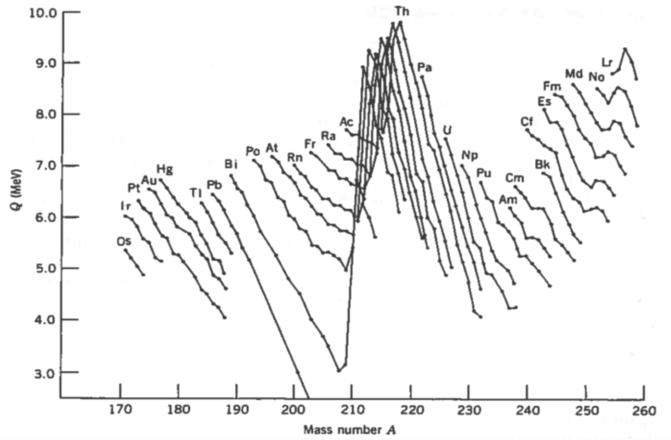
- Example:
  ^{232}Th → $Q = 4.08$ MeV; $T_{1/2} = 1.4 \times 10^{10}$ y
  ^{218}Th → $Q = 9.85$ MeV; $T_{1/2} = 1.0 \times 10^{-7}$ s **!**

- adding neutrons to heavy nuclei → Q-value decreases → … $T_{1/2}$ increases *(Geiger-Nuttal rule)* → more stable nuclei **!**

- discontinuity ~*A = 212 (N = 126)* → nuclear shell structure! 🙂



Geiger-Nuttal rule (even-Z, even-N)

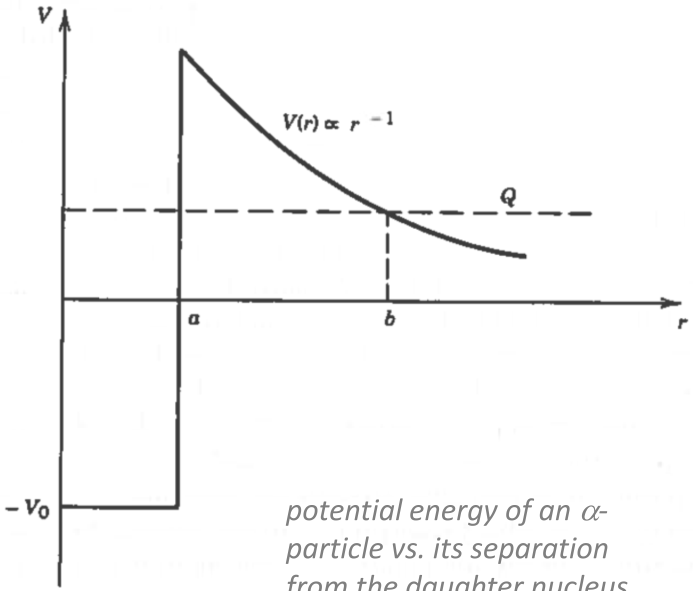


Q-values for various isotopic sequences

Alpha Decay *(theory)*

- Geiger-Nuttal → Gamow-Gurney quantum mechanical theory
- *one-body* model: α-particle preformed in the parent nucleus
- α-particle: moves in a spherical region determined by the daughter nucleus *(works well for even-even nuclei)*
- ***V(r)***: potential energy between α-particle and daughter nucleus
- ***Q***: disintegration energy
- ***a***: radius α-particle + radius daughter nucleus *(maximum value of Coulomb potential)*

- Three distinguished regions:
  - *i.* $r < a$: potential well V_0 *(α-particle moves with kinetic energy $Q + V_0$ but classically cannot escape)*
  - *ii.* $a < r < b$: potential barrier *(classically forbidden region from either directions)*
  - *iii.* $r > b$ *(classically permitted)*

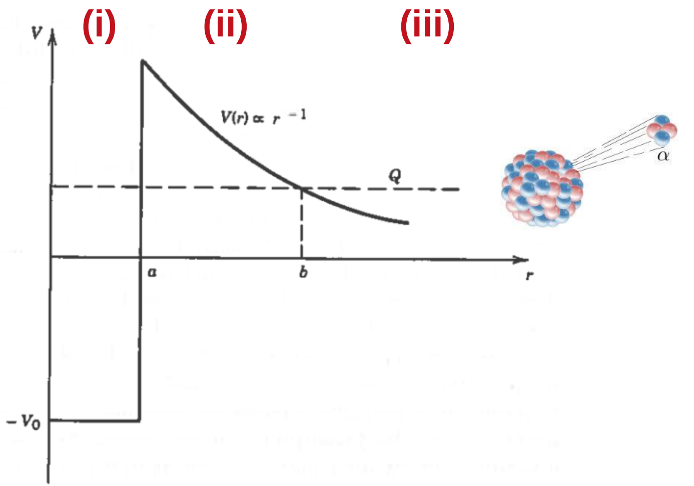


potential energy of an α-particle vs. its separation from the daughter nucleus

QUEEN'S UNIVERSITY BELFAST

Alpha Decay *(theory) cont.*

- classically: α-particle in (i) reverse its motion
- quantum mechanically: <u>tunnelling</u> through the barriers!!!
- Example:
  $^{238}U \rightarrow$ ~10^{38} tries to escape $\rightarrow 10^{9}$ y !!!



- disintegration constant/rate and transmission probability:
  (G: Gamow factor)

$$\boxed{\lambda = fP} \qquad \boxed{P = e^{-2G}}$$

Derive the equation for the Q-value of an α-decay reaction:

$$\boxed{Q = (m_X - m_{X'} - m_\alpha)c^2}$$

Conservation of energy:

$$m_X c^2 = m_{X'} c^2 + T_{X'} + m_\alpha c^2 + T_\alpha$$

$$(m_X - m_{X'} - m_\alpha)c^2 = T_{X'} + T_\alpha$$

$$Q = (m_X - m_{X'} - m_\alpha)c^2$$

Derive the equation for the kinetic energy of the α-particle emitted in α-decay:

$$\boxed{T_\alpha = Q\left(1 - \frac{4}{A}\right)}$$

Due to the conservation of linear momentum, X' and α must move with equal and opposite momenta:

$$\begin{cases} p_\alpha = p_{X'} \rightarrow p_\alpha^2 = p_{X'}^2 & (1) \\ Q = T_{X'} + T_\alpha & (2) \end{cases}$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2}\frac{m}{m} m v^2 = \frac{p^2}{2m}$$

$$(1) \rightarrow \cancel{2} m_\alpha T_\alpha = \cancel{2} m_{X'} T_{X'} \rightarrow T_{X'} = \frac{m_\alpha}{m_{X'}} T_\alpha$$

$$(2) \rightarrow Q = \frac{m_\alpha}{m_{X'}} T_\alpha + T_\alpha = T_\alpha\left(\frac{m_\alpha}{m_{X'}} + 1\right)$$

$$T_\alpha = \frac{Q}{1 + \frac{m_\alpha}{m_{X'}}} \underset{(A \gg 4)}{\cong} Q\left(1 - \frac{m_\alpha}{m_{X'}}\right) = \underline{Q\left(1 - \frac{4}{A}\right)}$$

QUEEN'S UNIVERSITY BELFAST

The α-decay reaction: $^{241}_{95}Am \rightarrow {}^{237}_{93}Np + {}^{4}_{2}He + 5.638\ MeV$
Calculate the kinetic energy of the α-particle and of the daughter nucleus

$$T_\alpha = Q\left(1 - \frac{4}{A}\right)$$

$$T_\alpha = 5.638\ [MeV] \times \left(1 - \frac{4}{241}\right) =$$

$$= 5.638 \times 0.983 = 5.544\ MeV$$

$$T_{Np} = Q - T_\alpha = 5.638 - 5.444 =$$

$$= 0.094\ MeV = \underline{94\ KeV\ (\simeq 1.7\ \%)}$$