Use lined, single-sided A4 paper
with a black or blue pen.
Write your student number
at the top of every page.

Any non-graphical calculator, except those with pre-programmable memory, may be used in this examination

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2004 - EXAM
Electricity, Magnetism and Optics
Wednesday, 27th May 2020, 9.30 AM - 13.30

Examiners: Prof S Matthews, Dr P van der Burgt
and the Internal Examiners
Dr J Greenwood (j.greenwood@qub.ac.uk)

Answer ALL TEN questions in Section A for 4 marks each.
Answer TWO questions in Section B for 20 marks each.
Answer ONE question in Section C for 20 marks.
You have FOUR hours to complete and upload this paper.

Contact the module coordinator if you have a query at g.sarri@qub.ac.uk and copy to mpts@qub.ac.uk

By submitting the work, you are declaring that:

1. The submission is your own original work and no part of it has been submitted for any other assignments;
2. You understand that collusion and plagiarism in an exam are major academic offences, for which a range of penalties may be imposed, as outlined in the Procedures for Dealing with Academic Offences.

THE QUEEN'S UNIVERSITY OF BELFAST
DEPARTMENT OF PHYSICS AND ASTRONOMY

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \text{ms}^{-1}$
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$ $\approx 1.26 \times 10^{-6}\ \text{Hm}^{-1}$
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{Fm}^{-1}$
Elementary charge	$e = 1.60 \times 10^{-19}\ \text{C}$
Electron charge	$= -1.60 \times 10^{-19}\ \text{C}$
Planck Constant	$h = 6.63 \times 10^{-34}\ \text{Js}$
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}\ \text{Js}$
Rydberg Constant for hydrogen	$R_{\infty} = 1.097 \times 10^{7}\ \text{m}^{-1}$
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}\ \text{kg}$ $1u = 931\ \text{MeV}$
1 electron volt (eV)	$= 1.60 \times 10^{-19}\ \text{J}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ \text{kg}$
Mass of proton	$m_p = 1.67 \times 10^{-27}\ \text{kg}$
Mass of neutron	$m_n = 1.67 \times 10^{-27}\ \text{kg}$
Molar gas constant	$R = 8.31\ \text{JK}^{-1}\text{mol}^{-1}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\ \text{JK}^{-1}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11}\ \text{Nm}^{2}\text{kg}^{-2}$
Acceleration of free fall on the Earth's surface	$g = 9.81\ \text{ms}^{-2}$

MATHEMATICAL IDENTITIES

In answering the questions on this paper you may make use of any of the following:

Divergence theorem $\int_V \nabla \cdot \underline{E}\, dV = \oint_S \underline{E} \cdot d\underline{S}$

Stoke's Theorem $\int_S \nabla \times \underline{E} \cdot d\underline{S} = \oint \underline{E} \cdot d\underline{\ell}$

Identities

$$\nabla \times \nabla (Scalar) = \underline{0}$$

$$\nabla \cdot \nabla \times (Vector) = 0$$

$$\nabla \times (\nabla \times \underline{E}) = \nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E}$$

$$\nabla(\psi \underline{E}) = \psi \nabla \cdot \underline{E} + \underline{E} \cdot \nabla \psi$$

$$\nabla \cdot (\underline{E} \times \underline{H}) = \underline{H} \cdot (\nabla \times \underline{E}) - \underline{E} \cdot (\nabla \times \underline{H})$$

Material Equations $\underline{J} = \sigma \underline{E} \quad \underline{B} = \mu \underline{H} \quad \underline{D} = \varepsilon \underline{E}$

Poynting Vector $\underline{S} = \underline{E} \times \underline{H}$

Trigonometric identities

$$cos(A \pm B) = cos\, A\, cos\, B \mp sin\, A\, sin\, B$$

$$sin(A \pm B) = sin\, A\, cos\, B \pm sin\, B\, cos\, A$$

$$cos\, A + cos\, B = 2\, cos\left(\frac{A+B}{2} \right) cos\left(\frac{A-B}{2} \right)$$

$$sin\, A - sin\, B = 2\, cos\left(\frac{A+B}{2} \right) sin\left(\frac{A-B}{2} \right)$$

Divergence in spherical coordinates: $\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta} \frac{\partial A_\phi}{\partial \phi}$

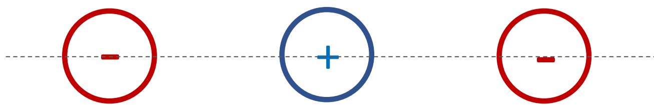
SECTION A

Use a section A answer book

Answer ALL 10 questions in this section

Full explanations of your answers are required to attain full marks

1. Consider two point-like negative charges equidistant from a central positive one (Fig. 1). All charges have the same amplitude. Draw the corresponding electric field lines and equipotential surfaces



[4]

Figure 1

2. Figure 2 shows charges in vacuum in the vicinity of a cubic surface. For each case, state the total flux of the electric field passing through the surface. **[4]**

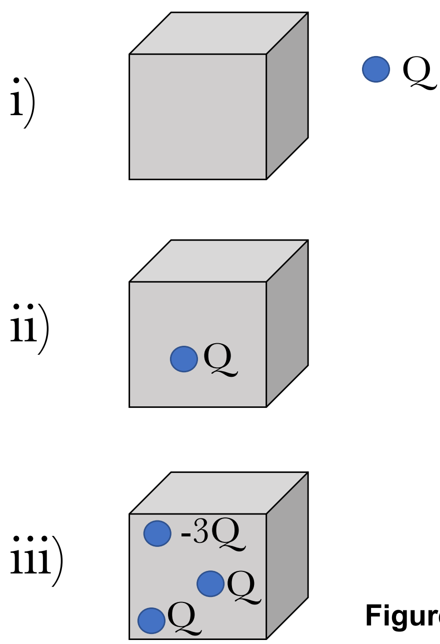


Figure 2

3. Define an electric dipole and show that the electric field it generates tends to zero when its size goes to zero. **[4]**

4. Consider the circuit in Figure 3:

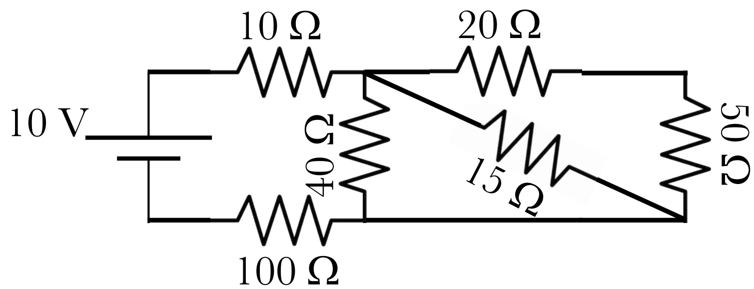


Figure 3

What is the equivalent resistance of the whole system? What is the current exiting the power supply? **[4]**

5. Consider a cylinder of paramagnetic material of radius a and relative permeability μ_r through which a uniform current I is steadily flowing, as in figure 4.

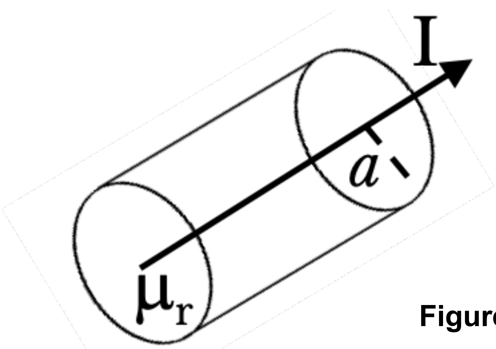


Figure 4

Calculate expressions for the amplitude of the ***B***, ***H***, and ***M*** fields *inside* the conductor. How would these quantities change if the current were to propagate in vacuum instead? **[4]**

6. Show that the two vector potentials $\vec{A} = \overrightarrow{A_0} \ln r$ and $\overrightarrow{A_1} = \overrightarrow{A_0} (\ln r + \nabla r^2)$, where *r* is the radial coordinate and $\overrightarrow{A_0}$ is the potential amplitude, are associated with the same magnetic field. **[4]**

7. Figure 5 shows a thin conducting ring. The ring sits in a uniform magnetic field, which is orthogonal to the plane of the ring. The magnetic field reduces linearly with time *t*, according to $B = B_o(1 - t/T)$, with *T* a constant.

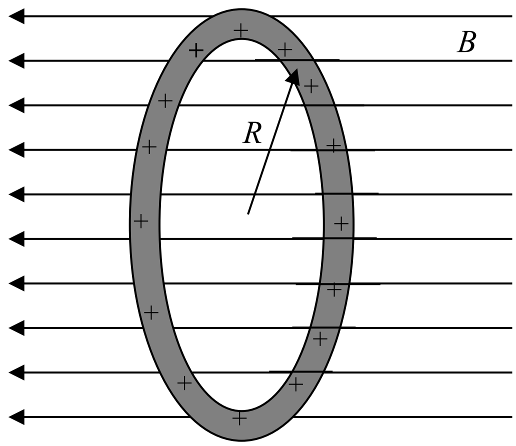


Figure 5

Indicate the direction of the electric field at the ring circumference induced by the magnetic field. Show that this field is given by: $E = \dfrac{B_0 R}{2T}$ **[4]**

8. A plane wave is propagating along the z-axis. The electric fields in the x-y plane have The following expressions:

$$E_x(t) = E_{x0}\cos(\omega t)$$

$$E_y(t) = E_{y0}\cos(\omega t + \delta)$$

Indicate the conditions for the plane wave to be linearly-polarised and left-circularly polarised, respectively. **[4]**

9. Describe the processes of stimulated light emission and the properties of the emitted light. Illustrate your answers with diagrams. **[4]**

10. The diagram below (Figure 6) shows the cross-section of an optical fibre, which is made of a glass core of $n_1 = 1.52$ and a cladding layer of $n_2 = 1.44$. Calculate the maximum incident angle θ_{max} that would result into total internal reflection inside the fibre. **[4]**

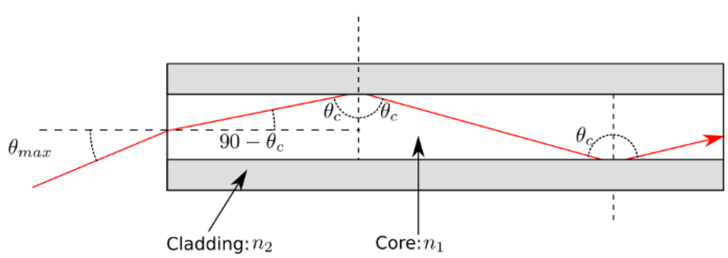


Figure 6

SECTION B

Use a Section B answer book

Answer TWO questions from this section

11. Two hollow and concentric spheres of conducting material and radii *a* and *b* ($a<b$) carry equal and opposite charge *Q* (positive on the inner conductor). The space between the two conductors is filled with a dielectric of relative permittivity ε_r (Figure 7).

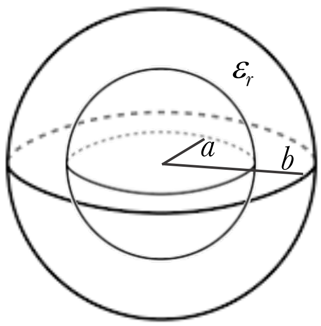


Figure 7

(a) Sketch the distribution of free and bound charges on the conductors and the dielectric. **[2]**

(b) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric displacement field ***D*** inside the inner conductor ($r < a$). **[2]**

(c) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric displacement field ***D*** in the dielectric ($b > r > a$). **[4]**

(d) Draw the direction of the three fields between the two conductors. **[3]**

(e) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric displacement field ***D*** outside the outer conductor ($r > b$). **[2]**

(f) Calculate the potential difference between the two conductors. **[3]**

(g) Calculate the capacitance of the whole system. **[1]**

(h) Calculate the volumetric and surface density of the bound charges. **[3]**

SECTION B

12. Two long concentric and hollow conducting cylinders of radii *a* and *b* (*a*<*b*) carry equal and opposite current I. Each cylinder has a negligible thickness L<<a. The space between the two conductors is filled with a diamagnetic material of relative permeability μ_r (Fig. 8).

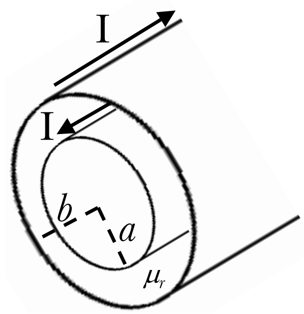


Figure 8

(a) Show that, in the approximation of L<<a,b , the areas traversed by the inner and outer currents are given, respectively, by $A_1 = 2\pi aL$ and $A_2 = 2\pi bL$. Calculate the corresponding current densities. **[3]**

(b) Draw the direction of the magnetic field generated by the inner current. **[1]**

(c) Calculate the amplitude of the ***B***, ***H***, and ***M*** fields for distances r < a. **[2]**

(d) Calculate the amplitude of the ***B***, ***H***, and ***M*** fields between the two conductors (*b* > *r* > *a*). **[5]**

(e) Calculate the amplitude of the ***B***, ***H***, and ***M*** fields outside of the whole system (r>b). **[2]**

(f) Discuss how the fields between the two conductors will change if the diamagnetic material is removed. **[2]**

(g) Discuss if wrapping the whole system in a paramagnetic material would change the magnetic field distribution generated by the system **[2]**

(h) In what direction should an external observer with the right velocity move in order not to observe any magnetic field between the two conductors? What type of field would the observer see for *r* > *b* ? **[3]**

SECTION B

13. Consider a disc of conducting material of radius *R*, with a total charge *Q* deposited on it. The disc rotates around its axis with angular frequency ω, as in Fig. 9.

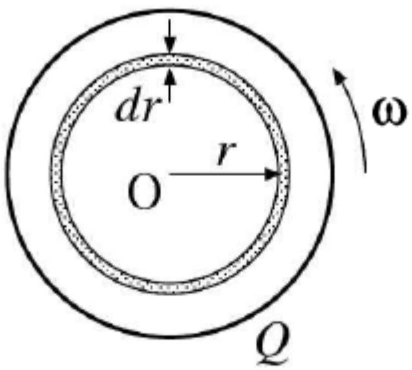


Figure 9

(a) Calculate the areal charge density on the disc. **[1]**

(b) Demonstrate that the infinitesimal area of a ring of width *dr* at a distance *r* from the centre is given by $dA=2\pi r\, dr$. **[2]**

(c) Show that the charge contained in the thin ring is: $dq = Q\, 2r\, dr / R^2$. **[2]**

(d) Show that the infinitesimal current generated by each of these thin circular rings once in motion with an angular frequency ω is $dI = Q\omega r / (\pi R^2)\, dr$ **[4]**

(e) Using Bio-Savart law ($d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{\ell} \times \vec{r}}{r^3}$), show that the amplitude of the magnetic field generated by a ring of radius *r* is, in its center, given by $B = \frac{\mu_0 I}{2r}$ **[5]**

(f) show that the magnetic field generated by each of these thin rings at the centre of the disc is then: $dB = \mu_0 \omega Q / (2\pi R^2) dr$ **[2]**

(g) Calculate the total magnetic field generated at the centre of the disc. What is the direction of the field? **[4]**

SECTION B

14. Figure 10 shows a short section of a long cylindrical conductor of resistivity ρ and radius r carrying a steady current I

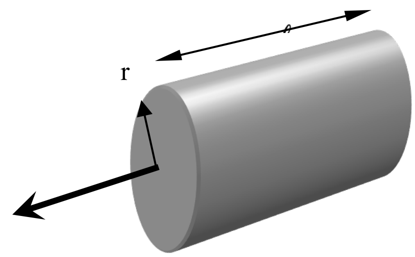


Figure 10

(a) Show that the magnetic field B at the surface is given by $B = \mu_0 I/(2\pi r)$ **[3]**

(b) Redraw the diagram and show clearly the direction of the $\underline{E}$ and $\underline{H}$ vectors. **[2]**

(c) Indicate the direction of the Poynting vector at the surface of the cylinder. **[1]**

(d) Calculate the current density flowing through the material. **[1]**

(e) Show that the flow of electromagnetic power per unit area across the curved surface is given by:

$$\frac{\rho I^2}{2\pi^2 r^3}$$ **[5]**

(f) Using the result obtained in question (e), show that the total power radiated by the cylinder is:

$$\frac{\rho I^2 \ell}{\pi r^2}$$ **[4]**

(g) Show that this result is consistent with Ohm's law **[4]**

SECTION C

Use a Section C answer book

Answer ONE question from this section

15. A plane wave of unpolarised light is incident from air ($n_1 = 1$) to a flat and smooth water surface ($n_2 = 1.33$).

(i) Calculate the reflectance of light at normal incidence. **[3]**

(ii) Calculate the Brewster angle. **[3]**

(iii) Indicate the polarisation states of the reflected beam at the Brewster angle and other incident angles, respectively. Provide rationales to your answers. **[4]**

(iv) Sketch the graphs of the reflectance of s- and p-polarisation light as a function of the incident angle, increasing from normal incidence to near 90 degrees, respectively. **[6]**

(v) Fishermen often wear polarised sunglasses to reduce the specular reflection of sun light at grazing angles on water surface. Explain how it works. **[2]**

(vi) If light is incident from water to air, which answers of (i-iii) would be different? Why? **[2]**

SECTION C

16.

(i) Describe how to produce linearly-polarised and circularly-polarised light from an unpolarised light source, respectively. Illustrate your answers with diagrams if necessary. **[4]**

(ii) Describe what is the Faraday effect and briefly explain the physical mechanisms of the effect. **[6]**

(iii) Consider the two-level system shown below (Figure 11) with $E_1 = -13.6$ eV and $E_2 = -3.4$ eV. Assume the Einstein coefficient $A_{21} \approx 6\times10^8\ s^{-1}$. What is the frequency (in unit of Hertz) of light emitted due to transitions from E_2 to E_1? Assuming the emission to have only natural broadening, what is the line width of the emission? What is the population ratio N_2/N_1 at T = 300 K? **[4]**

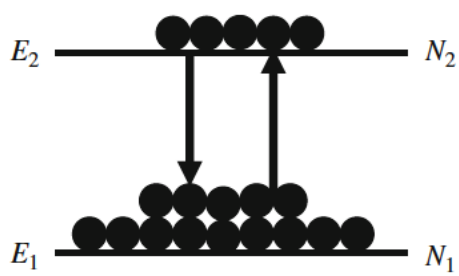


Figure 11

(iv) Explain what is the population inversion process and why it is essential in lasing process. Sketch a diagram and describe how this can be achieved in a 3-level lasing system. **[6]**

END OF THE EXAMINATION PAPER