QUEEN'S UNIVERSITY BELFAST

PHY2001

Exam Time Table Code PHY2001

Any calculator, except one with pre-programmable memory, may be used in this examination.

Answer Books A, B and C

LEVEL 2

EXAMINATION CONTRIBUTING TO THE DEGREES OF BACHELOR OF SCIENCE (BSc) AND MASTER IN SCIENCE (MSci)

PHY2001
Quantum and Statistical Physics

Duration: 3 Hours

Wednesday, 15th August 2018 9:30 AM - 12:30 PM

Examiners: Prof. P. Browning
Dr. P. van der Burgt
and the Internal Examiners

Answer ALL TEN questions in Section A for 4 marks each.
Answer TWO questions in Section B for 20 marks each.
Answer ONE question in Section C for 20 marks.

Use a separate answer book for each Section.
Follow the instructions on the front of the answer book. Enter your Anonymous Code number and Seat number, but NOT your name.

THE QUEEN'S UNIVERSITY OF BELFAST
SCHOOL OF MATHEMATICS & PHYSICS

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^8 \text{ ms}^{-1}$
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$ $\approx 1.26 \times 10^{-6} \text{ Hm}^{-1}$
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$
Elementary charge	$e = 1.60 \times 10^{-19} \text{ C}$
Electron charge	$= -1.60 \times 10^{-19} \text{ C}$
Planck Constant	$h = 6.63 \times 10^{-34} \text{ Js}$
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34} \text{ Js}$
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^7 \text{ m}^{-1}$
Unified atomic mass unit	$1u = 1.66 \times 10^{-27} \text{ kg}$ $1u = 931 \text{ MeV}$
1 electron volt (eV)	$= 1.60 \times 10^{-19} \text{ J}$
Mass of electron	$m_e = 9.11 \times 10^{-31} \text{ kg}$
Mass of proton	$m_p = 1.67 \times 10^{-27} \text{ kg}$
Mass of neutron	$m_n = 1.67 \times 10^{-27} \text{ kg}$
Molar gas constant	$R = 8.31 \text{ JK}^{-1}\text{mol}^{-1}$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$
Avogadro constant	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$
Acceleration of free fall on the Earth's surface	$g = 9.81 \text{ ms}^{-2}$

continued …

SECTION A

Use a section A answer book

Answer ALL 10 questions in this section

Full explanations of your answers are required to attain full marks

1 An electron is in an excited atomic state of an atom with a lifetime of approximately 5 ns. When it decays it emits a photon. What is the uncertainty in the energy of the emitted photon? **[4]**

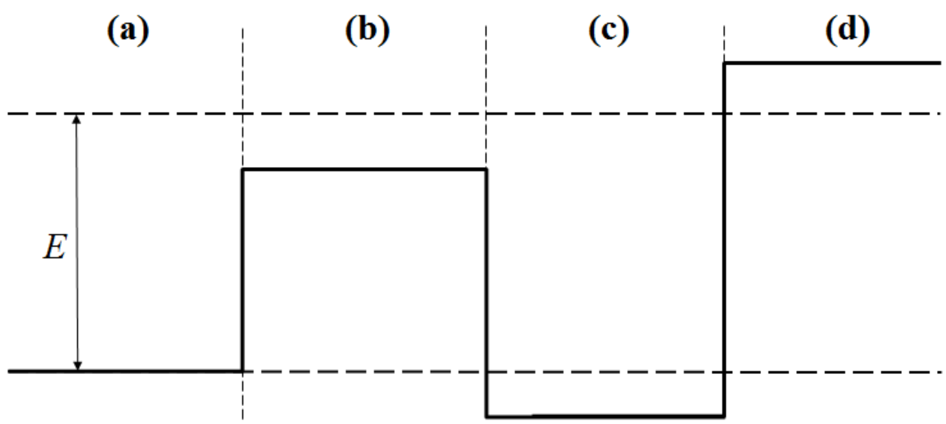
2 A particle is described by a wavefunction

$\psi = Ax(1+i)$ for $-1 < x < +1$

$\psi = 0$ for $x > +1,\ x < -1$

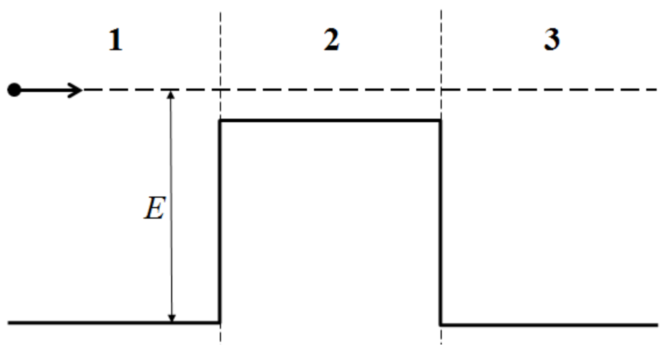
Find the value of the normalisation constant A. **[4]**

3 The diagram below shows a particle with energy E in four regions (a)-(d) with different potential energy (solid line). Which region has the shortest de Broglie wavelength?



[4]

4 A particle of energy E approaches a potential barrier as shown below. Under what conditions is there 100% probability that the particle is transmitted past the barrier?



[4]

continued …

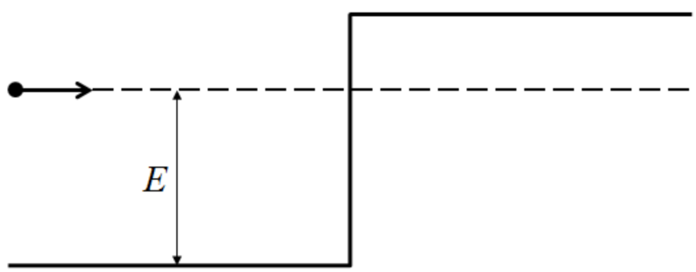
SECTION A

5 Consider the quantum energy levels of a

- Harmonic potential well
- Infinite potential well
- Coulomb potential well
- Finite potential well

Explain why the separation of the quantum levels in one of these wells decreases with increasing quantum number. **[4]**

6 The diagram shows a particle of total energy E approaching a barrier with a potential energy greater than E. Which of the following particles penetrates the greatest distance into the barrier and why?



(a) $E = 5$ eV electron

(b) $E = 5$ eV proton

(c) $E = 10$ eV electron

(d) $E = 10$ eV proton **[4]**

7 Draw an energy level diagram obtained from the solution of the Schrödinger equation for the H atom. Indicate the quantum numbers and associated degeneracies for the first 3 levels. **[4]**

8 Briefly explain what is meant by the terms *microstate*, *macrostate* and *distribution* in statistical mechanics. Your answer should include examples of these concepts for a classical ideal gas.

[4]

continued …

SECTION A

9 Consider a system of N weakly-interacting distinguishable particles. Each particle has access to four degenerate single-particle states. Give an expression for the total number of microstates available to this system. **[4]**

10 Describe the properties of fermions and bosons. Give at least one example particle of each. **[4]**

continued …

SECTION B

Use a Section B answer book

Answer TWO questions from this section

11 **(a)** Discuss the suitability of using plane wave to describe a particle in quantum mechanics. Include reference to the probability density and the velocity of the particle. **[4]**

(b) Explain how multiple plane waves can be used to provide a quantum mechanical representation of a particle and how this leads to the Uncertainty Principle. **[6]**

(c) An electron is confined in a one-dimensional infinite potential well of 1.0 nm width. According to the Uncertainty Principle what is the minimum energy of the electron. **[5]**

(d) Show how the Schrödinger equation was postulated using the concepts of a wavefunction and conservation of energy. **[5]**

continued …

SECTION B

12 **(a)** **(i)** A particle of mass m is trapped in a 1-dimensional potential well with $V(x) \to \infty$ at $x = \pm a/2$ and $V(x) = 0$ for $|x| < a/2$. Obtain the eigenfunctions of the time independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (V - E)\psi = 0$$

for this system and hence determine an expression for the allowed energy levels. **[8]**

(ii) An electron is confined within an infinite 1D potential well (V = 0 inside well) of width 0.4 nm. Show that there are two even parity eigenfunctions with energies less than 50 eV. **[3]**

(b) If the particle is trapped in a 3-dimensional potential where $|x|, |y|, |z| < a/2$ the energy levels are given by

$$E = \frac{\pi^2\hbar^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

(i) Explain the origin of n_x, n_y, n_z and their allowed values. **[3]**

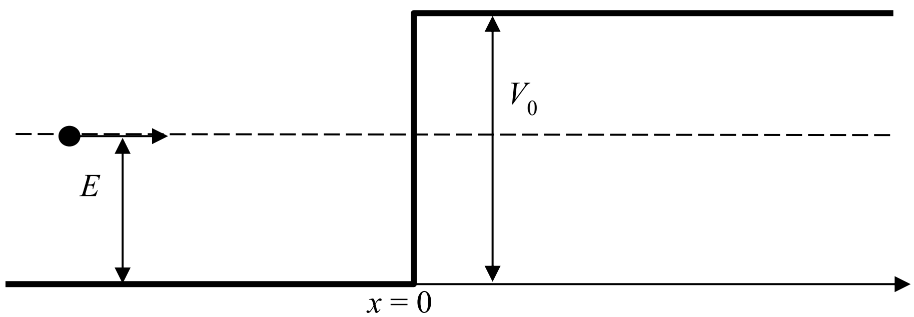
(ii) Define degeneracy and using this example explain how it is related to symmetry. **[3]**

(iii) Draw an energy level diagram comparing the first 5 allowed energy levels for the 1D and 3D cases. Indicate the degeneracy of each level. **[3]**

continued …

SECTION B

13 A particle with energy E approaches a potential step of height V_0 (where $E < V_0$) as shown below



(a) The time independent Schrödinger wave equation is given below, along with its solution ψ in the region $x > 0$.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (V - E)\psi = 0$$

$$\psi = D\exp(-\alpha x)$$

(i) Show how the expression for ψ can be obtained from a solution of the wave equation, and hence obtain an expression for α. **[4]**

(ii) In principle, explain how an expression for D in terms of the incident amplitude could be obtained. (It is not necessary to actually derive it). **[4]**

(b) The tip of a scanning tunnelling microscope is placed 0.2 nm above a conducting surface and a potential difference of δV = +0.04 V is applied to the surface relative to the tip. The tip is moved 0.01 nm further from the surface. If the tunnelling current is to remain the same, estimate the new value of δV required? The work function of the tip and the surface are both 4.2 eV. **[12]**

continued …

SECTION B

14 **(a)** The equation below is the radial Schrödinger wave equation for the hydrogen atom. Without going into full mathematical detail, explain how this equation is obtained from the full three-dimensional Schrödinger wave equation and explain what each of the terms inside the brackets represent. **[6]**

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{2\mu}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_o r} - \frac{l(l+1)\hbar^2}{2\mu r^2}\right)R = 0$$

(b) Explain how the quantum number l is related to the orbital angular momentum of the electron. **[3]**

(c) Explain the physical significance of the magnetic quantum number m_l and explain why it does not appear in the radial equation. **[5]**

(d) The symmetry of a hydrogen atom in the 4f state is broken by an external field along the z axis. Calculate the magnitude of the orbital angular momentum and its allowed spatial orientations. **[6]**

continued …

SECTION C

Use a Section C answer book

Answer ONE question from this section

15 **(a)** The Boltzmann distribution gives the probability of an energy level i being populated as

$$p_i = \frac{g_i}{Z} e^{-\varepsilon_i/kT}$$

(i) Give the meaning of each of the symbols in this equation and write down an expression for the partition function, Z. **[4]**

(ii) Discuss the physical conditions for which the Boltzmann distribution can be used. **[4]**

(b) The table below gives the energies and degeneracies of the lowest four states of the calcium atom.

(i) Use the data to calculate the partition function for the calcium atom at a temperature of 4000 K. **[4]**

(ii) Calculate the fraction of calcium atoms that would occupy Level 2 in a gas at 4000 K. **[2]**

(iii) Find the temperature at which the number of atoms in Level 2 will be 10% of the number in Level 1. **[6]**

Level	Energy (eV)	Degeneracy
1	0.00	1
2	1.89	9
3	2.52	15
4	2.71	5

continued …

SECTION C

16 **(a)** Draw a sketch to show the shape of the Fermi-Dirac distribution as a function of energy for a temperature much lower than the Fermi temperature (i.e. $T \ll T_F$). Briefly explain why the Fermi-Dirac function has this shape and give a definition of the Fermi energy, ε_F. **[6]**

(b) The density of states for a three-dimensional gas of spin-1/2 particles that occupy a volume V is given by

$$g(\varepsilon)d\varepsilon = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2}d\varepsilon$$

where m is the mass of each particle.

(i) If such a system has N particles, derive an expression for the Fermi energy ε_F. **[6]**

(ii) Show that, if $T \ll T_F$, the average energy per particle is given by $\frac{3}{5}\varepsilon_F$. **[5]**

(iii) How would the average energy per particle be related to the Fermi energy if the particles were confined to a two-dimensional surface rather than a three-dimensional volume? **[3]**

END OF EXAMINATION