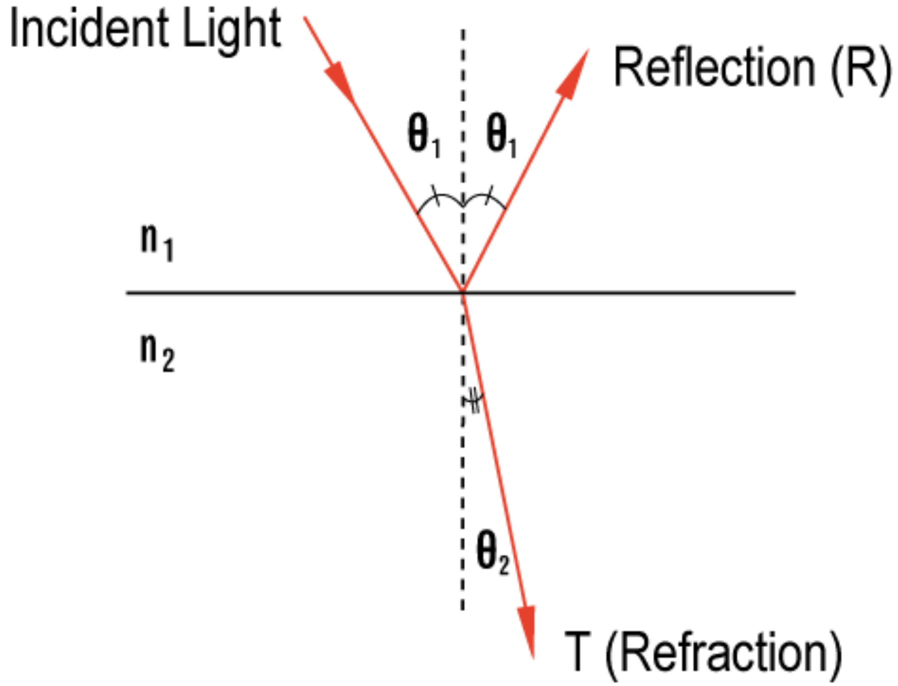
Topic 2

Reflection and refraction of light at a planar interface

Reflection and Refraction

When light is incident at the interface of two isotropic media, part of light will be reflected and part will transmit into the second medium (at a different angle) which is called refraction.



Snell's Law

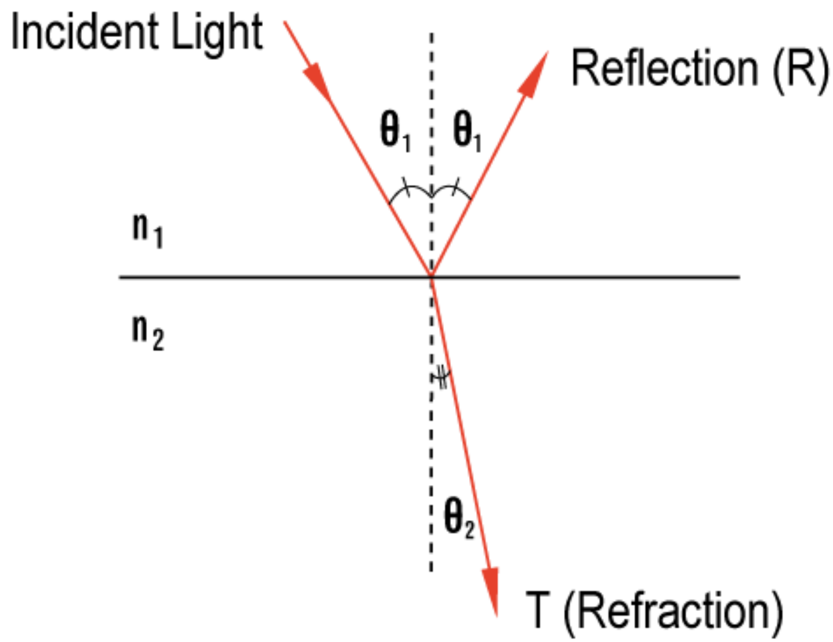
- Snell's law for refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

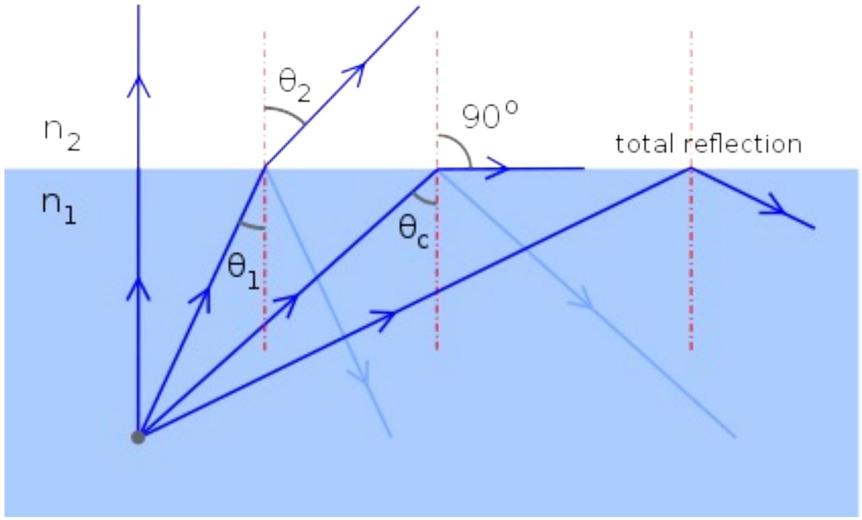
$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

If $n_1 > n_2$, then $\theta_2 > \theta_1$

If $n_1 < n_2$, then $\theta_2 < \theta_1$



Critical Angle



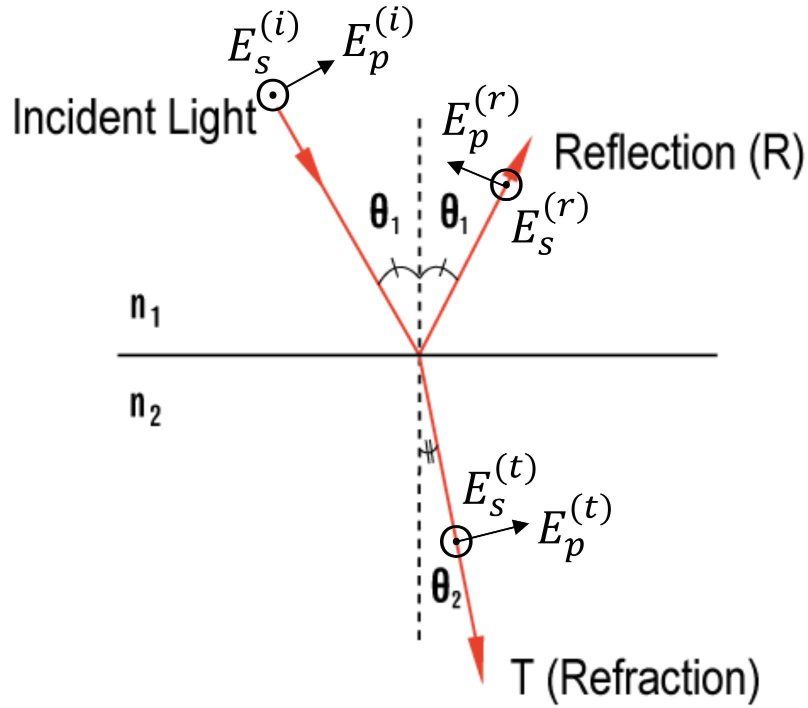
When light is incident from a medium of high refractive index to a medium of low refractive index, i.e., $\boldsymbol{n_1} > \boldsymbol{n_2}$, there exists a critical angle θ_c at which the refracted angle is 90°. Light incident at angles larger than θ_c will be 100% reflected. This is called **total internal reflection**.

$$\sin 90° = \frac{n_1}{n_2} \sin \theta_c = 1 \rightarrow \sin \theta_c = \frac{n_2}{n_1}$$

Reflectivity and Transmittivity

$$reflectivity = \frac{reflected\ field}{incident\ field}$$

$$transmittivity = \frac{transmitted\ field}{incident\ field}$$



Reflectivity and transmittivity are dependent on polarization.
p-polarization: also called transverse magnetic (TM) polarization, E-field is in the plane of incidence.
s-polarization: also called transverse electric (TE) polarization, E-field is perpendicular to the plane of incidence.

Reflectivity $\quad r_p = \frac{E_p^{(r)}}{E_p^{(i)}} \qquad r_s = \frac{E_s^{(r)}}{E_s^{(i)}}$

Transmittivity $\quad t_p = \frac{E_p^{(t)}}{E_p^{(i)}} \qquad t_s = \frac{E_s^{(t)}}{E_s^{(i)}}$

Fresnel Formulae

- p-polarization (TM-polarization)

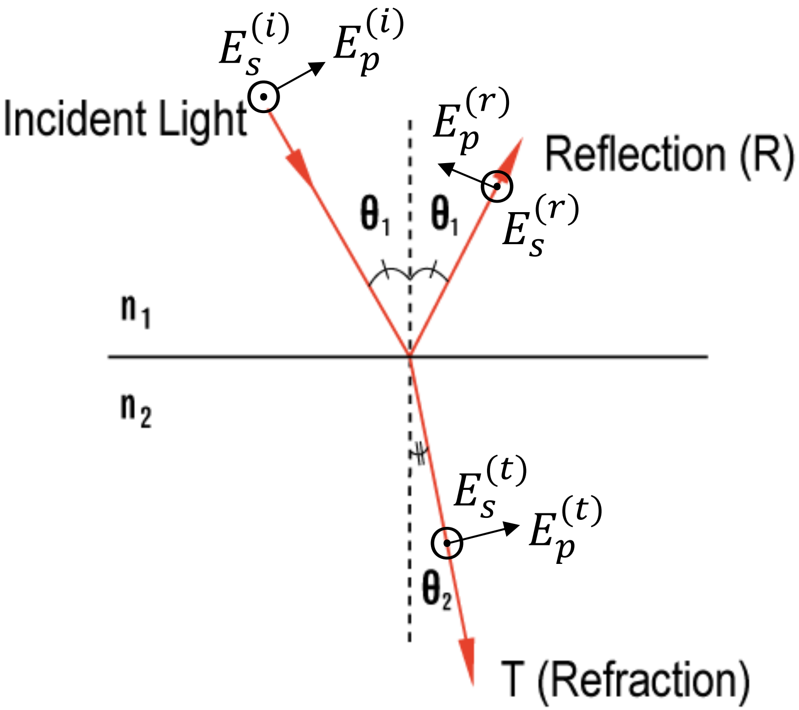
$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$t_p = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

- s-polarization (TE-polarization)

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

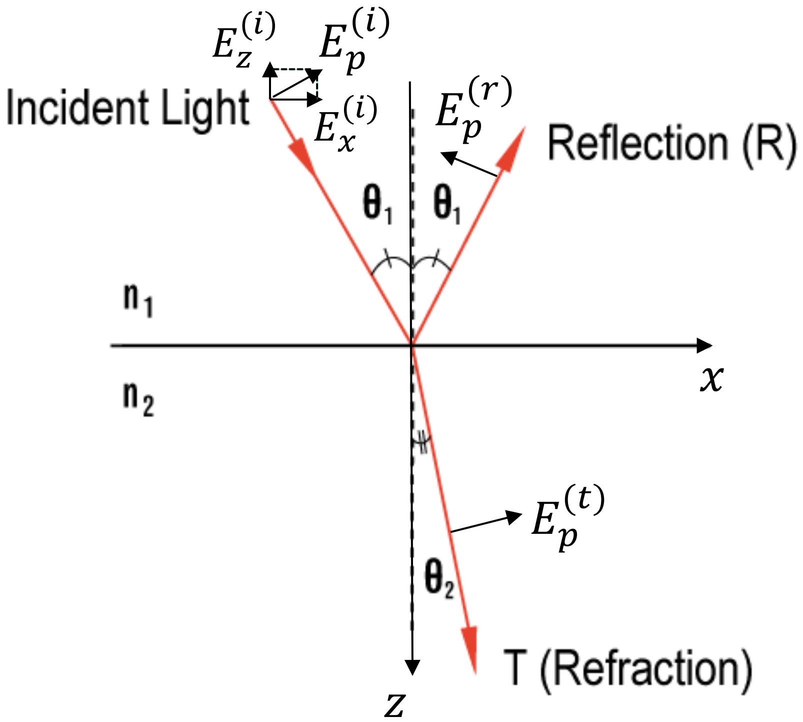
$$t_s = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$



Reflectivity and *Transmittivity* of s-polarization and p-polarization are **independent** from each other.

Derivation of Fresnel Formulae

(1) The case of p-polarization



$E_p^{(i)}$, $E_p^{(r)}$, $E_p^{(t)}$ represent the incident, reflected and transmitted fields, respectively. Each electric field can be decomposed into x- and z- components.

Incident field: $E_x^{(i)} = E_p^{(i)} \cos\theta_1$

$$E_z^{(i)} = -E_p^{(i)} \sin\theta_1$$

Reflected field: $E_x^{(r)} = -E_p^{(r)} \cos\theta_1$

$$E_z^{(r)} = -E_p^{(r)} \sin\theta_1$$

Refracted field: $E_x^{(t)} = E_p^{(t)} \cos\theta_2$

$$E_z^{(t)} = -E_p^{(t)} \sin\theta_2$$

Derivation of Fresnel Formulae

<u>Boundary Conditions</u>

1. Tangent components of E-field are continuous across the boundary

$$E_x^{(i)} + E_x^{(r)} = E_x^{(t)} \quad (1)$$

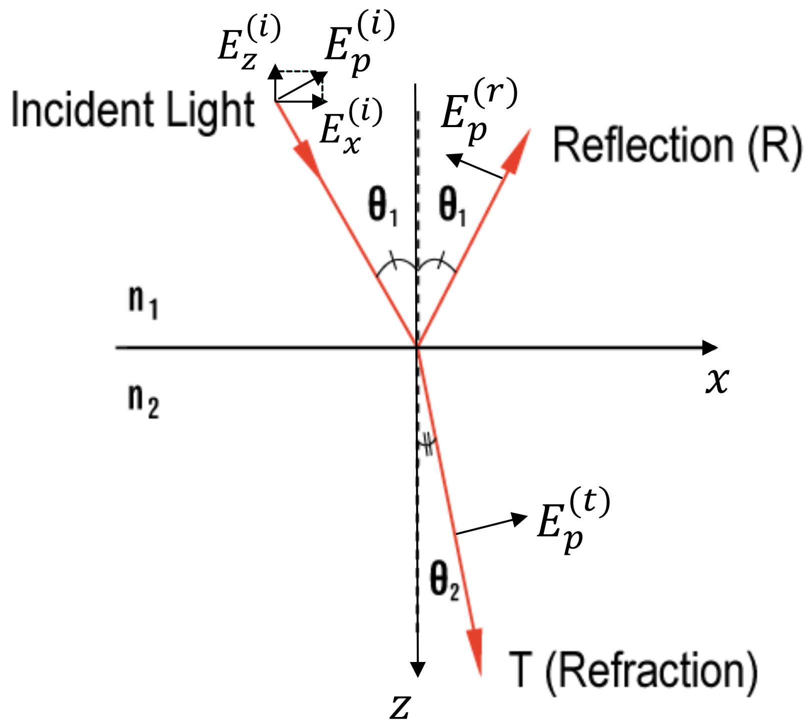
2. Normal components of D-field are continuous across the boundary

$$D_z^{(i)} + D_z^{(r)} = D_z^{(t)}$$

$$D = \varepsilon E = \varepsilon_r \varepsilon_0 E$$

Relative permittivity $\varepsilon_r = n^2$

$$\varepsilon_1 E_z^{(i)} + \varepsilon_1 E_z^{(r)} = \varepsilon_2 E_z^{(t)} \quad (2)$$



Derivation of Fresnel Formulae

Substituting the expressions of E_x and E_z in Eq.(1) and (2), we get

$$E_p^{(i)} \cos\theta_1 - E_p^{(r)} \cos\theta_1 = E_p^{(t)} \cos\theta_2 \tag{3}$$

$$-\varepsilon_1 E_p^{(i)} \sin\theta_1 - \varepsilon_1 E_p^{(r)} \sin\theta_1 = -\varepsilon_2 E_p^{(t)} \sin\theta_2 \tag{4}$$

To eliminate $E_p^{(t)}$, (3)$\times \varepsilon_2 \sin\theta_2$ + (4)$\times \cos\theta_2$, we get

$$\varepsilon_2 \sin\theta_2 \cos\theta_1 \left[E_p^{(i)} - E_p^{(r)}\right] - \varepsilon_1 \sin\theta_1 \cos\theta_2 \left[E_p^{(i)} + E_p^{(r)}\right] = 0$$

$$(\varepsilon_2 \sin\theta_2 \cos\theta_1 - \varepsilon_1 \sin\theta_1 \cos\theta_2) E_p^{(i)} = (\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2) E_p^{(r)}$$

$$r_p = \frac{E_p^{(r)}}{E_p^{(i)}} = \frac{\varepsilon_2 \sin\theta_2 \cos\theta_1 - \varepsilon_1 \sin\theta_1 \cos\theta_2}{\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2}$$

$$r_p = \frac{E_p^{(r)}}{E_p^{(i)}} = \frac{\varepsilon_2 \sin\theta_2 \cos\theta_1 - \varepsilon_1 \sin\theta_1 \cos\theta_2}{\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2} \qquad (5)$$

Substituting $\varepsilon_1 = n_1^2$ $\varepsilon_2 = n_2^2$

$$r_p = \frac{n_2^2 \sin\theta_2 \cos\theta_1 - n_1^2 \sin\theta_1 \cos\theta_2}{n_2^2 \sin\theta_2 \cos\theta_1 + n_1^2 \sin\theta_1 \cos\theta_2}$$

Applying Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$n_1^2 \sin\theta_1 = n_1(n_1 \sin\theta_1) = n_1(n_2 \sin\theta_2)$$

$$r_p = \frac{n_2 \sin\theta_2\,(n_2 \cos\theta_1 - n_1 \cos\theta_2)}{n_2 \sin\theta_2\,(n_2 \cos\theta_1 + n_1 \cos\theta_2)} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

Derivation of Fresnel Formulae

Transmittivity

$$E_p^{(i)} \cos\theta_1 - E_p^{(r)} \cos\theta_1 = E_p^{(t)} \cos\theta_2 \qquad (3)$$

Both sides divided by $E_p^{(i)}$

$$\cos\theta_1 - \frac{E_p^{(r)}}{E_p^{(i)}} \cos\theta_1 = \frac{E_p^{(t)}}{E_p^{(i)}} \cos\theta_2$$

$$\cos\theta_1 - r_p \cos\theta_1 = t_p \cos\theta_2$$

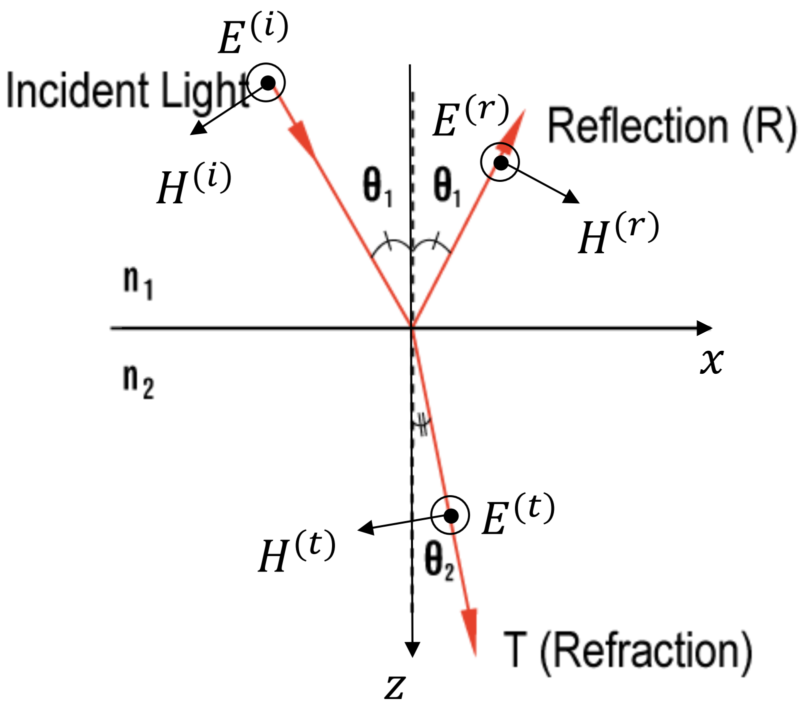
$$t_p = \frac{\cos\theta_1}{\cos\theta_2}(1 - r_p) = \frac{\cos\theta_1}{\cos\theta_2}\left(1 - \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}\right)$$

$$= \frac{\cos\theta_1}{\cos\theta_2} \times \frac{2n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

$$t_p = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}$$

Derivation of Fresnel Formulae

(2) The case of s-polarization



For s-polarization, E-field is perpendicular to the plane of incidence and H-field is in the plane of incidence, as shown in the diagram.

It is noted that the H-fields of s-polarization are analogue to the E-fields of p-polarization (but in opposite directions). If we make the following substitutions, the results of p-polarization can be used for s-polarization.

$$E \to -H$$
$$D = \varepsilon_r \varepsilon_0 E \to B = \mu_r \mu_0 H$$
$$\varepsilon_r \to \mu_r = 1$$

By making the following substitutions in Eq.(5) on slide 10,

$$E_p^{(r)} \to -H^{(r)}, E_p^{(i)} \to -H^{(i)}$$
$$\varepsilon_1 \to \mu_1 = 1, \varepsilon_2 \to \mu_2 = 1$$

we obtain $$\frac{-H^{(r)}}{-H^{(i)}} = \frac{\sin\theta_2\cos\theta_1 - \sin\theta_1\cos\theta_2}{\sin\theta_2\cos\theta_1 + \sin\theta_1\cos\theta_2} = -\frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)} \qquad (6)$$

According to Maxwell's equations, the magnitude of H-field is proportional to the magnitude of E-field through $H = \frac{E}{\eta}$, η is called impedance of the medium.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_r\varepsilon_0}} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0}}}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{n} \qquad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega\text{, impedance of vacuum}$$

So, $H^{(r)} = \frac{n_1 E^{(r)}}{\eta_0}$, $H^{(i)} = \frac{n_1 E^{(i)}}{\eta_0}$

The left side of Eq.(6) means $$\frac{H^{(r)}}{H^{(i)}} = \frac{\frac{n_1 E^{(r)}}{\eta_0}}{\frac{n_1 E^{(i)}}{\eta_0}} = \frac{E^{(r)}}{E^{(i)}} = r_s$$ Reflectivity of s-polarisation

Therefore $$r_s = -\frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)}$$

From Eq.(6) on previous slide

$$r_s = \frac{\sin\theta_2 \cos\theta_1 - \sin\theta_1 \cos\theta_2}{\sin\theta_2 \cos\theta_1 + \sin\theta_1 \cos\theta_2} = \frac{\dfrac{\sin\theta_2}{\sin\theta_1}\cos\theta_1 - \cos\theta_2}{\dfrac{\sin\theta_2}{\sin\theta_1}\cos\theta_1 + \cos\theta_2}$$

From Snell's law $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{n_1}{n_2}$, so $r_s = \dfrac{\dfrac{n_1}{n_2}\cos\theta_1 - \cos\theta_2}{\dfrac{n_1}{n_2}\cos\theta_1 + \cos\theta_2} = \dfrac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$

- <u>Transmittivity</u>

By making similar substitutions to eq.(3) on slide 11, we obtain

$$n_1 E^{(i)} \cos\theta_1 - n_1 E^{(r)} \cos\theta_1 = n_2 E^{(t)} \cos\theta_2 \qquad (7)$$

Both sides divided by $E^{(i)}$, we obtain

$$n_1 \cos\theta_1 - n_1 \cos\theta_1 \frac{E^{(r)}}{E^{(i)}} = n_2 \cos\theta_2 \frac{E^{(t)}}{E^{(i)}}$$

$$n_1 \cos\theta_1 \,(1 - r_s) = n_2 \cos\theta_2 \, t_s$$

$$t_s = \frac{n_1 \cos\theta_1 \,(1 - r_s)}{n_2 \cos\theta_2} = \frac{2 n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

Fresnel Formulae

Fresnel formulae can also be written in trigonometric forms by eliminating the refractive index n_1 and n_2 by applying Snell's law.

$$r_p = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

$$r_s = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}$$

$$\mathrm{t_p} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)}$$

$$\mathrm{t_s} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1 + \theta_2)}$$

Trigonometric Fresnel Formulae

$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{\cos\theta_1 - n_1/n_2 \cos\theta_2}{\cos\theta_1 + n_1/n_2 \cos\theta_2}$$

From Snell's law $\quad n_1/n_2 = \sin\theta_2/\sin\theta_1$

$$r_p = \frac{\cos\theta_1 - \dfrac{\sin\theta_2}{\sin\theta_1}\cos\theta_2}{\cos\theta_1 + \dfrac{\sin\theta_2}{\sin\theta_1}\cos\theta_2} = \frac{\sin\theta_1\cos\theta_1 - \sin\theta_2\cos\theta_2}{\sin\theta_1\cos\theta_1 + \sin\theta_2\cos\theta_2} = \frac{\sin(2\,\theta_1) - \sin(2\,\theta_2)}{\sin(2\,\theta_1) + \sin(2\,\theta_2)}$$

$$= \frac{2\sin(\theta_1 - \theta_2)\cos(\theta_1 + \theta_2)}{2\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

The other formulae can be obtained in similar ways

Brewster Angle

p−polarisation

$$r_p = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

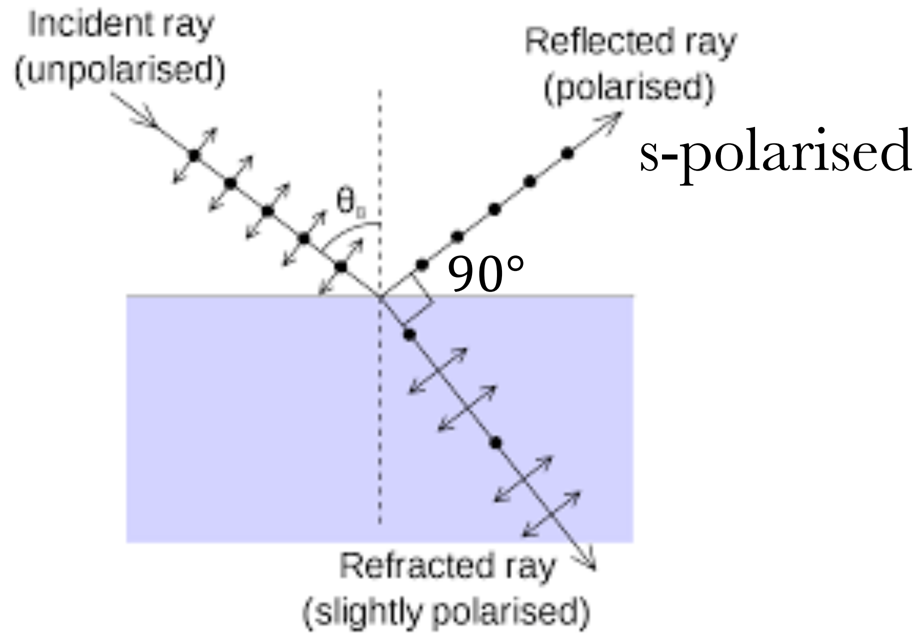
(a) Brewster angle

When $\theta_1 + \theta_2 = 90°$

$$\tan(\theta_1 + \theta_2) = \infty$$

$$r_p = 0$$

Zero-reflectance of the p-polarisation field. The reflected light is completely **s-polarised**



$$n_1 \sin\theta_b = n_2 \sin\theta_2 = n_2 \sin(90° - \theta_b) = n_2 \cos\theta_b$$

Brewster angle

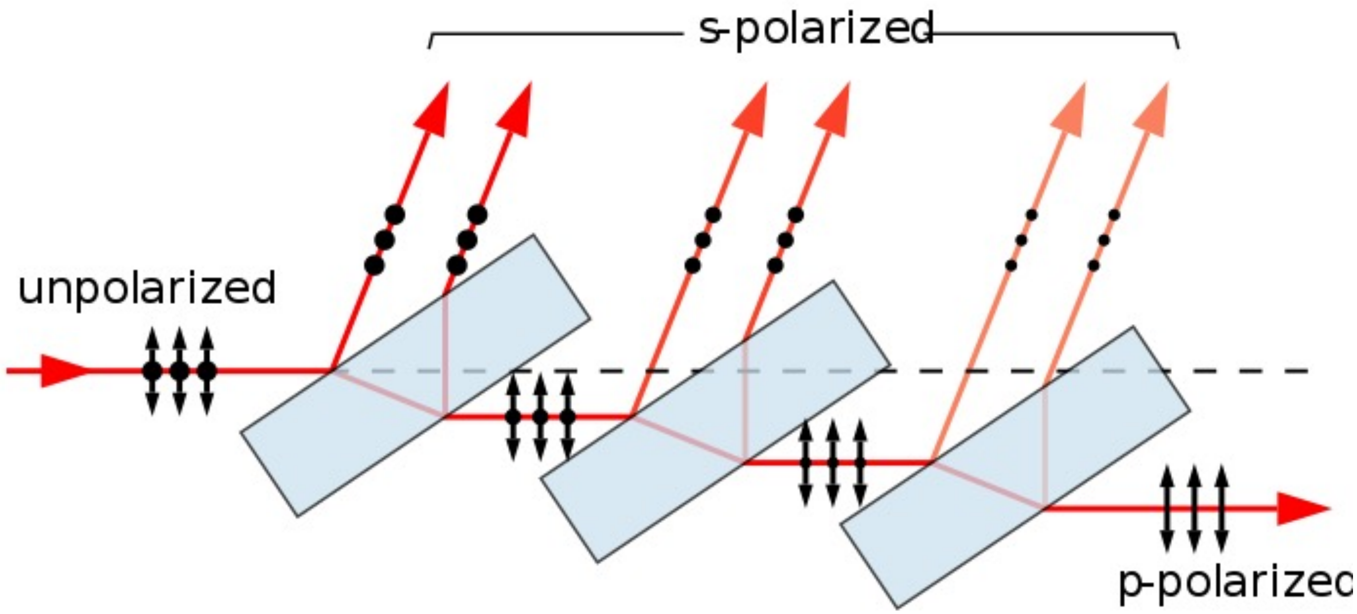
$$\tan\theta_b = \frac{n_2}{n_1}$$

Brewster Angle Polariser

Light incident on a stack of plates at the Brewster angle can be used to separate s- and p-polarisation light.

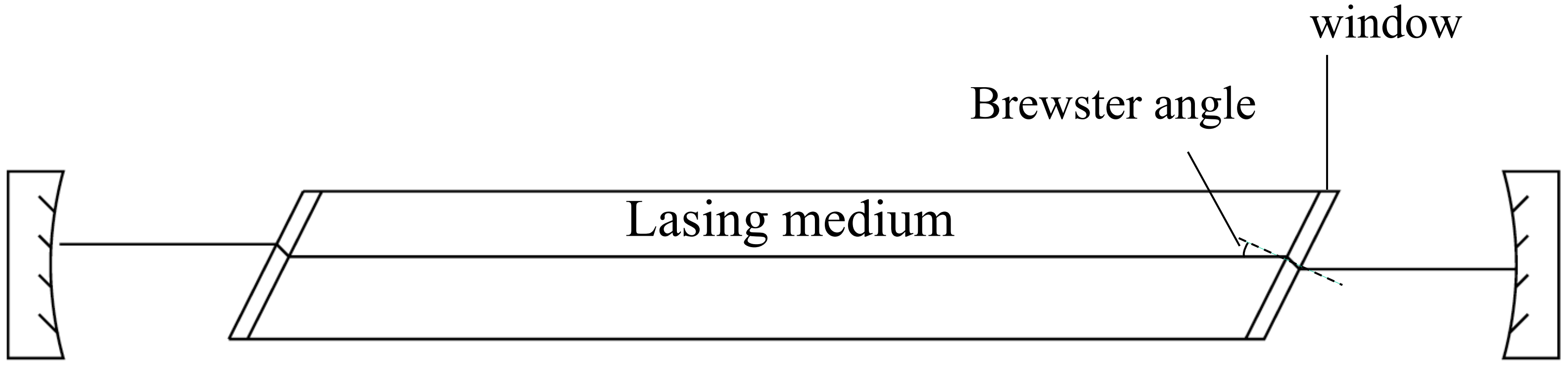
From air to glass surface

$$\tan\theta_b = 1.5 \Rightarrow \theta_b = 56.3°$$



Brewster Angle Cavity

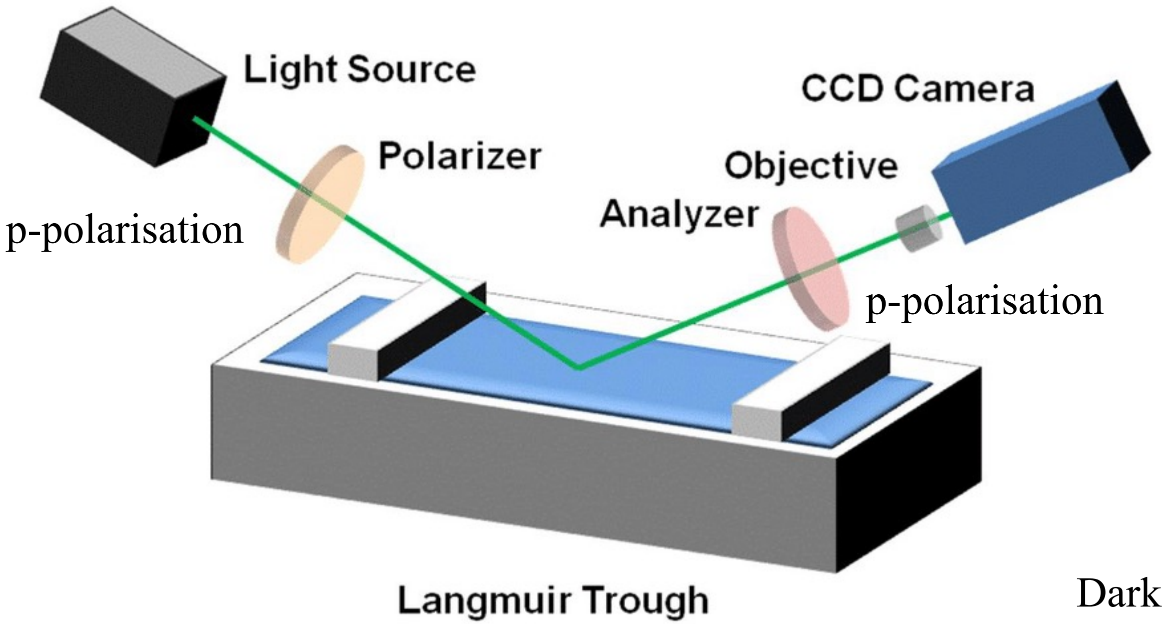
Brewster angle can be used to eliminate reflection. In a laser cavity, the window that seals the lasing medium (e.g., gases) is usually tilted with the Brewster angle so that p-polarized light can completely transmit through with the loss of reflection, while the s-polarization has loss due to reflection, so the emission of laser is linearly polarized with p-polarisation, which is desirable for many applications.

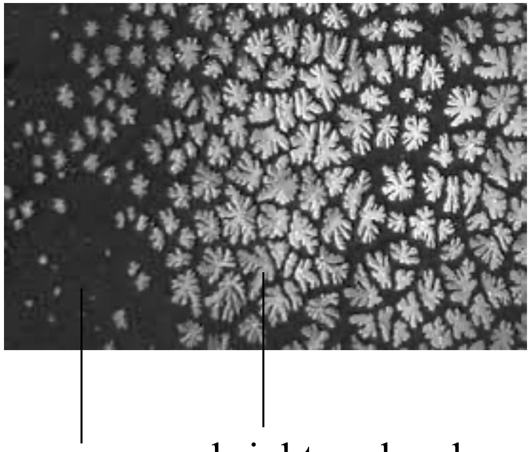


Brewster Angle Microscope

Brewster angle microscope: p-polarized light is incident at the Brewster angle to a liquid surface. A clean liquid surface has no reflection of p-polarized light, hence appears dark. When molecules or nanoparticles accumulate on the surface, the Brewster angle is different so that some portion of p-polarized light is reflected, therefore the molecules or nanoparticles will appear bright.

Brewster angle microscope is a powerful tool for investigating thin films (e.g., the Langmuir-Blodgett film) on liquid surface.





Reflectance

$$Reflectance = \frac{Reflected\ power}{Incident\ power}$$

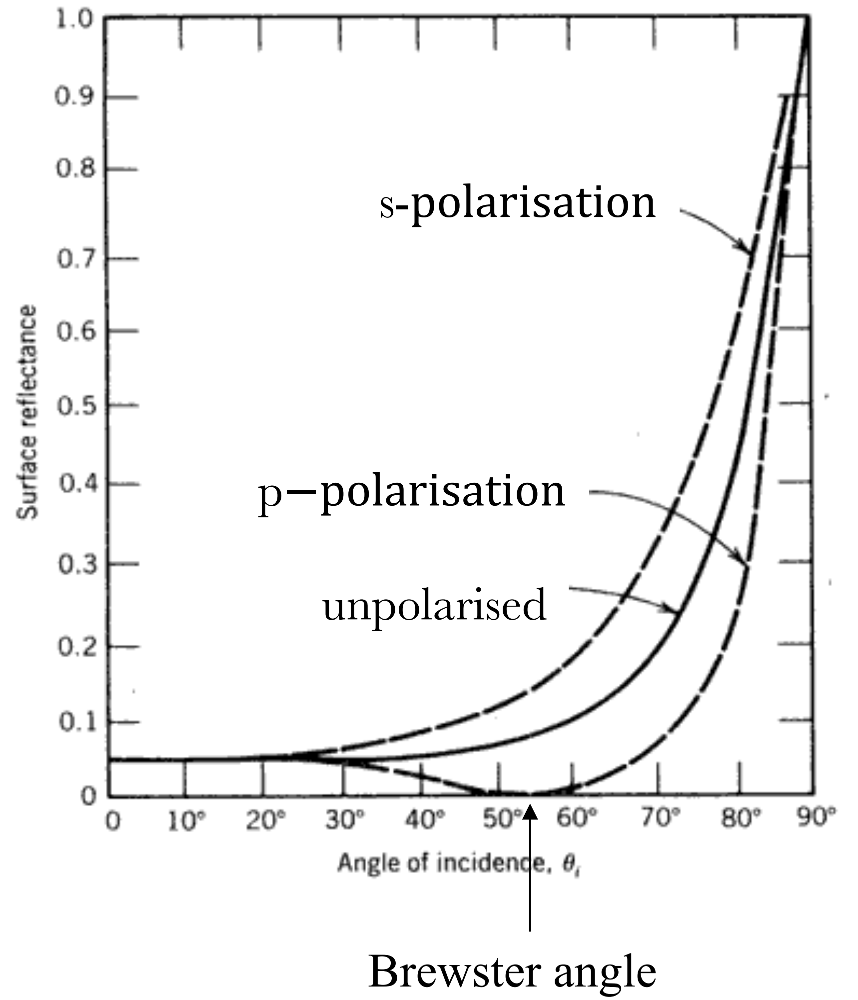
$$Reflectance = |reflectivity|^2$$

$$R = |r|^2$$

$$R_p = |r_p|^2 = \left|\frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}\right|^2$$

$$R_s = |r_s|^2 = \left|\frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}\right|^2$$

Reflectance of light incident from air to glass



Understanding Reflectance

(a) Normal incidence, $\theta_1 = \theta_2 = 0$

$$R_p = R_s = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2$$

(b) Grazing angle, $\theta_1 \to 90°$

$$\cos\theta_1 \approx 0$$

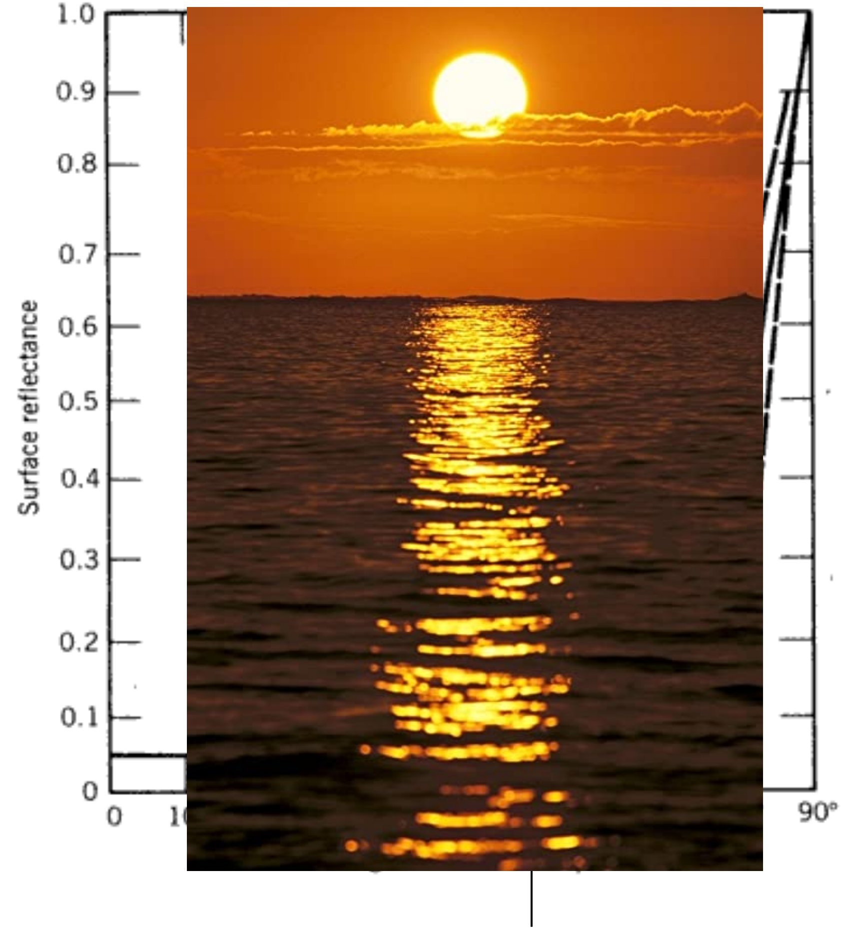
$$r_p = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2} \approx -1$$

$$r_s = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} \approx -1$$

$$R_p \approx R_s \to 1$$

Strong reflection at grazing angle, e.g., sunset reflection on water surface

Reflectance of light incident from air to glass



Brewster angle

Understanding Reflectance

(c) In general,

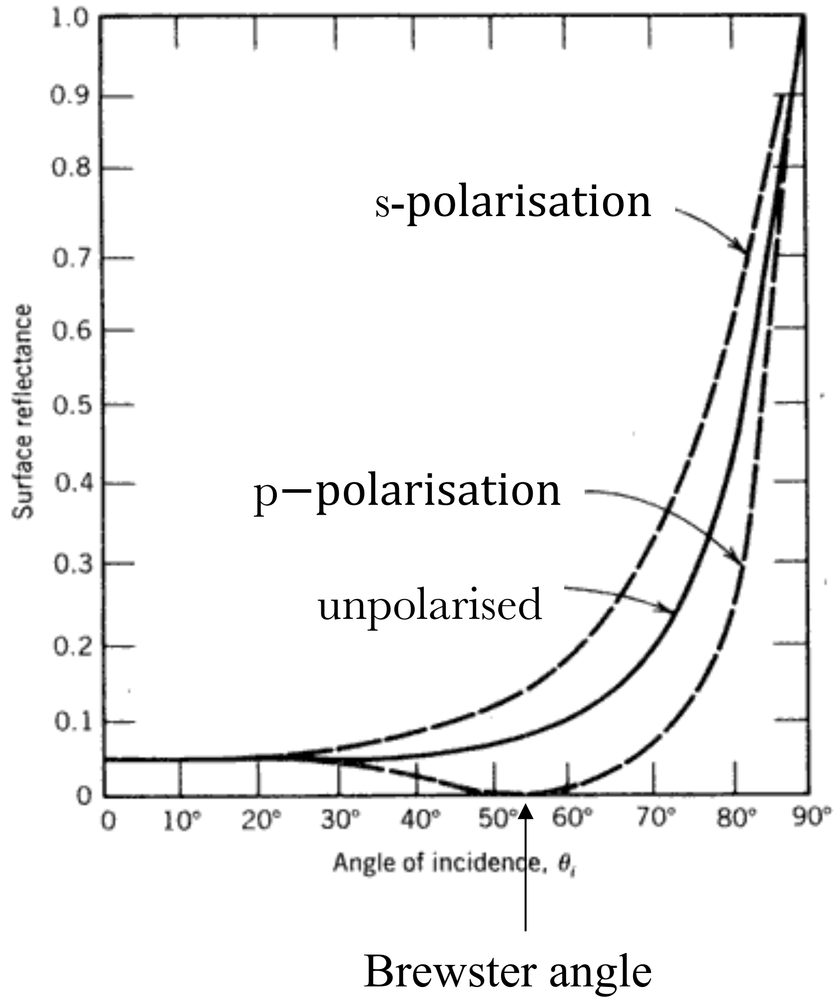
$$r_p = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

$$r_s = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}$$

$$\frac{R_s}{R_p} = \left|\frac{\cos(\theta_1 - \theta_2)}{\cos(\theta_1 + \theta_2)}\right|^2 \geq 1$$

Reflection of s-polarisation is generally greater than that of p-polarisation, especially near the Brewster angle.

Reflectance of light incident from air to glass

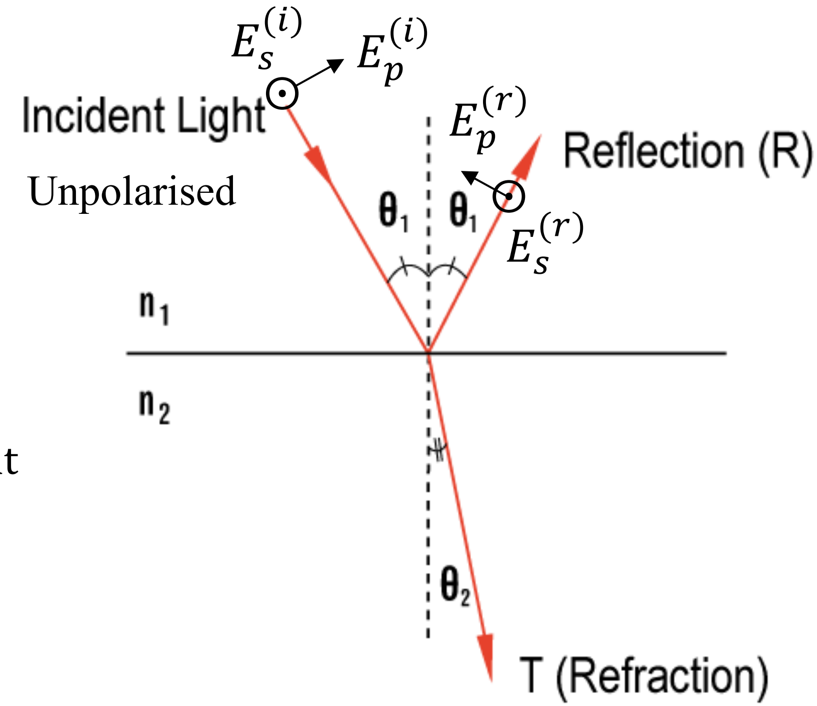


Degree of Polarisation of Reflected Light

Degree of polarisation

$$P = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

$$\langle E_x(t)E_y^*(t)\rangle$$
$$= \frac{1}{T}\int_0^T E_s^{(r)} e^{i(\omega t - kz + \delta_x(t))} E_p^{(r)} e^{-i(\omega t - kz + \delta_y(t))} dt$$
$$= \frac{E_s^{(r)} E_p^{(r)}}{T}\int_0^T e^{-i\delta(t)} dt = 0$$



Similarly, $\langle E_y(t)E_x^*(t)\rangle = 0$, therefore $S_2 = S_3 = 0$

$$S_0 = |E_s^{(r)}|^2 + |E_p^{(r)}|^2 = (R_s + R_p)E_0^2$$

$$S_1 = |E_s^{(r)}|^2 - \left|E_p^{(r)}\right|^2 = (R_s - R_p)E_0^2$$

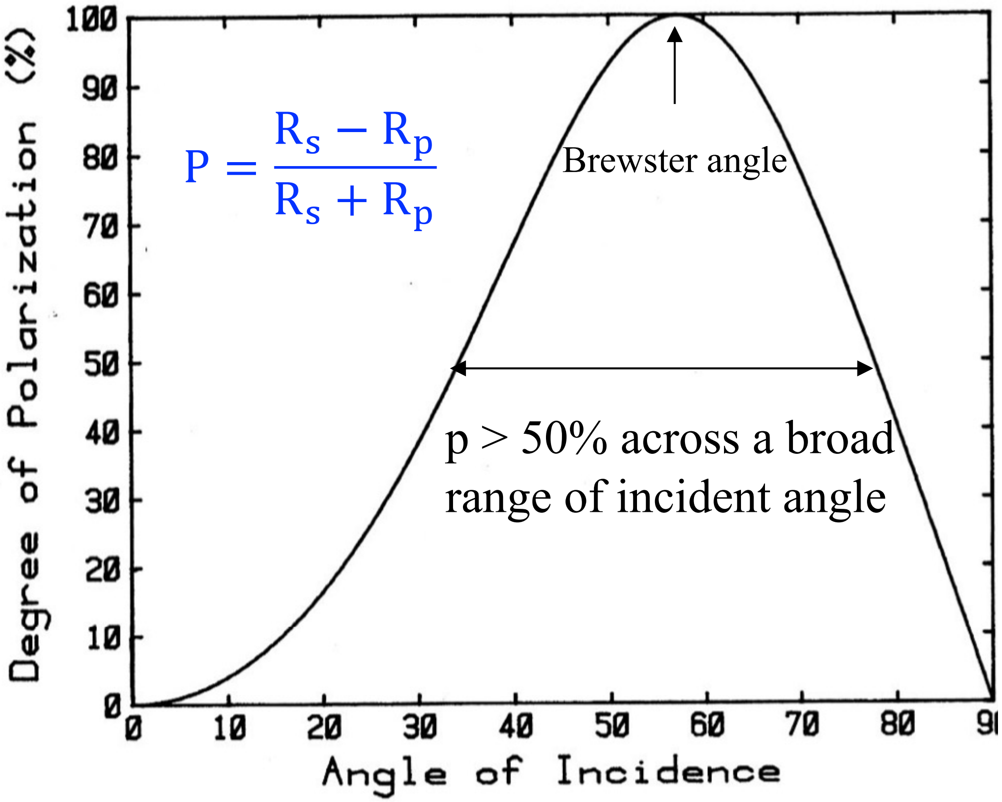
E_0 the magnitude of the s- and p-polarization component of incident light

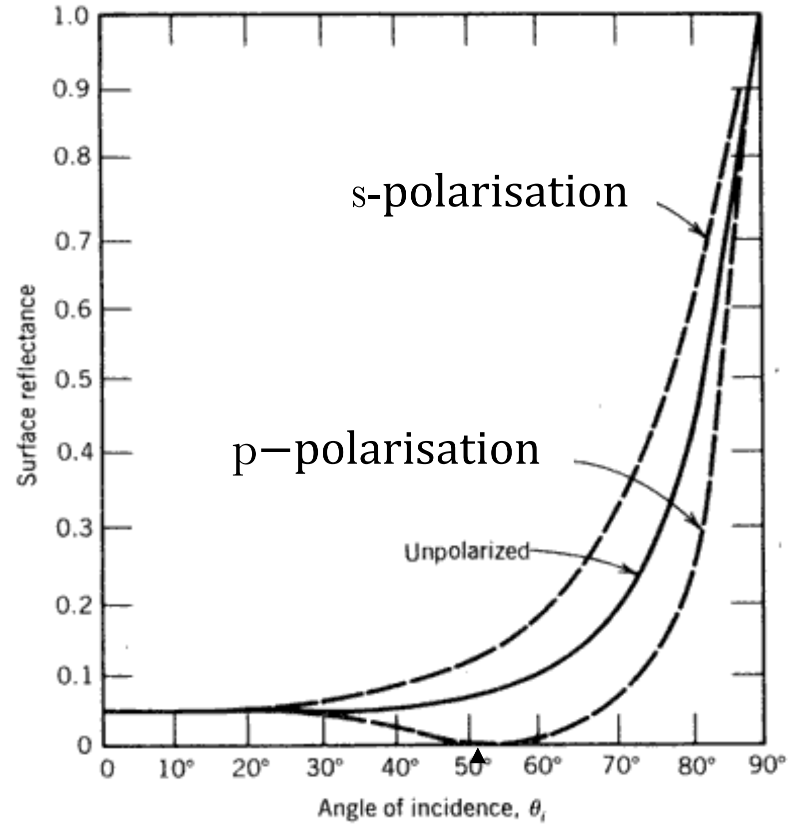
Degree of polarisation

$$P = \frac{S_1}{S_0} = \frac{R_s - R_p}{R_s + R_p}$$

Degree of Polarisation of Reflected Light

The degree of polarisation of reflected light at air/glass interface.





Upon reflection, unpolarized light becomes partially polarized. The degree of polarization is maximum (100% polarised) at Brewster angle and minimum (zero) near normal and grazing incident angles.

Polarised Sunglasses

Sunglasses with polarized filters can block most of glare light reflected from surface, therefore improving the visibility under strong reflection condition.

View without sunglasses

View with polarized sunglasses

Polarised Sunglasses

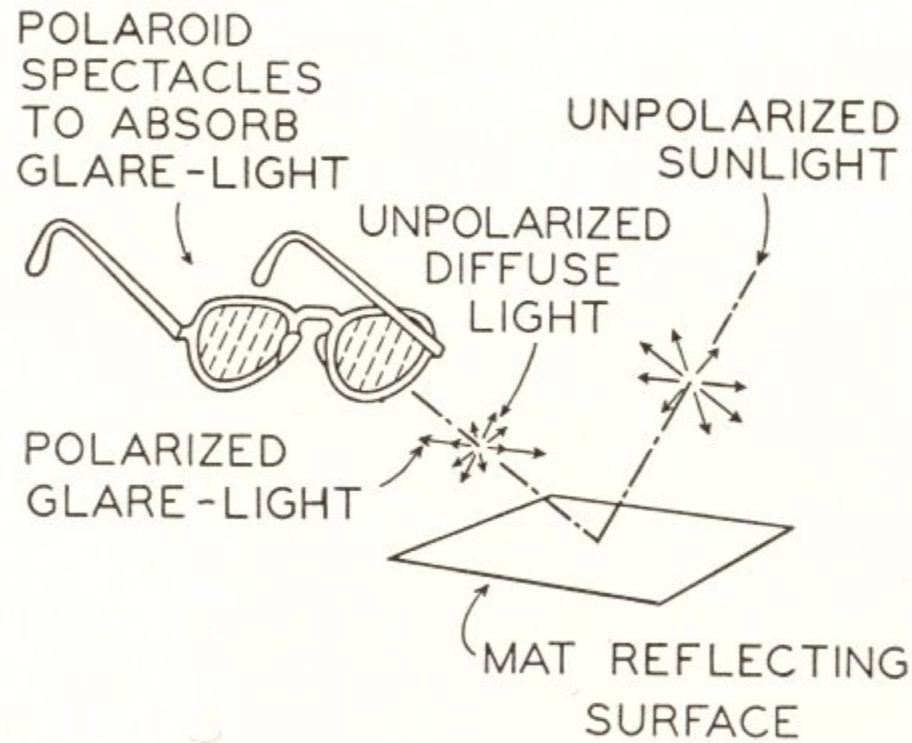
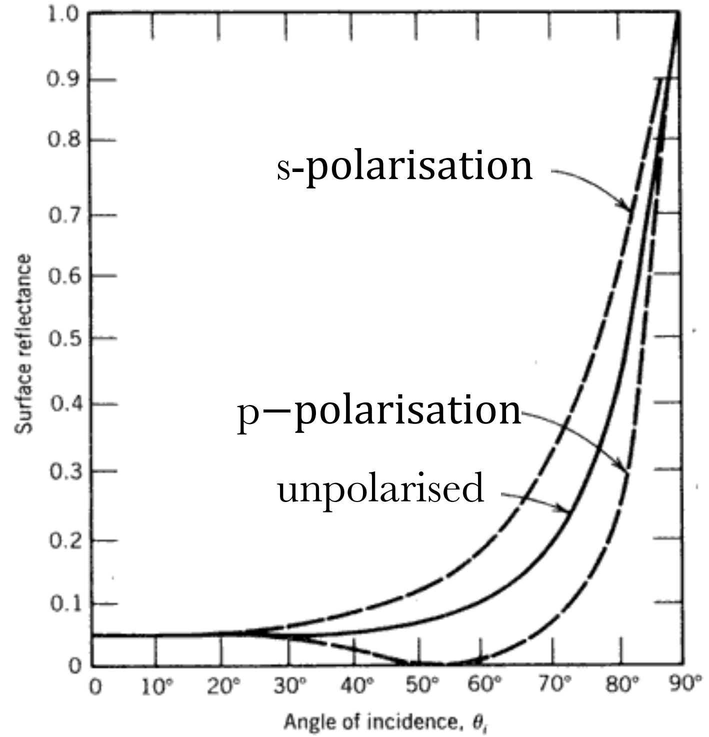


Fig. V-1. The Principle of Polarizing Sunglasses



Light from a horizontal surface is composed of unpolarized diffused light and specular reflection light which is mainly s-polarised. The specular reflection light can be blocked by sunglasses filters with vertical polarisation axis, therefore the view can be significantly improved.

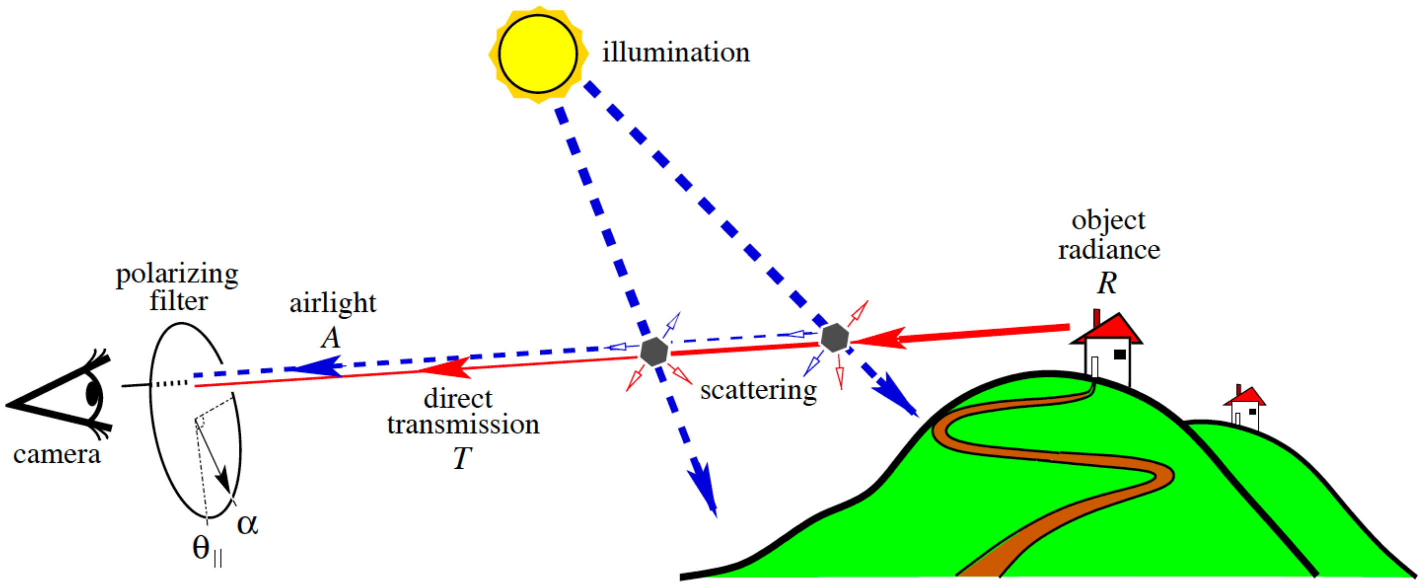
Dehazing Imaging

Polarised lens can significantly improve the image quality in a hazy day.

Original image

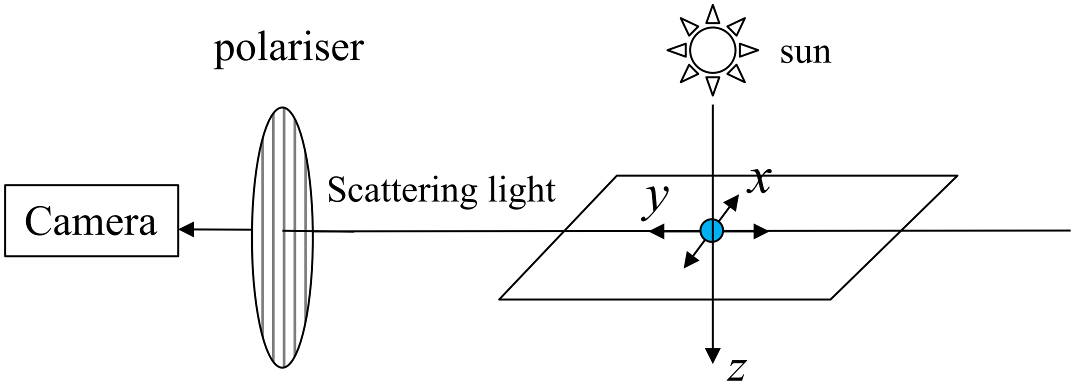
Dehazed image with polariser

Dehazing Imaging

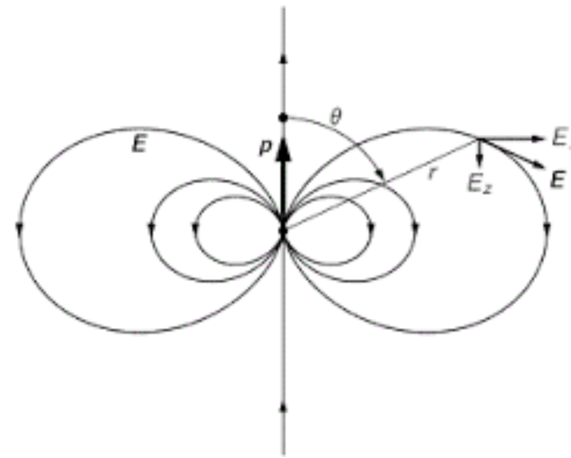


Light received by a camera is composed of two parts: direct transmission light from distant objects and the scattering light by small particles in air. The direct transmitted light is unpolarized while the scattering light is strongly polarized which can be blocked with a polarising filter, therefore the image appears clearer.

Dehazing Imaging



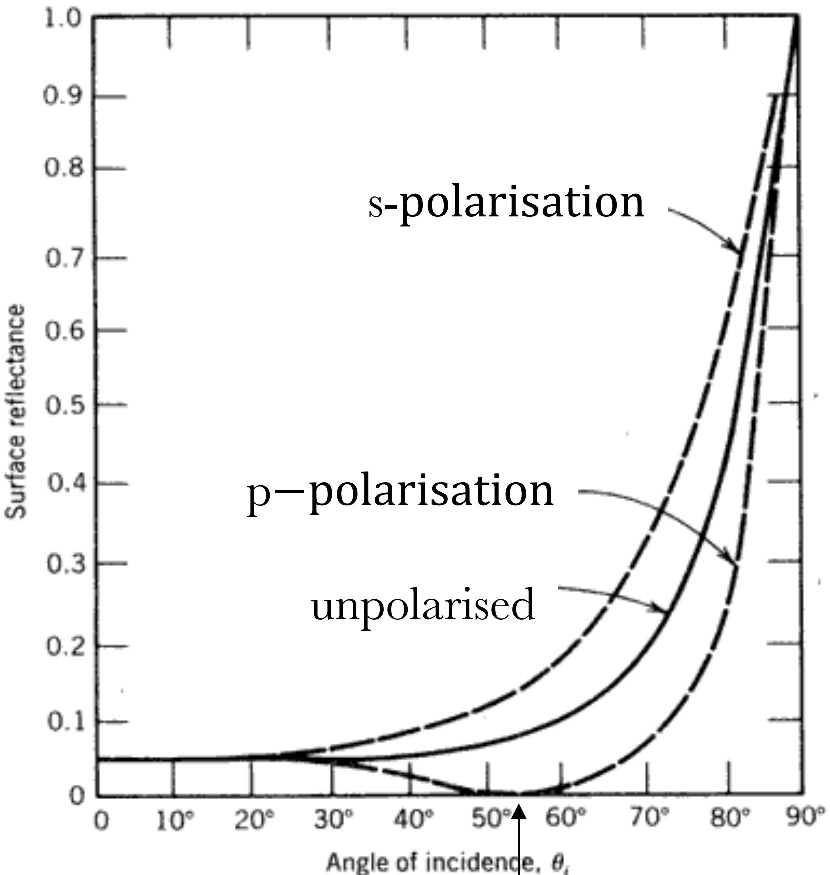
Angular dependent radiation of an electric dipole



- A small particle in air can be modelled as an electric dipole with dipole moments along the x- and y- direction, respectively.
- Electric dipole radiates strongly in the direction perpendicular to the dipole moment, but weakly along the dipole oscillating direction.
- The scattering light received by the camera is mainly from the x-dipole (as the y-dipole has no radiation towards the camera.), so it is mainly polarized along the x-direction, hence can be blocked with a polariser along the z-direction.

Recap of Previous Lecture

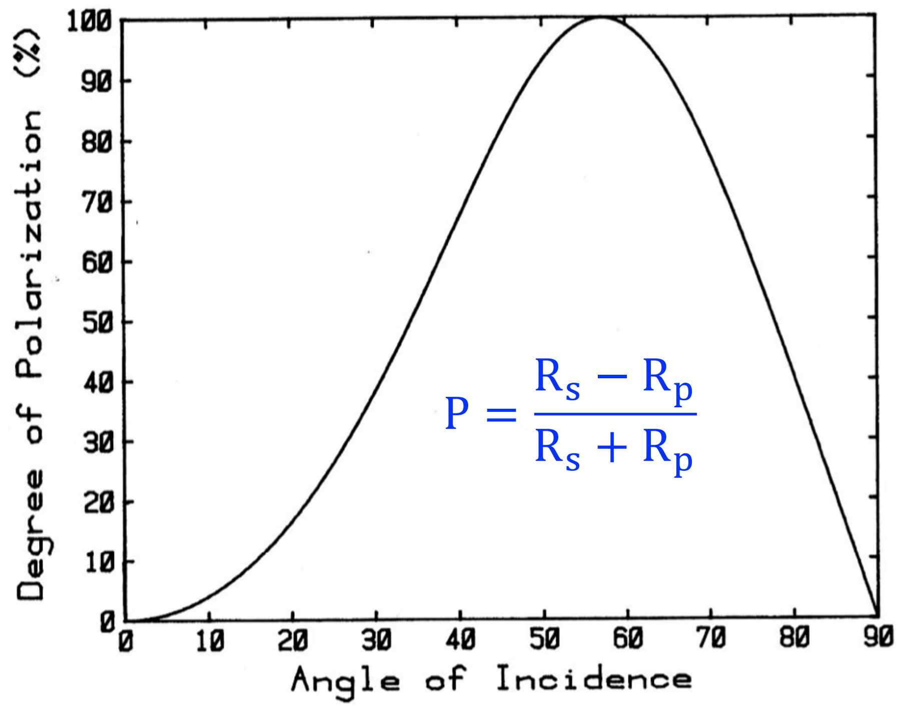
Reflectance of light incident from air to glass



Brewster angle

The degree of polarisation of reflected light at air/glass interface.

Brewster angle



$$P = \frac{R_s - R_p}{R_s + R_p}$$

Total Internal Reflection (TIR)

When light is incident from a medium of high refractive index to a medium of low refractive index, there exists a critical angle θ_c.

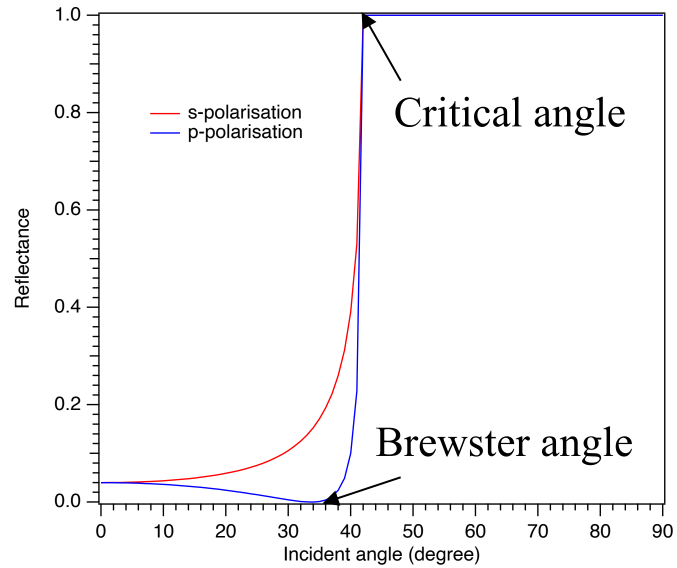
$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\text{For } \theta_1 > \theta_c,\ \ \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 > 1$$

No real solution

$$\cos\theta_2 = \sqrt{1-(\sin\theta_2)^2} = i\sqrt{(\sin\theta_2)^2 - 1}$$

Light incident from glass to air

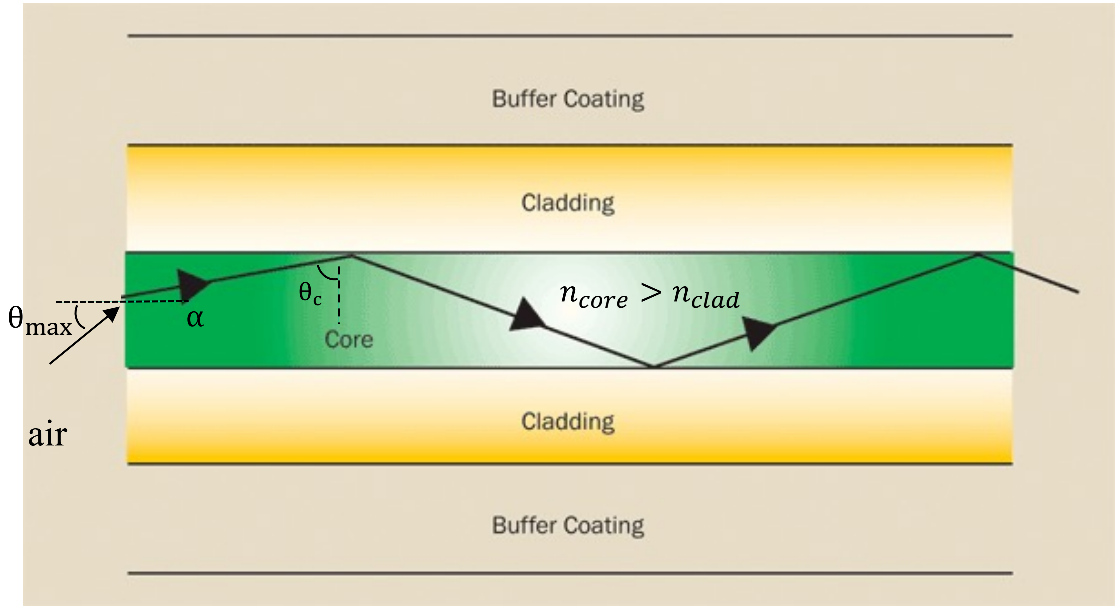


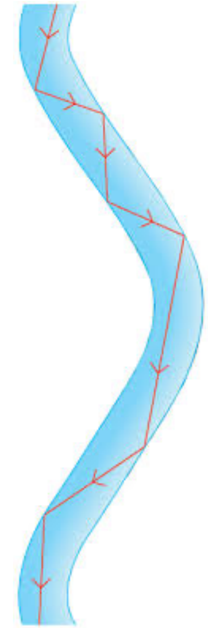
$$R_p = |r_p|^2 = |\frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2}|^2 = |\frac{n_2\cos\theta_1 - in_1\sqrt{(\sin\theta_2)^2-1}}{n_2\cos\theta_1 + in_1\sqrt{(\sin\theta_2)^2-1}}|^2 = 1$$

$$R_s = |r_s|^2 = |\frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}|^2 = |\frac{n_1\cos\theta_1 - in_2\sqrt{(\sin\theta_2)^2-1}}{n_1\cos\theta_1 + in_2\sqrt{(\sin\theta_2)^2-1}}|^2 = 1$$

Optical Fiber

Total internal reflection is exploited to guide light in conventional optical fiber





Maximum incident angle: $\sin\theta_{max} = n_{core}\sin\alpha = n_{core}\cos\theta_c \quad (\theta_c = 90° - \alpha)$

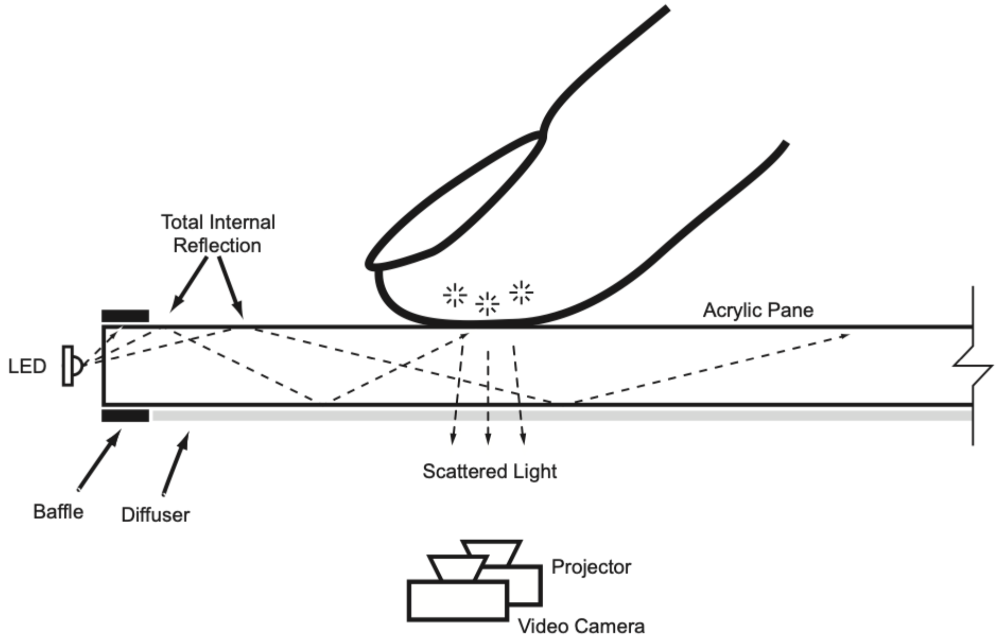
$$\boxed{\sin\theta_{max}} = n_{core}\sqrt{1-(\sin\theta_c)^2} = n_{core}\sqrt{1-\left(\frac{n_{clad}}{n_{core}}\right)^2} = \sqrt{n_{core}^2 - n_{clad}^2}$$

Numerical aperture of fiber

$$\sin\theta_c = \frac{n_{clad}}{n_{core}}$$

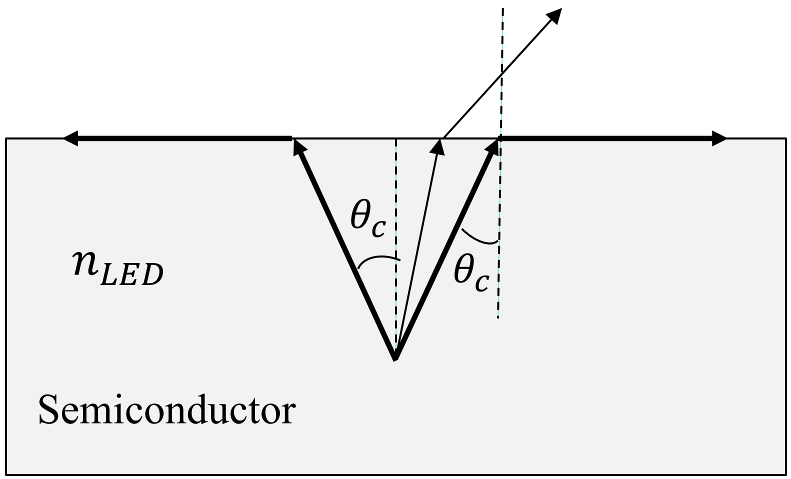
Fingerprint Recording

TIR is very sensitive to external stimuli, such as pressure, bending, touch, attachment of analyte molecules etc, therefore has been widely exploited in a broad range of sensing applications.

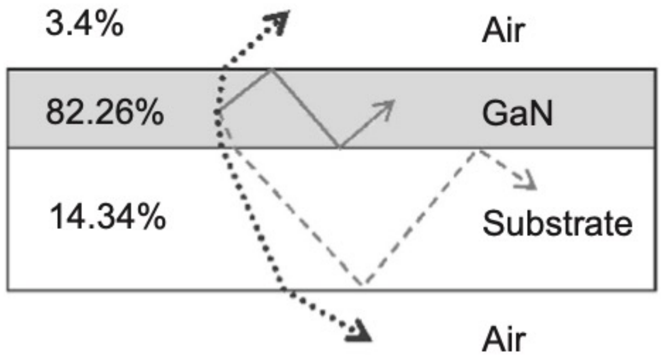


Finger print recording: the touch of finger breaks the TIR condition. Light is scattered from the finger and recorded by the camera underneath.

Light Extraction in LED

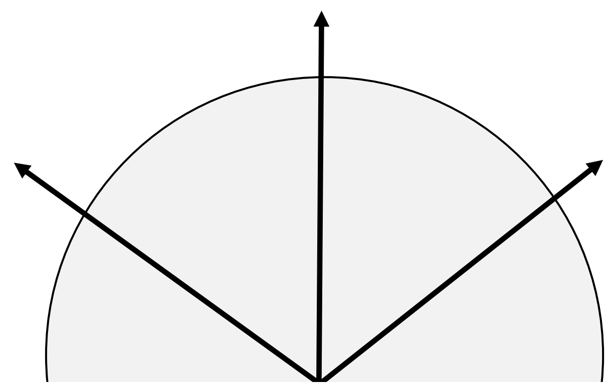


Planar LED



The refractive index of LED materials is typoically between 2.5~3.5.

$$n_{LED} = 3 \rightarrow \sin\theta_c = \frac{1}{3} \rightarrow \theta_c = 19.4°$$



Hemisphere LED

Light emission of a planar LED is restricted to a cone with semi angle equal to the critical angle. Curved surfaces (such as a hemisphere) have larger escape angles, hence can significantly increase the efficiency of LED.

Phase Change in Reflection

The E-field of reflected light could have different phases from that of incident light, i.e., reflection could cause a phase change of light.

Reflectivity $$r = \frac{E^{(r)}}{E^{(i)}} = \frac{E_0^{(r)} e^{i\varphi^{(r)}}}{E_0^{(i)} e^{i\varphi^{(i)}}} = \frac{E_0^{(r)}}{E_0^{(i)}} e^{i\Delta\varphi}$$

Phase change $$\Delta\varphi = \varphi^{(r)} - \varphi^{(i)}$$

(1) When r is real, $\Delta\varphi = 0$ for $r > 0$; $\Delta\varphi = \pi$ for $r < 0$.

(2) In the situation of total internal reflection $(\theta_1 > \theta_c)$, r is complex. $\Delta\varphi$ is dependent on incident angle.

Phase Change in Reflection

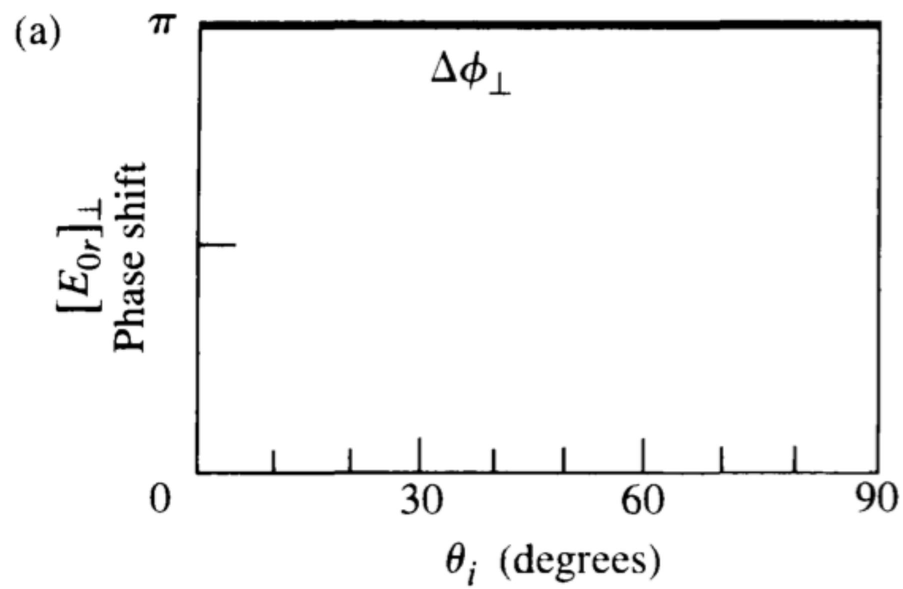
Case I: $n_1 < n_2, \quad \theta_1 > \theta_2$

Light is incident from low-index medium to high-index medium

(1) s-polarisation

$$r_s = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)} < 0$$

$$\varphi = \pi$$



Phase Change in Reflection

Case I: $n_1 < n_2, \quad \theta_1 > \theta_2$

(2) p-polarisation

$$r_p = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

(a) When $\theta_1 < \theta_b$,

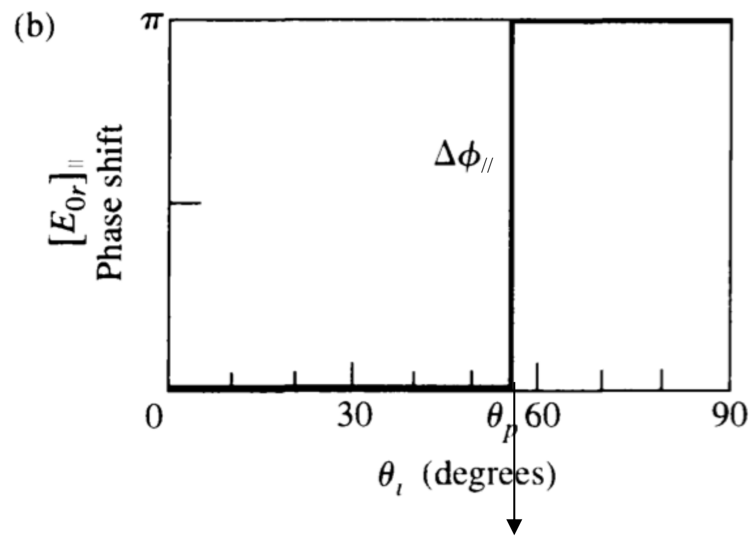
$$\theta_1 + \theta_2 < 90°$$

$$r_p > 0 \rightarrow \Delta\varphi = 0$$

(b) When $\theta_1 > \theta_b$,

$$\theta_1 + \theta_2 > 90°$$

$$r_p < 0 \rightarrow \Delta\varphi = \pi$$



Brewster angle 56.3^0 (from air to glass)

Phase Change in Reflection

Case II: $n_1 > n_2,\ \theta_1 < \theta_2$

Light is incident from high-index medium to low-index medium

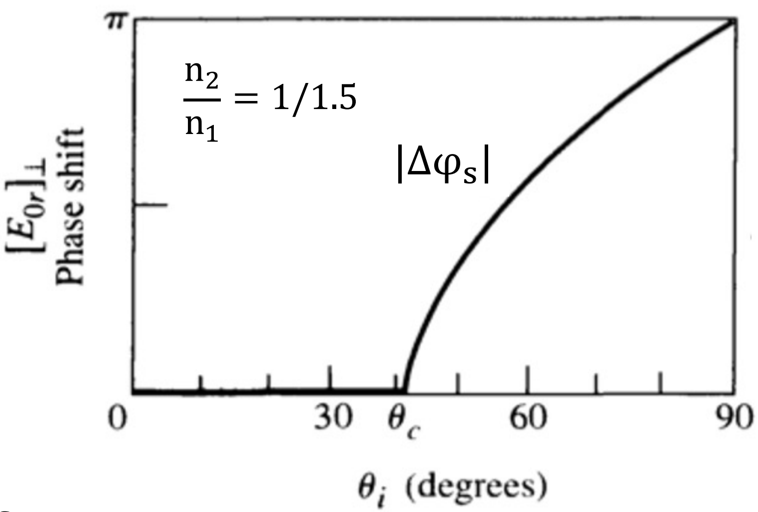
(1) s-polarisation $$r_s = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}$$

(a) When $\theta_1 < \theta_c$

$$r_s > 0 \rightarrow \Delta\varphi = 0$$

(b) When $\theta_1 > \theta_c$ (total internal reflection)

$$\mathrm{r_s} = \frac{n_1 \cos\theta_1 - i n_2\sqrt{(\sin\theta_2)^2 - 1}}{n_1 \cos\theta_1 + i n_2\sqrt{(\sin\theta_2)^2 - 1}} = \mathrm{e}^{-\mathrm{i}2\alpha_\mathrm{s}}$$



$$\Delta\varphi_\mathrm{s} = -2\alpha_\mathrm{s} = -2\tan^{-1}\left(\frac{n_2\sqrt{(\sin\theta_2)^2 - 1}}{\mathrm{n_1}\cos\theta_1}\right) = -2\tan^{-1}\left(\frac{\sqrt{(n_1\sin\theta_1)^2 - n_2^2}}{\mathrm{n_1}\cos\theta_1}\right)$$

Phase Change in Reflection

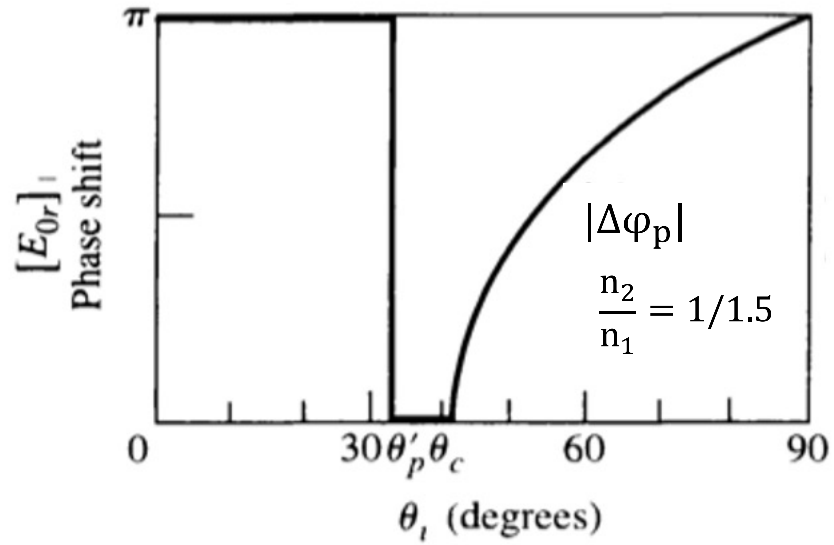
Case II: $n_1 > n_2,\ \theta_1 < \theta_2$

(2) p-polarisation $r_p = \dfrac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$

(a) When $\theta_1 < \theta_c$

When $\theta_1 < \theta_b$, $\quad r_p < 0 \rightarrow \Delta\varphi = \pi$

When $\theta_1 > \theta_b$, $\quad r_p > 0 \rightarrow \Delta\varphi = 0$



(b) When $\theta_1 > \theta_c$ (total internal reflection)

$$r_p = \frac{n_2 \cos\theta_1 - i n_1\sqrt{(\sin\theta_2)^2 - 1}}{n_2 \cos\theta_1 + i n_1\sqrt{(\sin\theta_2)^2 - 1}} = e^{-i2\alpha_p}$$

$$\Delta\varphi_p = -2\alpha_p = -2\tan^{-1}\left(\frac{n_1\sqrt{(\sin\theta_2)^2 - 1}}{n_2\cos\theta_1}\right) = -2\tan^{-1}\left(\frac{n_1\sqrt{(n_1\sin\theta_1)^2 - n_2^2}}{n_2^2\cos\theta_1}\right)$$

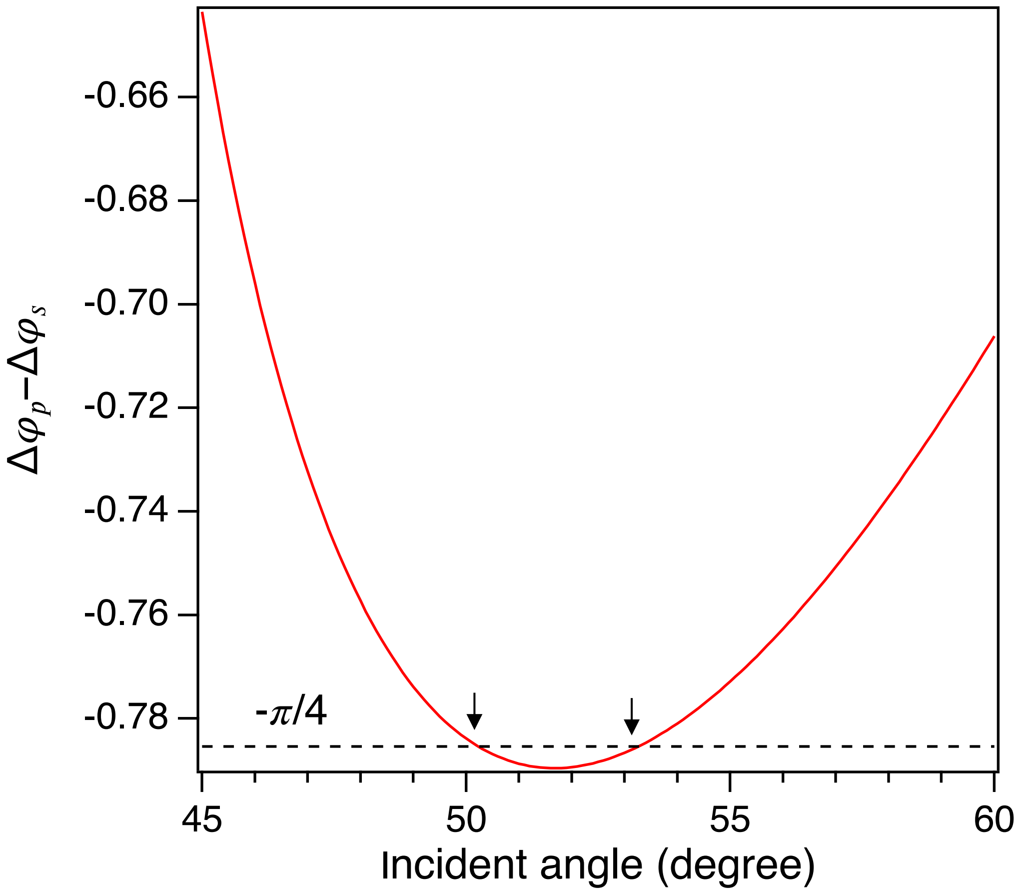
Phase Change in Transmission

$$t_p = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} > 0$$

$$t_s = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} > 0$$

Zero phase change of transmitted fields

45° Phase Difference



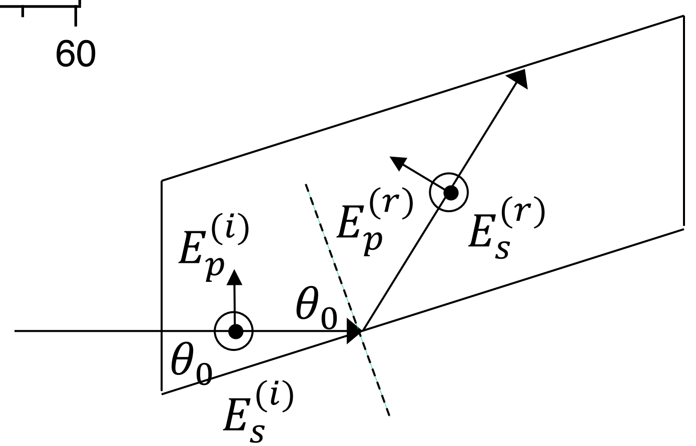
smission has significant
eflection coating,

s to construct phase
it angle θ_0 where the phase

$$: \pm\frac{\pi}{4}$$

$$\frac{{}_1\sqrt{(n_1 \sin\theta_0)^2 - n_2^2}}{n_2^2 \cos\theta_0}\Bigg) = \pm\frac{\pi}{4}$$

$\theta_0 = 50.2°$ or $53.3°$



Fresnel Rhomb

One TIR at θ_0 induces 45° phase change, two TIR will induce 90° phase change (equivalent to a quarter waveplate), …four TIR will induce 180° phase change (equivalent to a half waveplate).
A slab of crystal is cut to a tilt angle of θ_0. Light incident normal to the surface undergo TIR inside the crystal will induce the desired phase change. Such device is called Fresnel rhomb.

Quarter-wave Fresnel rhomb retarder

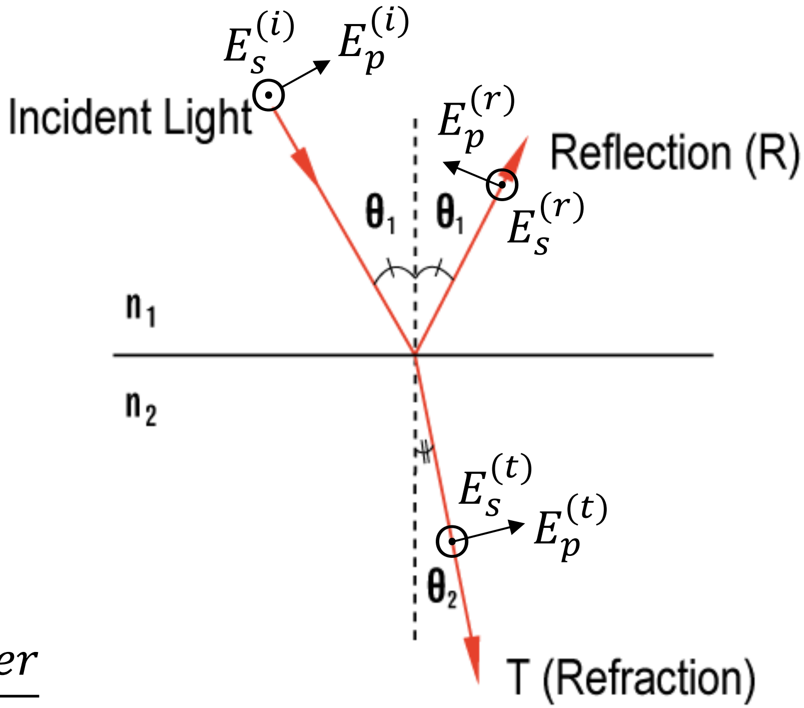
Half-wave Fresnel rhomb retarder

Transmittance

Transmittivity

$$t_p = \frac{E_p^{(t)}}{E_p^{(i)}} = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$t_s = \frac{E_s^{(t)}}{E_s^{(i)}} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$



$$transmittance(T) = \frac{transmitted\ power}{incident\ power}$$

$$transmittance(T) \neq |transmittivity|^2$$

Because the refracted angle θ_2 is different from that of incident angle θ_1, and the two media have different refractive indexes.

Transmittance

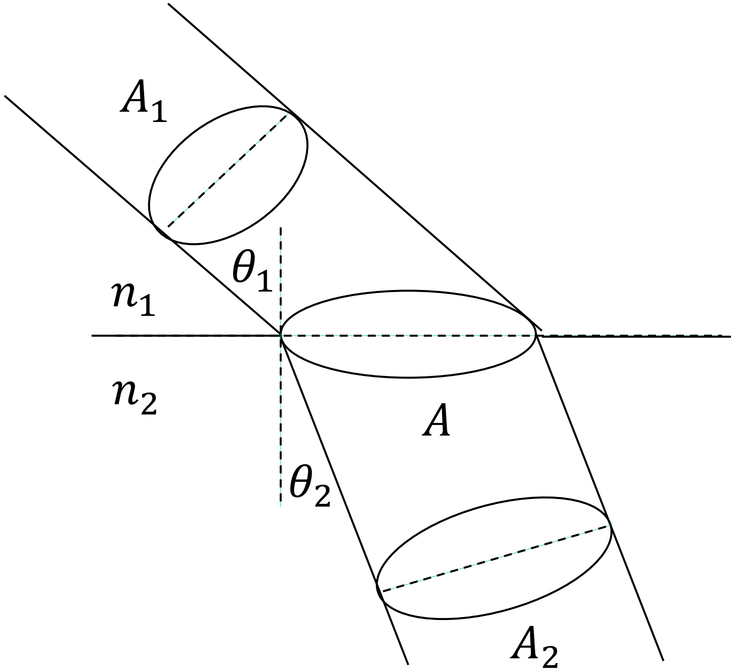
Poynting vector: energy flow across unit area in unit time

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

$$\mathrm{H} = \frac{nE}{\eta_0}$$

$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$, impedance of vacuum

$$\mathrm{S} = \frac{nE^2}{\eta_0}$$ Magnitude of Poynting vector



$A_1 = A\cos\theta_1 \qquad A_2 = A\cos\theta_2$

Incident power $P_i = S_1 A_1 = \frac{n_1 A \cos\theta_1 |E^{(i)}|^2}{\eta_0}$

Transmitted power $P_t = S_2 A_2 = \frac{n_2 \, A\cos\theta_2 |E^{(t)}|^2}{\eta_0}$

Transmittance

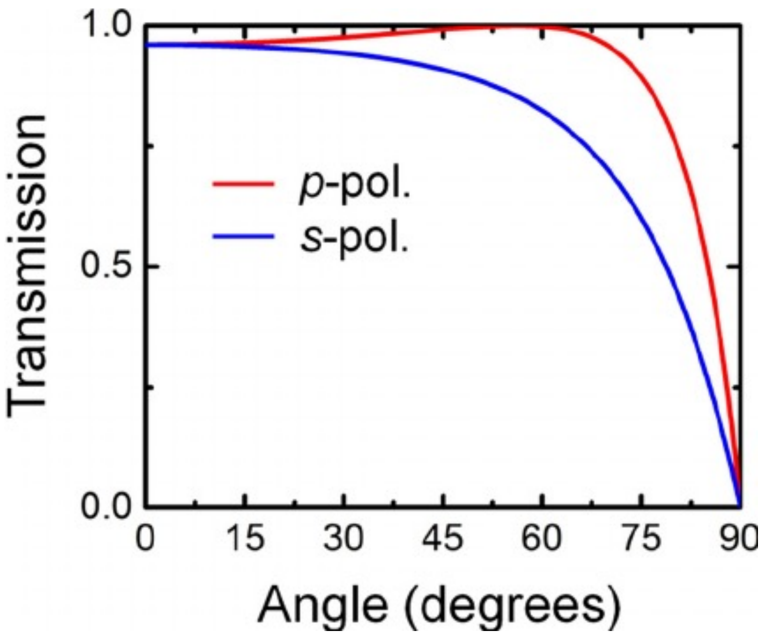
$$T = \frac{P_t}{P_i} = \frac{n_2\cos\theta_2}{n_1\cos\theta_1}|t|^2$$

Transmittance

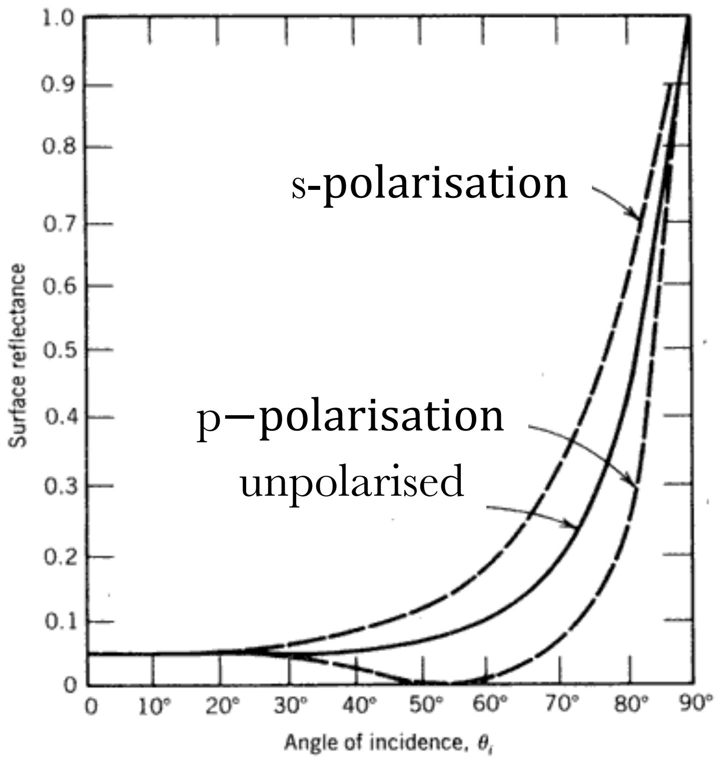
Energy conservation law requires:

$$reflectance\ (R) + transmittance(T) = 1$$

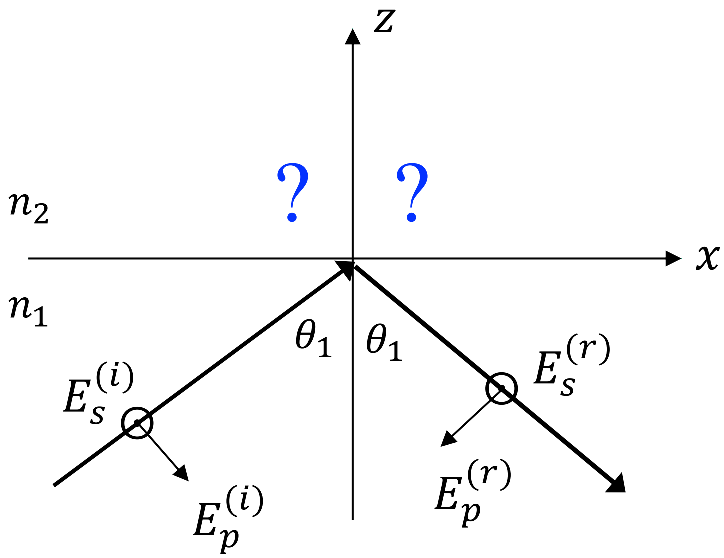
$$T = 1 - R$$



Transmittance of light through a dielectric with refractive index of 1.5 as a function of incident angle



Evanescent Field



Total internal reflection

Is there any field in medium 2?

Boundary conditions indicate that the tangent (normal) components of E and H (D and B) fields must be continuous across the interface.

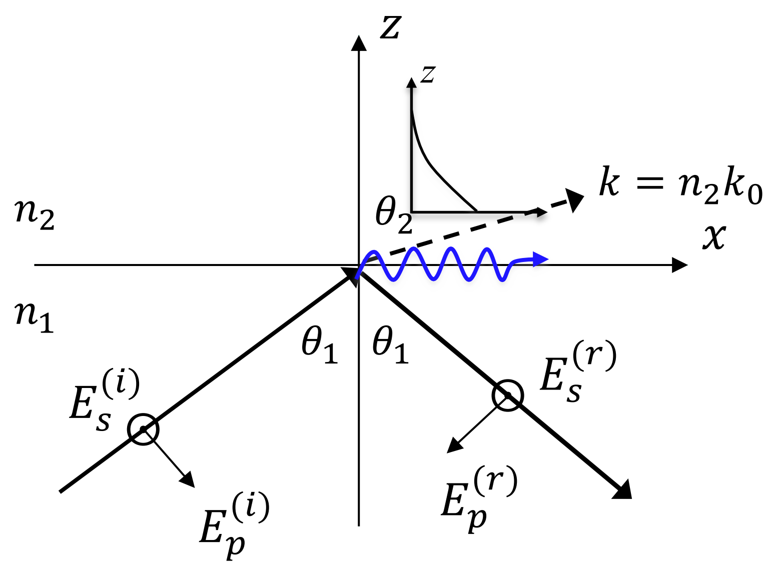
In medium 1, the total electric fields both along (tangent component) and normal to the boundary are nonzero.

$$E_x^{(i)} + E_x^{(r)} \neq 0$$

$$E_z^{(i)} + E_z^{(r)} \neq 0$$

Therefore, according to the Boundary Conditions, there must be electromagnetic fields in medium 2.

Evanescent Field



Total internal reflection

The general form of the electric fields in medium 2 can be written as

$$E = Ae^{i(k_x x + k_z z - \omega t)} \quad (1)$$

$k_x = k \sin\theta_2$ the x-component of wavevector k

$k_z = k \cos\theta_2$ the z-component of wavevector k

$$\cos\theta_2 = i\sqrt{(\sin\theta_2)^2 - 1}$$

$$k_z = n_2 k_0 \cos\theta_2 = i n_2 k_0 \sqrt{(\sin\theta_2)^2 - 1}$$
$$= i k_0 \sqrt{(n_2 \sin\theta_2)^2 - n_2^2} = i k_0 \sqrt{(n_1 \sin\theta_1)^2 - n_2^2}$$

$$k_x = n_2 k_0 \sin\theta_2 = n_1 \sin\theta_1 \, k_0$$

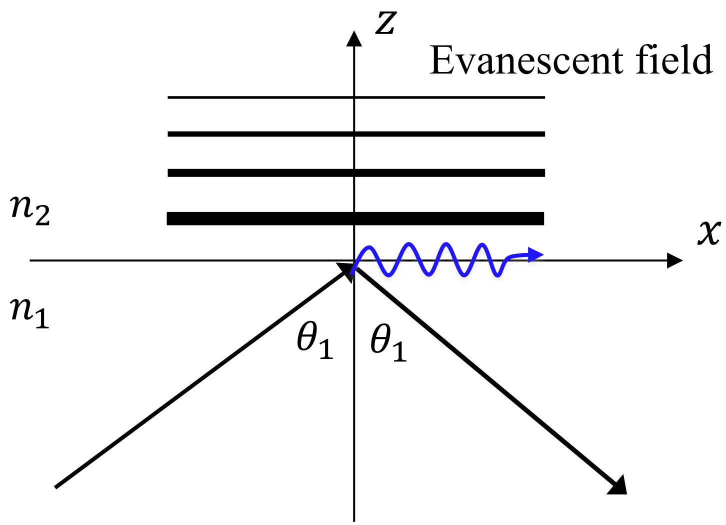
$$E = Ae^{i\left(n_1 \sin\theta_1 k_0 x + i k_0 z \sqrt{(n_1 \sin\theta_1)^2 - n_2^2} - \omega t\right)}$$
$$= Ae^{-k_0 z \sqrt{(n_1 \sin\theta_1)^2 - n_2^2}} e^{i(n_1 \sin\theta_1 k_0 x - \omega t)}$$

Exponentially decay along z-direction

Propagating along x-direction

Evanescent field

Evanescent Field



Total internal reflection

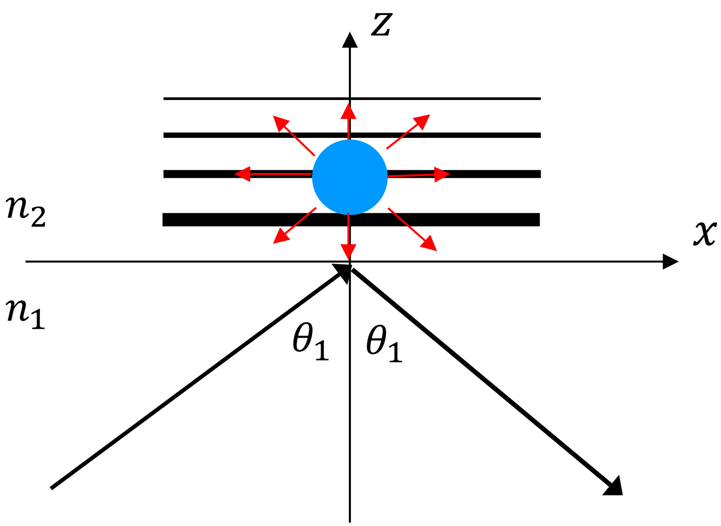
Poynting vector: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

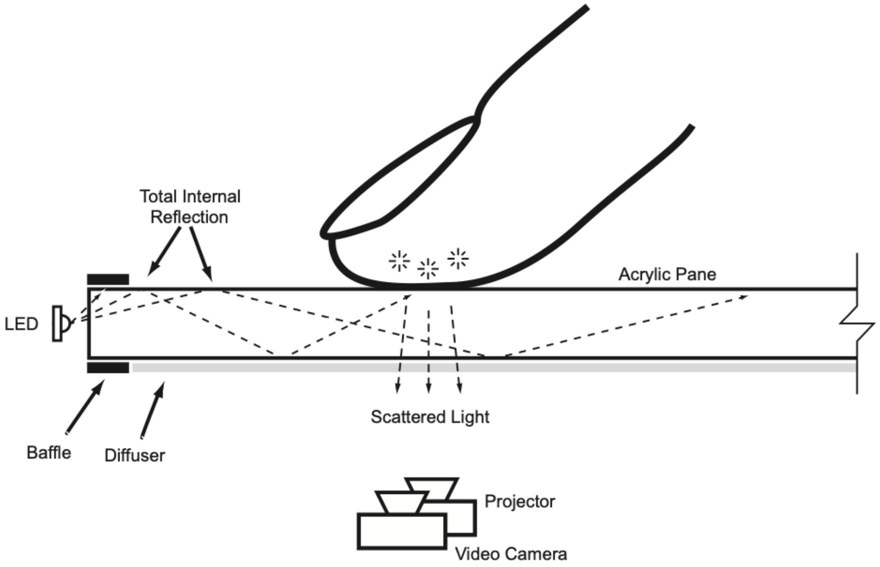
$\langle S_z(t) \rangle = 0$

For evanescent field, time averaged Poynting vector along the z-direction is zero, which means no optical energy is transported along the z-direction. All optical energy is transported to the reflected beam, so energy conservation law is preserved.

However, the electric (and magnetic) fields in medium 2 are nonzero. When an object is brought very close to the surface, it will experience the effects of the evanescent fields in a similar way to that of normal optical fields.

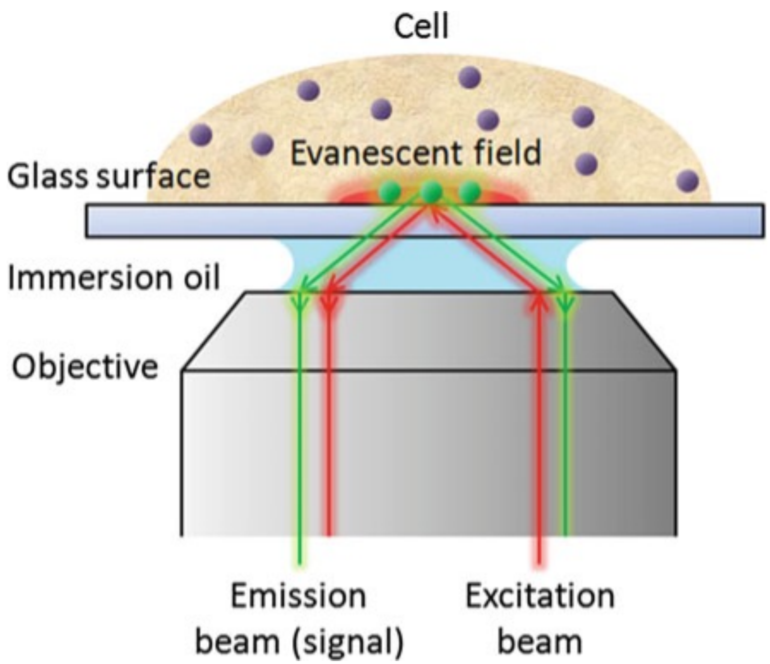
Evanescent Field

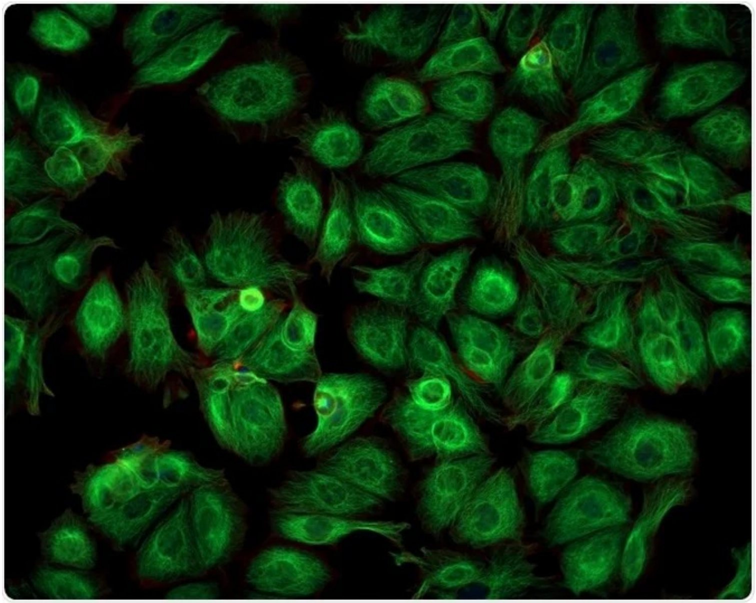




A particle or molecule placed near the surface will experience the evanescent fields and scatter or emit (e.g., fluorescence) light

Total Internal Reflection Fluorescence Microscope





Total internal reflection fluorescence microscope (TIRFM) ‘illuminates’ cells or molecules with evanescent fields via total internal reflection. There are several advantages of TIRFM in comparison to conventional widefield fluorescence microscope

- Short imaging depth: as evanescent fields decay exponentially with distance, only a thin layer (roughly <100 nm) of cell specimen very close to the surface is excited, which decrease the blurring effect due to out of focus effect often occurred in conventional fluorescence microscope.
- No background signal, so high contrast image.