Nuclear and Radiation Physics (PHY2005)

Lecture 8

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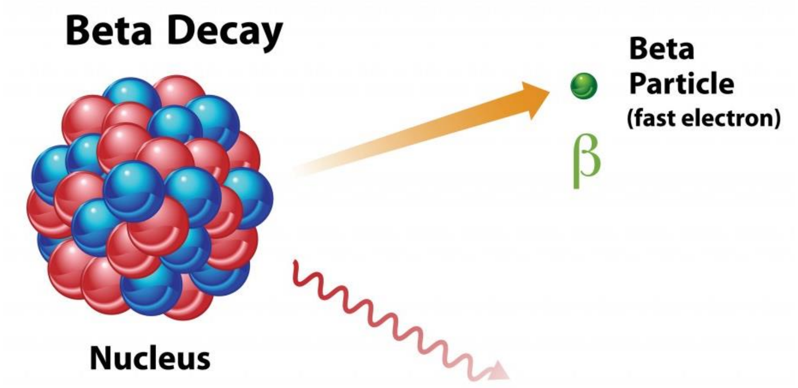
2021-2022



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Recap & Learning Goals

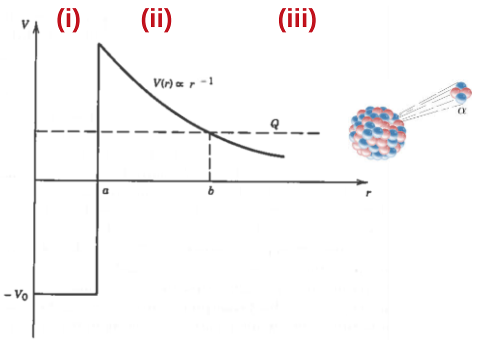
$$ {}^{A}_{Z}X_N \rightarrow {}^{A-4}_{Z-2}X'_{N-2} + \alpha $$

$$ Q = (m_X - m_{X'} - m_\alpha)c^2 $$

$$ T_\alpha = Q\left(1 - \frac{4}{A}\right) $$

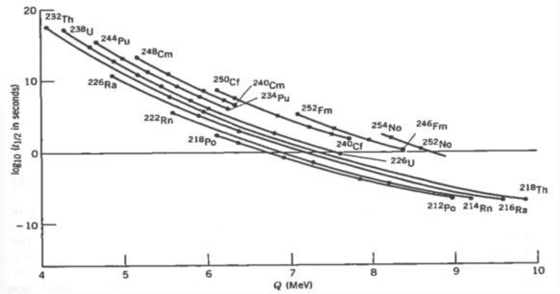
Summary of Lecture 7 *(Chap.4)*

- Alpha Decay
  - ✓ Properties (e.g. Geiger-Nuttal rule)
  - ✓ Key equations (Q-value, T_α)
  - ✓ Theory (Gamow-Gurney model, tunnelling)



Learning goals of Lecture 8 *(Chap.4)*

- Knowing the basic theory of *beta decay*
- Understanding physical reasoning behind *beta decay*
- Understanding physical reasoning behind *gamma decay*



4. Nuclear Decays
4.3 Beta Decay I

Beta Decay *(general properties)*

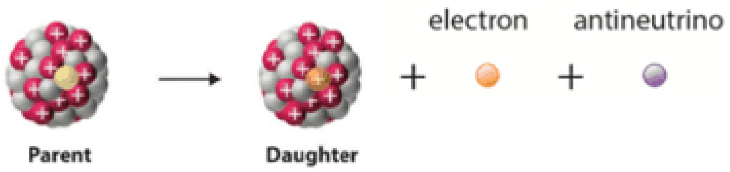
- β decay: proton → neutron, or neutron → proton
  $A = Z + N = constant$ ($Z \to Z \pm 1$, or $N \mp 1$)

- n → p + e⁻ negative beta decay (β^-)
  p → n + e⁺ positive beta decay (β^+)
  p + e → n orbital electron capture (ε)

- ***β^- decay*** → “creating” an electron from available decay energy
  (electron immediately rejected from the nucleus)

- ***β^+ and ε decays*** occur only for <u>protons bound in nuclei</u>

- <u>additional particle</u> must be involved in β decay processes (**?**)



$${}^{A}_{Z}X \to {}_{Z+1}^{A}Y + \beta^- + \bar{\nu}_e$$

Beta Decay *(general properties) cont.*

- β decay electron energy distribution → continuous from zero up to an endpoint energy (!)
- β decay is not a two-body process → ***neutrino*** *(carries missing energy)*
- conservation of electric charge → electrically neutral
- conservation of angular momentum → spin ½
- β^+ decay → neutrino (ν)
- β^- decay → antineutrino ($\bar{\nu}$)

typical β-decay processes

Decay	Type	Q (MeV)	$t_{1/2}$
$^{23}Ne \rightarrow {}^{23}Na + e^- + \bar{\nu}$	β^-	4.38	38 s
$^{99}Tc \rightarrow {}^{99}Ru + e^- + \bar{\nu}$	β^-	0.29	2.1×10^5 y
$^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$	β^+	3.26	7.2 s
$^{124}I \rightarrow {}^{124}Te + e^+ + \nu$	β^+	2.14	4.2 d
$^{15}O + e^- \rightarrow {}^{15}N + \nu$	ε	2.75	1.22 s
$^{41}Ca + e^- \rightarrow {}^{41}K + \nu$	ε	0.43	1.0×10^5 y

Beta Decay Energetics *(free-neutron decay)*

$n \rightarrow p + e^- + \bar{\nu}$

① $Q = (m_n - m_p - m_{e^-} - m_{\bar{\nu}})c^2$

② $Q = T_p + T_{e^-} + T_{\bar{\nu}} \simeq T_e + T_{\bar{\nu}}$ (max observe $T_e = 0.782 \pm 0.013$ MeV)

$T_p \sim 0.3$ keV (recoil)

① $Q = m_n c^2 - m_p c^2 - m_e c^2 - m_{\bar{\nu}} c^2 =$

$= 939.573 \text{ MeV} - 938.280 \text{ MeV} - 0.511 \text{ MeV} - m_{\bar{\nu}} c^2 =$

$= 0.782 \text{ MeV} - m_{\bar{\nu}} c^2 \longrightarrow m_{\bar{\nu}} \simeq 0$! (< 1 eV)

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Beta Decay Energetics *(β^- in a nucleus)*

$$Q_{\beta^-} = \left[m_N\left({}^A_Z X\right) - m_N\left({}^{A}_{Z+1}X'\right) - m_e\right]c^2$$

→ nucleon mass!

atomic mass!

$$m\left({}^A X\right)c^2 = m_N\left({}^A X\right)c^2 + Z m_e c^2 - \sum_{i=1}^{Z} B_i$$

B_i → binding energy of the i-electron

$$Q_{\beta^-} = \left\{\left[m\left({}^A X\right) - Z m_e\right] - \left[m\left({}^A X'\right) - (Z+1) m_e\right] - m_e\right\}c^2 +$$

$$+ \left\{\underbrace{\sum_{i=1}^{Z} B_i - \sum_{i=1}^{Z+1} B_i}_{\to\ \sim 0}\right\} = \underline{\left[m\left({}^A X\right) - m\left({}^A X'\right)\right]c^2}$$

EXAMPLE

${}^{210}Bi \rightarrow {}^{210}Po$

$$Q_{\beta^-} = \left[m\left({}^{210}Bi\right) - m\left({}^{210}Po\right)\right]c^2 = \underline{1.161\ MeV}$$

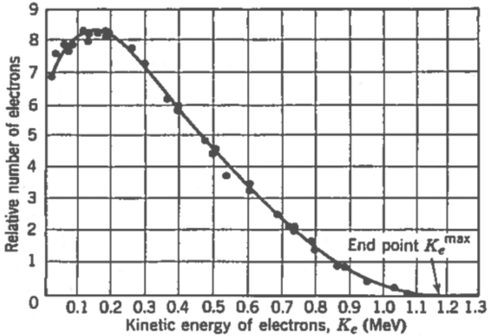
(end point of e⁻ energy distr.)

$${}^A_Z X_N \rightarrow {}^{A}_{Z+1}X'_{N-1} + e^- + \bar{\nu}$$

$$Q_{\beta^-} = \left[m\left({}^A X\right) - m\left({}^A X'\right)\right]c^2$$

$$\left(T_e\right)_{max} = \left(E_{\bar{\nu}}\right)_{max} = Q_{\beta^-}$$

electrons from β decay of ${}^{210}Bi$



Beta Decay *(β^+ and ε in a nucleus)*

- β+ decay *(<u>positron decay</u>)*:
  - continuous distribution of neutrino energies up to $Q_{\beta+}$

$$\boxed{{}^{A}_{Z}X_{N} \rightarrow {}^{\ \ \ A}_{Z-1}X'_{N+1} + e^{+} + \nu}$$

$$\boxed{Q_{\beta^+} = \left[m\left({}^{A}X\right) - m\left({}^{A}X'\right) - 2m_e\right]c^2}$$

- ε decay *(<u>electron capture</u>)*:
  - capture from inner atomic shell (K or L)
  - electron vacancy → electron filling → <u>X-ray(s)</u>
  - X-ray(s) energy = <u>atomic binding energy</u> of captured electron
  - atomic mass of X' (after decay) → greater than in its ground state by B_n *(binding energy of the captured n-shell electron)*
  - two-body final state → <u>monoenergetic neutrino</u> ($Q\varepsilon$)

$$\boxed{{}^{A}_{Z}X_{N} + e^{-} \rightarrow {}^{\ \ \ A}_{Z-1}X'_{N+1} + \nu}$$

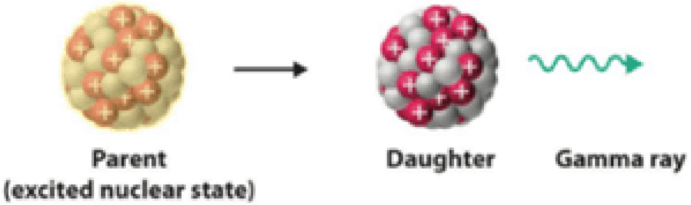
$$\boxed{Q_{\varepsilon} = \left[m\left({}^{A}X\right) - m\left({}^{A}X'\right)\right]c^2 - B_n}$$

4. Nuclear Decays
4.4 Gamma Decay I

Gamma Decay *(general properties)*

- γ decay: transitions from excited (nuclear) states to lower energy states
- typical energy range: *keV – MeV*
- can be preceded by β decay *(excited daughter nuclei)*
- … or by other reactions, e.g. neutron capture

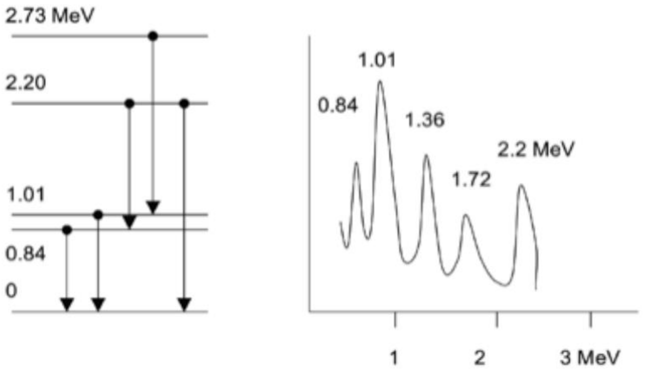
- γ-ray energy measurement → electron energy after Compton/photoelectric effect, or pair production
- measured γ-ray spectrum → energy of nuclear excited states



$$^{Am}_{Z}X \rightarrow {}^{A}_{Z}X + {}^{0}_{0}\gamma$$

EXAMPLE
5 MeV protons → ^{27}Al target → ^{27}Al* → characteristic γ-ray spectrum

Energy levels for ^{27}Al and γ-ray spectrum after bombardment with 5-MeV protons



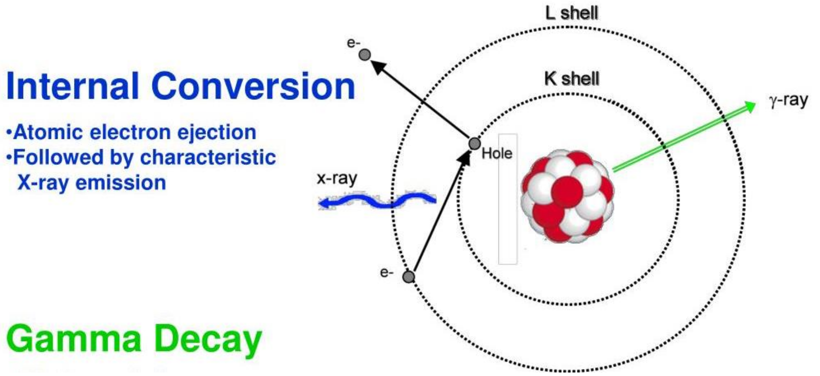
Gamma Decay *(energetics)*

- often γ-rays are accompanied by <u>monoenergetic electrons</u>
- Electron energy, ε, related to the decay energy, E:
- W: binding energy of a K-shell (or L-, M-shell)

$$\boxed{\varepsilon = E - W}$$

- ***internal conversion*** → direct transfer of energy between a nucleus and an electron of its atom through electromagnetic interaction
- nucleus decays to a lower state without producing a γ-ray (**!**)
- internal conversion coefficient, α_K
- total transition rate *(initial to final nuclear state)*, R_t
- *R : γ-ray emission rate*
- *R_{ic} : internal conversion rate*

$$\boxed{R_t = R + R_{ic}}$$



$$\boxed{R_t = R + \alpha_t R = R(1 + \alpha_t)}$$

- $\alpha_t = \alpha_K + \alpha_L + \alpha_M + \dots$

$$\boxed{T = \frac{1}{R_t} = \frac{1}{R(1 + \alpha_t)}}$$

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Calculate the binding energy of the captured electron in the following β-decay:

$^{41}Ca + e^- \rightarrow {}^{41}K + \nu$

$$Q_\varepsilon = [m({}^AX) - m({}^AX')]c^2 - B_n$$

[$m(^{41}Ca)$ = 40.962278 *u* ; $m(^{41}K)$ = 40.961825 *u* ; *Q* = *0.4216 MeV*]

$$m(^{41}Ca) - m(^{41}K) = 0.000453\,u = 0.000453 \times 931.5 = 0.4220\ MeV$$

$$B_n = Q - [m(^{41}Ca) - m(^{41}K)]c^2 = 0.4216 - 0.4220 =$$

$$= 0.4\ keV \quad (L\text{-shell})$$