# PHY2001: Statistical Mechanics Summary/Revision

*[Note: this document is intended as a complement to the lecture notes provided and your own notes taken at lectures. The aim is to summarise the main points and guide revision. It is not an exhaustive list of everything covered in the module nor everything required for the examinations. For definitions of the symbols used here, see the main lecture notes and/or the handout on notation.]*

## Section 1:

- **Definitions**: isolated system, weakly interacting system, macrostate, microstate, single-particle state, distribution, statistical weight
- **Counting microstates**
- **The central postulate of statistical mechanics** states that if $\Omega$ microstates are accessible, then the probability of being in each of the microstates is $1/\Omega$
- **Boltzmann's entropy formula** (entropy related to number of microstates):
$$S = k_B \ln \Omega$$

## Section 2:

- Applied to systems with many particles, the **central postulate** can be extended to conclude that the **average distribution is approximately the most probable distribution**
- **Deriving the Boltzmann distribution function:** obtain expression for statistical weight, differentiate the logarithm, find the maximum subject to constraints on total number of particles and total energy
- **The Boltzmann distribution:** $n_i = \frac{g_i N}{Z} e^{-\epsilon_i/k_B T}$
- **The partition function**: $Z = \sum_i g_i e^{-\epsilon_i/k_B T}$
- **Application of the Boltzmann distribution** to systems with few states

## Section 3:

- **Distinguishable** and **indistinguishable** particles
- **Bosons and Fermions**: symmetry of the wavefunction under particle exchange, the **exclusion principle**, **spin** and **spin degeneracy**
- **Many-level systems** and formulation in terms of **continuous range of energy levels**
- Deriving the **distribution functions for Fermi-Dirac, Bose-Einstein, and dilute gas statistics:**
$$f_{FD} = \frac{1}{C\exp(\epsilon/k_B T) + 1}\ ,\ f_{BE} = \frac{1}{C\exp(\epsilon/k_B T) - 1},\ f_{DG} = \frac{1}{C\exp(\epsilon/k_B T)}$$

## Section 4:

- **The density of states in a 3D cubic well:** derivation in $k$-space and transformation to other spaces (including energy):

$$g(k)dk = \frac{VG}{2\pi^2}k^2 dk \quad ; \quad g(\epsilon)d\epsilon = \frac{VG}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \epsilon^{1/2}d\epsilon$$

- **Use of the density of states to calculate number of states**: for a narrow interval, can use the density of states formulae directly. For a wide interval, integrate the density of states formula over appropriate range (e.g to calculate total number of states with energy less than $\epsilon_{\max}$, integrate $g(\epsilon)d\epsilon$ from 0 to $\epsilon_{\max}$).

## Section 5:

- **The distribution for a dilute gas and the Maxwell-Boltzmann speed distribution**, including the evaluation of the partition function.
- **Using the Maxwell-Boltzmann speed distribution** e.g. to calculate the number of particles in a particular speed range.
- **The degeneracy parameter,** $A = \frac{N}{V}\left(\frac{h^2}{2\pi m k_B T}\right)^{3/2}$, and its application
- **Fermi-Dirac gasses:** the **Fermi Energy/Fermi Level**, the **Fermi Temperature**, shape and interpretation of the **Fermi-Dirac distribution function**
- **Use of Fermi Temperature** to estimate when effects of Fermi-Dirac statistics will be significant
- **Degeneracy pressure in a Fermi-Dirac gas**
- **Blackbody radiation:** application of Bose-Einstein statistics to photon gas
- **Planck function** for black body photon energy density distribution