PHY2006 Assignment 2 – Partial Differentiation/PDEs

Deadline for Submission 6pm, Monday 4 Oct 2021

1. Consider the function

$$\psi(x,t) = A\,x\exp(-\gamma x^2)\exp\left(-\frac{iEt}{\hbar}\right)$$

Where A, γ and E and are constants.

Determine the following partial derivatives

(a) $\frac{\partial\psi}{\partial t}$ **[10]**

(b) $\frac{\partial^2\psi}{\partial x^2}$ **[20]**

(c) Show that $\psi(x,t)$ is a solution to the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}m\omega^2x^2\psi$$

and hence obtain expressions for γ and E.

[20]

2. In a spherical polar coordinates the Heat equation has the form

$$\frac{\partial T}{\partial t} = D\nabla^2 T$$

where $T(r,t)$ is the temperature, r is the distance from the centre and D is the heat diffusivity.

(a) If the system is spherically symmetric, i.e. T does not have an angular dependence, write down the right hand side of the equation in expanded form. **[15]**

(b) Show that the following function is a solution to this heat equation

$$T(r,t) = A\exp(-\lambda^2 Dt)\frac{\sin(\lambda r)}{r}$$

[35]

The Laplacian in spherical coordinates is

$$\nabla^2 u = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2}$$

Extra Question

Characterise the following partial differential equations in terms of the following categories. Make sure you clearly justify your answers, don't just write down the answer.

- Order
- Linear : Non-linear
- Homogeneous : Inhomogeneous
- Elliptical : Parabolic : Hyperbolic : Mixed : Undefined/Inapplicable

(a) Schrödinger's equation $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x,t)\psi = i\hbar\frac{\partial\psi}{\partial t}$

(b) Poisson's equation $\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} - \rho(x,y) = 0$

(c) Equation for light in a conducting material $\sigma\mu_0\frac{\partial \boldsymbol{E}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{\partial^2 \boldsymbol{E}}{\partial x^2}$ (σ, μ_0, c are constants)

(d) $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

(e) $\frac{\partial^2}{\partial x^2}\left(\frac{\partial}{\partial y}(yu)\right) + \left(\frac{\partial u}{\partial x}\right)^2 = 0$

(f) $x^2\frac{\partial^2 u}{\partial x^2} + 2y\frac{\partial^2 u}{\partial x\partial y} - \frac{\partial^2 u}{\partial y^2} = 0$