PHY2003 ASTROPHYSICS I

Lecture 2. Magnitudes and photon fluxes

Magnitudes

Ancient Greeks catalogued stars according to magnitude

$$\begin{aligned} \text{magnitude}\, 1.0 &= \text{brightest} \\ \text{magnitude}\, 6.0 &= \text{faintest} \end{aligned}$$

However, the eye responds roughly logarithmically to light. If f is the energy flux received from the star (J/sec/m^2) and m is the magnitude of the star, then the

$$\boxed{m = -2.5 \log f + C}$$

The constant C is the zero-point constant, to ensure that stars emitting enough flux in what ever units are used have $m = 0$.

Assume that two stars have magnitudes of m_1 and m_2 respectively. Then:

$$\begin{aligned} m_1 &= -2.5 \log f_1 + C \\ m_2 &= -2.5 \log f_2 + C \end{aligned}$$

Equating these by cancelling the constants, we get

$$m_1 - m_2 = -2.5 \log f_1 + 2.5 \log f_2 = 2.5 \log \frac{f_2}{f_1}$$

Inverting this, we find that

$$f_2 = f_1 10^{0.4(m_1 - m_2)}$$

Apparent Magnitudes

The apparent magnitude is a measurement of the total flux of radiation (light) f_λ reaching the Earth within a defined range of wavelengths.

f_λ can be measured in J cm^{-2} sec^{-1} Å^{-1} or J m^{-2} sec^{-1} nm^{-1}.

We use a filter of width $d\lambda$ centred on λ to cut out all other wavelengths. If T_λ is the transmission function of the filter as a function of wavelength, then the total amount of light (energy) observed is given by

$$f = \int_0^\infty f_\lambda T_\lambda d\lambda$$

converting to magnitudes:

$$m_\lambda = -2.5 \log f + C$$

f is measured in J m^{-2} sec^{-1} or ergs cm^{-2} sec^{-1}.

The UBV System

The most general and widely used set of filters is known as the UBV system, standing for Ultra-violet, Blue and Visible wavelengths respectively. The V filter closely matches the response of the human eye.

It has been extended into R (red) and I (infrared) wavelengths.

Filter	Central λ	FWHM
U	3500 Å	700 Å
B	4600 Å	1000 Å
V	5500 Å	900 Å
R	6500 Å	1200 Å
I	8000 Å	1000 Å

Thus we can define the magnitude for each of these filters:

$$m_{\rm U} = U = -2.5 \log f_{\rm U} + C_{\rm U}$$

$$m_{\rm B} = B = -2.5 \log f_{\rm B} + C_{\rm B}$$

$$m_{\rm V} = V = -2.5 \log f_{\rm V} + C_{\rm V}$$

By international agreement, the star α Lyrae (Vega) was to have an apparent magnitude of 0.00 at all wavelengths. Measuring the flux from Vega showed that Earth receives 3.2×10^{-8} J/sec/m^2 in the V-filter.

$$m_V(Vega) = 0.0 = -2.5 \log(3.2 \times 10^{-8}) + C_V$$

$$C_V = -18.74$$

$$\boxed{m_V = -2.5 \log f_{\rm V} - 18.74}$$

V-band magnitudes are still most common form of quoting the brightness of an astronomical object, Example: The human eye pupil is approximately 5mm in diameter when dark-adapted. How many photons/sec does it receive from the faintest star visible? (Faintest stars you can see with the naked eye are about 6th V magnitude)

Apparent Magnitudes

The apparent magnitude is the measured brightness of the source by the observer. Two stars that have the same luminosity but are located at different distances to Earth would have different measured apparent magnitudes. In this case, the star further away from the Earth would be fainter. We can use absolute magnitudes to compare the luminosities (or intrinsic brightnesses) of astronomical sources. The absolute magnitude (M) is how bright the star or galaxy would be if you could take it and move to it a distance of 10 parsecs from Earth. By considering stars/galaxies at a fixed distance, astronomers can easy compare sources and identify which have have larger or smaller luminosities.

For a star or galaxy with an apparent magnitude (m) and a distance (d) in parsecs from the Earth, its apparent magnitude is defined as:

$$M = m - 5\log(d) + 5$$

To use the formula above, make sure you put the star's/galaxy's distance d in parsecs (1 pc = 206265 au).

In planetary astronomy there is a slightly different definition for absolute magnitude called H. This is the magnitude that a Solar System body would have if it was placed at 1 au from the Sun. This allows the same type of comparisons, enabling estimates of the sizes of Solar System objects (related to the amount of light reflected by the objects). Unless specified as H, when astronomers refer to absolute magnitude, they are referring to the absolute magnitude M definition.

Optical Telescopes and Sensitivity

The basic function of a telescope is to gather more light than the human eye, and bring that light into the eye or to a focus in some kind of detector.

Telescopes can use lenses (refractors) or mirrors (reflectors) to gather light. All modern telescopes are reflectors.

Consider an object emitting L_λ photons/second isotropically (in all directions). This is the Luminosity of the object.

If it is a distance d from Earth, then the flux of photons at the Earth is

$$f_\lambda = L_\lambda/(4\pi d^2) \text{ photons/m}^2\text{/sec}$$

(λ because the photon flux and other quantities vary with wavelength, so we normally use a restricted range of wavelengths.)

The number of photons n_λ collected by your telescope of diameter D is given by

$$n_\lambda = \frac{\pi D^2}{4} \times \frac{L_\lambda}{4\pi d^2} = \frac{L_\lambda D^2}{16 d^2} \text{ photons/sec}$$

So if an object is twice as far away, you need a telescope twice as wide to collect the same number of photons.

Example: Roughly how much more sensitive is the 3.9-m Anglo-Australian Telescope than the 5-mm human eye?

In the Solar System, almost all light is reflected sunlight. Say the object is at a heliocentric distance from the Sun R_h. The Sun emits L_λ photons per second, so the number hitting an object of cross-section area C is

$$N_\lambda = C \frac{L_\lambda}{4\pi R_h^2} \text{ photons/sec}$$

Assuming 100% perfect reflection, these are the photons emitted by the object. These are not reflected in all directions, but we can still calculate an average number of photons $< n_\lambda >$ collected by a telescope a distance d away.

$$< n_\lambda >= \frac{N_\lambda D^2}{16d^2} = \frac{CD^2 L_\lambda}{64\pi R_h^2 d^2} \text{ photons/sec}$$

In reality, an object only reflects a fraction A_λ of light called the <u>Albedo</u>, so

$$\boxed{< n_\lambda >= \frac{A_\lambda CD^2 L_\lambda}{64\pi R_h^2 d^2} \text{ photons/sec}}$$