Assignment 2 : Quantum Mechanics

(Deadline : Friday, 12th Nov 2021, 10 pm)

To attain full marks, all answers must carefully explain/justify all steps.

1. **(a)** $\psi = 10\ \exp(ikx)$ represents the wave function of a stream of electrons of wavelength 1 nm. What is the probability flux of the electrons (number of electrons flowing per unit area, per unit time)?

[3]

(b) A beam of 5 eV electrons travels through a potential barrier of width 1 nm with no reflection. Calculate the potential at the barrier.

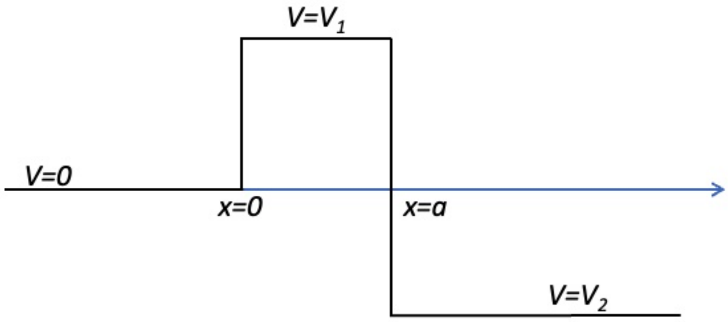
[3]

(c) An electron is completely confined in a cubic box with 0.8 nm sides. For energies less than 9.5 eV, what are the number of allowed energy levels and quantum states (ignore the spin of the electron)

[4]

CONTINUED...

2. Let us assume a stream of non-relativistic electrons of kinetic energy E travelling along the x-axis experiences an abrupt change in potential from 0 to V_1 at $x = 0$, where $E > V_1$. At $x = a$, the electrons experience another step change in potential from V_1 to V_2, as shown in the sketch below.



(a) Starting from the "time independent" Schrödinger equation for the particle wavefunctions in different regions (i.e. $x < 0$, $0 < x < a$ and $x > a$), write down the general solutions for the allowed eigenfunctions.

[5]

(b) Draw a suitably labelled and scaled diagram to represent appropriately the eigenfunctions in all regions.

[3]

(c) Using necessary boundary conditions to ensure the eigenfunctions are well behaved, show that there will be a 100% transmission of electrons to the region $x > a$ when $V_2 = 0$ and the wavelength of the eigenfunction in the region $0 < x < a$ is $2a$.

[12]

3. **(a)** μ^- is an elementary particle with charge $-e$ and mass that is 207 times that of an electron. If one replaces the electron in a hydrogen atom by a μ^-, calculate how the ionisation potential of the atom would change.

[3]

(b) Explain the physical significance of the magnetic quantum number m_l for an electron in a hydrogen atom.

[4]

(c) In a hydrogen atom, the radial wave function for an electron in its first excited state is given by

$$R(r) = \frac{1}{\sqrt{2a_0^3}} \left(1 - \frac{r}{2a_0}\right) \exp\left(-\frac{r}{2a_0}\right),$$

where a_0 is the Bohr radius.

(i) Calculate the distances (in units of the Bohr radius) of the electron from the proton where the radial probability distribution function has maxima.

[7]

(ii) Draw a suitably labelled sketch of the radial probability density distribution of the electron.

[4]

(iii) What is the orbital angular quantum number of the electron that this radial wavefunction corresponds to? Justify your answer.

[2]

END OF ASSIGNMENT