# Nuclear and Radiation Physics (PHY2005)

*Lecture 5*

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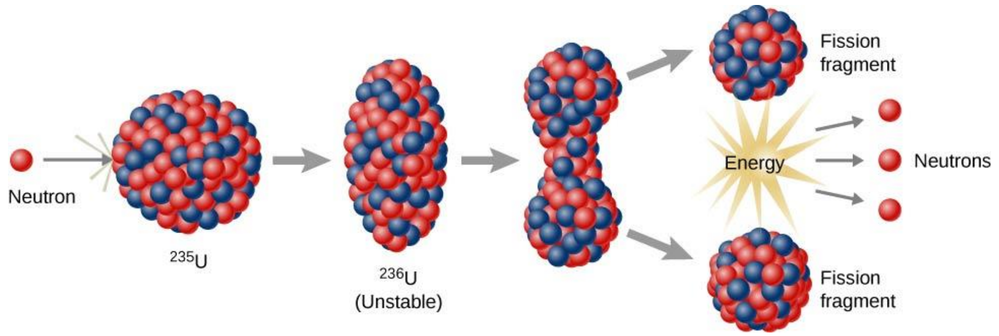
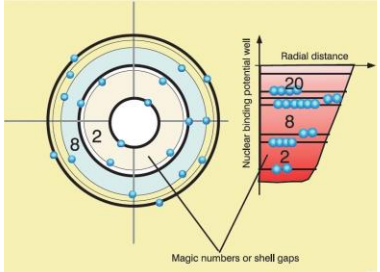
*2021-2022*



# Recap & Learning Goals

**Summary of Lecture 4** *(Chap.3)*

- The Nuclear Shell Model
  - ✓ Shell model potential and energy levels
  - ✓ Quantum states and magic numbers ☺
  - ✓ Magnetic dipole moments ☹
  - ✓ Electric quadrupole moments ☹




$$V(r) = \frac{-V_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

$J = L \pm ½$

*spin orbit degeneracy* → $2J + 1$

$$\langle Q_{sp}\rangle = -\frac{2J-1}{2(J+1)}\langle r^2\rangle$$

$$\langle Q\rangle = \langle Q_{sp}\rangle\left[1 - 2\frac{n-1}{2J-1}\right]$$

**Learning goals of of Lecture 5** *(Chap.3)*

- Understanding physical reasoning behind the *Liquid Drop Model*
- Understanding physical reasoning behind the *Collective Model*

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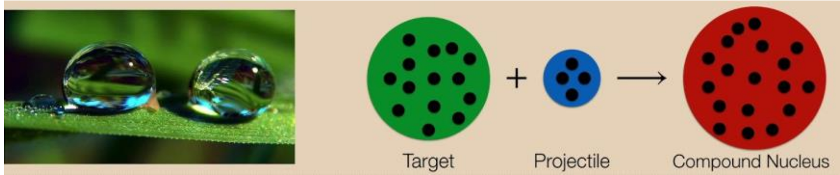
**Liquid Drop model** *(assumptions)*

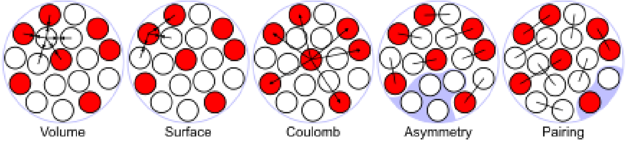
- interior mass densities of all nuclei is constant
  *(not valid for small-A nuclei)*
- total binding energies of all nuclei are proportional to their masses *(B/A ≈ constant)*
  - ✓ macroscopic liquid drops → interior densities constant
  - ✓ macroscopic liquid drops → heats of vaporization are proportional to the masses
- nucleus → sphere with density abruptly dropping to zero at its surface

*similarly to the behaviour of liquid drops, a small projectile can be added to a target (nucleus) forming a heavier compound nucleus*




**Semi-empirical mass formula**

$$\boxed{M_{Z,A} = f_0(Z,A) + f_1(Z,A) + f_2(Z,A) + f_3(Z,A) + f_4(Z,A) + f_5(Z,A)}$$




- $M_{Z,A}$ → mass of an atom whose nucleus has *Z* and *A* nucleons
- $f_0$ → mass term
- $f_1$ → volume term
- $f_2$ → surface term
- $f_3$ → Coulomb term
- $f_4$ → asymmetry term
- $f_5$ → pairing term

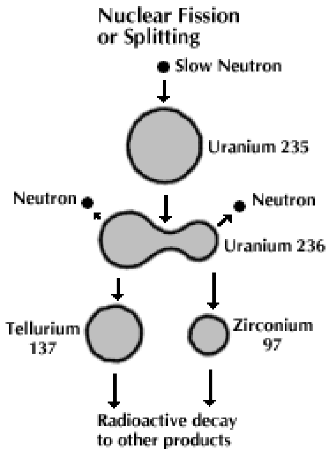
**Semi-empirical mass formula** *(terms and physical meaning)*

$$M_{Z,A} = f_0(Z,A) + f_1(Z,A) + f_2(Z,A) + f_3(Z,A) + f_4(Z,A) + f_5(Z,A)$$

- mass term ($f_0$) → mass of the atom constituents $f_0(Z,A) = 1.007825\,Z + 1.008665\,(A-Z)$
- remaining terms → mass correction for various effects influencing the nuclear binding energy
  - ✓ volume term ($f_1$) → constant binding energy per nucleon $f_1(Z,A) = -a_1 A$
    *(mass reduction → binding energy increase)*
  - ✓ surface term ($f_2$) → surface area of the nucleus ("surface tension energy") $f_2(Z,A) = a_2 A^{2/3}$
    *(mass increase → binding energy reduction)*
  - ✓ Coulomb term ($f_3$) → Coulomb repulsion between the protons $f_3(Z,A) = a_3 \dfrac{Z^2}{A^{1/3}}$
    *(mass increase → binding energy reduction)*
  - ✓ asymmetry term ($f_4$) → Z ~ N $f_4(Z,A) = a_4 \dfrac{\left(Z-\frac{A}{2}\right)^2}{A}$
    *(mass increase → binding energy reduction)*
  - ✓ pairing term ($f_5$) → even-Z and even-N (or odd-Z and odd-N)
    *(mass reduction → binding energy increase if both Z and N are even)*

- $\alpha_1$ to $\alpha_5$ → empirical fit of exp. masses
  $\alpha_1 = 0.001691$; $\alpha_2 = 0.001911$;
  $\alpha_3 = 0.000763$; $\alpha_4 = 0.10175$; $\alpha_5 = 0.012$

$$f_5(Z,A) = -a_5 A^{-\frac{1}{2}} \quad (Z\ even\ and\ N\ even)$$
$$f_5(Z,A) = 0 \quad (Z\ even\ and\ N\ odd, or\ N\ even\ and\ Z\ odd)$$
$$f_5(Z,A) = a_5 A^{-\frac{1}{2}} \quad (Z\ odd, N\ odd)$$

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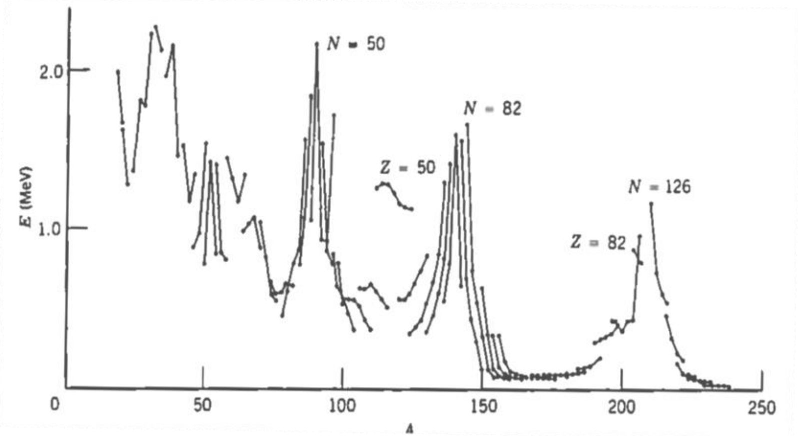
**Collective model** *(<u>nuclear vibrations</u>)*

- combination of Shell and Liquid Drop models
- nucleons in unfilled subshells move in a net nuclear potential (core of filled subshells), BUT the <u>potential</u> is not static (undergoes <u>deformations</u>)
- nuclear potential deformation → collective motion *(Liquid Drop model)*
- nucleons fill energy levels → same magic numbers *(Shell model)*

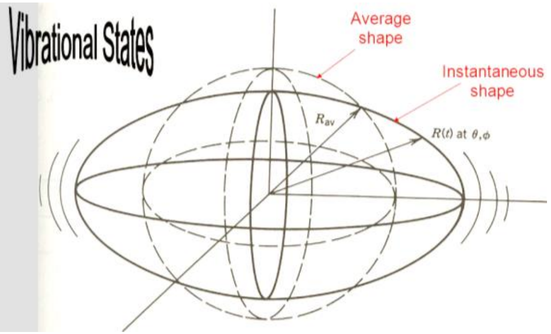
***EXAMPLE*** *(energies of lowest $2^+$ states for even-even nuclei)*
- *energy of excited states decreases smoothly (except for magic numbers)*
- *$150 \leq A \leq 190$ → small and constant E*




- ***A < 150*** → **vibrations** around a spherical equilibrium shape
- ***150 < A < 190*** → **rotations** of a non-spherical system

# 3. Nuclear Models
## *3.3. The Collective model II*

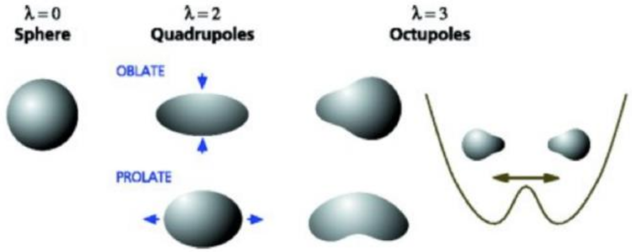
*vibrating nucleus with a spherical equilibrium shape*

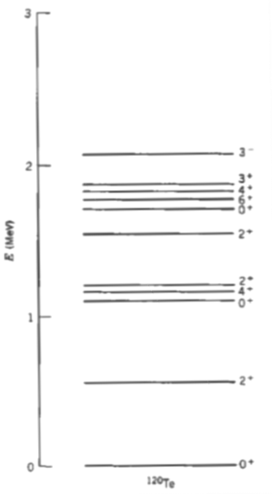



- Position of a point on the nuclear surface → spherical (instantaneous) coordinates
- Y: spherical harmonics
- $\alpha$ → amplitude of the spherical harmonics

$$\boxed{\mathrm{R}(t) = R_{av} + \sum_{L\geq 1} \sum_{m=-L}^{+L} \alpha_{Lm}(t) Y_{Lm}(\theta, \varphi)}$$

- *L* = *1* → dipole vibration
- *L* = *2* → quadrupole vibration
- *L* = *3* → octupole vibration
- <u>*phonon*</u>: quantum vibrational energy




*L=3 (one unit of octupole phonon)*

*L=2 (two units of quadrupole phonon)*

*L=2 (one unit of quadrupole phonon)*

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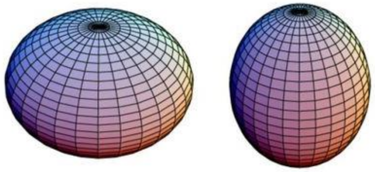
**Collective model** *(nuclear rotations)*

- Nuclei with non-spherical equilibrium shape → nuclear rotation motion *(deformed nuclei)*
- ***150 < A < 190*** and ***A > 220*** *(rare earths and actinides)*
- Typical shape → ellipsoid of revolution

  - deformed nuclei surface analytical expression: $\boxed{R(\theta,\varphi) = R_{av}[1+\beta Y(\theta,\varphi)]}$
  - $\beta > 0$ (prolate); $\beta < 0$ (oblate)
  - stable deformation → large electric quadrupole moment

*equilibrium (static) shape of deformed nuclei*



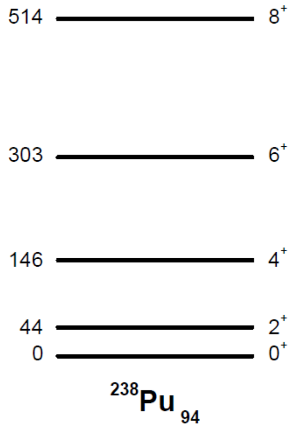
oblate deformation ($\beta<0$)    prolate deformation ($\beta>0$)

# 3. Nuclear Models
## *3.3. The Collective model IV*

**Nuclear rotation** *(cont.)*

- $\mathfrak{I}$: moment of inertia; *I*: angular momentum quantum number of the nucleus *(nuclear spin)*
- energies of a rotating nucleus: $$\boxed{E = \frac{\hbar^2}{2\mathfrak{I}} I(I+1)}$$
- increasing *I* → adding rotational energy *(rotational band)*

*ground rotational band of $^{238}$Pu (energies in keV)*




- vibrational and rotational collective motions → <u>magnetic dipole moment</u>:

$$\boxed{\mu(I) = I\frac{Z}{A}\mu_N}$$

*$Z/A \approx 0.5$ → $\mu(2) \approx +1\mu_N$*
*$Z/A \approx 0.4$ → $\mu(2) \approx +0.8\mu_N$* ☺

***Calculate the rotational energy levels of an even-Z, even-N nucleus, knowing that the mirror symmetry of even-even nuclei restricts the sequence of rotational states to even values of I ($0^+$, $2^+$, $4^+$, $6^+$, $8^+$)***

$$E = \frac{\hbar^2}{2\mathfrak{I}} I(I+1)$$

$$E(0^+) = \frac{\hbar^2}{2\mathfrak{I}}\, 0(0+1) = 0$$

$$E(2^+) = \frac{\hbar^2}{\cancel{2}\mathfrak{I}}\, \cancel{2}(2+1) = \frac{3\hbar^2}{\mathfrak{I}}$$

$$E(4^+) = \frac{\hbar^2}{2\mathfrak{I}}\, 4(4+1) = \frac{10\hbar^2}{\mathfrak{I}}$$

$$E(6^+) = \frac{\hbar^2}{2\mathfrak{I}}\, 6(6+1) = \frac{21\hbar^2}{\mathfrak{I}}$$

$$E(8^+) = \frac{\hbar^2}{2\mathfrak{I}}\, 8(8+1) = \frac{36\hbar^2}{\mathfrak{I}}$$

typically in [KeV or MeV]

***Calculate the rotational energy levels of $^{238}$Pu, knowing that the experimentally measured magnetic dipole moments are 1.6$\mu_N$ and 2.4$\mu_N$ , respectively.***

$$\boxed{\mu(I) = I\frac{Z}{A}\mu_N}$$

$$\boxed{E = \frac{\hbar^2}{2\mathfrak{I}}I(I+1)}$$

$A > 220!$ $A = 238$ ; $Z = 94 \rightarrow \frac{Z}{A} = \frac{94}{238} \simeq 0.4$

$\mu_1(I_1) = I_1 \frac{Z}{A}\mu_N = I_1 \times 0.4 \cancel{\mu_N} = 1.6 \cancel{\mu_N} \rightarrow I_1 = \frac{1.6}{0.4} = 4$

$E_1 = \frac{\hbar^2}{2\mathfrak{I}} I(I+1) = \frac{\hbar^2}{2\mathfrak{I}} 4(4+1) = \frac{20}{2}\frac{\hbar^2}{\mathfrak{I}} = \underline{\underline{10\frac{\hbar^2}{\mathfrak{I}}}}$

$\mu_2(I_2) = I_2 \times 0.4 \cancel{\mu_N} = 2.4 \cancel{\mu_N} \rightarrow I_2 = \frac{2.4}{0.4} = 6$

$E_2 = \frac{\hbar^2}{2\mathfrak{I}} I(I+1) = \frac{\hbar^2}{2\mathfrak{I}} \times 6(6+1) = \underline{21\frac{\hbar^2}{\mathfrak{I}}}$

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