Any calculator, except one with pre-programmable memory, may be used in this examination.

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2001
Quantum and Statistical Physics

Duration: 3 hours plus additional 1 hour for upload of work

Thursday 13th of May 2021
09:30 AM – 1:30 PM

Examiners: Prof S Matthews, Prof F. Peters
and the internal examiners
Dr S Sim (s.sim@qub.ac.uk)

Answer ALL questions in Section A for 4 marks each.
Answer TWO questions from Section B for 20 marks each.
Answer ONE question from Section C for 20 marks.

If you have any problems or queries, contact the School Office at mpts@qub.ac.uk or 028 9097 1907, and the module coordinator s.kar@qub.ac.uk

THE QUEEN'S UNIVERSITY OF BELFAST
DEPARTMENT OF PHYSICS AND ASTRONOMY

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.0 \times 10^{8}$ ms^{-1}
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}
	$\approx 1.26 \times 10^{-6}$ Hm^{-1}
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}
Elementary charge	$e = 1.6 \times 10^{-19}$ C
Electron charge	$= -1.6 \times 10^{-19}$ C
Planck Constant	$h = 6.63 \times 10^{-34}$ Js
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}$ Js
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}$ m^{-1}
Unified atomic mass unit	$1u = 931\ MeV = 1.66 \times 10^{-27}$ kg
1 electron volt (eV)	$= 1.6 \times 10^{-19}$ J
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.67 \times 10^{-27}$ kg
Molar gas constant	$R = 8.31$ JK^{-1}mol^{-1}
Boltzmann constant	$k = 1.38 \times 10^{-23}$ JK^{-1}
Avogadro constant	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}
Acceleration of free fall on the Earth's surface	$g = 9.81$ ms^{-2}

CONTINUED...

SECTION A

Answer ALL 10 questions from this section

Full explanations of your answers are required to attain full marks

1. Define the term blackbody. Does a blackbody always appear black? Sketch the spectral radiance against wavelength produced by two blackbodies, one at 3000 K and the other at 5000 K.

[4]

2. Consider an electron whose position is somewhere within an atom of radius $a_0 = 5.29\,\text{Å}$.

(i) What is the uncertainty in the electron's momentum?

(ii) Is this consistent with the ionisation potential (13.6 eV) of an electron in a hydrogen atom?

[4]

3. A particle is described by the eigenfunction

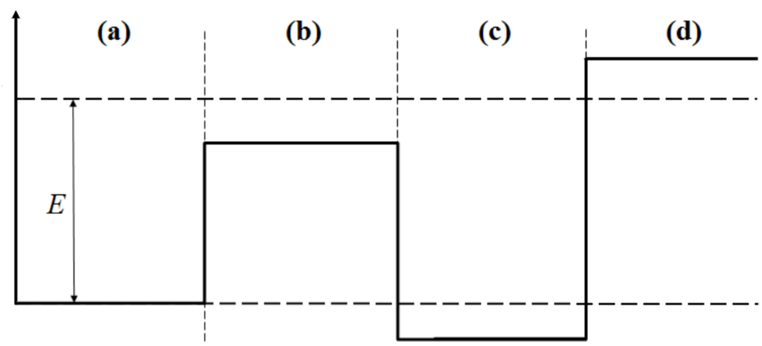
$\psi = A\sqrt{3x}$ $\quad 0 \leq x \leq 1$

$\psi = 0$ $\quad$ elsewhere

What is the probability of finding the particle in the range $0.25 \leq x \leq 0.5$?

[4]

4. The diagram below shows a particle with energy E in four regions (a)-(d) with different potential energy (solid line). Which region has the shortest de-Broglie wavelength? Justify your answer with reference to the properties of valid eigenfunctions and the time-independent Schrödinger equation.



[4]

CONTINUED...

5. A group of 100 particles is described by a wavefunction

$$\psi(x) = Ax(1+i) \qquad \text{if } -1 \leq x \leq 1$$
$$= 0 \qquad \text{elsewhere}$$

Find the value of the normalisation constant A.

[4]

6. An electron travelling with a speed of 2×10^6 m/s suddenly encounters a region of 20 V electric potential. Draw a suitably labelled sketch to show (qualitatively) the electron's wave function before and after the potential step. Justify your answer.

[4]

7. Define degeneracy. Write down all degenerate eigenstates in terms of their quantum numbers (for instance, in notation ψ_{nlm}) of an electron in its 3rd excited energy level in a hydrogen atom. Ignore the spin of the electron.

[4]

8. Explain the meaning of the terms *isolated* and *weakly interacting* as applied in statistical mechanics.

[4]

9. Consider a system of N weakly-interacting distinguishable particles. The particles have access to two single-particle states having energy 0 and ε. If the temperature of the system is T, give expressions for the entropy of this system

(a) for $kT \gg \varepsilon$

(b) for $kT \ll \varepsilon$

Carefully explain both your answers.

[4]

10. The number of particles with energy between ε and $\varepsilon + d\varepsilon$ in a system can be written as

$$n(\varepsilon)d\varepsilon = g(\varepsilon)f(\varepsilon)d\varepsilon$$

where $g(\varepsilon)$ is the density of states and $f(\varepsilon)$ is a distribution function that gives the number of particles per state. Briefly describe and explain the physical differences for both $g(\varepsilon)$ and $f(\varepsilon)$ between a photon gas and a gas of ^{4}He atoms.

[4]

CONTINUED...

SECTION B

Answer TWO questions from this section

11. **(a)** Give a short description of how Planck, Einstein, and Bohr developed models to explain experimental observations that were inconsistent with classical physics by using the idea of quantization.

[9]

(b) The Compton effect was the fourth experiment that was inconsistent with classical physics but could be explained using the ideas of quantisation.

(i) Briefly explain how the Compton effect was modelled.

[3]

(ii) Is the Compton effect observable using visible light? Justify your answer.

[2]

(c) The classical uncertainty principle states that there is a fundamental limit to the precision with which the position and wavenumber of a wavepacket may be known to, and is given by

$$\delta x \delta k \geq 1/2$$

(i) Show how a similar relationship may be obtained relating energy and time for a quantum state.

[4]

(ii) An atom absorbs a photon from a Nd:YAG laser ($\lambda = 532\,\mathrm{nm}$) to form a virtual excited state. Estimate the lifetime of this state.

[2]

12. The one-dimensional time-dependent Schrödinger equation for a single non-relativistic particle is;

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V(x,t)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

(a) Assuming that the potential is only a function of space, perform a separation of variables and clearly show (i.e. describe steps) how to obtain the time independent Schrödinger equation (TISE);

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+[V(x)-E]\psi(x)=0$$

[7]

(b) A particle of mass m is trapped in an infinite one-dimensional potential well in the region of $-a/2 \leq x \leq +a/2$. Obtain the odd and even parity eigenfunction solutions of the Schrödinger equation for this system and hence obtain an expression for the allowed energies.

[7]

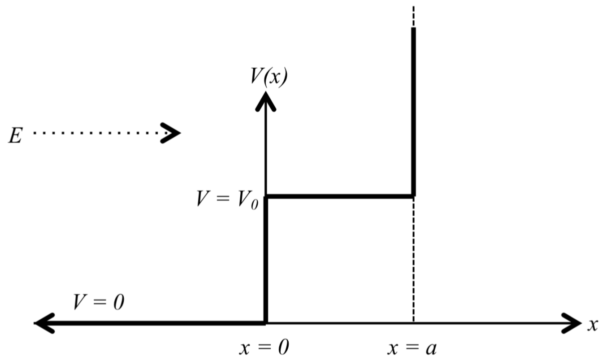
(c) Sketch the potential energy function for this well and add to it plots for the first three eigenfunctions. Clearly label the energies of these states in terms of the ground state energy E_0.

[2]

(d) Sketch the potential energy function and first three eigenfunctions for a finite well with the same width as the infinite well in (c), and of height to contain at least three bound states. Include dashed lines to show the corresponding energies for the infinite well in (c).

[4]

13. Let us assume a stream of non-relativistic electrons of kinetic energy E travelling along the x-axis experiences an abrupt change in potential from 0 to V_0 at x = 0, where $E > V_0$. At $x = a$, the electrons experience an infinitely high potential wall as shown in the figure below.



(a) Starting from the "time independent" Schrödinger equation for the particle wave-functions in different regions (i.e. $x < 0$, $0 < x < a$ and $x > a$), write down the general solutions for the allowed eigenfunctions.

[6]

(b) Using necessary boundary conditions to ensure the eigenfunctions are well behaved, show that the reflection coefficient at $x = 0$ is $R = 1$.

[12]

(c) What is the physical justification of $R = 1$ in this case?

[2]

CONTINUED...

14. **(a)** In the quantum mechanical description of single-electron systems (such as the hydrogen atom) using Schrödinger's equation, why and how has the effect of the finite nuclear mass been taken into account?

[4]

(b) μ^- is an elementary particle with charge $-e$ and mass that is 207 times that of an electron. If one replaces the electron in a hydrogen atom by a μ^-, calculate how the ionisation potential of the atom would change.

[3]

(c) In a hydrogen atom, the radial wave function for an electron in its first excited state is given by

$$R(r) = \frac{1}{\sqrt{2a_0^3}} \left(1 - \frac{r}{2a_0}\right) \exp\left(-\frac{r}{2a_0}\right),$$

where a_0 is the Bohr radius.

(i) Calculate the distances (in units of the Bohr radius) of the electron from the proton where the radial probability distribution function has maxima.

[7]

(ii) Draw a suitably labelled sketch of the radial probability density distribution of the electron.

[4]

(iii) What is the orbital angular quantum number of the electron that this radial wavefunction corresponds to? Justify your answer.

[2]

SECTION C

Answer ONE question from this section

15. **(a)** Consider an isolated system of 8 distinguishable particles with total energy 5 eV. The accessible energy levels of the system have energies and degeneracies as given in the table below.

(i) Identify all the possible distributions for the number of particles in each level, and calculate the statistical weight for each distribution.

[4]

(ii) Calculate the average distribution.

[3]

	Energy	Degeneracy
Level-1	0 eV	1
Level-2	1 eV	1
Level-3	5 eV	1

(b) For a system with non-degenerate energy levels, the Boltzmann distribution gives the fractional occupation of an energy level i as

$$p_i = \frac{1}{Z} e^{\beta \varepsilon_i}$$

where Z is the partition function, ε_i is the energy of the level and β is related to the temperature, T, via $\beta = -1/kT$.

(i) Calculate the partition function, Z, for a system with the same energy levels as in part (a) for kT=2.076 eV.

[4]

(ii) Calculate the fraction of the particles that will occupy each of the three energy levels at this temperature.

[3]

(iii) Calculate the average energy per particle at this temperature, and verify that it is close to that in part (a).

[3]

(iv) Comment on the comparison of the fraction of particles occupying each state you calculated in (b)(ii) to the average distribution you obtained in part (a). Why do the calculations in part (a) predict different fractions of the particles in each state compared to part (b)?

[3]

CONTINUED...

16. Throughout this question, you may use the fact that the density of states, $g(\varepsilon)$, for a three-dimensional system of non-relativistic electrons in volume V is given by

$$g(\varepsilon)d\varepsilon = \frac{V}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}\varepsilon^{1/2}d\varepsilon$$

(a) For a dilute electron gas at temperature T, the fraction of particles with energy between ε and $\varepsilon + d\varepsilon$ is given by

$$p(\varepsilon)d\varepsilon = \frac{2}{(kT)^{3/2}}\left(\frac{\varepsilon}{\pi}\right)^{1/2}\exp\left(-\frac{\varepsilon}{kT}\right)d\varepsilon$$

(i) Sketch a plot of $p(\varepsilon)$ versus ε.

[3]

(ii) Discuss the physical origin of the two energy dependent terms in this expression for $p(\varepsilon)$.

[4]

(b) A system of electrons at T=300 K is measured to have 3×10^{14} electrons with energy between 0.060 eV and 0.061 eV.

(i) Estimate the total number of electrons in the system, assuming that it can be treated as a dilute gas.

[5]

(ii) Estimate the minimum volume that the system must have for the dilute gas approximation to apply. Carefully explain your answer.

[5]

(iii) Make a sketch to show how the distribution of particles with energy would differ from the case you plotted in (a)(i), if the volume of the system were much smaller than the limiting volume you estimated in (b)(ii). Briefly justify your answer.

[3]

END OF EXAMINATION