QUEEN'S UNIVERSITY BELFAST

PHY2004
Exam Time Table
Code PHY2004

Answer Books A, B and C.

Any calculator, except one with pre-programmable memory, may be used in this examination.

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2004
Electricity, Magnetism and Optics

Wednesday, 15th May 2019 9:30 AM - 12:30 PM

Examiners: Professor P Browning
Dr P van der Burgt
and the Internal Examiners

Answer ALL TEN questions in Section A for 4 marks each.
Answer TWO questions in Section B for 20 marks each.
Answer ONE question in Section C for 20 marks.

Use a separate Answer Book for each Section.
You have THREE hours to complete this paper.

THE QUEEN'S UNIVERSITY OF BELFAST
SCHOOL OF MATHEMATICS AND PHYSICS

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \mathrm{ms^{-1}}$
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$ $\approx 1.26 \times 10^{-6}\ \mathrm{Hm^{-1}}$
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{Fm^{-1}}$
Elementary charge	$e = 1.60 \times 10^{-19}\ \mathrm{C}$
Electron charge	$= -1.60 \times 10^{-19}\ \mathrm{C}$
Planck Constant	$h = 6.63 \times 10^{-34}\ \mathrm{Js}$
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}\ \mathrm{Js}$
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}\ \mathrm{m^{-1}}$
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}\ \mathrm{kg}$ $1u = 931\ \mathrm{MeV}$
1 electron volt (eV)	$= 1.60 \times 10^{-19}\ \mathrm{J}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$
Mass of proton	$m_p = 1.67 \times 10^{-27}\ \mathrm{kg}$
Mass of neutron	$m_n = 1.67 \times 10^{-27}\ \mathrm{kg}$
Molar gas constant	$R = 8.31\ \mathrm{JK^{-1}mol^{-1}}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational constant	$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$
Acceleration of free fall on the Earth's surface	$g = 9.81\ \mathrm{ms^{-2}}$

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MATHEMATICAL IDENTITIES

In answering the questions on this paper you may make use of any of the following:

Divergence theorem $\int_V \nabla \cdot \underline{E}\, dV = \oint_S \underline{E} \cdot d\underline{S}$

Stoke's Theorem $\int_S \nabla \times \underline{E} \cdot d\underline{S} = \oint \underline{E} \cdot d\underline{\ell}$

Identities

$$\nabla \times \nabla \left(Scalar \right) = \underline{0}$$

$$\nabla \cdot \nabla \times \left(Vector \right) = 0$$

$$\nabla \times \left(\nabla \times \underline{E} \right) = \nabla \left(\nabla \cdot \underline{E} \right) - \nabla^2 \underline{E}$$

$$\nabla \left(y\, \underline{E} \right) = y\, \nabla \cdot \underline{E} + \underline{E} \cdot \nabla y$$

$$\nabla \cdot \left(\underline{E} \times \underline{H} \right) = \underline{H} \cdot \left(\nabla \times \underline{E} \right) - \underline{E} \cdot \left(\nabla \times \underline{H} \right)$$

Trigonometric identities

$$cos(A \pm B) = cos\, A\, cos\, B \mp sin\, A\, sin\, B$$

$$sin(A \pm B) = sin\, A\, cos\, B \pm sin\, B\, cos\, A$$

$$cos\, A + cos\, B = 2\, cos\left(\frac{A+B}{2} \right) cos\left(\frac{A-B}{2} \right)$$

$$sin\, A - sin\, B = 2\, cos\left(\frac{A+B}{2} \right) sin\left(\frac{A-B}{2} \right)$$

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SECTION A

Use a section A answer book

Answer ALL 10 questions in this section

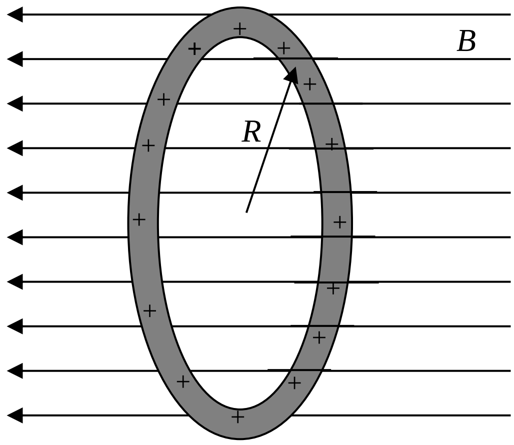
Full explanations of your answers are required to attain full marks

1 Using Gauss' law, derive the expression for the electrostatic field generated by a uniformly charged sphere (volumetric charge density ρ, dielectric constant ε_0) of radius a, outside ($r > a$) and inside ($r < a$) the sphere. Show that the electric field is continuous at $r = a$. **[4]**

2 Discuss how a temporally varying magnetic field generates a non-conservative electric field. **[4]**

3 Consider an infinitely long cylinder of radius a, relative permeability μ_r, and carrying a steady current I, surrounded by vacuum. Calculate the magnetization **M** outside ($r > a$) and inside ($r < a$) the cylinder. Is it continuous across the surface of the cylinder? **[4]**

4 The diagram below shows a thin insulating ring holding a charge per unit length λ on its circumference. The ring sits in a uniform magnetic field, which is orthogonal to the plane of the ring. The magnetic field reduces linearly with time t, according to $B = B_0(1 - t/T)$, over a period T.



Re-draw the diagram and indicate the direction of the electric field, at the ring circumference, induced by the magnetic field. Show that this field is given by

$$E = \frac{B_0 R}{2T}$$ **[4]**

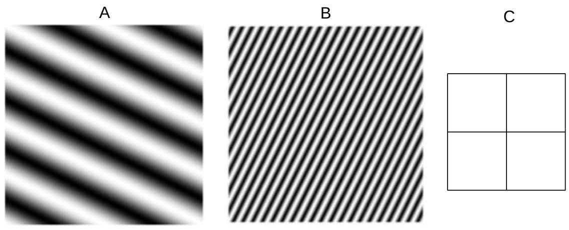
/continued …

SECTION A

5 By considering the electric and magnetic energy densities in a volume V, bounded by a surface S, the rate of change of the total electromagnetic energy W is described by the Poynting's Theorem $\int_S \underline{E} \times \underline{H} \cdot d\underline{S} + \int_V \underline{E} \cdot \underline{J} dV = -\frac{\partial W}{\partial t}$

Explain, in detail, the physical significance of the terms in the expression above and define the Poynting vector. What are the SI units of this vector? **[4]**

6 The pictures *A* and *B* show the intensity images of two sine gratings.



Re-draw the diagram of *C* and sketch the Fourier transform patterns of the two gratings on the frequency plane of *C*, and label them clearly. **[4]**

7 The permittivity of dielectrics is dependent on the plasma frequency ω_p of the material. Name the key factor that determines the plasma frequency and explain in which way it affects the plasma frequency. If a fictitious material has a plasma frequency with an equivalent energy of 0.1 eV, would you expect this material to be transparent or opaque for visible light? Give a rationale for your answer. **[4]**

8 The refractive index of a material is *complex* and *frequency-dependent*. Describe how the optical properties of this material would differ from that of an ideal dielectric material with a constant and real refractive index. Illustrate your answers with real-life optical phenomena. **[4]**

9 Describe the Kerr electro-optic effect and the Faraday magneto-optic effect, respectively. **[4]**

10 Describe the optical phenomenon linked to Brewster angle. For a plane wave incident from air to the planar surface of a glass with refractive index of $n = 1.5$, calculate the Brewster angle. **[4]**

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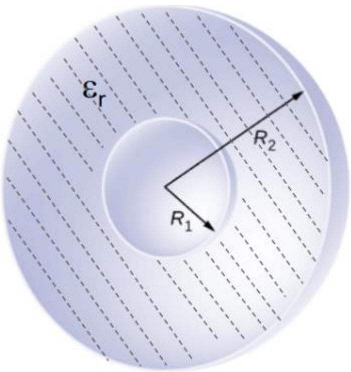
SECTION B

Use a Section B answer book

Answer TWO questions from this section

11 Two hollow and thin spherical sheets of conducting material, of radii R_1 and R_2, respectively, are concentric to each other and carry equal and opposite charge Q (positive on the inner conductor). The space between the two conductors is filled with a dielectric of relative permittivity ε_r (volume marked with dashed lines in the figure below).



(a) Sketch the distribution of charges on the conductors and the dielectric. **[2]**

(b) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric induction field ***D*** outside of the outer conductor ($r > R_2$). **[2]**

(c) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric induction field ***D*** in the dielectric ($R_2 > r > R_1$). **[5]**

(d) Calculate the volumetric and surface distribution of bound charges in the dielectric. **[4]**

Hint: you might need to use the divergence in spherical co-ordinates:

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

(e) Calculate the amplitude of the electric field ***E***, polarisation ***P***, and electric displacement field ***D*** inside the inner conductor ($r < R_1$). **[2]**

(f) Calculate the potential difference between the two conductors. **[3]**

(g) Calculate the capacitance of the whole system. **[2]**

/continued …

SECTION B

12 **(a)** Using Ampere's law, derive the magnetic field associated with an infinitely long and straight wire carrying a constant current. **[4]**

(b) By considering the electric and magnetic fields in the vicinity of a charging parallel-plate capacitor, discuss why the expression for the magnetic field given in question **12(a)** is incomplete. **[2]**

(c) It is known that $\nabla \cdot \underline{B} = 0$.

(i) Show how this equation implies the absence of magnetic monopoles. **[2]**

(ii) Show that from this expression we can deduce the concept of a magnetic vector potential $\underline{A}$. **[2]**

(iii) Show that $\underline{A}$ is not uniquely defined and explain why the arbitrary restriction $\nabla \cdot \underline{A} = 0$ makes this vector a useful quantity for solving problems in magnetostatics. **[2]**

(iv) Show that, for current carrying conductors, $\underline{A}$ may be expressed, assuming $\nabla \cdot \underline{A} = 0$, as

$$\underline{A} = \frac{\mu_0}{4\pi}\int \frac{\underline{J}dV}{r} \quad \text{or} \quad \underline{A} = \frac{\mu_0}{4\pi}\int \frac{\underline{I \cdot d\ell}}{r}.$$ **[4]**

(v) Demonstrate that the equations given in question **12(c)(iii)** are of similar form to those that would be obtained for the electrostatic potential as a function of charge density ρ.

[4]

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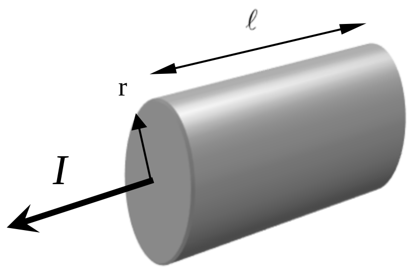
SECTION B

13 Fig. 4 shows a short section of a long cylindrical conductor of resistivity ρ and radius r carrying a steady current I



(a) Show that the magnetic field B at the surface is given by $B = \mu_0 l/(2\pi r)$. **[2]**

(b) Redraw the diagram and show clearly the direction of the $\underline{E}$ and $\underline{H}$ vectors. **[2]**

(c) Indicate the direction of the Poynting vector at the surface of the cylinder. **[2]**

(d) Calculate the current density flowing through the material. **[1]**

(e) Show that the flow of electromagnetic power per unit area across the curved surface is given by

$$\frac{\rho I^2}{2\pi^2 r^3}$$ **[3]**

(f) Using the result in Eq.4 show that the total power radiated by the cylinder is:

$$\frac{\rho I^2 \ell}{\pi r^2}$$ **[3]**

(g) Show that this result is consistent with Ohm's law. **[4]**

(h) Assume now that the cylinder is immersed in a paramagnetic material with a relative permeability μ_r. Write the new expression for the magnetic fields $\vec{B}$, $\vec{H}$, and $\vec{M}$ **[3]**

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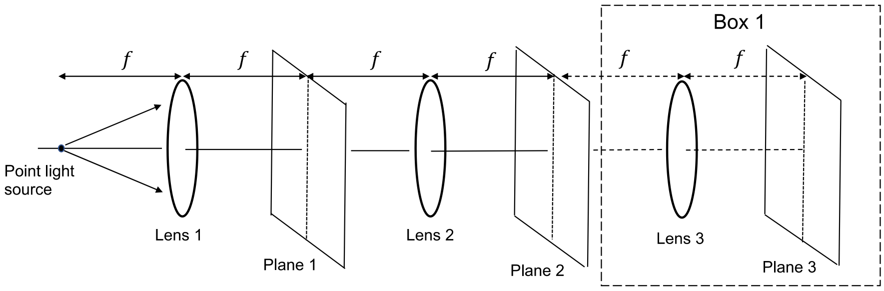
SECTION B

14 **(a)** A point light source is located at the front focal point of a $4f$ imaging system (excluding the third lens apparatus enclosed in Box 1), as shown below.

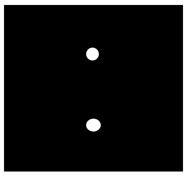


(i) Sketch the intensity pattern of light on Plane 1 and provide rationales of your answer. **[3]**

(ii) Sketch the intensity pattern on Plane 2 and give a rationale for your answer. **[3]**

(iii) If a third identical lens (Lens 3) is placed at a distance f after the Lens 2, sketch the image shown on Plane 3 and give a rationale for your answer. **[3]**

(iv) If a filter with two small holes as shown in the following diagram is placed at Plane 1, what would be the image appearing on Plane 2? Explain why. **[3]**



(b) **(i)** Explain why conventional optical microscope has limited resolution. **[3]**

(ii) Irish scientist Synge proposed an idea of near-field optical microscopy. Draw schematic diagrams to illustrate Synge's idea, describe the working principles of near-field optical microscopy and identify the key conditions to achieve super-resolution imaging and explain why. **[5]**

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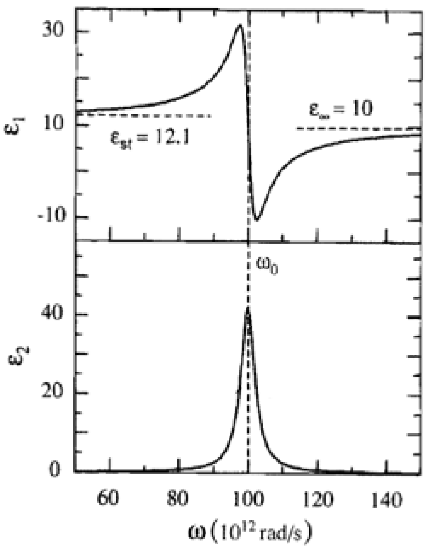
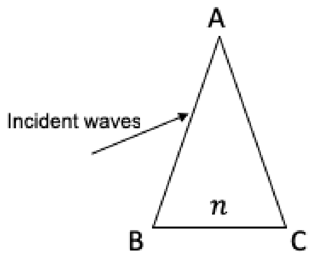
SECTION C

Use a Section C answer book

Answer <u>ONE</u> question from this section

15 The relative permittivity of a fictitious material near a resonant frequency $\omega_0 = 100 \times 10^{12}$ rad/s is shown below, which is described by $\varepsilon_r(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\gamma\omega}$.





(a) Derive the approximate values of ω_p and γ for the material. Show your working procedures. **[4]**

(b) Derive the formulae of the real (n_1) and imaginary (n_2) part of the refractive index in terms of the real (ε_1) and imaginary (ε_2) part of the permittivity. **[4]**

(c) Sketch the curves of n_1 and n_2 across the above frequency range **[4]**

(d) Three plane waves of frequency $\omega_1 = 80 \times 10^{12}$ rad/s, $\omega_2 = 90 \times 10^{12}$ rad/s and $\omega_3 = 105 \times 10^{12}$ rad/s are incident from air to a prism made of this material, as shown above. Name the order of the dispersed waves out of the AC side of the prism from top to bottom. If the incident beam is white light in the visible range, what are the answers. Provide rationales of your answers. **[4]**

(e) The group velocity of a wavepacket is given by $v_g = \frac{c}{\omega\frac{\partial n_1}{\partial\omega} + n_1}$ and the phase velocity is given by $v_p = \frac{c}{n_1}$. Specify the frequency regions in the permittivity graph for $v_g < v_p$, $v_g = v_p$, and $v_g > v_p$, respectively. Is there possibly a frequency region for $v_g > c$? Provide rationales of your answers. **[4]**

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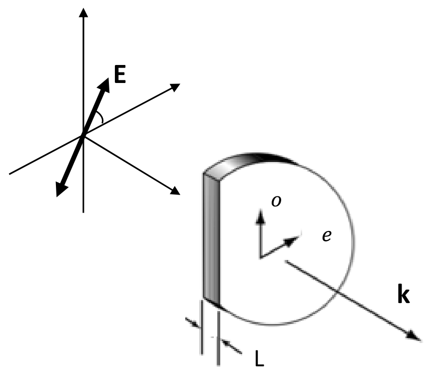
SECTION C

16 **(a)** The E-vector of a plane wave can be decomposed into two orthogonal components, described as $E_x = E_{0x}\cos(kz - \omega t)$ and $E_y = E_{0y}\cos(kz - \omega t + \delta)$. E_{0x} and E_{0y} are positive.

(i) Name the conditions for the plane wave to be linearly and circularly polarised, respectively. **[3]**

(ii) The E-field of a linearly-polarised plane wave (wavelength $\lambda = 500$ nm) is in the xy-plane with an angle of $\theta = 60°$ to the x-axis. A wave plate is perpendicular to the wave propagation direction, with the ordinary and extraordinary wave axis along the y- and x- axis, as shown in the diagram. $n_o = 1.46$, $n_e = 1.56$. To convert the E-field into a right-handed circular polarisation, what is the minimum thickness L of the wave plate and how it should be rotated? **[3]**

(iii) If the thickness of the above wave plate is then doubled, what would be the polarisation state and the orientation of the E-vector of the transmitted wave? **[2]**



(b) The reflectivity of a plane wave at the planar interface of two homogeneous materials is described by the Fresnel formulae $r_s = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$ and $r_p = \frac{n_1\cos\theta_2 - n_2\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1}$ for the s-polarisation and p-polarisation, respectively. Light of a plane wave is incident from medium 1 to medium 2. $n_1 = 1.5$, $n_2 = 1$.

(i) Calculate the reflectance of light at normal incidence. **[2]**

(ii) What is the angle of incidence above which light will be 100% reflected? **[2]**

(iii) Describe the reflectance of a p-polarisation plane wave for the incident angle θ_1 increasing from 0° to 89°. Sketch a graph to illustrate your answer. **[4]**

/continued …

SECTION C

(iv) For a plane wave of right-hand circular polarisation incident at $\theta_1 = 30°$, describe the polarisation state of the reflected wave. **[2]**

(v) For a plane wave incident at $\theta_1 = 45°$, describe the nature of the electromagnetic wave in medium 2. **[2]**

END OF EXAMINATION