PHY2003 ASTROPHYSICS I

Lecture 13. Solar system origins

Radionuclide dating

We can attempt to dat the Solar system by using radioactive decay where an unstable parent element (p) decays into a stable daughter element (d), with a mean lifetime τ.

If we start with $N_p(0)$ atoms of the parent, after time t we will have $N_p(t)$ atoms left where

$$N_p(t) = N_p(0)e^{-t/\tau}$$

We don't know $N_p(0)$, but we can measure $N_d(t)$, and if there were orginally no daughter atoms,

$$N_d(t) = N_p(0)(1 - e^{-t/\tau})$$

Combining:

$$\frac{N_d(t)}{N_p(t)} = \frac{1}{e^{-t/\tau}} - 1 = e^{t/\tau} - 1$$

$$\boxed{t = \tau \ln\left(\frac{N_d(t)}{N_p(t)} + 1\right)}$$

The unstable potassium isotope ^{40}K decays into the stable argon isotope ^{40}Ar with a mean lifetime of 1.3×10^9 yrs.

Measuring the ^{40}K and ^{40}Ar in oldest rocks found on Earth gives an age of 4.2×10^9 yrs.

Isochron dating

The problem is that most stable elements have some non-zero abundance at all times, and this initial abundance may change from place to place in a single rock because of the formation of different minerals.

$$
\begin{aligned}
N_d(t) &= N_d(0) + N_p(0)(1 - e^{-t/\tau}) \\
N_d(t) &= N_d(0) + N_p(t)e^{t/\tau}(1 - e^{-t/\tau}) \\
N_d(t) &= N_d(0) + N_p(t)(e^{t/\tau} - 1)
\end{aligned}
$$

So measuring $N_p(t)$ and $N_d(t)$ leaves two unknowns, $N_d(0)$ and t. Therefore in almost all cases it is necessary to measure the amount of *three* different isotopes, the parent, daughter and a stable "cousin" isotope of the daughter di.

$$
\boxed{\frac{N_d(t)}{N_{di}(t)} = \frac{N_d(0)}{N_{di}(t)} + \frac{N_p(t)}{N_{di}(t)}(e^{t/\tau} - 1)}
$$

By measuring these amounts in several different places in the rock, a graph is produced with slope of $(e^{t/\tau} - 1)$ and intercept of the original isotope value $\frac{N_d(0)}{N_{di}(t)}$.

The most common system used is ^{87}Rb (parent), ^{87}Sr (daughter) with $\tau = 7.04 \times 10^{10}$ years, and ^{86}Sr (stable isotope). The oldest meteorites are 4.56×10^9 yrs old.

Known facts

1. Planetary orbits are almost co-planar.

2. Planets orbit in same direction (counter-clockwise from North of the Solar system).

3. Most moons rotate in same direction as planets.

4. Inner planets terrestrial; outer planets gas giants.

5. Oldest meteorites and gas giant planets have similar composition to the Sun.

6. Old planetary surfaces are saturated by craters.

The Nebula/Accretion Theory

The solar system evolved from a slowly rotating cloud of interstellar gas and dust grains.

As the cloud collapsed due to self-gravity, a disk forms due to conservation of angular momentum.

Temperature will be warmest near forming Sun. Light elements will remain as gas, while heavier elements condense into solid grains. In outer disk, light elements remain in solid form.

Solids slowly accrete to form larger bodies. Accretion occurs by gravitational attraction and sticking.

Consider a microscopic test particle of mass m approaching a grain of radius r_d and mass M_d at velocity V_0. The small gravitational field will attract the test particle towards the grain. The *impact parameter* is the maximum transverse distance s the particle can have and still hit the grain.

It hits the grain with velocity V, so using conservation of angular momentum

$$m r_d V = m s V_0$$

Assume that originally grain is very distant so initial potential energy is minimal. Conservation of energy gives

$$mV_0^2/2 = mV^2/2 - GmM_d/r_d$$

$$V = \left(V_0^2 + 2GM_d/r_d\right)^{1/2}$$

$$V_0 s/r_d = \left(V_0^2 + 2GM_d/r_d\right)^{1/2}$$

So s is the sum of geometrical and gravitational terms:

$$\boxed{s^2 = r_d^2 + 2GM_d r_d/V_0^2}$$

The effective cross-section for gathering material is given by πs^2.

The rate of mass increase of grain will be

$\propto$ number of particles accreted per unit time $\propto s^2$

When particles are small gravitational forces are negligible.

$$s^2 \simeq r_d^2$$

$$dm/dt \propto s^2 \propto r_d^2$$

When particles are large, gravity dominates.

$$s^2 \simeq 2GM_d r_d/V_0^2$$

$$dm/dt \propto M_d r_d \propto r_d^4$$

So larger objects grow even faster. This leads to run-away accretion as long as the cloud of test particles remains.

At end of process would have had many small planetary-mass bodies in solar system. Collisions of these bodies would form the final planets we see today, but would have significant effects on the surface composition and possibly rotational properties of the planet.

Large impacts could explain origin of Earth's moon (likely), Venus spin (possible), Uranus tilt (who knows?).

Theoretical simulations imply that Earth was formed in $\sim 10^6$ years, Saturn in $\sim 10^7$ years.