4. NUCLEAR DECAYS AND REACTIONS

Nuclear decay can be divided into three categories. The α *decay* is the spontaneous emission of an α-particle (^{4}He nucleus) from a nucleus of large atomic number; this process is responsible for setting an upper limit on the atomic numbers of the chemical elements occurring in nature. The β *decay* is the spontaneous emission or absorption of an electron or a positron by a nucleus. The γ decay is the spontaneous emission of high-energy photons when a nucleus makes transitions from an excited state to its ground state.

Nuclear reactions provide information about excited states of nuclei since the residual nucleus in a reaction is typically formed in an excited state. Among the various reactions, nuclear fission and fusion are of high interest because they are used (or can be potentially used) as inexpensive sources of energy.

4.1 Decay Law

Unstable nuclei that originate from natural events are often called *radioactive*. The process occurring during a nuclear decay is typically called *radioactive decay*, or *radioactivity*. Radioactive decays are interesting for several reason, for instance because they provide key information about the origin of the universe. Let us consider a system containing many nuclei of the same species at some initial condition (time), and that the nuclei decay at a given decay rate *R*. If there are *N* undecayed nuclei at a time *t*, then the number of nuclei decaying in the following time interval *dt* can be written as *dN*. Since *R* is the probability that a particular nucleus will decay in 1 second, *Rdt* is the probability that it will decay during the time interval *dt*, and *NRdt* is the probability that the existing nuclei will decay in that interval. Thus, the average number of decaying nuclei is:

$$dN = -NRdt \tag{4.1}$$

where the minus sign account for the fact that *dN* is intrinsically negative since *N* decreases. Thus:

$$\frac{dN}{N} = -Rdt$$

$$\int_{N(0)}^{N(t)} \frac{dN}{N} = -R\int_0^t dt = -Rt$$

$$lnN(t) - lnN(0) = ln\frac{N(t)}{N(0)} = -Rt$$

$$\frac{N(t)}{N(0)} = e^{-Rt}$$

$$N(t) = N(0)e^{-Rt} \tag{4.2}$$

where *N(0)* is the number of undecayed nuclei at the initial time *0*, and *N(t)* is the number of undecayed nuclei at the subsequent time *t*. Since the calculations involve probabilities, its results are correct only in the average, however fluctuations are very small when the number of nuclei involved is very large. Eq. (4.2) is known as *exponential decay law* and is plotted in Fig.4.1, which also shows the *lifetime T* characteristic of the decay (average time a nucleus survives before it decays). *T* is inversely proportional to the decay rate *R*:

$$T = \frac{1}{R} \tag{4.3}$$

hence, N(T) = N(0)/e.

Fig.4.1 also shows the *half-life* $T_{1/2}$ (time required for the number of undecayed nuclei to decrease by a factor of 2), and:

$$T_{1/2} = (\ln 2)T = 0.693T \tag{4.4}$$

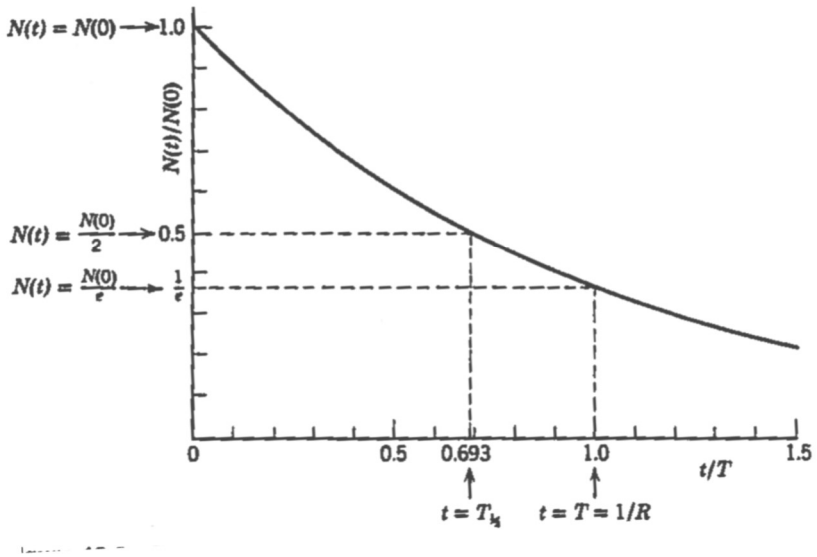

Figure 4.1 The exponential decay law (*from Eisberg and Resnick*).

As an example of applications of the nuclear decay law, it is worth mentioning Carbon-14 dating. This technique allows to date biological material fossils with high accuracy based on the β decay reaction of ^{14}C that has a characteristic half-life of 5730 ± 40 years. The moment of death of the living organism (e.g. a plant) can be retrieved by measuring the ^{14}C activity at the moment of the archaeological find, assuming the sample to have originally had the same $^{14}C/^{12}C$ ratio as in the atmosphere.

4.2 Alpha Decay

Alpha particles were first identified as the least penetrating radiation emitted by naturally occurring materials. In 1903 Rutherford measured their charge-to-mass ratio by deflecting α-particles from the decay of radium in electric and magnetic fields. Many heavy nuclei decay through α-emission, and only rarely any other spontaneous radioactive process results in the emission of nucleons. This is an indication of the existence of a special reason why nuclei chose α-emission over other possible decay modes.

Alpha decay is the result of a Coulomb repulsion effect that becomes increasingly important for heavy nuclei because the Coulomb force increases with size at a faster rate ($\sim Z^2$) than does the specific nuclear binding force ($\sim A$). Since α decay is a spontaneous process, the emission of kinetic energy during the process comes from a decrease in the mass of the system. For a typical α-emitter ^{232}U (72 y), it is possible to compute the energy release for various emitted particles by knowing the involved masses. Table 4.1 shows the calculated values indicating that among the considered particles, spontaneous decay is energetically possible only for the α-particle. Furthermore, if a nucleus is to be recognized as α-emitter, the decay rate must also not be too small to be detected (e.g. the half-life must be less than 10^{16} y).

Emitted Particle	Energy Release (MeV)	Emitted Particle	Energy Release (MeV)
n	−7.26	^{4}He	+5.41
^{1}H	−6.12	^{5}He	−2.59
^{2}H	−10.70	^{6}He	−6.19
^{3}H	−10.24	^{6}Li	−3.79
^{3}He	−9.92	^{7}Li	−1.94

[a] Computed from known masses.

Table 4.1 Q-values for various modes of decay of ^{232}U *(from Krane)*.

The spontaneous emission of an α-particle can be represented by the following process:

$$\boxed{{}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}X'_{N-2} + \alpha}$$

The α-particle, as shown by Rutherford, is a nucleus of ^{4}He, consisting of two neutrons and two protons. To understand the decay process, we must use the conservation of energy, as well as linear and angular momentum. Let us assume the initial decay nucleus *X* to be at rest. Thus, the total energy of the initial system is just the rest energy of *X*, $m_X c^2$. The final state will consist of *X'* and α, both in motion due to the conservation of linear momentum. Thus, the final total energy of the system is:

$$m_X c^2 = m_{X'} c^2 + T_{X'} + m_\alpha c^2 + T_\alpha$$

$$(m_X - m_{X'} - m_\alpha)c^2 = T_{X'} + T_\alpha \qquad (4.5)$$

The quantity on the left side of Eq. (4.5) is the net energy released in the decay, called the ***Q-value***:

$$Q = (m_X - m_{X'} - m_\alpha)c^2 \tag{4.6}$$

and the decay will occur spontaneously only if $Q > 0$. Q-values can be calculated from atomic mass tables because even though Eq. (4.6) represents a nuclear process, the electron masses will cancel in the subtraction. If the masses are in atomic mass units (u), the Q-values can be calculated directly in MeV, expressing c^2 as 931.50 MeV/u. The Q-value is also equal to the total kinetic energy carried by the decay fragments:

$$Q = T_{X'} + T_\alpha \tag{4.7}$$

If the original nucleus *X* is at rest, then its linear momentum is zero. Thus, due to the conservation of linear momentum, *X'* and α must move with equal and opposite momenta:

$$p_\alpha = p_{X'} \tag{4.8}$$

Since α decays typically release about 5 MeV of energy, it is possible to use nonrelativistic kinematics ($T << mc^2$) for both *X'* and α. Writing $T = P^2/2m$ and using (4.7) and (4.8), it is possible to express the kinetic energy of the α-particle in terms of the Q-value:

$$T_\alpha = \frac{Q}{1+m_\alpha/m_{X'}} \tag{4.9}$$

Because the mass ratio is small compared with 1 (*X'* is a heavy nucleus), it is sufficiently accurate to express this ratio simply as 4/(A-4), which gives, with A >> 4:

$$T_\alpha = Q\left(1 - \frac{4}{A}\right) \tag{4.10}$$

Typically, the α-particle carries about 98% of the Q-value, with the much heavier nuclear fragment *X'* carrying only about 2%. The kinetic energy of an α-particle can be measured directly with a magnetic spectrometer, and this would allow to determine experimentally the decay Q-value. This is particularly useful to measure the masses of short-lived X' nucleus that cannot be measured directly.

In 1911 Geiger and Nuttal noticed that α emitters with large disintegration energies (Q-values) had short half-lives and vice versa. The example of ^{232}Th (1.4×10^{10} y; Q = 4.08 MeV) and ^{218}Th (1.0×10^{-7} s; Q = 9.85 MeV), shows that a factor 2 in energy implies a factor 10^{24} in half life! The theoretical explanation of this effect was one of the first triumphs of quantum mechanics. A log-plot of the half-life of α decays against Q (the Geiger-Nuttal rule) for even-Z, even-N nuclei is shown in Fig.4.2. Another important systematic relationship for α emitters is shown in Fig.4.3. Considering only nuclei with A > 212, it is evident that adding neutrons to a nucleus reduces the disintegration energy, which in turn increase the half-life following the Geiger-Nuttal rule, i.e. the

nucleus becomes more stable. An abrupt discontinuity is present near A = 212 (N = 126). This is a clear evidence of the nuclear shell structure.

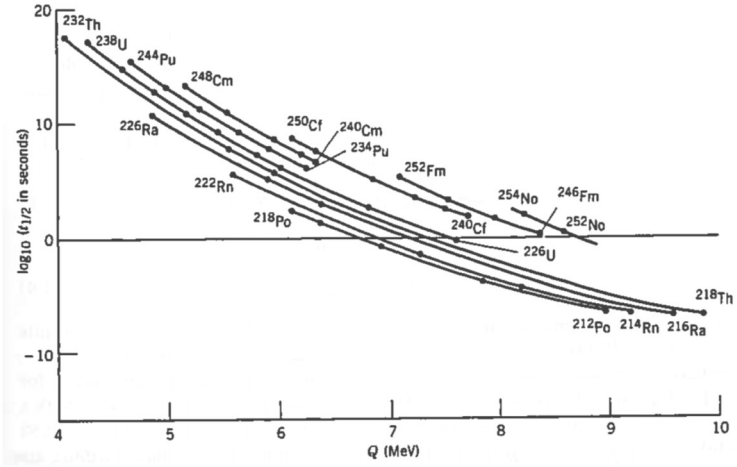

Figure 4.2 The inverse relation between α-decay half-life and decay energy, known as the Geiger-Nuttal rule. Only even-Z, even-N nuclei are shown *(from Krane)*.

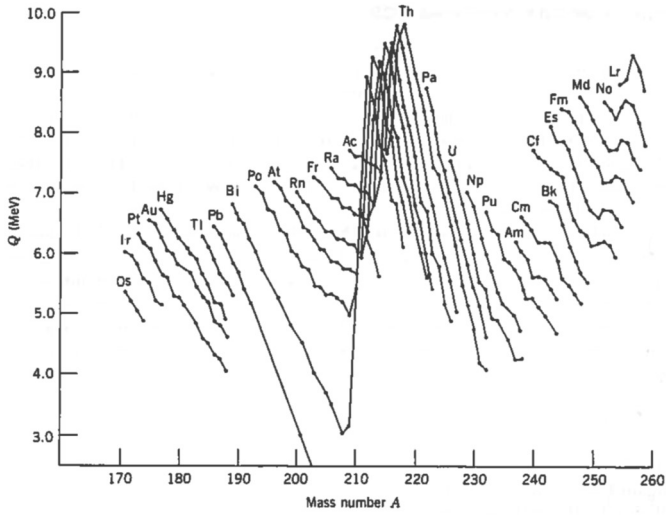

Figure 4.3 Q-values of α decay for various isotopic sequences of heavy nuclei *(from Krane)*.

The general features of the Geiger-Nuttal rule can be accounted for by a quantum mechanical theory that was developed by Gamow and by Gurney and Condon in 1928. Based on this theory, an α-particle is assumed to move in a spherical region determined by the *daughter* nucleus. The central feature of this *one-body model* is that the α-particle is preformed inside the parent nucleus. The theory works quite well, especially for even-even nuclei. However, one should note that the theory does not prove that α-particles are preformed, but merely that they behave as if they were. The basic principles of theory are described in Fig.4.4. The potential energy between the α-particle and the residual nucleus, *V(r)*, is indicated for various distances between their centres. The horizontal line *Q* is the disintegration energy. The Coulomb potential is extended

inward to a radius *a* that can be taken as the sum of the radius of the residual nucleus and of the α-particle. Three regions of interest can be distinguished. First, the spherical region inside the nucleus ($r < a$) characterized by a potential well $-V_0$, where V_0 is taken as a positive number (classically the α-particle can move in this region with a kinetic energy $Q + V_0$, but cannot escape from it). Second, the annular-shell region ($a < r < b$) forming a potential barrier coming from the fact that the potential energy is more than the total available energy *Q* (classically the α-particle cannot enter this region from either directions). Third, the region outside the barrier ($r > b$) that is classically permitted. From the classical point of view, an α-particle in the spherical potential well would sharply reverse its motion every time it tried to pass beyond $r = a$. However, quantum mechanically there is a chance of "tunnelling" through such a barrier. This is justified by the fact that α-unstable nuclei do not decay immediately, but the α-particle within the nucleus must present itself again and again at the barrier surface until it finally penetrates. For example, in ^{238}U the tunnelling probability is so small that the α-particle must make ~10^{38} tries (on the average) to escape (~10^9 years)! The disintegration constant of an α-emitter is given in the one-body-theory by:

$$\lambda = fP \tag{4.11}$$

where f is the frequency with which the α-particle presents itself at the barrier, and *P* is the probability of transmission through the barrier that can be expressed as:

$$P = e^{-2G} \tag{4.12}$$

where *G* is known as *Gamow factor*.

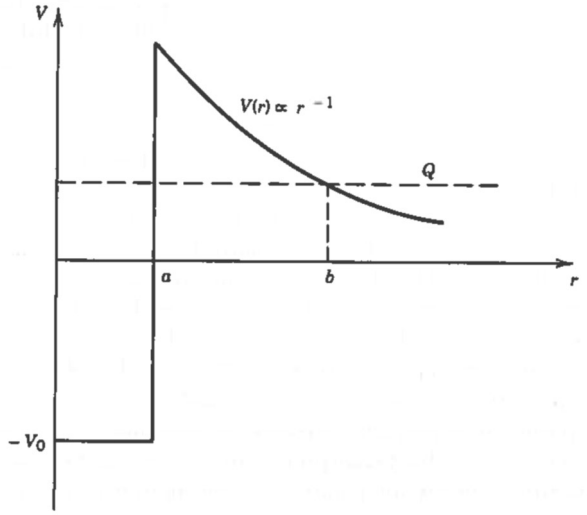

Figure 4.4 Relative potential energy of an α-particle as a function of its separation from the daughter-nucleus. The α-particle tunnels through the Coulomb barrier from *a* to *b*. *(from Krane)*.

4.3 Beta Decay

The emission of electrons from the nucleus is one of the earliest observed radioactive decay processes. The capture by a nucleus of an electron from its atomic orbital was observed in 1938 by Alvarez through the detection of the characteristic X-rays emitted in the filling of the electron vacancy. The Joliot-Curie in 1934 first observed the nuclear process of positron (positive electron) emission decay. These three nuclear processes are known as β decays. In β decay processes a proton is converted into a neutron, or vice versa, thus both *Z* and *N* are changed by one unit: $Z \rightarrow Z \pm 1$, $N \rightarrow N \mp 1$ (A = Z + N remains constant). The basic β decays are:

$n \rightarrow p + e^-$	negative beta decay (β^-)
$p \rightarrow n + e^+$	positive beta decay (β^+)
$p + e \rightarrow n$	orbital electron capture (ε)

The β^- decay process can be regarded as "creating" an electron from the available decay energy at the instant of decay. This electron is then immediately ejected from the nucleon. The three process above are not complete since there is yet another particle involved in each. One should note that the latter two processes occur only for protons bound in nuclei and are energetically forbidden for free protons.

β-decay electrons show a continuous energy distribution, from zero up to an upper limit (the endpoint energy) equal to the energy difference between the initial and the final states. This is because β decay is not a two-body process. In fact, a further particle is emitted in the decay process, the *neutrino*. The neutrino carries the missing energy. Conservation of electric charge requires the neutrino to be electrically neutral. Angular momentum conservation and statistical considerations require the neutrino to have a spin of ½. Two different kind of neutrino are observed experimentally, the neutrino (ν) and the antineutrino ($\bar{\nu}$). The antineutrino is emitted in β^- decay and the neutrino is emitted in β^+ decay and electron capture. Let us demonstrate β-decay energetics considering the decay of the free neutron:

$$n \rightarrow p + e^- + \bar{\nu}$$

We define the Q value to be the difference between the initial and the final nuclear mass energies:

$$Q = (m_n - m_p - m_e - m_{\bar{\nu}})c^2 \tag{4.12}$$

and for decays of neutrons at rest:

$$Q = T_p + T_e + T_{\bar{\nu}} \tag{4.13}$$

In a first approximation, we can ignore the proton recoil kinetic energy T_p (only 0.3 keV). The antineutrino and the electron will then share the decay energy, which accounts for the continuous electron spectrum. The maximum energy electrons correspond to minimum energy antineutrinos (and vice versa), and $Q \approx (T_e)_{max}$. The measured maximum electron energy is 0.782 ± 0.013 MeV. Furthermore, using the measured neutron, electron, and proton masses, it is possible to calculate the Q-value:

$$Q = m_n c^2 - m_p c^2 - m_e c^2 - m_{\bar{\nu}} c^2$$
$$= 939.573\ MeV - 938.280\ MeV - 0.511\ MeV - m_{\bar{\nu}} c^2$$
$$= 0.782\ MeV - m_{\bar{\nu}} c^2$$

Thus, within the precision of the measured maximum energy, it is possible to regard the antineutrino as massless, and for the following discussion we will consider the masses of the neutrino and antineutrino to be zero.

Because the neutrino is massless, it moves with the speed of light and its total relativistic energy E_ν is the same as its kinetic energy, thus we will use E_ν to represent neutrino energies. For the electron, we will use both its kinetic energy T_e, and its total relativistic energy E_e, which are related by $E_e = T_e + mc^2$. Decay energies are typically in the order of MeV, thus the nonrelativistic approximation is not valid for the decay electrons, and we must use relativistic kinematics. However, the nuclear recoil is of very low energy and can be treated non-relativistically.

Let us consider a typical negative β-decay process in a nucleus:

$$\boxed{{}^{A}_{Z}X_N \rightarrow {}^{A}_{Z+1}X'_{N-1} + e^- + \bar{\nu}}$$

The corresponding Q-value would be:

$$Q_{\beta^-} = \left[m_N\left({}^{A}_{Z}X\right) - m_N\left({}^{A}_{Z+1}X'\right) - m_e\right]c^2 \qquad (4.14)$$

where m_N indicates nuclear masses. To convert nuclear masses into the tabulated neutral atomic masses, we can use the following formula:

$$m({}^{A}X)c^2 = m_N\left({}^{A}X\right)c^2 + Zm_e c^2 - \sum_{i=1}^{Z} B_i \qquad (4.15)$$

where B_i represents the binding energy of the i^{th} electron. Thus, we can express Q_β in terms of atomic masses:

$$Q_{\beta^-} = \left\{\left[m\left({}^{A}X\right) - Zm_e\right] - \left[m\left({}^{A}X'\right) - (Z+1)m_e\right] - m_e\right\}c^2 + \left\{\sum_{i=1}^{Z} B_i - \sum_{i=1}^{Z+1} B_i\right\} \qquad (4.16)$$

The electron masses cancel, thus neglecting the differences in electron binding energies, we find:

$$\boxed{Q_{\beta^-} = \left[m\left({}^{A}X\right) - m\left({}^{A}X'\right)\right]c^2} \qquad (4.17)$$

where the masses are neutral atomic masses. Here the Q-value represents the energy shared by the electron and the neutrino:

$$(T_e)_{max} = (E_{\overline{\nu}})_{max} = Q_{\beta^-} \tag{4.18}$$

In the case of the ^{210}Bi → ^{210}Po decay, Eq. (4.17) gives:

$$Q_{\beta^-} = \left[m\left(^{210}Bi\right) - m\left(^{210}Po\right)\right]c^2 = 1.161\ MeV \tag{4.19}$$

Fig.4.5 shows $(T_e)_{max} = 1.16\ MeV$, which is in excellent agreement with the value expected from Q_β.

In the case of positron β decay, a typical decay process is:

$$^{A}_{Z}X_N \rightarrow\ ^{A}_{Z-1}X'_{N+1} + e^+ + \nu$$

and a calculation like the previous one shows:

$$Q_{\beta^+} = \left[m\left(^{A}X\right) - m\left(^{A}X'\right) - 2m_e\right]c^2 \tag{4.20}$$

where the electron masses do not cancel in this case.

For electron-capture processes, such as:

$$^{A}_{Z}X_N + e^- \rightarrow\ ^{A}_{Z-1}X'_{N+1} + \nu$$

The calculation of the Q-value must consider that the atom *X'* is in an atomic excited state immediately after the electron capture. Thus, if the capture takes place from an inner atomic shell (e.g. K-shell), an electronic vacancy will be present. The vacancy is quickly filled by an electron from higher atomic shells and one (or more) characteristic X-ray is emitted. The total X-ray energy is equal to the binding energy of the captured electron. Thus, the atomic mass of *X'* immediately after the decay is greater than the mass of *X'* in its atomic ground state by B_n, the binding energy of the captured *n*-shell electron (*n = K, L, …*). The corresponding Q-value is:

$$Q_\varepsilon = \left[m\left(^{A}X\right) - m\left(^{A}X'\right)\right]c^2 - B_n \tag{4.21}$$

In positron decay there is a continuous distribution of neutrino energies up to Q_{β^+}. In electron capture, however, the two-body final state results in unique values for the recoil energy and the neutrino energy. Thus, a monoenergetic neutrino with energy Q_ε is emitted. Table 4.2 shows some typical β-decay processes.

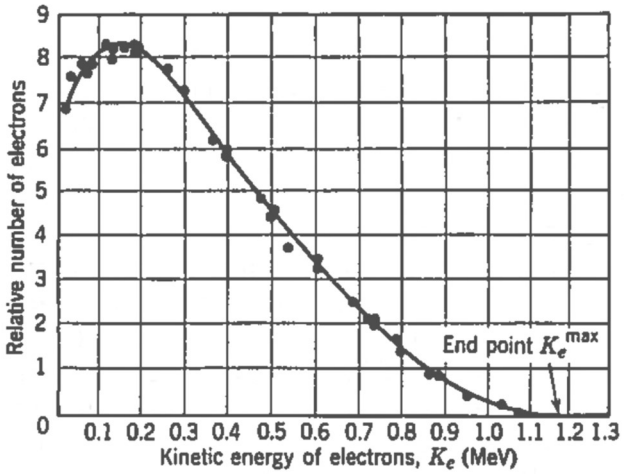

Figure 4.5 The spectrum of electrons emitted in the β decay of ^{210}Bi *(from Eisberg and Resnick)*.

Decay	Type	Q (MeV)	$t_{1/2}$
$^{23}Ne \rightarrow {}^{23}Na + e^- + \bar{\nu}$	β^-	4.38	38 s
$^{99}Tc \rightarrow {}^{99}Ru + e^- + \bar{\nu}$	β^-	0.29	2.1×10^5 y
$^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$	β^+	3.26	7.2 s
$^{124}I \rightarrow {}^{124}Te + e^+ + \nu$	β^+	2.14	4.2 d
$^{15}O + e^- \rightarrow {}^{15}N + \nu$	ε	2.75	1.22 s
$^{41}Ca + e^- \rightarrow {}^{41}K + \nu$	ε	0.43	1.0×10^5 y

Table 4.2 Typical β-decay processes *(from Krane)*.

4.4 Gamma Decay

Several radioactive nuclei emit γ-rays. These high energy photons carry away the excess energy when nuclei make γ-decay transitions from excited states to lower energy states. γ-rays have energies greater than ~10^{-3} MeV since nuclear excited states range upward ~10^{-3} MeV. Typically, γ decays arise when preceding β decay have produced some of the daughter nuclei in states of several MeV excitations. However, there are many other ways to produce nuclei in excited states, which subsequently γ decay. For instance, states of excitation energy around 7-8 MeV are produced following the capture of a low-energy neutron in a nucleus.

A widely used experimental technique to measure γ-ray energies is to let the photons transfer their energies to electrons by Compton effect, photoelectric effect, or pair production. The measured γ-ray energy spectrum is used to determine the energy of the nuclear excited states of the given transition. Another important information connected to γ decay is the transition rate *R* of each excited state. This can be measured by measuring the lifetime *T* of the excited state. An example of γ decay spectrum for ^{27}Al after bombardment with energetic protons is shown in Fig.4.6.

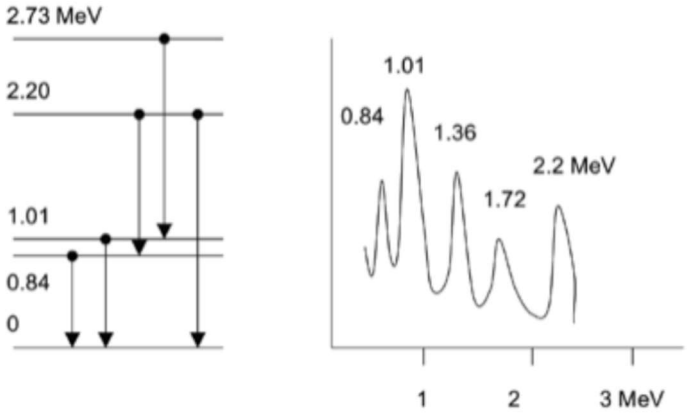

Figure 4.6 (left) Energy levels for ^{27}Al. Spectrum of γ-rays emitted after bombardment with energetic protons of ~5 MeV (from Krane)

In many γ decays, several groups of monoenergetic electrons are emitted along with the γ rays. In fact, if there is a preceding β decay, these groups will be superimposed on the continuous β-decay spectrum. The energy ε of these electrons is found to be related to the decay energy *E*:

$$\varepsilon = E - W \tag{4.22}$$

where *W* for the most prominent group equals the binding energy of a K-shell electron of the γ-decaying atom, and *W* for the other groups equal to the binding energies of electrons in the L, M, etc., atomic shells. The process involved is known as *internal conversion*. The latter consists of a direct transfer of energy through the electromagnetic interaction between a nucleus in an excited state and one of the electrons of its atom. The nucleus decays to a lower state, without ever producing a γ-ray. The internal conversion coefficient, α_K, is the ratio of the probability that a K-shell electron will be emitted, in a decay of its nucleus, to the probability that a γ-ray will be emitted. The two processes are independent alternatives, thus the total rate R_t for transition between the initial and final nuclear states is given by the following sum:

$$R_t = R + R_{ic} \tag{4.23}$$

where *R* and R_{ic} are the transition rates for γ-ray emission and internal conversion, respectively. This can also be written as:

$$R_t = R + \alpha_t R = R(1 + \alpha_t) \tag{4.24}$$

where $\alpha_t = \alpha_K + \alpha_L + \alpha_M + \ldots$ is the total internal conversion coefficient. Thus, we can write:

$$T = \frac{1}{R_t} = \frac{1}{R(1+\alpha_t)} \tag{4.25}$$

4.5 Nuclear Fission

In 1939 Meitner and Frisch proposed that the uranium nuclei following neutron capture are highly unstable and split nearly in half, thus called this effect *nuclear fission*. Fission results primarily from the competition between the nuclear and Coulomb forces in heavy nuclei. In fact, as already mentioned when describing the α-decay, the total nuclear binding energy roughly shows a linear increase with *A*, while the Coulomb repulsion energy of the protons increases faster (~Z^2). Like the α-decay case, we can consider heavy nuclei as residing very close to the top of the nuclear potential well, where the Coulomb barrier is very thin and easily penetrable. Thus, nuclear fission can occur spontaneously as a natural decay process, or it can be induced through the absorption of a relatively low-energy particle (e.g. a neutron, or a photon). Fission is an important process only for heavy nuclei (thorium and beyond) and can be applied for releasing a large total energy. This is possible thanks to the fact that every neutron-induced fission event produces, in addition to the heavy fragments, several neutrons that can in turn induce new fission events. This is known as fission "chain reaction" and can occur rapidly and without control (like in fission-based explosives), or slowly and under control (like in fission reactors).

The energetic preference for nuclei to undergo fission can be easily understood by considering the binding energy per nucleon. Let us consider the case of ^{238}U that shows a binding energy of about 7.6 MeV/nucleon. If ^{238}U were to divide into two equal fragments with A ≈ 119, their binding energy per nucleon would be about 8.5 MeV (more tightly bound system), thus energy must be released. The extra energy can appear in a variety of forms (neutrons, β and γ emission from the fragments), but it appears primarily as kinetic energy of the fragments since Coulomb repulsion drives them apart. While the fission spontaneous decay mode does exist, it is not very probable compared to α decay (^{238}U shows a $T_{1/2}$ = 4.5 x 10^9 y and ~10^{16} y for α decay and fission reactions, respectively), and it does not become an important decay process for nuclei with A < 250). The fission process is inhibited by the Coulomb barrier. In fact, if one divides ^{238}U into two identical fragments that are just touching at their surfaces, the corresponding Coulomb barrier is ~ 250 MeV, and this prevents the two fragments from separating, as shown in Fig. 4.7. However, the assumption that ^{238}U splits into two identical fragments is not very realistic, and there are certain nuclei for which the energy release puts the two fragments just below the Coulomb barrier, giving them a reasonably good change to penetrate it. These are nuclei undergoing *spontaneous fission*, and compete successfully with other radioactive decay processes. Theoretical calculations indicate that the Coulomb barrier against fission is zero at about *A = 300*. Other nuclei maybe far enough below the Coulomb barrier and spontaneous fission is not observed, but absorption of a relatively low amount of energy (e.g. from a neutron or a photon) forms an intermediate state

that is at (or above) the barrier, thus *induced fission* occurs, successfully competing with other decay modes. The ability of a nucleus to undergo induced fission will depend on the energy of the intermediate system. Absorption of thermal neutrons can be sufficient to push some nuclei over the Coulomb barrier, while for others, fast neutrons (~ MeV) may be required. A more realistic representation of the fission barrier for heavy nuclei is shown in Fig.4.8. Furthermore, a more detailed calculation of the energy needed to induce fission is shown in Fig.4.9, which provides the height of the fission barrier above the ground state, known as *activation energy*. Such calculation is based on the liquid-drop model that considers only average nuclear properties; however, the inclusion of effects based on the Shell model suggests that certain super-heavy nuclei (*A ~ 300*) may be more stable against fission.

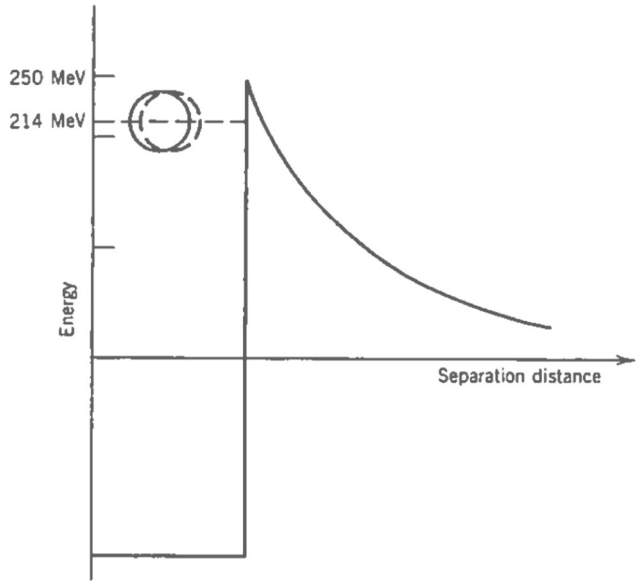

Figure 4.7 Nuclear potential well for two possible fragments of ^{238}U (^{119}Pd nuclei) *(from Krane)*

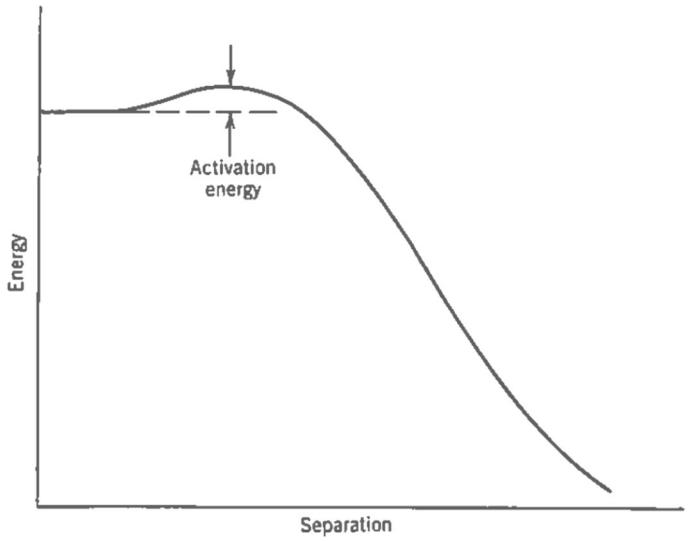

Figure 4.8 Smooth potential barrier opposing the spontaneous fission of ^{238}U *(from Krane)*

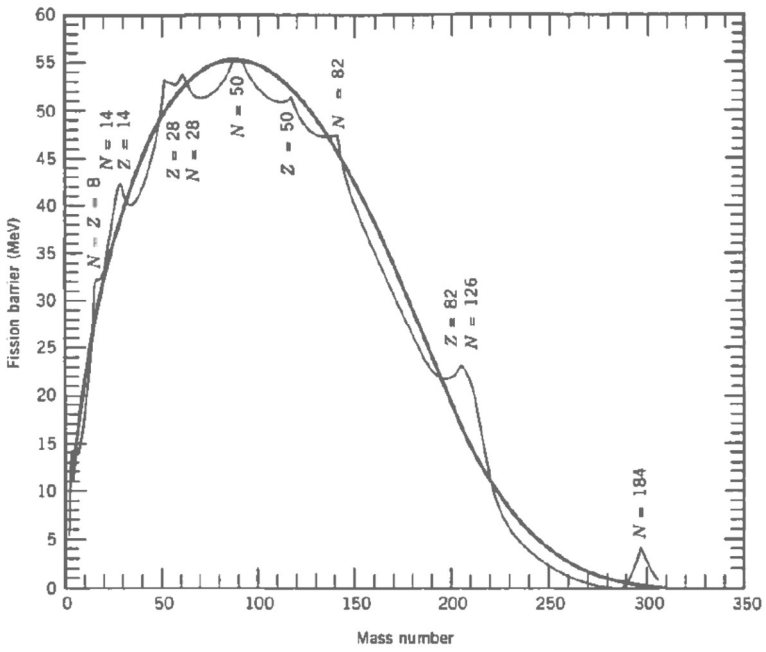

Figure 4.9 Variation of fission activation energy with mass number *(from Krane)*

A typical neutron induced fission reaction is:

$$^{235}\text{U} + \text{n} \rightarrow {}^{93}\text{Rb} + {}^{141}\text{Cs} + 2\text{n}$$

which is possible for incident neutrons of thermal energies. The fission products are not determined uniquely, and there is a distribution of masses of the two fission products of the form shown in Fig.4.10. The mass distribution is characteristic of low-energy fission processes. In contrast, fission induced by high-energy particles show mass distributions favouring equal-mass fragments. The fission fragments must share 92 protons, and the nuclei formed will be $^{95}_{37}Rb_{58}$ and $^{140}_{55}Cs_{85}$. These nuclei are extremely rich in neutrons. In fact, most stable nuclei in this region have Z/A ≈ 0.41, while for this fission products Z/A ≈ 0.39. The fission fragments compensate this neutron excess through emission of one, or more neutrons at the instant of fission (within 10^{-16} s), and these neutrons are known as *prompt neutrons*. The average number of prompt neutrons, indicated with ν, is characteristic of the given fission process. For thermal neutron induced fission, the experimentally observed values of ν are 2.48 for ^{233}U, 2.42 for ^{235}U, and 2.86 for ^{239}Pu. In addition to the prompt neutrons, *delayed neutrons* are often emitted in fission. These neutrons are emitted following the β decay of the fission fragments. Typical delay times are relatively short (of the order of seconds). They play a key role in the control of nuclear reactors. In fact, no mechanical system could respond rapidly enough to prevent statistical variations in the prompt neutrons from causing the reactor to run out of control, but it is indeed possible to achieve control using the delayed neutrons. The initial fission products are highly radioactive and decay towards stable isobars by emitting β and γ radiation, which ultimately contributes to the total energy

release in fission. These radioactive products represent the waste of nuclear reactors. Many of them decay very quickly, but others have long half-lives, especially near the stable members of the series. Fig.4.11 shows the cross sections for neutron-induced fission of ^{235}U and ^{238}U. The thermal region shows the typical *1/v* (here *v* is the incoming neutron velocity) dependence of the cross section for ^{235}U. The thermal cross section is 3 orders of magnitude larger than the cross section for fast neutrons. In fact, if one wants to use the fast (~ MeV) neutrons emitted in fission to induce new fission events, the neutrons must first be moderated to thermal energies to increase the cross section. For ^{238}U there is no fission occurring in the thermal region, and fission can occur only for fast neutrons. This clear difference results from the relationship between the excitation energy of the compound system and the activation energy needed to overcome the barrier.

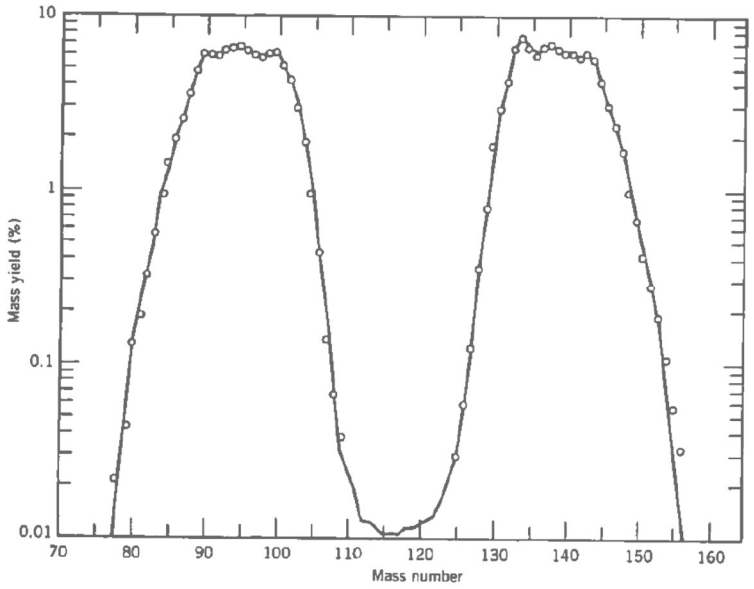

Figure 4.10 Mass distribution of fission fragments from thermal fission of ^{235}U *(from Krane)*

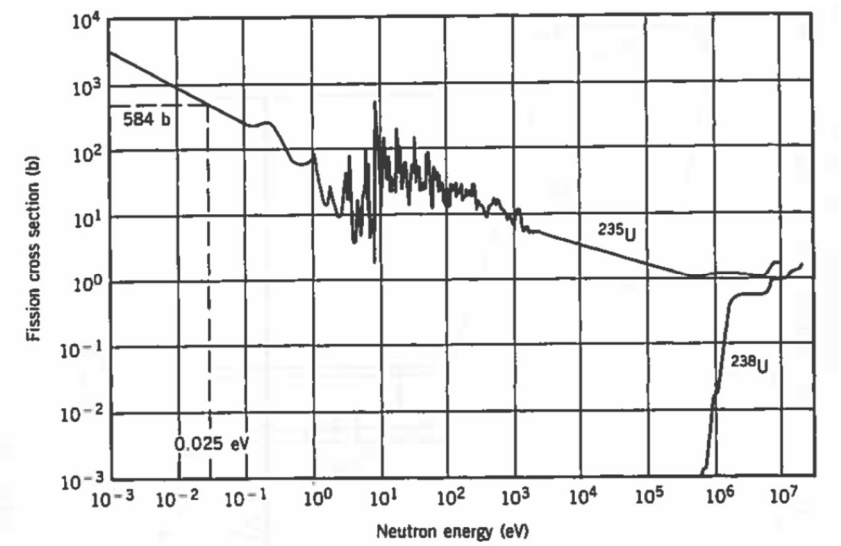

Figure 4.11 Cross-section for neutron-induced fission of ^{235}U and ^{238}U *(from Krane)*

When ^{235}U captures a neutron to form the compound state $^{236}U^*$, the excitation energy is:

$$E_{ex} = [m(^{236}U^*) - m(^{236}U)]\, c^2$$

The energy of the compound state can be found directly from the mass energies of ^{235}U and a neutron, assuming that the neutron's kinetic energy is negligible (thermal neutron):

$$m(^{236}U^*) = m(^{235}U) + m_n = (235.043924 + 1.008665)\ u = 236.052589\ u$$

Thus, we can estimate the excitation energy:

$$E_{ex} = (236.052589\ u - 236.045563\ u)\ 931.502\ MeV/u = 6.5\ MeV$$

The activation energy (energy to overcome the fission barrier) for ^{236}U is calculated also to be 6.2 MeV, thus the energy needed to excite ^{236}U into a fissionable state is exceed by the energy we get by adding a neutron to ^{235}U. This means that ^{235}U can be fissioned with "zero" energy neutrons, which is consistent with its the large fission cross section in the thermal region. A similar calculation for:

$$^{238}U + n \rightarrow {}^{239}U^*$$

gives $E_{ex} = 4.8$ MeV, which is much smaller than the calculated activation energy for ^{239}U (6.6 MeV). Neutrons of at least MeV energy are therefore required for fission of ^{238}U, which is consistent with the observed threshold for neutron-induced fission of ^{238}U.

The steps involved in fission are indicated schematically by the set of drawings in Fig.4.12. These define a parameter *s* which characterize the progress of the fission by specifying the elongation of the fissioning nucleus, and then the separation of the two fission fragments. Fig.4.13 illustrates the reason why the fission fragments tend to have relatively too many neutrons, thus undergo a succession of β decays, or in some case *evaporation* of two or three neutrons liberating several MeV kinetic energy.

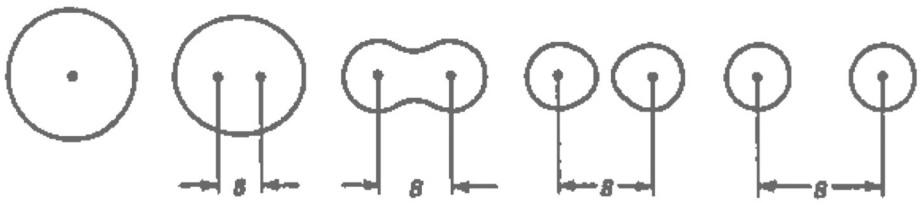

Figure 4.12 Schematic representation of the nuclear fission process *(from Eisberg & Resnick)*

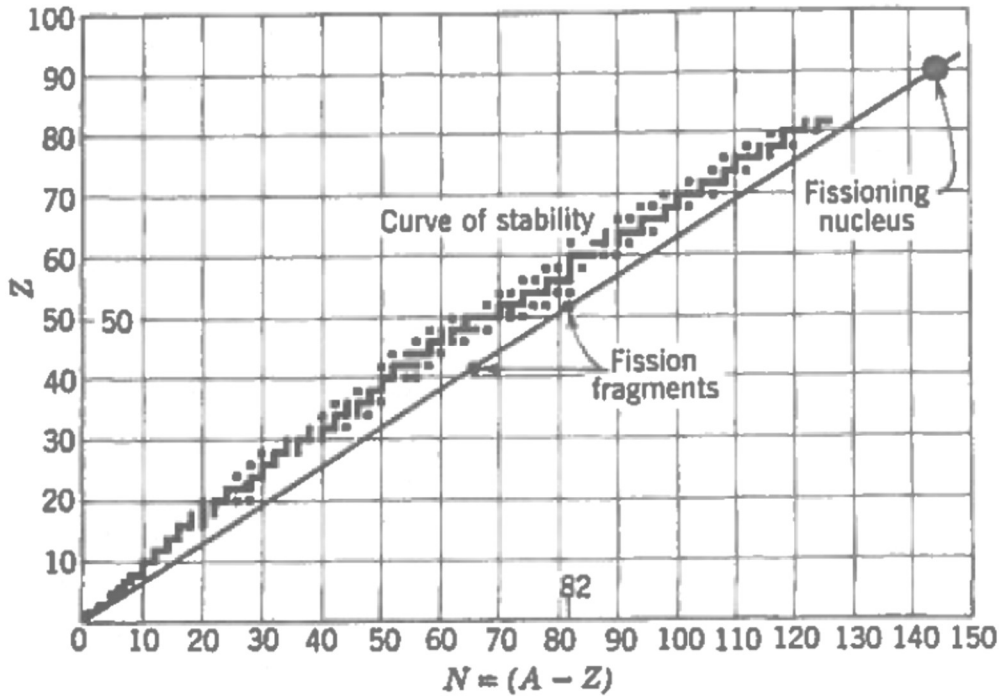

Figure 4.13 Fission fragments showing an excess of neutrons *(from Eisberg & Resnick)*

Let us consider an infinitely large mass of uranium in its natural isotopic composition (0.75% ^{235}U, 99.28% ^{238}U). A single fission event will produce about 2.5 neutrons on the average. Each of this "second-generation" neutrons can produce another fission event that, in turn, produces even more neutrons, and so on. This process is known as *chain reaction*. Each fission event releases about 200 MeV in the form of kinetic energy of heavy fragments, i.e. heat, and radiation. It is convenient to define the *neutron reproduction factor* k_∞ (for an infinite medium, i.e. ignoring loss of neutrons through leakage at the surface). The reproduction factor gives the net change in the number of thermal neutrons from one generation to the next. On the average, each thermal neutron produces k_∞ new thermal neutrons. For a chain reaction to continue, one must have $k_\infty \geq 1$. In fact, although we have an average of 2.5 neutrons emitted per fission, these are *fast neutrons*, for which the fission cross-section is typically small. Thus, it is convenient to moderate these neutrons to thermal velocities because of the large thermal cross sections. In the process, many neutrons can be absorbed, or lost, and the 2.5 fast neutrons per fission can easily become < 1 thermal neutron, effectively halting the reaction. Since neutrons lose energy in elastic collisions with nuclei, a popular choice for a *moderator* is carbon in the form of graphite blocks. In fact, the best choice for a moderator is the lightest nucleus, to which the neutron transfers the largest possible energy in an elastic collision. A lattice of blocks of uranium alternating with graphite is called a chain-reacting *pile*. If the reproduction factor k (for a finite pile) is exactly *1*, the pile is said to be *critical*; for $k < 1$, the pile is *subcritical*, and for $k > 1$ it is *supercritical*. To maintain a steadily release of energy one would like for the pile to be exactly critical.

In a nuclear reactor, fission proceeds at carefully controlled rate. A continuous source of power is obtained from the thermal energy produced when the fission fragments come to rest in the material of the reactor. All reactors consist of the same essential elements: (i) the *fuel*, or fissile material; (i) a *moderator* to thermalize the neutrons; (ii) a *reflector* surrounding the core (fuel elements plus moderator) to reduce neutron leakage and thereby the reactor size; (iv) a *containment* vessel to prevent the escape of radioactive fission products; (v) *shielding* to prevent neutrons and γ-rays from causing biological harms to the operating personnel; (vi) a *coolant* to remove heat from the core; (vii) a *control system* allowing the operator to control the power level and to keep it constant during normal operations; and (viii) various *emergency systems* designed to prevent runaway operations in the event of a failure of the control or coolant systems. After many years of technological development, nuclear reactors have become sources of power which are economically very competitive with coil and or oil. They are also source of unstable isotopes, not normally found in nature, that are used as tracers for diagnosing the operations of a variety of processes of interest to medicine, biology, chemistry, and engineering, or used for radiation therapy. The isotopes are produced in nuclear reactions induced by intense fluxes of neutrons present in a reactor.

4.6 Nuclear Fusion

Energy can be extracted from the nucleus in an alternative way to fission as suggested by the nuclear binding energy curve shown in Fig.4.14. In fact, it is possible to "climb" the binding energy curve toward the more stable nuclei by beginning with the very light nuclei, rather than the very heavy nuclei as in fission. Thus, if one combines two light nuclei into a nucleus below $A = 65$, energy is released. This process is called *nuclear fusion* because two light nuclei are fused into a heavier one. As an energy source, fusion has several advantages over fission. The light nuclei are abundant in nature and easy to obtain, and the end products of fusion are usually light, stable nuclei rather than heavy radioactive ones as in fission. However, nuclear fusion has a clear disadvantage over fission: to be combined, light nuclei must overcome their mutual Coulomb repulsion. On the other hand, fission induced by neutrons has practically no Coulomb barrier and very low energy incident particles can be used.

Let us consider the fusion of two ^{20}Ne nuclei to form a ^{40}Ca nucleus with a Q-value of 20.7 MeV, thus the reaction is energetically permitted. However, before the nuclear forces of the two ^{20}Ne nuclei can interact, we must move them close enough together so that their nuclear distributions begin to overlap. At the point when their surfaces are just touching, the Coulomb repulsion is 21.2 MeV. If we were to perform a nuclear reaction in which two ^{20}Ne nuclei were brought together

with a total kinetic energy of 21.2 MeV, the final energy of the system would be 41.9 MeV (the 21.2 MeV of potential energy associated with the nuclei Coulomb repulsion plus the 20.7 MeV energy released in the nuclear reaction). Therefore, the energy gain of such reaction is about a factor 2 (~ 21 MeV goes in and ~ 42 MeV comes out). Although accelerating ^{20}Ne to 21.2 MeV against a ^{20}Ne target can be easily done by particle accelerators, the overall efficiency of such induced nuclear physics experiment is not high. In fact, heavy-ion accelerators typically produce beams in the nA-μA range, and even in the totally unrealistic assumption that every particle in the beam were to react, the power output would be 20 W. This would not be enough even to run the accelerator laboratory! An alternative approach would be to heat a container of neon gas until the thermal energy is large enough to have a high probability of two nuclei approaching one another and colliding with 21.2 MeV of energy. This process is known as *thermonuclear fusion* and, in the case of Ne, would require an extremely high temperature of about 10^{11} K!

Despite the drawback mentioned above, nuclear fusion energy is currently a subject of intensive research to develop techniques allowing to heat fusible nuclei, and to increase their density to enhance the total number of fusion reactions. Fusion also powers the Sun and other stars, thus is ultimately responsible for the evolution of life on Earth. Understanding nuclear fusion is crucial to achieve a comprehensive knowledge about the end products of stellar reactions, when thermonuclear fuel is mostly exhausted and a star may pass through a nova or supernova stage, ending as an agglomerate of cosmic ash, or as a neutron star or a black hole.

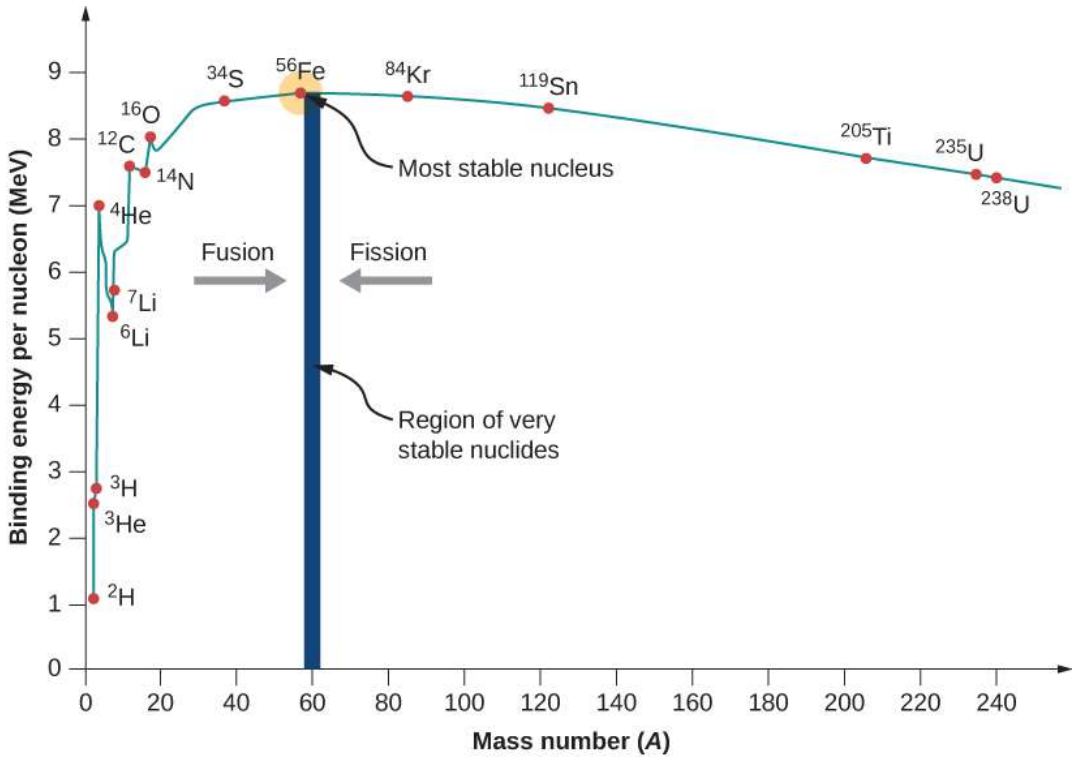

Figure 4.14 Nuclear binding energy curve

Differently from fission, nuclear fusion is not a spontaneous process on Earth because of the substantial limitations imposed by the Coulomb barrier. Typical reactions are based on *deuterium-deuterium (or D-D) fusion*:

$$^2H + ^2H \rightarrow ^3He + n \ (Q = 3.3 \ MeV)$$
$$^2H + ^2H \rightarrow ^3H + p \ (Q = 4.0 \ MeV)$$

The more stable the end product formed, the greater is the energy release in the reaction. In fact, a reaction that forms ^{4}He would be likely to show a large energy release, such as:

$$^2H + ^3H \rightarrow ^4He + n \ (Q = 17.6 \ MeV)$$

This reaction is known as *deuterium-tritium (or D-T) fusion*. Based on linear momentum conservation, and on the assumption that the incident particles have negligibly small kinetic energies, one can find that the ^{4}He and n share 17.6 MeV kinetic energy, and that a monoenergetic neutron with energy of 14.1 MeV emerges. Because of the large energy release, D-T fusion has been selected for its potential use in controlled fusion reactors. However, a disadvantage of such fusion reaction is that most of the energy is given to the neutron, thus it is not easy to extract. On the other hand, in nuclear fission only a small fraction of the energy is carried by the neutrons, thus the kinetic energy of the fission fragments is easy to extract.

The energy released in nuclear fusion can be estimated calculating the Q-value of the given reaction. Most of the fusion applications (including solar processes) are based on reacting particles with very small energies (1-10 keV) compared to the corresponding Q-values (several MeV). The final total energy of the product particles, again neglecting the initial motions, will then be equal to the Q-value:

$$\frac{1}{2}m_b v_b^2 + \frac{1}{2}m_Y v_Y^2 \approx Q \tag{4.26}$$

for product particles *b* and *Y*. Furthermore, the final momenta are equal and opposite:

$$m_b v_b \approx m_Y v_Y \tag{4.27}$$

and thus:

$$\frac{1}{2}m_b v_b^2 \approx \frac{Q}{1+m_b/m_Y} \tag{4.28}$$

$$\frac{1}{2}m_Y v_Y^2 \approx \frac{Q}{1+m_Y/m_b} \tag{4.29}$$

A consequence of this energy sharing is immediately apparent: the lighter product particle takes the larger share of the energy. The ratio of the kinetic energies can be calculated from Equation (4.27):

$$\frac{\frac{1}{2}m_b v_b^2}{\frac{1}{2}m_Y v_Y^2} \approx \frac{m_Y}{m_b} \tag{4.30}$$

Thus, in the D-T reaction case, the product neutron carries 80% of the total energy. In the DD reaction case, the product neutron (or proton) carries 75% of the available energy.

If R_a and R_X are the radii of the reacting particles, the Coulomb barrier is:

$$V_C = \frac{e^2}{4\pi\varepsilon_0}\frac{Z_a Z_X}{R_a + R_X} \tag{4.31}$$

when the particles just touch at their surface. The product Z_aZ_X will ultimately appear in an exponential barrier penetration probability. Therefore, the fusion probability decreases rapidly with Z_aZ_X, and the barrier is lowest for the hydrogen isotopes. For D-T fusion, $V_C = 0.4\ MeV$, thus it is still far above the typical incident particle energy (1-10 keV). However, just as in α decay, it is not necessary for the particles to be above the barrier since it is the barrier penetration probability that determines the outcome.

For particles reacting at thermal energies, it is likely that the fusion reaction will occur far from any resonance, and thus the energy dependence of the cross section comes mainly from two terms: i) the $1/v^2$ dependence, and ii) the partial reaction probability, which for two particles includes a barrier penetration factor of the form e^{-2G}, similarly to α decay:

$$\sigma \propto \frac{1}{v^2} e^{-2G} \tag{4.32}$$

where v represents the relative velocity of the reacting particles, and G is defined as:

$$G = \frac{e^2}{4\pi\varepsilon_0}\frac{\pi Z_a Z_X}{\hbar v} \tag{4.33}$$

Fig.4.15 shows the cross sections of three typical fusion reactions.

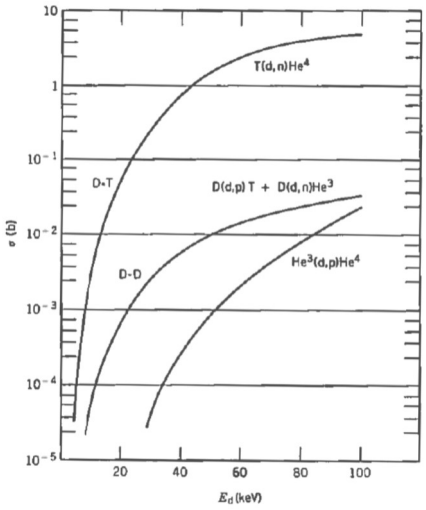

Figure 4.15 Cross sections for nuclear fusion reactions *(from Krane)*

The rate for a nuclear reaction depends on the product σv. In thermonuclear fusion there is a distribution of particle speeds described by a Maxwell-Boltzmann velocity distribution:

$$n(v) \propto e^{-mv^2/2kT} \tag{4.34}$$

In such a distribution of nuclei, it is appropriate to calculate σv averaged over all particle energies, E:

$$\langle \sigma v \rangle \propto \int_0^\infty e^{-2G} e^{-E/kT} dE \tag{4.35}$$

Fig.4.16 shows $<\sigma v>$ for several fusion reactions as a function of the temperature.

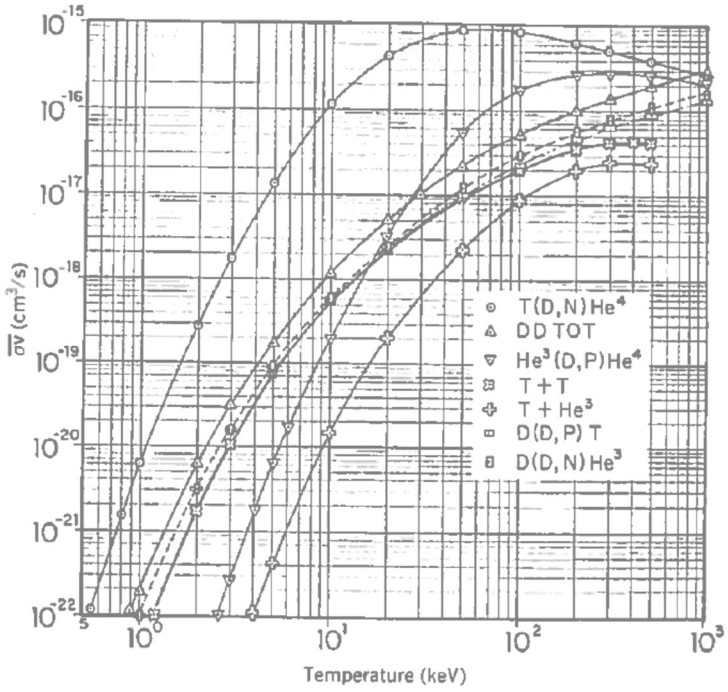

Figure 4.16 σv averaged over a Maxwell-Boltzmann energy distribution for various fusion reactions *(from Krane)*

The Sun is an extremely successful prototype of a self-sustaining thermonuclear reactor. In fact, based on fossil records on Earth, the Sun's output has been nearly constant over a time scale of more than 10^9 years. The basics nuclear process in the Sun is the fusion of hydrogen into helium (H is by far the most abundant element in the universe). All fusion reactions must be two-body processes since the simultaneous collision of three particles is too improbable to be significant. The first step in the ***solar fusion*** process must be the combination of two protons to form the only stable two-nucleon system:

$$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu \qquad (Q = 1.44\ MeV)$$

The presence of a neutrino in the final state signals a weak interaction process, which must occur to turn a proton into a neutron. The cross sections for weak interaction processes are very small,

and even at the high densities at the core of the Sun (~7.5 x 10^{25} protons/cm^3) the reaction rate is only about 5 x 10^{-18} s^{-1} per proton. However, the Sun keeps radiating thanks to the enormous number of reacting protons (~ 10^{56}) that allows to have a total reaction rate of about 10^{38} s^{-1}. This step in the solar fusion cycle is known as "bottleneck" because it is the slower and least probable one. Following deuteron formation, the following fusion reaction is very likely to occur:

$$^2H + ^1H \rightarrow ^3He + \gamma \qquad (Q = 5.49\ MeV)$$

At this stage, it is very unlikely to observe D-D reactions because of the small number of deuterons present (only one ^{2}H is formed every 10^{18} protons). Thus, deuterons are converted into ^{3}He nearly as rapidly as they are formed. Reactions of ^{3}He with ^{1}H or ^{2}H are not favourable, thus ^{3}He will interact only when it will find another ^{3}He:

$$^3He + ^3He \rightarrow ^4He + 2\,^1H + \gamma \qquad (Q = 12.86\ MeV)$$

The complete process is known as *proton-proton cycle* and is schematically shown in Fig.4.17. The net process is:

$$4\,^1H \rightarrow ^4He + 2e^+ + 2\nu \quad (Q = 26.7\ MeV)$$

The energy released in these nuclear reactions is converted to light in the *photosphere*, the outer region of the Sun. The visible light that reaches us from the Sun is characteristic of its surface and represents γ rays from reactions in the core that are scattered many thousands of times before reaching the surface. It can take millions of years for the radiation to emerge from the surface, hence the light that we see today results from solar processes that occurred millions of years ago. On the other hand, the neutrinos come to us directly from the core at the speed of light.

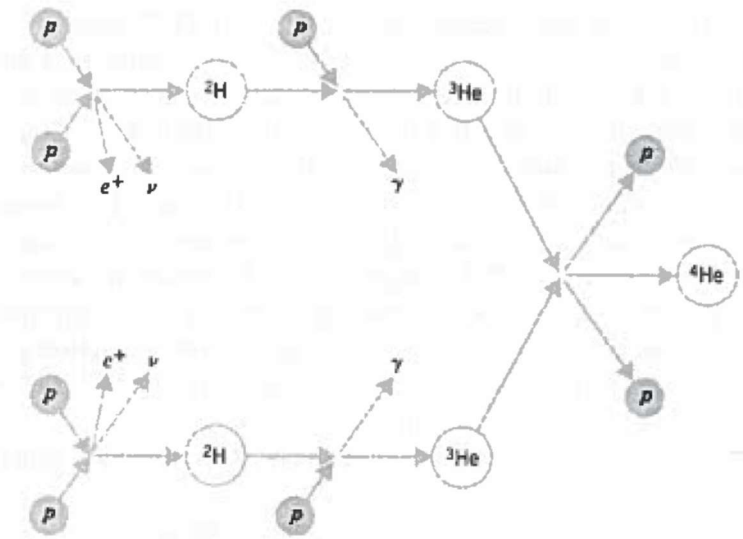

Figure 4.17 The proton-proton cycle of the Sun *(from Krane)*

If in addition to hydrogen and helium there are heavier elements present in the interior of a star, a different series of fusion reactions can occur, e.g. the *carbon (or CNO) cycle*:

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$$

$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu$$

$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$$

$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$$

$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu$$

$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$$

In this case ^{12}C is neither created nor destroyed, but act as a catalyst to aid in the fusion process. The net process is:

$$4\,{}^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\nu$$

Which is the same as in the proton-proton cycle, thus the Q-value is the same. However, the carbon cycle can proceed more rapidly than the proton-proton cycle because it does not have a process analogous to the deuterium bottleneck. Nevertheless, the Coulomb barrier is 6-7 times higher for proton reactions with carbon and nitrogen than for proton-proton reactions. Thus, the carbon cycle will be dominant at higher temperatures, where additional thermal energy is needed to increase the probability to penetrate the Coulomb barrier, as shown in Fig.4.18. Once a star has exhausted its hydrogen fuel, helium fusion reactions can take place with 3 $^{4}He \rightarrow {}^{12}C$ at higher temperature needed to penetrate the Coulomb barrier. Other reactions involving fusion of light nuclei can continue to release energy, until the process ends near ^{56}Fe, beyond which there is no energy gain in combining nuclei.

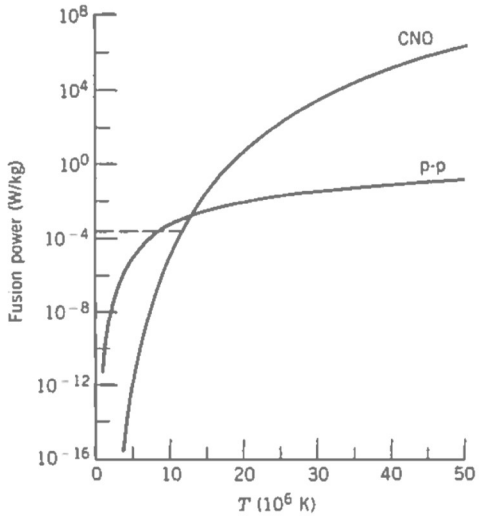

Figure 4.18 Power generation per unit mass of fuel for proton-proton and CNO processes. The dashed line indicates the Sun's power *(from Krane)*

Nuclear fusion can be considered as the most important phenomenon in nature. Fusion is the source of energy of the Sun and ultimately of all the natural physical and biological processes on the Earth. Furthermore, nuclear fusion is potential usable directly on Earth to produce energy in ***fusion reactors***. In fact, much of the Earth is covered by sea containing the hydrogen isotopes ^{1}H and ^{2}H, thus the fuel supply of low-A nuclei would be almost inexhaustible. However, an intense thermal fusion reactor of relatively small size would require reaching temperatures at least one order of magnitude higher than the internal temperature of the Sun. There are approaches that allow reaching such temperatures, however a "container" that would not be destroyed by the high temperature needs to be found. The Sun is so massive that gravitational fields provide a natural container. The essence of controlling fusion reactions and extracting usable energy is to heat a thermonuclear fuel to temperatures of the order of 10^8 K (mean particle kinetic energy of ~ 10 keV) while simultaneously maintaining a high enough density for a long enough time and allow to have a fusion reaction rate large enough to generate the desired power. At these temperatures the atoms rapidly become ionized (only 13.6 eV is needed to strip the electron from the hydrogen atom), and the fuel is a hot cloud of positive ions and negative electrons but electrically neutral overall. Such a system is called *plasma*. The electrostatic properties of a plasma determine a characteristic scale length called *Debye length*:

$$L_D = \sqrt{\left(\frac{4\pi\varepsilon_0}{e^2}\frac{kT}{4\pi n}\right)} \tag{4.36}$$

where *n* is the mean electron (or ion) density. For a rarefied plasma (~ 10^{22} m^{-3}) with a temperature of ~ 10 keV, the Debye length is ~ 10 μm, and the number of particles in a volume of dimension of one Debye length is ~ 10^7. It is important to mention that the physical size of the reacting plasma is much larger than the Debye length in dimension. One of the major problems is to be able to confine the plasma for a relatively long time. In fact, if the hot fuel exchanges energy with the walls of its container, it will cool down and the container will be melted. Currently there are two major schemes being explored to confine the thermonuclear fuel: *magnetic confinement fusion* and *inertial confinement fusion*. In magnetic confinement, the plasma is confined by an ad-hoc designed magnetic field system. In inertial confinement, a solid pellet is rapidly heated and compressed by being struck simultaneously from many directions with high peak-power laser beams. However, in both approaches there are many processes that cause the plasma to lose energy. The primary mechanism is *bremsstrahlung*, in which the Coulomb scattering of two particles produces an acceleration which in turn gives rise to emission of radiation. The largest accelerations are suffered by the lightest particles (electrons) but due to a local thermal equilibrium between ions and electrons, any loss by the electrons is also felt by the ions, which

become then less energetic and less effective in penetrating the Coulomb barrier. The power per unit volume radiated in bremsstrahlung is:

$$P_{br} = 0.5 \cdot 10^{-36} Z^2 n m_e (kT)^{\frac{1}{2}} W/m^3 \quad (4.37)$$

where *kT* is in keV. Comparing Equation (4.37) with the fusion power density in Fig.4.19, shows a temperature at which the fusion output will exceed the bremsstrahlung loss, which is about 4 keV for D-T and 40 keV for D-D reactions. This justifies the choice of D-T for fuel of the current nuclear fusion based research reactors. It is important to note that the bremsstrahlung losses increase as Z^2; therefore, fusion reactions using nuclei other than hydrogen have much larger bremsstrahlung losses, along with smaller reaction rates (due to the higher Coulomb barrier). Thus, the operating temperature of a fusion reactor is chosen to obtain a power gain where fusion generated power is higher than the bremsstrahlung losses.

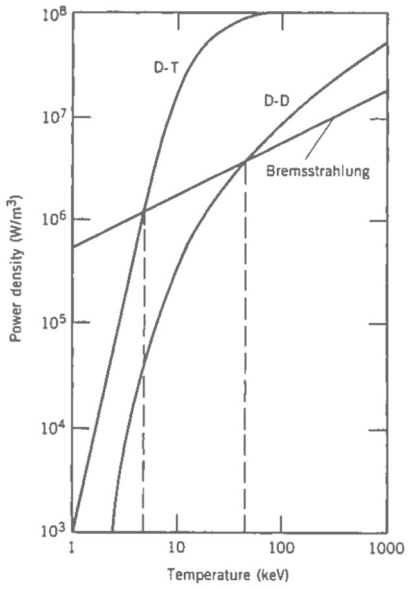

Figure 4.19 Comparison of bremsstrahlung losses with power outputs of D-D and DT nuclear fusion reactions, under the assumption of an io density of 10^{21} m^{-3} *(from Krane)*

The fusion reactor would have a net energy gain if the energy released in fusion reactions exceeds the radiation losses and the original energy investment in heating the plasma to the operating temperature. At temperature of 4-10 keV, the D-T fusion gain is greater than the radiation losses (see Fig.4.19), thus, in first approximation, we can neglect the loss in energy due to radiation. The energy released per unit volume from fusion reactions is:

$$E_f = \frac{1}{4} n^2 \langle \sigma v \rangle Q \tau \quad (4.38)$$

where we assume that the densities of D and T are each equal to ½*n* (total *n* is equal to n_e for D-T), *Q* is the energy released per reaction (17.6 MeV for D-T), and τ is the time duration the plasma

is confined during which reactions can occur. The thermal energy per unit volume needed to raise the plasma ions and electrons to a temperature T is $3/2 n_i kT$ (ions) and $3/2 n_e kT$ (electrons). If $n = n_i = n_e$ (local thermal equilibrium), the thermal energy is:

$$E_{th} = 3nkT \tag{4.39}$$

Thus, if we supply an energy of E_{th} to heat the plasma, and then we can confine it for a time τ, we can extract fusion energy E_f. The reactor would show a net energy gain if $E_f > E_{th}$:

$$\frac{1}{4} n^2 \langle \sigma v \rangle Q \tau > 3nkT \tag{4.40}$$

or:

$$\boxed{n\tau > \frac{12kT}{\langle \sigma v \rangle Q}} \tag{4.41}$$

This method allows to estimate the minimum necessary product of ion density and confinement time. This is known as *Lawson criterion* and represents the goal of a thermonuclear reactor design. For an operating temperature of 10 keV for D-T fusion, $<\sigma v> \approx 10^{-22}$ m^3/s, and thus $n\tau > 10^{20}$ s/m^3.

A reactor design called "Tokamak", using a magnetic confinement fusion approach, is one of the most promising candidates for the basic design of a fusion power reactor. Inertial confinement fusion uses a very different approach. A tiny pellet containing deuterium and tritium is rapidly struck with intense laser pulses (~ ns) that both hit and compress it to high density. The goal of this method is to achieve densities and temperatures that are high enough to allow efficient nuclear fusion before the pellet expands and blows apart. To make a rough estimate of the requirements for such a reactor, it is important to note that the time necessary for the compressed pellet to blow apart is determined by the propagation speed of mechanical waves in the plasma medium, which is of the same order of magnitude as the mean thermal speed of the particles in the medium. At a temperature of 10 keV, the mean thermal speed is ~ 10^6 m/s. If we consider a pellet compressed to a diameter of 0.1-1 mm, it is expected to blow apart in about 0.1-1 ns. Thus, applying the Lawson criterion for a D-T mixture, with a confinement time of 0.1-1 ns, we would need a density of at least $10^{29} – 10^{30}$ m^{-3}, which is orders of magnitude greater than ordinary liquid or solid densities for hydrogen. To heat a spherical pellet of diameter 1 mm to a mean thermal energy of 10 keV per particle, the total thermal energy that must be supplied is about:

$$E_{th} \approx 4/3\ \pi\ (0.5\ mm)^3 \times 10^{29}\ m^{-3} \times 10^4\ eV \approx 10^5\ J$$

This means we must supply an energy of the order of 0.1 MJ in about 1 ns, i.e. a net power of 10^{14} W! This is the best-case scenario considering that a substantial amount of energy supplied to the pellet is absorbed through various processes, thus the estimate of the total energy we must supply

to heat and compress the core of the pellet should be increased by a factor 10. Furthermore, lasers are usually inefficient to convert electrical energy into photons, and a 10% efficiency can be considered as an optimistic assumption. Therefore, the electrical power needed for the lasers can easily be 10^{16}-10^{17} W! Fortunately, this power would need to be provided only for a very short time interval (~ ns), but the estimated peak-power value is still extremely high if one considers that the entire electrical generating capacity of the world is ~ 10^{13} W. To run an inertial confinement reactor at a net energy gain, it is necessary to considerably exceed the Lawson criterion; compressions around 1000 times ordinary solid density and temperatures above 10 keV thermal energy per particle are required. The sequence of processes in laser-driven fusion can be summarised as follows: i) a pellet is injected into the machine and is simultaneously struck from many directions by intense laser pulses; ii) the outer layer of the solid pellet is immediately vaporized and forms a plasma, which continues to absorb the laser radiation; iii) the plasma itself is unconfined and rapidly "blows off", or *ablates*, which drives a compressional shock wave back into the remaining core of the pellet (according to Newton's third law); (iv) this shock wave compresses and heats the core to the point at which thermonuclear ignition can occur at the highest density region near the centre; (v) the α-particles resulting from the fusion events rapidly lose their energy in collisions with ions in the dense fuel, thus contributing to additional heating of the pellet; (vi) the thermonuclear burn propagates outwards, finally blowing the pellet apart and ending the nuclear fusion process. This sequence is schematically represented in Fig.4.20.

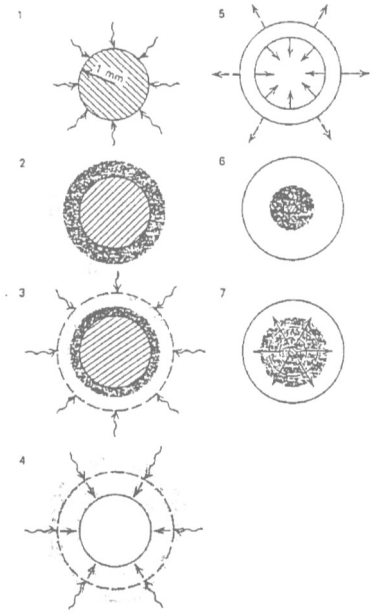

Figure 4.20 Sequence of stages in inertial confinement fusion: (1) pellet irradiation by lasers; (2) plasma formation; (3) further laser beam absorption; (4) ablation and resulting imploding shock wave; (5) shock wave compressing core; (6) ignition of core; (7) burn propagating outward *(from Krane)*

Unfortunately, nuclear fusion is also used for thermonuclear weapons. Once the first thermonuclear explosives were detonated by the United States and the USSR in the early 1950s, the fission explosives in the strategic arsenals of both nations were soon replaced with fusion-based explosive with energies 2-3 orders of magnitude greater than the early fission weapons, thus passing from a kiloton-range to a megaton-range energy. Fusion weapons includes a fission explosive as an initiator. The radiation from the fission explosion is responsible for heating and compressing the thermonuclear fuel. Present-day weapons use solid lithium deuteride as fuel. The neutrons released from the primary fission explosion (and subsequent fusions) convert the ^{6}Li into tritium:

$$^{6}Li + n \rightarrow {}^{3}H + {}^{4}He \qquad (Q = 4.78\ MeV)$$

Even for low-energy neutrons, the tritium carries enough energy (2.7 MeV) to easily penetrate the D-T Coulomb barrier and initiate the fusion reactions. Thus, similarly to the inertial confinement approach of controlled fusion, the heat, neutrons, and α-particles contribute to sustaining the reaction until the expansion of the fuel terminates the process. The fast neutrons released in the fusion can be used to add additional energy to the explosive by surrounding the fusion fuel with a casing of ^{238}U, which fissions with fast neutrons. The operation and energy released in a thermonuclear weapon are thus dependent on a fission-fusion-fission cycle.