

PHY2003 ASTROPHYSICS I

Lecture 6. Inside the Sun

Hydrostatic equilibrium

The Sun is trying to collapse from self-gravity. At a distance r from the center, the mass within r is given by

$$m(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$$

This gives an inward gravitational acceleration

$$a_g = -\frac{Gm(r)}{r^2}$$

Now consider the pressure on a parcel of gas at r , width Δr and area ΔA

$$F_p = [P_{outer} - P_{inner}] \Delta A$$

P_{inner} must be $> P_{outer}$, otherwise the star will collapse! So there must be a pressure gradient $dP/dr < 0$

The pressure on the outer surface of the gas parcel is

$$P_{outer} = P(r + \Delta r) = P(r) + \left(\frac{dP}{dr}\right) \Delta r$$

$$F_p = \left[P(r) + \left(\frac{dP}{dr}\right) \Delta r - P(r) \right] \Delta A = \left(\frac{dP}{dr}\right) \Delta r \Delta A$$

As the mass $\Delta M = \rho(r) \Delta V = \rho(r) \Delta A \Delta r$, acceleration is given by

$$a_p = \left(\frac{dP}{dr}\right)\Delta r\Delta A/\Delta M = \frac{dP}{dr} \frac{1}{\rho(r)}$$

In equilibrium, $a_p = a_g$

$$\frac{dP}{dr} \frac{1}{\rho(r)} = -\frac{Gm(r)}{r^2}$$

The Equation of Hydrostatic Equilibrium:

$$\boxed{\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}}$$

Example: From spectroscopic measurements, the photosphere has an average density of $\simeq 10^{-4} \text{ kg/m}^3$; what is the pressure gradient in the photosphere?

Average interior conditions

Take the EHE and multiply both sides by $4\pi r^3$ and integrate from center to surface:

$$\int_0^{R_\odot} 4\pi r^3 \frac{dP}{dr} dr = \int_0^{R_\odot} \frac{Gm(r)}{r^2} \rho(r) 4\pi r^3 dr$$

LHS Integrate by parts:

$$LHS = [P(r)4\pi r^3]_0^{R_\odot} - 3 \int_0^{R_\odot} P(r)4\pi r^2 dr$$

First term is effectively zero ($P \simeq 0$ at $r = R_\odot$)

For the Second term, substitute \bar{P} for $P(r)$

$$3 \int_0^{R_\odot} \bar{P}4\pi r^2 dr = 3\bar{P} \int_0^{R_\odot} 4\pi r^2 dr = 3\bar{P}V$$

where V is volume.

RHS is gravitational potential energy U_g of the Sun:

$$RHS = \int_0^{R_\odot} \frac{Gm(r)}{r^2} \rho(r)4\pi r^3 dr = \int_0^{R_\odot} \frac{Gm(r)}{r} \rho(r)4\pi r^2 dr$$

$$RHS = \int_0^{R_\odot} \frac{Gm(r)}{r} dm = U_g$$

$$0 - 3\bar{P}V = U_g$$

$$\boxed{\bar{P} = -\frac{U_g}{3V}}$$

(This is one form of the *Virial Theorem*, which gives a relationship between the kinetic and potential energies of a gravitationally bound system of particles).

Using the ideal gas equation $\bar{P} = \bar{n}k\bar{T}$, $\bar{n} = \bar{\rho}/\bar{m}$. \bar{m} (the mean molecular weight of the Sun) = $0.61 \cdot m_{\text{Hydrogen}}$.

$$\bar{T} = \frac{\bar{P}\bar{m}}{k\bar{\rho}}, \bar{P} = -\frac{U_g}{3V}, U_g \simeq -\frac{GM_\odot^2}{R_\odot}$$

$$\boxed{\bar{T} \simeq \frac{GM_\odot\bar{m}}{3kR_\odot}}$$

Example: Calculate the average pressure and temperature of the Sun.

Plasma values at the Solar core

From the EHE:

$$\frac{dP}{dr} = -\rho(r) \frac{Gm(r)}{r^2} = -\rho(r) \frac{G}{r^2} \frac{4}{3} \pi r^3 \bar{\rho}$$

Approximate $\rho(r)$ by $\bar{\rho}$:

$$\frac{dP}{dr} \simeq -\bar{\rho}^2 G \frac{4}{3} \pi r$$

Integrate this from core to surface:

$$\int_{P_c}^0 dP \simeq -\bar{\rho}^2 G \frac{4}{3} \pi \int_0^{R_\odot} r dr$$

$$-P_c \simeq -\bar{\rho}^2 G \frac{4}{3} \pi \frac{R_\odot^2}{2}$$

$$P_c \simeq \frac{2}{3} \pi G \bar{\rho}^2 R_\odot^2$$

To estimate the temperature, use ideal gas equation:

$$T_c = \frac{P_c}{n_c k} \simeq \frac{P_c}{\bar{n} k}$$

$$T_c \simeq \frac{P_c \bar{m}}{\bar{\rho} k}$$

The Sun's structure: The Core - $0 \leq r \leq 0.25 R_\odot$.

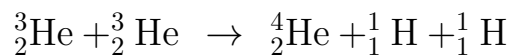
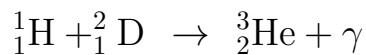
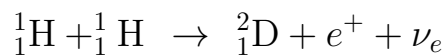
Gravitational collapse cannot generate the Sun's luminosity: The gravitational PE of the Sun is

$$U_\odot \simeq GM_\odot^2/R_\odot \equiv 3.8 \times 10^{41} \text{ J}$$

So the lifetime of the Sun under this process would be

$$t_\odot \simeq U_g/L_\odot \simeq 10^{15} \text{ sec} \simeq 3 \times 10^7 \text{ years. But the Sun is } \sim 4.5 \times 10^9 \text{ years old.}$$

The most likely explanation for the Sun's luminosity is nuclear fusion, where $H \rightarrow He + \gamma$. The most common form of this reaction is the proton-proton ($p-p$) chain.



The result of this reaction is that 4 hydrogen atoms create one helium atom, with 2 neutrinos and 6 photons carrying an associated release of energy of 26 MeV $\equiv 4 \times 10^{-12} \text{ J}$

Therefore the number of p-p chains per second occurring is $\sim 3.8 \times 10^{26} J / 4 \times 10^{-12} J$ or $\simeq 10^{38}$.

In one second 4×10^{38} atoms of Hydrogen are transformed into Helium, or $4 \times 10^{38} \times 1.66 \times 10^{-27} = 6.6 \times 10^{11}$ kg. Therefore the Sun has so far used $6.6 \times 10^{11} \times 4.5 \times 10^9 \times 3.2 \times 10^7 \simeq 9 \times 10^{28}$ kg of H.

This is $\sim 5\%$ of its present H-mass. Therefore this is a viable energy source, and implies that the Sun has not grossly changed since formation.

Example: Estimate the flux of solar neutrinos through each square cm of your body.

The Sun's structure: The Radiative Zone

$$0.25 \leq r \leq 0.7 R_{\odot}$$

Region in which most energy is transported through radiation.

Photons follow a random-walk through scattering by ions.

D is distance traveled, N is number of steps, l is length of one step:

$$D^2 = Nl^2$$

For the Sun

$$N = R_{\odot}^2/l^2$$

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The time taken for a photon to move one step is l/c .

The time taken for a photon to leave the Sun is

$$T = Nl/c = R_{\odot}^2/lc$$

The Sun is very opaque, $l \simeq 10^{-3}\text{m}$, so $T \simeq 50,000$ years!

The Sun's structure: The Convective Zone

$$0.7 \leq r \leq 1.0R_{\odot}$$

Region in which most energy is transported through convection.

Top of the convective zone is the photosphere - granules are individual convective bubbles of gas (but remember size!).

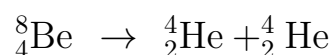
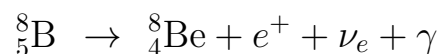
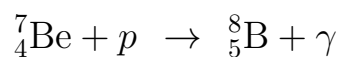
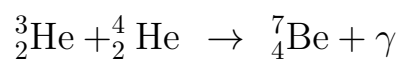
Observational tests

(a) Solar Oscillations

Regions of the photosphere are seen to rise and fall on a 5-minute period. This is caused by granulation sending sound waves throughout Sun, setting up standing waves and resonances. Astronomers have now measured thousands of different wave frequencies to accuracies of $\pm 0.01\%$. The resulting passage of sound waves allows determination of internal properties – helioseismology.

(b) Solar Neutrinos

The neutrinos released by the p-p chain are of low energy and are difficult to detect. However, a small amount of ${}^3_2\text{He}$ undergoes a different reaction that releases energetic neutrinos:



For 40 years, experiments detecting neutrinos only detected 25-50% of predicted

numbers. In 2001, this was solved by experiments that found the total number of electron neutrinos leaving the Sun is as predicted, but roughly half change into a different type.