Nuclear and Radiation Physics (PHY2005)

Lecture 4

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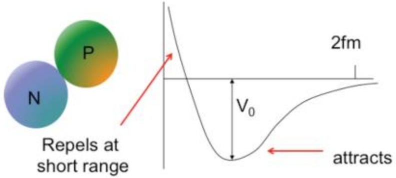
Recap & Learning Goals

$$\boxed{\begin{array}{lll} V(r) = +\infty & r < R_{core} & (R_{core} \sim 0.5 \text{ fm}) \\ V(r) = -V_0 & R_{core} \le r \le R & \\ V(r) = 0 & r > R & \end{array}}$$



Summary of Lecture 3 *(Chap.2)*

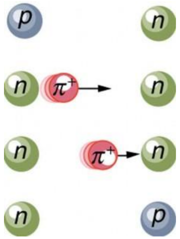
- Properties of the nuclear force
  - ✓ nucleon-nucleon interaction
  - ✓ spin-orbit interaction
- Exchange force model
  - ✓ virtual meson exchange

$$\boxed{P = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}}$$

$$\boxed{R = \frac{200MeV \cdot fm}{m_x c^2}}$$

n-p scattering experiments (T ~100 MeV)

$$\boxed{\theta = \frac{V_0}{2T} \; (\theta \le 10°)}$$



Learning goals of of Lecture 4 *(Chap.3)*

- Knowing the terminology and notation of the nuclear *Shell model*
- Understanding physical reasoning behind the *Shell model*

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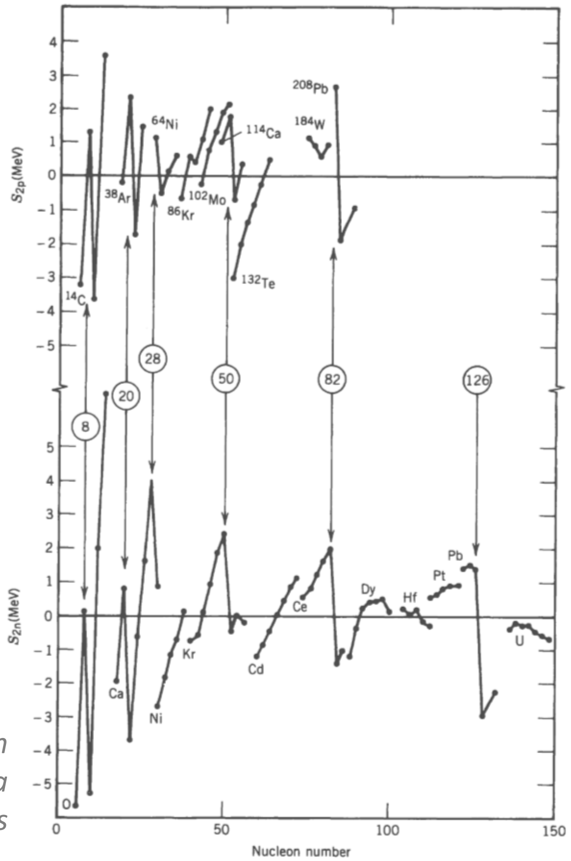
Nuclear Shell model

- heavy nuclei (many-body problem) ☹ → simplified nuclear models ☺
- nuclear <u>Shell model</u> → valence nucleons experience a potential created by the nucleons themselves
- nucleons are large (size of nucleus) → collisions (?)

- experimental evidence → nuclear shells
- <u>magic numbers</u> → filled major shells
  (Z or N = 2, 8, 20, 28, 50, 82, and 126)
- nucleons orbit as if they were transparent to one another

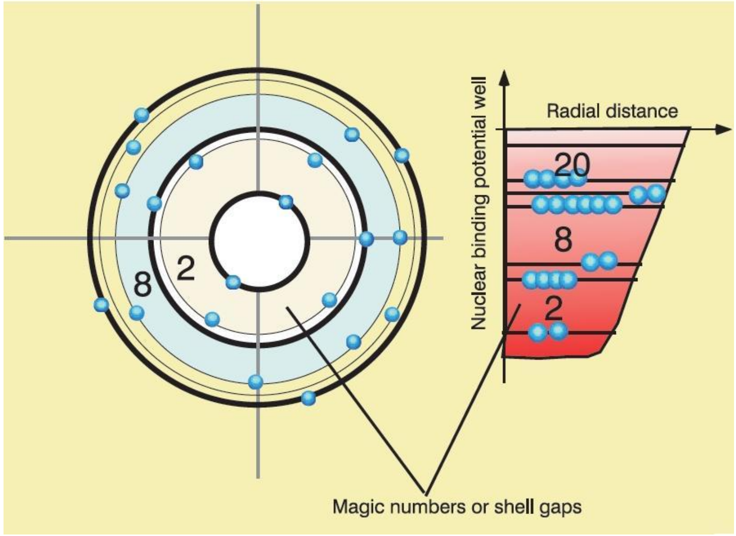


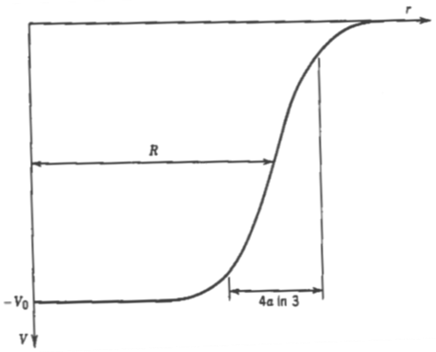
two-proton/neutron separation energies of a sequence of isotones/isotopes

Nuclear Shell model

- ***Shell model potential*** → <u>*Woods-Saxon potential*</u>

$$V(r) = \frac{-V_0}{1+\exp\left(\frac{r-R}{a}\right)}$$





Quantum states and magic numbers

- level degeneracy (Pauli principle) → *2(2L + 1)*
  $m_L \rightarrow (2L+1)$; $m_S \rightarrow 2$

- Nuclear spectroscopic notation → *n* is not the principal quantum number (!)

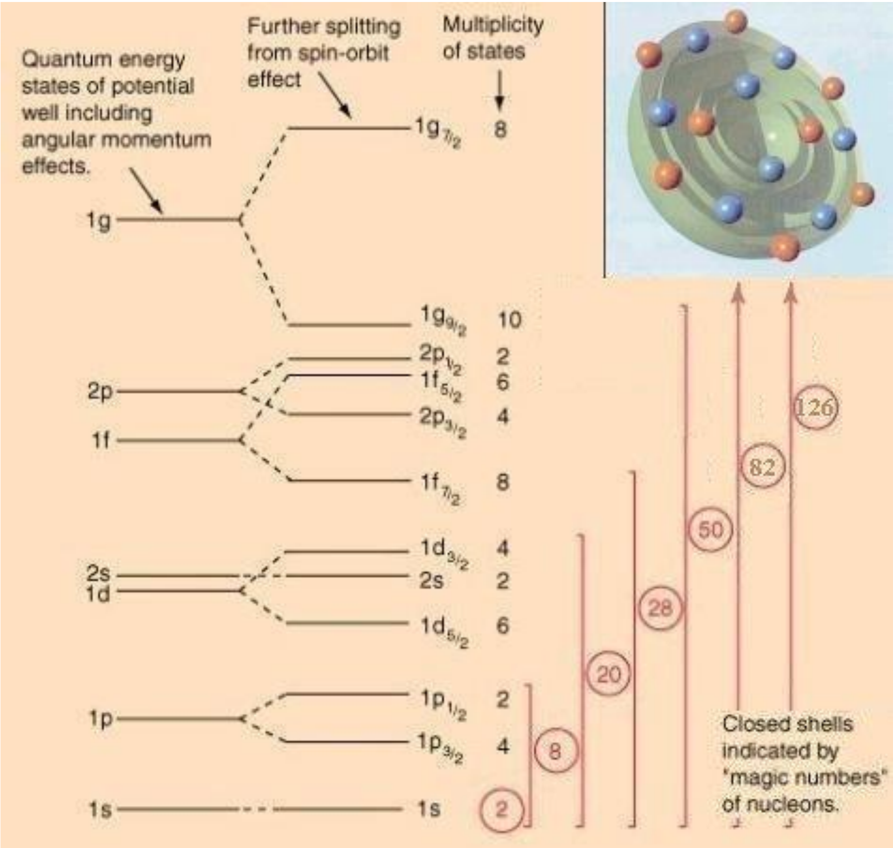
- OK <u>only</u> for magic numbers 2, 8, and 20 ☹

- spin-orbit potential → OK for all magic numbers! ☺

- $\mathbf{J} = \mathbf{L} + \mathbf{S}$ → $J = L \pm \frac{1}{2}$ (for a single nucleon $s = \frac{1}{2}$)

- … except for $L = 0$ → only $J = 1/2$ is allowed

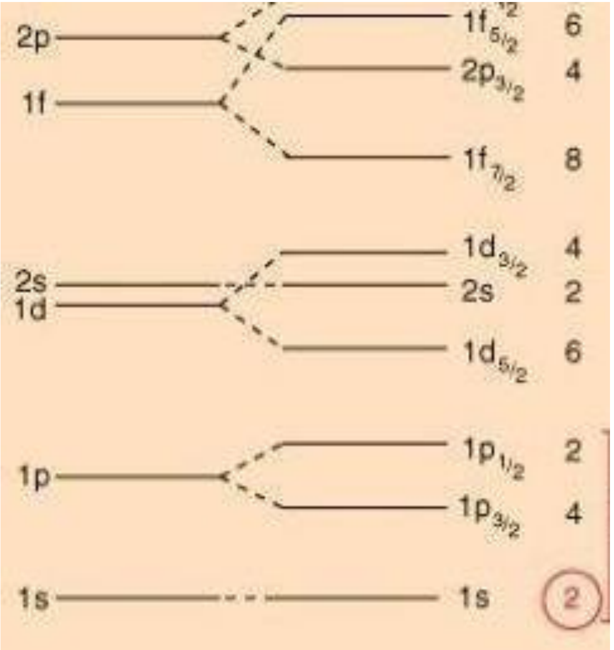
- level degeneracy (spin-orbit) → *(2J + 1)*



3. Nuclear Models

3.1. The Shell model IV

- ✓ ***1s*** *(L = 0)*
  - *L degeneracy (no spin-orbit): 2(2L + 1) =* ***2***
  - *possible J values (L ± ½): 1/2 →* $\boldsymbol{1s_{1/2}}$
  - *J degeneracy (spin-orbit): 2J + 1 =* ***2***
- ✓ ***1p*** *(L = 1)*
  - *L degeneracy (no spin-orbit): 2(2L + 1) =* ***6***
  - *possible J values (L ± ½): 3/2, 1/2 →* $\boldsymbol{1p_{3/2}}$ ***;*** $\boldsymbol{1p_{1/2}}$
  - *J degeneracy (no spin-orbit): 2J + 1 =* ***4 ; 2***
- ✓ ***1d*** *(L = 2)*
  - *L degeneracy (no spin-orbit): 2(2L + 1) =* ***10***
  - *possible J values (L ± ½): 5/2, 3/2 →* $\boldsymbol{1d_{5/2}}$ ***;*** $\boldsymbol{1d_{3/2}}$
  - *J degeneracy (spin-orbit): 2J + 1 =* ***6 ; 4***
- ✓ ***1f*** *(L = 3)*
  - *L degeneracy (no spin-orbit): 2(2L + 1) =* ***14***
  - *possible J values (L ± ½): 7/2, 5/2 →* $\boldsymbol{1f_{7/2}}$ ***;*** $\boldsymbol{1f_{5/2}}$
  - *J degeneracy (spin-orbit): 2J + 1 =* ***8 ; 6***



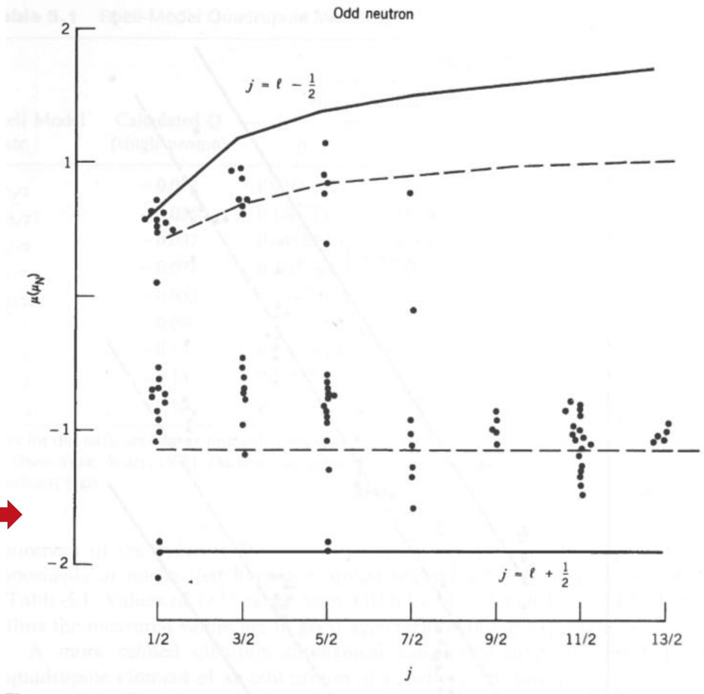
Magnetic Dipole Moments *(Shell model)*

- Shell model → not exact agreement with measured magnetic dipole moments ☹
- expectation value of magnetic moment operator of odd-A nucleon (state with max z-projection of L):
  g_L = +1 (proton)
  g_L = 0 (neutron)
  g_S = +5.6 (proton)
  g_L = -3.8 (neutron)

$$\mu = \frac{\mu_N (g_L L_z + g_S S_z)}{\hbar}$$

- even-A nuclei (paired) → $\langle\mu\rangle = 0$
- odd-A nuclei (unpaired) → $\langle\mu\rangle \neq 0$

experimental vs. calculated values of the magnetic moments of odd-A nuclei



- *theoretical vs. experimental data of odd-A nuclei*
- *exp. data are smaller in magnitude ☹*
- *wrong assumption on g_s (μ-nucleon = μ-nucleus)*
- *meson cloud of bounded nucleon ≠ meson cloud of free nucleon*
- *the shell-model theory oversimplifies μ calculation!*

Electric Quadrupole Moments *(Shell model)*

- electric quadrupole moment → $3z^2 - r^2$ operator
- expectation value of electric quadrupole operator of odd-A nuclei (state with max z-projection of L, i.e. $m_J = J$)
- *single-particle electric quadrupole moment of an odd proton* (state *J*):

$\langle r^2\rangle = 3/5\, R^2 = 3/5\, R_0^2\, A^{2/3}$

$$\boxed{\langle Q_{sp}\rangle = -\frac{2J-1}{2(J+1)}\langle r^2\rangle}$$

- *electric quadrupole moment* (subshell with more than a single-particle):

  n: number of nucleons in the subshell
  ($1 \le n \le 2J$)

$$\boxed{\langle Q\rangle = \langle Q_{sp}\rangle\left[1 - 2\frac{n-1}{2J-1}\right]}$$

- $n = 2J$ (subshell lacking only one nucleon) → $Q = -Q_{sp}$ ("hole" states)

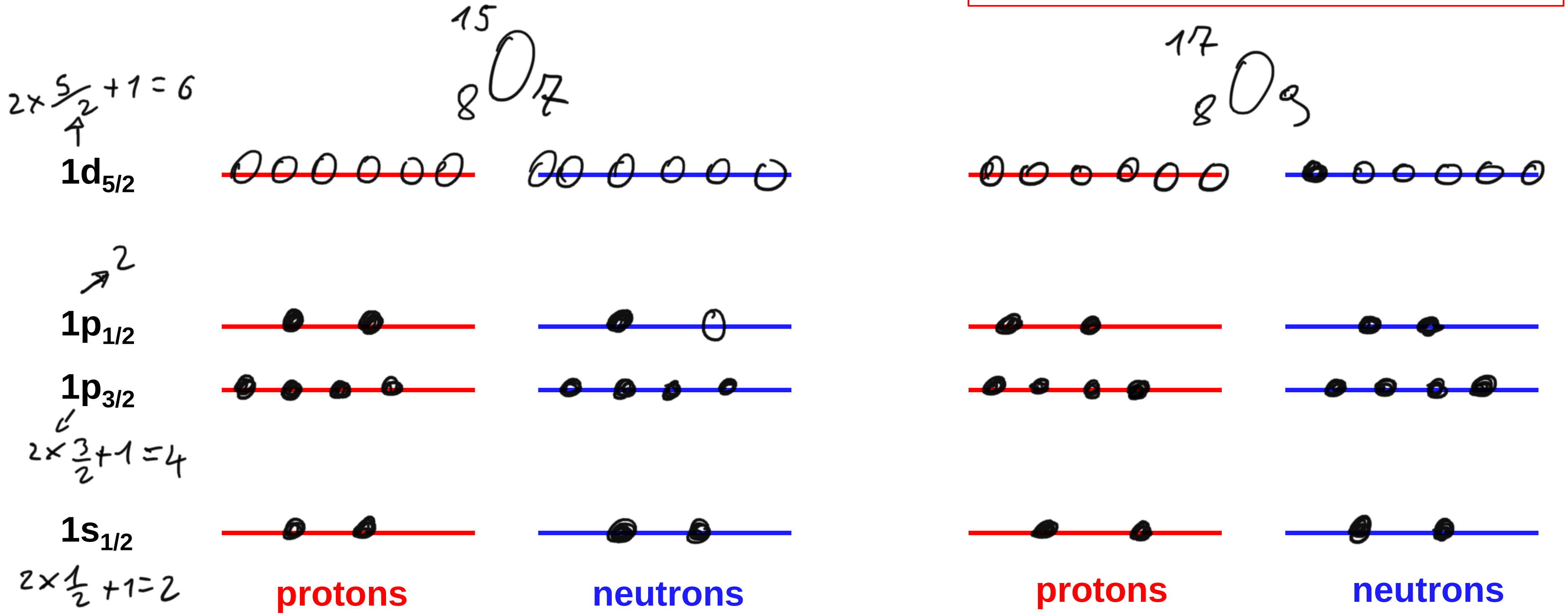
- Experiments → electric quadrupole moment of an odd-n nucleus ≠ 0 !!! ☹
- Shell model fails to predict *Q* of heavy nuclei ☹
- *the shell-model theory oversimplifies μ calculation!*

3. Nuclear Models

Example 3.1

Fill the given quantum states with protons and neutrons for the two isotopes ^{15}O and ^{17}O, according to the Nuclear Shell model rules for the level degeneracy in each state.

degeneracy → *2J + 1*   *J = L ± ½*



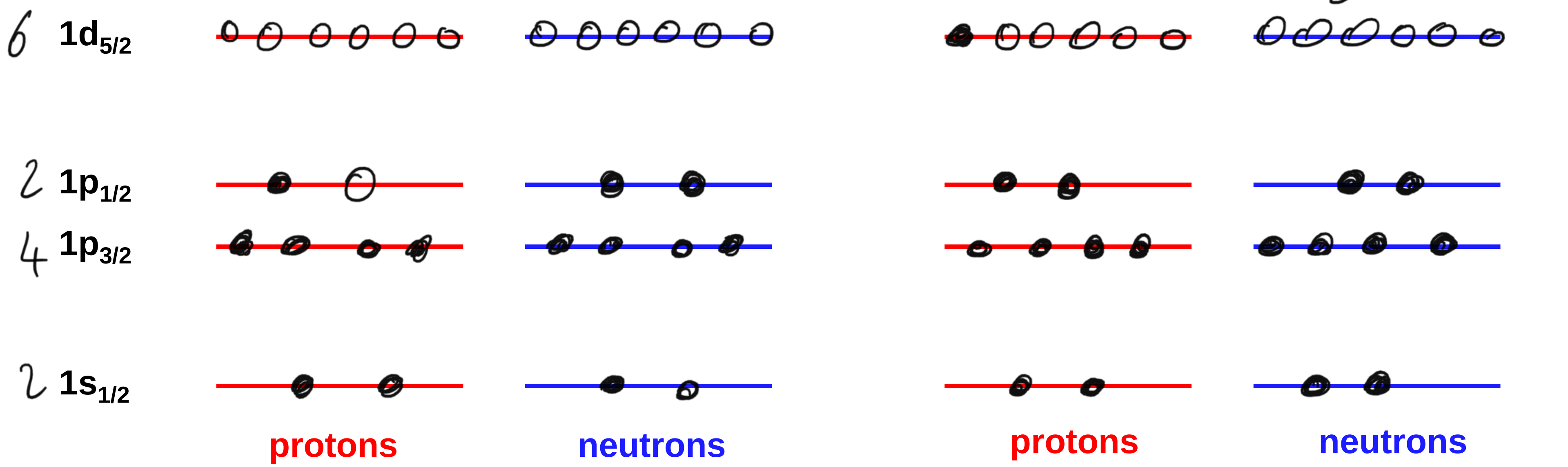
Fill the given quantum states with protons and neutrons for the two nuclei ^{15}N and ^{17}F, and specify their nuclear spin and parity.

degeneracy → $2J + 1$ $J = L \pm \frac{1}{2}$

$\pi \to (-1)^L$

$^{15}_{7}N_8 \to \frac{1}{2}^-$

$^{17}_{9}F_8 \to \frac{5}{2}^+$



Example 3.3

Calculate the theoretical electric quadrupole moments of the nuclei ^{15}O, ^{17}O, ^{15}N, and ^{17}F, according to the Nuclear Shell model assumptions.

$$\boxed{\langle Q_{sp}\rangle = -\frac{2J-1}{2(J+1)}\langle r^2\rangle}$$

$\langle r^2\rangle = 3/5\, R_0^2\, A^{2/3}$

$$\boxed{\langle Q\rangle = \langle Q_{sp}\rangle\left[1-2\frac{n-1}{2J-1}\right]}$$

$^{15}_{8}O_{7}$ protons in complete sub-shells $\rightarrow \underline{\langle Q\rangle = 0}$

$^{17}_{8}O_{9}$ neutrons do not contribute $\rightarrow \underline{\langle Q\rangle = 0}$

$^{15}_{7}N_{8}$ 1 proton in $1p_{1/2} \rightarrow \langle Q_{sp}\rangle = -\frac{2\frac{1}{2}-1}{2(\frac{1}{2}+1)}\langle r^2\rangle = \underline{0}$ $(L=0!)$

$^{17}_{9}F_{8}$ 1 proton in $1d_{5/2} \rightarrow \langle Q_{sp}\rangle = -\frac{2\frac{5}{2}-1}{2(\frac{5}{2}+1)}\langle r^2\rangle =$

$= -\frac{4}{7}\times\frac{3}{5}\times(1.2\ \mathrm{fm})^2\times(10^{-15})^2\times 17^{2/3} = \underline{3.26\times10^{-30}\ \mathrm{C\,m^2}}$