QUEEN'S UNIVERSITY BELFAST

PHY2001
Exam Time Table
Code PHY2001

Answer Books A, B and C.

Any calculator, except one with pre-programmable memory, may be used in this examination.

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2001
Quantum and Statistical Physics

Thursday, 2nd May 2019 9:30 AM - 12:30 PM

Examiners: Professor P Browning
Dr P van der Burgt
and the Internal Examiners

Answer ALL TEN questions in Section A for 4 marks each.
Answer TWO questions in Section B for 20 marks each.
Answer ONE question in Section C for 20 marks.
Use a separate answer book for each Section.

You have THREE HOURS to complete this paper.

THE QUEEN'S UNIVERSITY OF BELFAST

SCHOOL OF MATHS AND PHYSICS

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \mathrm{ms}^{-1}$
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Hm}^{-1}$ $\approx 1.26 \times 10^{-6}\ \mathrm{Hm}^{-1}$
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{Fm}^{-1}$
Elementary charge	$e = 1.60 \times 10^{-19}\ \mathrm{C}$
Electron charge	$= -1.60 \times 10^{-19}\ \mathrm{C}$
Planck Constant	$h = 6.63 \times 10^{-34}\ \mathrm{Js}$
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}\ \mathrm{Js}$
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}\ \mathrm{m}^{-1}$
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}\ \mathrm{kg}$ $1u = 931\ \mathrm{MeV}$
1 electron volt (eV)	$= 1.60 \times 10^{-19}\ \mathrm{J}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$
Mass of proton	$m_p = 1.67 \times 10^{-27}\ \mathrm{kg}$
Mass of neutron	$m_n = 1.67 \times 10^{-27}\ \mathrm{kg}$
Molar gas constant	$R = 8.31\ \mathrm{JK}^{-1}\mathrm{mol}^{-1}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11}\ \mathrm{Nm}^{2}\mathrm{kg}^{-2}$
Acceleration of free fall on the Earth's surface	$g = 9.81\ \mathrm{ms}^{-2}$

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SECTION A

Use a section A answer book

Answer ALL 10 questions in this section

Full explanations of your answers are required to attain full marks

1 A nanoparticle confines an electron so that when excited, a transition from the first excited state to the ground state results in emission of a photon at a wavelength of 1000 nm. If the diameter of the nanoparticle is halved, what is the wavelength of the transition now? **[4]**

2 A particle is described by the wavefunction

$\psi = A\sqrt{x} \quad 0 \leq x \leq 1$

$\psi = 0 \qquad x < 0, x > 1$

What is the probability of the particle being found in the range $0 \leq x \leq 0.5$? **[4]**

3 A particle is described by the wavefunction (normalised using periodic boundary conditions):

$\psi = \exp i\left(-\frac{x}{2} - \omega t\right)$

What is the average momentum of the particle (momentum operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$) ? **[4]**

4 Consider the quantum energy levels of a

- Harmonic potential well
- Infinite potential well
- Coulomb potential well
- Finite potential well

Explain why the separation of the quantum levels in one of these wells decreases with increasing quantum number. **[4]**

5 $\psi(x) = 10 \exp(ikx)$ represents the wave function of a stream of electrons of wavelength 1 nm. What is the probability flux of the electrons (number of electrons flowing per unit area, per unit time)? **[4]**

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SECTION A

6 If an electron of 7 eV energy is incident on a potential step of 12 eV, what is the penetration depth of its wave function in the classically excluded region? **[4]**

7 What are the degeneracies of the 1st and 2nd excited states of an electron trapped in a three-dimensional, infinite potential well of side lengths 0.1a, 0.2a, 0.2a? **[4]**

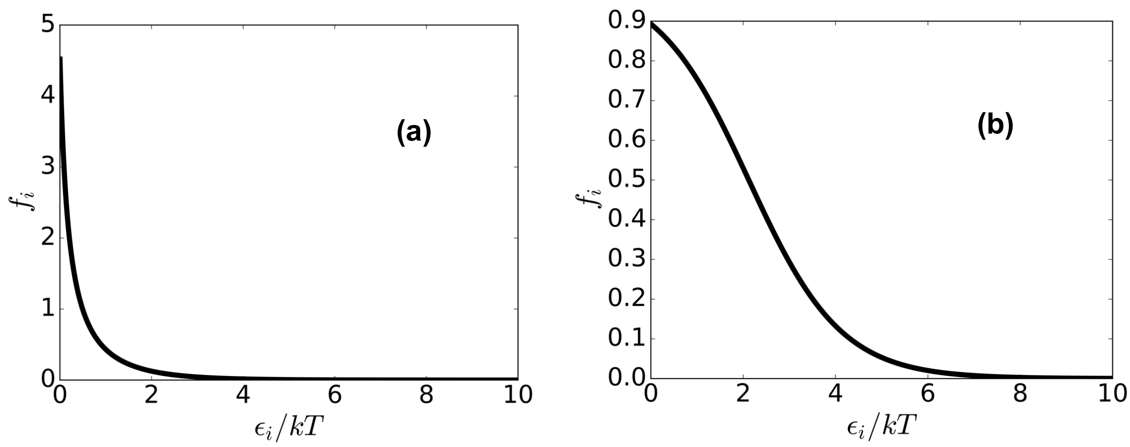
8 In elementary statistical mechanics, systems are often approximated as *isolated* and *weakly interacting*. Explain the physical meaning of these two terms as used in statistical mechanics. **[4]**

9 Consider a system with three single-particle states having energies $\varepsilon_1 = -1.3 \times 10^{-19}$ J, $\varepsilon_2 = 0$ and $\varepsilon_3 = 1.3 \times 10^{-19}$ J, respectively. For a population of classical distinguishable particles at a temperature of $T = 8000$ K, calculate the partition function of this system and estimate the fraction of particles that will occupy the middle ($\varepsilon_2 = 0$) state. **[4]**

10 In statistical mechanics, the distribution function can be defined as

$$f_i = \frac{n_i}{g_i}$$

where n_i is the number of particles occupying an energy level that has degeneracy g_i and energy ε_i. The figures below show f_i as a function of ε_i/kT for two different systems of particles with temperature T. For both cases state whether the system is governed by Fermi-Dirac, Bose-Einstein or dilute-gas statistics and explain your reasoning. **[4]**



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SECTION B

Use a Section B answer book

Answer TWO questions from this section

11 **(a)** Give a short account of why and how Planck, Einstein and Bohr introduced quantization to help explain experimental observations of blackbody radiation, the photoelectric effect and atomic spectra. **[9]**

(b) By considering the representation of a particle by a series of plane waves:

(i) Explain the origin of the Heisenberg Uncertainty Principle in terms of the particle's position and momentum. **[5]**

(ii) Show how a similar expression can be obtained relating energy and time. **[3]**

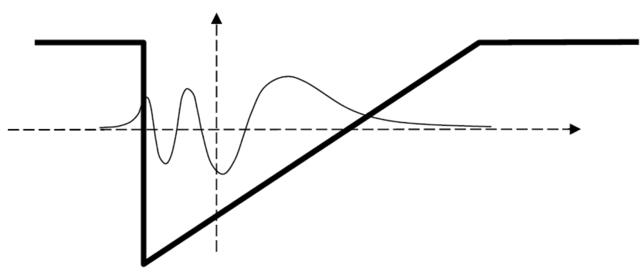
(iii) An atom absorbs a photon from a laser operating at 10.6 μm to form a virtual excited state. How long can this state exist for? **[3]**

12 **(a)** Explain how the time dependent Schrödinger wave equation (shown below), is formulated by describing a particle as a wave and ensuring conservation of energy.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\,\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$ **[7]**

(b) If a particle is in a static (time independent) potential, derive the time independent Schrödinger wave equation. **[5]**

(c) A particle is trapped in a potential well given by the thick line below. An allowed eigenfunction for this well is drawn as the thin line. Explain why the wavelength and amplitude of this eigenfunction vary as shown. Also determine the quantum number and parity associated with this state. **[8]**



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SECTION B

13 (a) A particle of total energy E is incident on a potential barrier of potential V_0 located between $x = 0$ and $x = a$. Considering the problem in one dimension:

(i) Starting from the “time independent” Schrödinger equation for the particle wavefunctions in the regions $x < 0$, $0 < x < a$ and $x > a$, write down the general solutions for the allowed eigenfunctions when the barrier height is less than the particle energy ($V_0 < E$). Show that the wavelength inside the barrier is larger than outside. **[8]**

(ii) What are the necessary boundary conditions to be applied to ensure the eigenfunctions are well behaved? **[2]**

(b) The transmission coefficient (ratio between transmitted and incident probability fluxes) for the particle can be obtained by solving the Schrödinger equation, which can be expressed as

$$\mathrm{T} = \left[1 + \frac{\sin^2(ka)}{4\frac{E}{V_0}\left(\frac{E}{V_0}-1\right)}\right]^{-1}, \text{ where } k = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

How does this transmission coefficient, obtained quantum mechanically, differ from what we know from classical mechanics? Discuss briefly the experimental results leading to the so-called ‘Ramsauer and Townsend effect’ and explain the results on the basis of the formula for the transmission coefficient shown above. **[7]**

(c) If the particle energy were less than the height of the potential barrier, i.e. $E < V_0$, sketch a labelled diagram showing the eigenfunctions for the three regions ($x < 0$, $0 < x < a$ and $x > a$). **[3]**

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SECTION B

14 **(a)** In the quantum mechanical description of single-electron systems (such as the hydrogen atom) using Schrödinger equation, why and how has the effect of the finite nuclear mass been taken into account? **[4]**

(b) Due to the spherical symmetry of the Coulomb potential, the Schrödinger equation for the hydrogen atom was solved in a spherical polar coordinate system assuming a general form of the eigenfunction as

$$\psi(r,\theta,\ \phi) = R(r) \times\ \Theta(\theta) \times \Phi(\phi)$$

Using the separation of variables method, the azimuthal part of the Schrödinger wave equation can be written as

$$\frac{d^2\Phi(\phi)}{d\phi^2} + \alpha\,\Phi(\phi) = 0$$

Derive a well-behaved solution for the azimuthal part of the wave equation. Identify the origin of the quantum number associated with this and explain its physical significance. **[10]**

(c) The radial wavefunction for the 2p quantum state of hydrogen is

$$R_{2p} = A\,r\exp\left(-\frac{r}{2a_0}\right)$$

Show that, for this state, the most probable and the average distances of the electron from the proton are $4a_0$ and $5a_0$ respectively.

Hint: $\int_0^\infty x^n \exp\left(-kx\right) dx = \frac{n!}{k^{n+1}}$ **[6]**

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SECTION C

Use a Section C answer book

Answer ONE question from this section

15 **(a)** Explain what is meant by a *microstate* and *macrostate* in statistical mechanics. Include examples appropriate for the classical ideal gas to illustrate your answer. **[4]**

(b) A weakly interacting system of 3 distinguishable particles has total energy 4ε. The allowed single-particle states for the system have equally spaced energies: 0, ε, 2ε, 3ε, 4ε, etc.

(i) Identify all the possible distributions of particles in the system. Present you results in tabular form. **[6]**

(ii) Calculate the average distribution of particles among the single-particle states. **[4]**

(c) Consider a system containing a large number, N, of distinguishable particles that have access to two single-particle states that have energies 0 and E, respectively.

(i) Explain why, at high temperature ($T \gg E/k$) the entropy is given by

$$S = Nk\ln 2$$ **[4]**

(ii) What would the entropy be at low temperatures ($T \ll E/k$)? Explain your answer. **[2]**

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SECTION C

16 **(a)** The *distribution function* for photons at temperature T can be written

$$f(\varepsilon) = \frac{1}{e^{\varepsilon/k_B T} - 1}$$

where ε is photon energy and k_B is the Boltzmann constant. Explain how, and why, this form of the Bose-Einstein distribution function differs from that which applies to other systems of boson, such as ^{4}He particles. **[4]**

(b) The density of states in wavenumber space, $g(k)$, for a three-dimensional system occupying a volume V is given by

$$g(k)\,\mathrm{d}k = G\frac{Vk^2}{2\pi^2}\,\mathrm{d}k$$

where G is a degeneracy factor. Use this relation, together with results from above, to prove that the number of photons with wavelength between λ and $\lambda + \mathrm{d}\lambda$ is given by

$$n(\lambda)\mathrm{d}\lambda = \frac{8\pi V}{\lambda^4\left(e^{hc/\lambda k_B T} - 1\right)}\mathrm{d}\lambda$$ **[8]**

(c) **(i)** Show that, for long wavelengths, $n(\lambda) \propto \lambda^{-3}$. **[3]**

(ii) Radiation in the atmospheres of stars is well described as an equilibrium photon gas. One way to estimate the temperature of a star is by measuring its spectrum and comparing the ratio of the number of photons emitted at different wavelengths to the predictions of the result in part (b). Explain why this method will not work well if measurements are only made at λ = 600nm and 800nm and the star is hotter than around $T = 20{,}000$ K. **[5]**

END OF EXAMINATION