PHY2006 Assignment 1 – Ordinary Differential Equations Revision

Deadline for Submission 6pm, Monday 27 Sept 2021

1. A red dwarf star has internal heat energy U, heat capacity C, and radius R . The power radiated by a blackbody at a temperature T per unit surface area is σT^4 (Stefan-Boltzmann law, $\sigma = 5.67 \times 10^{-8}$ $Wm^{-2}K^{-4}$).

(a) Using the definition for heat capacity in thermodynamics, show/justify that the temperature of the star is governed by the following 1st order ODE

$$\frac{dT}{dt} = -\frac{4\pi R^2 \sigma}{C} T^4$$

[10]

(b) Obtain a solution to this equation if the initial temperature is T_0. **[20]**

(c) The nearest star to the Sun, Proxima Centauri, is a red dwarf star with parameters $T_0 = 3000$ K, $R = 1.0 \times 10^8$ m, $C = 1.0 \times 10^{35}$ JK^{-1}. How long will it take to reach a temperature of 300K. **[5]**

2. By substituting a trial solution $u = e^{mx}$ into the following homogeneous 2nd order, linear ODE, obtain an expression for the general solution

$$5\frac{d^2u}{dx^2} + 4\frac{du}{dx} + 4u = 0$$

[20]

3. A capacitor C, inductor L, resistor R are connected in series in a closed circuit.

(a) Using Kirchhov's 2nd law, show that the charge Q on the capacitor is described by

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$

[15]

(b) Obtain a general solution to this equation when $C < \frac{4L}{R^2}$ and hence describe the current flow in the circuit as a function of time.

[30]

Extra Question

Solve the following 2nd order differential equation using a power series solution.

$$x\frac{d^2u}{dx^2} + u = 0$$

$$u(x) = \sum_{n=0}^{\infty} a_n x^n$$

Determine the coefficients a_n up to $n = 4$.