PHY20003 ASTROPHYSICS

Lecture 15 Stellar Distances and Magnitudes

15.1 Trigonometric Parallax

The parallax is the shift in the position of an object when the position of the observer changes over a given distance. The symbol π is used for the parallax.

As the Earth is moving around the Sun, we can use the Earth's motion to measure the position of a star at two different positions along the Earth's orbit, and we can get a baseline equal to twice the Earth-Sun distance, *a*, to measure the distance between the Sun and the star, *d*. The parallax in radians is given by

$$\pi = \frac{a}{d} \tag{15.1}$$

One radian is equal to 57.296° which in turn is equal to 206,265 arcsec.

Thus when the parallax π = 1 arcsec, the distance is equal to 206,265 AU. This distance is called a parsec (pc). Therefore equation 15.1 can be rewritten as:

$$\pi(\text{arcsec}) = \frac{1}{d(pc)} \tag{15.2}$$

Using telescopes from the ground, it is typically possible to measure parallaxes to π ≈ 0.02 arcsec, and therefore it can only determine distances to *d* ≈ 50 pc.

The European Space Agency's *Hipparcos* mission performed the astrometry of 120,000 stars to an accuracy of ≈ 0.001 arcsec (1 mas). Distance measurements were therefore limited to stars within ≈ 1,000 Pc (1 kPc).

ESA's *GAIA* mission (which launched in December 2013) has achieved and astrometric accuracy of $\approx 10^{-5}$ arcsec (10 microarcseconds), and has determined the distances of $\approx 10^{9}$ stars out to 10^{5} pc.

Table 15.1 shows the parallax and distance to some stars in the Solar Neighbourhood.

Table 15.1: Parallax and distance for some stars in the Solar Neighbourhood

Star	π (arcsec)	d (pc)
Proxima Centauri	0.772 ± 0.002	1.30 ± 0.00
Sirius	0.379 ± 0.002	2.64 ± 0.01
51 Peg	0.065 ± 0.002	15.38 ± 0.24

15.2 Magnitudes, Absolute Magnitudes, and the Distance Modulus

If F is the flux measured near the star, and $f(d)$ is the flux measured at a distance d parsecs further away, then

$$f(d) \propto \frac{F}{d^2} \tag{15.3}$$

In astronomy, we mainly use magnitudes, and if we convert equation 15.3 to magnitudes we get:

$$\begin{aligned} m_\lambda &= -2.5\log f_\lambda + C \\ m_\lambda &= -2.5\log F_\lambda + 2.5\log d^2 + C + E \\ m_\lambda &= -2.5\log F_\lambda + 5\log d + C + E \end{aligned}$$

Where E is a constant that is given by a standard distance set by consensus, 10 pc.

The absolute magnitude, M_λ, of a star is the magnitude it would have at a distance of 10 pc.

As the inverse-square law does not depend on wavelength, we can measure the magnitudes through any filter, usually the V-band.

Therefore, $M_V = -2.5\log F_\lambda + C$ at 10 pc, and this is equal to the apparent magnitude at that distance. We therefore find that (for d measured in pc):

$$m_V = M_V = M_V + 5\log 10 + E$$

$$-5\log 10 = E$$

$$E = -5$$

The relation between the apparent magnitude, m_V, and absolute magnitude, M_V is given as

Table 15.2: Distance and distance modulus for some objects

Object	d	DM
Proxima Centauri	1.3 pc	-4.4
Betelgeuse	440 pc	8.2
47 Tucanae	4.0 kpc	13.0
Small Magellanic Cloud	53 kpc	18.6
Andromeda Galaxy (M31)	0.7 Mpc	24.2
Virgo Cluster	8.4 Mpc	29.6

$$m_V = M_V + 5\log d - 5 \tag{15.4}$$

Which can be rewritten as

$$(m_V - M_V) = 5\log d - 5 \tag{15.5}$$

Where $(m_V - M_V)$ is called the distance modulus (DM) of an object. Note that the distance modulus is *independent* of wavelength.

Table 15.2 shows the distances to a number of objects as well as the corresponding distance modulus.

Example: Calculate the absolute magnitude M_V of the Sun, and calculate how bright it would be when viewed from 100 pc away.

15.3 Luminosities and Bolometric Magnitudes

If a star is approximated by a black-body of effective temperature T_{eff}, then the luminosity of the star is given by

$$L = 4\pi\, R_*^2\, \sigma\, T_{eff}^4 \tag{15.6}$$

Note that the luminosity is integrated over all wavelengths. The total flux reaching Earth, l, from a star a distance d pc away is

$$l = \frac{L}{4\pi d^2} = \left(\frac{R_*}{d}\right)^2 \sigma T_{eff}^4 \tag{15.7}$$

The <u>*bolometric magnitude*</u>*, M_{bol}, is defined as the total flux received at the Earth from the star at a distance of 10 pc:*

$$M_{bol} = -2.5 \log l + C \tag{15.8}$$

The zero-point C of the bolometric magnitude can be set using the Sun.

Typically, it is (almost) impossible to measure the flux from a star across all wavelengths. Instead, the flux in one specific waveband (typically V-band) is measured, and subsequently multiplied by a factor to get the total luminosity across all wavelengths:

$$-2.5 \log l = -2.5 \log F_V K = -2.5 \log F_V - 2.5 \log K \tag{15.9}$$

And we find

$$M_{bol} = M_V + BC \tag{15.10}$$

Where BC is a constant called the <u>Bolometric Correction</u>. This constant will depend on the temperature of the star.

For the Sun we have that $M_V = +4.83$ and $BC = -0.08$, so $M_{bol} = +4.75$.

For any other star, if we can measure M_V and the colour (which gives us the temperature), we can calculate BC and therefore determine M_{bol}.

As the bolometric magnitude is directly related to the luminosity of the star, we also find that

$$M_{bol,\odot} - M_{bol,*} = -2.5 \log\left(\frac{L_{\odot}}{L_*}\right) \tag{15.11}$$

Which can be rewritten as

$$\log\left(\frac{L_*}{L_{\odot}}\right) = 1.89 - 0.4 M_{bol,*} \tag{15.12}$$