Nuclear and Radiation Physics

Dr D. Margarone

2021-2022

PRELIMINARIES

A. THE IMPORTANCE OF NUCLEAR PHYSICS

Nuclear physics, i.e. the study of the nucleus of the atom, is a fundamental discipline to understand the universe. Nuclear physics provides key answers both to problems from the infinitely small and the extremely large. Few examples are listed below:

- The nuclear strong force is one of the fundamental forces in nature.
- Nuclear fusion powers the stars, such as the Sun.
- Radioactivity warms the core of the Earth and has implication in the delaying the cooling process of the Earth and protection from solar wind (Earth magnetic field).
- Nuclear power plants, both fission and (potentially) fusion, are important energy production methods.
- Radiotherapy is important in cancer treatment and cancer diagnostics (imaging).
- Carbon dating, based on radioactive decay, is very useful in geology and palaeontology.

B. OBJECTIVES AND OUTCOMES

The objectives and learning outcomes for the nuclear physics part of the module are:

- Knowing the terminology and notation of nuclear physics.
- Understanding physical reasoning behind models of the nucleus.
- Understanding processes such as radioactive decay, fission, fusion.
- Becoming aware of applications of nuclear physics in science, technology, and medicine.

C. SYLLABUS

1. Nuclear Properties
2. The Inter-Nucleon Potential
3. Nuclear Models
4. Nuclear Decays and Reactions
5. Interaction of Radiation with Matter
6. Applications of Nuclear Physics

D. SUGGESTED READING

Eisberg and Resnick: Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles (Wiley 2nd Edition):
Chapter 15; Chapter 16, sections 16-1,16-2,16-3,16-5,16-9 and 16-10
KS Krane: Introductory Nuclear Physics (Wiley):
Chapter 1; Chapter 3; Chapter 4, sections 4.1 and 4.4; Chapter 5, sections 5.1 and 5.2; Chapter 7 up to section 7.6 inclusive; Chapter 8, sections 8.1-8.4; Chapter 9, section 9.1; Chapter 13, sections 13.1-13.3 and sections 13.5,13.6; Chapter 14, Chapters 19 and 20.

1. NUCLEAR PROPERTIES

1.1 General Nuclear Properties

Although *nuclear forces* are known, a comprehensive theory of nuclei able to explain their properties in terms of the nuclear forces acting between their protons and neutrons is not available yet. Therefore, several models are used to explain a certain limited range of nuclear properties, using arguments that do not involve all the details of nuclear forces.

A pronounced difference between experimental studies of atoms and of nuclei arises from the difference between their characteristic energies. The characteristic energy of nuclei is of the order of 1 MeV, while for atoms it is of the order of 1 eV. Thus, nuclei require very special circumstances to be excited because of their very high characteristic energy. In fact, in our environment atomic nuclei typically exist only in their ground state, and most of the interesting nuclear phenomena occur only under special conditions created by accelerating machines, or in certain region of the universe such as the centre of a star.

To a considerable extend, we can describe a nucleus by a relatively small number of parameters: electric charge, radius, binding energy, angular momentum, parity, magnetic dipole and electric quadrupole moments, and characteristic energies of excited states. These are known as *static properties* of nuclei. However, additional nuclear properties exist such as decay, fission and fusion, and are known as *dynamic properties*.

Nuclei are composed of two types of nucleons: protons and neutrons. The *neutron* is an uncharged particle nearly the same mass as the *proton*. A nucleon with mass number A and atomic number Z contains A nucleons of which Z are protons and A - Z are neutrons. Some years before its discovery, Rutherford suggested the existence of a particle having the properties of what we now call neutron. The neutron, being uncharged, was difficult to detect since it does not easily ionize atoms when it passes through matter, and most devices for detecting particles depend on ionization. In 1932 Chadwick succeeded in detecting neutrons emitted from beryllium nuclei when bombarded with α-particles produced from a radioactive source, as illustrated in Fig.1.1. For this measurement he used a Geiger counter behind a layer of paraffin. The neutrons collide with protons in the paraffin and transfer a fraction of their kinetic energy to the protons. The protons then penetrate the Geiger counter, where they are detected with high efficiency since they are charge particles.

The first artificially produced nuclear reaction discovered by Rutherford in 1919 was the following:

$$^{4}He + ^{14}N \rightarrow ^{17}O + ^{1}H \qquad (1.1)$$

A bombarding ^{4}He (α-particle) interacts with a target nucleus ^{14}N to produce a residual nucleus ^{17}O and a product particle ^{1}H (proton). The 7.7 MeV α-particles were produced from a radioactive source. In general, mass and energy are not separately conserved in nuclear reactions. Instead, there is a conservation of total relativistic energy, *E + K = mc²*, where *K* is kinetic energy and *m* is used here for rest mass. Thus, for the general case:

$$a + A \rightarrow B + b \tag{1.2}$$

the conservation of total relativistic energy in the laboratory frame of reference reads:

$$(K_a + m_a c^2) + m_A c^2 = (K_B + m_B c^2) + (K_b + m_b c^2) \tag{1.3}$$

After the reaction takes place, the product particle *b* is emitted at the angle θ, and the residual nucleus *B* recoils in such a way that momentum is conserved.

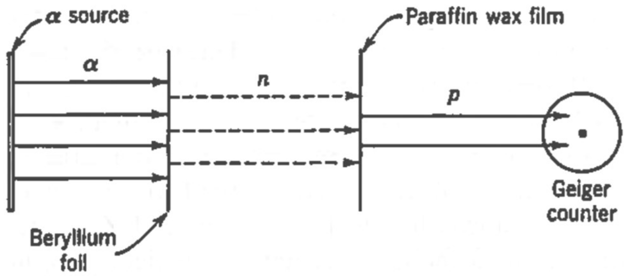

Figure 1.1 A schematic description of the experimental arrangement used Chadwick in the discovery of the neutron (*from Eisberg & Resnick*)

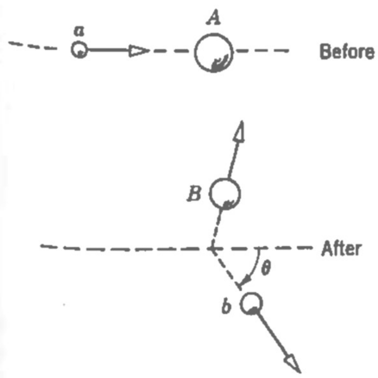

Figure 1.2 Schematic representation of a nuclear reaction (*from Eisberg & Resnick*)

1.2 The Nuclear Radius and Density

Like the radius of an atom, the radius of a nucleus is not a precisely defined quantity. Neither atoms nor nuclei are solid spheres with abrupt boundaries. It is relatively natural to characterize the nuclear shape with two parameters: the *mean radius*, where the density is half its central value, and the *skin thickness*, over which the density drops from near its maximum to near its minimum. However, the density distribution that is measured depends on the kind of experiment that is performed. In some experiment, such as high-energy electron scattering, we measure the Coulomb interaction of charged particles with the nucleus, thus we determine the ***distribution of nuclear charge*** (primarily the distribution of protons but also involving the distribution of neutrons because of their internal constituents). In other experiments, such as Rutherford scattering (with α-particles), we measure the strong nuclear interaction of nuclear particles, thus we determine the distribution of nucleons (protons and neutrons), called ***distribution of nuclear matter***.

The usual means to determine the size and shape of an object is to examine the radiation scattered from it, similarly to a diffraction pattern generated by a plane wave incident into a circular object. To see an object and its details, the wavelength of the radiation must be smaller than the dimensions of the object. For nuclei with diameter of about 10 fm, we require $\lambda < 10\ fm$, i.e. $p > 100\ MeV/c$. Beams of electrons with energies of 0.1-100 MeV can be easily produced with a conventional linear accelerator, and can be analysed with a precise energy spectrometer to select only those electrons that are elastically scattered from the nuclear target. A result from this kind of experiment is shown in Fig.1.3. The first minimum in the diffraction-like pattern can be clearly distinguished. For diffraction by a circular disk of diameter *D*, the first minimum should appear at $\boldsymbol{\theta = \sin^{-1}(1.22/D)}$. Fig.1.4 shows the result of elastic electron scattering from a heavy nucleus, ^{208}Pb. Several minima in the diffraction pattern can be seen. They do not fall to zero like diffraction minima seen with light incident on an opaque disk, because the nucleus does not have a sharp boundary.

The central nuclear charge density is nearly the same for all nuclei. Nucleons do not seem to congregate near the centre of the nucleus, but instead have a fairly constant distribution out to the surface. Thus, the number of nucleons per unit volume is roughly constant:

$$\frac{A}{\frac{4}{3}\pi R^3} \sim constant \tag{1.4}$$

where *R* is the mean nuclear radius. Thus, defining the proportionality constant R_0 gives:

$$\boxed{R = R_0 A^{1/3}} \tag{1.5}$$

where $R_0 \approx 1.2$ fm.

Fig.1.5 shows how diffuse the nuclear surface appears to be. The charge density is roughly constant out to a certain point and then drops relatively slowly to zero. The distance over which this occurs is nearly independent of the size of the nucleus, thus is usually assumed to be constant. The skin thickness parameter can be precisely defined as the distance over which the charge density falls from 90% of its central value to 10% (typically ≈ 2.3 fm).

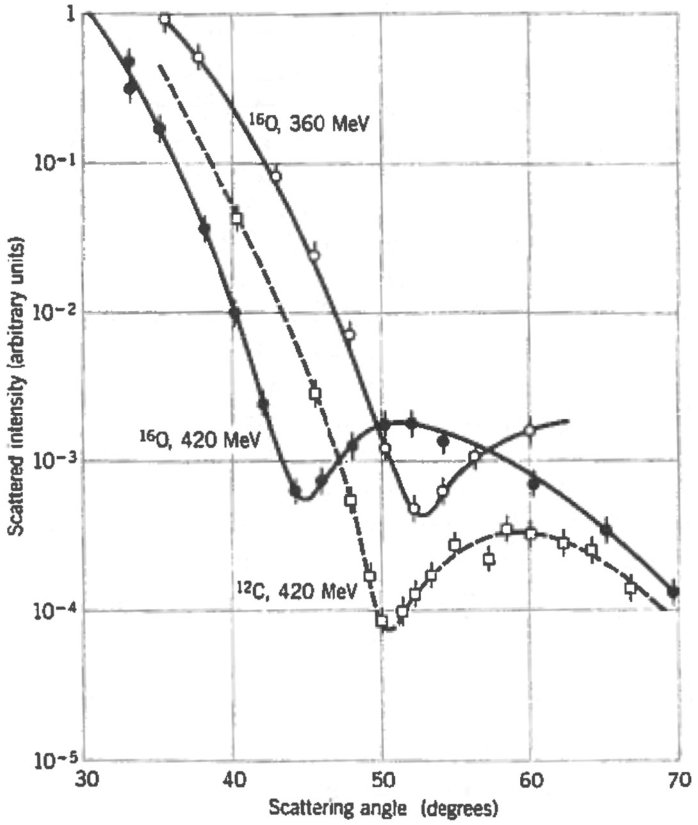

Figure 1.3 Elastic electron scattering from ^{16}O and ^{12}C (*from Krane*)

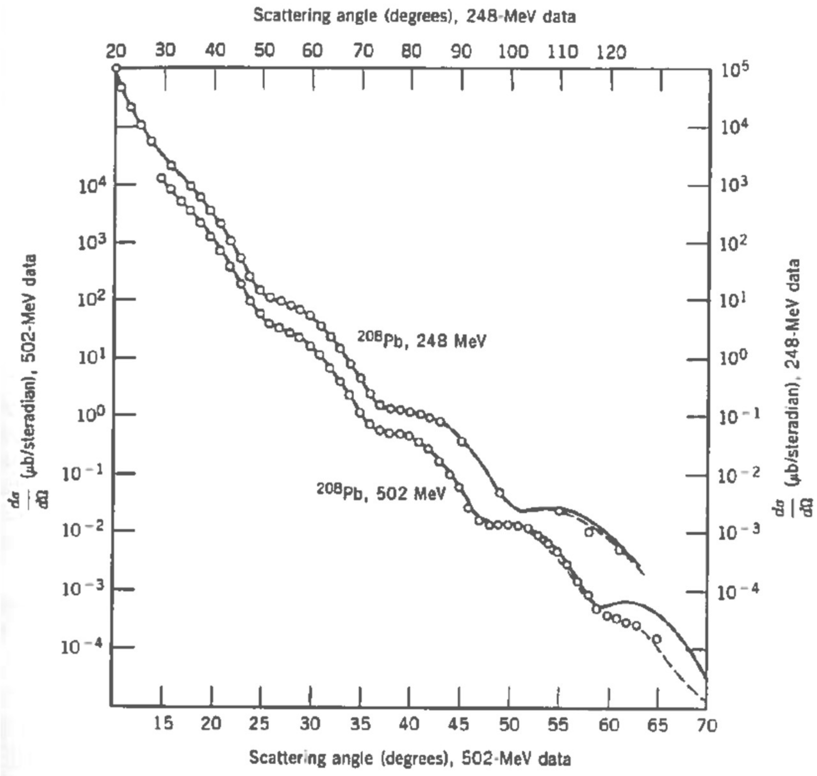

Figure 1.4 Elastic electron scattering from ^{208}Pb (*from Krane*)

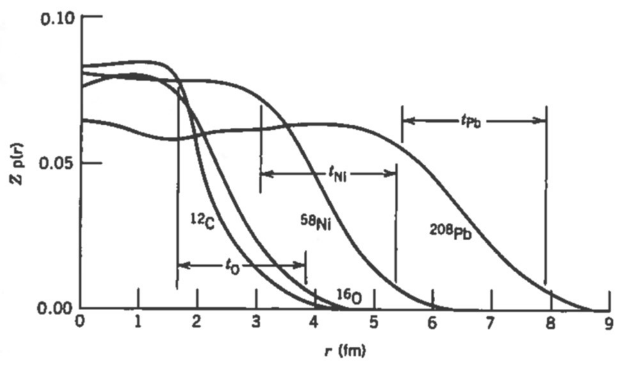

Figure 1.5 Radial charge distribution of several nuclei determined from electron scattering (*from Krane*)

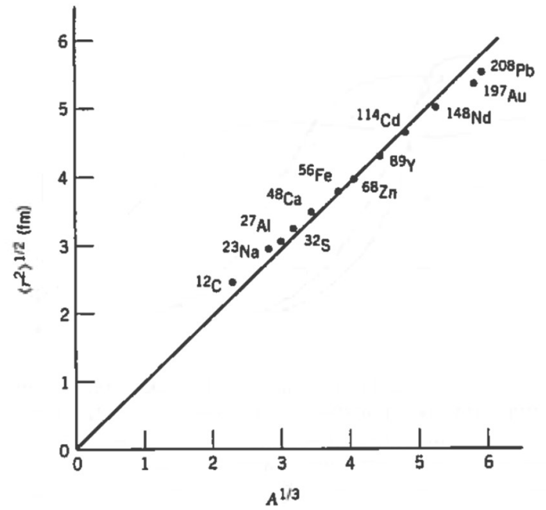

Figure 1.6 Nuclear radius determined from electron scattering experiments (*from Krane*)

Fig.1.6 shows a more quantitative determination of eq. (1.5), based on electron scattering results. The root mean square (rms) radius, $<r^2>^{1/2}$, is deduced directly from the distribution of scattered electrons. The slope of the straight line gives R_0 = 1.23 fm (the error bars are smaller than the size of the experimental points).

An experiment that involves the nuclear force between two nuclei will often provide a measure of the nuclear radius. The determination of the spatial variation of the force between nuclei enables the calculation of the nuclear radii. In this case the radius is characteristic of the nuclear, rather than the Coulomb, force. This radius therefore reflects the distribution of all nucleons in a nucleus, not only the protons. An example of experiments that can be performed to determine the size of the nuclear matter distribution is the scattering of an α-particle (^{4}He nucleus) from a much heavier target of ^{197}Au. If the separation between the two nuclei is always greater than the sum of their radii, each is always beyond the range of the other's nuclear force, thus only the Coulomb force acts (this setup is known as Rutherford scattering). The probability of scattering at a certain angle depends on the energy of the incident particle exactly as predicted by the Rutherford formula, when the energy of the incident particle is below a certain value. As the energy of the incident α-particle is increased, the Coulomb repulsion of the nuclei is overcome, and they may approach close enough to allow the nuclear force to act. In this case the Rutherford formula no longer holds, as shown in Fig.1.7.

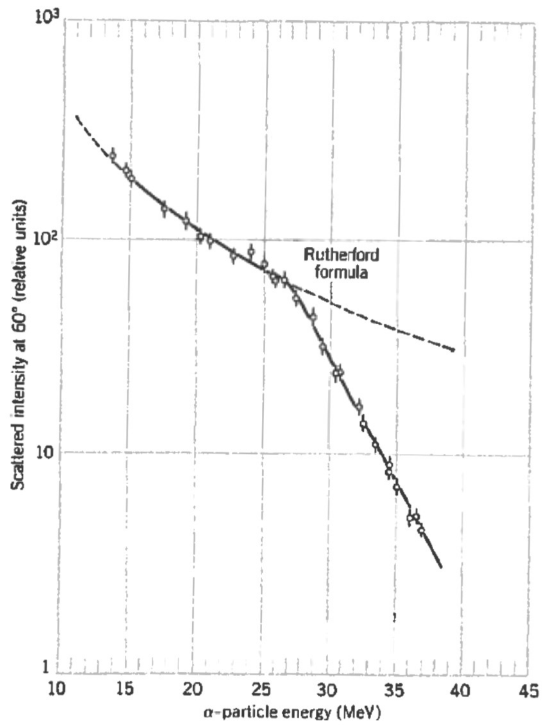

Figure 1.7 Breakdown of the Rutherford scattering formula (*from Krane*)

1.3 Mass and Abundance of Nuclides

Measured values of the masses and abundances of neutral atoms of various stable and radioactive nuclei are tabulated and available from different sources. Even though we must analyse the energy balance in nuclear reactions and decays using nuclear masses, it is conventional to tabulate the masses of neutral atoms. It may therefore be necessary to correct for the mass and binding energy of the electrons.

As we probe deeper into the constituents of matter, the nuclear binding energy becomes greater in comparison with the rest energy of the bound system. In a simple nucleus, such as deuterium (one proton and one neutron), the binding energy of 2.2 MeV is 1.2×10^{-3} of the total mass energy. The deuterium is weakly bound and thus this number is rather low compared with other nuclei, for which the fraction will be more like 8×10^{-3}. It is therefore not possible to separate a discussion of nuclear mass from a discussion of nuclear binding energy.

To determine the nuclear masses and relatively abundances in a sample of ordinary matter, which even for a pure element may be a mixture of different isotopes, we must have a way to separate the isotopes from one another by their masses. Typically, instruments known as mass spectrograph or mass spectrometers (see Fig.1.8) are used to separate masses with high precision (order of 10^{-6}), and this allowed to map the entire scheme of stable isotopes. All mass spectroscopes begin with an ion source, which produces a beam of ionized atoms. Often a vapour of a material under study is bombarded with electrons to produce the ions, or they can be produced as a result of a spark discharge between electrodes coated with the material. Ions emerging from the source have a broad range of velocities, as might be expected for a thermal distribution, and different mass range. The next element is a velocity selector, consisting of perpendicular electric (E) and magnetic (B) fields. The E field would exert a force qE that would tend to divert the ions upward (see Fig.1.8); the B field would exert a downward force qvB. Ions pass through undeflected if the forces cancel, thus:

$$qE = qvB \tag{1.6}$$

$$v = \frac{E}{B} \tag{1.7}$$

The final element is a momentum selector, which is essentially a uniform magnetic field that bends the beam into a circular path with radius r determined by the momentum:

$$mv = qBr \tag{1.8}$$

$$r = \frac{mv}{qB} \tag{1.9}$$

Since q, B, and v are uniquely determined, each different mass m appears at a particular r.

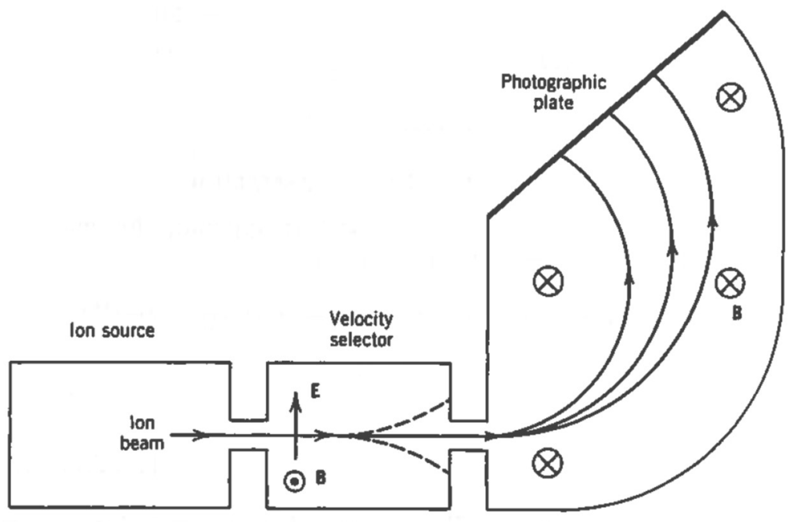

Figure 1.8 Schematic diagram of a mass spectrograph (*from Krane*)

Mass spectrometry also allows us to measure the relative abundances of the various isotopes of an element. In fact, as we scan the mass range by varying E or B, we can measure the current passing through an exit slit, thus we can reconstruct a mass-spectrum with different peaks corresponding to different masses. A two-dimensional graph of isotopes of the elements, in which one axis represents the number of neutrons (symbol N) and the other represents the number of protons (atomic number, symbol Z) in the atomic nucleus, is known as Segrè chart (see Fig.1.9). The distribution of stable nuclei for even and odd $A/N/Z$, where $A = N + Z$, is shown in Table 1.1. At high Z we have $N > Z$ such that $Z/A \sim 2/5$.

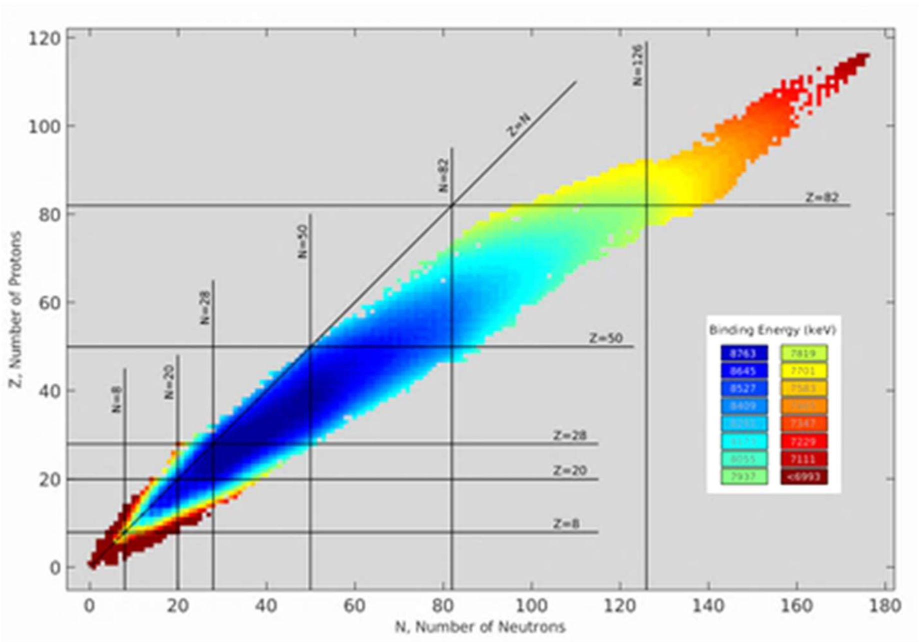

Figure 1.9 The Segrè chart: two-dimensional graph of isotopes of the elements, in which one axis represents the number of neutrons and the other represents the number of protons in the atomic nucleus

A	N	Z	Number of Stable Nuclei
Even	Even	Even	166
	Odd	Odd	8
Odd	Even	Odd	57
	Odd	Even	53

Table 1.1 The distribution of stable nuclei (*from Eisberg & Resnick*)

1.4 Nuclear Binding Energy

The mass energy $m_N c^2$ of a certain nuclide is its atomic mass energy $m_A c^2$ less the total mass energy of Z electrons and the total electronic binding energy:

$$m_{Nucl}c^2 = m_A c^2 - Z m_e c^2 + \sum_{i=1}^{Z} B_i \quad (1.10)$$

where B_i is the binding energy of the i^{th} electron. Electronic binding energies are of the order of 10-100 keV in heavy atoms, while atomic mass energies are of the order *A* x 1 GeV, thus to a precision of about 10^{-6} we can neglect the last term of eq. (1.10). Furthermore, since in nuclear physics we usually work with "differences" in mass energies, the effect of electron binding energies tends to cancel in such differences.

The *binding energy B* of a nucleus is the difference in mass energy between a nucleus ${}^{A}_{Z}X_N$ and its constituents *Z* protons and *N* neutrons:

$$B = \{Zm_p + Nm_n - [m({}^{A}X) - Zm_e]\}c^2 \tag{1.11}$$

where we have dropped the subscript from m_A since from now on, we will be dealing with atomic masses. Proton and neutron masses are similar but not identical. The masses are often expressed in MeV and this is shorthand for the mass-energy, which is given by mc^2. Grouping the *Z* proton and electron masses into *Z* neutral hydrogen atoms, we can rewrite eq. (1.11) as:

$$\boxed{B = [Zm({}^{1}H) + Nm_n - m({}^{A}X)]c^2} \tag{1.12}$$

With the masses generally given in atomic mass units, it is convenient to include the unit conversion factor in $c^2 = 931.50\ MeV/u$.

We occasionally find atomic mass tables in in which, rather than $m({}^{A}X)$, what is given is the mass defect $\Delta = (m - A)c^2$. Thus, given the mass defect it is possible to use eq. (1.12) to deduce the atomic mass.

Other interesting and useful properties that are often tabulated are the neutron and proton separation energies. The *neutron separation energy* S_n is the amount of energy that is needed to remove a neutron from a nucleus ${}^{A}_{Z}X_N$, equal to the difference in binding energies between ${}^{A}_{Z}X_N$ and ${}^{A-1}_{Z}X_{N-1}$:

$$\begin{aligned} S_n &= B({}^{A}_{Z}X_N) - B({}^{A-1}_{Z}X_{N-1}) \\ &= [m({}^{A-1}_{Z}X_{N-1}) - m({}^{A}_{Z}X_N) + m_n]c^2 \end{aligned} \tag{1.13}$$

In a similar way we can define the *proton separation energy* S_p as the energy needed to remove a proton:

$$\begin{aligned} S_p &= B({}^{A}_{Z}X_N) - B({}^{A-1}_{Z-1}X_N) \\ &= [m({}^{A-1}_{Z-1}X_N) - m({}^{A}_{Z}X_N) + m({}^{1}H)]c^2 \end{aligned} \tag{1.14}$$

The neutron and proton separation energies are analogous to the ionization energies in atomic physics. They tell us about the binding of the outermost or valence nucleons. Just like the atomic ionization energies, they show evidence for nuclear shell structure that is similar to atomic shell structure. Table 1.2 reports the mass defects and separation energies for some nuclides.

Since the binding energy increases more or less linearly with *A*, it is general practice to show the average binding energy per nucleon, *B/A*, as a function of *A* (see Fig.1.10). The curve is relatively constant except for the very light nuclei. The average binding energy of most nuclei is, to within 10%, about 8 MeV per nucleon. Furthermore, it is important to note that the curve reaches a peak near *A = 60*, where the nuclei are most tightly bound (e.g. maximum stability). This suggests that we can "gain" (i.e. release) energy into two ways: (i) *below A = 60* by assembling lighter nuclei into heavier nuclei, or (ii) *above A = 60* by breaking heavier nuclei into lighter nuclei. In either case we "climb the curve" of binding energy and liberate nuclear energy. The first method is known as *nuclear fusion*, while the second as *nuclear fission*.

Nuclide	Δ (MeV)	S_n (MeV)	S_p (MeV)
^{16}O	−4.737	15.66	12.13
^{17}O	−0.810	4.14	13.78
^{17}F	+1.952	16.81	0.60
^{40}Ca	−34.847	15.64	8.33
^{41}Ca	−35.138	8.36	8.89
^{41}Sc	−28.644	16.19	1.09
^{208}Pb	−21.759	7.37	8.01
^{209}Pb	−17.624	3.94	8.15
^{209}Bi	−18.268	7.46	3.80

Table 1.2 Mass defects and separation energies of some nuclides (*from Krane*)

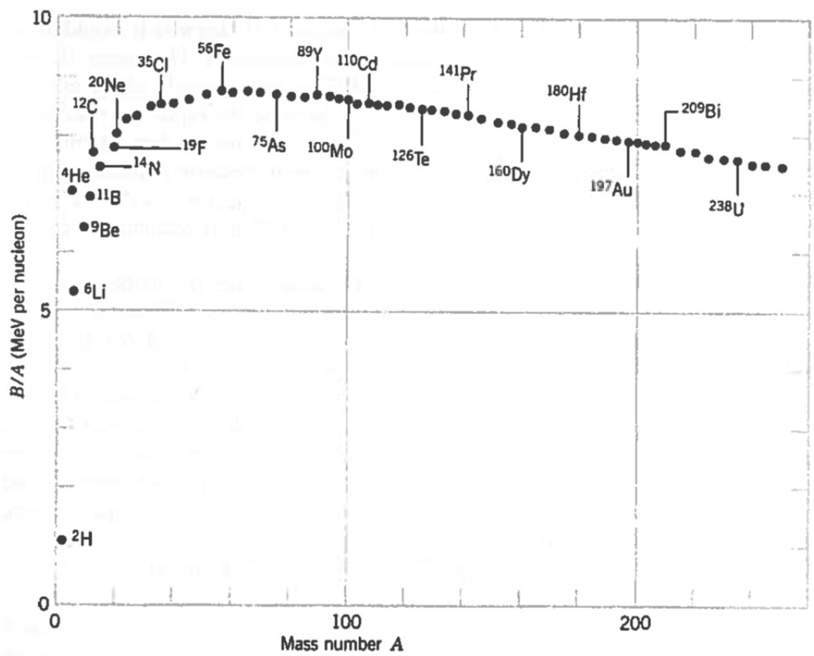

Figure 1.10 The binding energy per nucleon (*from Krane*)

1.5 Nuclear Angular Momentum and Parity

The coupling of orbital angular momentum ***L*** and spin ***S*** gives a total angular momentum ***J*** (***J*** = ***L*** + ***S***). In the quantum mechanical sense, we can therefore label every nucleon with the corresponding quantum numbers *L*, *S*, *J*. The total angular momentum of a nucleus containing *A* nucleons would then be the vector sum of the angular momenta of all the nucleons. This total angular momentum is usually called the *nuclear spin* and is represented by the symbol ***I***. The angular momentum ***I*** has all the usual properties of quantum mechanical angular momentum vectors: $\boldsymbol{I}^2 = \hbar I(I+1)$ and $I_z = m\hbar$ $(m = -I, \ldots, +I)$. For many applications involving angular momentum, the nucleus behaves as if it were a single entity with an angular momentum of *I*. To avoid confusion, we will use *I* to denote the nuclear spin (total angular momentum of a nucleus containing *A* nucleons) and *J* to represent the total angular momentum of a single nucleon. It will often be the case that a single valence particle determines all the nuclear properties, thus in that case *I* = *J*. In other cases, it may be necessary to consider two valence particles, thus $\boldsymbol{I} = \boldsymbol{J_1} + \boldsymbol{J_2}$, and several different resultant values of *I* may be possible.

One important restriction of the allowed values of *I* comes from considering the possible z components of the total angular momentum of the individual nucleons. Each *J* must be half-integral (*1/2, 3/2, 5/2,* ...) and thus its only possible z-components are likewise half-integral ($\pm 1/2\hbar$, $\pm 3/2\hbar$, $\pm 5/2\hbar$, …). If we have an even number of nucleons, there will be an even number of half-integral components, with the result that the z-component of the total *I* can take only integral values. This requires that *I* itself be an integer. If the number of nucleons is odd, the total z-component must be half-integral and so must the total *I*. Therefore, the following rules are valid:

odd-A nuclei → *I* half-integer

even-A nuclei → *I* integer

The measured values of the nuclear spin can tell us a great deal about the nuclear structure. For example, of the hundreds of known (stable and radioactive) even-Z, even-N nuclei, all have spin-0 ground states. This is evidence for a nuclear pairing: the nucleons couple together in spin-0 pairs, giving a total *I* of zero. Consequently, the ground state spin of an odd-A nucleus must be equal to the *J* of the odd proton or neutron. Therefore, along with the nuclear spin, the *parity*, π, is also used to label nuclear states. The parity can take either + (even) or – (odd) values. Like the spin *I*, we regard the parity π as an "overall" property of the whole nucleus. It can be directly measured using a variety of techniques of nuclear decays and reactions. It is usually denoted by a *+* or a – superscript to the nuclear spin (e.g. 0^+, 2^-, $½^-$, $5/2^+$). There is no direct theoretical relation between *I* and π, thus for any value of *I*, it is possible to have either $\pi = +$, or $\pi = -$.

1.6 Nuclear Electromagnetic Moments

Much of what we know about nuclear structure comes from studying not the strong nuclear interaction of nuclei with their surroundings, but instead the much weaker electromagnetic interaction. In fact, the strong nuclear interaction establishes the distribution and motion of nucleons in a nucleus, and we probe that distribution with the electromagnetic interaction. In doing so, we can use electromagnetic fields that have less effect on the motion of nucleons than the strong force of the nuclear environment, thus our measurements do not seriously distort the object we are trying to measure.

Electromagnetic theory gives us a recipe for calculating the various electric and magnetic multiple moments, and the same recipe can be carried over into the nuclear regime using quantum mechanics, by treating the multiple moments in operator form and calculating their expected values for various nuclear states. These expected values can then be directly compared with the experimental values measured in the laboratory. The simplest distributions of charges and currents give the lowest order multiple fields. A spherical charge distribution gives only a monopole (Coulomb) field, and the higher order fields all vanish. A circular current loop gives only a magnetic dipole field. In general, if a simple, symmetric structure is possible, then nuclei tend to acquire that structure. It is therefore usually necessary to measure or calculate only the lowest order multiple moments to characterize the electromagnetic properties of the nucleus.

A restriction on the multiple moments comes from the symmetry of the nucleus and is directly related to the parity of the nuclear states. Each electromagnetic multiple moment has a parity. The parity of electric moments is $(-1)^L$, where L is the order of the moment ($L = 0$ for monopole, $L = 1$ for dipole, $L = 2$ for quadrupole, etc.). For magnetic moments the parity is $(-1)^{L+1}$. All odd-parity static multiple moments must vanish (electric dipole, magnetic quadrupole, electric octupole, etc.).

The monopole electric moment is just the net nuclear charge Ze. The next nonvanishing moment is the ***magnetic dipole moment*** μ. A circular loop carrying current i and enclosing area A has a magnetic moment of magnitude $|\mu| = iA$. If the current is caused by a charge e, moving with speed v in a circle of radius r (with period $2\pi r/v$), then:

$$|\boldsymbol{\mu}| = \frac{e}{2\pi r/v}\pi r^2 = \frac{evr}{2} = \frac{e}{2m}|\boldsymbol{L}| \qquad (1.15)$$

where $|\boldsymbol{L}|$ is the classical angular momentum mvr. In quantum mechanics we define the observable magnetic moment to correspond to the direction of greatest component of $\boldsymbol{L}$, thus we can take eq. (1.15) directly into the quantum regime by replacing $\boldsymbol{L}$ with the expectation value relative to the axis where it has maximum projection, which is $m_L\hbar$ with $m_L = + L$. Thus:

$$\boxed{\mu = \frac{e\hbar}{2m} L} \tag{1.16}$$

where now L is the angular momentum quantum number of the orbit.

The quantity $e\hbar/2m$ is called a "magneton". For atomic motion we use the electron mass and obtain the Bohr magneton, $\mu_B = 5.7884x10^{-5}\ eV/T$. Putting in the proton mass we have the nuclear magneton, $\mu_N = 3.1525x10^{-8}\ eV/T$. Note that $\mu_N \ll \mu_B$ owing to the different in the masses, thus under most circumstances atomic magnetism has much larger effect than nuclear magnetism.

Protons and neutrons, like electrons, also have intrinsic (or spin) magnetic moments. The spin quantum number is indicated as S, where $S = ½$ for protons, neutrons, and electrons. Interestingly, proton and neutron both have finite magnetic dipole:

Proton: $\mu_p = +2.79\ \mu_N$

Neutron: $\mu_n = -1.91\ \mu_N$

Thus, the uncharged neutron has a nonzero magnetic moment! Here is our first evidence that the nucleons are not elementary point particles like the electron, but have an internal structure, which must be due to charged particles in motion, whose resulting currents give the observed spin magnetic moments.

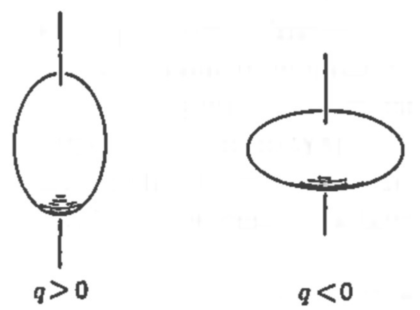

Figure 1.11 Prolate (left) and oblate (right) charge distributions for nucleons (*from Eisberg and Resnick*)

In nuclei, the pairing force favours the coupling of nucleons so that their orbitals angular momentum and spin angular momentum each add to zero. Thus, the pair nucleons do not contribute to the magnetic moment, and we need only consider a few valence nucleons. If this were not so, we might expect on statistical grounds alone to see a few heavy nuclei with a very large magnetic moment; however, no nucleon has been observed with a magnetic dipole moment larger than about $6\mu_N$.

The next nonvanishing moment is the ***electric quadrupole moment***, Q_0. For a classical point charge e, this is of the form $e(3z^2 - r^2)$. If the nuclear particle moves with spherical symmetry, then the quadrupole moment vanishes. If the nucleus does not have a spherical shape, then it will have an electric quadrupole moment. Examples of prolate (positive quadrupole moment) and oblate (negative quadrupole moment) charge distributions of nucleons are shown in Fig.1.11. In general, the electric quadrupole moment of a nucleus is expressed as:

$$\boxed{Q_0 = \int \rho(r)(3z^2 - r^2)dV} \tag{1.17}$$

where ρ is the charge density distribution as a function of position *r*. Note that *r* has components in the x, y and z directions; thus, if the distribution is isotropic, the quadrupole moment is zero as $3z^2$ and r^2 average out to zero.

1.7 Nuclear Excited States

Just as we learn about atoms by studying their excited states, we study nuclear structure partially through the properties of nuclear excited states. In fact, in analogy to atomic excited states, nuclear excited states are unstable and decay rapidly to the ground state. In atoms, we make excited states by moving individual electrons to the higher energy orbits, and we can do the same with individual nucleons. Thus, the excited states of the nucleus can reveal something about the orbit of individual nucleons. Nuclei possess both single-particle and collective structure, thus we can produce excited states by adding energy to the core of paired nucleons. This energy can take the form of collective rotation or vibrations of the entire core, or it might even break one of the pairs, thereby adding two additional valence nucleons.

Part of the goal of nuclear spectroscopy is to observe the possible excited states and to measure their properties. The experimental techniques available include radioactive decay and nuclear reaction studies. Among the properties we should like to measure for each excited state there are: energy of excitation, lifetime and modes of decay, spin and parity, magnetic dipole moment, and electric quadrupole moment. A schematic representation of the potential and total energy of a nucleon is shown in Fig.1.12. The potential extends beyond the nuclear mass distribution by about the range of the nuclear force, and then it rapidly goes to zero. A few examples of level schemes showing the excited states of different nuclei are represented in Fig.1.13. Some nuclei, such as ^{209}Bi, show great simplicity, while others, such as ^{182}Ta, show great complexity.

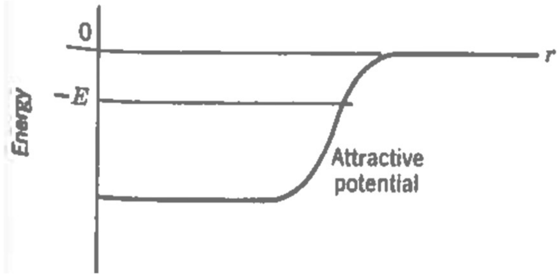

Figure 1.12 Schematic representation of the potential and total energy of a nucleon (*from Eisberg and Resnick*)

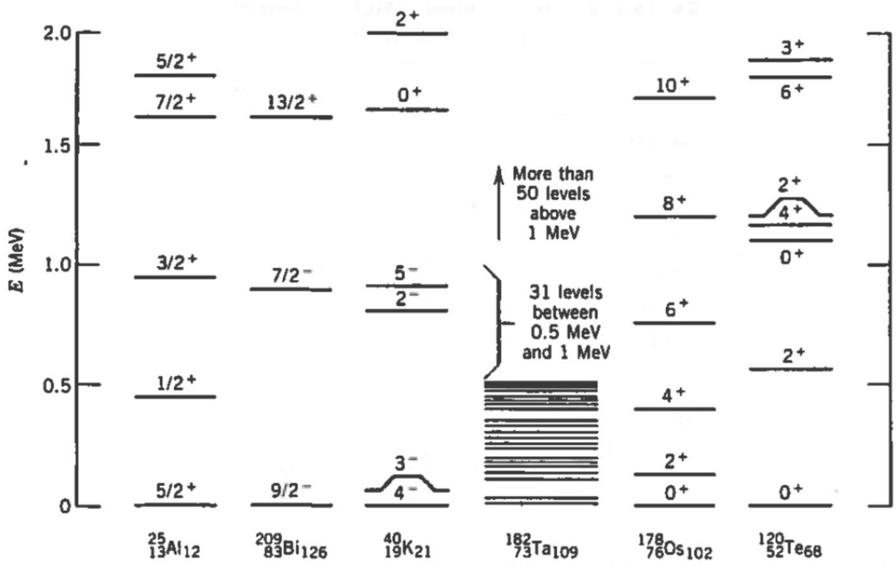

Figure 1.13 Examples of level schemes showing the excited states of different nuclei (*from Krane*)

2. THE INTER-NUCLEON FORCE

Based on intuitive considerations, we can list a few properties expected from the nucleon-nucleon force:

- It is stronger than the Coulomb force at short distances, i.e. the nuclear force can overcome the Coulomb repulsion of protons in the nucleus.
- It is negligible at long distances (order of atomic sizes), i.e. the interaction among nuclei in a molecule is based only on the Coulomb force.
- Some particles (e.g. electrons) do not feel the nuclear force.
- The nucleon-nucleon force is nearly independent of whether the nucleons are neutrons or protons (*charge independence*).
- The nucleon-nucleon force depends on whether the spins are parallel or antiparallel.
- The nucleons are kept at a certain average separation thanks to a repulsive term of the nucleon-nucleon force.
- The nucleon-nucleon force has a noncentral (tensor) component that does not conserve orbital angular momentum.

2.1 The Deuteron

A *deuteron* (^2H nucleus) consists of a neutron and a proton (a neutral atom of ^2H is called *deuterium*). This is the simplest bound state of nucleons; therefore, it is an ideal system for studying the nucleon-nucleon interaction. This problem is equivalent to the hydrogen atom in atomic physics.

The **binding energy** of the deuteron is a very precisely measured quantity, which can be determined using three different methods. Spectroscopy allows directly determining the mass of the deuteron through eq. (1.12):

$$B = \left[m(\,^1H) + m_N - m(\,^2H)\right]c^2 = 2.22463 \pm 0.00004\ MeV$$

We can also determine this binding energy directly by bringing a proton and a neutron together to form ^2H and measuring the energy of the γ-ray photon that is emitted:

$$^1H + n \rightarrow\, ^2H + \gamma$$

The deduced binding energy, which is equal to the observed energy of the photon less a small recoil correction, is *2.224589 ± 0.000002 MeV*, thus in excellent agreement with the mass spectroscopic value. A third method uses the reversed reaction, known as *photodissociation*, in which a γ-ray photon breaks apart a deuteron:

$$\gamma + {}^{2}H \rightarrow {}^{1}H + n$$

The minimum γ-ray energy that accomplishes this process is equal to the binding energy (again corrected for the recoil of the final products). In this case, the observed value is 2.224 ± 0.002 MeV, which is in good agreement with the previous findings. As discussed in Chapter 1, the average binding energy per nucleon is about 8 MeV. Thus, the deuteron is very weakly bound compared to typical nuclei. In fact, there are no excited states of the deuteron but unbound systems consisting of a free proton and neutron.

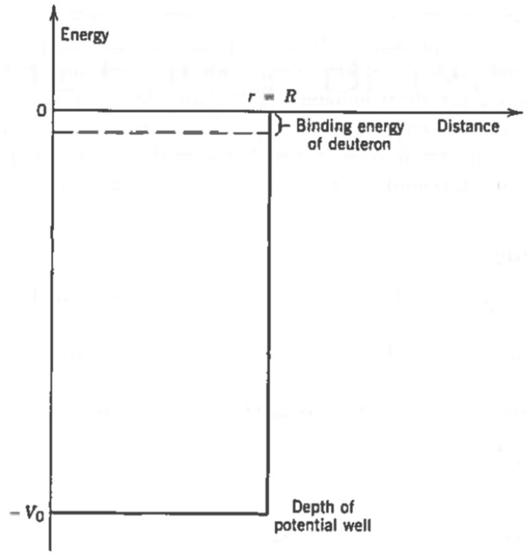

Figure 2.1 The spherical square-well potential

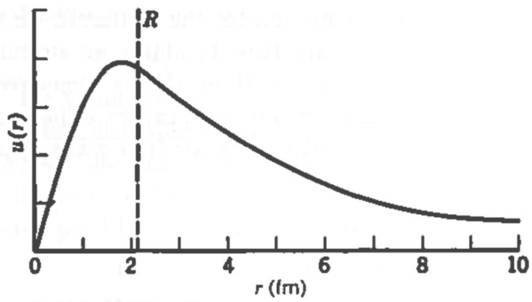

Figure 2.2 The deuteron wave function for R = 2.1 fm (the exponential joins smoothly the sine at r = R)

To simplify the analysis of the deuteron, we will assume the nucleon-nucleon potential as a three-dimensional square well, as shown in Fig.2.1:

$$V(r) = -V_0 \qquad \text{for } r < R$$
$$= 0 \qquad \text{for } r > R \tag{2.1}$$

This is an oversimplification, but it is sufficient for qualitative conclusions. Here *r* represents the separation between the proton and the neutron, and R is a measure of the deuteron diameter.

Furthermore, we will assume that the lowest energy state of the deuteron has a spherical symmetry, i.e. the angular momentum is zero (*L = 0*). The next step is to solve the Schrödinger equation:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi = E\Psi \tag{2.2}$$

by defining the radial part of Ψ as *u(r)/r*, hence:

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V(r)u(r) = Eu(r) \tag{2.3}$$

For $r < R$, the solution to eq. (2.3) can be written as:

$$u(r) = Asink_1r + Bcosk_1r \tag{2.4}$$

with $k_1 = \sqrt{2m(E + V_0/\hbar^2}$.

For r > R, the solution can be written as:

$$u(r) = Ce^{-k_2r} + De^{+k_2r} \tag{2.5}$$

with $k_2 = \sqrt{-2mE/\hbar^2}$ ($E < 0$ for bound states). To keep the wave function finite for $r \to \infty$ we must have $D = 0$, and to keep it finite for $r \to 0$ we must have B = 0 (Ψ depends on *u(r)/r*, hence as $r \to 0$, *u(r)* also must go to zero). Applying the continuity conditions on *u* and *du/dr* at *r = R*, we obtain:

$$k_1cotk_1R = -k_2 \tag{2.6}$$

This equation provides a relation between V_0 and *R*. The deuteron wave function is shown in Fig.2.2. We have mentioned in Chapter 1 that from electron scattering experiments we can estimate the rms charge radius of the deuteron (~ *2.1 fm*), which provides a reasonable estimate for *R*. Solving eq. (2.6) numerically, a nuclear potential V_0 of about 35 MeV is calculated. This is a quite reasonable estimate of the strength of the nucleon-nucleon potential, even in more complex nuclei.

Fig.2.1 shows how close the deuteron is to the top of the well. If the nucleon-nucleon force were just a bit weaker, the deuteron bound state would not exist and we would not be here to discuss it! In fact, the formation of the deuteron from hydrogen is the first step not only in the proton-proton cycle of fusion by which our sun makes its energy, but also in the formation matter from primordial hydrogen that filled the early stages of universe.

The **nuclear spin** (total angular momentum) ***I*** of the deuteron can be expressed as the sum of three components: the individual spins $\boldsymbol{S_n}$ and $\boldsymbol{S_p}$ of the neutron and proton (each equal to ½), and the orbital angular momentum $\boldsymbol{L}$ of the nucleons as they move about their common centre of mass:

$$\boldsymbol{I} = \boldsymbol{S_n} + \boldsymbol{S_p} + \boldsymbol{L} \tag{2.7}$$

The experimentally measured nuclear spin of the deuteron is *I = 1*. Since the neutron and proton spins can be either parallel (for a total of 1) or antiparallel (for a total of zero), there are four ways to couple $\boldsymbol{S_n}$, $\boldsymbol{S_p}$ and $\boldsymbol{L}$ to get a total I of 1:

- $\boldsymbol{S_n}$ and $\boldsymbol{S_p}$ parallel with a $L = 0$
- $\boldsymbol{S_n}$ and $\boldsymbol{S_p}$ antiparallel with a $L = 1$
- $\boldsymbol{S_n}$ and $\boldsymbol{S_p}$ parallel with a $L = 1$
- $\boldsymbol{S_n}$ and $\boldsymbol{S_p}$ parallel with a $L = 2$

The experimentally measured **parity** of the deuteron is *even*. In general, the parity associated with orbital motion is $(-1)^L$. Thus, even parity is obtained for *L = 0* (*s-state*) or *L = 2* (*d-state*), and odd parity for *L = 1* (*p-state*). The observed even parity allows us to eliminate the combinations of spins that include *L = 1*, hence leaving only *L = 0* and *L = 2* as possibilities. The spin and parity of the deuteron are therefore consistent with *L = 0* as assumed earlier, but the possibility of *L = 2* cannot be excluded yet.

If the L = 0 assumption is correct, there should be no orbital contribution to the **magnetic dipole moment** in the deuteron. Thus, we can assume the total magnetic moment to be simply the combination of the neutron and proton magnetic moments:

$$\boldsymbol{\mu} = \boldsymbol{\mu_n} + \boldsymbol{\mu_p} \tag{2.8}$$

If we take the observed magnetic moment to be the *z* component of μ, when the spins have their maximum value (*+ ½ ħ*), the calculated value for μ will be 0.879804 μ_N. This is in good but not quite exact agreement with the observed value (0.8574376 ±0.0000004 μ_N). The small discrepancy can be ascribed to different contributions, such as mesons exchange between the neutron and proton, and the fact that the deuteron wave function should be expressed as a mixture of d-state (*L = 2*) and s-state (*L = 0*), thus the assumption of a pure *L = 0* state is pretty good but not quite exact.

The bare neutron and proton have no **electric quadrupole moment**, therefore any nonzero value for the quadrupole moment of the deuteron must be due to the orbital motion. The pure *L = 0* deuteron wave function would have a vanishing quadrupole moment. However, the experimentally observed electric quadrupole moment is *Q = 0.00288 ± 0.00002 b*, which is obviously very small but not zero.

2.2 Properties of the Nuclear Force

This section provides an overview of the main features of the internucleon force.

*The interaction between two nucleons can be approximated (to lower order) to an **attractive central potential**.* For convenience, we have previously represented this potential with a square-well form, which simplifies the calculations and reproduces the observed data well. The key characteristics of this potential is that it depends only on the internucleon distance *r* and, in fact, we represent this central term as $V_C(r)$. To study $V_C(r)$ experimentally, one would measure the energy dependence of nucleon-nucleon parameters such as scattering phase shifts, and then try to choose the form for $V_C(r)$ that best reproduces those parameters.

*The nucleon-nucleon interaction is strongly **spin dependent***. This observation follows from the failure to observe a singlet bound state of the deuteron, and from the measured differences between the singlet and triplet cross sections. Obviously, the additional term to the potential that accounts for this effect must depend on the spins of the two nucleons S_1 and S_2, but not all possible combinations of S_1 and S_2 are permitted.

*The internucleon potential includes a noncentral term, known as **tensor potential***. Evidence from the tensor force comes primarily from the observed quadrupole moment of the deuteron ground state. In fact, an s-state ($L = 0$) wave function is spherically symmetric, and the corresponding quadrupole moment vanishes. On the other hand, wave functions with mixed *L* states must results in noncentral potentials. The tensor potential must be of the form $V(\mathbf{r})$, instead of $V(r)$. For a single nucleon, the choice of a certain direction in space is obviously arbitrary (nucleons do not distinguish north from south, or east from west). We can express the tensor contribution to the internucleon potential as follows:

$$S_{12} = \frac{3(\boldsymbol{s_1} \cdot \boldsymbol{r})(\boldsymbol{s_2} \cdot \boldsymbol{r})}{r^2} - \boldsymbol{s_1} \cdot \boldsymbol{s_2} \quad (2.9)$$

which averages to zero over all angles.

*The nucleon-nucleon force is **charge symmetric***. This means that the proton-proton interaction is identical to the neutron-neutron interaction, after the correction for the Coulomb force in the proton-proton system. Here the term “charge” refers to the character of the nucleon and not to the electric charge. This is supported by the equality of the pp and nn scattering lengths and effective ranges.

*The nucleon-nucleon force is nearly **charge independent***. This means that the three nuclear forces nn, pp, and pn are nearly identical, again correcting for the pp Coulomb force. We note the existence of “mirror nuclei” which have the same odd number of nucleons but swap a proton for a neutron, e.g. $^{17}O^{8}$ and $^{17}F^{9}$ ($Z = 8$ and $N = 9$, or $Z = 9$ and $N = 8$, respectively). The first few energy levels for these nuclei are shown in Table 2.1. In fact, there is a great similarity that demonstrate the independence of the inter-nucleon force.

^{17}**O**^{8} [MeV]	^{17}**F**^{9} [MeV]
5.08	5.10
4.55	4.69
3.85	3.86
3.06	3.10
0.87	0.50
0	0

Table 2.1 Some energy levels for $^{17}O^{8}$ and $^{17}F^{9}$

The nucleon-nucleon interaction becomes ***repulsive at short distances***. This conclusion follows from qualitative considerations of the nuclear density. In fact, as we add more nucleons, the nucleus grows in such a way that its central density remains roughly constant, and thus something is keeping the nucleons from crowding too closely together. Nucleon-nucleon scattering can be quantitively studied at high particle energies. Fig.2.3 shows the deduced singlet s-wave phase shift for neutron-proton scattering up to 500 MeV (at these energies, phase shifts from higher partial waves are also present). At about 300 MeV, the s-wave phase shift becomes negative, corresponding to a change from an attractive to a repulsive force. To account for the repulsive core, we could choose a square-well form to simplify the calculation as follows:

$$\begin{aligned} V(r) &= +\infty & r &< R_{core} \\ V(r) &= -V_0 & R_{core} \leq\ & r \leq R \\ V(r) &= 0 & r &> R \end{aligned} \tag{2.10}$$

This is schematically represented in Fig.2.4. The value of $R_{core} \sim 0.5$ fm gives agreement with the observed phase shift.

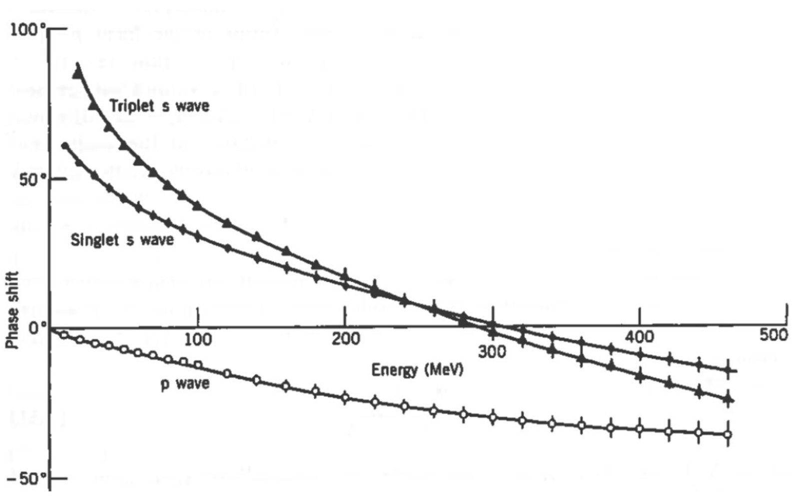

Figure 2.3 The phase shift from neutron-proton scattering at medium energies

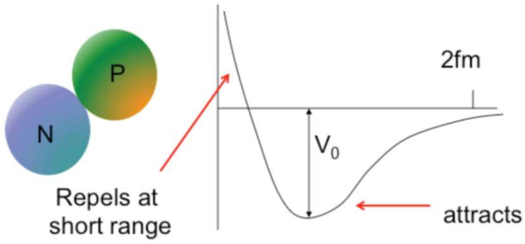

Figure 2.4 Sketch of possible potential well for internucleon force

In summary, the intern-nucleon potential is much more complicated than the Coulomb potential and has several components. This is due to the relatively complex structure of the nucleons that are, in fact, made up of other particles, quarks.

The ***spin-orbit interaction*** is an additional inter-nucleon interaction coming from the observation that scattered neutrons can have their spins aligned, or *polarized*, in certain directions. The polarization of the nucleons in a beam (or in a target) is defined as:

$$P = \frac{N(\uparrow)-N(\downarrow)}{N(\uparrow)+N(\downarrow)} \tag{2.11}$$

where *N(↑)* and *N(↓)* refer to the number of nucleons with their spins pointed up and down, respectively. Values of *P* range from +1 (100% spin-up polarized beam) to -1 (100% spin-down polarized beam). An unpolarized beam (*P = 0*) has equal numbers of nucleons with spins pointing up and down. Let us consider the nucleon scattering experiment in Fig.2.5, in which an unpolarized beam (shown as a mixture of spin-up and spin-down nucleons) is incident on a spin-up target nucleon. The nucleon-nucleon interaction causes the incident spin-up nucleons to be scattered to the left at angle θ, and the incident spin-down nucleons to be scattered to the right at angle -θ.

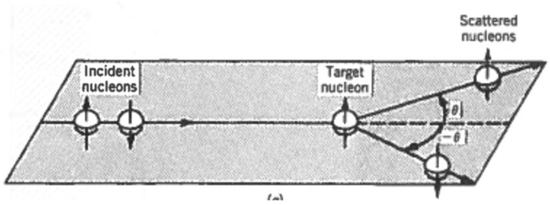

Figure 2.5 An unpolarized beam (mixture of spin-up and spin-down nucleons) is scattered from a target having spin-up.

2.3 The exchange force model

There are two principal arguments in support of the presence of an *exchange force* in nuclei. The first come from the saturation of nuclear forces. The experimental support for the saturation comes from the relatively constant nuclear density and binding energy per nucleon as we go to heavier nuclei. A given nucleon seems to attract only a small number of neighbours, but it also repels at small distance to keep those neighbours from getting too close.

Another argument in favour of the exchange force model comes from the study of np scattering at high energies. Fig.2.6 shows the np differential cross section, where a strong peak is present at forward angles near 0°, corresponding to a small momentum transfer between the projectile and the target. In principle, we can estimate the extent of this forward peak by calculating the theoretical maximum momentum transfer. For small deflection angles, $\sin\theta \approx \theta = \Delta p/p$, where p is the momentum of the incident particle and Δp is the transverse momentum added during the collision. If F is the average force that acts during the collision time Δt, then $\Delta p = F\Delta t$. The force F is $-dV/dr$ and, in a first approximation, the average force should be of the order of V_0/R, where V_0 is the depth of the nucleon-nucleon square-well potential and R is its range. The collision time Δt should be of the order of R/v, where v is the projectile velocity. Thus:

$$\theta = \frac{\Delta p}{p} = \frac{F\Delta t}{p} = \frac{V_0}{pv} = \frac{V_0}{2T} \tag{2.12}$$

where T is the kinetic energy of the projectile. For the energies shown in Fig.2.6, this gives values of θ in the range of 10° or smaller; hence we do not expect to see a peak at 180°! In fact, even if it is tempting to regard this backward peak in the centre of mass frame as the result of a head-on collision, our theoretical estimate above indicates such and explanation is not likely to be correct.

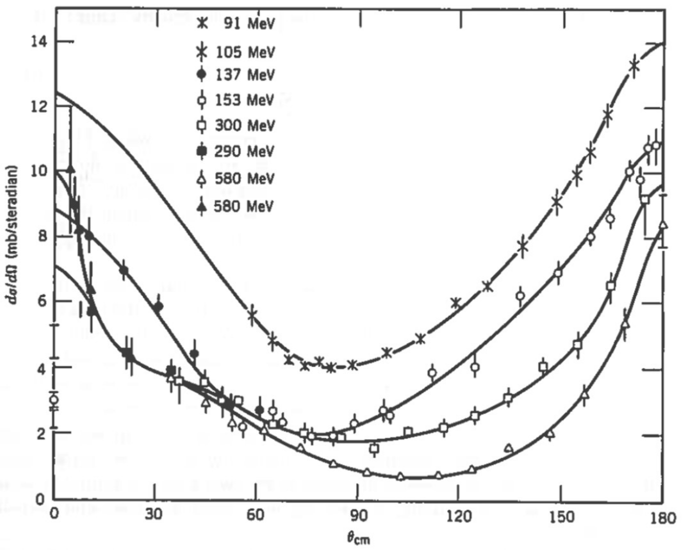

Figure 2.6 The neutron-proton differential cross section at medium energies: the strong forward-scattering peak (near 0°) is expected, while the equally strong backward-scattering peak (near 180°) is evidence for the exchange force.

A more successful explanation can be found in the exchange model if, during the collision, the neutrons and protons exchange places. In other words, the forward-moving neutron becomes a proton, and the backward-moving proton (in the centre-of-mass system) becomes a neutron. The incident nucleon (a neutron) then reappears in the laboratory as a forward moving nucleon (now a proton), thus consistent with our estimate of the small deflection angle in nucleon-nucleon scattering.

In summary, exchange forces explain both the saturation of nuclear forces and the strong backward peak in np scattering. In the first case, "something" is exchanged between nucleons to produce a sort of saturated bond. In the second case, "something" is exchanged between nucleons and actually changes their character. According to quantum field theory, the first object (a nucleon in our case) does not setup a classical field throughout space but instead emits field quanta. The second object can then absorb those field quanta (and potentially reemit them back to the first object). The two objects interact directly with the exchanged field quanta and therefore indirectly with each other. Thus, it is natural to associate the "something" that is exchanged in the nucleon-nucleon interaction with quanta of the nuclear field. For a neutron with spin ½ to turn into a proton with spin ½, it is clear that the exchanged particle must have integral spin (0 or 1) and must carry electric charge. In addition, if we wish to apply the same exchange force concept to nn and pp interactions, there must also be an uncharged variety of the exchanged particle. Furthermore, based on the observed range of the nuclear force, we can estimate the mass of the exchanged particle. Let us assume that a nucleon (N) emits a particle X, and a second nucleon absorbs it:

$$N_1 \rightarrow N_1 + X$$

$$X + N_2 \rightarrow N_2$$

The fact that a nucleon emits a particle of mass $m_x c^2$ and remains a nucleon without violating the conservation of energy can be admitted only if the emission and reabsorption take place within a short enough time, Δt, that makes us unaware of energy conservation violation. This is possible since the limits of our ability to measure a given energy are restricted by the uncertainty principle; hence, if $\Delta t < \hbar/(m_x c^2)$, we will be unaware that energy conservation has been violated by an amount $\Delta E = m_x c^2$. The maximum range of the force is determined by the maximum distance that the particle X can travel in the time Δt. If it moves at speeds of the order of c, then the range R can be at most:

$$\boxed{R = c\Delta t = \frac{\hbar c}{m_x c^2} = \frac{200 MeV \cdot fm}{m_x c^2}} \tag{2.13}$$

Eq. (2.13) gives a useful relation between the mass energy of the exchanged particle and the range of the force. For nuclear forces with a range of about 1 fm, the exchanged particle should have a mass energy of the order of 200 MeV.

Such particles that exist only for a very short time and allow us to violate conservation of energy are known as *virtual particles*. Thus, we can observe the force resulting from the exchange of virtual particles, but we cannot observe the particle themselves during the exchange. The exchanged particles that carry the nuclear force are called ***mesons*** (from the Greek "meso" meaning middle, because the predicted mass was between the masses of the electron and the nucleon). Mesons have been found in particle physics experiments in 1947. They are also found at high altitude where cosmic rays with high energy interact with the atmosphere. The lightest of the meson, known as ***pion***, is responsible for the major portion of the longer range (1-1.5 fm) part of the inter-nucleon potential. To satisfy all the varieties of the exchanges needed in the two-nucleon system, there must be three pions with the electric charges of +1 (π^+), 0 (π^0), and -1 (π^-). The pions have spin 0, odd parity, and rest energies of 139.6 MeV (for $\pi^\pm$) and 135.0 MeV (for π^0), i.e. 273m_e and 264m_e, respectively. At shorter ranges (0.5-1 fm), two pion exchanges is responsible for the nuclear binding. At much shorter ranges (0.25 fm), the exchange of ω *mesons* (mc^2 = 783 MeV) may contribute to the repulsive core, whereas the exchange of ρ *mesons* (mc^2 = 769 MeV) may provide the spin-orbit part of the interaction. The differing masses for the charged and neutral pions may explain the possible small violation of charge independence previously mentioned. The single pion that is exchanged between two identical nucleons must be a π^0:

$$n_1 \rightarrow n_1 + \pi^0 \qquad \pi^0 + n_2 \rightarrow n_2$$

$$p_1 \rightarrow p_1 + \pi^0 \qquad \pi^0 + p_2 \rightarrow p_2$$

The neutron-proton interaction can be carried by charged as well as neutral pions:

$$n_1 \rightarrow n_1 + \pi^0 \qquad \pi^0 + p_2 \rightarrow p_2$$

$$n_1 \rightarrow p_1 + \pi^- \qquad \pi^- + p_2 \rightarrow n_2$$

The exchange force model enjoyed a remarkable success in accounting for the properties of the nucleon-nucleon system. The forces are based on the exchange of virtual mesons, all of which can be produced in the laboratory and studied directly. The pion is the lightest of the mesons and therefore has the longest range. Exploring the neutrons with higher energy probes allows us to study phenomena that are responsible for the finer details of the nuclear structure, such as those that occur only over very short distances. These phenomena are interpreted as arising from the exchange of heavier mesons. On the other hand, particle physicists are able to observe a large variety of mesons, including new particles, from high-energy collisions done with large particle accelerators. Nuclear physicists are then able to choose from this list candidates for the mesons exchanged in various details of the inter-nucleon interaction.