QUEEN'S UNIVERSITY BELFAST

Any calculator, except one with pre-programmable memory, may be used in this examination.

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2004
Electricity, Magnetism and Optics

Duration: 3 hours plus additional 1 hour for upload of work

Friday 28th of May 2021
09:30 AM – 1:30 PM

Examiners: Prof S Matthews, Prof F. Peters
and the internal examiners
Dr S Sim (s.sim@qub.ac.uk)

Answer ALL questions in Section A for 4 marks each.
Answer TWO questions from Section B for 20 marks each.
Answer ONE question from Section C for 20 marks.

If you have any problems or queries, contact the School Office at mpts@qub.ac.uk or 028 9097 1907, and the module coordinator g.sarri@qub.ac.uk

THE QUEEN'S UNIVERSITY OF BELFAST
DEPARTMENT OF PHYSICS AND ASTRONOMY

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}$ ms^{-1}
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1} $\approx 1.26 \times 10^{-6}$ Hm^{-1}
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}
Elementary charge	$e = 1.60 \times 10^{-19}$ C
Electron charge	$= -1.60 \times 10^{-19}$ C
Planck Constant	$h = 6.63 \times 10^{-34}$ Js
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}$ Js
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}$ m^{-1}
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}$ kg $1u = 931$ MeV
1 electron volt (eV)	$= 1.60 \times 10^{-19}$ J
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.67 \times 10^{-27}$ kg
Molar gas constant	$R = 8.31$ JK^{-1}mol^{-1}
Boltzmann constant	$k = 1.38 \times 10^{-23}$ JK^{-1}
Avogadro constant	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Gravitational constant	$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}
Acceleration of free fall on the Earth's surface	$g = 9.81$ ms^{-2}

MATHEMATICAL IDENTITIES

In answering the questions on this paper you may make use of any of the following:

Divergence theorem $\int_V \nabla \cdot \underline{E}\, dV = \oint_S \underline{E} \cdot d\underline{S}$

Stoke's Theorem $\int_S \nabla \times \underline{E} \cdot d\underline{S} = \oint \underline{E} \cdot d\underline{\ell}$

Identities

$$\nabla \times \nabla (Scalar) = \underline{0}$$

$$\nabla \cdot \nabla \times (Vector) = 0$$

$$\nabla \times (\nabla \times \underline{E}) = \nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E}$$

$$\nabla(\psi\, \underline{E}) = \psi\, \nabla \cdot \underline{E} + \underline{E} \cdot \nabla \psi$$

$$\nabla \cdot (\underline{E} \times \underline{H}) = \underline{H} \cdot (\nabla \times \underline{E}) - \underline{E} \cdot (\nabla \times \underline{H})$$

Material Equations $\underline{J} = \sigma \underline{E} \quad \underline{B} = \mu \underline{H} \quad \underline{D} = \varepsilon \underline{E}$

Poynting Vector $\underline{S} = \underline{E} \times \underline{H}$

Trigonometric identities

$$cos(A \pm B) = cos\, A\, cos\, B \mp sin\, A\, sin\, B$$

$$sin(A \pm B) = sin\, A\, cos\, B \pm sin\, B\, cos\, A$$

$$cos\, A + cos\, B = 2\, cos\left(\frac{A+B}{2} \right) cos\left(\frac{A-B}{2} \right)$$

$$sin\, A - sin\, B = 2\, cos\left(\frac{A+B}{2} \right) sin\left(\frac{A-B}{2} \right)$$

SECTION A

Answer ALL 10 questions in this section

Full explanations of your answers are required to attain full marks

1. Derive the expression for the electrostatic field generated by a uniformly charged sphere (volumetric charge density ρ, dielectric constant ε_0) of radius a, outside ($r > a$) and inside ($r < a$) the sphere. Show that the electric field is continuous at $r = a$. **[4]**

2. Consider an infinitely long cylinder of radius a, relative permeability μ_r, and carrying a steady current I, surrounded by vacuum. Calculate the magnetization **M** outside ($r > a$) and inside ($r < a$) the cylinder. Is it continuous across the surface of the cylinder? **[4]**

3. Figure 1 below shows a thin insulating ring holding a charge per unit length λ on its circumference. The ring sits in a uniform magnetic field, which is orthogonal to the plane of the ring. The magnetic field reduces linearly with time t, according to $B = B_o(1 - t/T)$, over a period T.

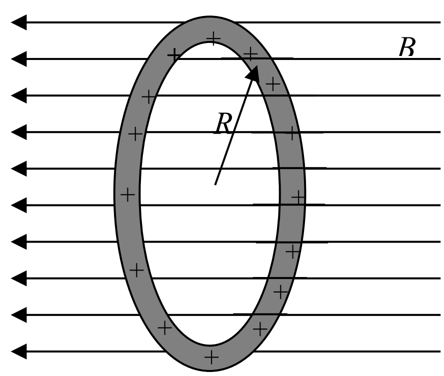


Fig. 1

Re-draw the diagram and indicate the direction of the electric field at the ring circumference. Calculate the amplitude of the electric field. **[4]**

4. Consider the circuit in Figure 2:

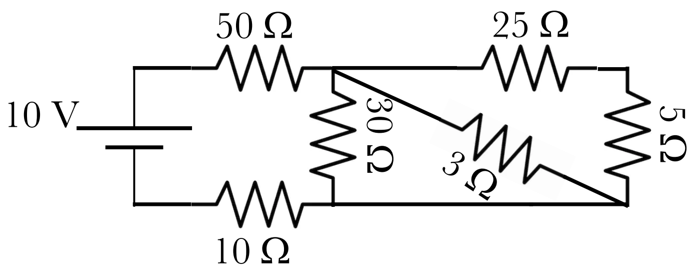


Fig. 2

What is the equivalent resistance of the whole circuit? What is the power dissipated? **[4]**

5. Discuss the implications of the electrostatic field being irrotational. Discuss why a magnetic field does not allow for a scalar potential and define the vector potential **A**. **[4]**

6. Fig. 3 shows the electric field lines generated by three point-like charges, q_1, q_2, and q_3.

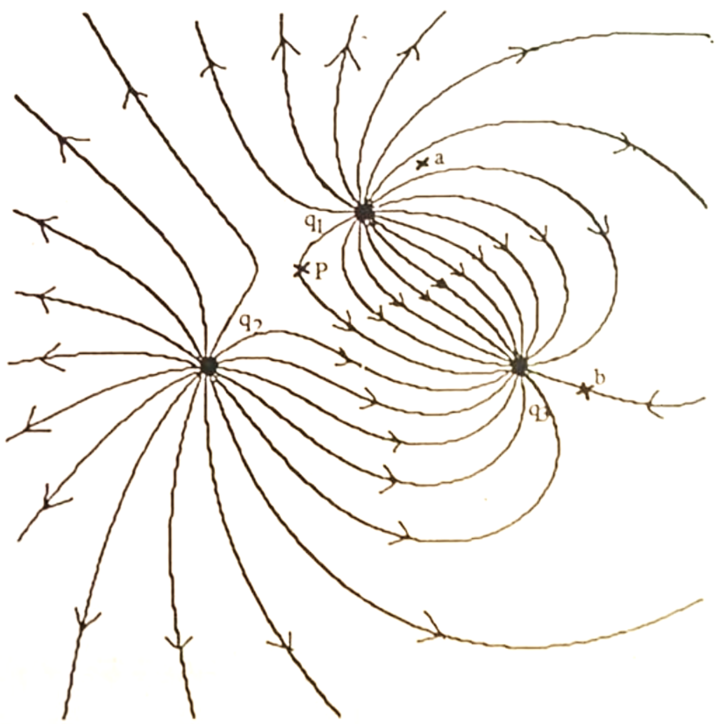


Fig. 3

What do field lines suggest about the signs and approximate relative magnitudes of the three charges? Explain your reasoning.

Point a is the same distance from q_1 as b is from q_3. How does the magnitude of the net electric field at a compare to the magnitude of the net electric field at b? Explain your reasoning. **[4]**

[SECTION A CONTINUED OVERLEAF]

7. Figure 4 shows the cross section of an air-cored solenoid of radius *R*, loop density *n*, and carrying a current *I* and a number of circular lines indicating the vector potential field.

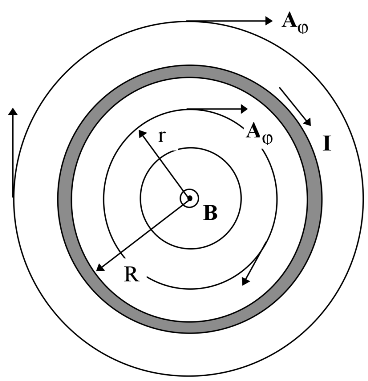


Fig. 4

Show that the vector potential A_φ is given by the following expressions:

$$A_\varphi = \frac{\mu_0 n I r}{2} \qquad r \le R, \qquad\qquad A_\varphi = \frac{\mu_0 n I R^2}{2r} \qquad r \ge R$$

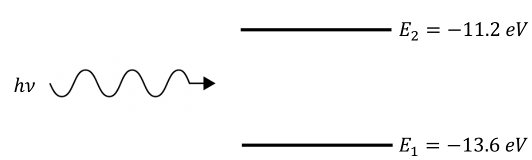
[4]

8. The electric field of a linearly-polarised plane wave is described by $\vec{\mathbf{E}} = \mathrm{E}_0 \sin(kz - \omega t)\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{x}}$ is the unit vector along the x axis. The linearly-polarised plane wave can be decomposed into two circularly-polarised waves. Show the expressions of the two circularly-polarised waves. **[4]**

9. A beam of unpolarised light is incident from a medium with refractive index of $n_1 = 1.5$ to a planar interface of a medium with refractive index of $n_2 = 2.0$. Calculate the incident angle at which the reflected beam will be linearly-polarised and the corresponding refracted angle. **[4]**

10. Figure 5 shows a two-level energy system of atoms. Describe the processes that will happen when the system is excited by incoming photons with energy $h\nu = E_2 - E_1$. At the equilibrium state, what is the population ratio of atoms distributed at the energy level 2 and 1 at room temperature? **[4]**

Fig. 5



[END OF SECTION A]

SECTION B

Answer <u>TWO</u> questions from this section

Full explanations of your answers are required to attain full marks

11. A conducting hollow sphere of radius *R* is filled with a dielectric material of relative permittivity ε_r. A total charge of *Q* is homogeneously distributed over the dielectric material and the total charge on the conducting sphere is -*Q*. The sphere is surrounded by air.

(i) Calculate the electric fields **D**, **E** and **P** inside the dielectric material $(r < R)$. **[6]**

(ii) Calculate **D**, **E** and **P** in air $(r > R)$. **[3]**

(iii) Are **D**, **E** and **P** and their first derivatives in radial distance continuous across the whole system? Explain your reasoning. **[2]**

(iv) Calculate the bound *volume* charge density inside the dielectric. **[6]**

[*You may assume, that in spherical polar co-ordinates* (r,θ,ϕ)

$$\nabla \cdot \underline{P} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 P_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(P_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial P_\phi}{\partial\phi}$$]

(v) Assume the total charge on the conducting sphere is doubled to -*2Q*. Calculate **D**, **E** and **P** in air $(r > R)$. **[3]**

SECTION B

12. Let us assume a magnetized cylinder (magnetization **M**) of radius *R*, height *h*, and mass *m*, aligned along the vertical axis *z*, as in Figure 6.

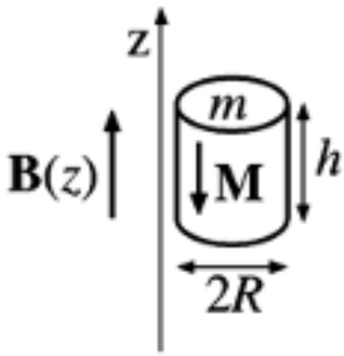


Fig. 6

The cylinder is in the presence of an external magnetic field aligned along *z* of the form (assume *a* and *b* > 0):

$$\vec{B} = \frac{a}{b+z}\hat{z}$$

(i) Calculate the magnetic moment of the magnet $\vec{m}$. By using the fact that the force felt by a magnet in an external magnetic field is given by: $\vec{F} = \nabla\left(\vec{m}\cdot\vec{B}\right)$, calculate the force felt by the magnet. What is the direction of the force? **[8]**

(ii) By assuming that the magnet is also subject to gravity, pointing downwards, calculate the equilibrium position of the magnet. **[8]**

(iii) Is the equilibrium stable or unstable? Justify your answer. **[4]**

SECTION B

13. Half of a rectangular circuit of resistance *R* and sides *a* and 4*a* is immersed in an external constant magnetic field, as in Figure 7.

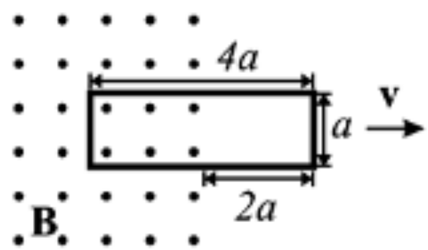


Fig. 7

The circuit is pulled away from the region of magnetic field with a constant velocity *v*.

(i) Show that the temporal derivative of the flux of the magnetic field is given by:

$$\frac{\partial\Phi(B)}{\partial t} = -Bav$$

and calculate the current circulating in the circuit. In what direction does the current flow? **[7]**

(ii) Using the result in part (i), calculate the overall force felt by the circuit as it is pulled away. What is the direction of the force for each arm of the circuit? **[7]**

(iii) What is the work done to extract the circuit completely from the magnetic field region? **[6]**

SECTION B

14. By considering the electric and magnetic energy densities in a volume V, bounded by a surface S, the rate of change of the total electromagnetic energy W is described by the Poynting's Theorem

$$\int_S \underline{E} \times \underline{H} \cdot d\underline{S} + \int_V \underline{E} \cdot \underline{J}\, dV = -\frac{\partial W}{\partial t}$$

(a) Explain, in detail, the meaning of each term in the equation, and define the Poynting vector. What are the SI units of this vector? **[5]**

(b) Figure 8 shows a short section of a long cylindrical conductor of conductivity σ and radius *R* carrying a steady current I uniformly distributed within the conductor.

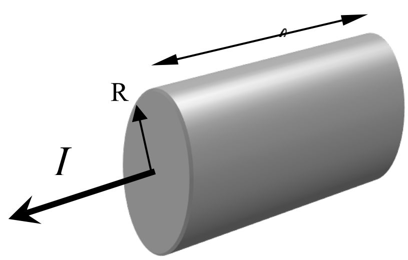


Figure 8

(i) Calculate the amplitude of the magnetic field intensity **H** inside and outside of the cylinder and show its direction. **[4]**

(ii) Calculate the amplitude of the electric field **E** in the cylinder and show its direction. **[4]**

(iii) Indicate the direction of the Poynting vector at the surface of the cylinder. **[1]**

(iv) Show that the flow of electromagnetic energy per unit area per second across the curved surface is given by:

$$\frac{I^2}{2\pi^2 R^3 \sigma}\ .$$ **[4]**

(v) Hence, calculate the total electromagnetic *power* in the system. **[2]**

[END OF SECTION B]

SECTION C

Answer ONE question from this section

Full explanations of your answers are required to attain full marks

15 The reflectivity of a plane wave at the planar interface of two homogeneous media is given by $r_{\parallel} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$ and $r_{\perp} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$ for TM- and TE-polarised light, respectively.

(a) Prove $|r_{\parallel}| \leq 1$ and $|r_{\perp}| \leq 1$, and state the scenario under which the equal conditions are fulfilled. $|r_{\parallel}|$ and $|r_{\perp}|$ denote the absolute magnitudes of $r_{\parallel}$ and $r_{\perp}$. **[4]**

(b) Show that when $\theta_i + \theta_t = 90°$, then $r_{\parallel} = 0$. **[2]**

(c) Show that $r_{\perp}$ cannot be zero. **[2]**

(d) For the case of $n_1 = 2.0$ and $n_2 = 1.5$, sketch the plots of $|r_{\parallel}|$ and $|r_{\perp}|$ as a function of the incident angle θ_i from 0 to 90 degrees. **[4]**

(e) For the conditions indicated in (d), sketch the graphs of phase changes for $r_{\parallel}$ and $r_{\perp}$ as a function of the incident angle θ_i from 0 to 90 degrees. **[4]**

(f) For the conditions indicated in (d), a beam of unpolarised plane wave with intensity I_0 is incident at an angle of $\theta_i = 30°$. Calculate the intensities of the TM- and TE- polarisation components in the transmitted beam. **[4]**

SECTION C

16(a) The diagram below (Figure 9) shows an experimental setup of Faraday effect. Two polarisers are inserted in the optical path of the setup. A magnetic field is applied along the light propagation direction. A mirror is placed after the second polariser. A beam of unpolarised light with intensity I_0 is incident onto the first polariser. It is found when the magnetic field is increased to a value of B_0, rotating polariser 2 to an angle of $10°$ with regard to the first polariser will allow 100% transmission of light. The polarisers are assumed to have 100% absorption of light with polarisation perpendicular to their axes. The mirror is assumed to have 100% reflection.

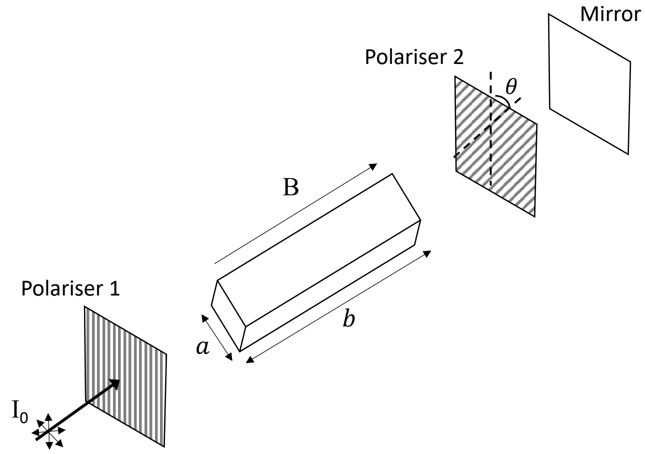


Fig. 9

(i) Keeping the magnetic field at value B_0, when polariser 2 is rotated to an angle of $30°$, what is the intensity of light transmitting through polariser 2? **[2]**

(ii) In the case of (i), what is the intensity of the reflected light transmitting through polariser 1? **[2]**

(iii) Keeping polariser 2 at $30°$, to what value should the magnetic field be increased in order to have a zero transmission of the reflected light through polariser 1? **[2]**

(iv) The above setup is similar to that of Faraday rotator, except that in Faraday rotator the axis of polariser 2 is set to be $45°$ to that of polariser 1. Explain why the axis of polariser 2 is set to $45°$ in a Faraday rotator. **[2]**

(v) Briefly explain the role of magnetic field in Faraday effect. **[2]**

[QUESTION 16 CONTINUED OVERLEAF]

[QUESTION 16 CONTINUED]

(b) Explain how polarised sunglasses can help to reduce specular reflection from the road or water surface, and polarised filters can help to take clear pictures in a hazy day. **[4]**

(c) The diagram below (Figure 10) shows the energy structure of a 4-level laser system.
$E_1 = 0$ eV, $E_2 = 1$ eV, $E_3 = 3$ eV, $E_4 = 4$ eV.

(i) What is the emission wavelength of the laser? **[2]**

(ii) Explain the working processes of the 4-level laser system. **[4]**

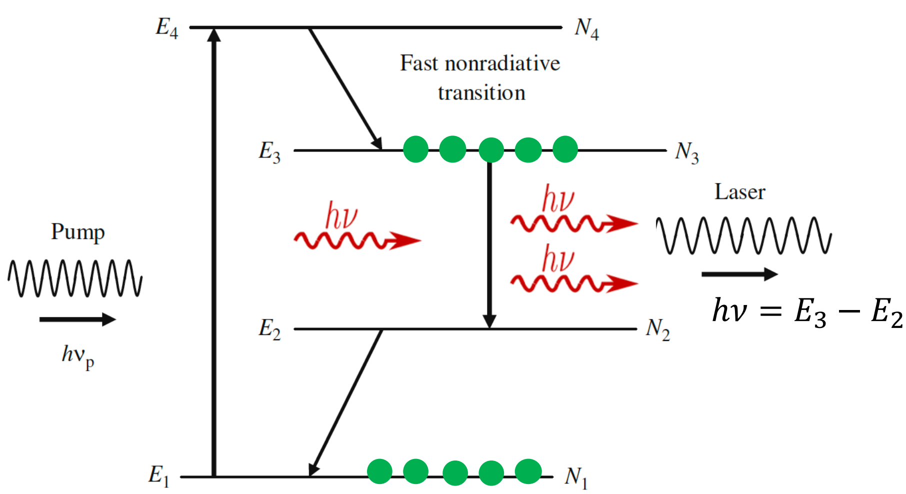


Fig. 10

END OF EXAMINATION