Use lined, single-sided A4 paper
with a black or blue pen.
Write your student number
at the top of every page.

Any non-graphical calculator, except those with pre-programmable memory, may be used in this examination

LEVEL 2
Examination contributing to the Degrees of Bachelor of Science (BSc) and Master in Science (MSci)

PHY2005 - EXAM
Atomic and Nuclear Physics
Tuesday, 12th May 2020, 9.30 AM - 13.30

Examiners: Prof S Matthews, Dr P van der Burgt
and the Internal Examiners
Dr J Greenwood (j.greenwood@qub.ac.uk)

Answer ALL TEN questions in Section A for 4 marks each.
Answer ONE question in Section B for 30 marks.
Answer ONE question in Section C for 30 marks.
You have FOUR hours to complete and upload this paper.

Contact the module coordinator if you have a query at s.sim@qub.ac.uk and copy to mpts@qub.ac.uk

By submitting the work, you are declaring that:

1. The submission is your own original work and no part of it has been submitted for any other assignments;
2. You understand that collusion and plagiarism in an exam are major academic offences, for which a range of penalties may be imposed, as outlined in the Procedures for Dealing with Academic Offences.

QUEEN'S UNIVERSITY BELFAST
SCHOOL OF MATHEMATICS AND PHYSICS

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \text{ms}^{-1}$
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$ $\approx 1.26 \times 10^{-6}\ \text{Hm}^{-1}$
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{Fm}^{-1}$
Elementary charge	$e = 1.60 \times 10^{-19}\ \text{C}$
Electron charge	$= -1.60 \times 10^{-19}\ \text{C}$
Planck Constant	$h = 6.63 \times 10^{-34}\ \text{Js}$
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}\ \text{Js}$
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}\ \text{m}^{-1}$
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}\ \text{kg}$ $1u = 931\ \text{MeV} / \text{c}^2$
1 electron volt (eV)	$= 1.60 \times 10^{-19}\ \text{J}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ \text{kg}$
Mass of proton	$m_p = 1.67 \times 10^{-27}\ \text{kg}$
Mass of neutron	$m_n = 1.67 \times 10^{-27}\ \text{kg}$
Molar gas constant	$R = 8.31\ \text{JK}^{-1}\text{mol}^{-1}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\ \text{JK}^{-1}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$
Acceleration of free fall on the Earth's surface	$g = 9.81\ \text{ms}^{-2}$

SECTION A

Use a section A answer book

Answer <u>ALL</u> 10 questions in this section

Full explanations of your answers are required to attain full marks

1 Briefly discuss the successes and shortcomings of the Bohr model in accounting for atomic spectra. **[4]**

2 Consider a two-electron atom. If the total orbital angular momentum quantum number of the atom is $L = 2$, give all the allowed values for J, the total angular momentum quantum number for the atom. Also give all the allowed values for the M_J quantum number. **[4]**

3 Give the *electron configuration* and the *spectroscopic term* for the ground state of neutral neon. Neon has atomic number $Z = 10$. **[4]**

4 Outline the concept of electron screening in atomic physics and briefly discuss how it affects the energies associated with orbitals in multi-electron atoms. **[4]**

5 For each of the following transitions, indicate if the transition is allowed. If not allowed briefly explain why.

(a) 4p $^2P_{1/2} \rightarrow$ 2p $^2P_{1/2}$

(b) 3d $^2D_{5/2} \rightarrow$ 2p $^2P_{3/2}$

(c) $1s^2\,2s\,2p\ ^3P_0 \rightarrow 1s^2\,2s\,2p\ ^1P_1$ **[4]**

6 A neutron is absorbed by a nucleus of $^{235}_{92}\mathrm{U}$ and fission occurs to produce $^{90}_{36}\mathrm{Kr}$ and $^{144}_{56}\mathrm{Ba}$ and 2 neutrons. Using your knowledge of the binding energy curve, estimate roughly how much binding energy, in MeV, is released in this process. Explain qualitatively why there are usually free neutrons created in such fission decays. **[4]**

7 Explain qualitatively why we do not see nuclei with masses much above A=260. *Hint: consider the competition between surface tension and Coulomb repulsion.* **[4]**

continued …

SECTION A

8 A ^{40}K nucleus decays by β^+ emission with an end-point energy of 1.311 MeV. Calculate the maximum, classically expected, angular momentum transfer to the daughter nucleus in units of $\hbar$. **[4]**

9 A proton of kinetic energy 350 MeV is passing through water with refractive index, n = 1.33. Explain whether or not Cerenkov radiation is emitted. **[4]**

10 Briefly explain the origin of the Bragg peak in stopping of heavy charged particles in matter. **[4]**

[END OF SECTION A]

continued …

SECTION B

Use a Section B answer book

Answer ONE question from this section

11(a) Write down a Bohr-model formula for the energy of an electron orbital with principal quantum number n if the nuclear charge is $+Ze$. Use your expression to calculate the energy of orbitals when $Z = 2$ for

(i) $n = 1$

(ii) $n = 2$

You may assume that the reduced mass can be approximated by the electron mass. **[5]**

(b) Briefly explain why the total energy of a neutral helium atom with a 1s 2s configuration is not accurately given by the sum of the two energies from part (a). Your answer should explain the underlying physics and state whether the true atomic states will have more or less binding energy than suggested by the calculations in part (a). **[4]**

[Helium has atomic number $Z = 2$.]

(c) Six-times ionized oxygen (O^{6+}) is a two-electron system.

(i) The total energy associated with the 1s 2p $^1P_1$ state of O^{6+} is -1037 eV. Note that this energy relates to the total net binding energy of the state (i.e. indicates the energy required to fully unbind both electrons from the system). Using suitably modified versions of your calculations above, or otherwise, use this total energy to estimate the mean separation of the two electrons in the 1s 2p $^1P_1$ state. **[8]**

(ii) Comment on the physical plausibility of the answer you obtained in part (c)(i). **[4]**

[Oxygen has atomic number $Z = 8$.]

(d) Consider each of the four transitions listed below. Give a list of these transitions ordered by wavelength (starting with the shortest wavelength). Carefully justify your answer.

(i) The $n = 2$ to $n = 1$ transition of neutral hydrogen.

(ii) The 1s 2p $^1P_1 \rightarrow 1s^2$ $^1S_0$ transition of neutral helium.

(iii) The $1s^2$ 4d $^2D_{3/2} \rightarrow 1s^2$ 2p $^2P_{3/2}$ transition of singly-ionized Beryllium.

(iv) The 1s 2p $^1P_1 \rightarrow 1s^2$ $^1S_0$ transition of six-times ionized oxygen (O^{6+}). **[9]**

[Beryllium has atomic number $Z = 4$.]

continued …

SECTION B

12(a) The energy of interaction between a magnetic dipole moment, $\underline{\mu}$, and a magnetic field, $\underline{B}$ can be written

$$\Delta E = -\underline{\mu} \cdot \underline{B}$$

Discuss how this relationship can be applied to understand

(i) the *normal Zeeman effect* and

(ii) fine-structure in spectra due to the *spin-orbit interaction*.

You do not need to give detailed calculations but you should comment on the nature of the magnetic dipole moment and magnetic field in both cases and discuss how the interaction gives rise to splitting and/or fine structure in spectral lines, as appropriate. **[14]**

(b) Justify why the energy shift due to the spin-orbit interaction can be expressed in terms of the quantum numbers L, S and J

$$\Delta E = K\,[J(J+1) - L(L+1) - S(S+1)]$$

where K is a proportionality constant (you do <u>not</u> need to derive an expression for K). **[5]**

(c) Transitions between energy levels of the $1s^2\,2s^2\,2p\;^2P$ term and the $1s^2\,2s^2\,3d\;^2D$ term of neutral Boron show fine-structure splitting due to the spin-orbit interaction.

(i) Explain why the spin-orbit interaction splits the $1s^2\,2s^2\,2p\;^2P \rightarrow 1s^2\,2s^2\,3d\;^2D$ transition into three spectral lines. Draw an energy level diagram that illustrates the spin-orbit splitting of the energy levels and indicates the three spectral lines. Your diagram should include values for the J quantum number of each split level. **[6]**

(ii) Adopting $K = 6.3 \times 10^{-4}$ eV for the $1s^2\,2s^2\,2p\;^2P$ levels and $K = 3.6 \times 10^{-6}$ eV for the $1s^2\,2s^2\,3d\;^2D$ levels, estimate the differences in photon frequency between the three fine-structure components. **[5]**

[Boron has atomic number $Z = 5$.]

[END OF SECTION B]

continued …

SECTION C

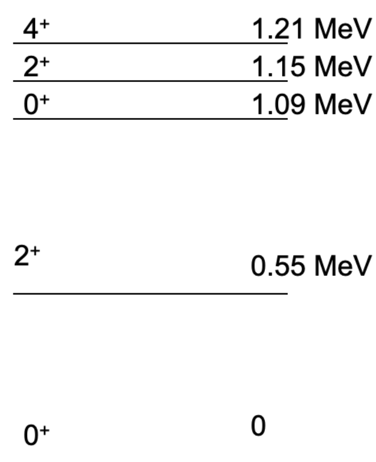
Use a Section C answer book

Answer ONE question from this section

13(a) (i) Explain briefly why at low atomic mass the stable nuclei tend to have similar neutron and proton numbers i.e. N ~ Z. Why does the stability veer towards N > Z for high atomic mass nuclei? **[3]**

(ii) Give a brief explanation of why the experimentally measured nuclear quadrupole moments are not accurately predicted by the single particle shell model, especially when the number of protons and neutrons are far from magic numbers. **[5]**

(iii) The diagram below shows low lying energy levels for ^{114}Cd with approximate energy level values. Is this level structure likely to be due to rotational or vibrational modes of the nucleus? Briefly justify your answer. **[5]**



Q13 CONTINUED ON NEXT PAGE

continued …

SECTION C

Q13 CONTINUED

(b) (i) For a Rutherford scattering experiment α-particles with kinetic energy 5.5 MeV are fired at a gold foil (Z=79). Calculate the classical distance of closest approach between the α-particles and the gold nuclei. State whether we are likely to see effects of the nuclear force, giving calculations and any assumptions made. Estimate the kinetic energy for the α-particles that would be needed before nuclear effects are seen in the scattering, again stating all assumptions. **[7]**

(ii) It is believed that the inter-nucleon force is mediated by *mesons*, one of which, the π^0, has mass of $264.1m_e$ where m_e is the electron mass. Based on this, estimate the range of the inter-nucleon force. **[4]**

(iii) Electrons with kinetic energy of 420 MeV are scattered from target nuclei and the first minimum in the resulting Airy pattern is at 17.7 degrees. Estimate the atomic mass number of the target nuclei, stating all assumptions and equations used. **[6]**

continued …

SECTION C

14(a) (i) The liquid drop model gives the binding energy per nucleon, B as

$$B = a_V - \frac{a_s}{A^{\frac{1}{3}}} - \frac{a_C Z^2}{A^{\frac{4}{3}}} - \frac{a_{as}}{A^2}(A - 2Z)^2 - \frac{\delta(A,Z)}{A^{7/4}}$$

where A is the atomic mass, Z is the atomic number and in MeV units we have;

$$a_V = 15.76$$
$$a_s = 17.81$$
$$a_C = 0.711$$
$$a_{as} = 23.702$$

The value for $\delta(A,Z)$ is zero for even-odd and odd-even nuclei, it is -34 MeV for even-even nuclei and +34 MeV for odd-odd nuclei. Using this model calculate the kinetic energy of an alpha particle emitted by decay of ^{257}Fm (Z=100) to ^{253}Cf. Assuming the ^{253}Cf is in the ground state, estimate how much of the released energy is in recoil energy of the daughter nucleus. Show all working and state assumptions.

[9]

(ii) Explain, briefly, why in β decay there is a spread in the energy of the emitted electron or positron. Define what is meant by a Fermi decay and a Gamow-Teller decay. What is the principal difference between an allowed and a forbidden decay? **[4]**

(iii) Derive an equation for the Q value (kinetic energy released) of β^- decay in terms of atomic masses. State any assumptions made. **[8]**

(b) A nucleus of mass M decays from an excited state by emitting a gamma ray of energy E_γ. Derive a relationship between the gamma-ray energy and the difference in energy for the initial and final state of the nucleus. If the nucleus is ^{72}Se and emits a gamma ray of 1.317 MeV then calculate the recoil energy of the nucleus. State briefly how recoil affects the chances of making a gamma ray laser. **[9]**

END OF EXAMINATION