Nuclear and Radiation Physics (PHY2005)

Lecture 2

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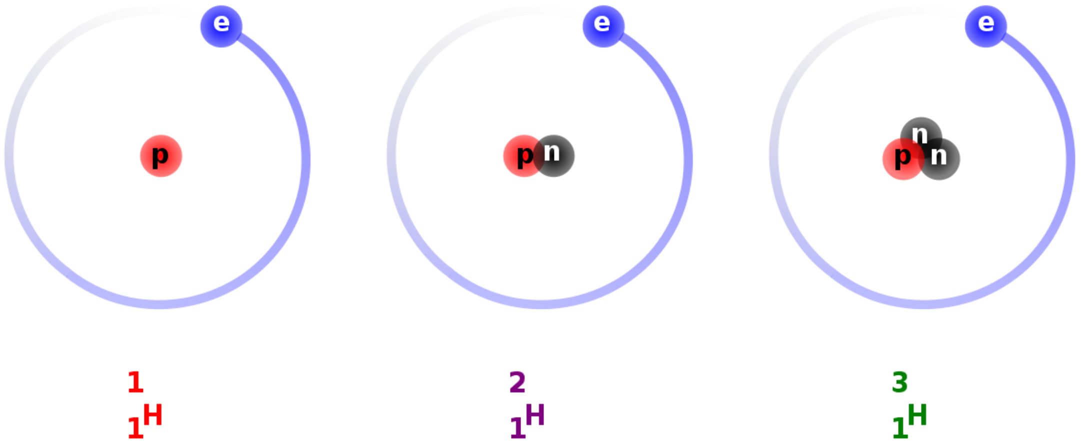

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Recap & Learning Goals

Summary of Lecture 1 *(Chap.1)*

- General nuclear properties
- Nuclear radius and density
- Mass and abundance of nuclei
- Nuclear binding energy

$$\theta = sin^{-1}\left(\frac{1.22}{D}\right)$$

$$R = R_0 A^{1/3}$$

$$B = \left[Zm(\,^1H) + Nm_N - m(\,^AX)\right]c^2$$

Learning goals of of Lecture 2 *(Chap.1-2)*

- Knowing the terminology and notation of nuclear angular momentum and parity
- Understanding physical reasoning behind nuclear electromagnetic moments
- Understanding physical reasoning behind the model of the deuteron

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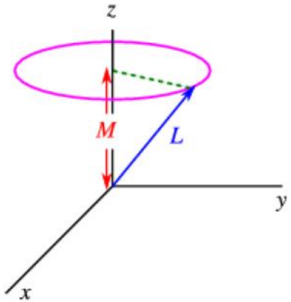
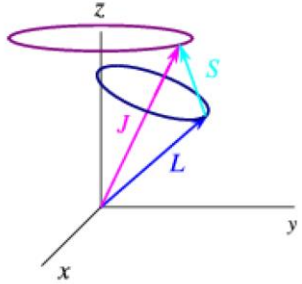
Angular momentum

- ***J*** is the total angular momentum of a single *nucleon* (***J*** = ***L*** + ***S***)
- ***I*** is the total angular momentum of a *nucleus (A nucleons)*
- ***I*** is usually called "**nuclear spin**"
- in each nucleus there are many quantum states *(quantum numbers L, S, J, m), and* $I_z = m\hbar$ *(m = -I, …, +I)*
- the energy of a quantum state is independent of the *m* quantum number *(unless we apply a magnetic field)*
- each *J* must be half-integer *(1/2,3/2, 5/2,…),* thus $J_z = \pm 1/2\hbar, \pm 3/2\hbar\ldots$
- odd-A nuclei → I_Z half-integer → *I* half-integer
- even-A nuclei → I_Z integer → *I* integer



"Quantum precession" of an angular momentum of fixed length L and projection m



Vector addition of spin and orbital angular momentum

Parity

- hundreds of known nuclei with even-Z and even-N have spin-0 ground state *(evidence of nuclear pairing!)*
- nucleons couple together in spin-0 pairs
- the ground state *I* of an odd-A nucleus is equal to the *J* of the unpaired nucleon (proton or neutron)
- **parity** can take either even *(**+**)* or odd *(**-**)* values, and is usually denoted as a superscript *(0^+, 2^-, $1/2^-$, $5/2^+$)*

1. Nuclear Properties

1.6. Electromagnetic Moments

Electric and Magnetic multiple moments

- restriction on multiple moments coming from the symmetry of the nucleus *(parity of the nuclear states)*
- **magnetic dipole moment** *(charge moving in a circle)*, with *L* corresponding to the angular momentum quantum number of the orbit *(μ_N nuclear magneton)*

$$\boxed{\mu = \frac{e\hbar}{2m}L}$$

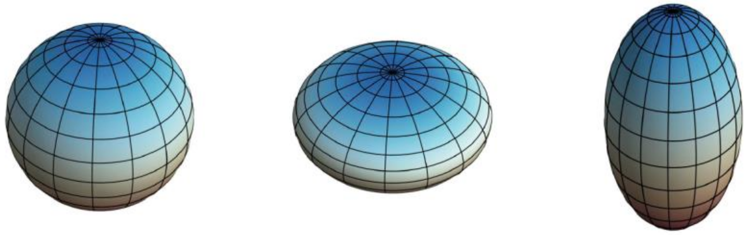
$$\boxed{\mu_{\mathrm{N}} = \frac{e\hbar}{2m} = 3.1x10^{-8}\ \frac{eV}{T}\ \textit{(for a proton)} << \mu_{\mathrm{B}} = 5.8x10^{-5}\frac{eV}{T}\ (for\ an\ electron)}$$

- protons and neutrons (like electrons) have intrinsic spin *(s = 1/2)*, and magnetic dipole:
  $\mu_p = +2.79\ \mu_N$
  $\mu_n = -1.91\ \mu_N$ *(the uncharged neutron has a nonzero magnetic moment!)*
- paired nucleons do not contribute to the magnetic moment *(only valence nucleons to be considered)*

- **electric quadrupole moment** *(nuclei with no spherical shape)*
- prolate nuclei → positive el. quad. mom.
- oblate nuclei → negative el. quad. mom
- for an isotropic distribution → zero

$$\boxed{Q_0 = \int \rho(r)(3z^2 - r^2)dV}$$

$\rho(r)$ is the nuclear charge density distribution



Sphere $Q_I = 0$ | Oblate Spheroid $Q_I < 0$ | Prolate Spheroid $Q_I > 0$

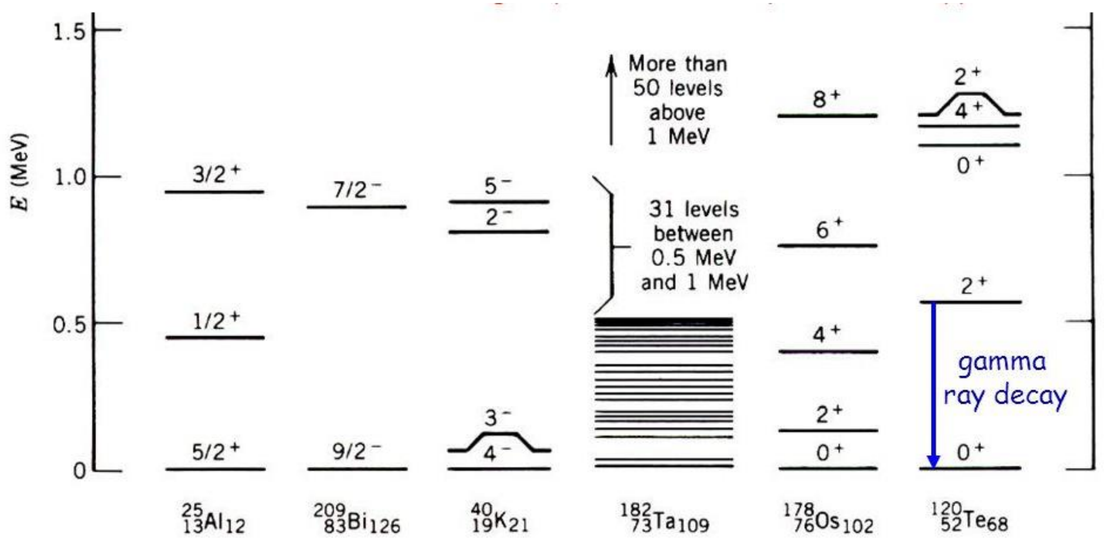
1. Nuclear Properties

1.7. Nuclear Excited States

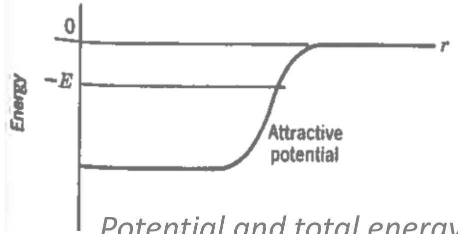
Excited states

- nuclear excited states are <u>unstable</u> and decay "rapidly" to the ground state
- nuclear spectroscopy allows to observe possible excited states *(e.g. radioactive decay or nuclear reactions)*
- … and measure nuclear properties *(e.g. excitation energy, lifetime, spin and parity, electromagnetic moments)*

- <u>nuclear potential</u> *(extending beyond the nuclear mass distribution – range of the nuclear force)* and <u>total energy of a nucleon</u> *(E)*
- nuclei possess both single and collective structures, thus excited states can be produced also by adding energy to the core of paired nucleons



Excited states of different nuclei



Potential and total energy of a nucleon

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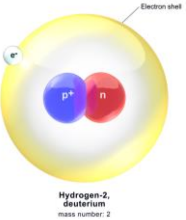
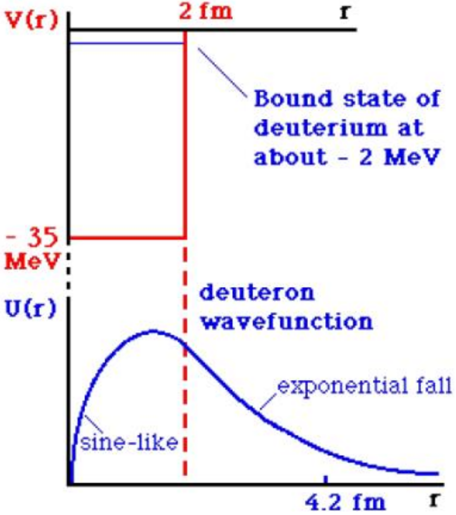


The deuteron (^2H)

- simplest bound system of nucleons (p + n)
- binding energy can be precisely measured by:
  (i) mass spectrometry *(2.22463 ± 0.00004 MeV)*
  (ii) γ-photon energy from $^1H + n \rightarrow {^2H} + \gamma$ *(2.224589 ± 0.000002 MeV)*
  (iii) *photodissociation* $\gamma + {^2H} \rightarrow {^1H} + n$ *(2.224 ± 0.002 MeV)*

- ^2H is very <u>weakly bound</u> *(~1 MeV/u)* compared to heavier nuclei *(~8 MeV/u)*
- ^2H has no excited states *(just free proton and neutron)*
- ^2H inter-nucleon *(p-n)* potential can be roughly approximated as a 3D square well of the following form: $V(r) = -V_0\ (r < R);\ V(r) = 0\ (r > R)$; R: deuteron diameter
- ^2H wave function *(assuming L = 0)*:

$$u(r) = A\sin k_1 r \qquad k_1 = \sqrt{2m(E+V_0/\hbar^2} \qquad r < R$$
$$u(r) = Ce^{-k_2 r} \qquad k_2 = \sqrt{-2mE/\hbar^2} \qquad r > R$$
$$k_1 \cot k_1 R = -k_2 \qquad R \sim 2.1\ fm$$
$$V_0 \sim 35\ MeV$$



The deuteron spherical square well and wave function

Deuteron nuclear properties

- ^2H nuclear spin (p and n intrinsic spins and orbital angular momentum around their common centre of mass)
- experimentally measured nuclear spin: $I = 1$ $(L = 0, 1, 2)$
- experimentally measured parity is *even*, and it is given by $(-1)^L$, thus states with $L = 1$ *(p-state)* are not permitted, and only or $L = 0$ *(s-state)* or $L = 2$ *(d-state)* can be considered

$$\boxed{I = S_n + S_p + L}$$

L = 0 or
L = 2 ??

- assuming the only possibility for ^2H is $L = 0$, no contribution from orbital angular momentum to magnetic dipole moment
- …theoretical value $(0.879804\ \mu_N)$ shows a small discrepancy with the experimentally measured magnetic dipole moment $(0.8574376 \pm 0.0000004\ \mu_N)$: the deuteron wave function is a mixture of d-state $(L = 2)$ and s-state $(L = 0)$
- … also confirmed by experimental measurements of the electric quadrupole moment: $Q_0 = 0$, if $L = 0$ (p and n have no intrinsic electric quadrupole moments); however experimentally $Q_0 = 0.00288 \pm 0.00002\ b$

$$\boxed{\mu = \mu_n + \mu_p}$$

Example 2.1

Calculate the magnetic dipole moment of <u>rigid</u> (light) nuclei with:

$$\boxed{\mu_N = \frac{e\hbar}{2m} = 3.1x10^{-8}\ \frac{eV}{T}}$$

$$\boxed{\mu = \frac{e\hbar}{2m} L}$$

- ***L = 0***

$\mu = \mu_L \times L = 0 \; eV/T$

- ***L= 1***

$\mu = 3.1 \times 10^{-8} \times 1 = 3.1 \times 10^{-8} \; eV/T$

- ***L=2***

$\mu = 3.1 \times 10^{-8} \times 2 = 6.2 \times 10^{-8} \; eV/T$

Calculate the mass defect and the binding energy of the deuteron, and compare the result with the energy required to liberate an electron bound to a hydrogen atom

$m(^1H) = 938.28\ MeV/c^2$
$m_n = 939.57\ MeV/c^2$
$m(^2H) = 1875.61\ MeV/c^2$

$$\Delta m = m_p + m_n - m_D =$$
$$= 938.28\ MeV/c^2 + 939.57\ MeV/c^2 - 1875.61\ MeV/c^2 =$$
$$= 2.24\ MeV/c^2$$

$$B(^2H) = \Delta m c^2 = 2.24\ \frac{MeV}{\cancel{c^2}}\ \cancel{c^2}$$

$$E_i \simeq 10\ eV$$

$$\frac{B(^2H)}{E_i} \simeq \frac{2.24\times 10^6\ eV}{10\ eV} \simeq \underline{2\times 10^5}$$

2. The Inter-nucleon Force

Example 2.3

Calculate the binding energy per nucleon of the tritium nucleus (^3H)

$m(^3H) = 3.016049$ u; $m(^1H) = 1.007825$ u;
$m(n) = 1.008665$ u
1 u = 931.50 MeV/c^2

$$B = \left[Zm(\,^1H) + Nm_N - m(\,^AX)\right]c^2$$

$$B(^3H) = \left[1 \times m(^1H) + 2\, m_N - m(^3H)\right]c^2 =$$

$$= (1 + 1.007825\,u + 2 \times 1.008665\,u - 3.016049\,u)\,c^2 =$$

$$= 0.009106 \times 931.5\ \frac{\mathrm{MeV}}{\cancel{c^2}}\ \cancel{c^2} = \underline{8.48\ \mathrm{MeV}}$$

$$\frac{B(^3H)}{A} = \frac{8.48}{3} = \underline{\underline{2.83\ \mathrm{MeV}}}$$