Nuclear and Radiation Physics (PHY2005)

Lecture 3

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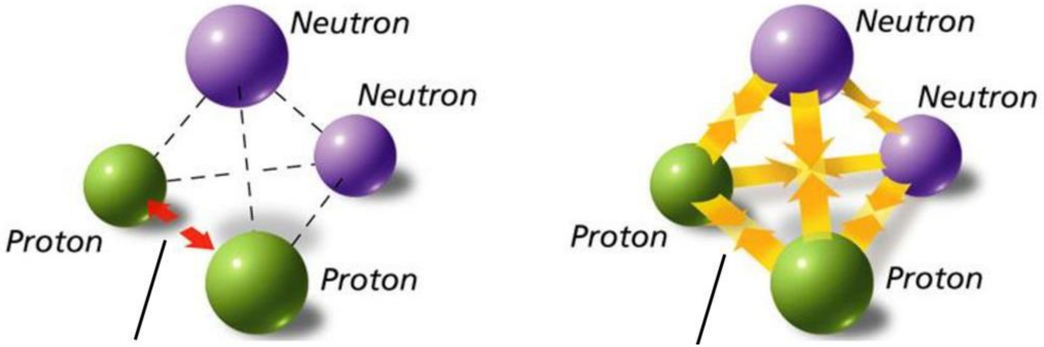

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Recap & Learning Goals

Summary of Lecture 2 *(Chap.1-2)*

- Nuclear angular momentum (*spin*) and parity
- Electromagnetic moments
- Nuclear excited states
- The deuteron

$$\boxed{\mu = \frac{e\hbar}{2m} L}$$

$$\boxed{Q_0 = \int \rho(r)(3z^2 - r^2)dv}$$

$$\boxed{\boldsymbol{I} = \boldsymbol{S_n} + \boldsymbol{S_p} + \boldsymbol{L} \rightarrow L=0,\ L=2}$$

$$\cancel{\boxed{\begin{array}{c} L = 0: \\ \boldsymbol{\mu} = \boldsymbol{\mu_n} + \boldsymbol{\mu_p} \\ \boldsymbol{Q_0 = 0} \end{array}}}$$

Learning goals of Lecture 3 *(Chap.2)*

- Understanding physical reasoning behind nucleon-nucleon forces
- Understanding physical reasoning behind the exchange force model

2. The Inter-nucleon Force
2.2. Properties of the Nuclear Force I

Nuclear force properties

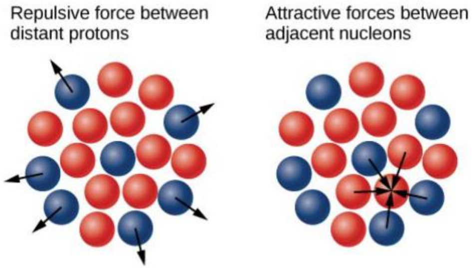
- nucleon-nucleon interaction → <u>attractive central potential</u>, $V_C(r)$,
  (r is the internucleon distance)

- nucleon-nucleon interaction → strongly <u>spin dependent</u>
  (depends on the intrinsic spins of the nucleons and their orientation)

- internucleon potential has a non-central term → <u>tensor potential</u>, $V(\mathbf{r})$,
  (quadrupole moment observed in deuteron)

- nucleon-nucleon force is <u>charge symmetric</u> → p-p ≡ n-n
  ("charge" is intended as character of the nucleon)

- nucleon-nucleon force is nearly <u>charge independent</u> → p-p ≡ n-n ≡ p-n
  ("mirror nuclei" with the same odd-A but different Z and N show very similar energy levels)

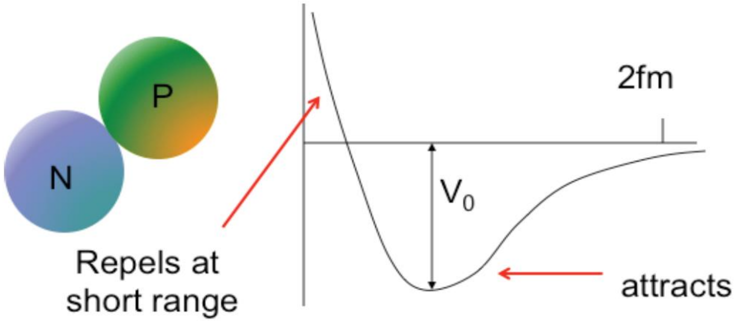


$^{17}O^8$ [MeV]	$^{17}F^9$ [MeV]
5.08	5.10
4.55	4.69
3.85	3.86
3.06	3.10
0.87	0.50
0	0

Some energy levels for $^{17}O^8$ and $^{17}F^9$

Nuclear force properties (cont.)

- nucleon-nucleon interaction → <u>repulsive at short distances</u> *(nuclear central density is constant)*



Possible potential well for internucleon force

Simplified square-well potential description

$$\begin{array}{lll} V(r) = +\infty & r < R_{core} & (R_{core} \sim 0.5\text{ fm}) \\ V(r) = -V_0 & R_{core} \leq r \leq R & \\ V(r) = 0 & r > R & \end{array}$$

2. The Inter-nucleon Force

2.2. Properties of the Nuclear Force III

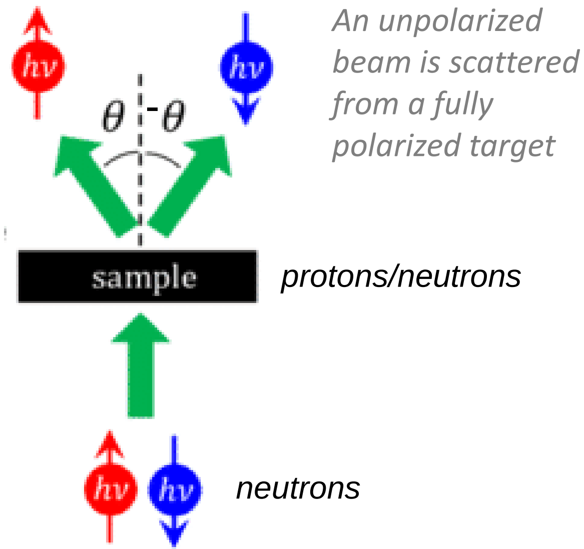
Spin-orbit interaction

- spin-orbit is an additional inter-nucleon interaction causing a shift in nucleons energy levels *(discovered through neutron scattering experiments)*
- polarization of a nucleon in a <u>beam</u> (or target/sample) can be aligned (<u>polarized</u>) in certain directions:

  N(↑), spin up
  N(↓), spin down
  -1 < P < 1

$$\boxed{P = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}}$$

- unpolarized beam *(P = 0)* → equal number of nucleons with spins pointing up and down
- nucleon scattering experiments on a fully polarized target *(P = 1, or P = -1)* can reveal the polarization of an incident beam



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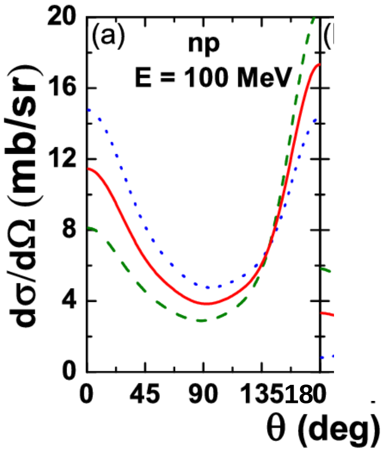
Exchange force model

- high energy n-p scattering → presence of an exchange force in nuclei
- n-p differential cross section → two strong peaks (~0° and ~180°)
- expected scattering angle *(centre-of-mass frame)*: $\theta \approx V_0/2T$ *(V_0: depth of the nucleon-nucleon square potential, T: projectile kinetic energy)* → $\theta \leq 10°$ *!!!*
- during collision neutrons and protons exchange places
- … forward moving *n* → *p*
- … and backward *(centre-of-mass frame)* moving *p* → becomes *n*
- something is exchanged between nucleons and change their character



Neutron-proton scattering differential cross-section

2. The Inter-nucleon Force

2.3. The Exchange force Model II

Virtual particles

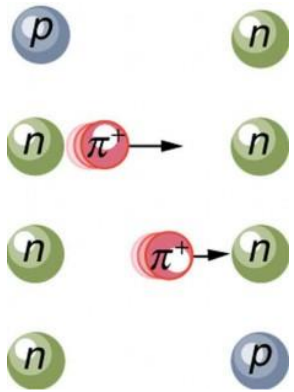
- "nuclear field quanta" (quantum field theory) are exchanged between nucleons during n-p scattering: the first nucleon emits field quanta, and the second nucleon absorbs them *(the two nucleons interact indirectly with each other)*
- $S_n = ½ \rightarrow S_p = ½$: exchanged particle must have $S = 0$ or $S = 1$, and must carry electric charge
- additionally, for n-n or p-p exchange there must be an uncharged virtual particle
- let's consider an exchange between two neutrons through a particle X of mass m_x; energy conservation is violated, unless the exchange occurs in a very short time *(uncertainty principle)*:

$$N_1 \rightarrow N_1 + X$$
$$X + N_2 \rightarrow N_2$$

$$\Delta t < \hbar/\Delta E = \hbar/(m_x c^2)$$

$$\boxed{R = c\Delta t \cong \frac{\hbar c}{m_x c^2} = \frac{200 MeV \cdot fm}{m_x c^2}}$$

- for nuclear forces R ~1 fm, hence m_x ~200 MeV
- these are virtual particles and are called **mesons** *(discovered in 1947)*
- **pions** are the lightest mesons and have different electric charge: +1 (π^+), 0 (π^0), -1 (π^-)
- 1-pion exchange occurs at longer ranges (1-1.5 fm)
- 2-pion exchange occurs at shorter ranges (0.5-1 fm)
- **ω-mesons** exchange occurs at much shorter ranges (~0.25 fm)



Schematic representation of virtual meson exchange

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2. The Inter-nucleon Force

Example 2.2

Calculate the rest energy for Π^+ (Pion), K^+ (Kaon), and D $^+$ (D-meson) particles, assuming that their range is 1.433 fm, 0.405 fm, and 0.107 fm, respectively.
Compare the calculated masses with the electron and proton mass.

$$\boxed{R = c\Delta t \cong \frac{\hbar c}{m_x c^2} = \frac{200MeV \cdot fm}{m_x c^2}}$$

1) $m(\pi^+) = \frac{200\,[MeV \times fm]}{R(\pi^+)\,c^2} = \frac{200}{1.433}\frac{[MeV\,fm]}{[f]c^2} = \underline{140\ MeV/c^2}$

2) $m(K^+) = \frac{200}{0.405} = \underline{494\ MeV/c^2}$

3) $m(D^+) = \frac{200}{0.107} = \underline{1869\ MeV/c^2}$

1) $\frac{m(\pi^+)}{m_e} = \frac{140\ MeV}{0.511\ MeV} \cong 274 \rightarrow m(\pi^+) \cong \underline{274\ m_e}$

$\frac{m(\pi^+)}{m_p} = \frac{140}{938.28} = 0.15 \rightarrow m(\pi^+) \simeq \underline{0.15\ m_p}$

2) $m(K^+) \simeq \underline{967\ m_e}$

$m(K^+) \simeq \underline{0.53\ m_p}$

3) $m(D^+) \simeq \underline{3657\ m_e}$

$m(D^+) \simeq \underline{1.99\ m_p}$

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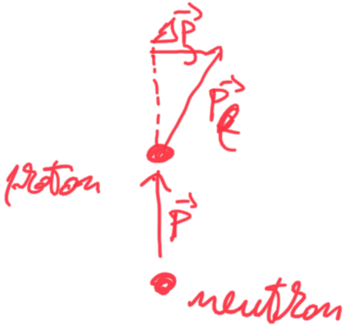
2. The Inter-nucleon Force

Derivation/Example 2.3

A n-p scattering experiment is carried out with fast neutrons of kinetic energy of 100 MeV and protons at rest. Assuming that the average force acting during the collision is of the order of V_0/R (V_0 is the depth of the neutron-proton square potential and R is its range):

(i) Derive the following equation: $\boxed{\theta = \dfrac{V_0}{2T}}$

(ii) Calculate the maximum scattering angle of the neutron beam



(i) for small θ : $\sin\theta \simeq \theta = \dfrac{\Delta P}{P}$ → transferring momentum

$\Delta P = F\,\Delta t$ → collision time

P → momentum of the incident neutron

$$\theta = \frac{F\Delta t}{P}$$

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2. The Inter-nucleon Force

Derivation/Example 2.3

$$F = \frac{V_0}{R} \rightarrow \theta = \frac{V_0}{R}\frac{\Delta t}{p} = \frac{V_0}{v p}$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2} p v \rightarrow \theta = \frac{V_0}{2T}$$

(ii) $V_0 \rightarrow$ internucleon potential energy

$$V_0 = 35 \text{ MeV}$$

$$T = 100 \text{ MeV}$$

$$\theta = \frac{V_0}{2T} = \frac{35\,[\text{MeV}]}{100\,[\text{MeV}]} = 0.17 \text{ rad} \simeq 10^\circ$$

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