# PHY2004: Electromagnetism and Optics

## Part 2: Optics

Dr. Fumin Huang

f.huang@qub.ac.uk

# Course Contents

- Polarisation of light
    - Types of polarisation
    - Linear and circular polarisation conditions
    - Stokes polarisation parameters
      Jones Vector
    - Faraday effect
- Light reflection and refraction at a planar interface
    - Fresnel formulae
    - Brewster angle, critical angle
    - Total internal reflection
    - Phase changes during reflection
- Principles of lasers
    - Einstein coefficients
    - Population inversion
    - Three energy level and four energy level laser systems

# Suggested Textbooks

1. Polarized light, 3rd Edition, Dennis H Goldstein.
2. Principles of Optics, Max Born & Emil Wolf
3. Lasers : theory and applications, K. Thyagarajan and A.K. Ghatak

# Please submit your attendance at:

https://go.qub.ac.uk/mpattendance

Or Scan the QR code below with a mobile device by:
Open the camera app on your device
Point the camera towards the QR code
Login with your QUB credentials

# Part 2: Optics

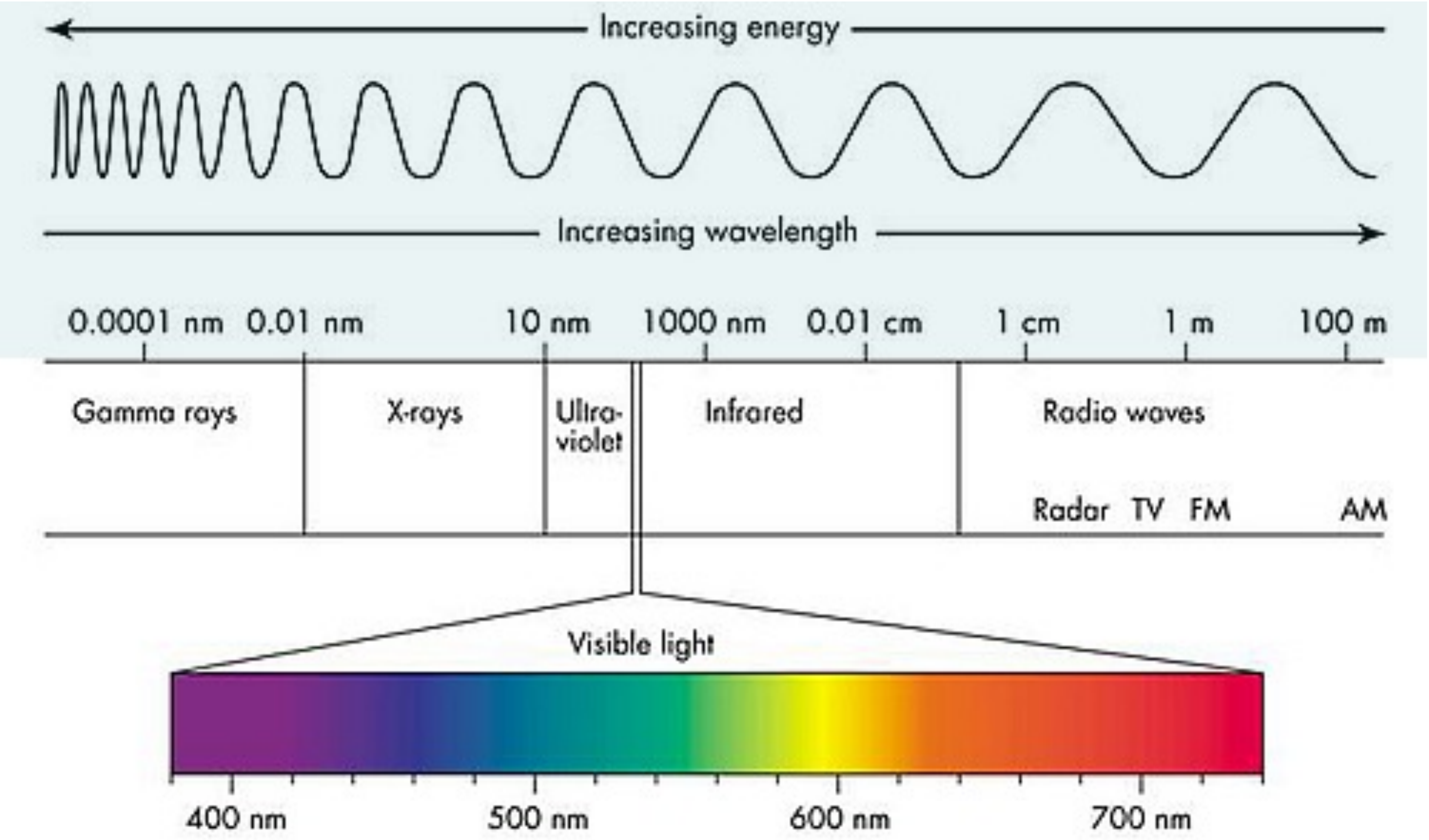
## Lecture 1

## Polarisation of Light

# Electromagnetic Spectrum

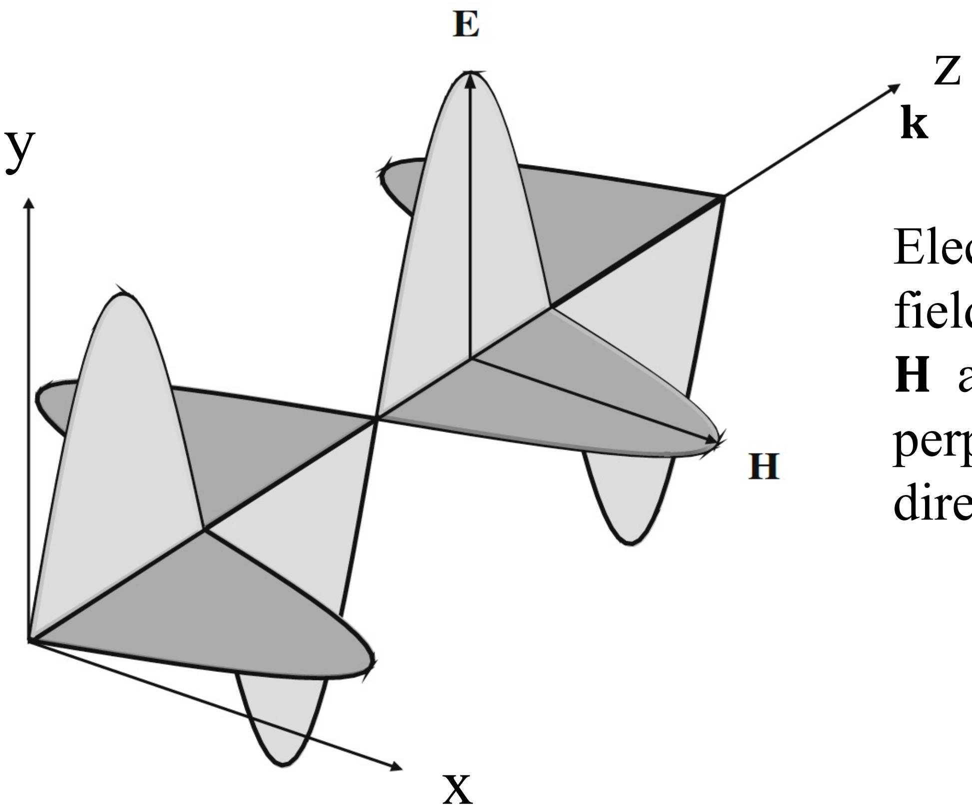
Light is an electromagnetic wave ($\lambda \sim$ 400-700 nm).

$$\text{Energy } E = h\upsilon = \frac{h}{T} = \frac{hc}{\lambda}$$



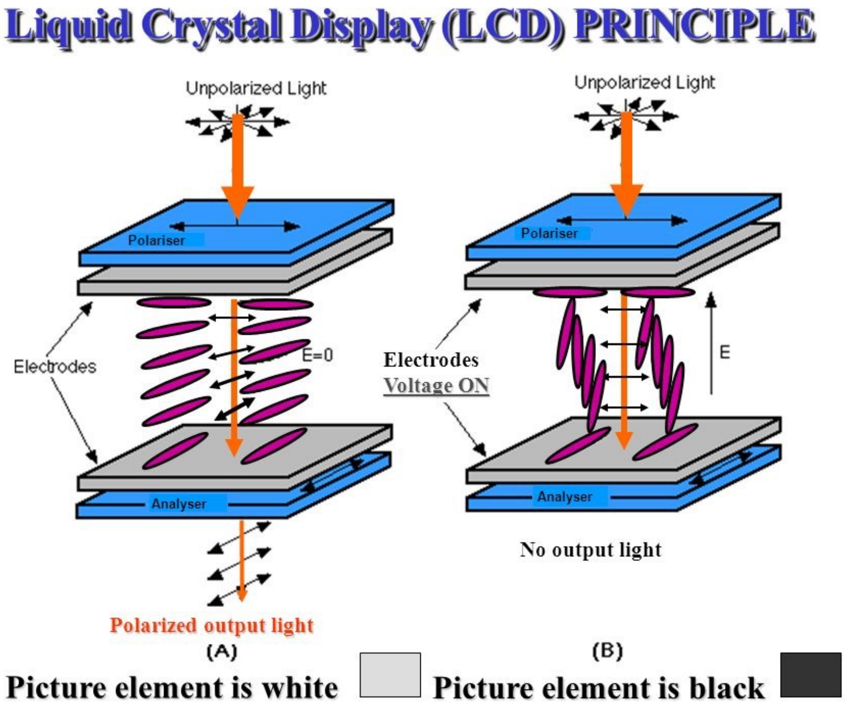
# What is Polarisation?

**Polarisation**: a fundamental property of EM field, specifying the oscillating orientation of the **E-field** in space and how it evolves with time.



Electromagnetic wave is a **transverse** field: electric field **E**, magnetic field **H** are both oscillating in a plane perpendicular to the wave propagation direction (**k**).

# Why Need to Know Polarisation?

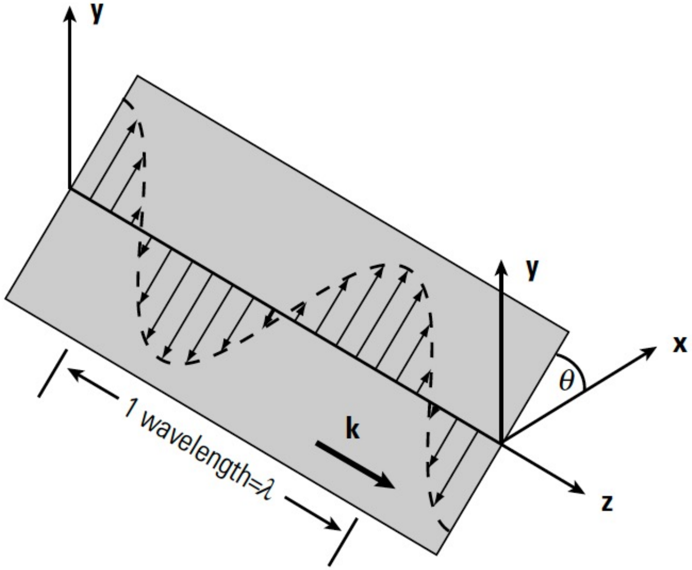


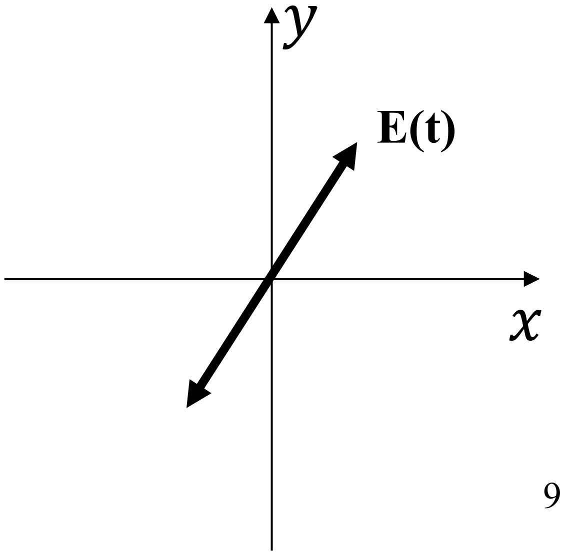
# Types of Polarisation

Based on the trajectory of the tips of E-field, polarisation can be classified as:

1. Linear polarisation
2. Circular polarisation
   - Right-handed circular polarisation
   - Left-handed circular polarisation
3. Elliptic polarisation

<u>Linear polarisation</u>: E-field oscillates along a *fixed* orientation, which remains unchanged in *time* (could vary in space).





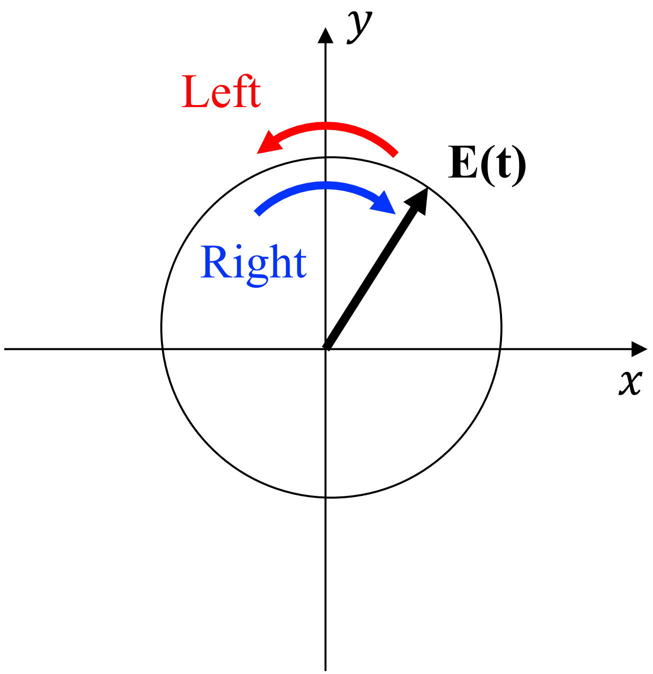
# Circular Polarisation

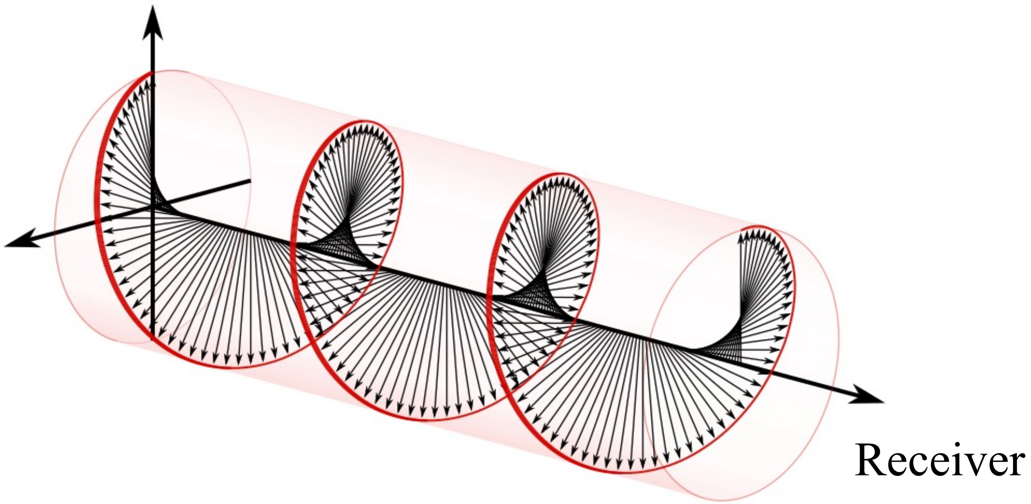
<u>Circular polarisation</u>:

- The *orientation* of E-field rotates circularly with *time*.
- The *magnitude* of the field remains constant.

Right/Left-handed circular light: E-vector rotates clockwise/anticlockwise with time (viewed by the source).

From the Receiver's view, the tip of the E-vector of right-handed circular polarisation forms a shape like a right-handed screw.

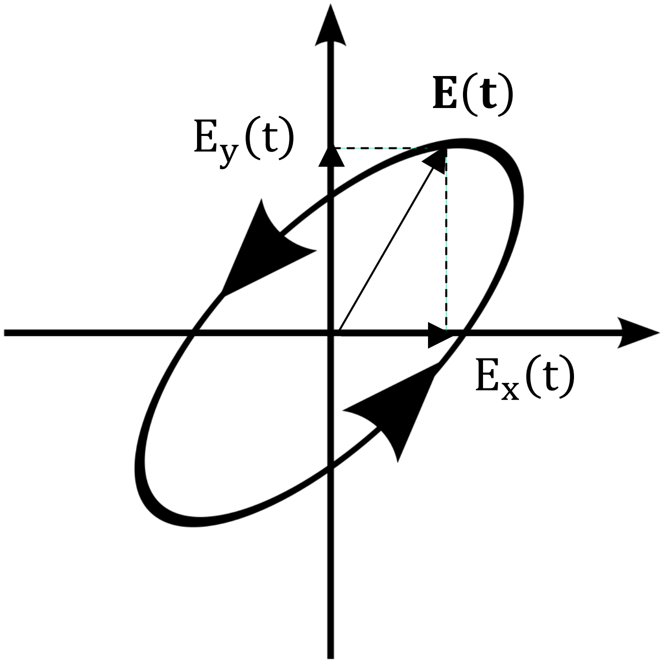




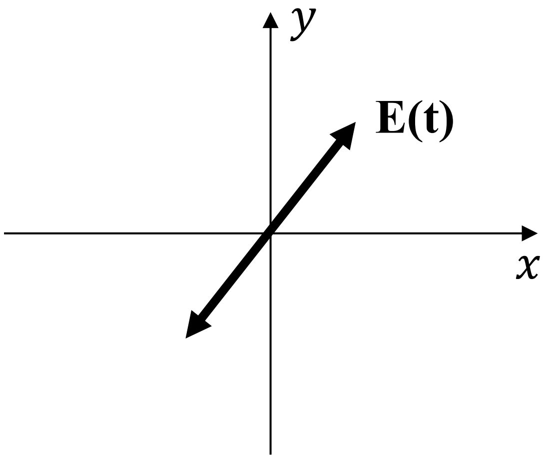
Right-handed circular polarisation

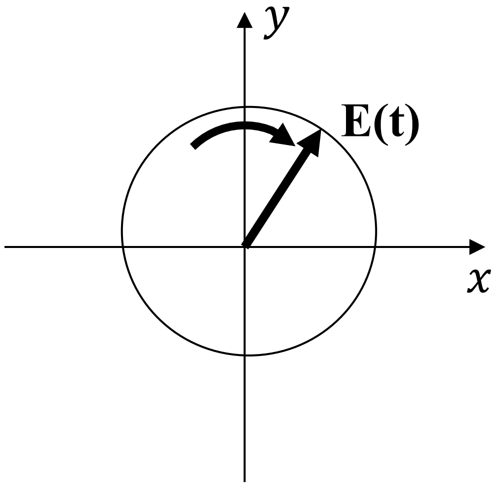
# Elliptic Polarisation

<u>Elliptic polarisation</u>: in the general case, the tip of E-field moves along an ellipse.

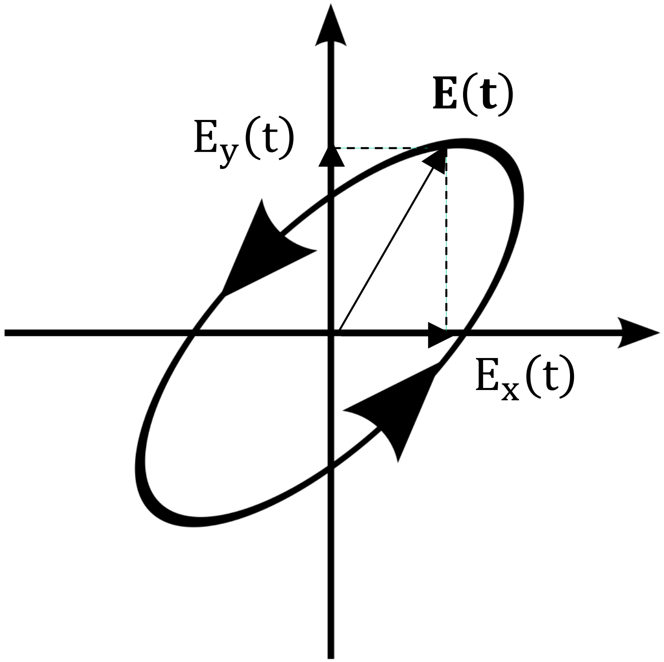


Linear polarisation and circular polarisation are the special cases of elliptic polarisation (as line and circle are special shapes of ellipse)





# Polarisation Ellipse



$$\mathbf{E}(\mathbf{t}) = E_x(t)\hat{\mathbf{x}} + E_y(t)\hat{\mathbf{y}}$$

$$E_x(t) = E_{0x}\cos(\omega t - kz + \delta_x)$$

$$E_y(t) = E_{0y}\cos\left(\omega t - kz + \delta_y\right)$$

↓ magnitude ↓ phase

Ellipse Equation

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos\delta = \sin^2(\delta)$$

$\delta = \delta_y - \delta_x$ Phase difference

# Linear Polarisation

$$\frac{\mathrm{E}_x^2}{\mathrm{E}_{0x}^2}+\frac{\mathrm{E}_y^2}{\mathrm{E}_{0y}^2}-2\,\frac{\mathrm{E}_x}{\mathrm{E}_{0x}}\frac{\mathrm{E}_y}{\mathrm{E}_{0y}}\cos\delta=\sin^2(\delta)$$

What are the conditions of linear polarisation?

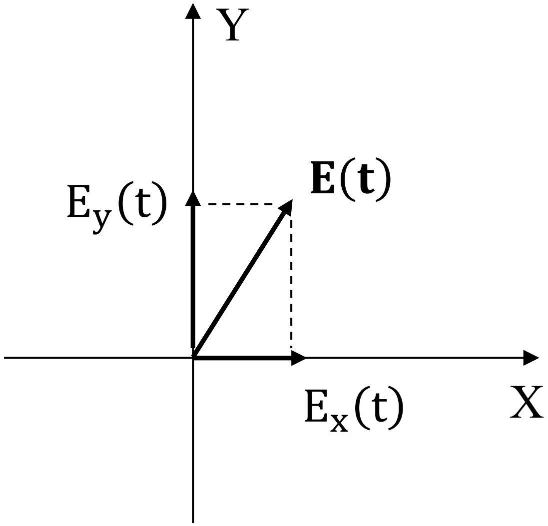
1. Ordinary cases: $\mathrm{E}_{0y}=0$, $\mathbf{E(t)}=\mathrm{E}_x(t)\hat{\mathbf{y}}$, Linear horizontal polarisation

or $\mathrm{E}_{0x}=0$, $\mathbf{E(t)}=\mathrm{E}_y(t)\hat{\mathbf{y}}$, Linear vertical polarisation

2. $\frac{\mathrm{E}_y}{\mathrm{E}_x}=\text{const.}$ This can be achieved when $\delta=0$ or $\pi$

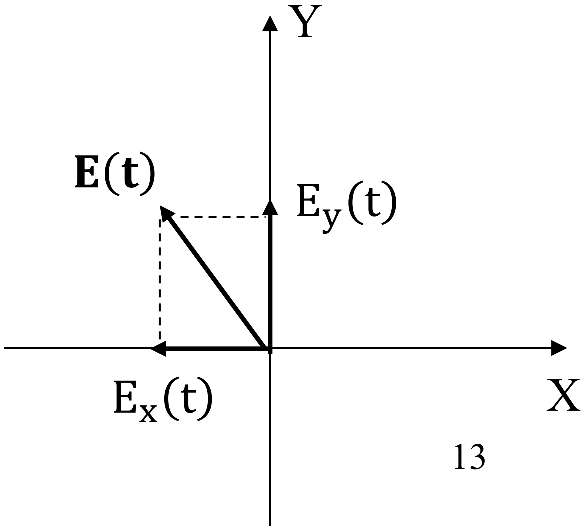
(a) $\delta=0$

$$\frac{\mathrm{E}_y}{\mathrm{E}_x}=\frac{\mathrm{E}_{0y}}{\mathrm{E}_{0x}}$$



(b) $\delta=\pi$

$$\frac{\mathrm{E}_y}{\mathrm{E}_x}=-\frac{\mathrm{E}_{0y}}{\mathrm{E}_{0x}}$$



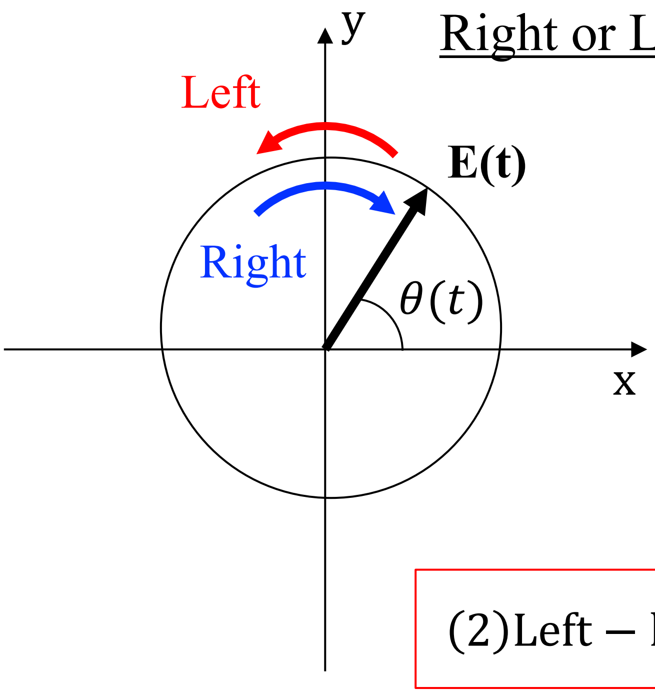
# Circular Polarisation

Conditions for circular polarisation:

$$\delta = \pm\frac{\pi}{2} \quad \text{AND} \quad E_{0x} = E_{0y} = E_0$$

$$\frac{E_x^2}{E_0^2} + \frac{E_y^2}{E_0^2} = 1 \qquad \underline{\text{circle}}$$

Right or Left?



(1) Case: $\delta = \delta_y - \delta_x = +\frac{\pi}{2}$

$$E_y(t) = E_0 \cos\left(\omega t - kz + \delta_x + \frac{\pi}{2}\right)$$
$$= E_0 \sin\left[\frac{\pi}{2} - \left(\omega t - kz + \delta_x + \frac{\pi}{2}\right)\right]$$
$$= E_0 \sin(kz - \delta_x - \omega t)$$

$$E_x(t) = E_0 \cos(\omega t - kz + \delta_x)$$
$$= E_0 \cos(kz - \delta_x - \omega t)$$

$$\boxed{\theta(t) = kz - \delta_x - \omega t}$$

$$E_x(t) = E_0 \cos\theta(t)$$

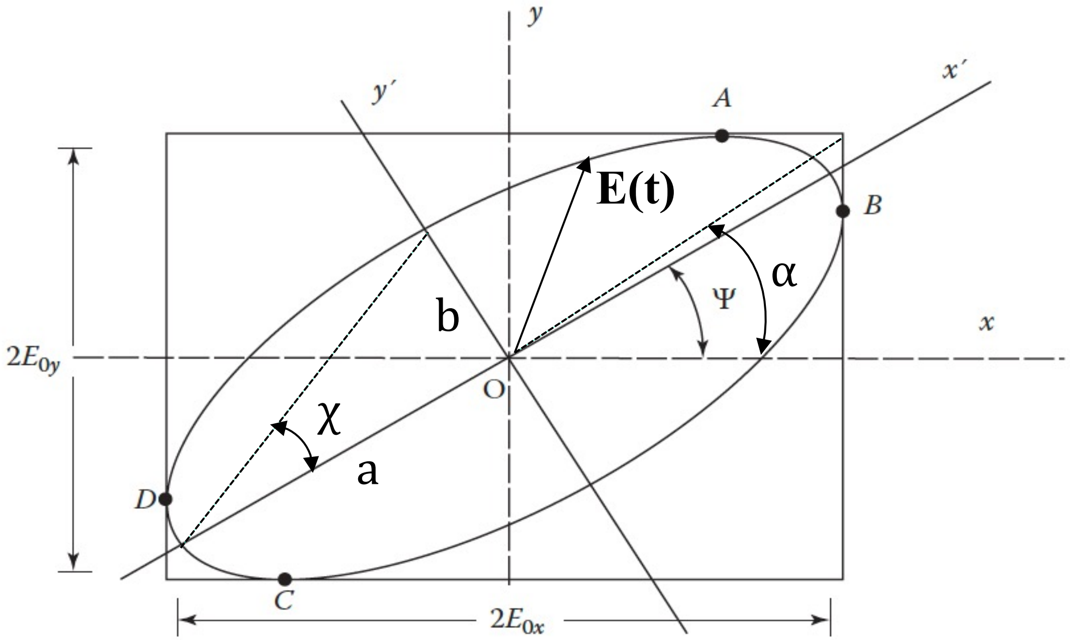
$$E_y(t) = E_0 \sin\theta(t)$$

$\theta(t)$ decreases with time, *clockwise rotation*:
**right-handed** circular polarisation

$$\boxed{(2)\text{Left} - \text{handed circular: } \delta = \delta_y - \delta_x = -\frac{\pi}{2}}$$

# Polarisation Ellipse

$$\frac{\mathrm{E}_{\mathrm{x}}^2}{\mathrm{E}_{0\mathrm{x}}^2}+\frac{\mathrm{E}_{\mathrm{y}}^2}{\mathrm{E}_{0\mathrm{y}}^2}-2\frac{\mathrm{E}_{\mathrm{x}}}{\mathrm{E}_{0\mathrm{x}}}\frac{\mathrm{E}_{\mathrm{y}}}{\mathrm{E}_{0y}}\cos\delta=\sin^2(\delta)$$



The ellipse can be described by the orientation Ψ and ellipticity χ angles.

$$0\le\Psi<\pi,\ -\frac{\pi}{4}\le\chi\le\frac{\pi}{4}$$

$$\tan(2\Psi)=\tan(2\alpha)\cos\delta$$

$$\sin(2\chi)=\sin(2\alpha)\sin\delta$$

$$\mathrm{a}^2+\mathrm{b}^2=\mathrm{E}_{0\mathrm{x}}^2+\mathrm{E}_{0\mathrm{y}}^2$$

$$\alpha=\tan^{-1}\left(\frac{\mathrm{E}_{0\mathrm{y}}}{\mathrm{E}_{0\mathrm{x}}}\right)$$

Ellipticity angle χ: $\tan\chi=\pm\frac{\mathrm{b}}{\mathrm{a}}$

- Line: $\chi=0$

(1) $\alpha=0\ (\mathrm{E}_{0\mathrm{y}}=0),\ \frac{\pi}{2}\ (\mathrm{E}_{0x}=0)$

**or**

(2) $\delta=0,\pi$

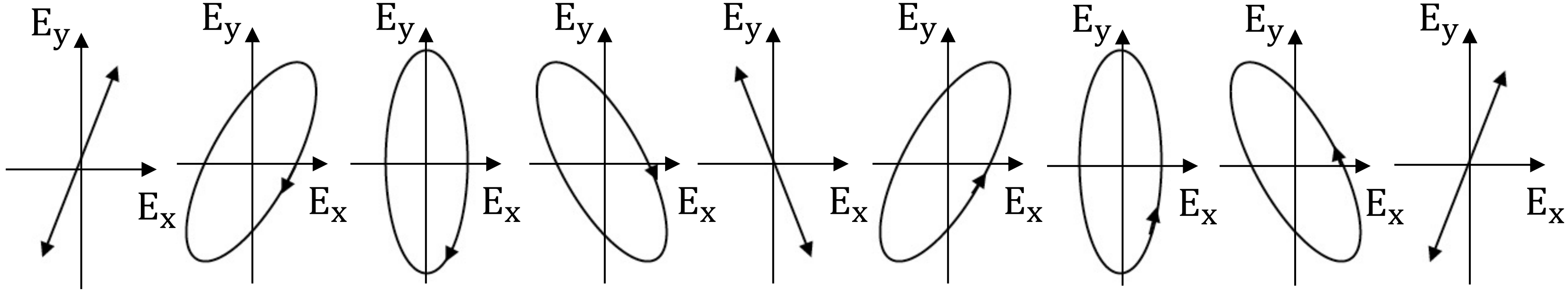
- Circle: $\chi=\pm\frac{\pi}{4}$

(1) $\alpha=\frac{\pi}{4}\ (\mathrm{E}_{0\mathrm{x}}=\mathrm{E}_{0y})$ **and**

(2) $\delta=\pm\frac{\pi}{2}$

# Polarisation Ellipse

- Linear polarisation
  1) $E_{0x} = 0$ or $E_{0y} = 0$, **or**
  2) $\delta = 0$ or $\pi$
- Circular polarisation
  1) $E_{0x} = E_{0y}$, **and**
  2) $\delta = \frac{\pi}{2}$ (right), or $\delta = -\frac{\pi}{2}$ (left),
- Other conditions: elliptic polarisation



$\delta = 0$ $\delta = \pi/4$ $\delta = \pi/2$ $\delta = 3\pi/4$ $\delta = \pi$ $\delta = 5\pi / 4$ $\delta = 3\pi/2$ $\delta = 7\pi / 4$ $\delta = 2\pi$

# Examples of Questions

1. $E_x(t) = A\cos(kz - \omega t)$, $E_y(t) = A\sin(\omega t - kz + \frac{\pi}{4} + \delta)$. Describe the conditions of $\delta$ $(0 \le \delta < 2\pi)$ for linear, right- and left-circular polarisation, respectively.

2. Determine the polarisation states of the following cases $(A > 0)$

(a) $E_x(t) = A\cos(\omega t - kz)$, $E_y(t) = 2A\cos(\omega t - kz)$

(b) $E_x(t) = A\sin(\omega t - kz)$, $E_y(t) = A\sin(\omega t - kz + \frac{\pi}{2})$

(c) $E_x(t) = A\cos(\omega t - kz)$, $E_y(t) = A\sin(\omega t - kz)$

(d) $E_x(t) = A\sin(\omega t - kz)$, $E_y(t) = A\cos(\omega t - kz)$

(e) $E_x(t) = A\cos(\omega t - kz)$, $E_y(t) = 2A\sin(\omega t - kz)$

# Stokes Polarisation Parameters

## How to measure and characterize the polarisation state of light?

In 1852, Lord George Stokes discovered that any polarisation state of light can be completely described by four ***measurable*** real quantities ($S_0$, $S_1$, $S_2$, $S_3$), which are called Stokes polarisation parameters.

$$S_0 = \langle E_x(t)E_x^*(t)\rangle + \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 + E_{0y}^2$$

$$S_1 = \langle E_x(t)E_x^*(t)\rangle - \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 - E_{0y}^2$$

$$S_2 = \left\langle E_x(t)E_y^*(t)\right\rangle + \left\langle E_y(t)E_x^*(t)\right\rangle = 2E_{0x}E_{0y}\cos\delta$$

$$S_3 = i\left(\left\langle E_x(t)E_y^*(t)\right\rangle - \left\langle E_y(t)E_x^*(t)\right\rangle\right) = 2E_{0x}E_{0y}\sin\delta$$

$\langle\quad\rangle$ means average, $\left\langle E_i(t)E_j^*(t)\right\rangle = \frac{1}{T}\int_0^T E_i(t)E_j^*(t)dt$

T: Period of EM field; *: complex conjugate

<u>Only three of the four parameters are independent</u>

$$E_x(t) = E_{0x}e^{i(\omega t - kz + \delta_x)},\ E_y(t) = E_{0y}e^{i(\omega t - kz + \delta_y)}$$

$$\langle E_x(t)E_x^*(t)\rangle = E_{0x}^2, \quad \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0y}^2$$

$$S_0 = \langle E_x(t)E_x^*(t)\rangle + \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 + E_{0y}^2,\ S_1 = \langle E_x(t)E_x^*(t)\rangle - \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 - E_{0y}^2$$

$$S_2 = \left\langle E_x(t)E_y^*(t)\right\rangle + \left\langle E_y(t)E_x^*(t)\right\rangle = 2E_{0x}E_{0y}\cos\delta$$

$$\left\langle E_x(t)E_y^*(t)\right\rangle = \frac{1}{T}\int_0^T E_{0x}e^{i(\omega t - kz + \delta_x)}E_{0y}e^{-i(\omega t - kz + \delta_y)}dt$$

$$= \frac{E_{0x}E_{0y}}{T}\int_0^T e^{-i\delta}dt = E_{0x}E_{0y}e^{-i\delta}$$

$$\left\langle E_y(t)E_x^*(t)\right\rangle = E_{0x}E_{0y}e^{i\delta}$$

$$S_2 = \left\langle E_x(t)E_y^*(t)\right\rangle + \left\langle E_y(t)E_x^*(t)\right\rangle = E_{0x}E_{0y}\left(e^{-i\delta} + e^{i\delta}\right) = 2E_{0x}E_{0y}\cos\delta$$

$$S_3 = i\left(\left\langle E_x(t)E_y^*(t)\right\rangle - \left\langle E_y(t)E_x^*(t)\right\rangle\right) = 2E_{0x}E_{0y}\sin\delta$$

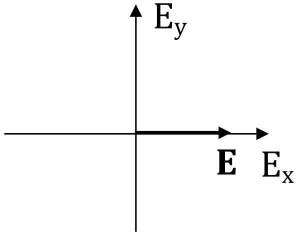
# Stokes Polarisation Parameters

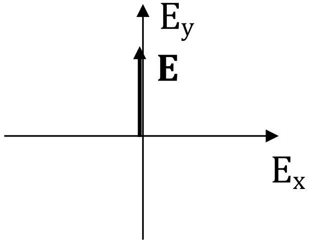
1. Linear horizontal polarisation (LHP): $E_{0y} = 0$

$$S_0 = S_1 = E_{0x}^2, S_2 = S_3 = 0$$

2. Linear vertical polarisation (LVP): $E_{0x} = 0$

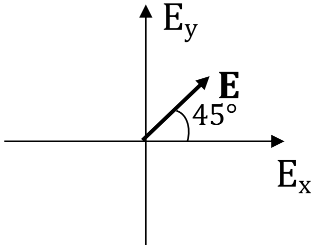
$$S_0 = S_1 = E_{0y}^2, S_2 = S_3 = 0$$

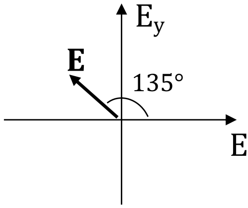




3. Linear +45° polarisation: $E_{0x} = E_{0y} = E_0, \delta=0$

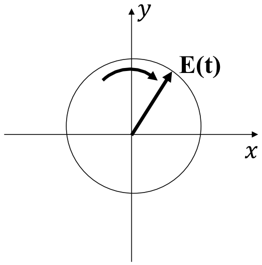
$$S_0 = 2E_0^2, S_1 = 0, S_2 = 2E_0^2, S_3 = 0$$

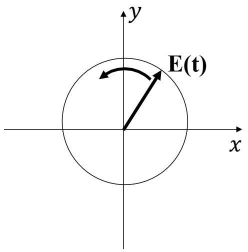




4. Linear -45° polarisation: $E_{0x} = E_{0y} = E_0, \delta=\pi$

$$S_0 = 2E_0^2, S_1 = 0, S_2 = -2E_0^2, S_3 = 0$$





5. Right-handed circular polarisation (RCP): $E_{0x} = E_{0y} = E_0, \delta = \frac{\pi}{2}$

$$S_0 = 2E_0^2, S_1 = 0, S_2 =0, S_3 = 2E_0^2$$

6. Left-handed circular polarisation (LCP): $E_{0x} = E_{0y} = E_0, \delta = -\frac{\pi}{2}$

$$S_0 = 2E_0^2, S_1 = 0, S_2 =0, S_3 = -2E_0^2$$

# Unpolarised Light

$$E_x(t) = E_0 e^{i[\omega t - kz + \delta_x(t)]}$$

Random phases

$$E_y(t) = E_0 e^{i[\omega t - kz + \delta_y(t)]}$$

$$\langle E_x(t) E_y^*(t) \rangle = \frac{1}{T}\int_0^T E_0 e^{i(\omega t - kz + \delta_x(t))} E_0 e^{-i(\omega t - kz + \delta_y(t))} dt$$

$$= \frac{E_0^2}{T} \overset{= 0}{\boxed{\int_0^T e^{-i\delta(t)} dt}}$$

$\delta(t) = \delta_y(t) - \delta_x(t)$
random function of t

$\langle E_x(t) E_y^*(t) \rangle = 0$ Similarly, $\langle E_y(t) E_x^*(t) \rangle = 0$

$$S_2 = S_3 = 0$$

As $\langle E_x(t) E_x^*(t) \rangle = \langle E_y(t) E_y^*(t) \rangle = E_0^2$

So $S_1 = \langle E_x(t) E_x^*(t) \rangle - \langle E_y(t) E_y^*(t) \rangle = E_{0x}^2 - E_{0y}^2 = 0$

# Degree of Polarisation

In general: $S_0^2 \geq S_1^2 + S_2^2 + S_3^2$

Intensity of polarized light: $I_p = \sqrt{S_1^2 + S_2^2 + S_3^2}$

Intensity of unpolarized light: $I_u = S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2}$

$$I_{tot} = S_0 = I_p + I_u$$

Degree of polarisation: $P = \dfrac{I_p}{I_{tot}} = \dfrac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$ $(0 \leq P \leq 1)$

$P = 1$ Completely polarised light

$P = 0$ Unpolarised light

$0 < P < 1$ Partially polarised light

# Stokes Polarisation Parameters

$$S_0 = \langle E_x(t)E_x^*(t)\rangle + \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 + E_{0y}^2$$

$$S_1 = \langle E_x(t)E_x^*(t)\rangle - \left\langle E_y(t)E_y^*(t)\right\rangle = E_{0x}^2 - E_{0y}^2$$

$$S_2 = \left\langle E_x(t)E_y^*(t)\right\rangle + \left\langle E_y(t)E_x^*(t)\right\rangle = 2E_{0x}E_{0y}\cos\delta$$

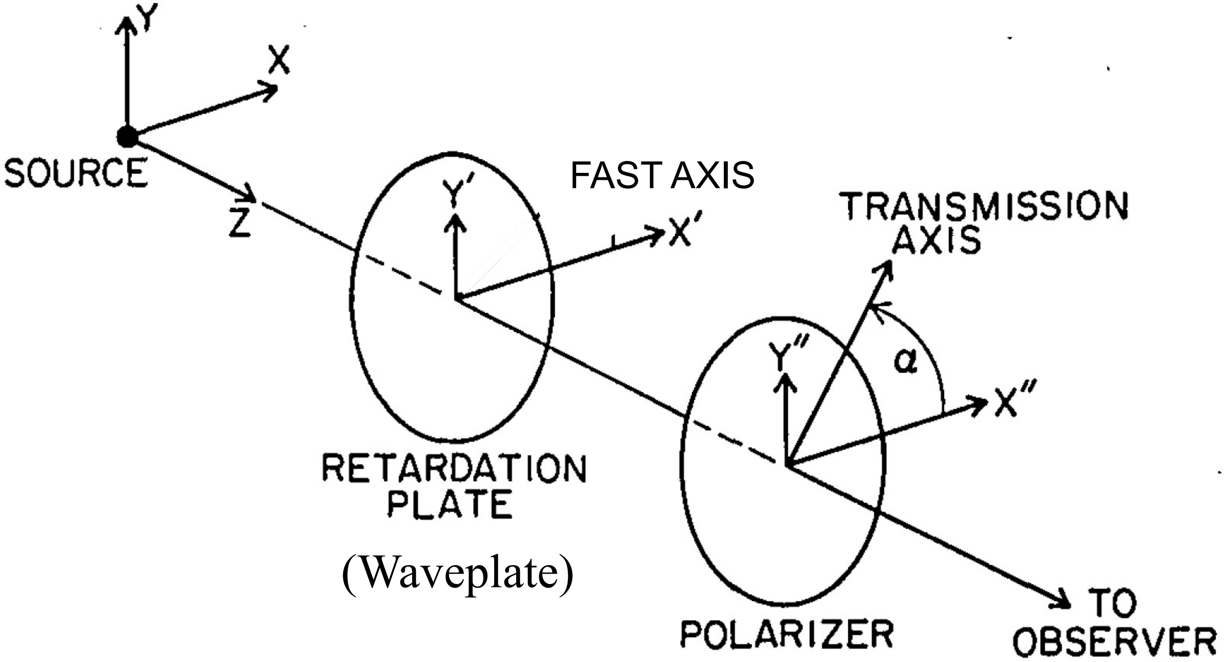
$$S_3 = i\left(\left\langle E_x(t)E_y^*(t)\right\rangle - \left\langle E_y(t)E_x^*(t)\right\rangle\right) = 2E_{0x}E_{0y}\sin\delta$$

Intensity of polarized light: $I_p = \sqrt{S_1^2 + S_2^2 + S_3^2}$

Intensity of unpolarized light: $I_u = S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2}$

Degree of polarisation: $P = \frac{I_p}{I_{tot}} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$

# Measurement of Stokes Parameters



The retardation plate (a waveplate) provides a phase difference of $\phi$ for E-field vectors parallel and perpendicular to its *fast axis*. The intensity of transmitted beam after the polarizer is:

$$I(\alpha, \phi) = \frac{1}{2}[S_0 + S_1 \cos 2\alpha + (S_2 \cos\phi + S_3 \sin\phi) \sin 2\alpha]$$

# Measurement of Stokes Parameters

The E-field component measured by the detector is:

$$E_D(t) = E'_x \cos\alpha + E'_y \sin\alpha = E_x e^{i\phi}\cos\alpha + E_y \sin\alpha$$

<u>Measured intensity</u>

$$I_D(\alpha,\phi) = \langle E_D(t)E_D^*(t)\rangle$$

$$= \frac{1}{2}\begin{bmatrix} \left\langle E_x(t)E_x^*(t) + E_y(t)E_y^*(t)\right\rangle + \left\langle E_x(t)E_x^*(t) - E_y(t)E_y^*(t)\right\rangle \cos 2\alpha \\ +\left\langle E_x(t)E_y^*(t) + E_y(t)E_x^*(t\right\rangle \cos\phi \sin 2\alpha + i\left\langle E_x(t)E_y^*(t) - E_y(t)E_x^*(t\right\rangle \sin\phi \sin 2\alpha \end{bmatrix}$$

$$= \frac{1}{2}(S_0 + S_1\cos 2\alpha + S_2\cos\phi\sin 2\alpha + S_3\sin\phi\sin 2\alpha)$$

$$I_D(0°,0°) = \frac{1}{2}(S_0 + S_1) \qquad S_0 = I_D(0°,0°) + I_D(90°,0°)$$

$$I_D(45°,0°) = \frac{1}{2}(S_0 + S_2) \qquad S_1 = I_D(0°,0°) - I_D(90°,0°)$$

$$I_D(90°,0°) = \frac{1}{2}(S_0 - S_1) \qquad S_2 = 2I_D(45°,0°) - I_D(0°,0°) - I_D(90°,0°)$$

$$I_D(45°,90°) = \frac{1}{2}(S_0 + S_3) \qquad S_3 = 2I_D(45°,90°) - I_D(0°,0°) - I_D(90°,0°)$$

# Determine the polarisation states from Stokes parameters

1. Intensity of unpolarized light: $I_u = S_0 - \sqrt{S_1^2 + S_2^2 + S_3^2}$

   Intensity of polarized light: $I_p = \sqrt{S_1^2 + S_2^2 + S_3^2}$

   Degree of polarisation: $P = \frac{I_p}{I_{tot}} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$

2. For the completely polarised component

$$I_p = \sqrt{S_1^2 + S_2^2 + S_3^2} = E_{0x}^2 + E_{0y}^2$$

$$S_1 = E_{0x}^2 - E_{0y}^2$$

$$S_2 = 2E_{0x}E_{0y}\cos\delta$$

$$S_3 = 2E_{0x}E_{0y}\sin\delta$$

$$E_{0x} = \sqrt{\frac{\sqrt{S_1^2 + S_2^2 + S_3^2} + S_1}{2}}$$

$$E_{0y} = \sqrt{\frac{\sqrt{S_1^2 + S_2^2 + S_3^2} - S_1}{2}}$$

$$\tan\delta = \frac{S_3}{S_2}$$

$E_{0x}$, $E_{0y}$ and $\delta$ can be completely determined, hence the polarisation ellipse.

# Waveplate

A e
a
re

- Cry
refr
orie
*fast*
call
($n_e$

)

<u>Pha</u>

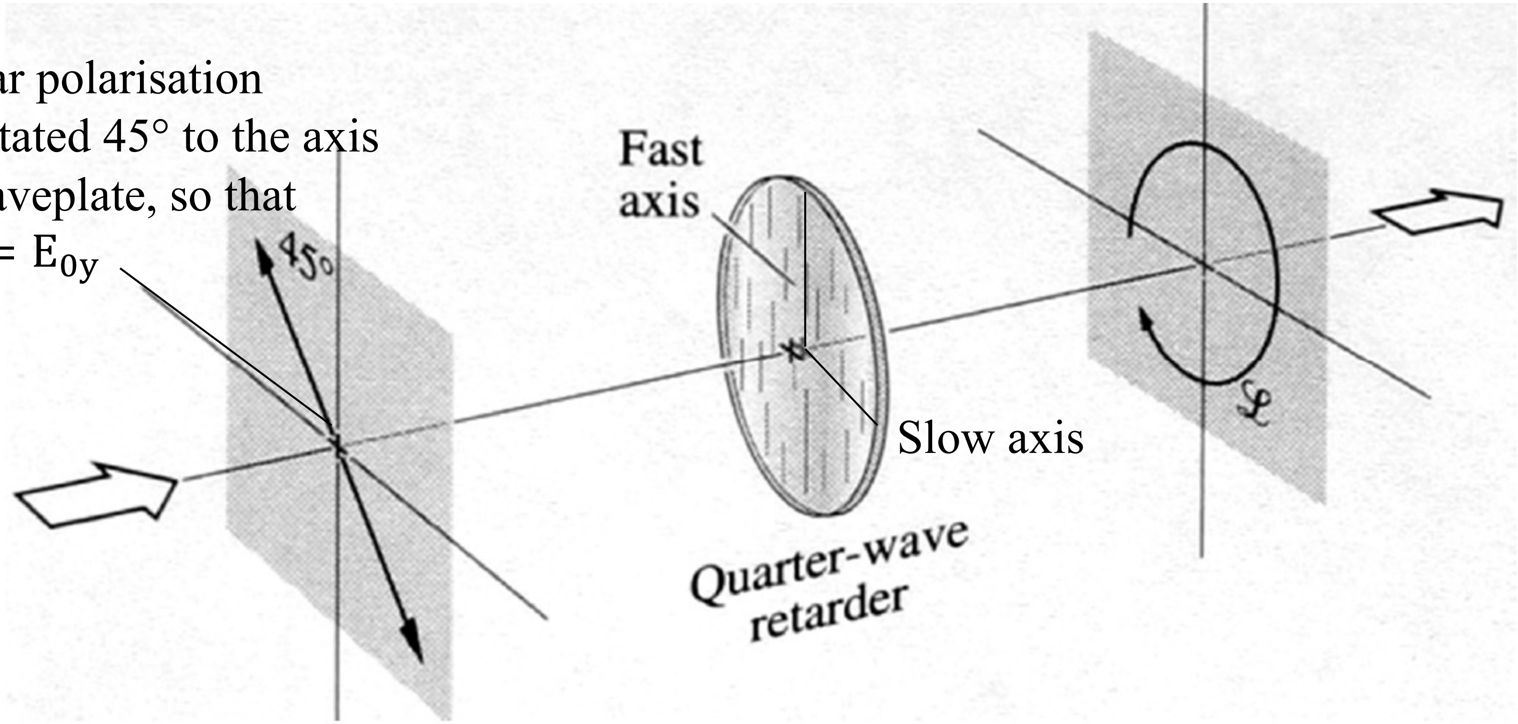
Qu

Ha

# Polarisation Transformation

Linear polarization ⇔ Circular polarisation

$$\delta = 0, \pi \iff \delta = \pm\frac{\pi}{2} \text{ And } E_{0x} = E_{0y}$$



Linear polarisation orientated 45° to the axis of waveplate, so that $E_{0x} = E_{0y}$

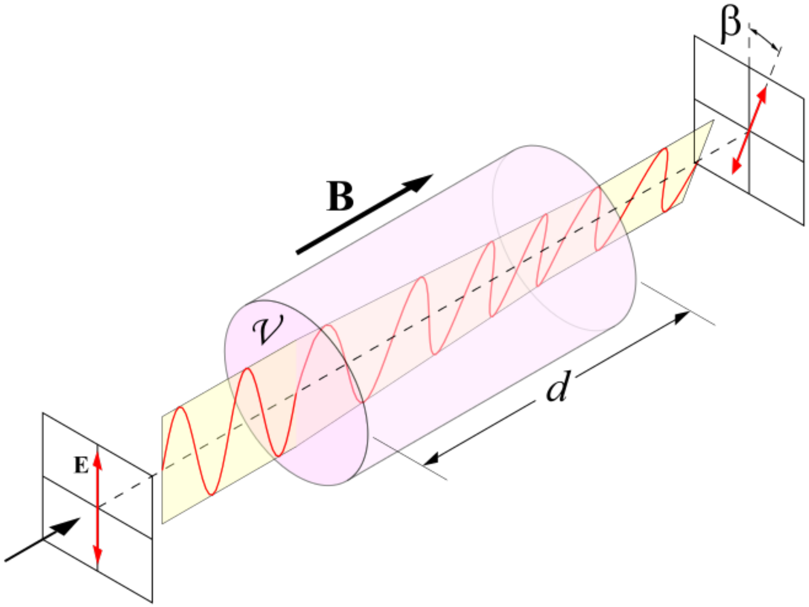
Quarter wave plate introduces $\frac{\pi}{2}$ phase change between $E_y$ and $E_x$

# The Faraday Effect

The **Faraday effect** or **Faraday rotation** is a magneto-optical phenomenon discovered by Faraday in 1845.

A linearly-polarised plane wave will rotate its plane-of-polarisation after passing through a piece of material, when a magnetic field along the light propagation direction is applied .

The rotation is **linearly proportional** to the magnetic field and the thickness of the material.



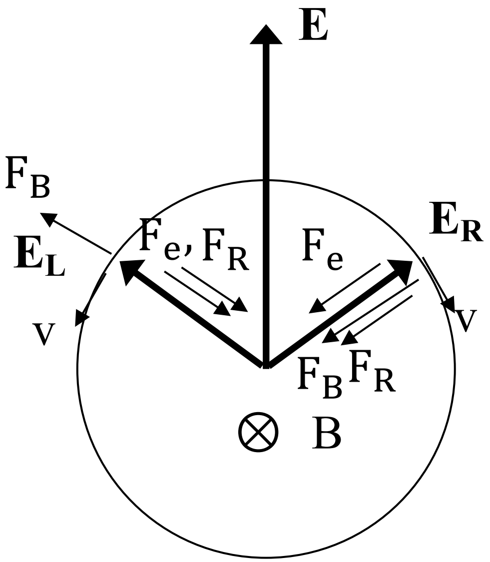
Rotated polarisation angle

$$\theta = VBd$$

V: Verdet constant

# Magneto-Optic Effect

A linear polarisation can be decomposed into equal proportion of right- and left-circular polarisation



Assuming the forces pointing towards the circle centre is positive

## Forces experienced by electrons

(1) E-field force $F_e = -e(-E) = eE$

(2) Restoring force $F_R = kr = m\omega_0^2 r$

(3) Lorentz force $\mathbf{F_B} = -e\mathbf{v}\times\mathbf{B}$

$$F_B^{R,L} = \pm evB = \pm eB\omega r$$

$$eE_{R,L} \pm e\omega Br + m\omega_0^2 r = m\omega^2 r$$

$$r_{R,L} = \frac{eE_{R,L}/m}{\omega^2 - \omega_0^2 \mp e\omega B/m}$$

With applied magnetic field, electrons move on circles of different radius for right and left circular polarisations

# Circular Birefringence

Total induced dipole moment per unit volume

Number of electrons per unit volume

$$P = \overset{\nearrow}{N}er = \frac{Ne^2 E_{R,L}/m}{\omega^2 - \omega_0^2 \mp e\omega B/m}$$

Relative permittivity

$$\varepsilon_r^{R,L} = 1 + \frac{P}{\varepsilon_0 E_{R,L}} \quad = 1 + \frac{\frac{Ne^2}{m\varepsilon_0}}{\omega^2 - \omega_0^2 \mp e\omega B/m}$$

$$= 1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 \mp e\omega B/m}$$

$$\omega_p = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$ Plasma frequency

Refractive index

$$n = \sqrt{\varepsilon_r}$$

$$n_R = \sqrt{1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 - e\omega B/m}}$$

$$n_L = \sqrt{1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 + e\omega B/m}}$$

$$n_R \neq n_L$$

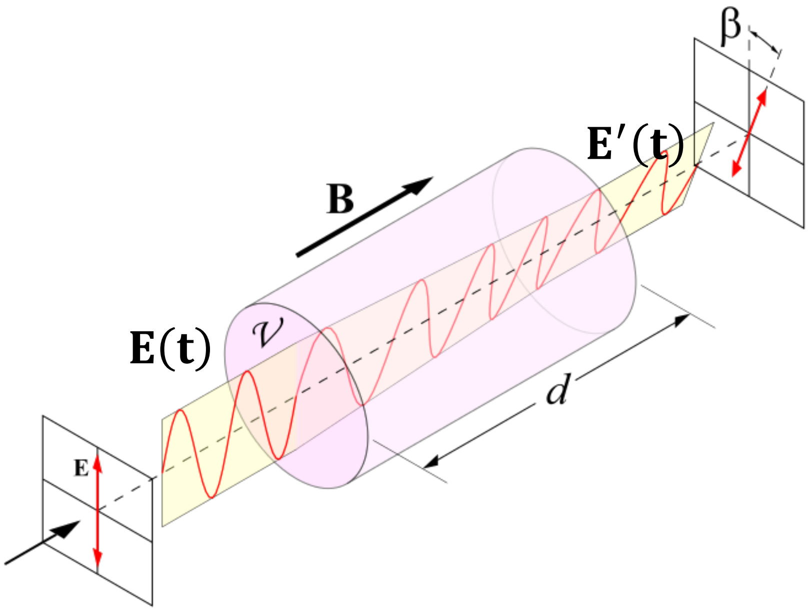
The refractive indices of right and left-circular polarisation are different, which is called **circular birefringence**.

# Polarisation Rotation

$$\mathbf{E}_\mathbf{R}'(\mathbf{t}) = \frac{1}{2}\mathrm{E}_0\left\{\cos(\omega t - k_R d)\,\hat{\mathbf{x}} + \cos\left(\omega t - k_R d + \frac{\pi}{2}\right)\hat{\mathbf{y}}\right\}$$

$$= \frac{1}{2}\mathrm{E}_0\left\{\cos(\omega t - n_R k_0 d)\,\hat{\mathbf{x}} + \cos\left(\omega t - n_R k_0 d + \frac{\pi}{2}\right)\hat{\mathbf{y}}\right\}$$

$$\mathbf{E}_\mathbf{L}'(\mathbf{t}) = \frac{1}{2}\mathrm{E}_0\left\{\cos(\omega t - n_L k_0 d)\,\hat{\mathbf{x}} + \cos\left(\omega t - n_L k_0 d - \frac{\pi}{2}\right)\hat{\mathbf{y}}\right\}$$



$$\mathbf{E}'(\mathbf{t}) = \mathbf{E}_\mathbf{R}'(\mathbf{t}) + \mathbf{E}_\mathbf{L}'(\mathbf{t})$$

$$= \mathrm{E}_0 \cos\left(\omega t - \frac{k_R + k_L}{2} d\right)\left\{\cos\left(\frac{n_R - n_L}{2} k_0 d\right)\hat{\mathbf{x}} + \sin\left(\frac{n_R - n_L}{2} k_0 d\right)\hat{\mathbf{y}}\right\}$$

$$\theta = \frac{n_R - n_L}{2} k_0 d = \mathrm{VBd}$$

$$\mathrm{V} = \frac{n_R - n_L}{2\mathrm{B}} k_0 = \frac{\pi(n_R - n_L)}{\mathrm{B}\lambda}$$

# Verdet Constant

$$V = \frac{n_R - n_L}{2B} k_0$$

$$n_{R,L}(\omega) = \sqrt{1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 \mp e\omega B/m}}$$

$$\approx \sqrt{1 + \frac{\omega_p^2}{\left(\omega \mp \frac{eB}{2m}\right)^2 - \omega_0^2}} = n(\omega \mp \Delta\omega)$$

$$\Delta\omega = \frac{eB}{2m} \qquad n(\omega) = \sqrt{1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2}}$$

$$n(\omega \mp \Delta\omega) \approx n(\omega) \mp \Delta\omega \frac{dn}{d\omega}$$

$$n_R - n_L = -2\Delta\omega \frac{dn}{d\omega} = -\frac{eB}{m}\frac{dn}{d\omega}$$

$$V = \frac{\pi(n_R - n_L)}{B\lambda} = -\frac{e\pi}{m\lambda}\frac{dn}{d\omega}$$

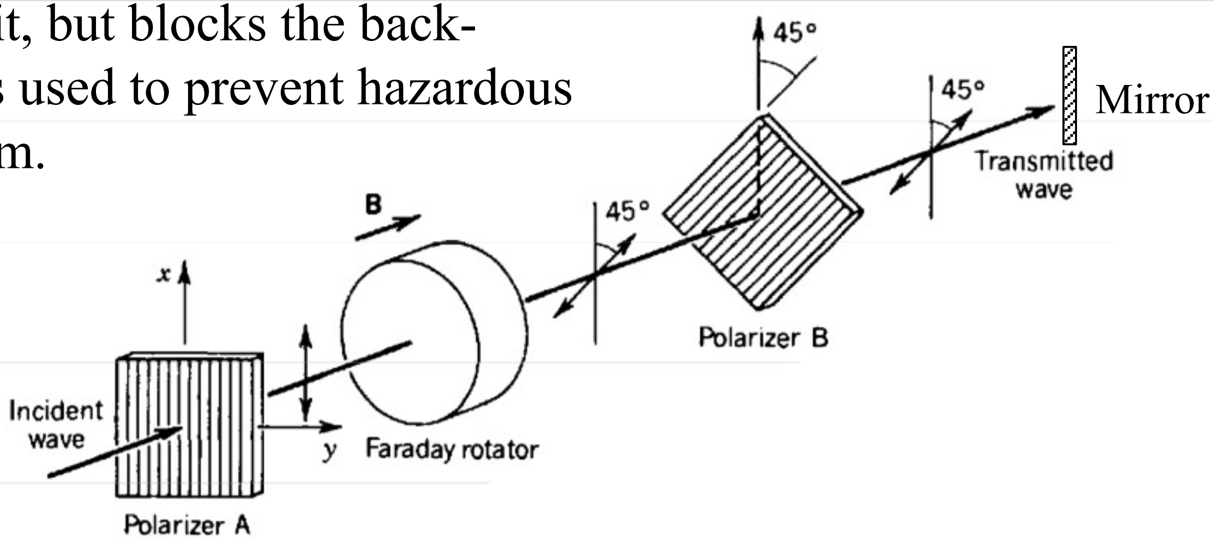
$$\omega = \frac{2\pi c}{\lambda}$$

$$\frac{dn}{d\omega} = \frac{\frac{dn}{d\lambda}}{\frac{d\omega}{d\lambda}} = -\frac{\lambda^2}{2\pi c}\frac{dn}{d\lambda}$$

$$V = \frac{e\lambda}{2mc}\frac{dn}{d\lambda}$$

# Optical Isolator

Optical isolator is an optical element which allows light transmit, but blocks the back-reflection light. It is used to prevent hazardous back-reflection beam.



- Two polarisers are orientated 45° to each other. A magnetic field is applied to rotate the polarisation of linearly-polarized light by 45° so that light can completely transmit through the second polariser.
- When the reflected light passes through the Faraday rotator, its polarisation is rotated by another 45° so that it is 90° to the axis of the first polariser, hence is blocked.