QUEEN'S UNIVERSITY BELFAST

PHY2001

Exam Time Table Code PHY2001

Any calculator, except one with pre-programmable memory, may be used in this examination.

Answer Books A, B and C

LEVEL 2

EXAMINATION CONTRIBUTING TO THE DEGREES OF BACHELOR OF SCIENCE (BSc) AND MASTER IN SCIENCE (MSci)

PHY2001
Quantum and Statistical Physics

Duration: 3 Hours

Tuesday, 8th May 2018 2:30 PM - 5:30 PM

Examiners: Prof. P. Browning
Dr. P. van der Burgt
and the Internal Examiners

Answer ALL TEN questions in Section A for 4 marks each.
Answer TWO questions in Section B for 20 marks each.
Answer ONE question in Section C for 20 marks.

Use a separate answer book for each Section.
Follow the instructions on the front of the answer book. Enter your Anonymous Code number and Seat number, but NOT your name.

THE QUEEN'S UNIVERSITY OF BELFAST

SCHOOL OF MATHS AND PHYSICS

PHYSICAL CONSTANTS

Speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \text{ms}^{-1}$
Permeability of a vacuum	$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$ $\approx 1.26 \times 10^{-6}\ \text{Hm}^{-1}$
Permittivity of a vacuum	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{Fm}^{-1}$
Elementary charge	$e = 1.60 \times 10^{-19}\ \text{C}$
Electron charge	$= -1.60 \times 10^{-19}\ \text{C}$
Planck Constant	$h = 6.63 \times 10^{-34}\ \text{Js}$
Reduced Planck Constant	$\hbar = 1.05 \times 10^{-34}\ \text{Js}$
Rydberg Constant for hydrogen	$R_\infty = 1.097 \times 10^{7}\ \text{m}^{-1}$
Unified atomic mass unit	$1u = 1.66 \times 10^{-27}\ \text{kg}$ $1u = 931\ \text{MeV}$
1 electron volt (eV)	$= 1.60 \times 10^{-19}\ \text{J}$
Mass of electron	$m_e = 9.11 \times 10^{-31}\ \text{kg}$
Mass of proton	$m_p = 1.67 \times 10^{-27}\ \text{kg}$
Mass of neutron	$m_n = 1.67 \times 10^{-27}\ \text{kg}$
Molar gas constant	$R = 8.31\ \text{JK}^{-1}\text{mol}^{-1}$
Boltzmann constant	$k = 1.38 \times 10^{-23}\ \text{JK}^{-1}$
Avogadro constant	$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11}\ \text{Nm}^{2}\text{kg}^{-2}$
Acceleration of free fall on the Earth's surface	$g = 9.81\ \text{ms}^{-2}$

SECTION A

Use a section A answer book

Answer ALL 10 questions in this section

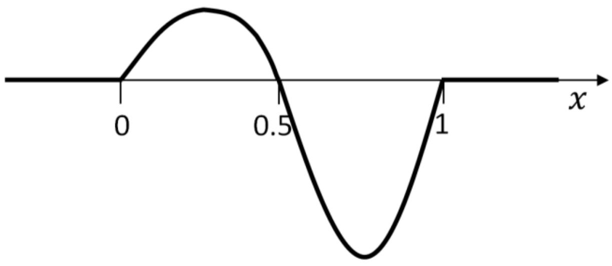
Full explanations of your answers are required to attain full marks

1 According to the Uncertainty Principle, the vacuum is not empty but is filled with virtual particles. Estimate how long a virtual electron-positron pair would exist for. **[4]**

2 A wavefunction $\psi = A \text{ for } -1 < x < +1 \text{ and } \psi = 0$ elsewhere.

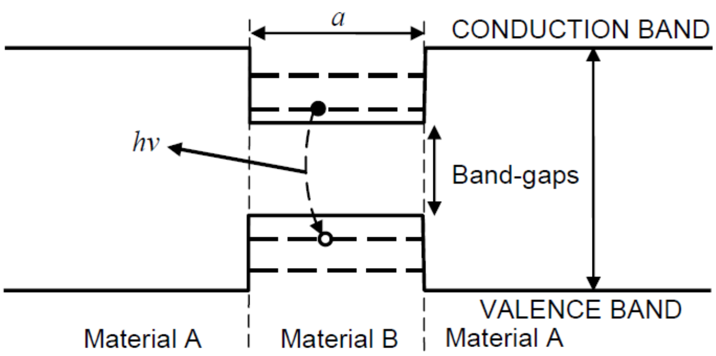
What is the value of A? **[4]**

3 Define the expectation value for a particle's position $\langle x \rangle$. If a particle is described by the following wavefunction, qualitatively estimate $\langle x \rangle$?



[4]

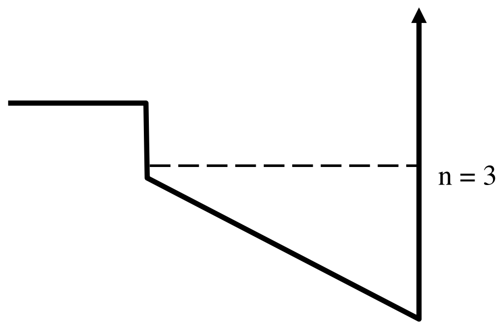
4 In a quantum well semiconductor a thin piece of material B is sandwiched between two pieces of material A (as shown below). If an electron in material B is excited to the conduction band it can decay with the emission of a photon. If the **bandgap of material A increases**, does the photon energy increase, decrease, or stay the same?



[4]

SECTION A

5 Reproduce the diagram below in your answer booklet and sketch the $n = 3$ wavefunction for this potential well.



[4]

6 Why are the energies of alpha particles emitted by a nucleus strongly related to the half-life? Given the alpha decay energies of ^{241}Am and ^{238}U are 5.64 MeV and 4.27 MeV respectively, which one has the shortest half-life? **[4]**

7 The allowed energies of a particle in a three dimensional square potential well are given by $E = C\left(n_x^2 + n_y^2 + n_z^2\right)$. Determine the energy and degeneracy of the first six energy levels.

[4]

8 Explain what is meant by the terms *microstate* and *macrostate* as used in statistical mechanics. Illustrate your answer by describing how these concepts apply to a classical ideal gas. **[4]**

9 Consider a system with two non-degenerate energy levels having energies $\varepsilon_1 = 0$ and $\varepsilon_2 = 2 \times 10^{-20}$ J, respectively. For a population of classical (distinguishable) particles at a temperature of $T = 6500$ K, calculate the fraction of particles that will occupy the lower energy level. **[4]**

SECTION A

10 Draw a sketch showing the Fermi-Dirac distribution function, $f_{FD}(\varepsilon)$, versus energy for a system in which the temperature, T, is much lower than the Fermi temperature. Clearly label your axes and mark the value of the Fermi energy on your sketch. Briefly explain the shape of the distribution with reference to the Pauli exclusion principle.

[4]

SECTION B

Use a Section B answer book

Answer <u>TWO</u> questions from this section

One dimensional time independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+(V-E)\psi=0$$

11(a) A particle of mass m is trapped in an infinite 1-dimensional potential well in the region $-\frac{a}{2}<x<+\frac{a}{2}$. Obtain the odd and even parity eigenfunction solutions of the Schrödinger equation for this system and hence obtain an expression for the allowed energies. **[9]**

(b) The quantum states of a finite, square potential well of depth V_0 and width a have energies E determined by

$$k\tan\frac{ka}{2}=(\beta^2-k^2)^{1/2}$$

where

$$k^2=\frac{2mE}{\hbar^2}\qquad\beta^2=\frac{2mV_0}{\hbar^2}$$

(i) By sketching the functions on either side of this equation as a function of k, explain how the energies of the finite well differ from an infinite well. **[8]**

(ii) As the potential well becomes very shallow, i.e. $V_0\to 0$, how many bound states are there, if any? **[3]**

SECTION B

12 $\psi = Cx\,\exp(-\gamma x^2)$ is an eigenstate of the simple harmonic oscillator potential well $V = \frac{1}{2}m\omega^2 x^2$

(a) By substituting this solution into the Schrödinger equation, obtain an expression for γ and hence the energy of this eigenstate. **[10]**

(b) Draw the probability density of this state and compare it with what would be expected from a classical description. **[6]**

(c) Determine the expectation value for the momentum of the particle in this state.
(momentum operator $\hat{p} = -i\hbar\frac{d}{dx}$) **[4]**

13 A particle of total energy E is incident on a potential step at $x = 0$, of height $V_0 (E < V_0)$. The allowed eigenfunctions are

$$\psi(x < 0) = \frac{D}{2}\left(1 + i\frac{\alpha}{k}\right)\exp(ikx) + \frac{D}{2}\left(1 - i\frac{\alpha}{k}\right)\exp(-ikx)$$

$$\psi(x > 0) = D\,\exp(-\alpha x)$$

(a) Explain how these eigenfunctions are obtained and obtain expressions for k and α. **[8]**

(b) Show that the particle is reflected with 100% probability as predicted classically. **[4]**

(c) Give, with the aid of rough sketches, a brief qualitative explanation of what happens when

(i) the step is replaced by a thin barrier

(ii) $E > V_0$ **[8]**

SECTION B

14(a) Without going into full mathematical detail, explain how the Schrödinger wave equation for the hydrogen atom can be solved. Clearly identify the origin of the quantum numbers generated and explain their physical significance. **[12]**

(b) The radial part of the wavefunction for the 2p state of the hydrogen atom is

$$R_{2p} = A\, r \exp\left(-\frac{r}{2a_0}\right)$$

where a_0 is the Bohr radius. Show that the average radial position is $5a_0$. **[8]**

$$\int_0^\infty x^n \exp(-kx) = \frac{n!}{k^{n+1}}$$

SECTION C

Use a Section C answer book

Answer ONE question from this section

15(a) State the central postulate of statistical mechanics and outline how it can be used to determine the macrostate properties of a system of particles, such as a classical gas, if the set of accessible microstates is known. **[4]**

(b) A weakly interacting system of 4 distinguishable particles has total energy 5ε. The allowed single-particle states for the system lie at equally spaced energies: 0, ε, 2ε, 3ε, 4ε, 5ε etc.

(i) Identify all the possible distributions of particles in the system and calculate their statistical weights. Present you results in tabular form. **[7]**

(ii) Calculate the average distribution of particles among the single-particle states and verify that it is consistent with the expected total energy and total number of particles. **[4]**

(iii) Calculate the Boltzmann entropy for this system. **[2]**

(c) If the particles in part (b) were indistinguishable bosons, what would be the entropy of the system? **[3]**

SECTION C

16(a) Explain why the allowed quantum states of particles confined in a three-dimensional cubic box with sides of length a are separated by intervals of $\Delta k = \pi/a$ along each of the k-space axes. **[4]**

(b) For a three-dimensional system of weakly interacting spin-0 particles occupying a volume, V, the density of states in wavenumber space, $g(k)$, is given by

$$g(k)\,\mathrm{d}k = \frac{Vk^2}{2\pi^2}\,\mathrm{d}k$$

Show, for non-relativistic particles of mass m, that the density of states in energy is given by

$$g(\varepsilon)\,\mathrm{d}\varepsilon = \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\varepsilon^{1/2}\,\mathrm{d}\varepsilon$$

[6]

(c) Consider a box of volume $1\ \mathrm{m}^3$ containing helium atoms at standard temperature and pressure.

(i) Estimate the number of states that have energy below 0.02 eV. **[5]**

(ii) Using your answer to (c)(i) explain whether this system of helium atoms can be described as a *dilute gas*. **[5]**

[Standard temperature and pressure may be taken as 273 K and 10^5 Pa, respectively. You may assume that helium atoms have mass of 4 a.m.u. each.]

END OF EXAMINATION