Lecture 7:
Electric currents and magnetic fields

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Recap on electrostatics

In electrostatics, we assume the charges to be at rest (or slowly moving) so that we can neglect the $\vec{v} \times \vec{B}$ term in Lorentz equation.

If this is the case, the curl of the electrostatic field is always 0, which **implies** the existence of a scalar potential:

$$\boxed{\nabla \times \vec{E} = 0 \qquad \rightarrow \qquad \vec{E} = -\nabla V}$$

The source of the electric field is the charge density:

$\nabla \cdot \vec{E} = \rho / \varepsilon_0$, which is equivalent to say that:

the flux of the electric field through a surface is equal to the density of charges contained in it divided by ε_0 (Gauss' law)

Recap on electrostatics

In a conductor, the charges can only be uniformly distributed on the surfaces and there cannot be any electric field inside. The external electric field always points outward, perpendicular to the surface

In a dielectric, the situation is different, since it can get **polarised**

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$

There is a similar Gauss'law for the vector *D*: $\nabla \cdot \vec{D} = \rho_F$
which is equivalent to say that:

the flux of the induction field (D) through a surface is equal to the density of free charges contained in it (Gauss' law in media)

Electric current

What if charges are now moving?

In this case, the first thing to define is how fast they are moving.
The rate of flow of charges through a surface (number of charges passing through the surface per unit time) is called the **electric current**

$$I = \frac{dq}{dt}$$

Another useful quantity is the **electric current density** defined as the current passing through a surface (A) divided by the surface itself:

$$J = \frac{I}{A} \quad \text{or, more rigorously,} \quad I = \int_A \vec{J} \cdot d\vec{s}$$

Conservation of charges

It is an empirical fact that **charge is conserved locally**.
If we then have a volume containing a charge density, this must increase/decrease according to the amount of charge escaping through the surface enclosing this volume (the current):

$$\therefore \int_S \underline{J} \, . \, \underline{dS} = -\int_V \frac{d\rho}{dt} \, dV$$

if we now use the divergence theorem:

$$\int_V \nabla \, . \, \underline{\mathrm{J}} \; \mathrm{dV} = -\int \frac{\mathrm{d}\rho}{\mathrm{dt}} \, \mathrm{dV} \quad \rightarrow \quad \boxed{\frac{d\rho}{dt} + \nabla \cdot \vec{J} = 0}$$

This equation is called **the continuity equation**

Magnetostatics

Magnetostatics is an approximation of electromagnetism.

While electrostatics assumed static charges, **magnetostatics assumes steady currents**

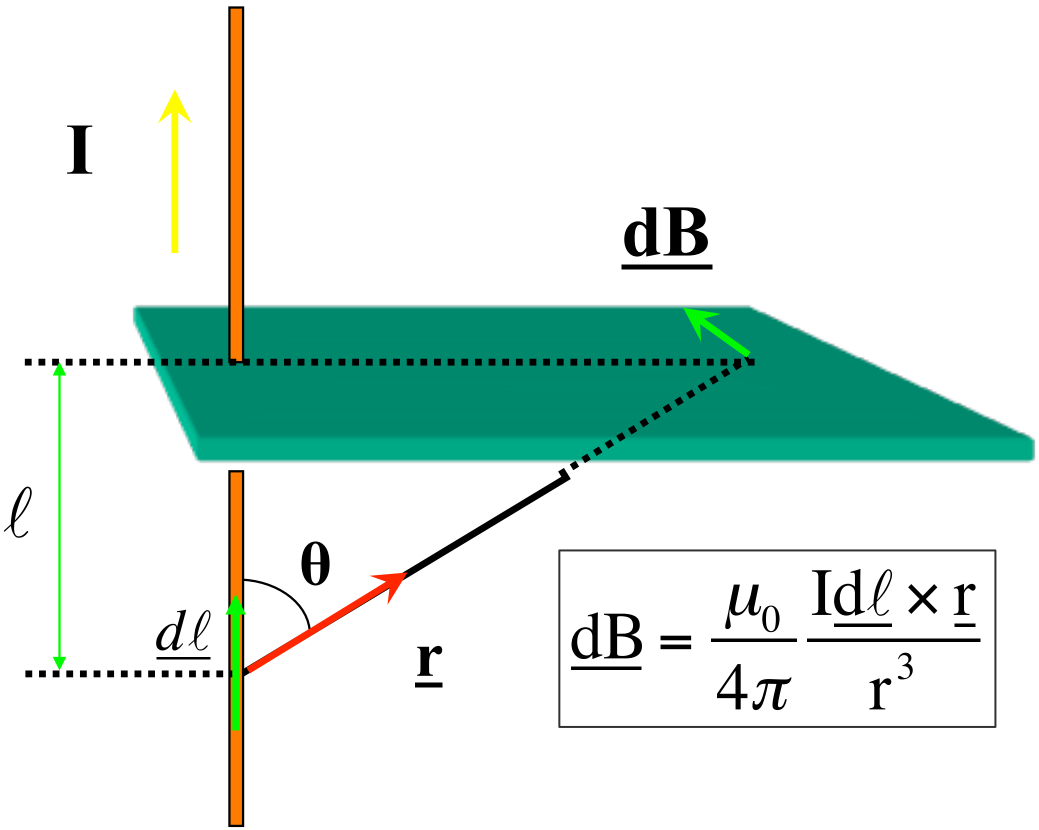
$$\boxed{\nabla \cdot \underline{J} = -\frac{\partial \rho}{\partial t} = 0}$$

How can we generate magnetic fields?

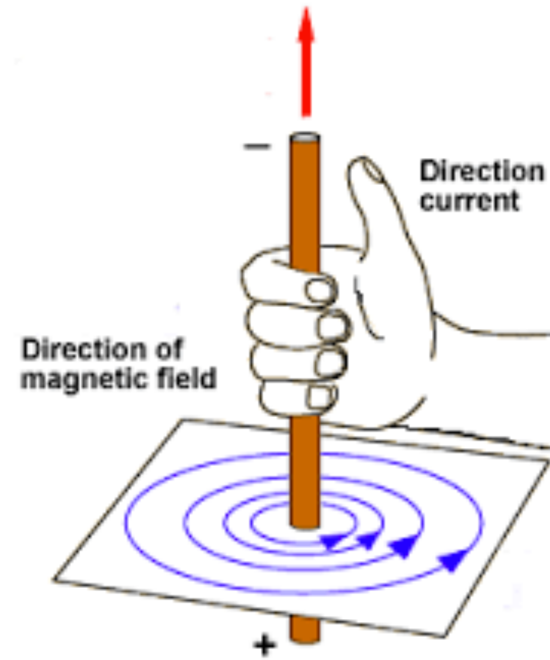
- permanent magnets
- currents
- variable electric fields *(later in the module)*

Biot-Savart law

The first useful relation that was empirically found is the so-called **Biot-Savart law,** which relates a steady current to the amount of magnetic field that it generates:



$$\underline{dB} = \frac{\mu_0}{4\pi}\frac{I d\underline{\ell} \times \underline{r}}{r^3}$$

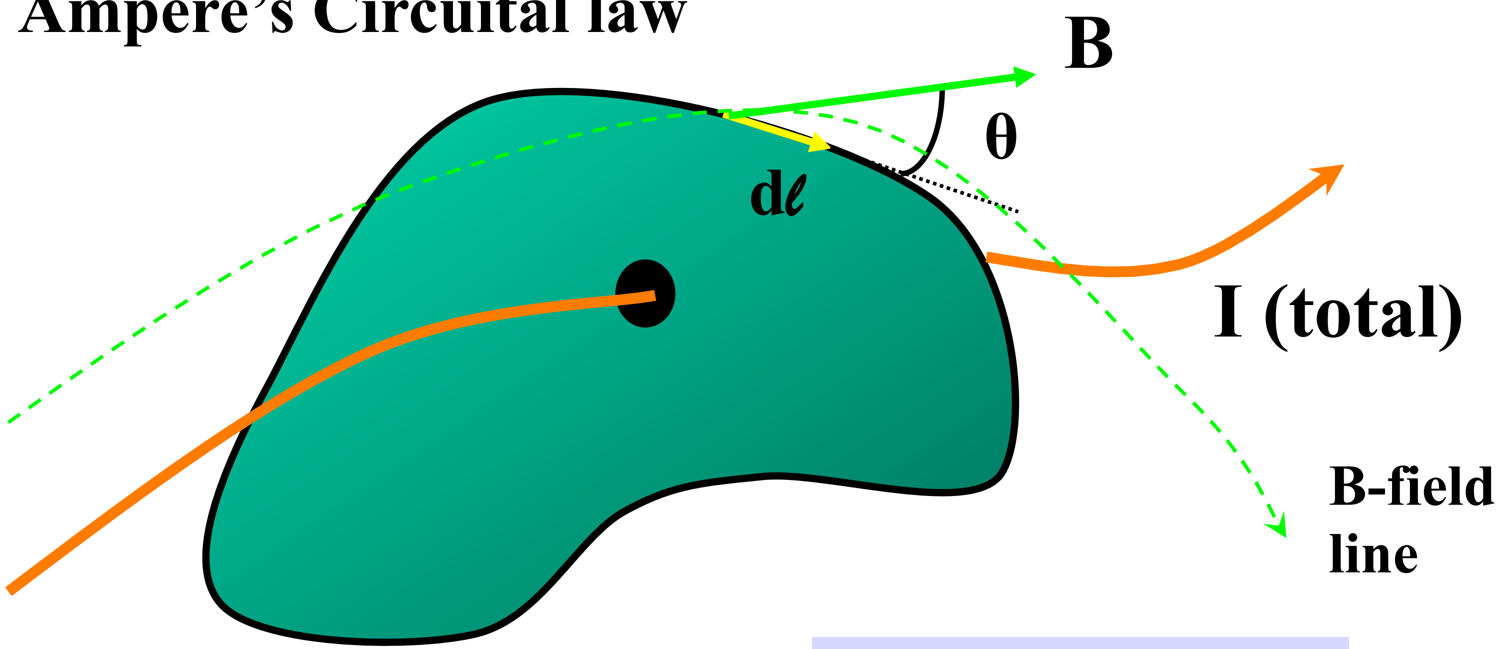


where $\mu_0 = 4\pi \times 10^{-7}$ Vs/Am is called **the permeability of free space**

Ampere’s law

The Ampere’s law is a generalisation of the Biot-Savart law, which is valid in any geometry:

Ampère’s Circuital law



Imaginary circuit

$$\int \underline{B}.\underline{d\ell} = \mu_0 I$$

Ampere's law

This law is somehow similar to Gauss' law to calculate electric fields

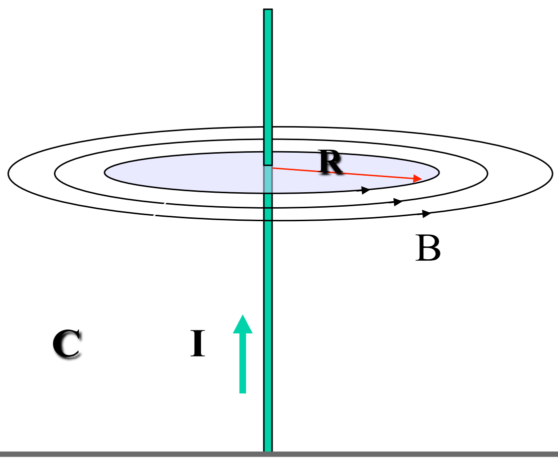
Gauss' law

$$\int_A \vec{E}\cdot d\vec{s} = Q/\varepsilon_0$$

Ampere's law

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$$

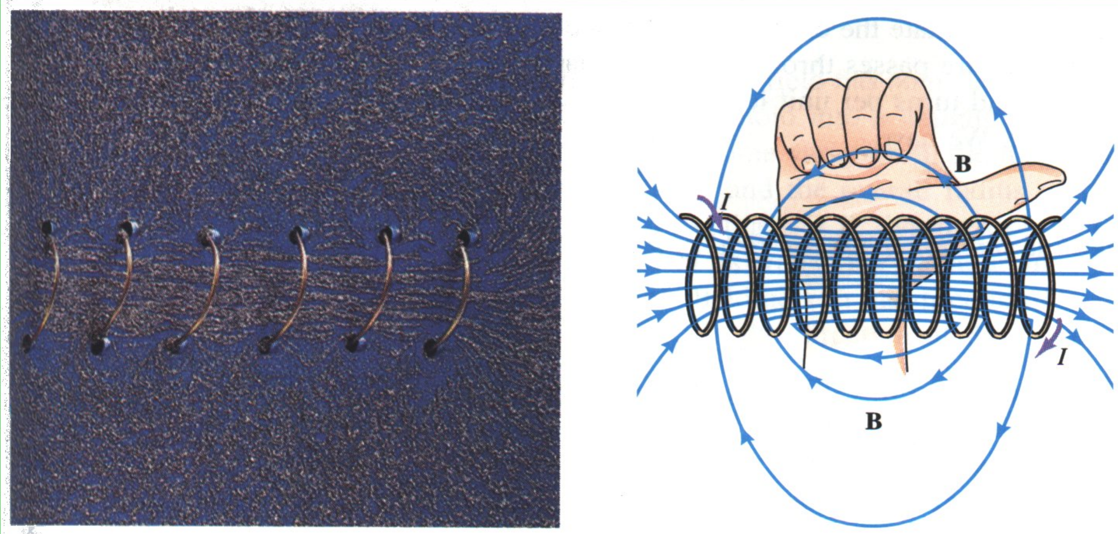
In the magnetic case though, we need to multiply the field by the line enclosing a surface, and not multiply by the surface enclosing a volume!



$$\oint_C \underline{B}.\underline{d\ell} = B_R 2\pi R = \mu_0 I$$

$$\therefore B_R = \mu_0 I / 2\pi R$$

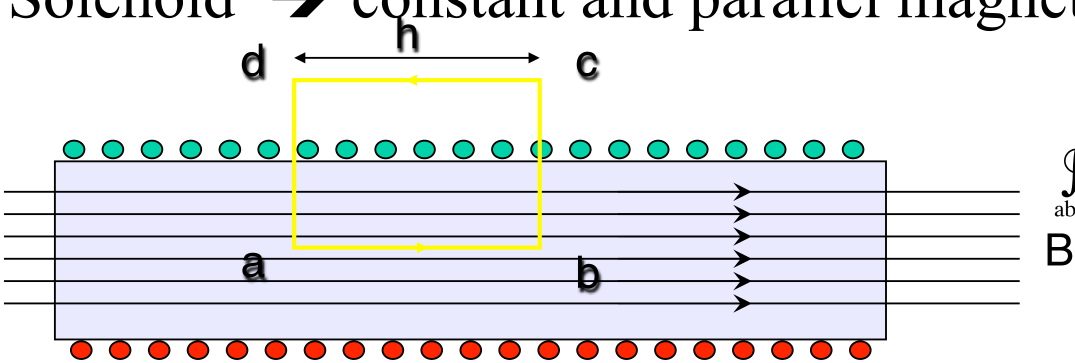
The solenoid



(a) (b)

Capacitor ➔ constant and parallel electrostatic field

Solenoid ➔ constant and parallel magnetic field



n turns/unit length each carries current I.

$$\oint_{abcd} \underline{B}.\underline{d\ell} = (Bh)_{ab} + (0)_{bc} + (0)_{cd} + (0)_{da} = \mu_0 nhI$$

$$\therefore B = \mu_0 nI$$