# PHY20003 ASTROPHYSICS

## Lecture 16 Colours and the H-R diagram

### 16.1 Magnitudes and colours

The colour of a light emitting object is determined by the relative fractions of light emitted at different wavelengths.

This can be expressed as a difference in magnitude (remember, a difference in magnitude is a ratio of fluxes). For example, in the UBV system previously discussed, we can write

$$m_U - m_B = -2.5\log f_U + \mathrm{C_U} + 2.5\log f_B - \mathrm{C_B} \quad (16.1)$$

$$(U - B) = -2.5\log\frac{f_U}{f_B} + C_{UB} \quad (16.2)$$

Similarly, we get

$$m_B - m_V = -2.5\log f_B + \mathrm{C}_B + 2.5\log f_V - \mathrm{C}_V \quad (16.3)$$

$$(B - V) = -2.5\log\frac{f_B}{f_V} + C_{BV} \quad (16.4)$$

Where we express the colour as $(U - B)$ and $(B - V)$, and these are sometimes known as the colour index.

Vega is frequently used as the primary standard, and all magnitudes for Vega are therefore defined as 0.00. The colours of Vega are therefore also 0.00, i.e. $(U - B) = 0.00$ and $(B - V) = 0.00$. (Although note that some magnitude systems, such as the AB magnitude system, use a different zeropoint)

Similarly, for observations in the Sloan filters (*u, g, r, i* & z) we can define e.g. $(g - r)$, $(r - i)$ etc.

### 16.2 Temperatures

If we approximate the star by a black body, then Wien's Displacement Law tells us that

$$\lambda_{max}T = constant \quad (16.5)$$

Thus, as a black body gets hotter, most of their flux gets emitted at shorter wavelengths. *This is also true for stars*, i.e. a hotter star will have bluer colours than a cooler star.

Consider a star with a a surface hotter than Vega. Its radiation will generally be emitted at shorter wavelengths in the blue region of the spectrum. Therefore, more light will pass through the B filter, and this implies tha $m_B$ is smaller. Less light will pass through the V-filter, and this implies that $m_V$ is larger. Thus, stars hotter than Vega have $(B - V) < 0.00$.

On the other hand, for stars that are cooler than Vega, less light passes through the B-filter, and therefore $m_B$ is larger, while more light will arrive through the V filter, and therefore $m_V$ is smaller. Therefore, stars cooler than Vega have $(B - V) > 0.00$.

For example, we know that the Sun has a surface temperature of almost 6000 K, and $(B - V)_\odot = 0.67$, we therefore know that the Sun is cooler than Vega, and $T_{eff,Vega} > 6000$ K.

## 16.3 The Hertzsprung-Russell Diagram

From photometry of a single star, we can measure two fundamental properties:

a) Luminosity $(M_{bol}, M_V)$ (this requires a distance measurement as well)
b) Temperature ( $(B - V), T_{eff}$ )

Hertzsprung and Russell were the first astronomers to plot these two quantities against each other. Instead of Finding a uniform spread across their graph, they found that stars were lying within well-defined areas.

About 90% of the stars appear to lie in a relatively thin band diagonally across the diagram, going from hot and bright blue stars to cool and dim red stars.

As the diagram is based on stars with random ages (except when looking at star-clusters, where stars form at the same time), this implies that either

a) 90% of the stars are born, live and die in this band
b) Stars will move across the diagram to different regions, but spend 90% of their lives in this band.

This diagonal band is known as the <u>main sequence</u>.

Typically, a distinction is made between the observational H-R diagram, which has the colour on the x-axis and the absolute magnitude on the y-axis, and the theoretical H-R diagram, which has the (log of the) temperature on the x-axis and the (log of the) luminosity on the y-axis. It should be noted that both show similar information.

The observational H-R diagram can be constructed directly from observation, and, in principle, it requires 3 measurements: photometry of the star in the 2 passbands for the colour and the parallax for the distance. For star clusters where stars are at (almost) the same distance, the parallax is not even required (unless you need to compare with detailed models), and the apparent magnitude can be used.

The theoretical H-R diagram relies on determining the intrinsic properties of the star – its effective temperature and its luminosity – to be determined. This requires more detailed observations and modelling.

## 16.4 Interpreting the H-R Diagram

The stars that lie above and below the main sequence have smaller or larger absolute magnitudes, respectively, corresponding to higher or lower luminosities.

However, they have the same $(B - V)$ colour as the main sequence stars, implying that they have the same temperature.

As the stars have the same temperature, their surface brightnesses $(\sigma T^4)$, are the same. To explain the differences in luminosity their radii must be different:

$$L_1 = 4\pi R_1^2 \sigma T_1^4 \qquad L_2 = 4\pi R_2^2 \sigma T_2^4 \tag{16.6}$$

If $T_1 = T_2$:

$$\frac{L_1}{L_2} = \frac{R_1^2}{R_2^2} \tag{16.7}$$

And therefore

$$R_1 = R_2 \sqrt{\frac{L_1}{L_2}} \tag{16.8}$$

Stars lying above the main sequence have radii larger than stars on the main sequence, they are therefore known as giant stars.

For giant stars:

$$L_* \sim 100 L_\odot \rightarrow R_* \sim 10 R_\odot$$

A few stars (by percentage) appear over 3-4 magnitudes brighter than the giant stars. These are called supergiants.

For supergiants:

$$L_* \sim 10{,}000 L_\odot \rightarrow R_* \sim 100 R_\odot$$

To differentiate them from giant stars (and continue the ‘sizest’ classification), main sequence stars are called dwarf stars. For example, our Sun is a yellow dwarf, and Proxima Centauri is a red dwarf star.

When investigating the HR diagram of nearby stars, we can see that there is a group of stars that in intrinsically faint, about 10-12 magnitudes below the main sequence. These stars have

$$L_* \sim 10^{-4} L_\odot \rightarrow R_* \sim 0.01 R_\odot \sim 1 R_\oplus$$

i.e. they are roughly the same size as the Earth. These stars are known as white dwarfs.