

# On cosmic and blackbody radiation.

Brian Rogers

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## 1 Abstract

This report aims to address two main objectives. Firstly, to verify the Stefan-Boltzman law, an underpinning equation of blackbody radiation theory. By a chi square test of fit method, we achieve a power law value of  $x = 4.02 \pm 0.04$  and an implied Stefan-Boltzman constant,  $\sigma = 5.75 \cdot 10^{-8} W m^2 K^{-4}$ . These results are consistent with existing consensus values.

Secondly, we measure the temperature of the cosmic background radiation. These photons permeate the universe and provide key evidence of the 'big bang' theory. By analysis of the data collected during the COBE experiment, we estimate that the background radiation follows a blackbody distribution with a temperature of  $T = 2.73 K \pm 0.00546 K$ . This is consistent within the uncertainties with measured by the COBE experiment.

## 2 Introduction

Blackbody radiation has played an integral role in modern physics. Inconsistencies with a theoretical blackbody and classical electromagnetic theory led Max Planck to consider the quantization of energy and laid the ground work for the oncoming "Quantum Revolution".[2] For his work, Planck received the Nobel prize in 1919. In addition to its role on the quantum scale, blackbody radiation has had an important contribution to astrophysics. Penzias and Wilson detected cosmic microwave background radiation and found it consistent with a blackbody at temperature  $T=3K$ . This was confirmed by the COBE satellite launched in 1989 which confirmed that the CMBR had a perfect blackbody spectrum with  $T=2.73K$ . Both scientific endeavours earned the Nobel prize in 1978 and 2006 respectively.

The radiance for a blackbody as a function of wavelength,  $\lambda$  and temperature,  $T$  is as follows:

$$I_{\lambda}(\lambda, T) = \frac{2hc^2}{\lambda^5 (e^{\frac{hc}{\lambda kT}} - 1)} \quad (1)$$

Simulating a theoretical blackbody spectrum at wavelength intervals of 10nm you can generate the following curve for  $T=5780K$ .

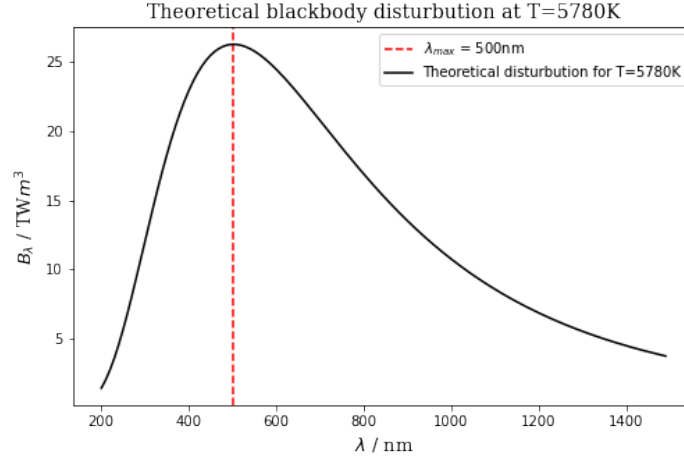


Figure 1: Example theoretical plot of a blackbody emission distribution for  $T=5780K$

The corresponding maximum wavelength,  $\lambda_{max} = 500nm$ . Comparing this to Wien's law which states:

$$\lambda_{max} = \frac{b}{T} \quad (2)$$

Where  $b = 2.90 \cdot 10^{-3} \text{ mK}$ . Wien's law yields  $\lambda_{max} = 501nm$ . This confirms our expectations for the theoretical spectrum.

Using a halogen lamp as a source and a spectrograph connected to the Spectralab software we compare our theoretical blackbody simulation to a physical approximation of a blackbody.

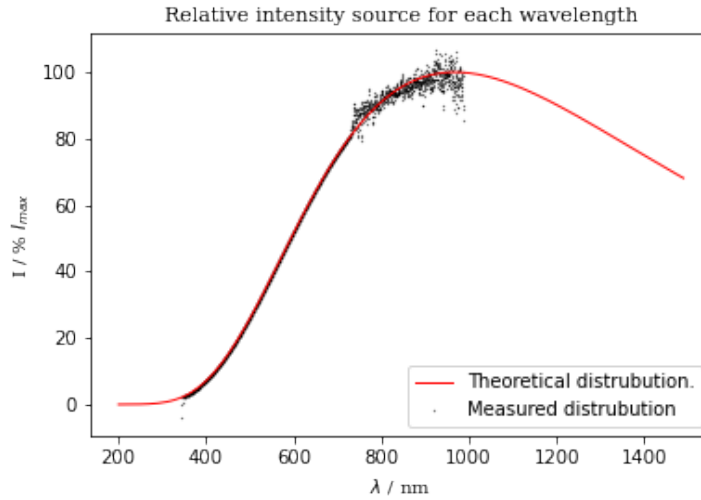


Figure 2: Comparison of theoretical blackbody spectrum and a spectrum produced by a blackbody at  $T=3000K$

The spectrograph had a detection limit of  $360nm < \lambda < 1000nm$ . However within these limits clear agree-

ment between the theoretical and experimental curves is demonstrated. With confidence in the theoretical simulation, we proceeded to address the key objectives of our experiment.

### 3 Verification of the Stefan-Boltzman law

#### 3.1 Methodology

We used a halogen lamp as a light source powered by a variable power supply. Using a connected voltmeter and ammeter, we calculated the power by the source to be:

$$P = IV \quad (3)$$

We used voltage values ranging from 12V to 6V at 1V intervals. After accounting for background effects and incorporating the sensitivity of the spectrograph for the relevant exposure time, we used the fitting software on Spectralab to record the temperature of the source. We repeated this for both wavelength and frequency regimes to gain additional temperature data and calculate an average temperature value for specific power values.

Once the data was collected we could begin to analyse the results. Starting with the Stefan-Boltzman law with the unknown exponent:

$$P = \sigma T^x \quad (4)$$

where  $\sigma$  is the Stefan-Boltzman constant and  $x$  is the constant to be determined.

Converting this equation into linear form we get:

$$\ln(P) = \ln(\sigma) + x \ln(T) \quad (5)$$

A plot of  $\ln(P)$  on  $\ln(T)$  yields a linear curve with gradient  $x$  and y-intercept of  $\ln(\sigma)$

#### 3.2 Results and analysis

Using our theoretical blackbody model, it was possible to compare observed values for the radiance to the expected theoretical values. A chi square method was used to calculate the optimal value of  $x = 4.02 \pm 0.04$ . The plot belows shows the optimal value occuring at the minimum of the graph.

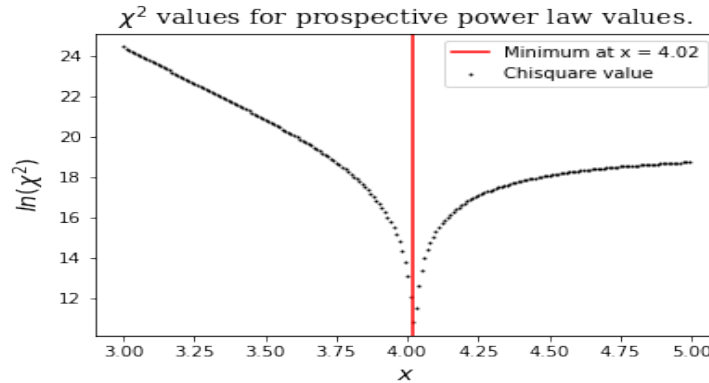


Figure 3: Iterations of  $\chi^2$  value for a range of temperature values used with equation (5)

The experimental results are as follows with associated line of best fit determined from the chi square fit process from above.

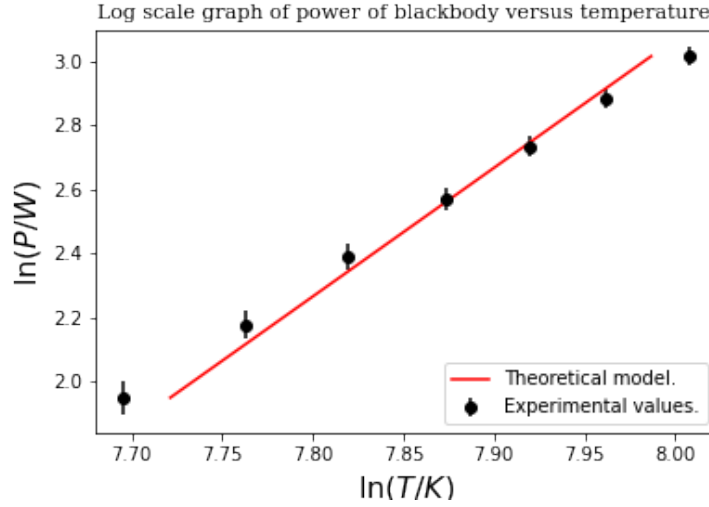


Figure 4: Power on Temperature on a natural log scale

Interpreting the y-intercept value as  $\ln(\sigma)$ , we get  $\sigma = 5.75 \cdot 10^{-8} W m^{-2} K^{-4}$

### 3.3 Discussion

The experimental values are in broad agreement with the currently accepted values. The exponent value of  $x = 4.02 \pm 0.04$  agrees with derived value of  $x = 4$  [4]. Whilst the implied Stefan-Boltzman constant has an error of +1.39%. [5] Limitations in our experimental method can explain in part why our values deviate above the accepted values.

During the experiment, noise detected on the intensity of the higher wavelengths increased significantly, making it more difficult to obtain a stable value for the temperature. We attempted to use a higher integration time but this often caused the software to crash. By using a computer with higher processing power we could obtain a higher resolution spectrum.

Further, error may be induced by not incorporating the efficiency of the lamp. An efficiency of less than 1 would mean that power values used are overestimated. This is visible by close up inspection of a figure 2, where the intensity values are lower for each frequency. By changing equation (3) used to calculate power to:

$$P = \eta IV \quad (6)$$

Where  $\eta$  is the efficiency of the lamp, we could obtain a more accurate value for  $x$  and  $\sigma$ .

We could also attribute error to fluctuations in the background radiation. By increasing the number of times we correct for background radiation, we could improve our measurements. An alternative would be to enclose the lamp in opaque box and remove any background fluctuations in the visible region.

## 4 Measurement of the temperature of the cosmic background radiation

### 4.1 Methodology

We used a Chi square fit method to determine the optimal temperature value for the COBE dataset, again utilising our theoretical simulation for the blackbody for a range of temperature values. The minimum chi square value corresponded to a temperature of  $T=2.71\text{K}$ . It was then possible to generate the best fit curve for the COBE dataset.

### 4.2 Results and analysis

The chi square values can be shown for various temperature values.

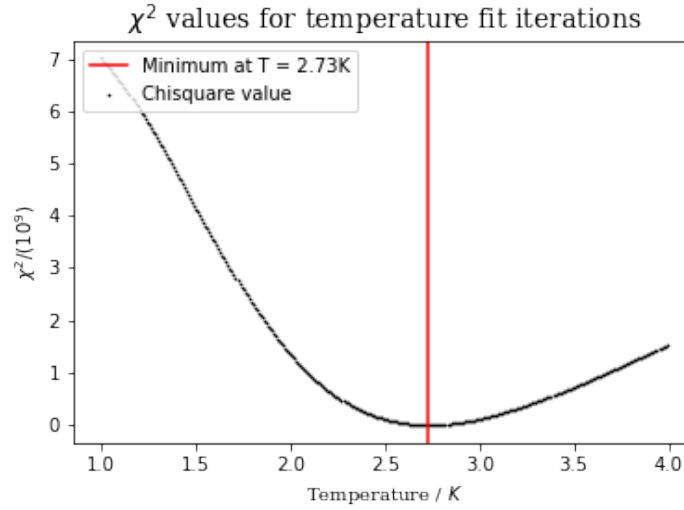


Figure 5: Plot of Chi square values obtained from comparing our COBE dataset and theoretical blackbody simulation.  $\chi^2_{min}$  yields  $T = 2.71\text{K}$

The corresponding best curve fit can then be plotted:

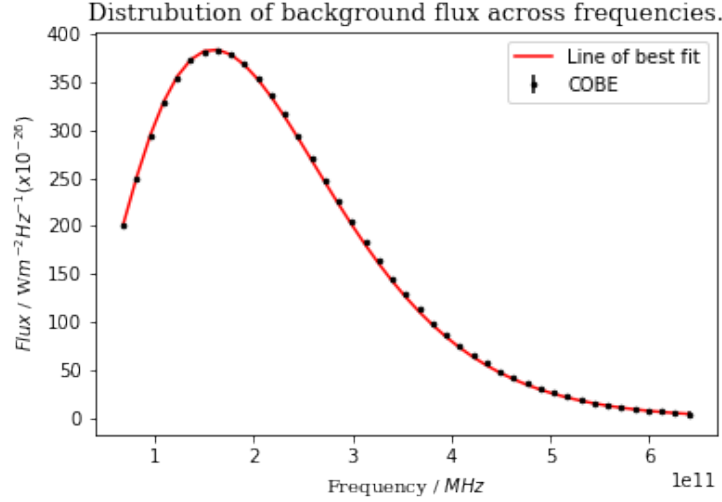


Figure 6: COBE experimental data with line of best fit obtained by the chi square method.

### 4.3 Discussion

In comparison with the Mather et. al our value in is broad agreement with their result of  $T=2.725 \pm 0.06K$  [1]. However, we lack the accuracy of their measurement. This is primarily due to the fact that the subset of that we use has 42 measurements. By obtaining and using a larger dataset, we could improve the accuracy of our measurement. [3]

## 5 Conclusion

We have addressed both our objectives: to verify the Stefan-Boltzman law and to measure the cosmic microwave background radiation. Although broad agreement has been demonstrated with the currently accepted measurements, we have suggested a number of improvements to our experimental and analytical methods for future experimentation.

## References

- [1] Mather et. al. "A Preliminary Measurement of the Cosmic Microwave Background Spectrum by the Cosmic Background Explorer (COBE) Satellite". In: *Astrophysical Journal Letters* v.354 (1990).
- [2] J. Baggot. "The Quantum Story". In: Oxford University Press, 2011. Chap. 1, p. 15.
- [3] *Blackbody report notebook*. <https://www.github.com/>. 2021.
- [4] L. Boltzman. "Ableitung des Stefan'schen Gesetzes, betreffend die Abhängigkeit der Wärmestrahlung von der Temperatur aus der electromagnetischen Lichttheorie". In: *Annalen der Physik* 258.6 (1884).
- [5] T. J. Quinn and J. E. Martin. "A Radiometric Determination of the Stefan-Boltzmann Constant and Thermodynamic Temperatures between -40 degrees C and +100 degrees C". In: *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences* 316.1536 (1985).