On cosmic and blackbody radiation.

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1 Abstract

This report aims to address two main objectives. Firstly, to verify the Stefan-Boltzman law, an underpinnnig equation of blackbody radiation theory. By a chi square fit method, we achieve a power law value of $x = 4.02 \pm 0.04$ and an implied Stefan-Boltzan constant, $\sigma = 5.75 \cdot 10^{-8} W m^2 K^{-4}$. These results are consistent with existing the consensus values. [4] [6]

Secondly, we measure the temperature of the cosmic background radiation. These photons permeate the universe and provide key evidence of the 'big bang' theory. By analysis of the data collected during the COBE experiment, we estimate that the background radiation follows a blackbody distrubution with a temperature of $T = 2.73K \pm 0.00546K$. This is consistent within the uncertainties with the measurements produced by the COBE experiment. [1]

2 Introduction

Blackbody radiation has played an integral role in modern physics. Inconstitencies between a theoretical blackbody and classical electromagnetic theory led Max Planck to consider the quantization of energy and laid the ground work for the oncoming "quantum revolution".[3] For his work, Planck recieved the Nobel prize in 1919. In addition to it's role in quantum physics, blackbody radiation has had important contributions to astrophysics. Penzias and Wilson first detected cosmic microwave background radiation and found it consistent with a blackbody at temperature T=3.5K. [5] This result was refined by the COBE satellite launched in 1989, showing CMBR follwed a perfect blackbody spectrum with T=2.73K. [1]. Both scientific endeavours earned the Noble prize in 1978 and 2006 respectively.

The radiance for a blackbody as a function of wavelength, λ and temperature, T is as follows:

$$I_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5 (e^{\frac{hc}{\lambda k T}} - 1)} \tag{1}$$

Simulating a theoretical blackbody spectrum at wavelength intervals of 10nm you can generate the following curve for T=5780K.

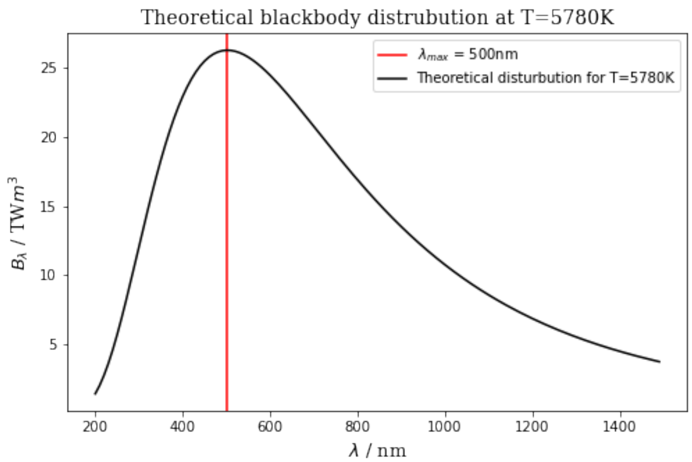

Figure 1: Example theoretical plot of a blackbody emission distrubution for T=5780K

The corresponding maximum wavelength, λ_{max} = 500nm. Comapring this to Wien's law which states:

$$\lambda_{max} = \frac{b}{T} \tag{2}$$

Where $b = 2.90 \cdot 10^-3$ mK . Wien's law yields λ_{max} = 501nm. This confirms our expectations for the theoretical spectrum.

Using a halogen lamp as a source and a spectrograph connected to the Spectralab software we compare our theoretical blackbody simulation to a physical approximation of a blackbody.

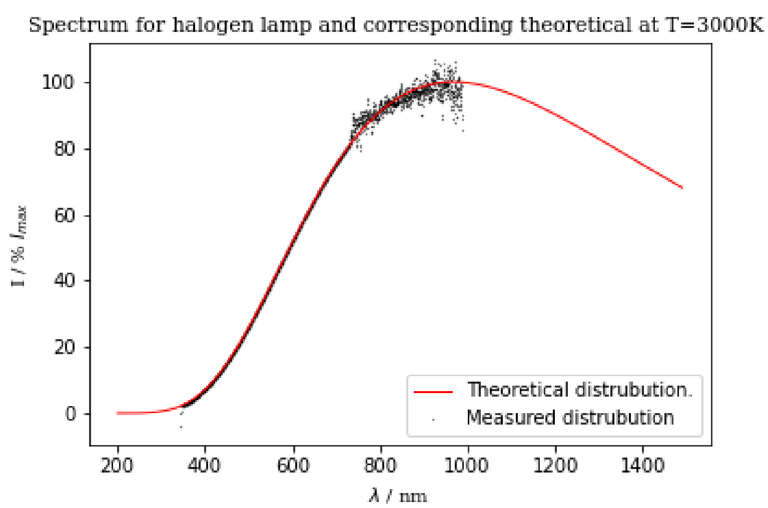

Figure 2: Comparison of theoretical blackbody spectrum and a spectrum produced by a blackbody at T=3000K

The spectrograph had a dection limit of $360nm < \lambda < 1000nm$. However within these limits clear aggrement between the theoretical and experimental curves is demonstrated. With confidence in the theoretical model, we preceeded to the address the key objectives of our experiment.

3 Verification of the Stefan-Boltzman law

3.1 Methodology

We used a halogen lamp as a light source powered by a variable power supply. Using a a connected voltmeter and ammeter, we calculated the power by the source to be:

$$P = IV \tag{3}$$

We used voltage values ranging from 12V to 6V at 1V intervals. After accounting for background effects and incorporating the sensitivity of the spectrogrpah for the relevant exposure time, we used the fitting software on Spectralab to record the temperature of the source. We repeated this for both wavelength and frequency regimes to gain additional temperature data and calculate an average temperature value for specific power values.

Once the data was collected we could begin to analyse the results. Starting with the Stefan-Boltzman law with the unknown exponent:

$$P = \sigma T^x \tag{4}$$

where σ is the Stefan-Boltzman constant and x is the constant to be determined.

Converting this equation into linear form we get:

$$\ln(P) = ln(\sigma) + x\ln(T) \tag{5}$$

A plot of $\ln(P)$ on $\ln(T)$ yields a linear curve with gradient x and y-intercept of $\ln(\sigma)$.

3.2 Results and analysis

Using our theoretical blackbody model, it was possible to compare observed values for the radiance to the expected theoretical values. A chi square method was used to calculate the optimal value of $x = 4.02 \pm 0.04$. The plot belows shows the value of x which yields the minimum chi square value.

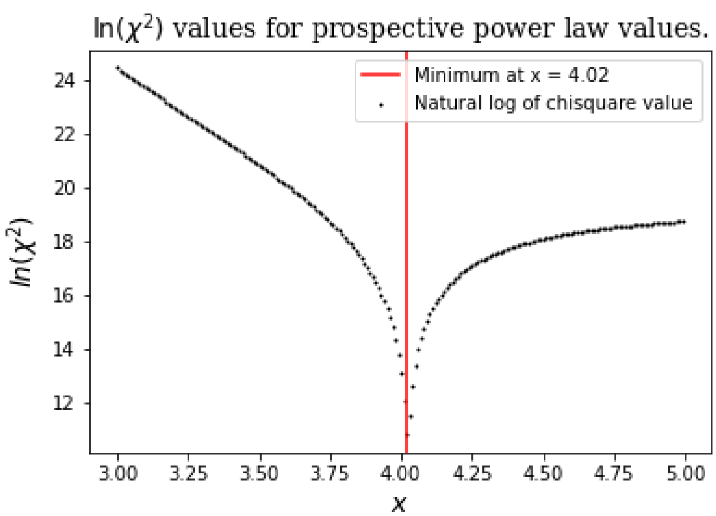

Figure 3: Iterations of χ^2 value for a range of temperature values used with equation (5)

The experimental results are as follows with associated line of best fit determined from the chi sqaure fit process from above.

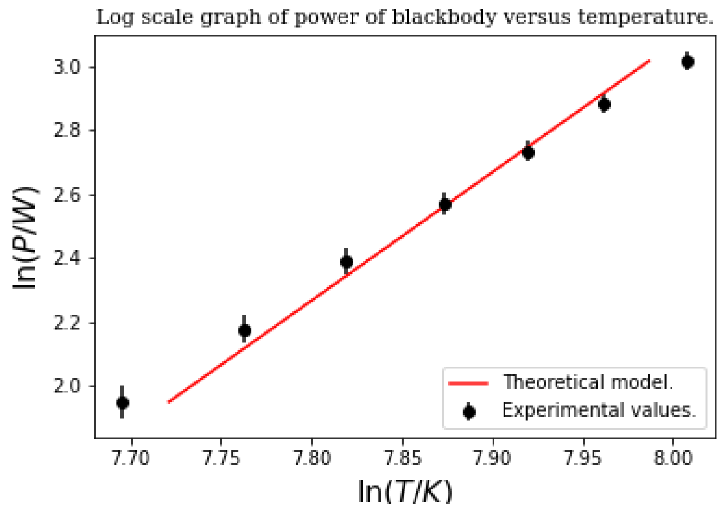

Figure 4: Power on Temperature on a natural log scale

Interpretting the y-intercept value as $\ln(\sigma)$, we get $\sigma = 5.75 \cdot 10^{-8} W m^{-2} K^{-4}$

3.3 Discussion

The experimental values are in broad agreement with currently accepted values. The exponent value of x=4.02 ±0.04 agrees with derived value of x=4 [4]. Whilst the implied Stefan-Boltzman constant has an error of +1.39%. [6] Limitations in our experimental method can explain in part why our values deviate above the accepted values.

During the experiment, noise detected on the intensity of the higher wavelengths increased significantly, making it more diificult to obtain a stable value for the temperature. We attempted to use a higher integration time but this often caused the software to crash. By using a computer with higher processing power we could obtain a higher resolution spectrum.

Further, error may be induced by not incorporating the efficiency of the lamp. An efficiency of less than 1 would mean that power values used are overestimated. This is visible by close up inspection of a figure 2, where the intensity values are lower for each frequency. By changing equation (3) used to calculate power to:

$$P = \eta I V \tag{6}$$

Where η is the efficiency of the lamp, we could obtain a more accurate value for x and σ.

We could also attribute error to fluctuations in the background radiation. By increasing the number of times we correct for background radiation, we could improve our measurements. An alternative would be to enclose the lamp in opaque box and remove any background fluctuations in the visible region.

4 Measurement of the temperature of the cosmic background radiation

4.1 Methodology

We used a Chi square fit method to determine the optimal temperature value for the COBE dataset, again utilising our theoretical simulation for the blackbody for a range of temperature values. The minimum chi square value corresponded to a temperature of T=2.73K ± 0.00546K. It was then possible to generate the best fit curve for the COBE dataset. [2]

4.2 Results and analysis

The chi square values can be shown for various temperature values.

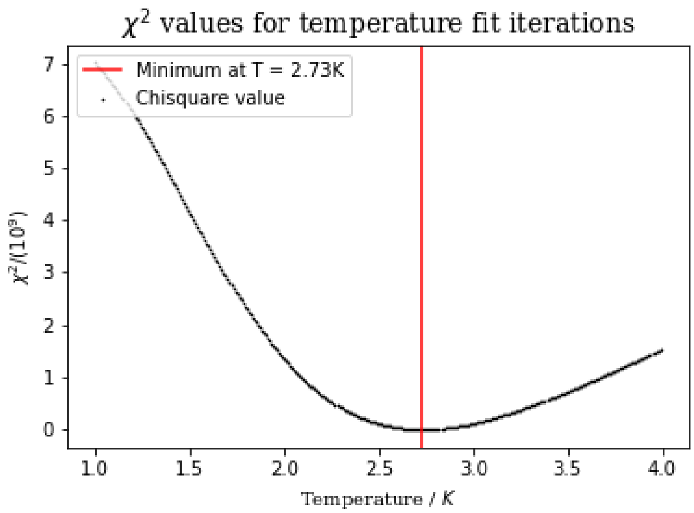

Figure 5: Plot of Chi square values obtained from comparing our COBE dataset and theoretical blackbody simulation. χ^2_{min} yields T = 2.73K

The corresponding best curve fit can then be plotted:

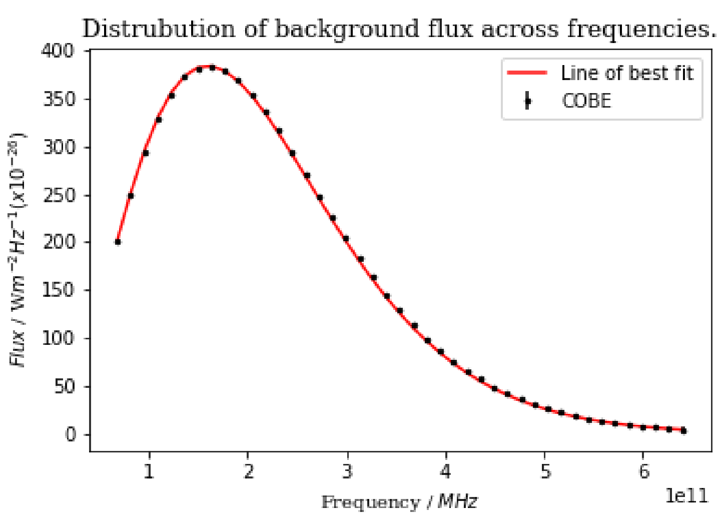

Figure 6: COBE experimental data with line of best fit obtained by the chi square method.

4.3 Discussion

In comparison with the Mather et. al our value in is broad agreement with their result of T=2.725 $\pm$ 0.06K [1] within our uncertainties. However, we lack the precision of their measurement. This is primarily due to the fact that the subset of that we use has only 42 measurements. By obtaining and using a larger COBE dataset, we could improve the precision of our calculation.

5 Conclusion

We have addressed both our objectives: to verify the Stefan-Boltzman law and to measure the cosmic microwave background radiation. Although broad agreement has been demonstrated with currently accepted values, we have suggested a number of improvements to our experimental and analytical methods for future experimentation.

References

[1] Mather et. al. "A Preliminary Measurement of the Cosmic Microwave Background Spectrum by the Cosmic Background Explorer (COBE) Satellite". In: *Astrophysical Journal Letters* v.354 (1990).

[2] *Analysis available here if required - Blackbody report notebook*. `https://bit.ly/3FNF7RH`. 2021.

[3] J. Baggot. "The Quantum Story". In: Oxford University Press, 2011. Chap. 1, p. 15.

[4] L. Boltzman. "Ableitung des Stefan'schen Gesetzes, betreffend die Abhängigkeit der Wärmestrahlung von der Temperatur aus der electromagnetischen Lichttheorie". In: *Annalen der Physik* 258.6 (1884).

[5] A. A. Penzias and R. W. Wilson. "A Measurement of Excess Antenna Temperature at 4080 Mc/s." In: *Astrophysical Journal* 142 (1965).

[6] T. J. Quinn and J. E. Martin. "A Radiometric Determination of the Stefan-Boltzmann Constant and Thermodynamic Temperatures between -40 degrees C and +100 degrees C". In: *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences* 316.1536 (1985).