Agenda

1) Addition & Multiplication Rule
2) Permutation Basics
3) Combination & Properties
4) Pascal Triangle
5) Nth column title.

Addition & Multiplication Rule

Q1 In a class → 10 girls & 7 boys.

Find the total no. of ways to form a couple.

Boy	Girl
B_1	$\{G_1, G_2 \dots G_9, G_{10}\}$
B_2	$\{G_1, G_2 \dots G_9, G_{10}\}$
⋮	
B_7	$\{G_1, G_2 \dots G_9, G_{10}\}$

Boy		Girl
7	×	10

$= \underline{\underline{70}}$

AND

To form a couple $\Rightarrow$ Select a boy & Select a girl.

$$(7) \times (10)$$

OR

$B1 \quad \{G_1, G_2 \dots G_9, G_{10}\} = 10$

OR

$B2 \quad \{G_1, G_2 \dots G_9, G_{10}\} = 10$

OR

$B3 \quad \{G_1, G_2 \dots G_9, G_{10}\} = 10$

$\vdots \qquad\qquad\qquad\qquad \vdots$

OR

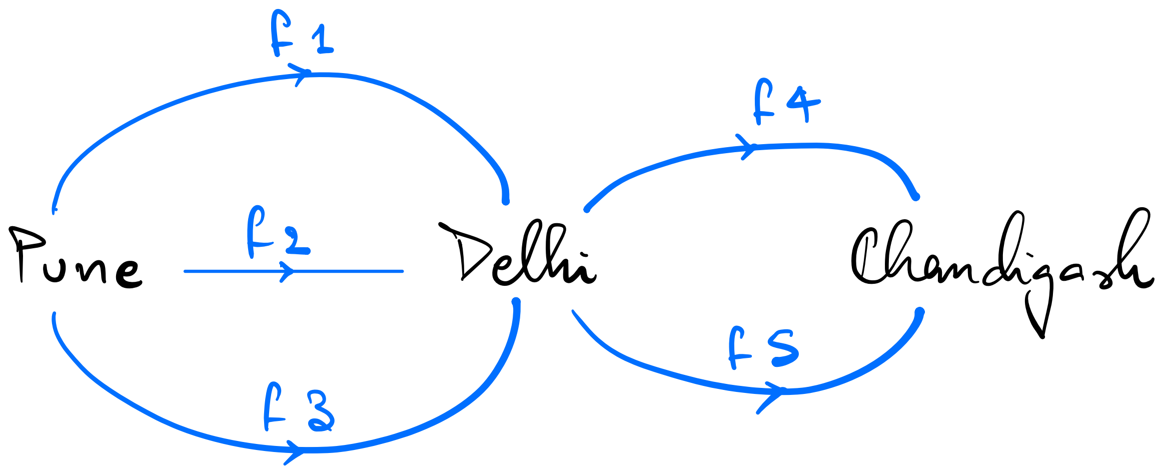
$37 \quad \{G_1, G_2 \dots G_9, G_{10}\} = 10$

$\underline{\underline{70}}$

AND $\Rightarrow$ Multiply

OR $\Rightarrow$ Add.

Q2



Total no. of different ways to reach Chandigarh

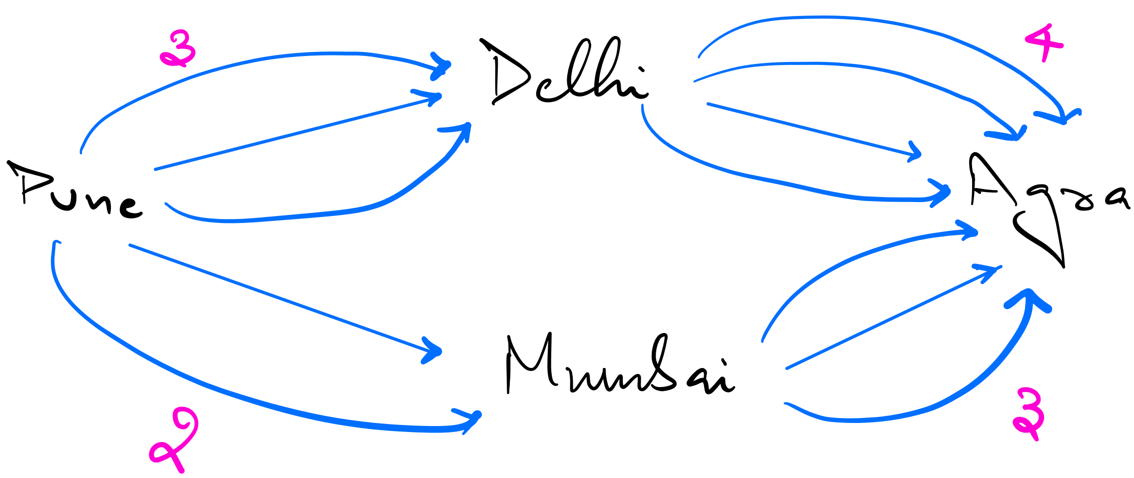
Pune → Delhi & Delhi to Chand.

$$3 \times 2$$

$$= 6 \text{ ways.}$$

f1 f4   f2 f4   f3 f4

f1 f5   f2 f5   f3 f5



Goal : Travel from Pune to Agra.

via Delhi **or** via Mumbai

$P \to D$ & $D \to A$

$3 \times 4 = 12$

+

$P \to M$ & $M \to A$

$2 \times 3 = 6$

$= 18$ ways.

Permutation

Arrangement of objects where the order matters.

Q Given 3 different characters. In how many ways can you form a string of size 3??

S = "a b c"

$\underline{3} \times \underline{2} \times \underline{1} = 6 = 3!$

a → b → c abc

a → c → b acb

$b \rightarrow a \rightarrow c$ bac

$b \rightarrow c \rightarrow a$ bca

$c \rightarrow b \rightarrow a$ cba

$c \rightarrow a \rightarrow b$ cab.

N Distinct Elements & N positions

$$\underline{N} \times \underline{N-1} \times \underline{N-2} \;\ldots\ldots\; \underline{1} = N!$$

for 4 distinct characters $= 4!$

Q = Given N distinct char but only R positions where $(R <= N)$

figure out the total no. of ways to arrange R out of these N characters.

Eg = date $N = 4$ $\underline{4} \times \underline{3} = 12$ $R = 2$

Let's generalise this.

N Elements $\qquad$ R positions.

$$\underset{0}{\underline{N}} \times \underset{1}{\underline{(N-1)}} \times \underset{2}{\underline{(N-2)}} \cdots\cdots \underset{R-2}{\underline{(N-R+2)}} \; \underset{R-1}{\underline{N-(R-1)}}$$

R position

$$^{N}P_{R}$$

$$\|$$

$$N \times (N-1) \times (N-2) \ldots\ldots (N-R+2) \times (N-R+1)$$

$$N! = N \times (N-1) \times (N-2) \ldots (N-R+2) \times (N-R+1) \times \underbrace{(N-R) \times (N-R-1) \ldots\ldots 1}_{(N-R)!}$$

$$^{N}P_{R} = N \times (N-1) \times (N-2) \ldots\ldots (N-R+1)$$

$$^{N}P_{R} = \frac{N!}{(N-R)!}$$

$\Downarrow$

No. of ways to arrange R out of N elements.

Break 10:20 PM.

Combination $(N \geq R)$

No. of ways to select something

Order of selection does not matter

$(i, j) = (j, i)$

Ex Sport with a team of 3 players. (3×100 m relay race)

$4: \{P_1, P_2, P_3, P_4\}$

$\Downarrow$

Select 3 out of 4 players.

(P_4)	P_1	P_2	P_3
(P_3)	P_1	P_2	P_4
(P_2)	P_1	P_3	P_4
(P_1)	P_2	P_3	P_4

} 4 ways.

$$^4C_3 = 4$$

Select $\underset{\text{Ele}}{R}$ out of $\underset{\text{Ele}}{N}$ $= {}^NC_R$

Arrangement

$P_1\ P_2\ P_3$	$P_1\ P_2\ P_4$	$P_1\ P_3\ P_4$	$P_2\ P_3\ P_4$
$P_1\ P_2\ P_3$			
$P_1\ P_3\ P_2$			
$P_2\ P_1\ P_3$			
$P_2\ P_3\ P_1$			
$P_3\ P_1\ P_2$			
$P_3\ P_2\ P_1$			

24 ways.

$$^{N}C_R \times (R!) = {}^{N}P_R$$

$$^{N}C_R \times R! = \frac{N!}{(N-R)!}$$

$$\boxed{^{N}C_R = \frac{N!}{(N-R)! \times (R!)}}$$

Properties of Combination

$0! = 1$

1) $^{N}C_0 = 1$

$$\frac{N!}{0! \times (N-0)!} = \frac{\cancel{N!}}{\cancel{N!}} = 1$$

2) $${}^{N}C_{N} = 1$$ (Selecting N out of N elements)

$$\frac{\cancel{N!}}{\cancel{N!} \times \cancel{(N-N)!}^{\,1}} = 1$$

3) $${}^{N}C_{R} = {}^{N}C_{N-R}$$

put $R = (N-R)$

$${}^{N}C_{R} = \frac{N!}{(R!)\,(N-R)!}$$

$R = N-R$

$${}^{N}C_{N-R} = \frac{N!}{(N-R)! \times (N-(N-R))!} = \frac{N!}{(N-R)! \times R!}$$

---

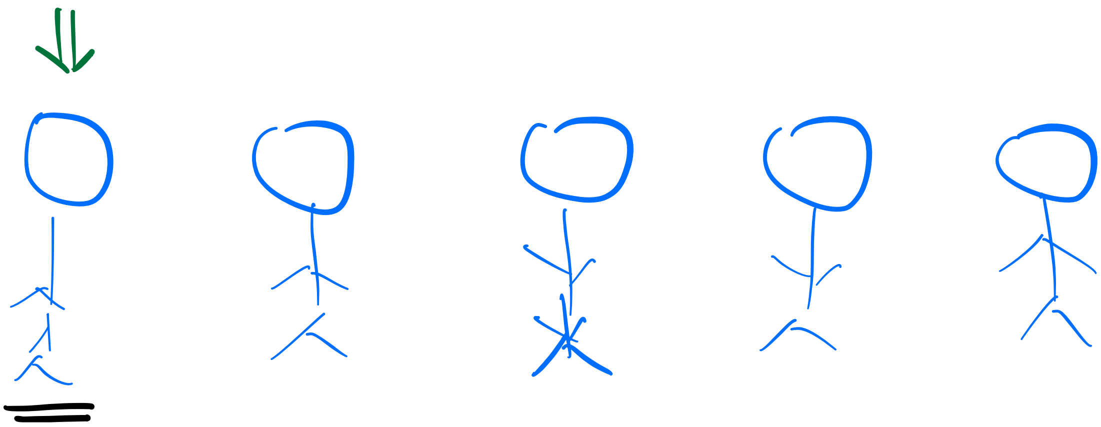
4) Given N elements. Select R out of these N elements

$\underline{1}$ $\underline{2}$ $\underline{3}$ . . . . $\underline{N-2}$ $\underline{N-1}$ $\underline{N}$

Select or Not select.

$$ {}^{N}C_R = {}^{N-1}C_{R-1} + {}^{N-1}C_R $$

$$ \boxed{{}^{N}C_R = {}^{N-1}C_{R-1} + {}^{N-1}C_R} $$



Stop / Not Stop

$$ {}^{5}C_3 = {}^{4}C_2 + {}^{4}C_3 $$

Q Pascal's Triangle

Given an integer N. Generate the pascal triangle.

$N = 4$

$$
\begin{array}{c}
{}^{0}C_{0} \\
{}^{1}C_{0} \quad {}^{1}C_{1} \\
{}^{2}C_{0} \quad {}^{2}C_{1} \quad {}^{2}C_{2} \\
{}^{3}C_{0} \quad {}^{3}C_{1} \quad {}^{3}C_{2} \quad {}^{3}C_{3} \\
{}^{4}C_{0} \quad {}^{4}C_{1} \quad {}^{4}C_{2} \quad {}^{4}C_{3} \quad {}^{4}C_{4}
\end{array}
$$

5 x 5 matrix

	0	1	2	3	4
0	$^{0}C_{0}$				
1	$^{1}C_{0}$	$^{1}C_{1}$			
2	$^{2}C_{0}$	$^{2}C_{1}$	$^{2}C_{2}$		
3	$^{3}C_{0}$	$^{3}C_{1}$	$^{3}C_{2}$	$^{3}C_{3}$	
4	$^{4}C_{0}$	$^{4}C_{1}$	$^{4}C_{2}$	$^{4}C_{3}$	$^{4}C_{4}$

$$^{N}C_{0} = {}^{N}C_{N} = 1$$

$$M[i][j] = {}^{i}C_{j} = \frac{i!}{j! \times (i-j)!}$$

	0	1	2	3	4
0	1				
1	1	1			
2	1	2	1		
3	1	3	3	1	
4	1	4	6	4	1

[1, 1]

[1, 2]

[1, 3]

ith → [1, i-1)

$${}^{i}C_{j} = {}^{i-1}C_{j-1} + {}^{i-1}C_{j}$$

$$M[i][j] = M[i-1][j-1] + M[i-1][j]$$

Code

```
int[][] pascalTriangle (int N) {
    int M [N+1][N+1] = {0};
    for (i = 0; i <= N; i++) {
        M[i][0] = 1;
        M[i][i] = 1
        for (j = 1; j < i; j++) {
            M[i][j] = M[i-1][j-1] + M[i-1][j];
        }
    }
    return M;
}
```

	0	1	2	3	. . . - -											
0	0															
1	0	1														
2	0	1	1	0												
3	0	1	1	0	1	0	0	1								
4	0	1	1	0	1	0	0	1	1	0	0	1	0	1	1	0
	0	1	2	3	4	5										

i $\quad$ $2 \times i$ $\quad$ $(2 \times i + 1)$

$0 \rightarrow 0, \ 1$

$1 \rightarrow 2, 3$

$2 \rightarrow 4, 5$

$3 \rightarrow 6, 7$

$i/2 =$

i