

Decimal Number System

Digits $\Rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Symbols

Total no. of unique symbols = Base = 10
in any number system

$$342 = 300 + 40 + 2 = 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$
$$2563 = 2000 + 500 + 60 + 3 = 2 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 3 \times 10^0$$

Octal No. System = $\{0, 1, 2, 3, 4, 5, 6, 7\}$
(Base = 8)

$$(1067)_8 = 1 \times 8^3 + 0 \times 8^2 + 6 \times 8^1 + 7 \times 8^0$$

=

Hexadecimal No. System.
(16 Base)

$\{0-9, A, B, C, D, E, F\}$

732 logs of wood.

↓ Bundles of 10

73 bundles & 2 logs

↓ Packet of 10 bundles

7 packets & ~~3~~⁴ bundles & 2 logs.

$$\begin{array}{r} + 9 \\ \hline 11 \end{array} (10 + 1)$$

Binary Number System

Base = 2

Unique Symbols = Bits = $\{0, 1\}$

$$\begin{array}{cccc} \textcolor{violet}{3} & \textcolor{violet}{2} & \textcolor{violet}{1} & \textcolor{violet}{0} \\ \left(\begin{array}{cccc} 1 & 0 & 1 & 1 \end{array} \right)_2 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ 2^3 \quad 2^2 \quad 2^1 \quad 2^0 \end{array} = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$
$$= 8 + 0 + 2 + 1$$
$$= 11$$

Binary to Decimal Conversion

$$\textcircled{1} \quad \begin{array}{cccccc} \textcolor{violet}{4} & \textcolor{violet}{3} & \textcolor{violet}{2} & \textcolor{violet}{1} & \textcolor{violet}{0} \\ (10101)_2 \end{array} = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 16 + 0 + 4 + 0 + 1$$

$$= 21$$

$$(1011010)_2 = 1 \times 2^6 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1$$

$$= 64 + 16 + 8 + 2$$

$$= 90$$

Decimal To Binary Conversion

2	20		→	0
2	10		→	0
2	5		→	1
2	2		→	0
2	1		→	1
	<u>0</u>			

$= (10100)_2$

$$20 - 2^4 = 4 - 2^2 = 0$$

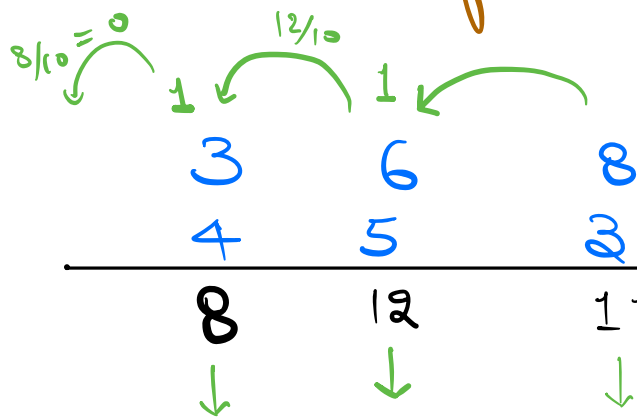
$$(10100)_2$$

Binary representation of 45

2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

$(101101)_2$

Addition of Decimal Numbers



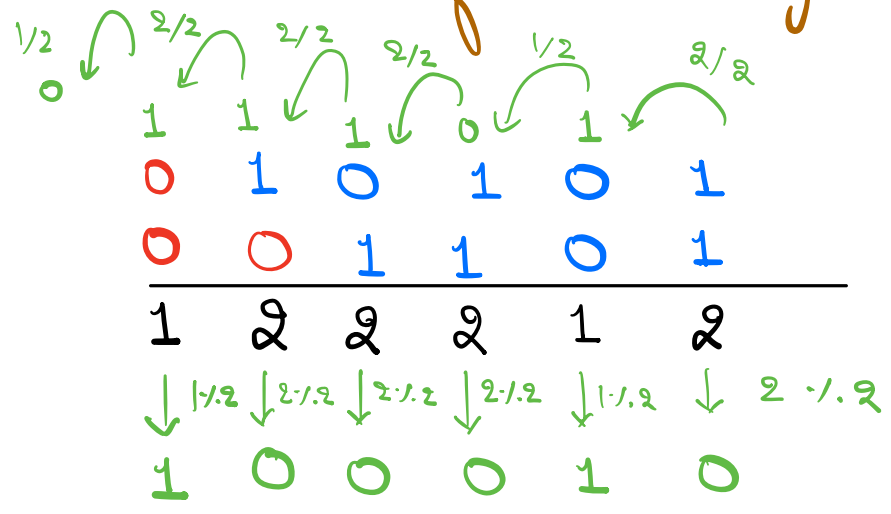
$\frac{11}{10} \Rightarrow \text{Carry}$
 $= \frac{\text{Result}}{\text{Base}}$

8	12	11
↓	↓	↓
$8 \div 10 = 8$	$12 \div 10 = 2$	$11 \div 10 = 1$

$11 \div 10 \Rightarrow \text{Answer}$
 $\underline{1} = \text{Result} \div \text{Base}$

Addition of Binary Numbers

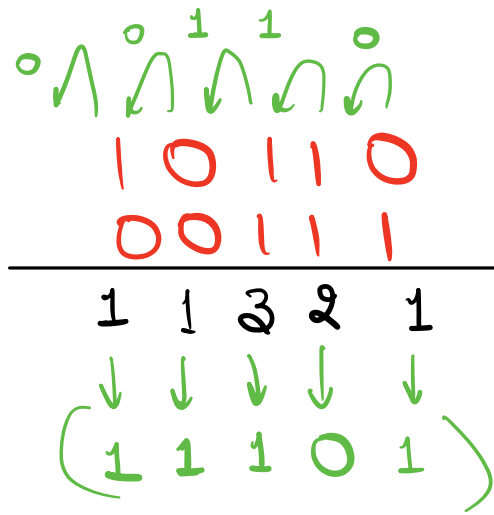
Numbers
 (Base = 2)



21
 13

34

Quiz



Transistors \Rightarrow Measure the amount of voltage



(Decay in Transistors over time)



{0, 1, 2, 3, 4}

Bitwise Operators.

AND, OR, XOR, NOT.
(&), (|), (^), (~)

Bit

0 \rightarrow False / Unset

1 \rightarrow True / Set

1) NOT (\sim / !)

$$\begin{aligned}\sim 0 &= 1 \\ \sim 1 &= 0\end{aligned}$$

Bits $A \neq B$

A	B	AND (&)	OR ()	XOR (^)
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

2) AND (&)

Result of & operator is 1 (True) when all the bits are 1

$$0 \& 0 \& 1 \& 0 \& 1 \& 1 = 0$$

$$1 \& 1 \& 1 \& 1 \& 1 = 1$$

3) OR (|)

OR is True when any one bit is True

4) XOR (^) \Rightarrow Exclusive OR

$a \wedge b$ is True only when exactly one bit is True.

Examples

1) $5 \& 6 = 4$

	5	\rightarrow	² 1	¹ 0	⁰ 1
$\&$	6	\rightarrow	1	1	0
	4	\Leftarrow	1	0	0

```
int x = 5;  
int y = 6;  
print (x & y);
```

2) $20 \& 45 = 4$

		5	4	3	2	1	0
20 \Rightarrow	0	1	0	1	0	0	
45 \Rightarrow	1	0	1	1	0	1	
	<hr/>						
	0	0	0	1	0	0	
				\uparrow			

3) $20 | 45 = 61$

		5	4	3	2	1	0
20 \Rightarrow	0	1	0	1	0	0	
OR 45 \Rightarrow	1	0	1	1	0	1	
	<hr/>						
	$(1\ 1\ 1\ 1\ 0\ 1)_2 = 61$						

4) $20 \wedge 45$

		5	4	3	2	1	0
20 \Rightarrow	0	1	0	1	0	0	
XOR 45 \Rightarrow	1	0	1	1	0	1	
	<hr/>						
	$(1\ 1\ 1\ 0\ 0\ 1)_2 = 57$						

Binary Representation of -ve no.

Integers = 32 bits

2^{31}	2^{30}	2^{29}	2^{28}	...	2^3	2^2	2^1	2^0
31	30	29	28	...	3	2	1	0

int x = 5;
int x = -5;

NOTE: for simplicity let's assume integers have 5 bits.



Sign Bit

0	→	+ve no.
1	→	-ve no.

	10	:	1	0	1	0	1	0
-	10	:	1	1	0	1	0	
			X	0	0	1	0	0

$-0 = \begin{matrix} 0 & : & 0 & 0 & 0 & 0 & 0 \\ 0 & : & 1 & 0 & 0 & 0 & 0 \end{matrix} > 2 \text{ representations of } 0$

2's Complement Representation

10 : 0 1 0 1 0

↓ Invest all bits

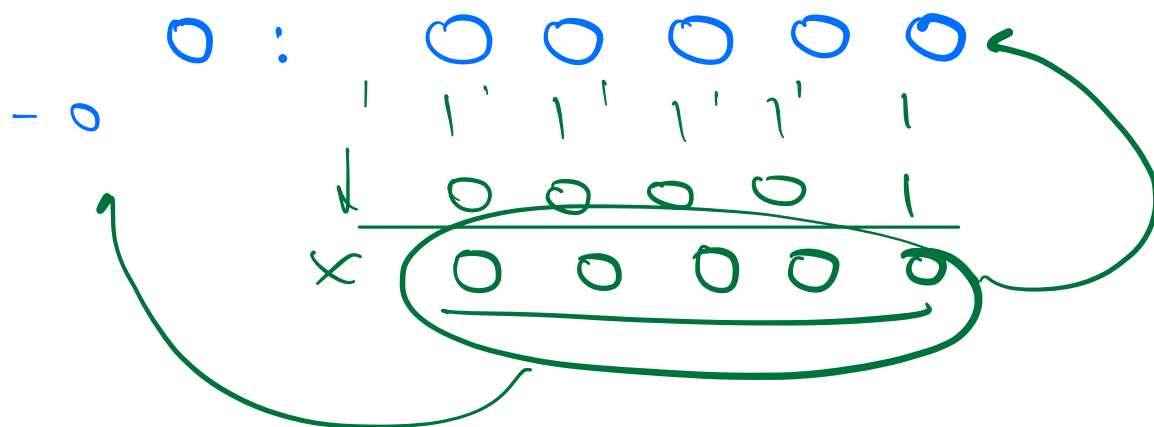
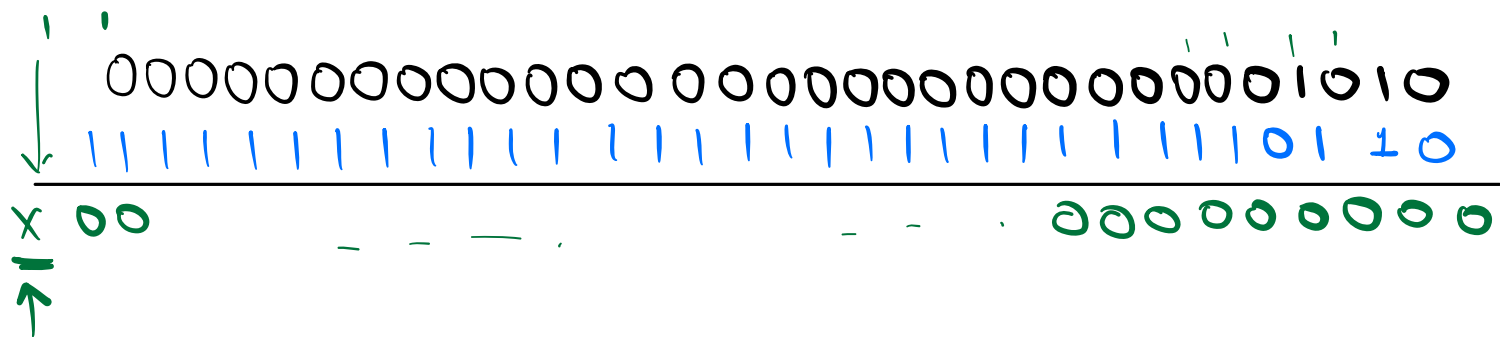
Add 1 ↓

Diagram illustrating a binary operation (likely XOR) on two 5-bit numbers:

1	0	1	0	1
0	0	0	0	1
<hr/>				
1	0	1	1	0

Binary representation of -10

$$\begin{array}{r} 10 : 01010 \\ -10 \quad \downarrow \quad 10000 \\ \hline \quad \quad \times \quad 00000 \end{array}$$
[illegible]



Range of Data Types

$$\begin{array}{ccccc} -2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \hline 4 & 3 & 2 & 1 & 0 \end{array}$$

↑
Sign
Bit

→ +ve ⇒ 0
→ -ve ⇒ 1

4 Bits will contribute to value

$$10 : \begin{array}{ccccc} 4 & 3 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ -2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$$= 0 \times (-2^4) + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 10$$

$$\begin{array}{cccccc}
 & 4 & 3 & 2 & 1 & 0 \\
 -10 : & 1 & 0 & 1 & 1 & 0 \\
 & -2^4 & 2^3 & 2^2 & 2^1 & 2^0
 \end{array}
 = 1 \times (-2^4) + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= -16 + 4 + 2 = \underline{\underline{-10}}$$

32 Bit System

$$\begin{array}{cccccccc}
 -2^{31} & 2^{30} & 2^{29} & 2^{28} & \dots & 2^3 & 2^2 & 2^1 & 2^0 \\
 \hline
 31 & 30 & 29 & 28 & \dots & 3 & 2 & 1 & 0
 \end{array}$$

↓
Sign Bit

No. of Bits	Unsigned	Signed.
2 ↓ $2^2 = 4$	$ \begin{array}{cc} \overline{2^1} & \overline{2^0} \\ \text{Max} = (11)_2 = 3 \\ \text{Min} = (00)_2 = 0 \\ [0, 3] \end{array} $	$ \begin{array}{cc} \overline{-2^1} & \overline{2^0} \\ \text{Max} = (01)_2 = 1 \\ \text{Min} = (10)_2 = -2 \\ [-2, 1] \end{array} $
8 ↓ 2^8	$ \begin{array}{ccccccc} \overline{2^7} & \dots & \dots & \dots & \dots & \overline{2^1} & \overline{2^0} \\ \text{Max} = 11111111 = 2^8 - 1 \\ \text{Min} = 00000000 = 0 \end{array} $	$ \begin{array}{ccccccc} \overline{-2^7} & \overline{2^6} & \overline{2^5} & \dots & \dots & \overline{2^1} & \overline{2^0} \\ \text{Max} = 01111111 = 2^7 - 1 \\ \text{Min} = 10000000 = -2^7 \end{array} $

	$[0, 2^8 - 1]$	$[-2^7, 2^7 - 1]$
N Bit no. ↓ 2^N	<div style="text-align: center;"> ----- ··· ----- 2^{N-1} 2^1 2^0 Max = 1111...11 = $2^N - 1$ Min = 000000...0 = 0 $[0, 2^N - 1]$ </div>	<div style="text-align: center;"> ↓ ----- ··· ----- -2^{N-1} 2^{N-2} ... 2^1 2^0 Max : 0111...11 = $2^{(N-1)} - 1$ Min : 10000...0 = -2^{N-1} $[-2^{N-1}, 2^{(N-1)} - 1]$ </div>

$$2^{N-2} + 2^{N-3} + 2^{N-4} + \dots + 2^2 + 2^1 + 2^0$$

$$(2^{(N-1)} - 1)$$

Integer $(N=32) \Rightarrow 2^{32}$

$$\left[\begin{array}{cc} -2^{31} & 2^{31}-1 \end{array} \right]$$

$$\begin{array}{cc} s & e \end{array}$$

$$(2^{31} - 1) - (-2^{31}) + 1$$

$$2 \times 2^{31} = 2^{32}$$

Friday at 9:00 PM

9:05 → 10:30

↓
1hr 25min

H.W.

String Class function