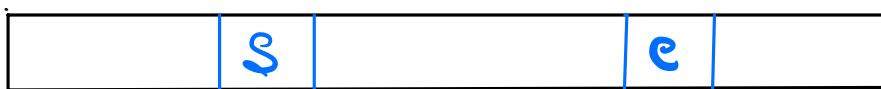


Intermediate Content \Rightarrow Rettempt 2

\Downarrow
Sat & Sunday.

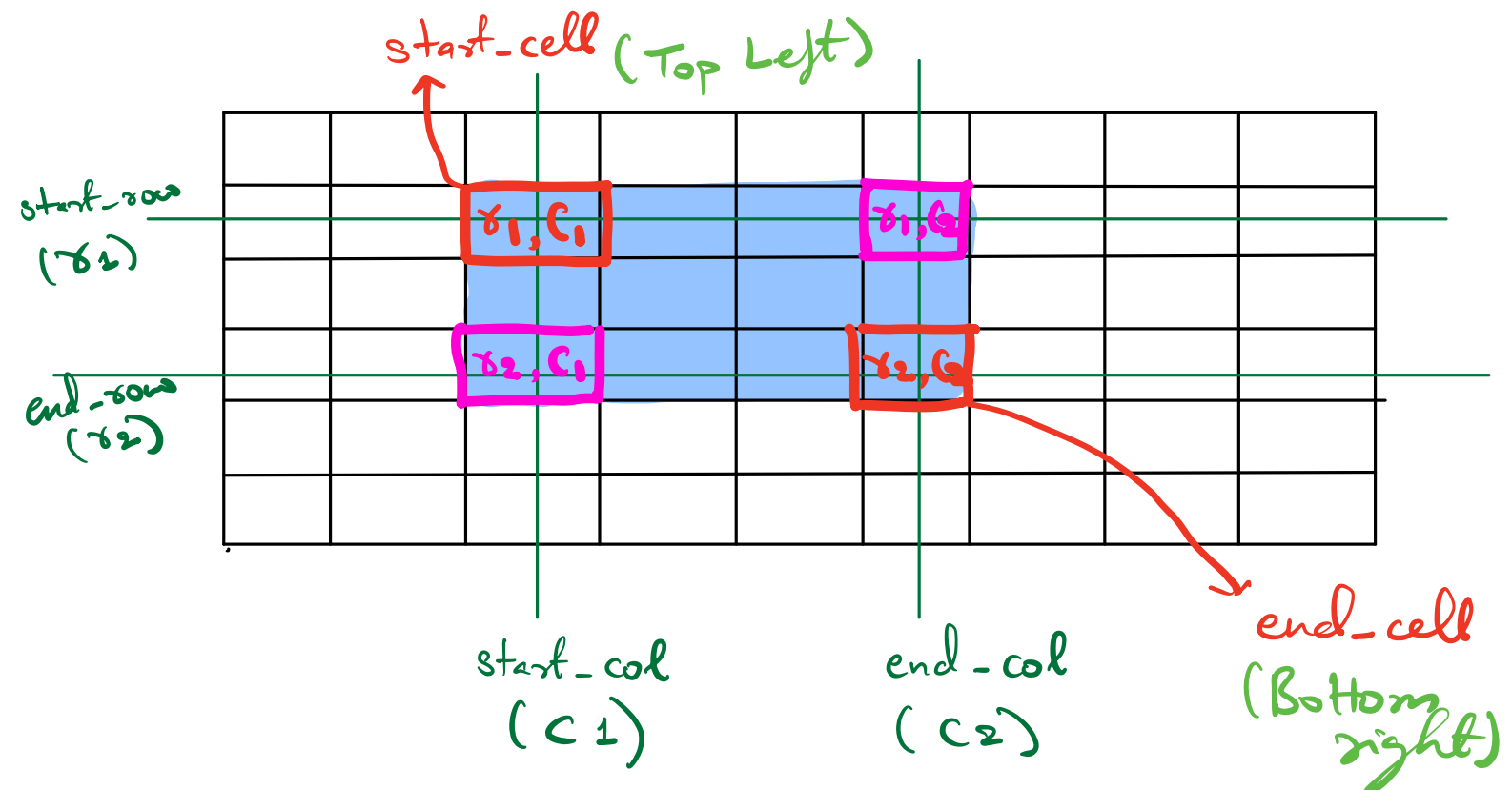
Subarrays \Rightarrow Array.

$(s \leq e)$



2 Elements

Submatrix \Rightarrow Matrix



Submatrix $\Rightarrow (r_1, c_1)$ to (r_2, c_2)

Given a matrix of size $N \times M$

Print a given submatrix from (r_1, c_1)
 \downarrow
 (r_2, c_2)

		c_1		c_2					
	0	1	(2)	3	4	(5)	6	7	8
0	2	3	5	1	0	6	7	-1	-12
δ_1 (1)	6	5	3	1	0	2	1	6	7
2	4	2	1	4	7	6	2	-5	11
δ_2 (3)	13	6	9	5	4	8	0	1	9
4	8	8	7	6	5	3	1	9	6
5	0	3	-6	-8	-1	0	7	5	2

$$\begin{bmatrix} 3, & 1, & 0, & 2 \\ 1, & 4, & 7, & 6 \\ 9, & 5, & 4, & 8 \end{bmatrix}$$

Code

```
for (i = r1, i <= r2; i++)  
    for (j = c1, j <= c2; j++)  
        print (M[i][j]);
```

```
    print (" /M'");
```

$$T.C. = O(N \times M)$$

Sum of a Given Submatrix

(r_1, c_1) to (r_2, c_2)

sum = 0;

```
for (i = r1, i <= r2; i++) {  
    for (j = c1, j <= c2; j++) {
```

sum = sum + M[i][j];



Sum of All Submatrices.

Sum of All Subarrays \Rightarrow Contribution Technique



Given a matrix of size $N \times M$.
Find the sum of all submatrix sum.

1) How to find out all the submatrices.

(r_1, c_1)

↓

	0	1	2	3	4	5	6	7	8
0	2	3	5	1	0	6	7	-1	-12
1	6	5	3	1	0	2	1	6	7
2	4	2	1	4	7	6	2	-5	11
3	13	6	9	5	4	8	0	1	9
4	8	8	7	6	5	3	1	9	6
5	0	3	-6	-8	-1	0	7	5	2

(r_2, c_2)

↓

$$(r_1 \leq r_2)$$

$$(c_1 \leq c_2)$$

Top Left [for ($r_1 = 0$; $r_1 < N$; r_1++) d
for ($c_1 = 0$; $c_1 < M$; c_1++) d

Bottom Right [for ($r_2 = r_1$; $r_2 < N$; r_2++) d
for ($c_2 = c_1$; $c_2 < M$; c_2++) d

logic

$$T.C. = O(N^2 M^2)$$

2) Sum of All Submatrices Sum

1) Brute force

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{bmatrix} 1, 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 \end{bmatrix} \quad \begin{bmatrix} 3 \end{bmatrix} \quad \begin{bmatrix} 1, 2, 3 \end{bmatrix}$$

$$\begin{bmatrix} 2, 3 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \quad \begin{bmatrix} 2, 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2, 4, 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 \end{bmatrix} \quad \begin{bmatrix} 4, 6 \end{bmatrix} \quad \begin{bmatrix} 6 \end{bmatrix}$$

ans = 0;

Top Left [for ($r_1 = 0$; $r_1 < N$; r_1++) &
 for ($c_1 = 0$; $c_1 < M$; c_1++) &

Bottom Right [for ($r_2 = r_1$; $r_2 < N$; r_2++) &
 for ($c_2 = c_1$; $c_2 < M$; c_2++) &

sum = 0;

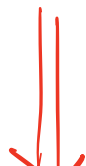
for ($i = r_1$, $i \leq r_2$; $i++$) &
 for ($j = c_1$, $j \leq c_2$; $j++$) &

sum = sum + M[i][j];

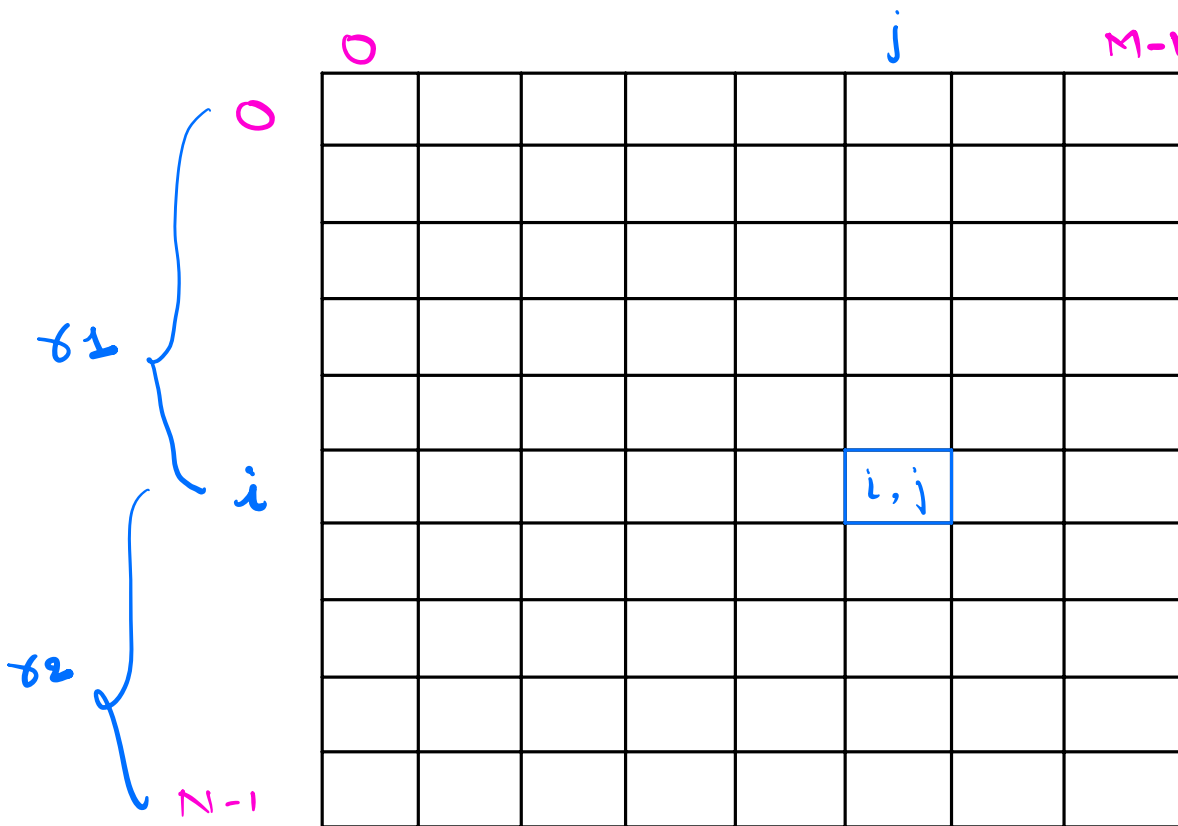
ans = ans + sum;

T.C. = $O(N^3 \times M^3)$

Contn of
 ith elements = $\left(\begin{array}{l} \# \text{ Subarray} \\ \text{element on index} \\ i \text{ is a part of} \end{array} \right) \times \left(\begin{array}{l} \text{Value of the} \\ \text{element} \end{array} \right)$



Contri of $M[i][j]$ towards the sum = $\left(\begin{array}{l} \# \text{ submatrix} \\ M[i][j] \text{ is a} \\ \text{part of} \end{array} \right) \times (M[i][j])$



r_1, c_1, r_2, c_2

$$\begin{aligned}
 r_1 : [0, i] &= (i+1) \\
 r_2 : [i, N-1] &= (N-i) \\
 c_1 : [0, j] &= (j+1) \\
 c_2 : [j, M-1] &= (M-j)
 \end{aligned}$$

Submatrices
 $MS[i][j]$ would
be a part of $= (i+1)(N-i)(j+1)(M-j)$

Code

```
ans = 0;
```

```
for (i = 0; i < N; i++)  
    for (j = 0; j < M; j++)
```

```
        count = (i+1)(N-i)(j+1)(M-j)
```

```
        contri = count * MS[i][j]
```

```
        ans = ans + contri;
```

```
    }
```

```
}
```

T.C. = $O(N \times M)$

S.C. = $O(1)$