

- 1) Count Sort
- 2) Merge Sort

Q Find the smallest no. that can be formed by rearranging the digits of no. given in the form of an array.

Return in form of array

Eg $A = \langle 6, 3, 4, 2, 7, 2, 1 \rangle = 6342721$

O/P = $\langle 1, 2, 2, 3, 4, 6, 7 \rangle = 1223467$

Solⁿ 1) Sort the array & return it.

$$T.C. = O(N \log N)$$

$$\Downarrow O(N)??$$

Solⁿ 2)

OBS: $0 \leq A[i] \leq 9$

format of O/P = $\langle 0, 0, \dots, 1, 1, \dots, 2, 2, \dots, 3, \dots, 4, \dots, 9, 9 \rangle$

I/P: $\langle 4, 2, 3, 2, 9, 0, 1, 6, 0, 1, 2, 5 \rangle$

⇒ Store the freq of all elements

freq[10]: [2, 2, 3, 1, 1, 1, 1, 0, 0, 1]
0, 1, 2, 3, 4, 5, 6, 7, 8, 9
i i i i i i i i i i
S.C. = $O(10) = O(1)$

I/P: {0, 4, 2, 3, 2, 9, 0, 2, 1, 6, 0, 1, 2, 5}
0 1 2 3 4 5 6 7 8 9 10 11
j j j j j j j j j j j j j j

"Count Sort"

Code

```
int freq[10] = {0};
```

```
for (i = 0; i < N; i++) {  
    freq[A[i]]++;  
}
```

} $O(N)$

```
int index = 0;
```

```
for (i = 0; i < 10; i++) {
```

for (j=0; j < freq[i]; j++) ~~1~~

A[index] = i;
index++;

}

}

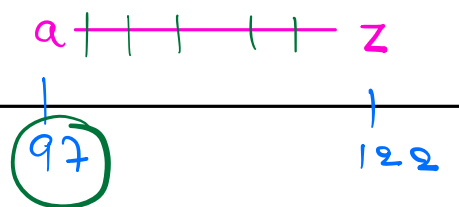
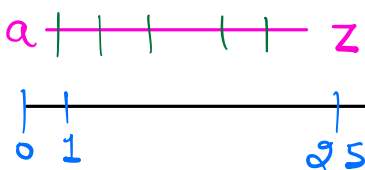
T.C. = $O(N)$
S.C. = $O(1)$

Q Will Count Sort work for large values

A[i] > 10⁹ ??
↑

Count sort can typically be applied
for A[i] $\leq 10^6$

Q How to implement count sort for Ex.
containing -ve no's as well.



$$\begin{aligned}
 a &= 97 - 97 = 0 \\
 b &= 98 - 97 = 1 \\
 c &= 99 - 97 = 2 \\
 &\vdots \\
 z &= 122 - 97 = 25
 \end{aligned}$$

Approach

1) Find the range of elements (min & max)

The freq of any element X will be stored on the index $(X - \min)$

$$\text{Size of freq array} = \underbrace{[\min, \max]}_{\downarrow}$$

$$\# \text{ elements} = \max - \min + 1$$

Eg

$$A = \left[\overset{0}{-2}, \overset{1}{3}, \overset{2}{8}, \overset{3}{3}, \overset{4}{-2}, \overset{5}{3}, \overset{6}{0}, \overset{7}{4} \right]$$

$$\begin{aligned}
 \min &= -2 \\
 \max &= 8
 \end{aligned}
 > \text{Size} = (8 - (-2) + 1) = 11$$

$$\text{freq} = \left[\overset{0}{2}, \overset{1}{0}, \overset{2}{1}, \overset{3}{0}, \overset{4}{0}, \overset{5}{3}, \overset{6}{1}, \overset{7}{0}, \overset{8}{0}, \overset{9}{0}, \overset{10}{1} \right]$$

$$X - (\min) = \underline{\text{index}}$$

+ min

Q Given an integer array where the odd valued elements are relatively sorted & even valued elements are relatively sorted.

Sort the entire array.

$A = [2, 5, 4, 8, 11, 13, 10, 15, 21]$

Solⁿ

i) Use the default sorting algo.

$$T.C. = O(N \log N)$$

$$\downarrow O(N) ??$$

Idea: Split the array into 2 arrays

Even [ε]

Count of
Even
elements

Odd [o]

Count of
odd elements

~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~
0 1 2 3 4 5 6 7 8 9
A = [2, 5, 4, 8, 11, 13, 10, 14, 15, 21]

~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~
Odd[s] = [5, 11, 13, 15, 21]

~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~ ~~i~~
Even[s] = [2, 4, 8, 10, 14]

} O(N)

~~ind~~ ~~ind~~ ~~ind~~ ~~ind~~ ~~ind~~ ~~ind~~ ~~ind~~ ~~ind~~ ~~ind~~ ~~ind~~
0 1 2 3 4 5 6 7 8 9
A = [2, 4, 5, 8, 10, 11, 13, 14, 15, 21]

Code

- 1) Split s/w Odd & Even
- 2) Merge back into A.

```
int Od = 0;
```

```
for (i=0; i<N; i++) {  
    if (A[i] % 2 != 0) {  
        Od++;  
    }  
}
```

```
int E = N - Od;
```

// split

```
ind_even = 0;
```

```
ind_odd = 0;
```

```
Even[E];
```

```
Odd[Od];
```

```
for (i=0; i<N; i++) {
```

```
    if (A[i] % 2 == 0) {
```

```
        Even[ind_even] = A[i];
```

```
        ind_even++;
```

```
    } else {
```

```
        Odd[ind_odd] = A[i];
```

```
        ind_odd++;
```

```
    }
```

```
}
```

// Merge

```
i = j = 0;
```

ind = 0;

while ((i < Odd) && (j < Even)) {

if (Odd[i] < Even[j]) {

A[ind] = Odd[i];

i++;

ind++;

}
else {

A[ind] = Even[j];

j++;

ind++;

}

}

if (i < Odd) {

while (i < Odd) {

A[ind] = Odd[i];

i++;

ind++;

}

else {

while (j < Even) {

A[ind] = Even[j];

j++;

ind++;

b

b

iteration = $2 \times \underline{\underline{N}}$.

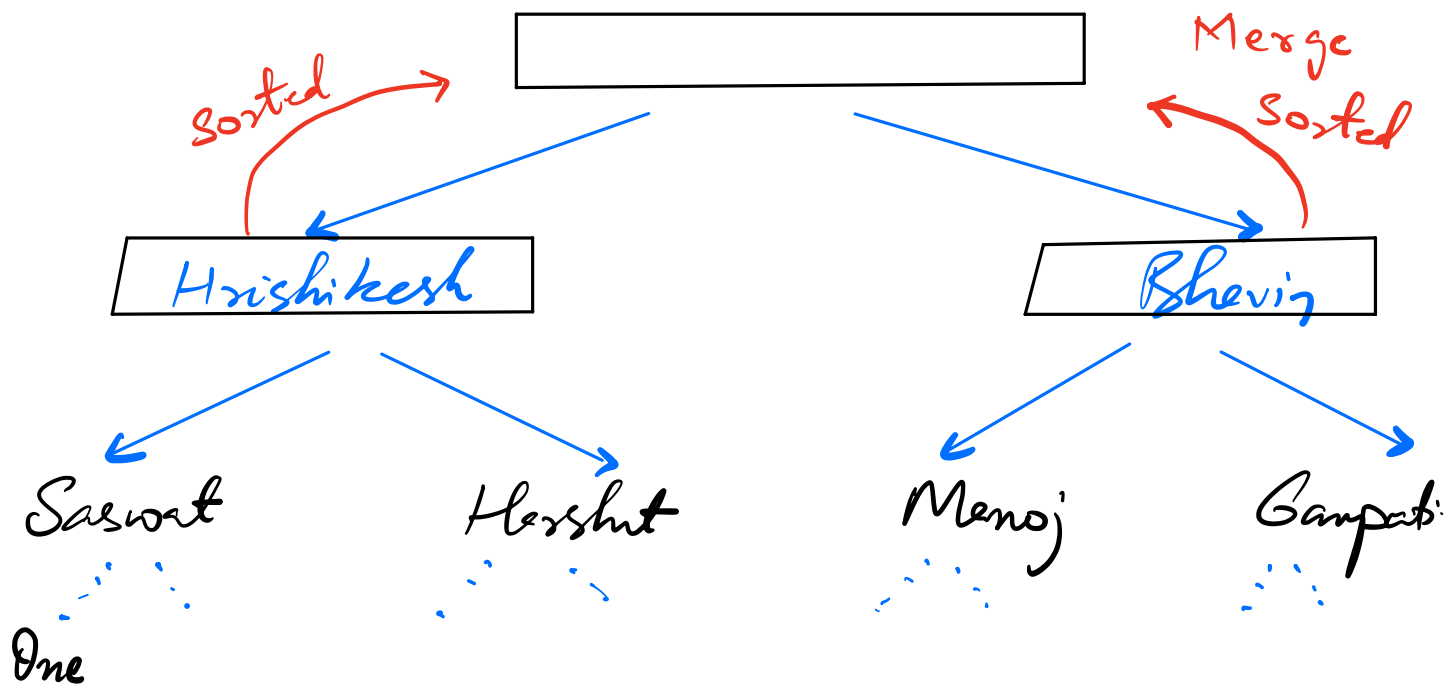
T.P. = $O(N)$

S.P. = $\underline{\underline{O(N)}}$

Merge Sort

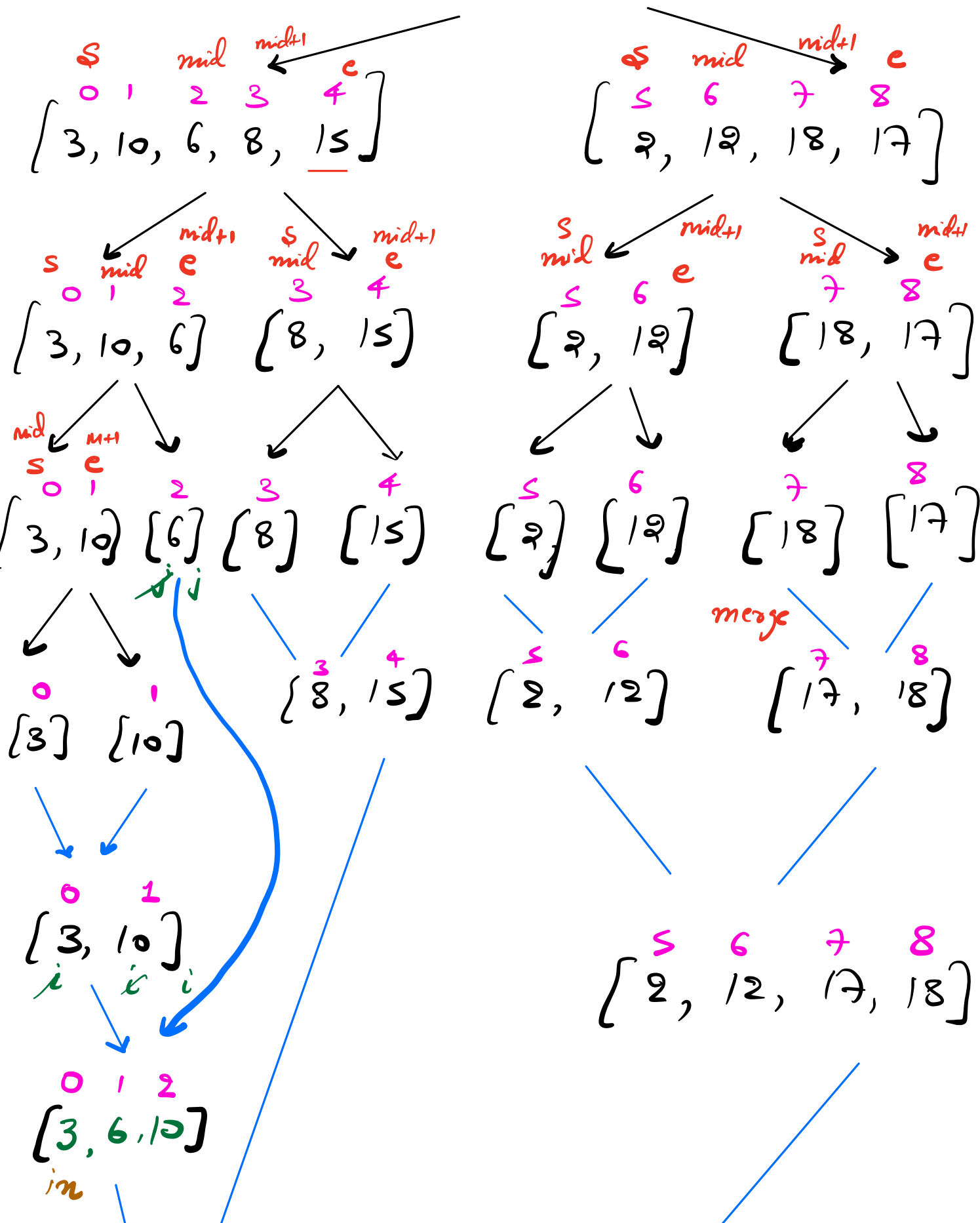
Teacher \Rightarrow Mark Sheet of all students
random order

Goal: Arrange in sorting order



mergesort

$$A = [3, 10, 6, 8, \underline{15}, 2, 12, 18, 17]$$



0 1 2 3 4
 $\{3, 6, 8, 10, 15\}$

0 1 2 3 4 5 6 7 8
 $\{2, 3, 6, 8, 10, 12, 15, 17, 18\}$

Code

void mergeSort ($A[]$, s, e) {

if ($s > e$) return; }

int mid = $(s+e)/2$ $\left(s + \frac{(e-s)}{2} \right)$

mergeSort (A , s , mid); $A[s, mid]$ is sorted

mergeSort (A , mid+1, e); $A[mid+1, e]$ is sorted

merge (A , s , mid, e); // $O(n)$

}

```
void merge (A, s, mid, e) {
```

```
    int n1 = mid - s + 1;    [s, mid]
```

```
    int n2 = e - mid;        [mid+1, e]
```

```
    int A1[n1];
```

```
    int A2[n2];
```

```
    int ind = 0;
```

```
    for (i = s; i ≤ mid; i++) {
```

```
        A1[ind] = A[i];
        ind++;
```

```
    }
```

```
    ind = 0;
```

```
    for (i = mid + 1; i ≤ e; i++) {
```

```
        A2[ind] = A[i];
        ind++;
```

```
    }
```

```
// Merge
```

```
ind = s;
```

```
i = j = 0;
```

```
while (i < n1 || j < n2) {
```

```
if (A1[i] < A2[j]) <
    A[ind] = A1[i];
    i++;
    ind++;
```

```
    }
    else <
        A[ind] = A2[j];
        j++;
        ind++;
    }
```

```
    }
    if (i < n1) <
        while (i < n1) <
            A[ind] = A1[i];
            i++;
            ind++;
```

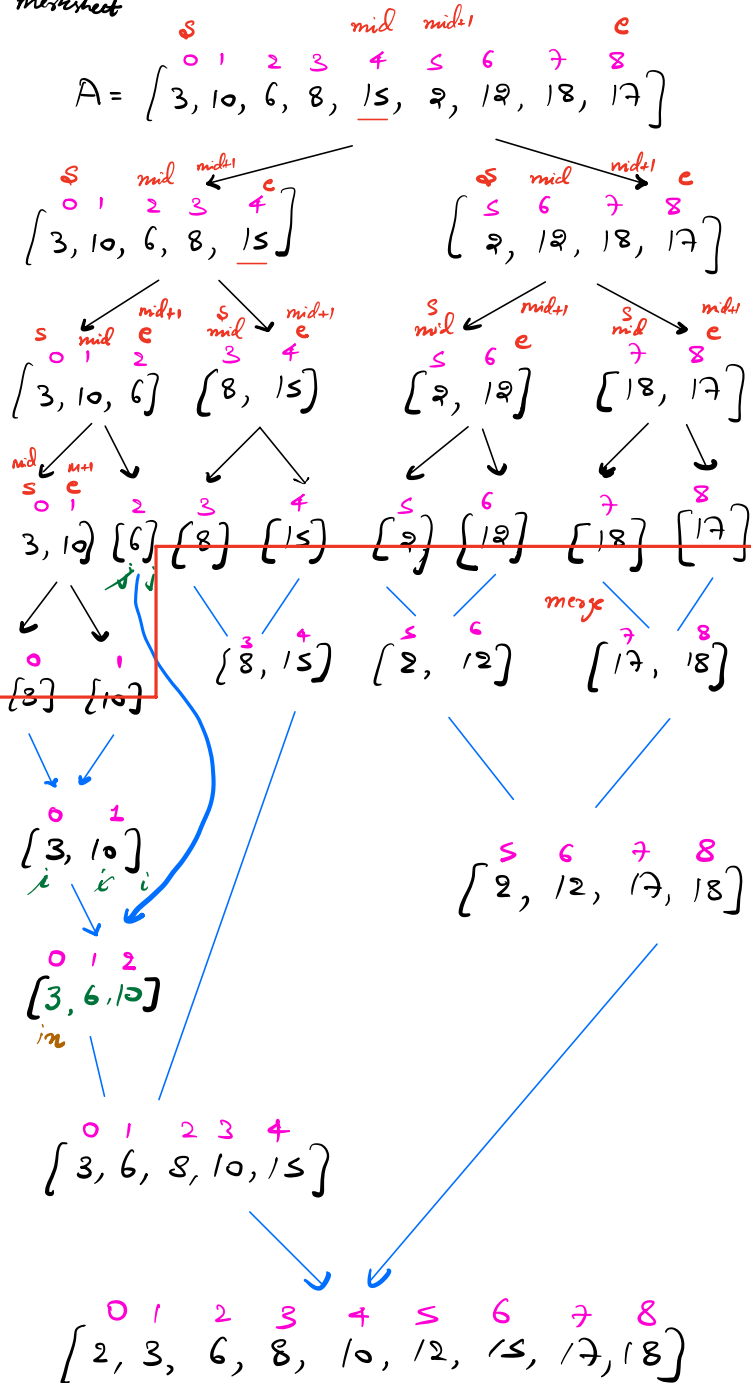
```
    } else <
        while (j < n2) <
            A[ind] = A2[j];
            j++;
            ind++;
    }
```

```
    }
```

```
}
```

T.C. =

mergesort



Divide ($\log_2 N$ steps)

$O(1)$

Merge ($\log_2 N$ steps)

$O(N)$

$O(\log N + N \times \log N)$

T.C. = $O(N \log N)$

S.C. = $O(N)$

{ Divide & Conquer }

$$T(N) = 2T\left(\frac{N}{2}\right) + N$$

Assign

⇒

Inversion
Count

- 1) Brute force
- 2) Doy
- 3) Hint

Modify
merge funⁿ
✓ merge sort

A = [2, 3, 4, 5, 7, 8, 11, 13, 10, 15, 14, 21]

Indices: 0 1 2 3 4 5 6 7 8 9 10 11

Annotations:
- Index 0: 2 (red), i (blue)
- Index 1: 3 (red), i (blue)
- Index 2: 4 (blue)
- Index 3: 5 (blue), j (blue)
- Index 4: 7 (blue)
- Index 5: 8 (red)
- Index 6: 11 (red)
- Index 7: 13 (red)
- Index 8: 10 (red)
- Index 9: 15 (red)
- Index 10: 14 (red)
- Index 11: 21 (red)