Computer Programming

Methods

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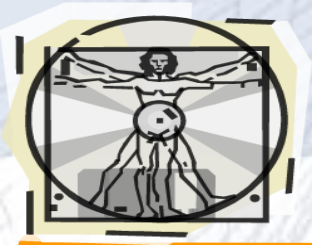
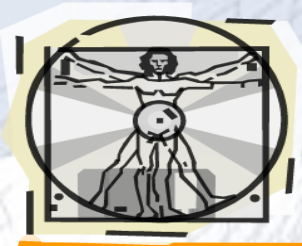
Computer Engineering Department



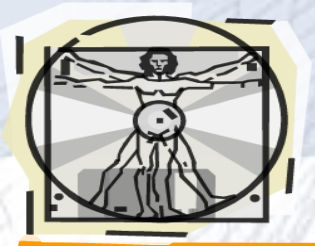
Opening Problem

- Find the sum of integers from 1 to 10, from 20 to 30, and from 35 to 45, respectively.



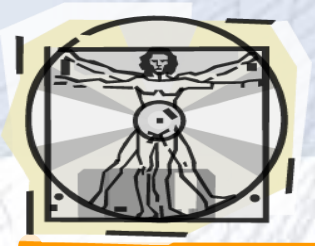
Problem

```
int sum = 0;
for (int i = 1; i <= 10; i++)
  sum += i;
System.out.println("Sum from 1 to 10 is " + sum);

sum = 0;
for (int i = 20; i <= 30; i++)
  sum += i;
System.out.println("Sum from 20 to 30 is " + sum);

sum = 0;
for (int i = 35; i <= 45; i++)
  sum += i;
System.out.println("Sum from 35 to 45 is " + sum);
```



Problem

```
int sum = 0;
for (int i = 1; i <= 10; i++)
  sum += i;
System.out.println("Sum from 1 to 10 is " + sum);

sum = 0;
for (int i = 20; i <= 30; i++)
  sum += i;
System.out.println("Sum from 20 to 30 is " + sum);

sum = 0;
for (int i = 35; i <= 45; i++)
  sum += i;
System.out.println("Sum from 35 to 45 is " + sum);
```

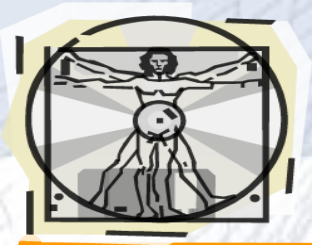


Solution

```
public static int sum(int i1, int i2) {
  int sum = 0;
  for (int i = i1; i <= i2; i++)
    sum += i;
  return sum;
}
```

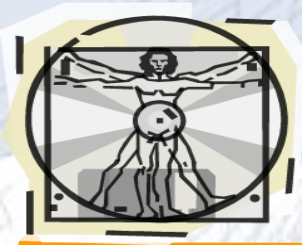
MethodDemo Run

```
public static void main(String[] args) {
  System.out.println("Sum from 1 to 10 is " + sum(1, 10));
  System.out.println("Sum from 20 to 30 is " + sum(20, 30));
  System.out.println("Sum from 35 to 45 is " + sum(35, 45));
}
```



Objectives

- To define methods with formal parameters (§6.2).
- To invoke methods with actual parameters (i.e., arguments) (§6.2).
- To define methods with a return value (§6.3).
- To define methods without a return value (§6.4).
- To pass arguments by value (§6.5).
- To develop reusable code that is modular, easy to read, easy to debug, and easy to maintain (§6.6).
- To write a method that converts hexadecimals to decimals (§6.7).
- To use method overloading and understand ambiguous overloading (§6.8).
- To determine the scope of variables (§6.9).
- To apply the concept of method abstraction in software development (§6.10).
- To design and implement methods using stepwise refinement (§6.10).



Defining Methods

- A method is a collection of statements that are grouped together to perform an operation.

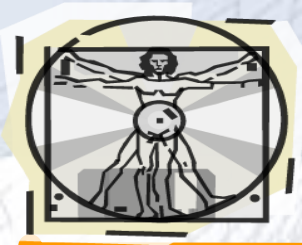
Define a method

```
public static int max(int num1, int num2) {

   int result;

   if (num1 > num2)
     result = num1;
   else
     result = num2;

   return result;
}
```

Invoke a method

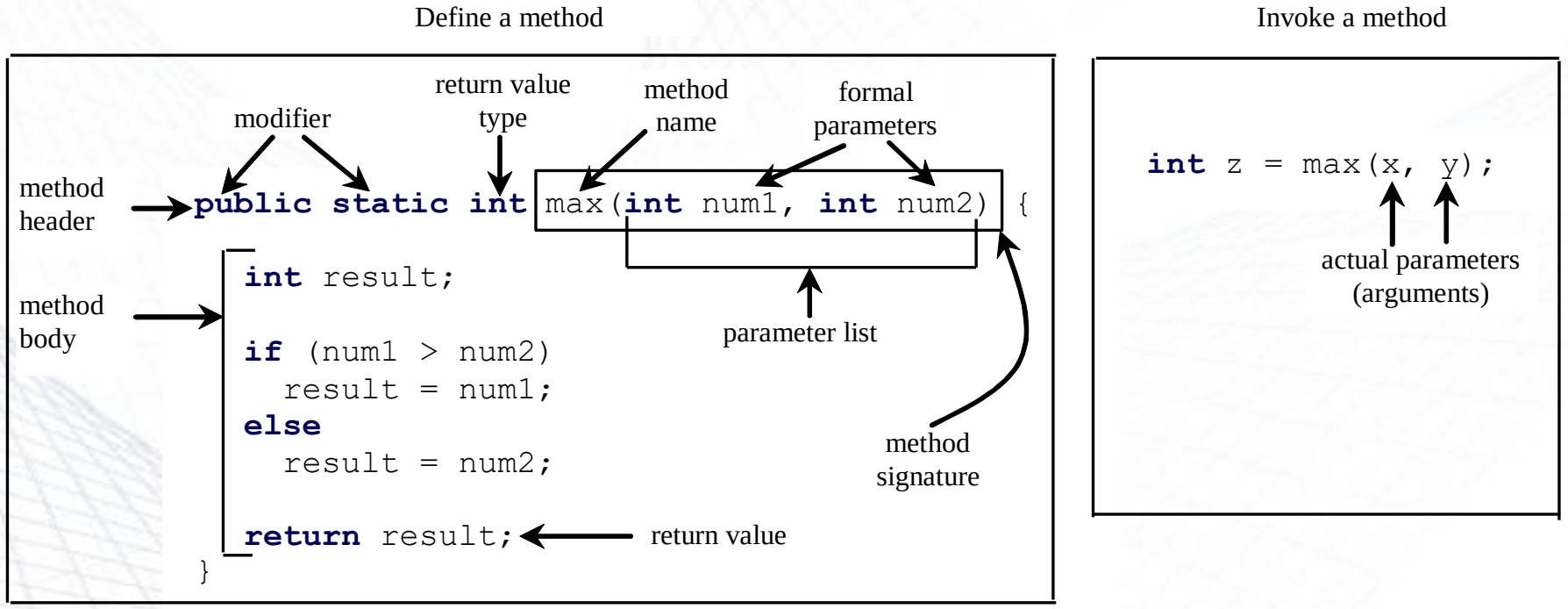
```
int z = max(x, y);
```

actual parameters (arguments)



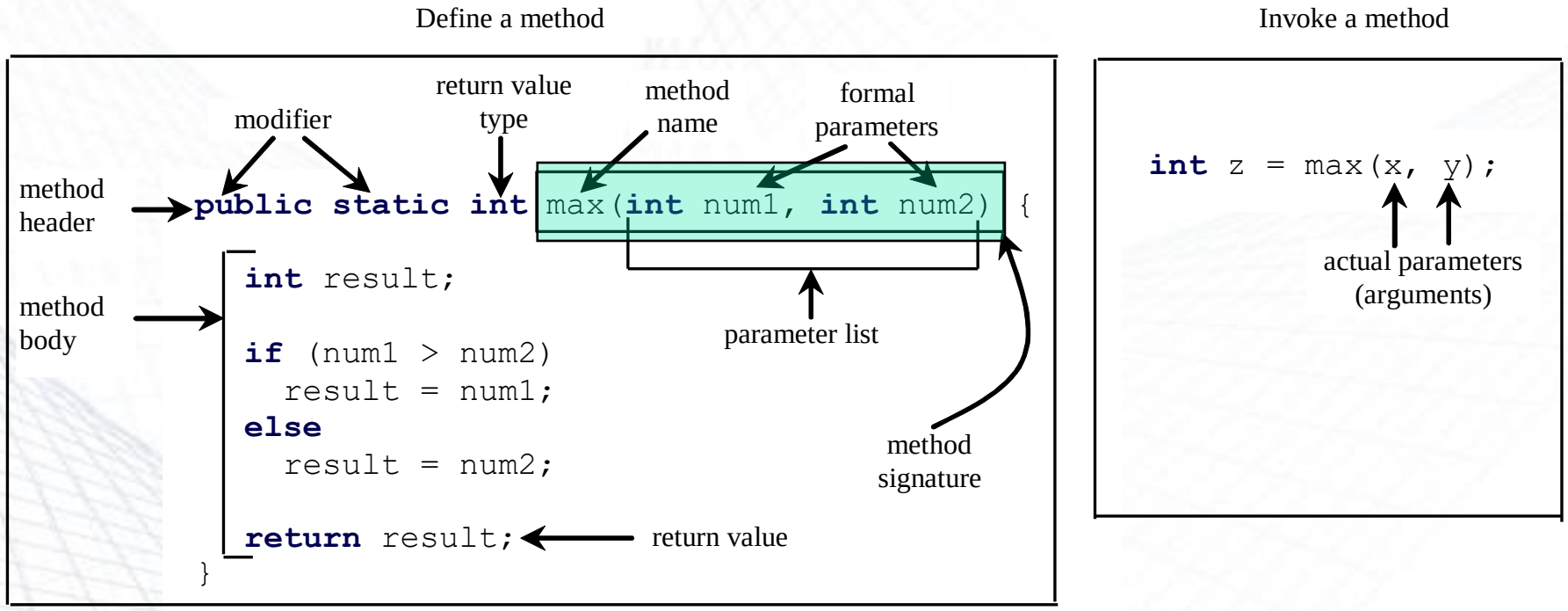
Defining Methods

- A method is a collection of statements that are grouped together to perform an operation.



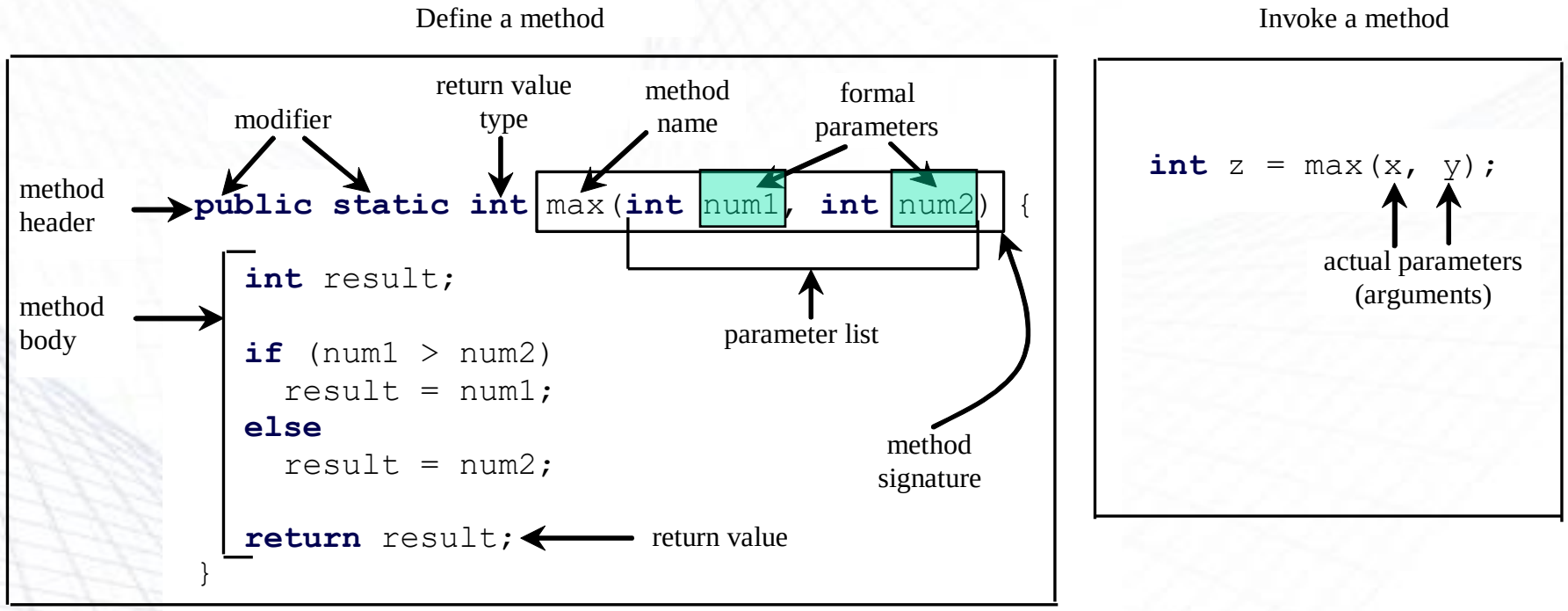
Method Signature

- Method signature is the combination of the method name and the parameter list.



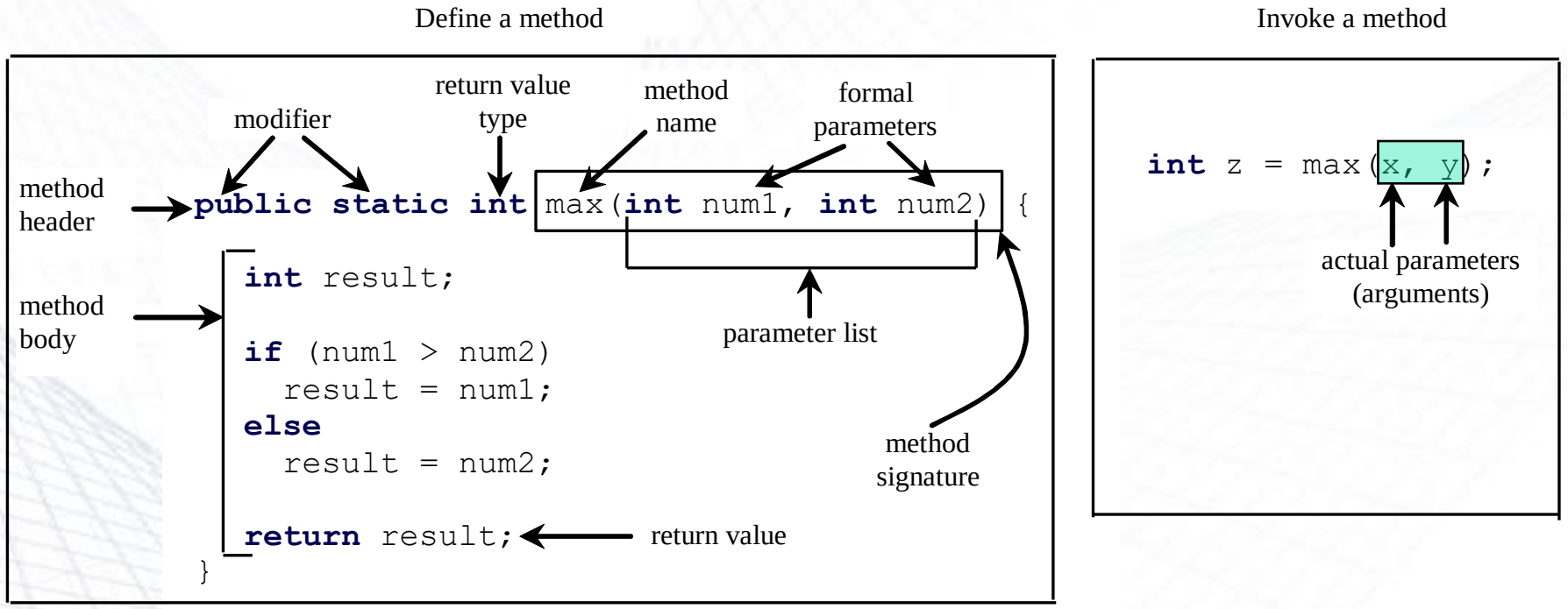
Formal Parameters

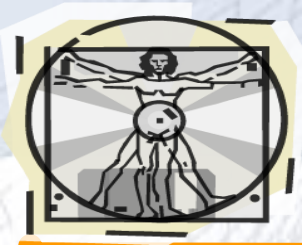
- The variables defined in the method header are known as formal parameters.



Actual Parameters

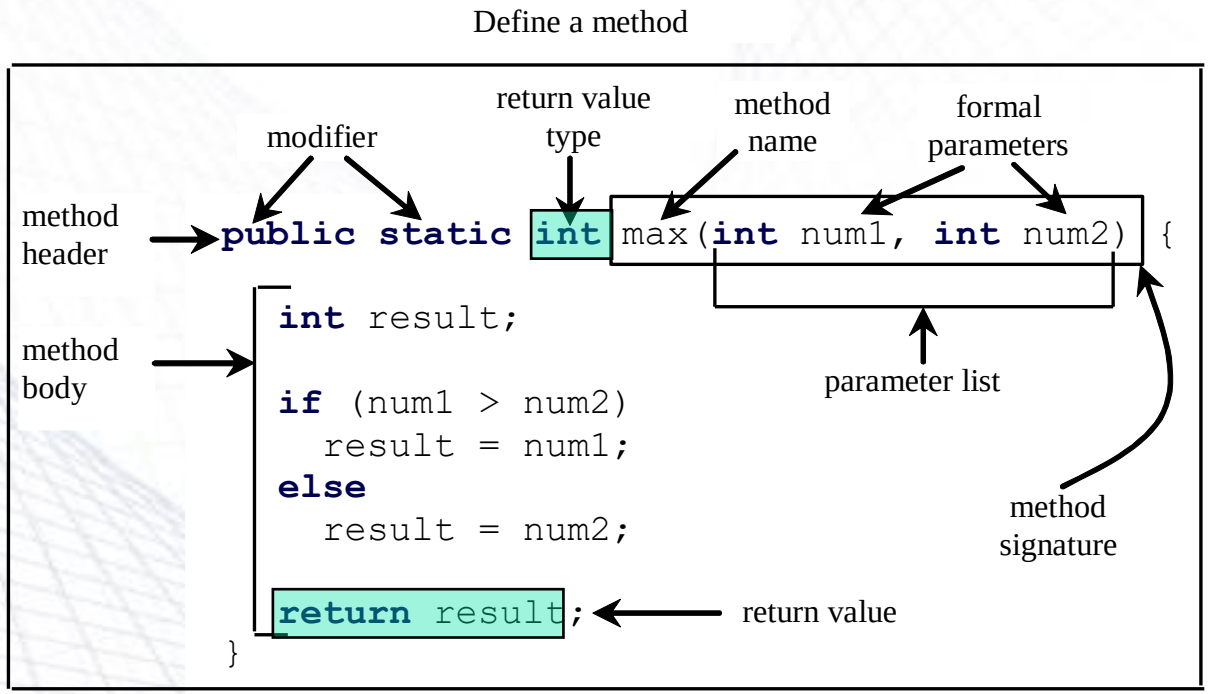
- When a method is invoked, you pass a value to the parameter. This value is referred to as actual parameter or argument.

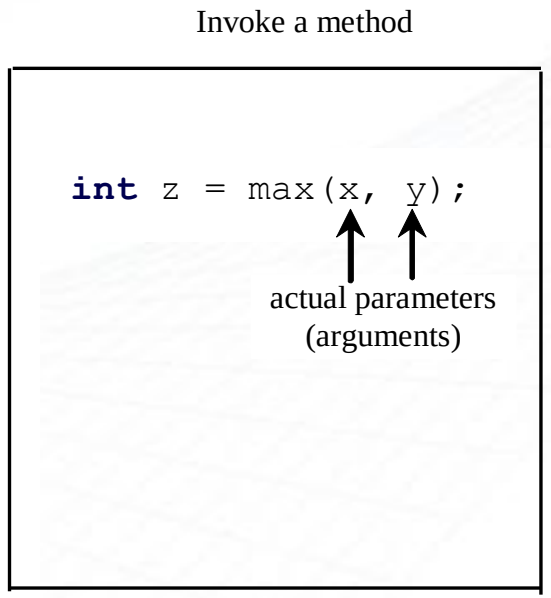


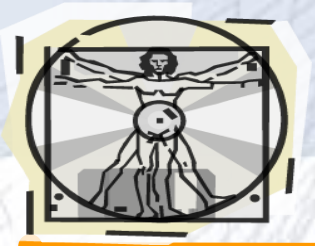


Return Value Type

A method may return a value. The returnValueType is the data type of the value the method returns. If the method does not return a value, the returnValueType is the keyword void. For example, the returnValueType in the main method is void.



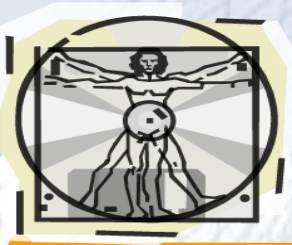




Calling Methods

- Testing the max method
- This program demonstrates calling a method max to return the largest of the int values

TestMax Run



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Calling Methods, cont.

pass the value of i

pass the value of j

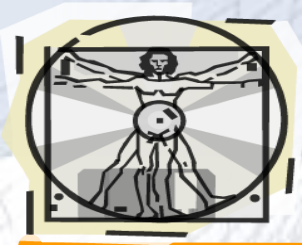
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



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Trace Method Invocation

i is now 5

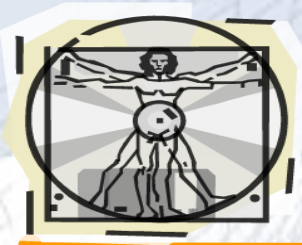
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



animation

Trace Method Invocation

j is now 2

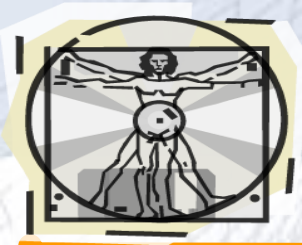
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



animation

Trace Method Invocation

invoke max(i, j)

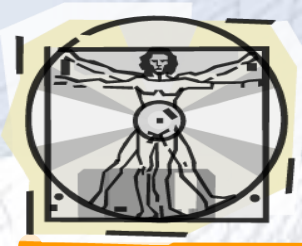
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



animation

Trace Method Invocation

invoke max(i, j)
Pass the value of i to num1
Pass the value of j to num2

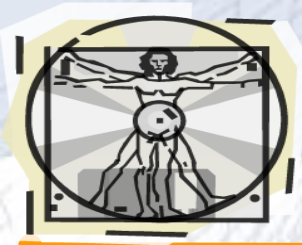
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



Trace Method Invocation

declare variable result

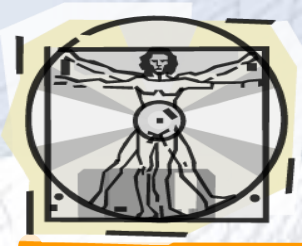
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



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Trace Method Invocation

(num1 > num2) is true since num1 is 5 and num2 is 2

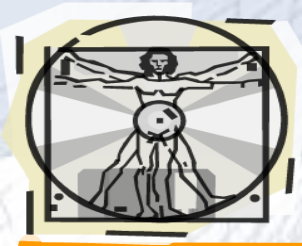
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



Trace Method Invocation

result is now 5

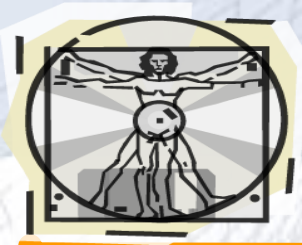
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



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Trace Method Invocation

return result, which is 5

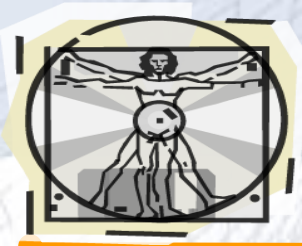
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
pu      tatic int max(int num1, int num2) {
        esult;

    num1 > num2)
    esult = num1;
   se
    result = num2;

  return result;
}
```



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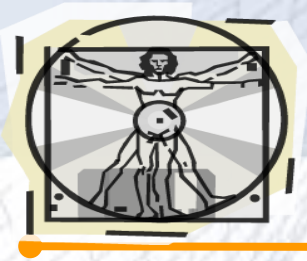
Trace Method Invocation

return max(i, j) and assign the return value to k

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```

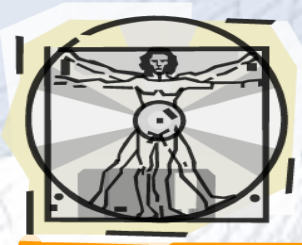


Trace Method Invocation

Execute the print statement

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



CAUTION

A return statement is required for a value-returning method. The method shown below in (a) is logically correct, but it has a compilation error because the Java compiler thinks it possible that this method does not return any value.

```
public static int sign(int n) {
  if (n > 0)
    return 1;
  else if (n == 0)
    return 0;
  else if (n < 0)
    return -1;
}
```

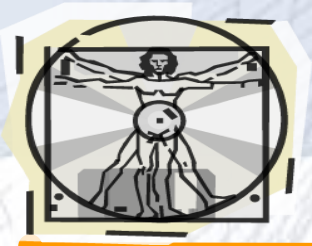
(a)

Should be

```
public static int sign(int n) {
  if (n > 0)
    return 1;
  else if (n == 0)
    return 0;
  else
    return -1;
}
```

(b)

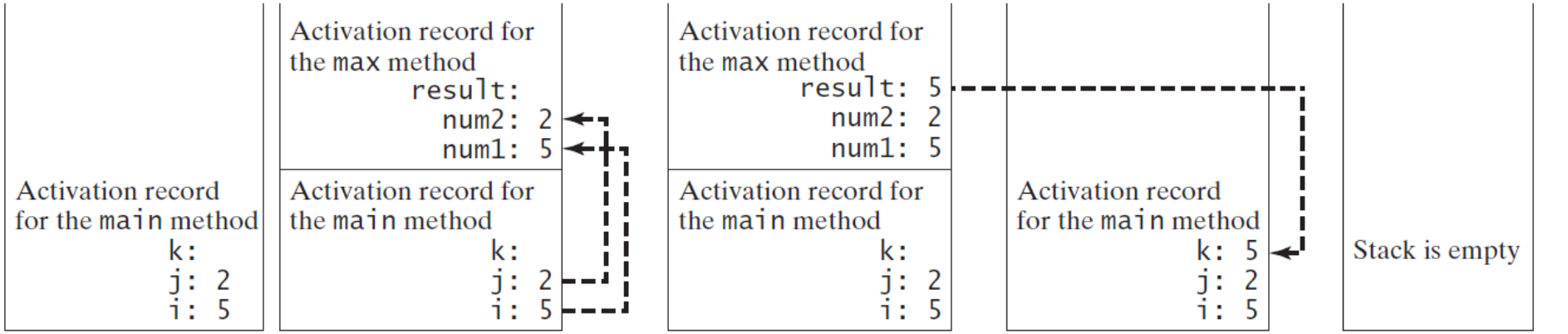
To fix this problem, delete *if (n < 0)* in (a), so that the compiler will see a return statement to be reached regardless of how the if statement is evaluated.



Reuse Methods from Other Classes

NOTE: One of the benefits of methods is for reuse. The max method can be invoked from any class besides TestMax. If you create a new class Test, you can invoke the max method using ClassName.methodName (e.g., TestMax.max).

Call Stacks



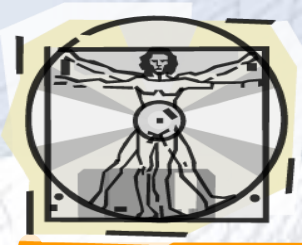
(a) The `main` method is invoked.

(b) The `max` method is invoked.

(c) The `max` method is being executed.

(d) The `max` method is finished and the return value is sent to `k`.

(e) The `main` method is finished.



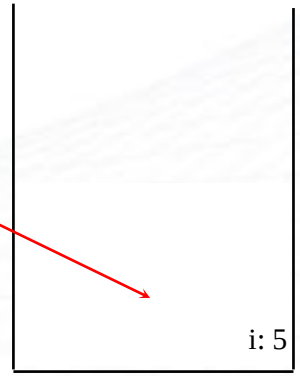
animation

Trace Call Stack

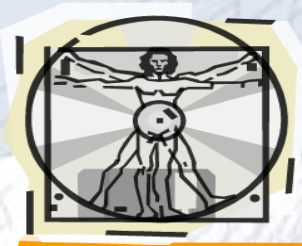
i is declared and initialized

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



The main method is invoked.

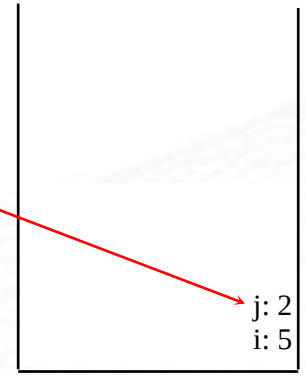


Trace Call Stack

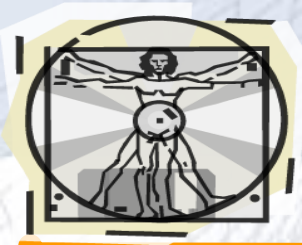
j is declared and initialized

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



The main method is invoked.

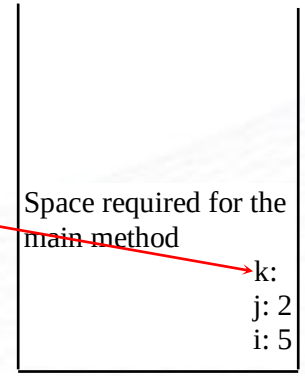


Trace Call Stack

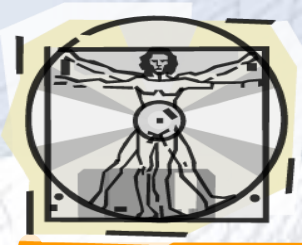
Declare k

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



The main method is invoked.



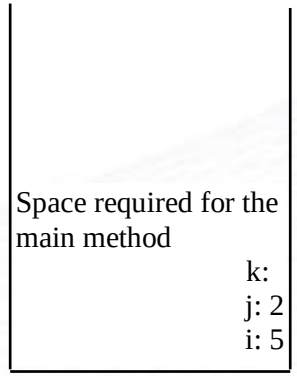
animation

Trace Call Stack

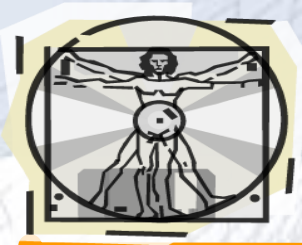
Invoke max(i, j)

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



The main method is invoked.

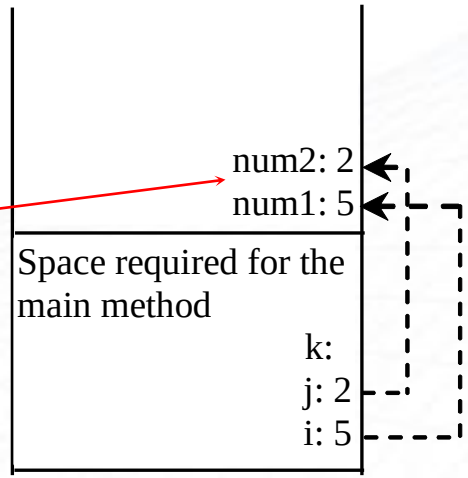


Trace Call Stack

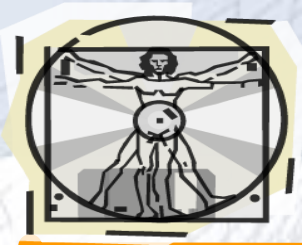
pass the values of i and j to num1 and num2

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



The max method is invoked.

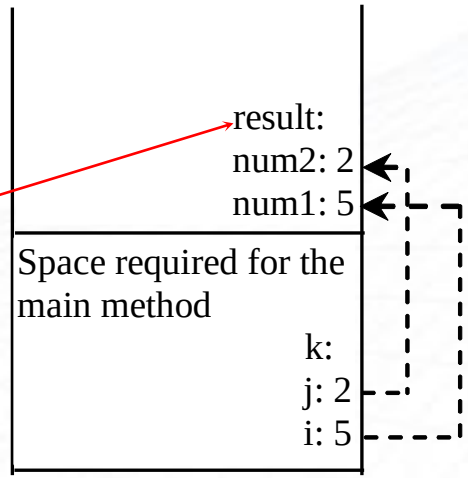


Trace Call Stack

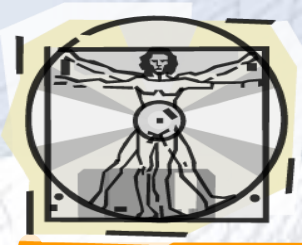
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```

Declare result



The max method is invoked.



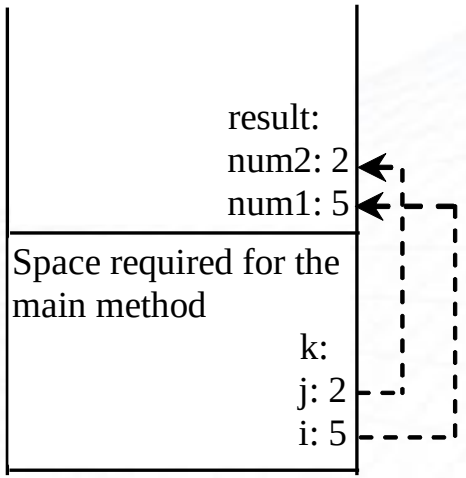
animation

Trace Call Stack

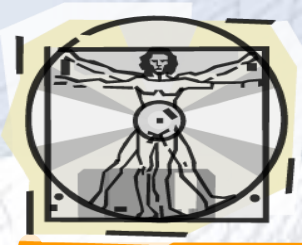
(num1 > num2) is true

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



The max method is invoked.



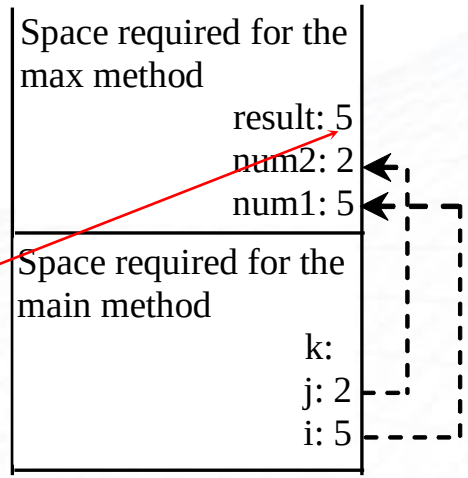
animation

Trace Call Stack

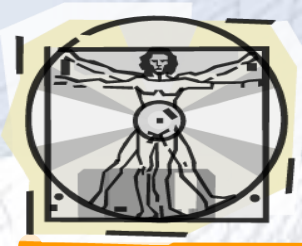
```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2)
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```

Assign num1 to result



The max method is invoked.



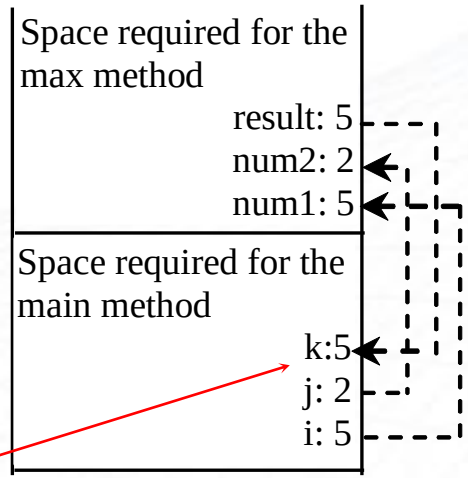
animation

Trace Call Stack

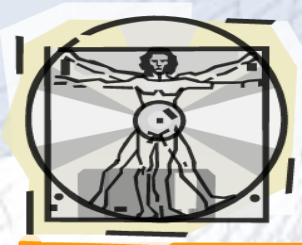
Return result and assign it to k

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```



The max method is invoked.



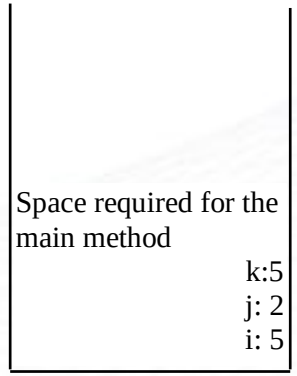
animation

Trace Call Stack

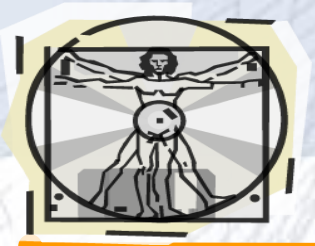
Execute print statement

```
public static void main(String[] args) {
  int i = 5;
  int j = 2;
  int k = max(i, j);

  System.out.println(
   "The maximum between " + i +
   " and " + j + " is " + k);
}
```

```
public static int max(int num1, int num2) {
  int result;

  if (num1 > num2)
    result = num1;
  else
    result = num2;

  return result;
}
```

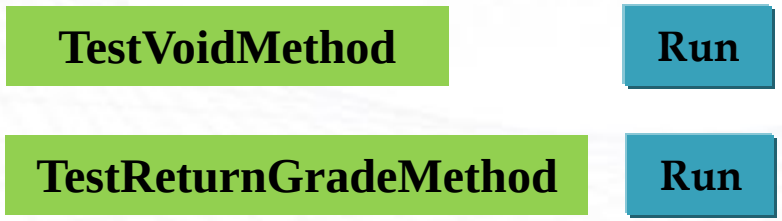


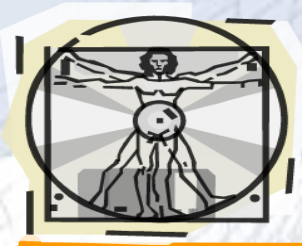
The main method is invoked.



void Method Example

- This type of method does not return a value. The method performs some actions.





Passing Parameters

```
public static void nPrintln(String message, int n) {
  for (int i = 0; i < n; i++)
    System.out.println(message);
}
```

Suppose you invoke the method using

nPrintln("Welcome to Java", 5);

What is the output?

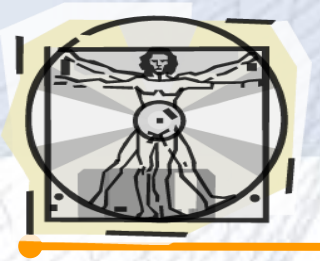
Suppose you invoke the method using

nPrintln("Computer Science", 15);

What is the output?

Can you invoke the method using

nPrintln(15, "Computer Science");

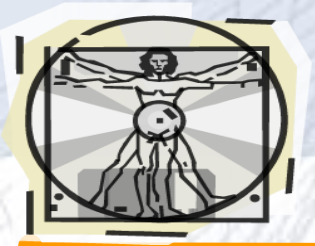


Pass by Value

- This program demonstrates passing values to the methods.

```
public class Increment {
  public static void main(String[] args) {
    int x = 1;
    System.out.println("Before the call, x is " + x);
    increment(x);
    System.out.println("After the call, x is " + x);
  }

  public static void increment(int n) {
    n++;
    System.out.println("n inside the method is " + n);
  }
}
```

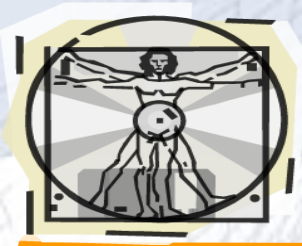
Increment Run



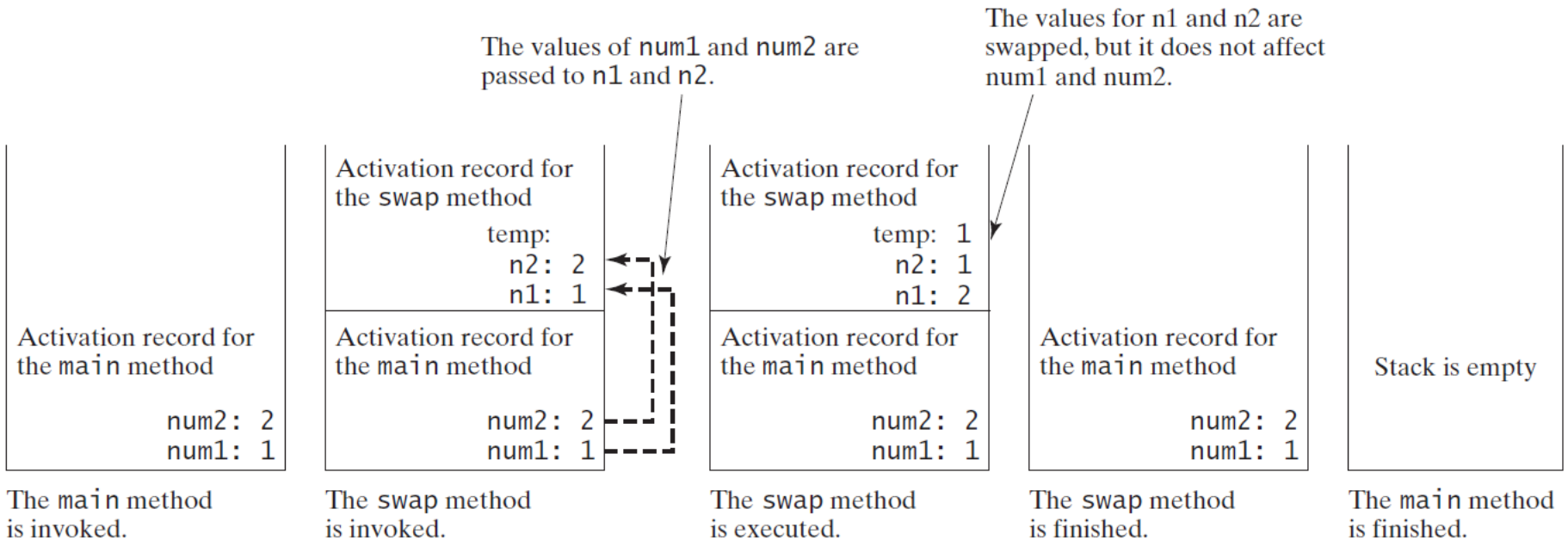
Pass by Value

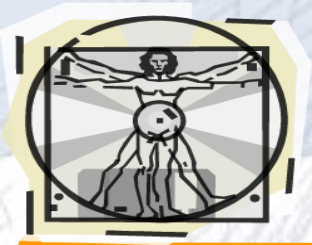
- Testing Pass by value
- This program demonstrates passing values to the methods.

TestPassByValue **Run**



Pass by Value, cont.



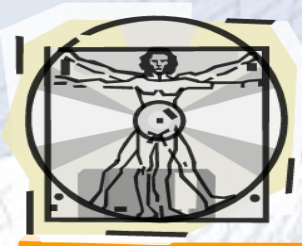


Modularizing Code

Methods can be used to reduce redundant coding and enable code reuse. Methods can also be used to modularize code and improve the quality of the program.

GreatestCommonDivisorMethod **Run**

PrimeNumberMethod **Run**



Case Study: Converting Hexadecimals to Decimals

- Write a method that converts a hexadecimal number into a decimal number.

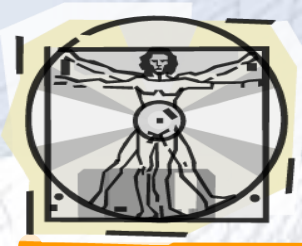
ABCD =>

A*16^3 + B*16^2 + C*16^1+ D*16^0

= ((A*16 + B)*16 + C)*16+D

= ((10*16 + 11)*16 + 12)*16+13 = ?

Hex2Dec **Run**



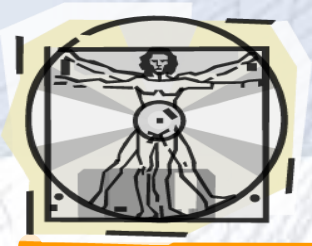
Overloading Methods

Overloading the `max` Method

```
public static double max(double num1, double num2) {
   if (num1 > num2)
      return num1;
   else
      return num2;
}
```

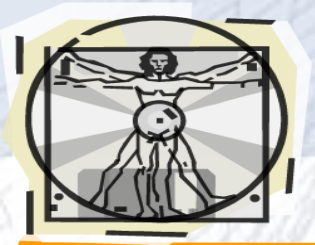
TestMethodOverloadin g

Run



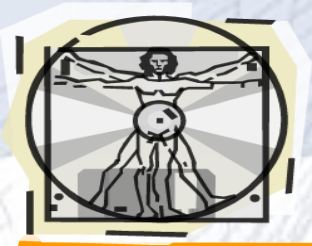
Ambiguous Invocation

Sometimes there may be two or more possible matches for an invocation of a method, but the compiler cannot determine the most specific match. This is referred to as *ambiguous invocation*. Ambiguous invocation is a compile error.



Ambiguous Invocation

```
public class AmbiguousOverloading {
  public static void main(String[] args) {
    System.out.println(max(1, 2));
  }

  public static double max(int num1, double num2) {
    if (num1 > num2)
      return num1;
    else
      return num2;
  }

  public static double max(double num1, int num2) {
    if (num1 > num2)
      return num1;
    else
      return num2;
  }
}
```

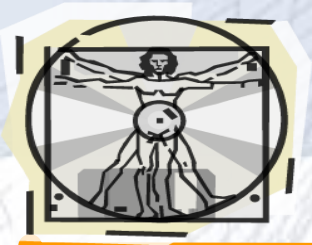


Scope of Local Variables

A local variable: a variable defined inside a method.

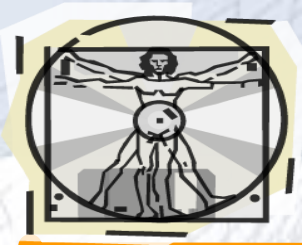
Scope: the part of the program where the variable can be referenced.

The scope of a local variable starts from its declaration and continues to the end of the block that contains the variable. A local variable must be declared before it can be used.



Scope of Local Variables, cont.

You can declare a local variable with the same name multiple times in different non-nesting blocks in a method, but you cannot declare a local variable twice in nested blocks.



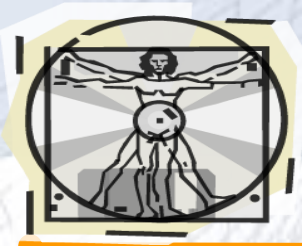
Scope of Local Variables, cont.

A variable declared in the initial action part of a for loop header has its scope in the entire loop. But a variable declared inside a for loop body has its scope limited in the loop body from its declaration and to the end of the block that contains the variable.

```
public static void method1() {
  .
  .
  for (int i = 1; i < 10; i++) {
    .
    .
    int j;
    .
    .
    .
  }
}
```

The scope of i

The scope of j



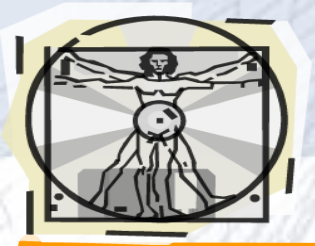
Scope of Local Variables, cont.

It is fine to declare i in two non-nesting blocks

```
public static void method1() {
  int x = 1;
  int y = 1;

  for (int i = 1; i < 10; i++) {
    x += i;
  }

  for (int i = 1; i < 10; i++) {
    y += i;
  }
}
```

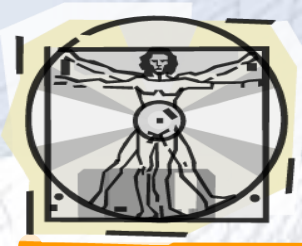
It is wrong to declare i in two nesting blocks

```
public static void method2() {

  int i = 1;
  int sum = 0;

  for (int i = 1; i < 10; i++)
    sum += i;
  }
}
```



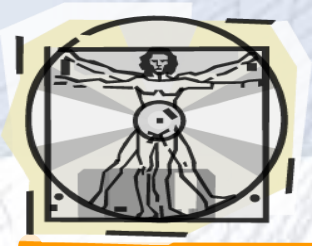
Scope of Local Variables, cont.

```
// Fine with no errors
public static void correctMethod() {
   int x = 1;
   int y = 1;
   // i is declared
   for (int i = 1; i < 10; i++) {
      x += i;
   }
   // i is declared again
   for (int i = 1; i < 10; i++) {
      y += i;
   }
}
```



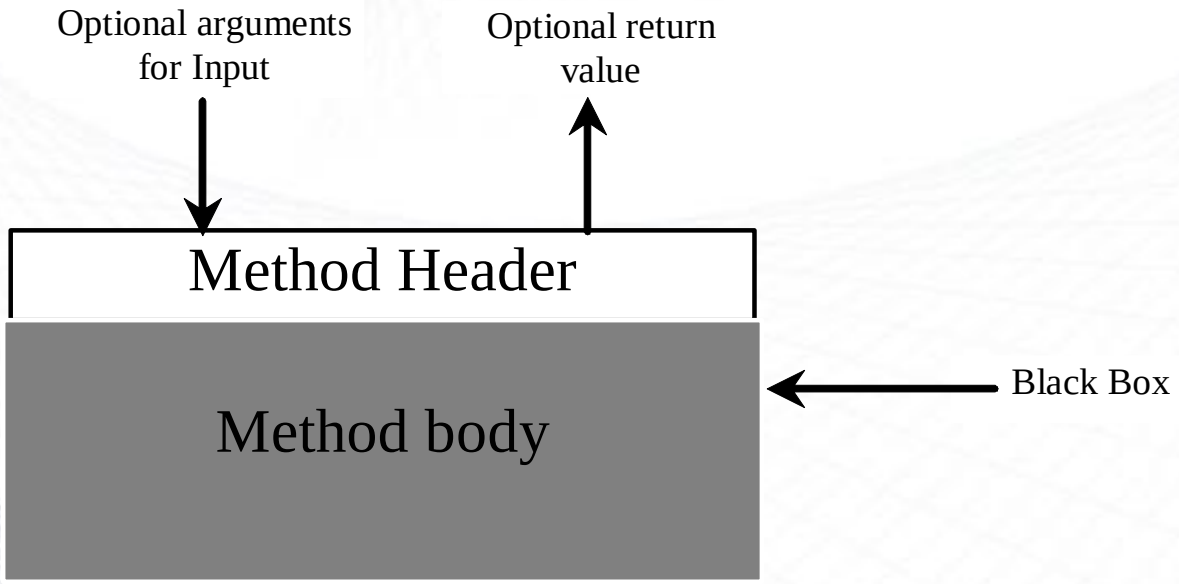
Scope of Local Variables, cont.

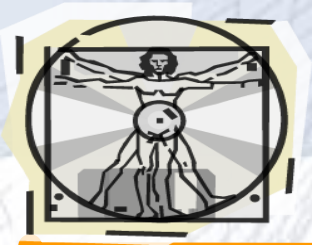
```
// With errors
public static void incorrectMethod() {
   int x = 1;
   int y = 1;
   for (int i = 1; i < 10; i++) {
      int x = 0;
      x += i;
   }
}
```



Method Abstraction

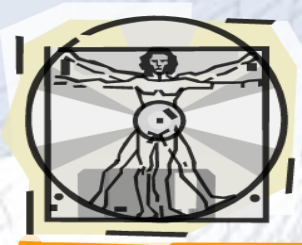
You can think of the method body as a black box that contains the detailed implementation for the method.





Benefits of Methods

- Write a method once and reuse it anywhere.
- Information hiding. Hide the implementation from the user.
- Reduce complexity.

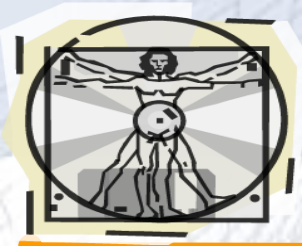


Case Study: Generating Random Characters

Computer programs process numerical data and characters. You have seen many examples that involve numerical data. It is also important to understand characters and how to process them.

As introduced in Section 4.3, each character has a unique Unicode between 0 and FFFF in hexadecimal (65535 in decimal). To generate a random character is to generate a random integer between 0 and 65535 using the following expression: (note that since 0 <= Math.random() < 1.0, you have to add 1 to 65535.)

```
(int)(Math.random() * (65535 + 1))
```



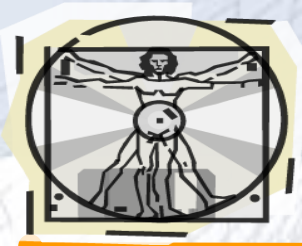
Case Study: Generating Random Characters, cont.

Now let us consider how to generate a random lowercase letter. The Unicode for lowercase letters are consecutive integers starting from the Unicode for 'a', then for 'b', 'c', ..., and 'z'. The Unicode for 'a' is

```
(int)'a'
```

So, a random integer between (int)'a' and (int)'z' is

```
(int)((int)'a' + Math.random() * ((int)'z' - (int)'a' + 1)
```



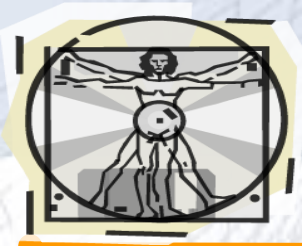
Case Study: Generating Random Characters, cont.

Now let us consider how to generate a random lowercase letter. The Unicode for lowercase letters are consecutive integers starting from the Unicode for 'a', then for 'b', 'c', ..., and 'z'. The Unicode for 'a' is

```
(int)'a'
```

So, a random integer between (int)'a' and (int)'z' is

```
(int)((int)'a' + Math.random() * ((int)'z' - (int)'a' + 1)
```



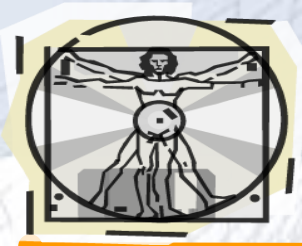
Case Study: Generating Random Characters, cont.

As discussed in Chapter 2, all numeric operators can be applied to the char operands. The char operand is cast into a number if the other operand is a number or a character. So, the preceding expression can be simplified as follows:

```
'a' + Math.random() * ('z' - 'a' + 1)
```

So a random lowercase letter is

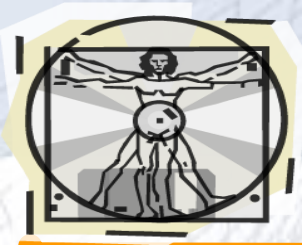
```
(char)('a' + Math.random() * ('z' - 'a' + 1))
```



Case Study: Generating Random Characters, cont.

To generalize the foregoing discussion, a random character between any two characters ch1 and ch2 with ch1 < ch2 can be generated as follows:

```
(char)(ch1 + Math.random() * (ch2 – ch1 + 1))
```



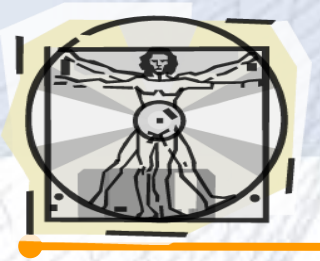
The RandomCharacter Class

```
// RandomCharacter.java: Generate random characters
public class RandomCharacter {
  /** Generate a random character between ch1 and ch2 */
  public static char getRandomCharacter(char ch1, char ch2) {
    return (char)(ch1 + Math.random() * (ch2 - ch1 + 1));
  }

  /** Generate a random lowercase letter */
  public static char getRandomLowerCaseLetter() {
    return getRandomCharacter('a', 'z');
  }

  /** Generate a random uppercase letter */
  public static char getRandomUpperCaseLetter() {
    return getRandomCharacter('A', 'Z');
  }

  /** Generate a random digit character */
  public static char getRandomDigitCharacter() {
    return getRandomCharacter('0', '9');
  }

  /** Generate a random character */
  public static char getRandomCharacter() {
    return getRandomCharacter('\u0000', '\uFFFF');
  }
}
```

RandomCharacter

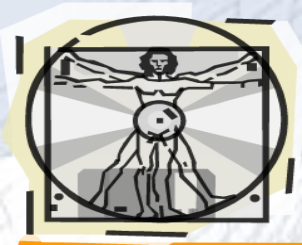
TestRandomCharacter

Run



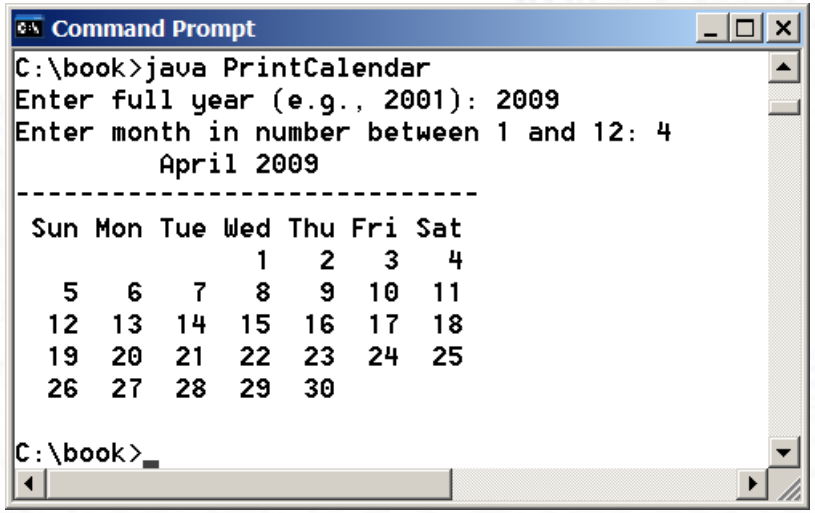
Stepwise Refinement (Optional)

The concept of method abstraction can be applied to the process of developing programs. When writing a large program, you can use the "divide and conquer" strategy, also known as *stepwise refinement*, to decompose it into subproblems. The subproblems can be further decomposed into smaller, more manageable problems.

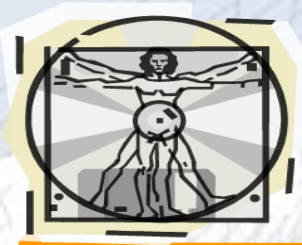


PrintCalender Case Study

Let us use the PrintCalendar example to demonstrate the stepwise refinement approach.

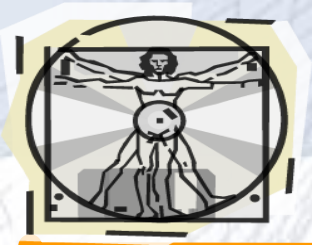


PrintCalendar **Run**

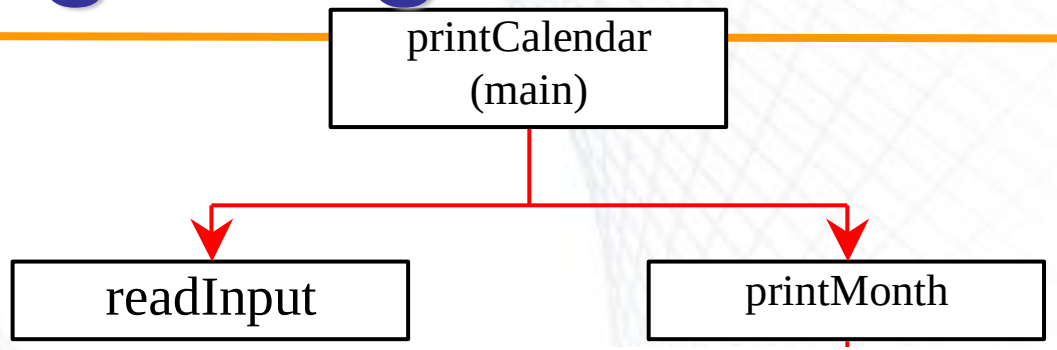


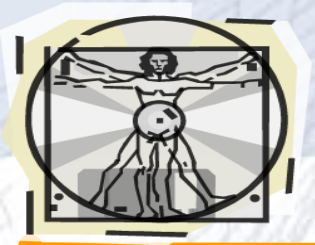
Design Diagram

printCalendar
(main)

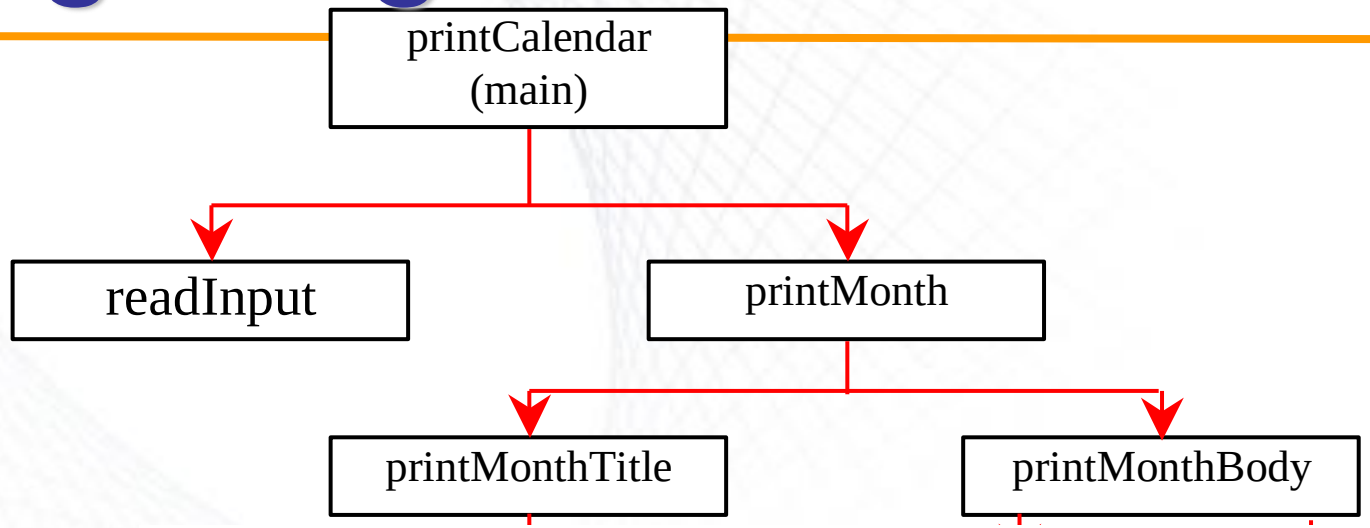


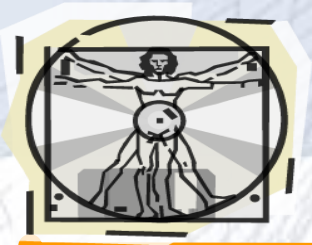
Design Diagram



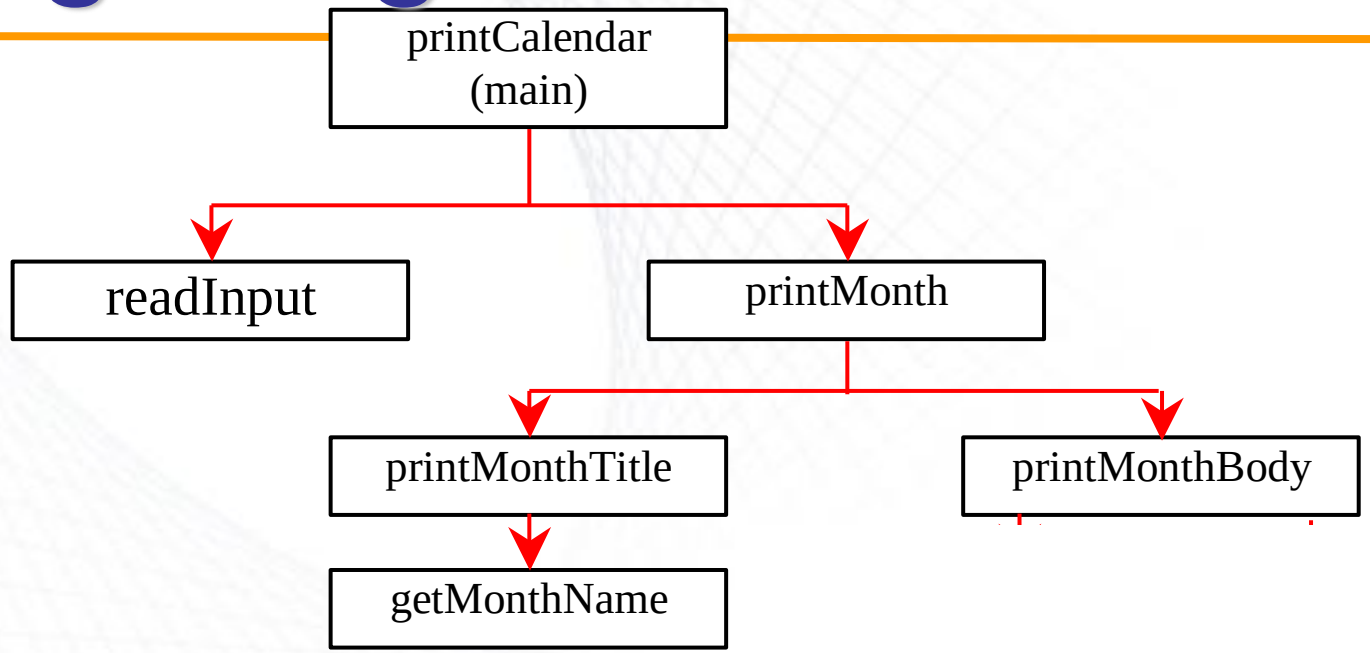


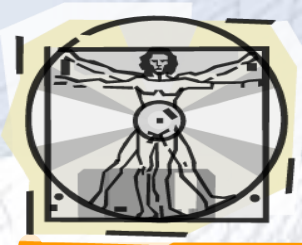
Design Diagram



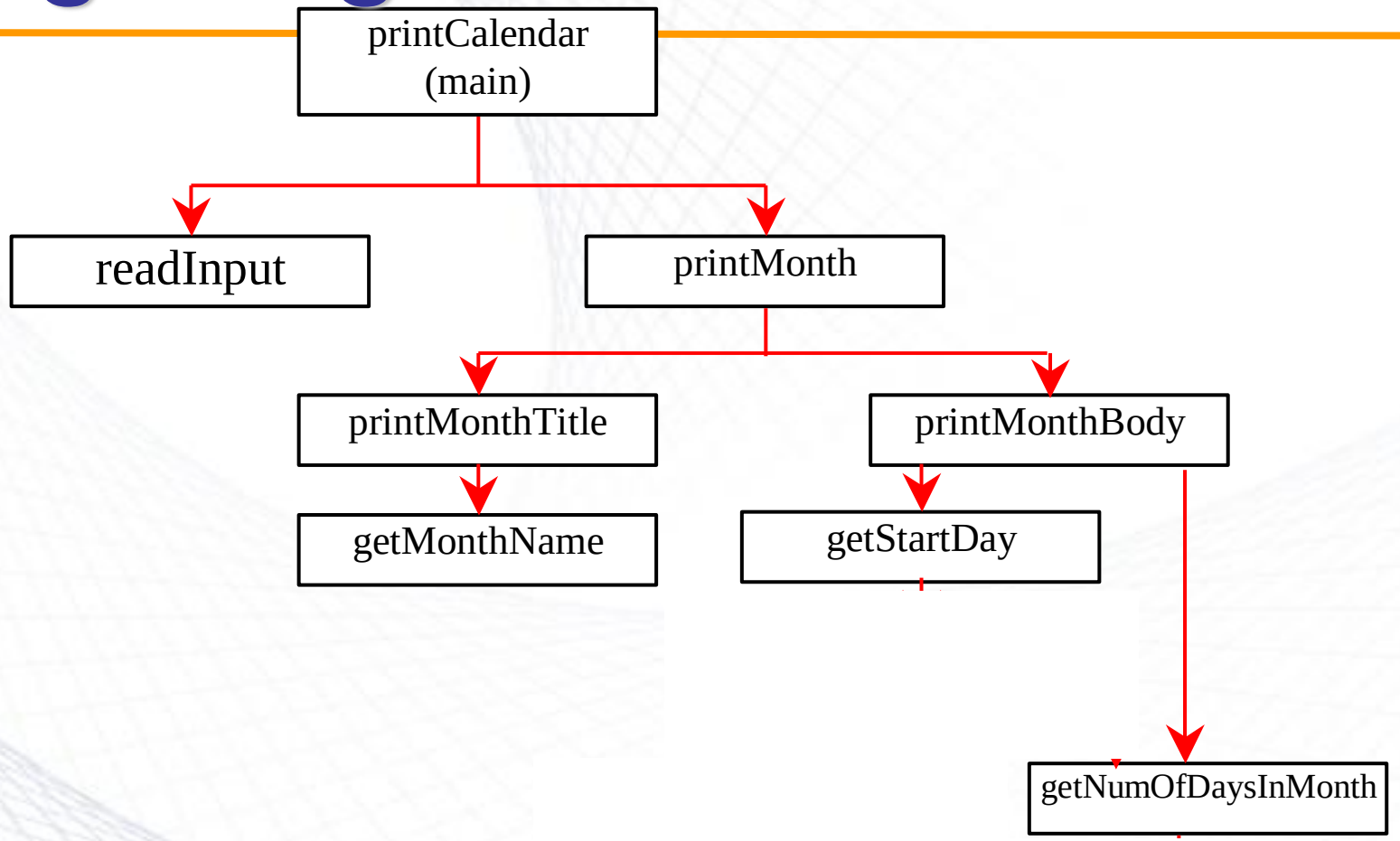


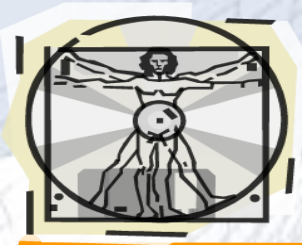
Design Diagram



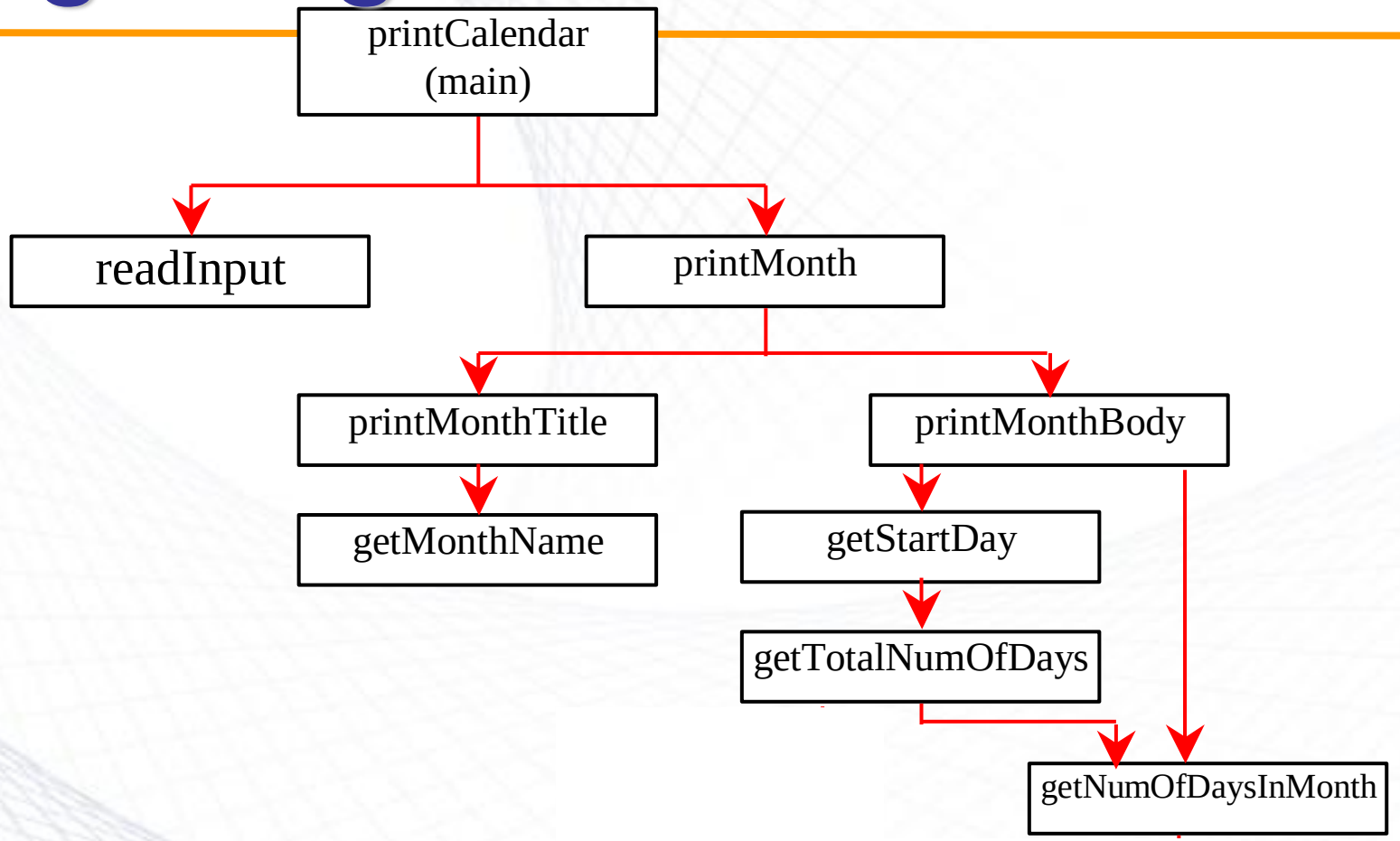


Design Diagram

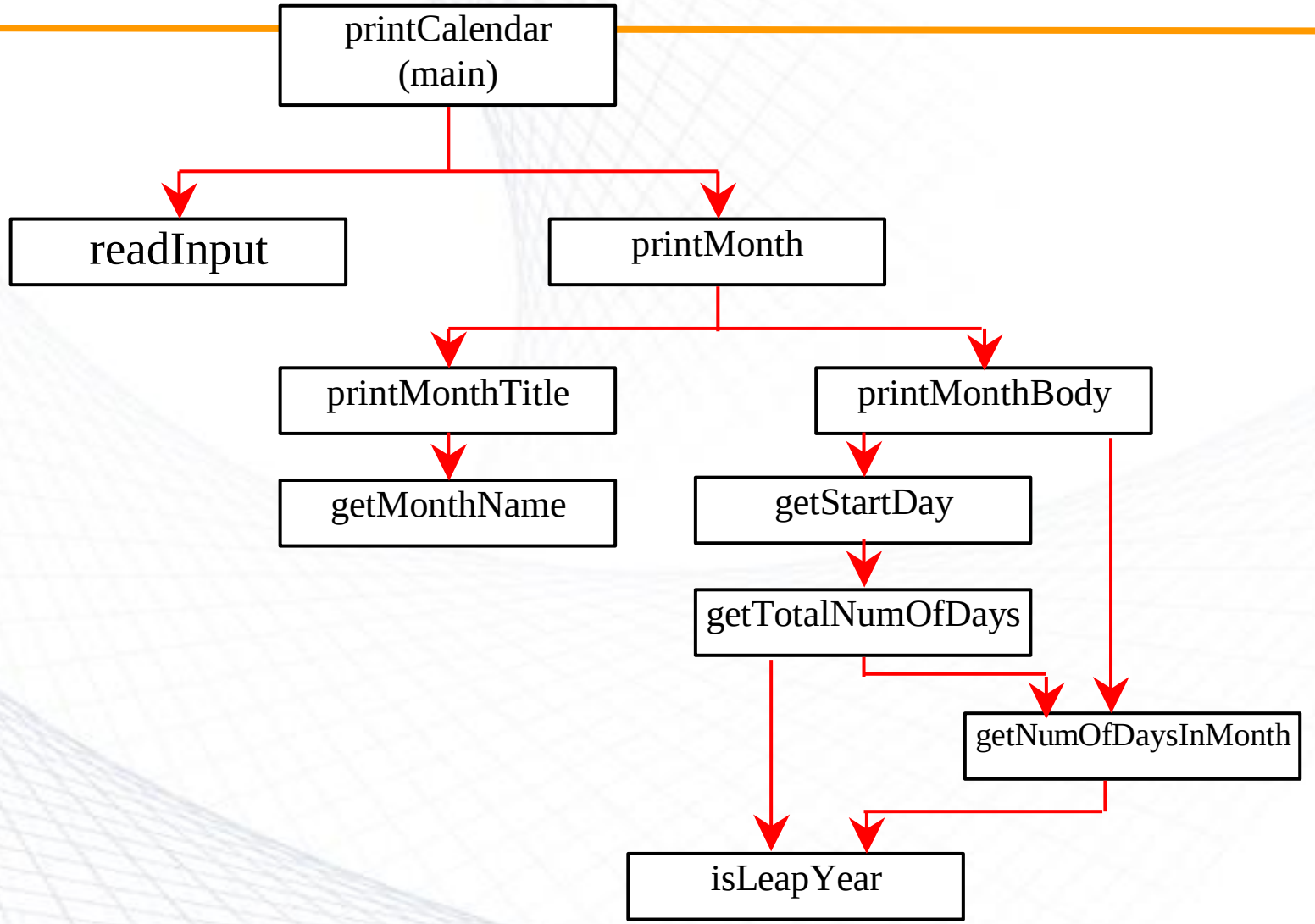


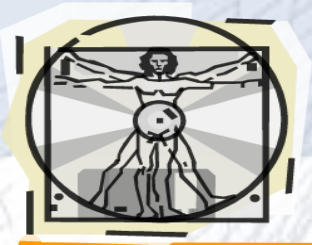


Design Diagram



Design Diagram

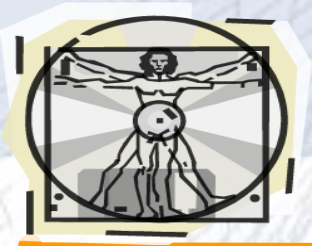




Implementation: Top-Down

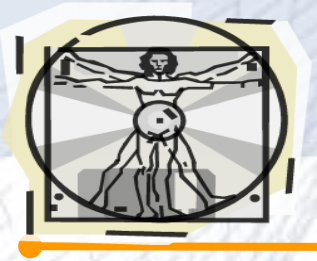
- Top-down approach is to implement one method in the structure chart at a time from the top to the bottom. Stubs can be used for the methods waiting to be implemented. A stub is a simple but incomplete version of a method. The use of stubs enables you to test invoking the method from a caller. Implement the main method first and then use a stub for the printMonth method. For example, let printMonth display the year and the month in the stub. Thus, your program may begin like this:

A Skeleton for printCalendar



Implementation: Bottom-Up

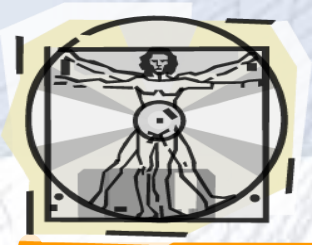
- Bottom-up approach is to implement one method in the structure chart at a time from the bottom to the top. For each method implemented, write a test program to test it. Both top-down and bottom-up methods are fine. Both approaches implement the methods incrementally and help to isolate programming errors and makes debugging easy. Sometimes, they can be used together.



Benefits of Stepwise Refinement

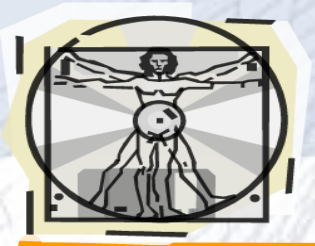
- Simpler Program
- Reusing Methods
- Easier Developing, Debugging, and Testing
- Better Facilitating Teamwork

Recursion



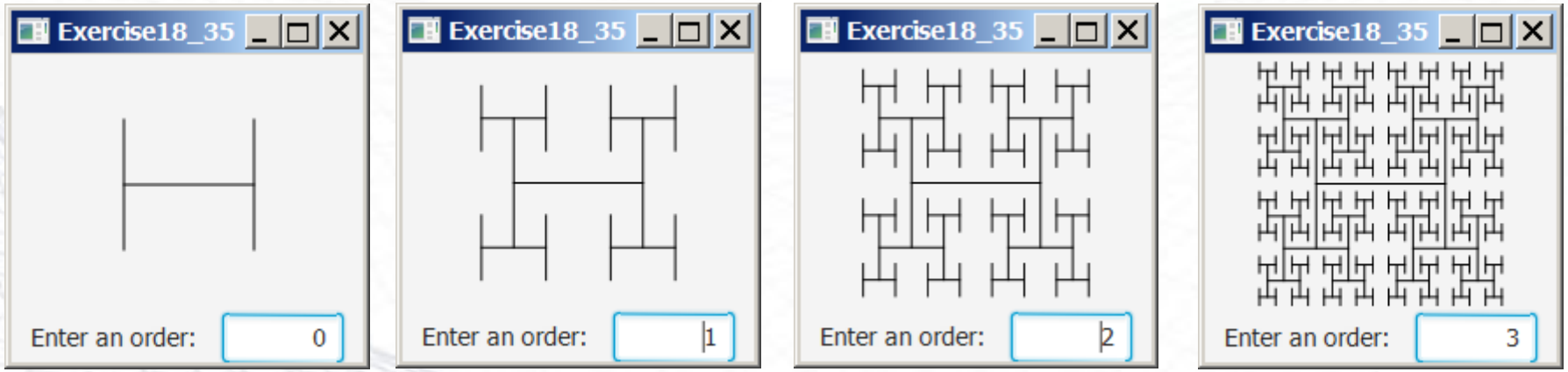
Motivations

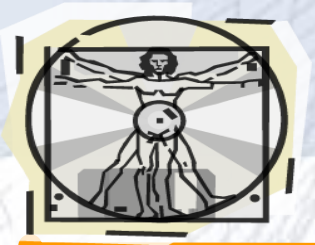
Suppose you want to find all the files under a directory that contains a particular word. How do you solve this problem? There are several ways to solve this problem. An intuitive solution is to use recursion by searching the files in the subdirectories recursively.



Motivations

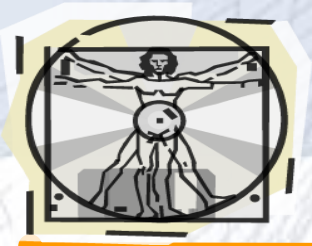
H-trees, depicted in Figure 18.1, are used in a very large-scale integration (VLSI) design as a clock distribution network for routing timing signals to all parts of a chip with equal propagation delays. How do you write a program to display H-trees? A good approach is to use recursion.





Objectives

- To describe what a recursive method is and the benefits of using recursion (§18.1).
- To develop recursive methods for recursive mathematical functions (§§18.2–18.3).
- To explain how recursive method calls are handled in a call stack (§§18.2–18.3).
- To solve problems using recursion (§18.4).
- To use an overloaded helper method to derive a recursive method (§18.5).
- To implement a selection sort using recursion (§18.5.1).
- To implement a binary search using recursion (§18.5.2).
- To get the directory size using recursion (§18.6).
- To solve the Tower of Hanoi problem using recursion (§18.7).
- To draw fractals using recursion (§18.8).
- To discover the relationship and difference between recursion and iteration (§18.9).
- To know tail-recursive methods and why they are desirable (§18.10).



Computing Factorial

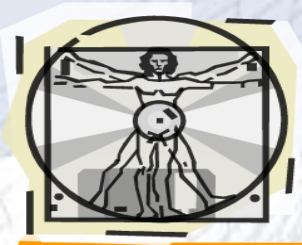
factorial(0) = 1;

factorial(n) = n*factorial(n-1);

n! = n * (n-1)!

0! = 1

ComputeFactorial Run

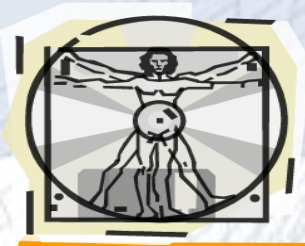


Computing Factorial

factorial(4)

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

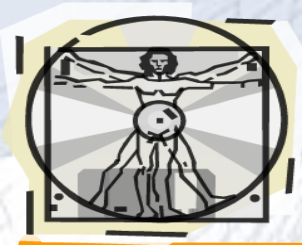


Computing Factorial

factorial(4) = 4 * factorial(3)

factorial(0) = 1;

factorial(n) = n*factorial(n-1);



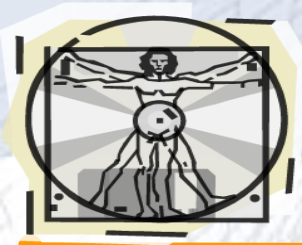
Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)



Computing Factorial

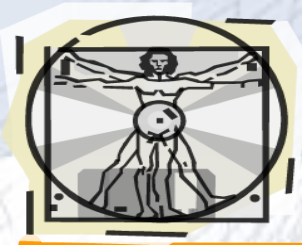
factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)

= 4 * 3 * (2 * factorial(1))



animation

Computing Factorial

factorial(0) = 1;

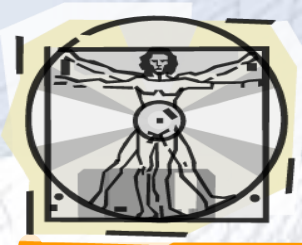
factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)

= 4 * 3 * (2 * factorial(1))

= 4 * 3 * (2 * (1 * factorial(0)))



animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

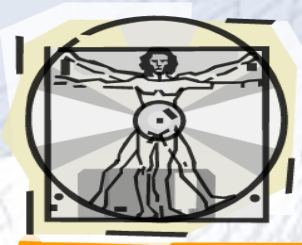
factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)

= 4 * 3 * (2 * factorial(1))

= 4 * 3 * (2 * (1 * factorial(0)))

= 4 * 3 * (2 * (1 * 1)))



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Computing Factorial

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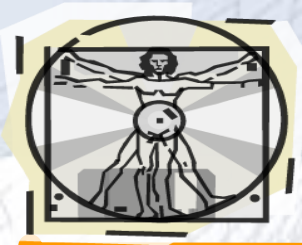
= 4 * 3 * (2 * factorial(1))

= 4 * 3 * (2 * (1 * factorial(0)))

= 4 * 3 * (2 * (1 * 1)))

= 4 * 3 * (2 * 1)



animation

Computing Factorial

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factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * 3 * factorial(2)

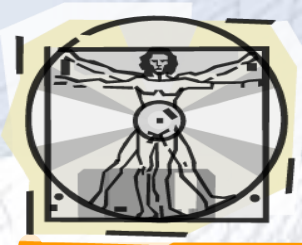
= 4 * 3 * (2 * factorial(1))

= 4 * 3 * (2 * (1 * factorial(0)))

= 4 * 3 * (2 * (1 * 1)))

= 4 * 3 * (2 * 1)

= 4 * 3 * 2



animation

Computing Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

factorial(4) = 4 * factorial(3)

= 4 * (3 * factorial(2))

= 4 * (3 * (2 * factorial(1)))

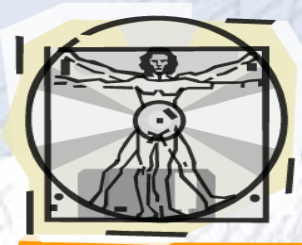
= 4 * (3 * (2 * (1 * factorial(0))))

= 4 * (3 * (2 * (1 * 1))))

= 4 * (3 * (2 * 1))

= 4 * (3 * 2)

= 4 * (6)



Computing Factorial

factorial(0) = 1;

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factorial(4) = 4 * factorial(3)

= 4 * (3 * factorial(2))

= 4 * (3 * (2 * factorial(1)))

= 4 * (3 * (2 * (1 * factorial(0))))

= 4 * (3 * (2 * (1 * 1))))

= 4 * (3 * (2 * 1))

= 4 * (3 * 2)

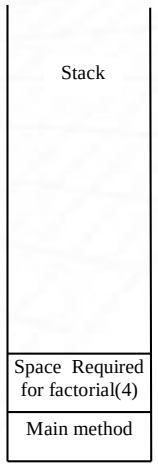
= 4 * (6)

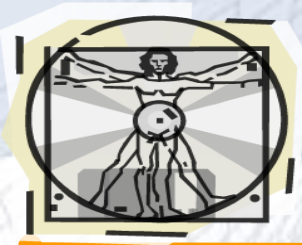
= 24

Trace Recursive factorial

Executes factorial(4)

factorial(4)





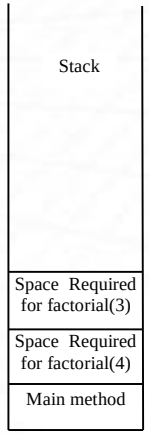
Trace Recursive factorial

factorial(4)

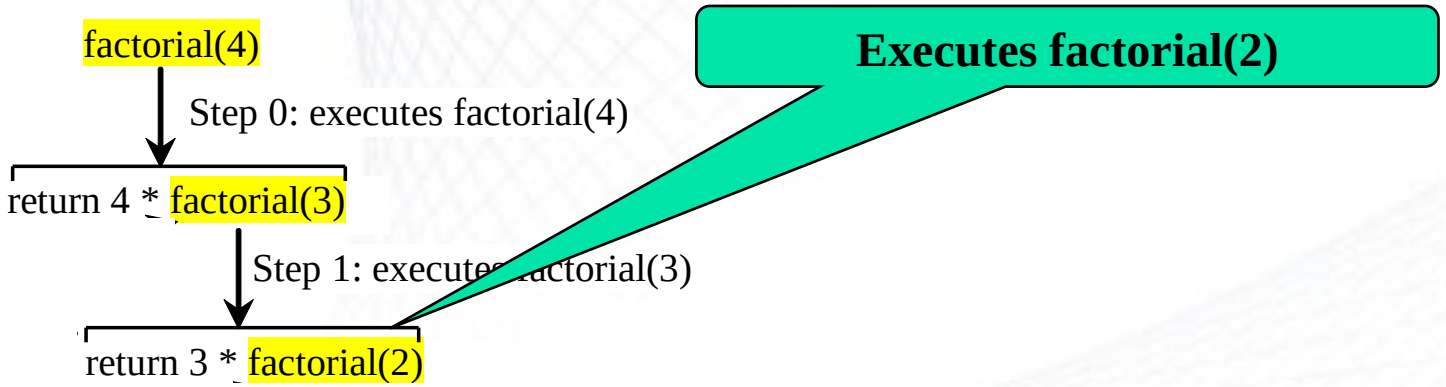
Step 0: executes factorial(4)

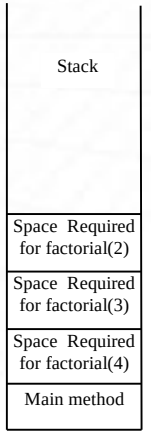
return 4 * factorial(3)

Executes factorial(3)

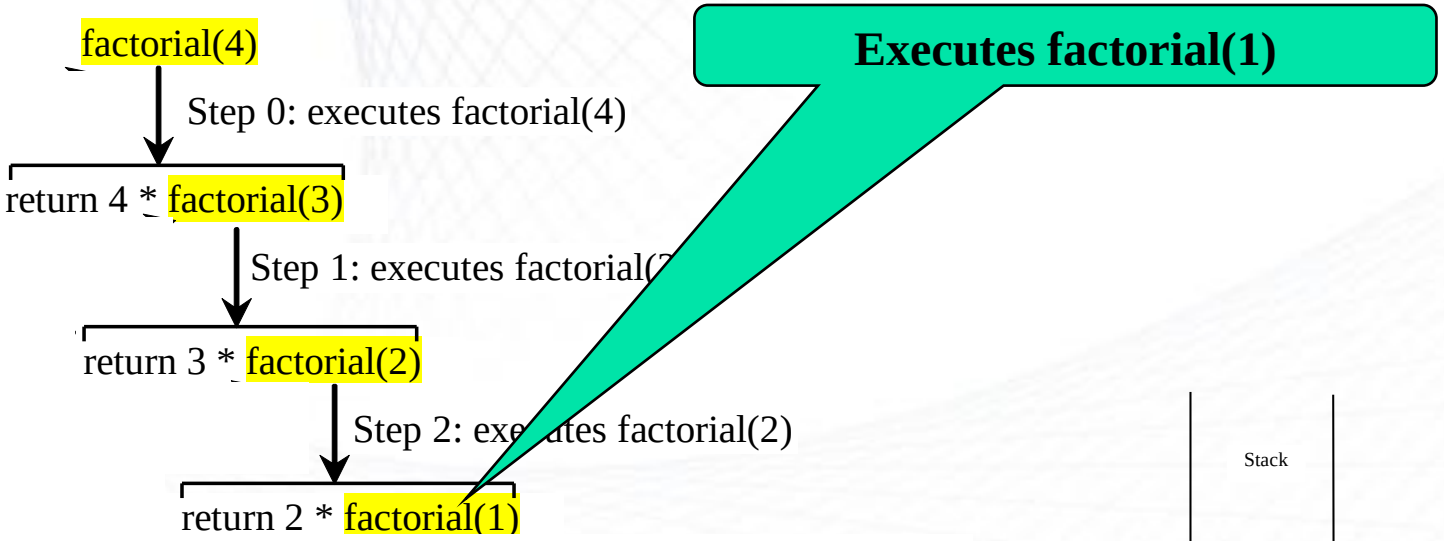


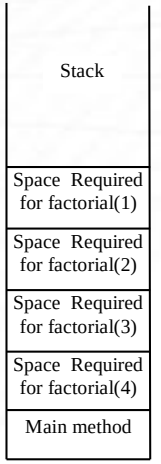
Trace Recursive factorial

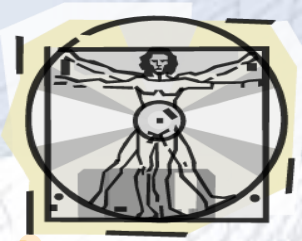




Trace Recursive factorial

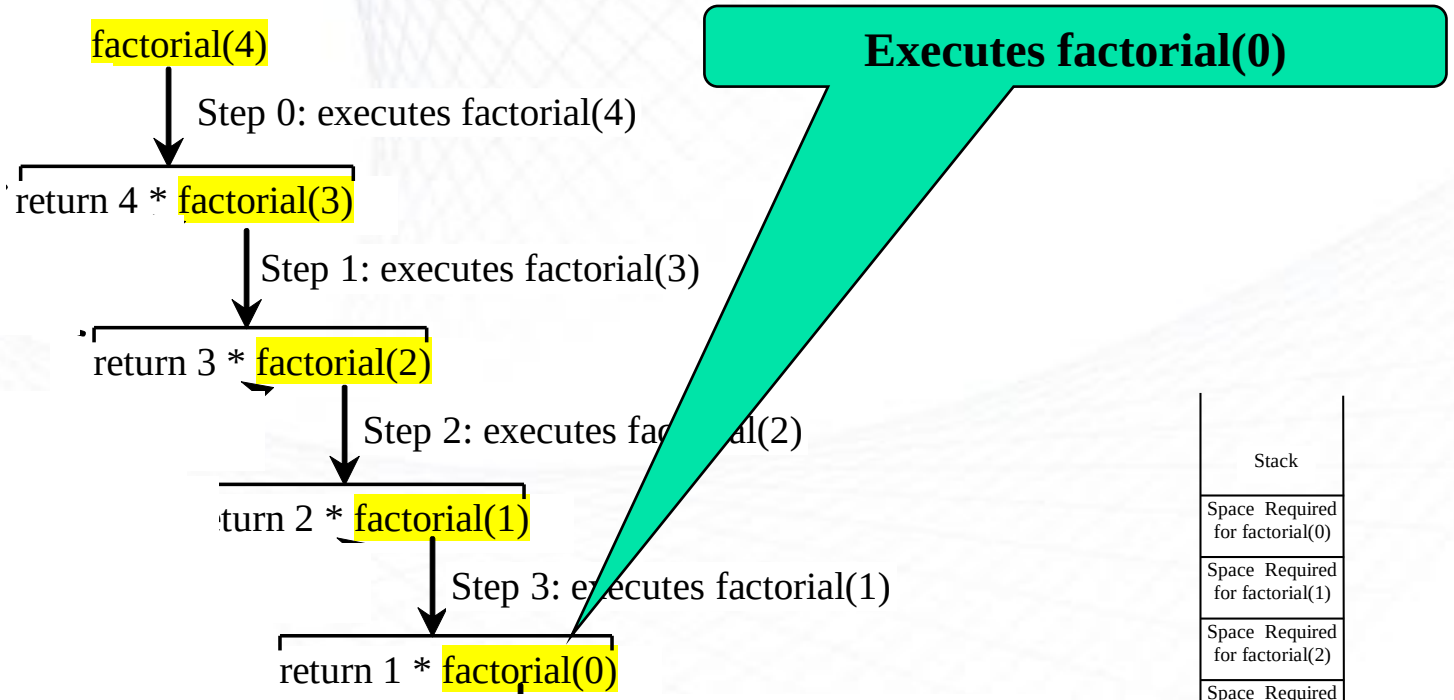


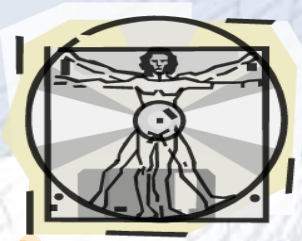




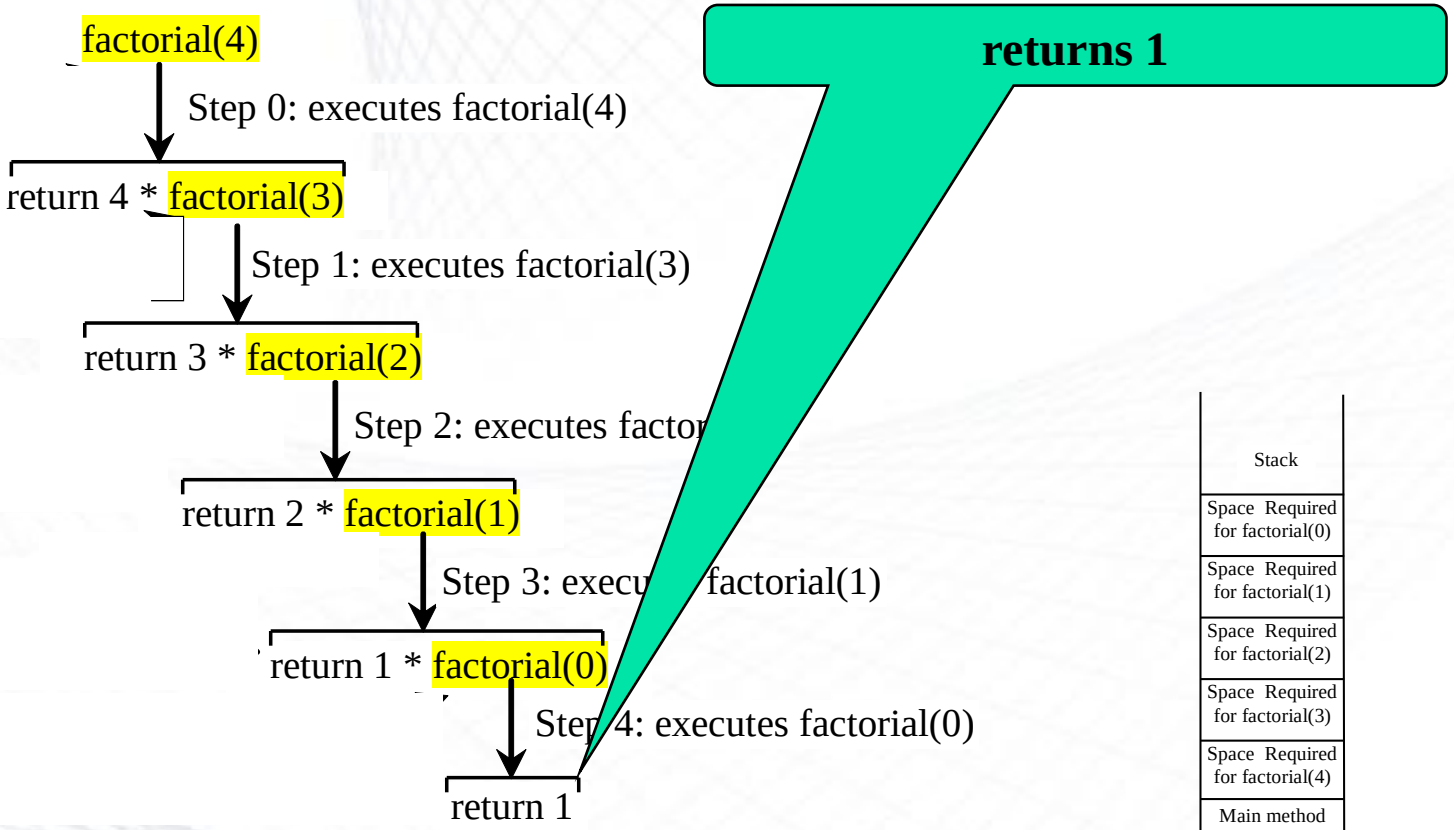
animation

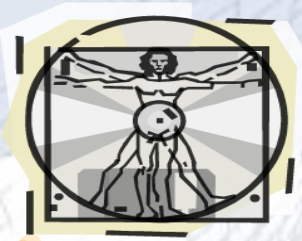
Trace Recursive factorial





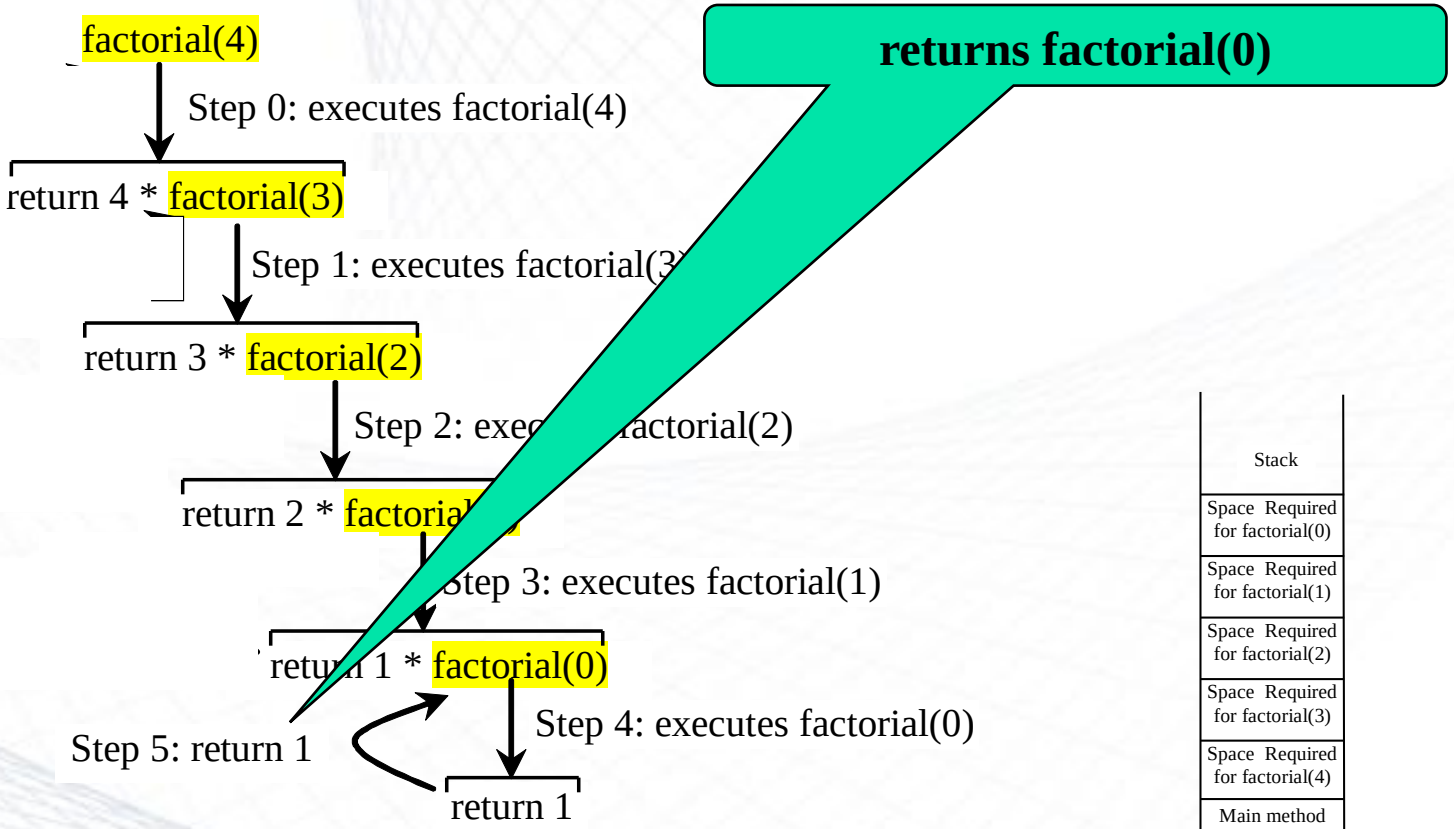
Trace Recursive factorial



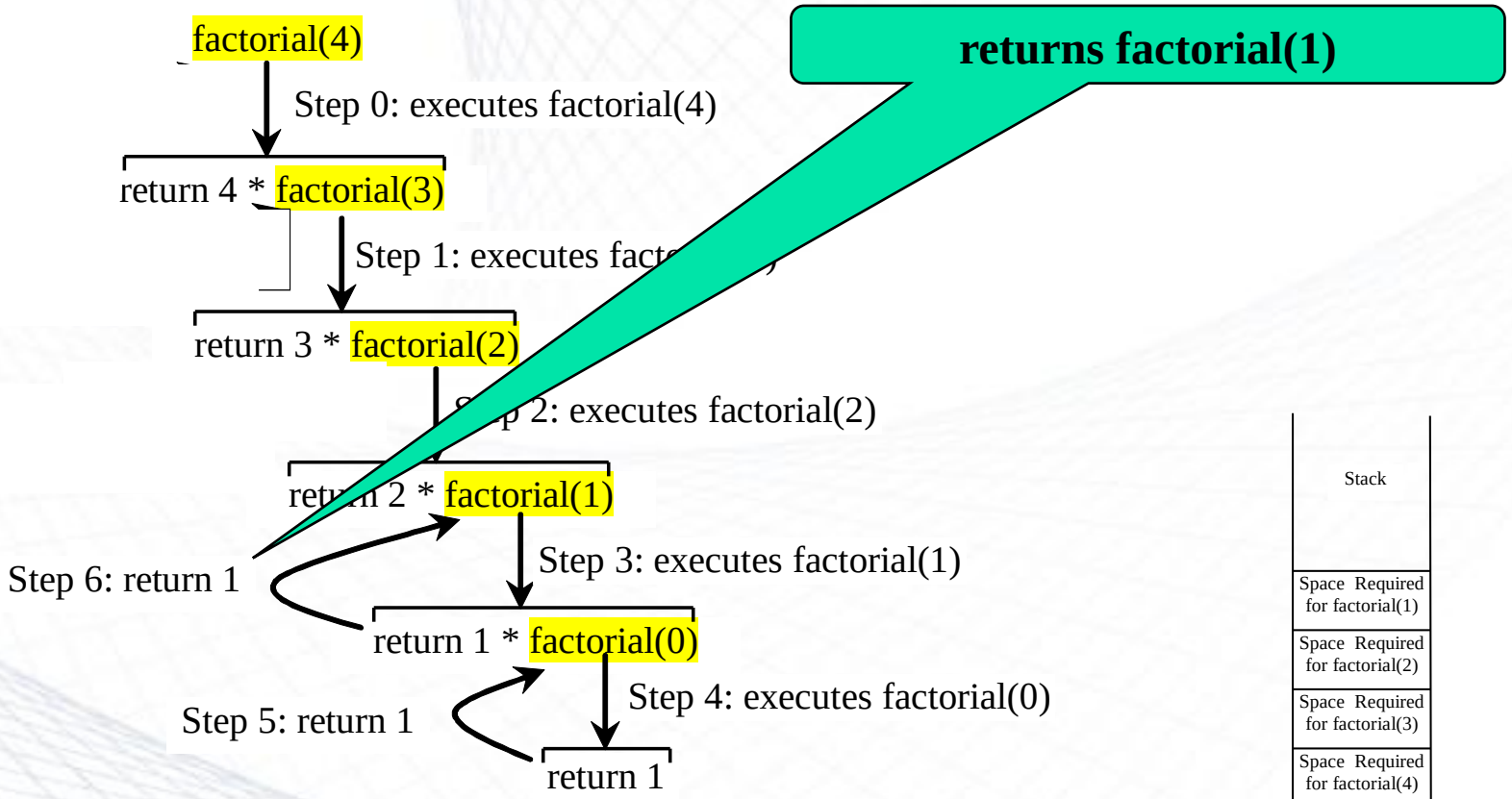


animation

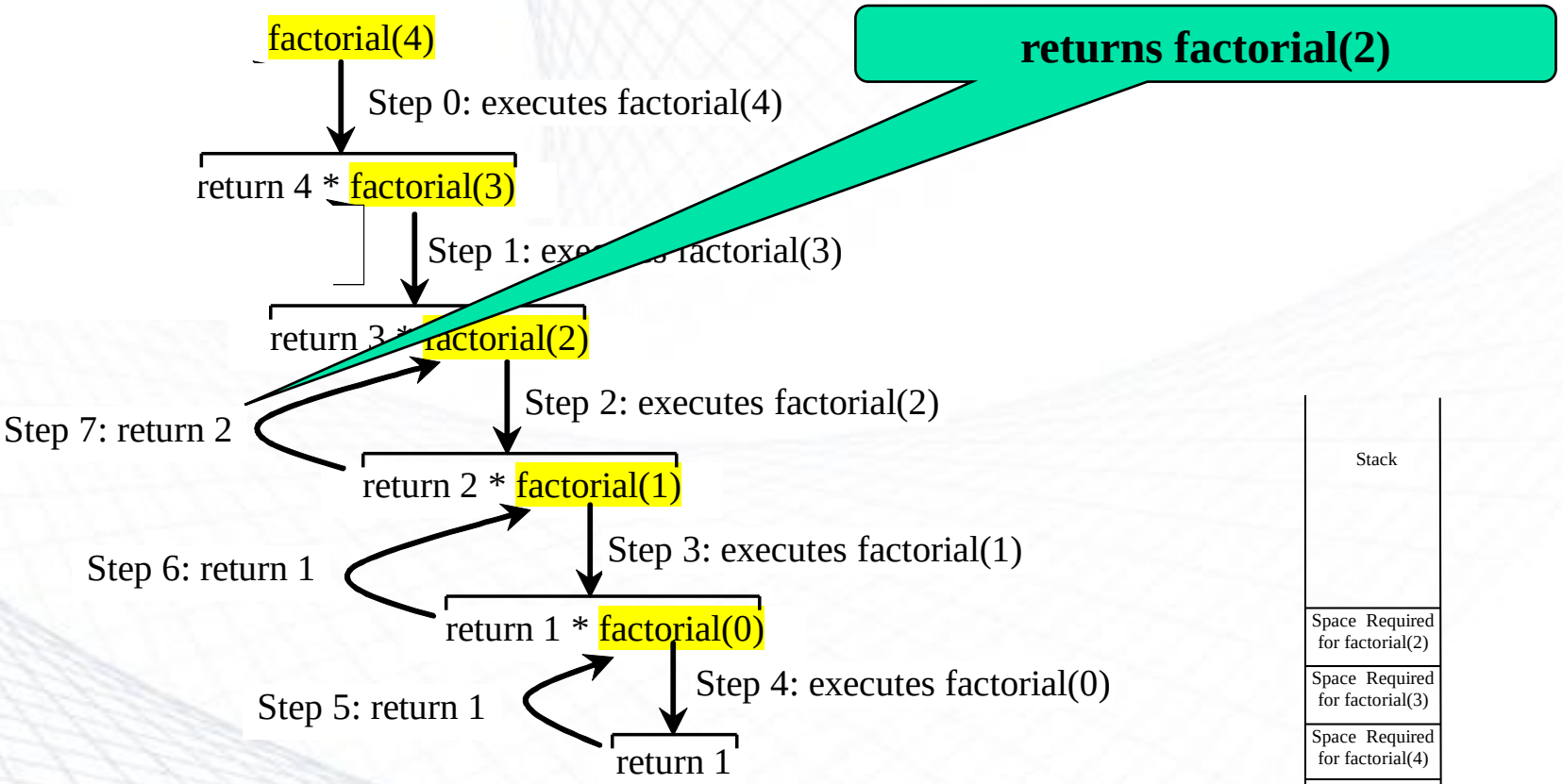
Trace Recursive factorial



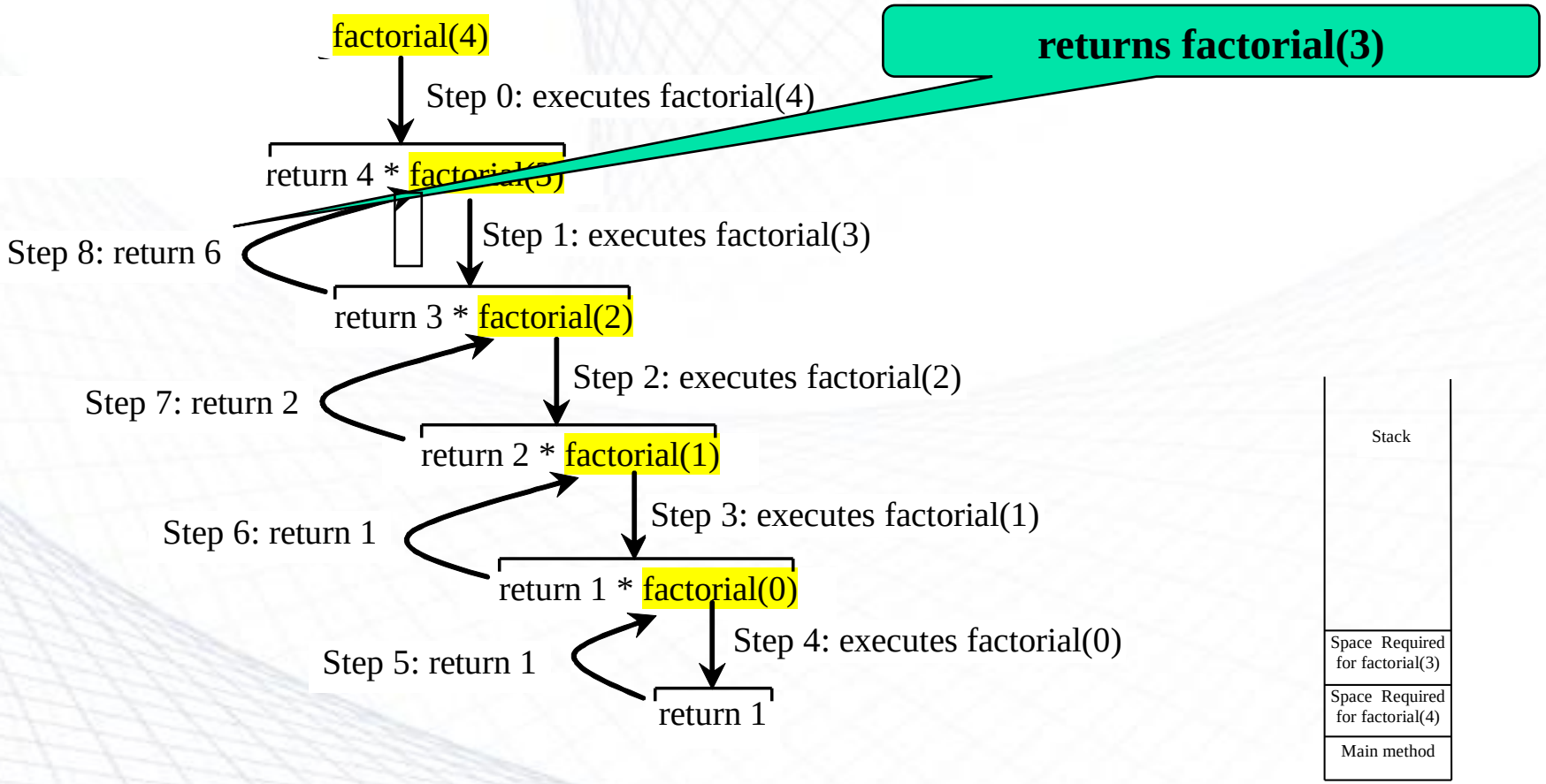
Trace Recursive factorial

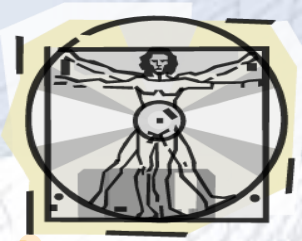


Trace Recursive factorial

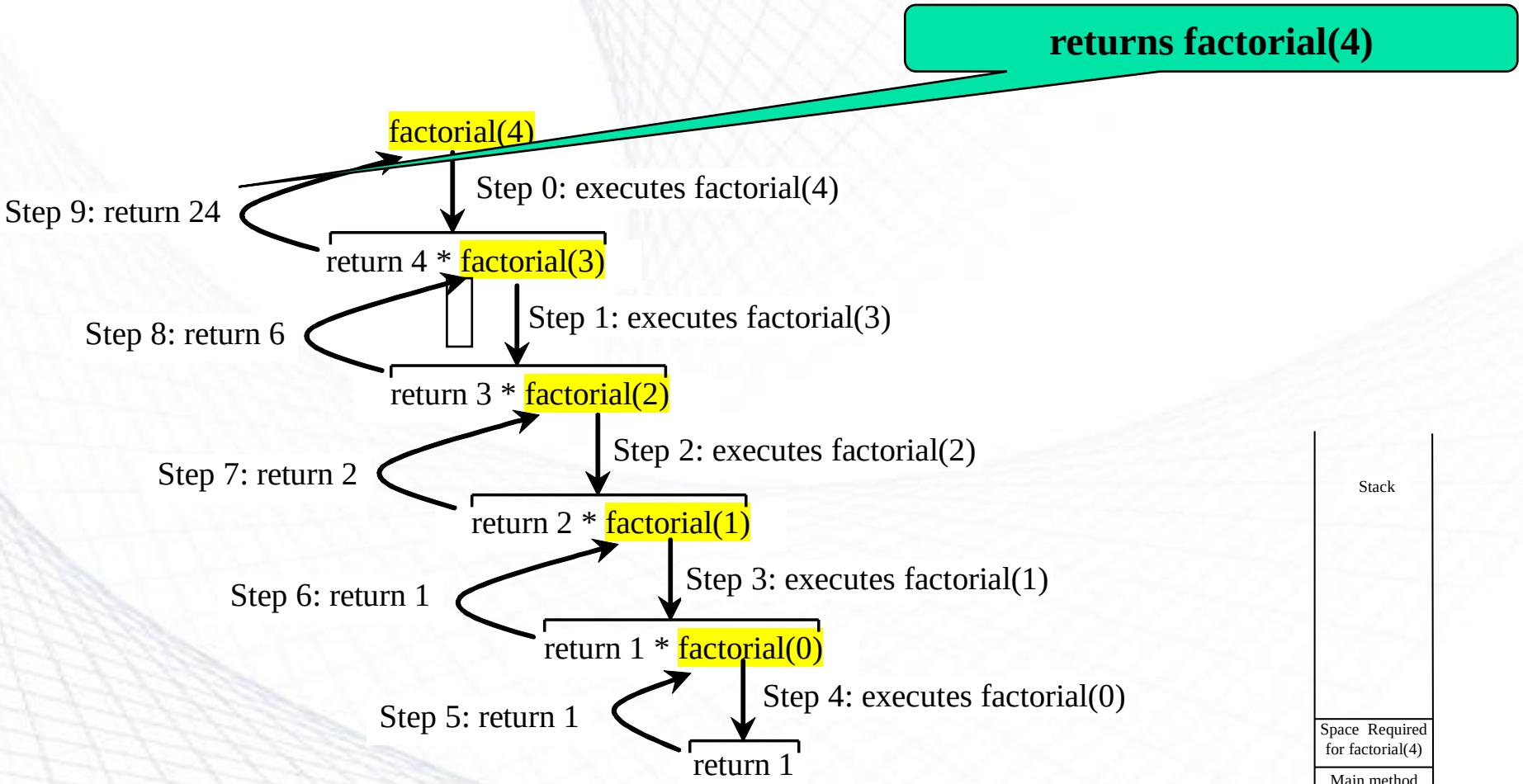


Trace Recursive factorial

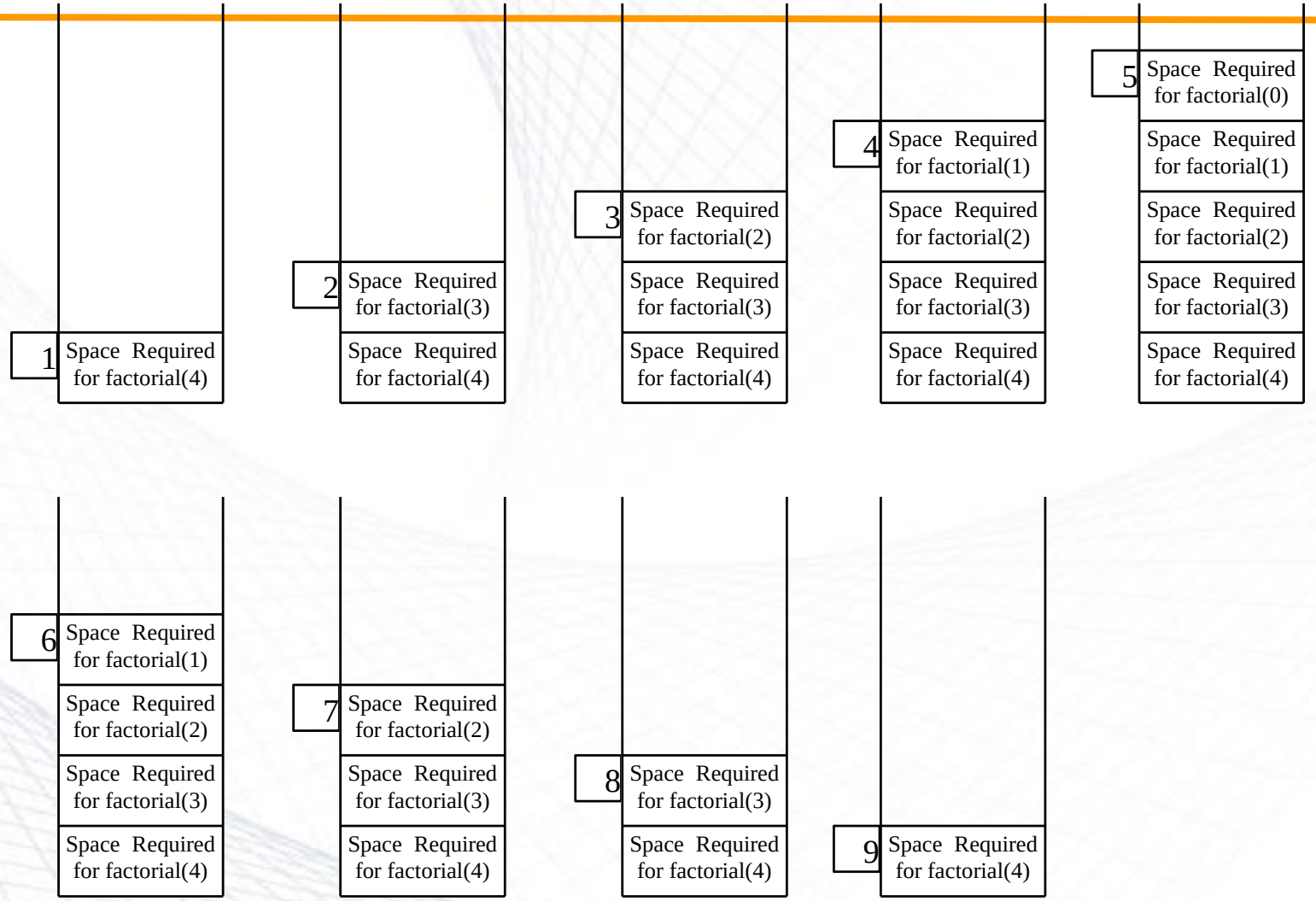


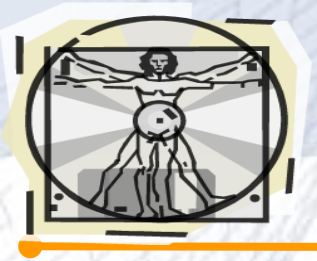


Trace Recursive factorial



factorial(4) Stack Trace

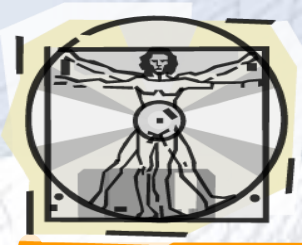




Other Examples

$f(0) = 0;$

$f(n) = n + f(n-1);$



Fibonacci Numbers

```
Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89…

         indices: 0 1 2 3 4 5 6 7  8  9  10 11
```

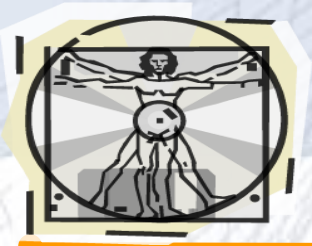
fib(0) = 0;

fib(1) = 1;

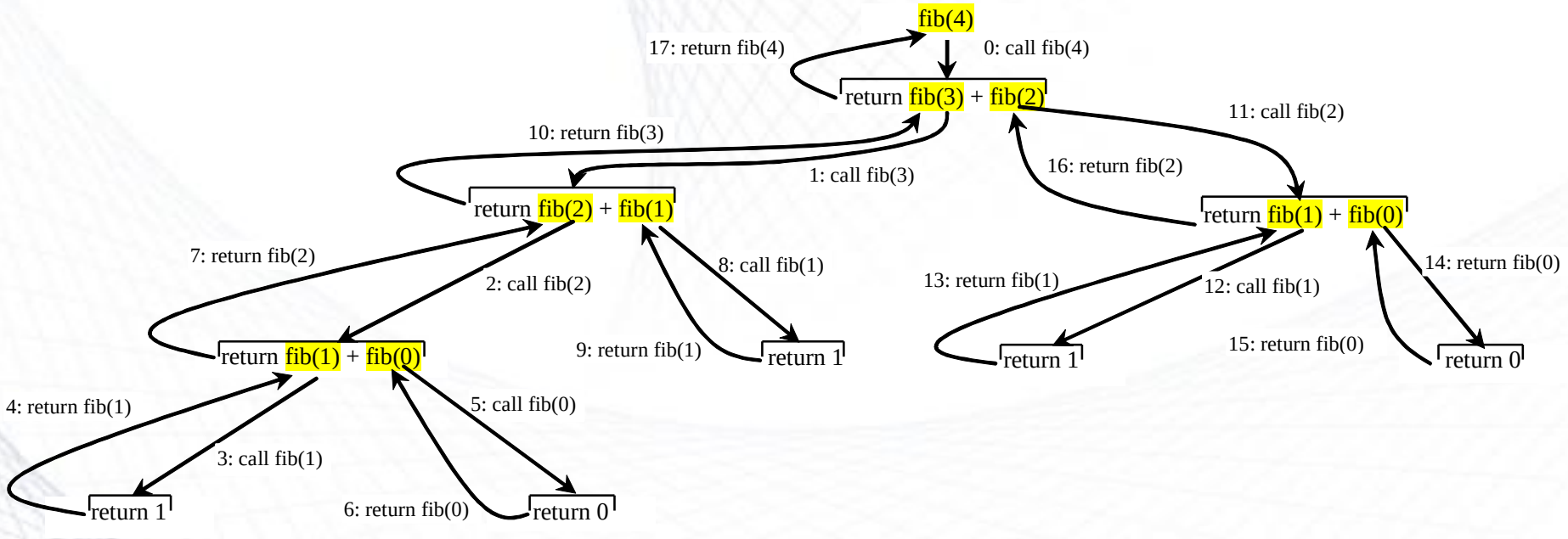
fib(index) = fib(index -1) + fib(index -2); index >=2

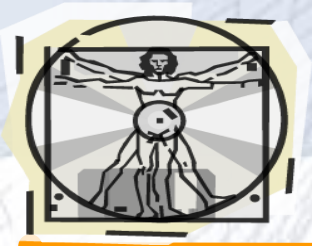
fib(3) = fib(2) + fib(1) = (fib(1) + fib(0)) + fib(1) = (1 + 0) +fib(1) = 1 + fib(1) = 1 + 1 = 2

ComputeFibonacci **Run**



Fibonnaci Numbers, cont.



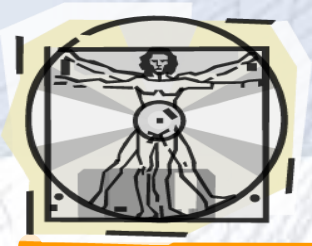


Characteristics of Recursion

All recursive methods have the following characteristics:

- One or more base cases (the simplest case) are used to stop recursion.
- Every recursive call reduces the original problem, bringing it increasingly closer to a base case until it becomes that case.

In general, to solve a problem using recursion, you break it into subproblems. If a subproblem resembles the original problem, you can apply the same approach to solve the subproblem recursively. This subproblem is almost the same as the original problem in nature with a smaller size.

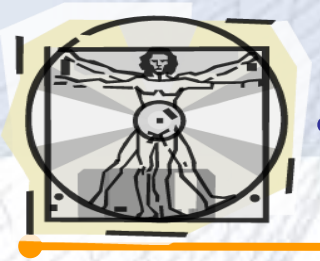


Problem Solving Using Recursion

Let us consider a simple problem of printing a message for n times. You can break the problem into two subproblems: one is to print the message one time and the other is to print the message for n-1 times. The second problem is the same as the original problem with a smaller size. The base case for the problem is n==0. You can solve this problem using recursion as follows:

nPrintln("Welcome", 5);

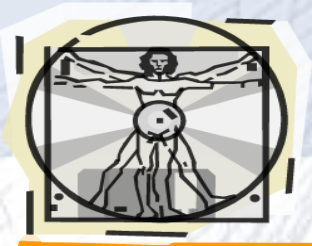
```
public static void nPrintln(String message, int times) {
  if (times >= 1) {
    System.out.println(message);
    nPrintln(message, times - 1);
  } // The base case is times == 0
}
```



Think Recursively

Many of the problems presented in the early chapters can be solved using recursion if you *think recursively*. For example, the palindrome problem can be solved recursively as follows:

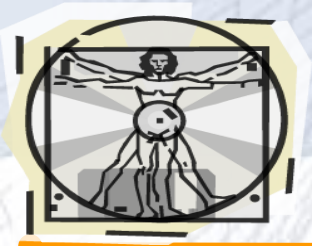
```
public static boolean isPalindrome(String s) {
  if (s.length() <= 1) // Base case
    return true;
  else if (s.charAt(0) != s.charAt(s.length() - 1)) // Base case
    return false;
  else
    return isPalindrome(s.substring(1, s.length() - 1));
}
```



Recursive Helper Methods

The preceding recursive isPalindrome method is not efficient, because it creates a new string for every recursive call. To avoid creating new strings, use a helper method:

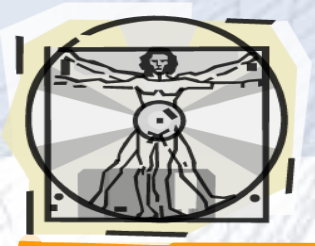
```
public static boolean isPalindrome(String s) {
  return isPalindrome(s, 0, s.length() - 1);
}
public static boolean isPalindrome(String s, int low, int high) {
  if (high <= low) // Base case
    return true;
  else if (s.charAt(low) != s.charAt(high)) // Base case
    return false;
  else
    return isPalindrome(s, low + 1, high - 1);
}
```



Recursive Selection Sort

1. Find the smallest number in the list and swaps it with the first number.
2. Ignore the first number and sort the remaining smaller list recursively.

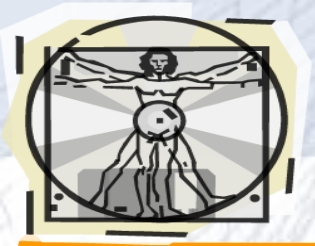
RecursiveSelectionSort



Recursive Binary Search

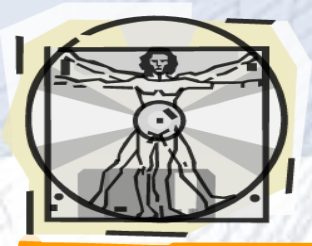
1. Case 1: If the key is less than the middle element, recursively search the key in the first half of the array.
2. Case 2: If the key is equal to the middle element, the search ends with a match.
3. Case 3: If the key is greater than the middle element, recursively search the key in the second half of the array.

RecursiveBinarySearch



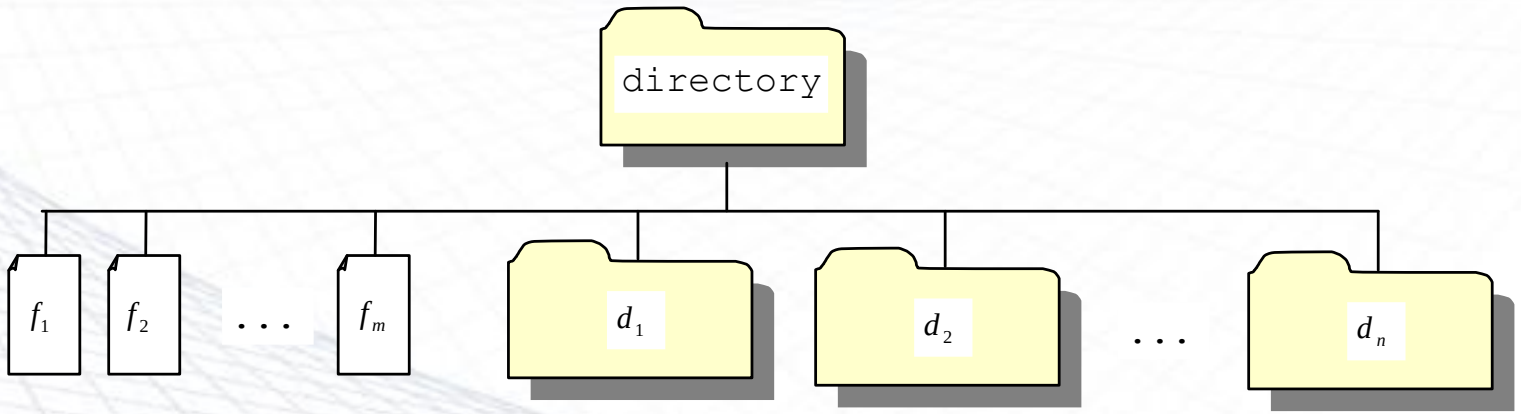
Recursive Implementation

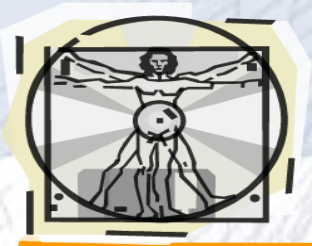
```
/** Use binary search to find the key in the list */
public static int recursiveBinarySearch(int[] list, int key) {
  int low = 0;
  int high = list.length - 1;
  return recursiveBinarySearch(list, key, low, high);
}

/** Use binary search to find the key in the list between
    list[low] list[high] */
public static int recursiveBinarySearch(int[] list, int key,
  int low, int high) {
  if (low > high)  // The list has been exhausted without a match
    return -low - 1;

  int mid = (low + high) / 2;
  if (key < list[mid])
    return recursiveBinarySearch(list, key, low, mid - 1);
  else if (key == list[mid])
    return mid;
  else
    return recursiveBinarySearch(list, key, mid + 1, high);
}
```



Directory Size

The preceding examples can easily be solved without using recursion. This section presents a problem that is difficult to solve without using recursion. The problem is to find the size of a directory. The size of a directory is the sum of the sizes of all files in the directory. A directory may contain subdirectories. Suppose a directory contains files , , ..., , and subdirectories , , ..., , as shown below.

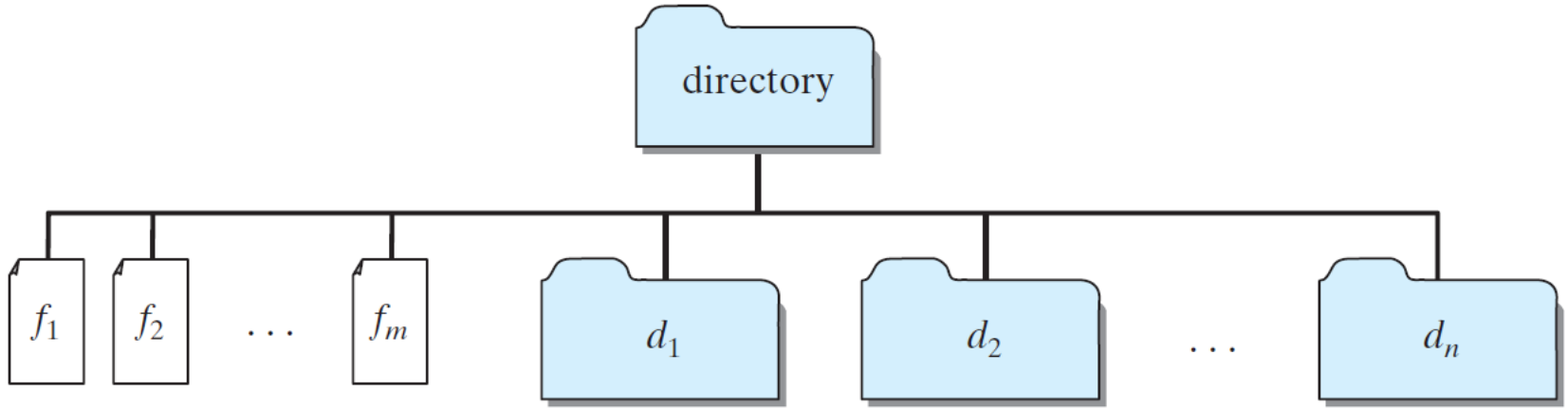




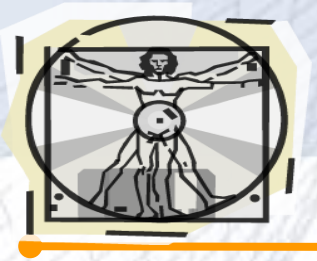
Directory Size

The size of the directory can be defined recursively as follows:

$$size(d) = size(f_1) + size(f_2) + \ldots + size(f_m) + size(d_1) + size(d_2) + \ldots + size(d_n)$$

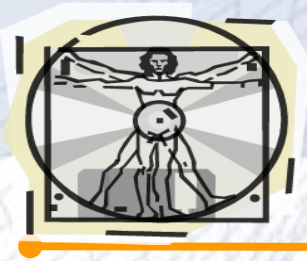


DirectorySize **Run**

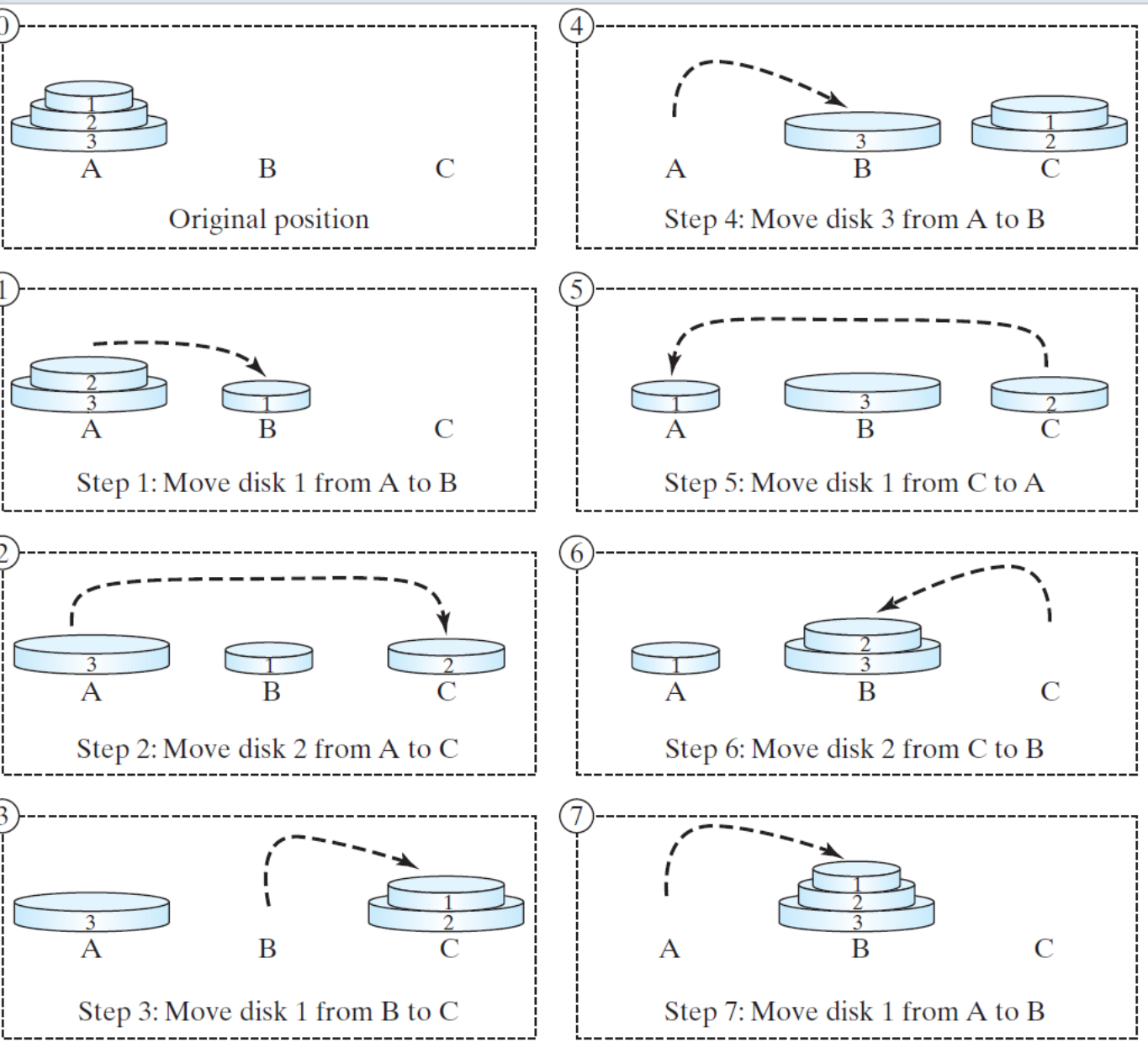


Tower of Hanoi

- There are *n* disks labeled 1, 2, 3, . . ., *n*, and three towers labeled A, B, and C.
- No disk can be on top of a smaller disk at any time.
- All the disks are initially placed on tower A.
- Only one disk can be moved at a time, and it must be the top disk on the tower.

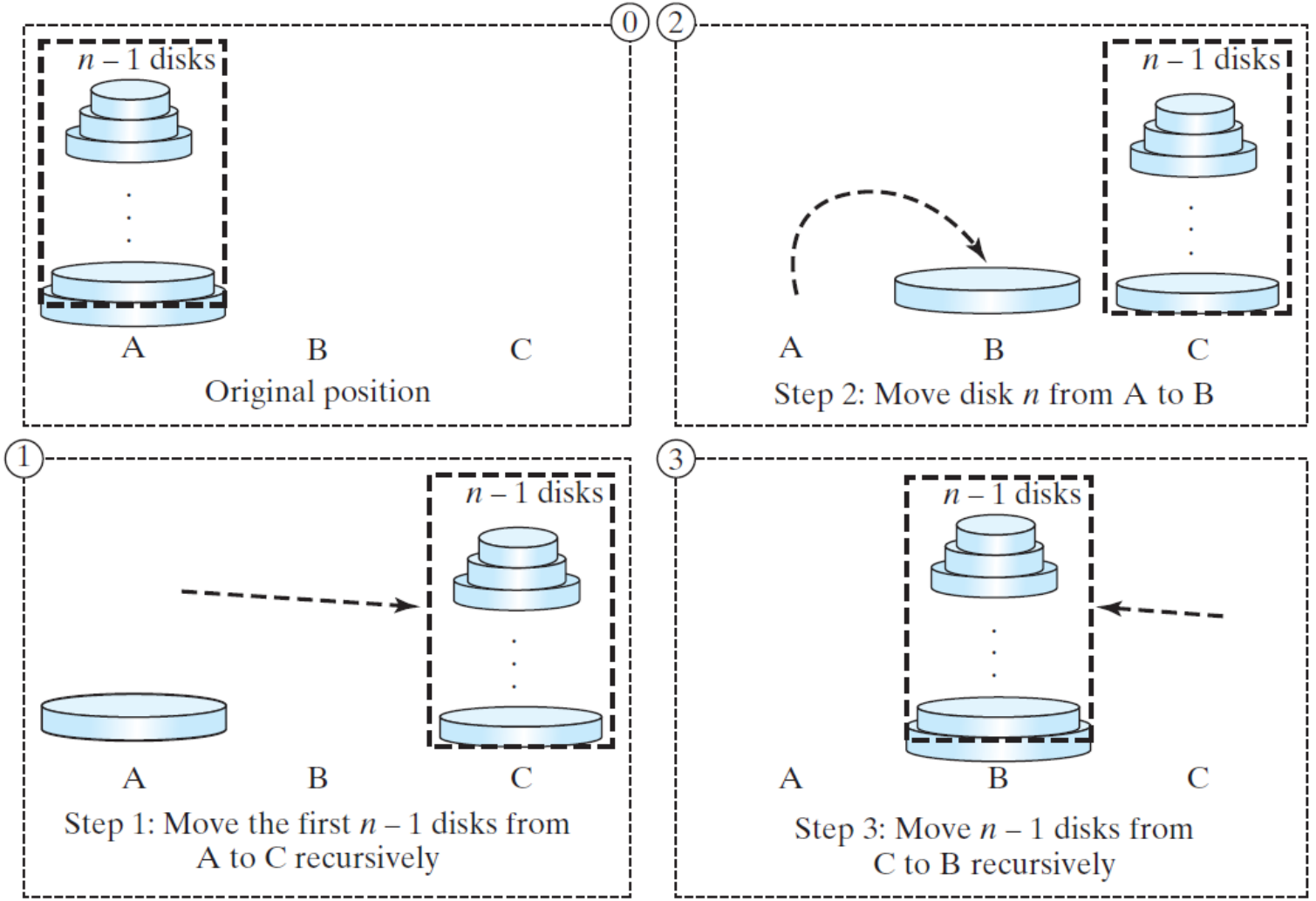


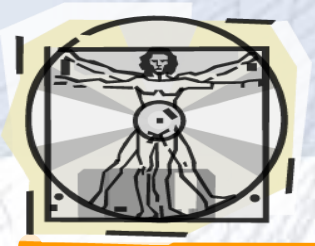
Tower of Hanoi, (



Solution to Tower of Hanoi

The Tower of Hanoi problem can be decomposed into three subproblems

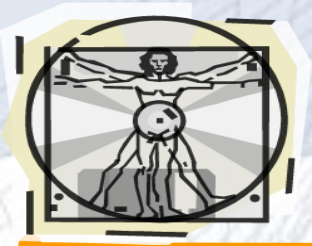




Solution to Tower of Hanoi

- Move the first n - 1 disks from A to C with the assistance of tower B.
- Move disk n from A to B.
- Move n - 1 disks from C to B with the assistance of tower A.

TowerOfHanoi Run



Exercise 18.3 GCD

gcd(2, 3) = 1

gcd(2, 10) = 2

gcd(25, 35) = 5

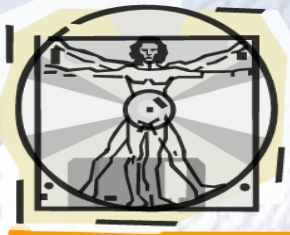
gcd(205, 301) = 5

gcd(m, n)

Approach 1: Brute-force, start from min(n, m) down to 1, to check if a number is common divisor for both m and n, if so, it is the greatest common divisor.

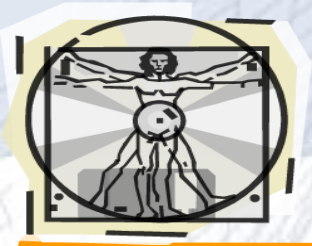
Approach 2: `Euclid's` algorithm

Approach 3: Recursive method



Approach 2: Euclid's algorithm

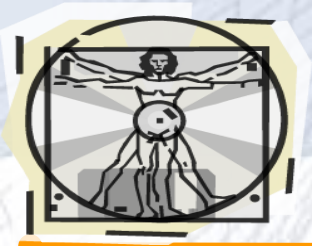
```
// Get absolute value of m and n;
t1 = Math.abs(m); t2 = Math.abs(n);
// r is the remainder of t1 divided by t2;
r = t1 % t2;
while (r != 0) {
  t1 = t2;
  t2 = r;
  r = t1 % t2;
}

// When r is 0, t2 is the greatest common
// divisor between t1 and t2
return t2;
```



Approach 3: Recursive Method

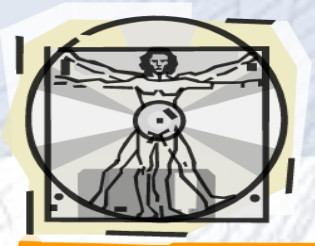
gcd(m, n) = n if m % n = 0;

gcd(m, n) = gcd(n, m % n); otherwise;



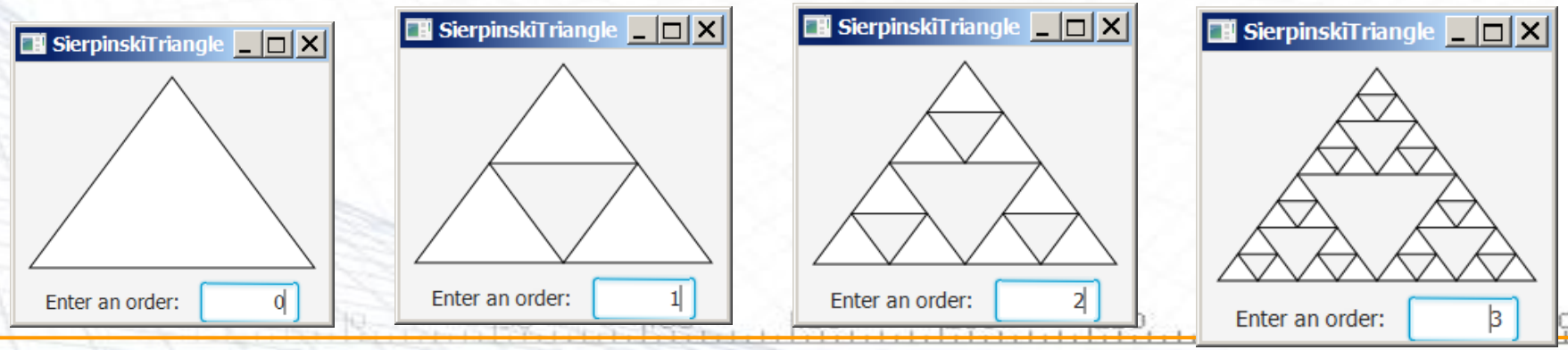
Fractals?

A fractal is a geometrical figure just like triangles, circles, and rectangles, but fractals can be divided into parts, each of which is a reduced-size copy of the whole. There are many interesting examples of fractals. This section introduces a simple fractal, called *Sierpinski triangle*, named after a famous Polish mathematician.

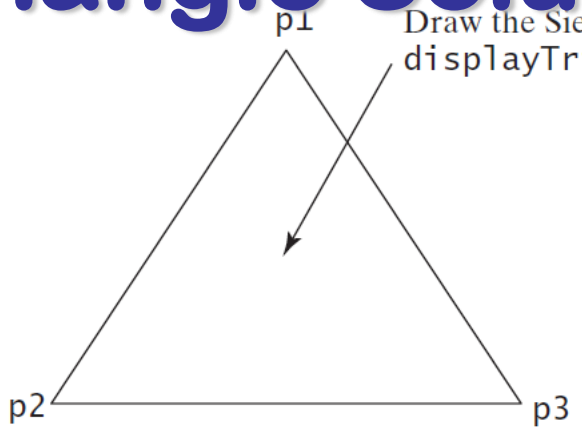


Sierpinski Triangle

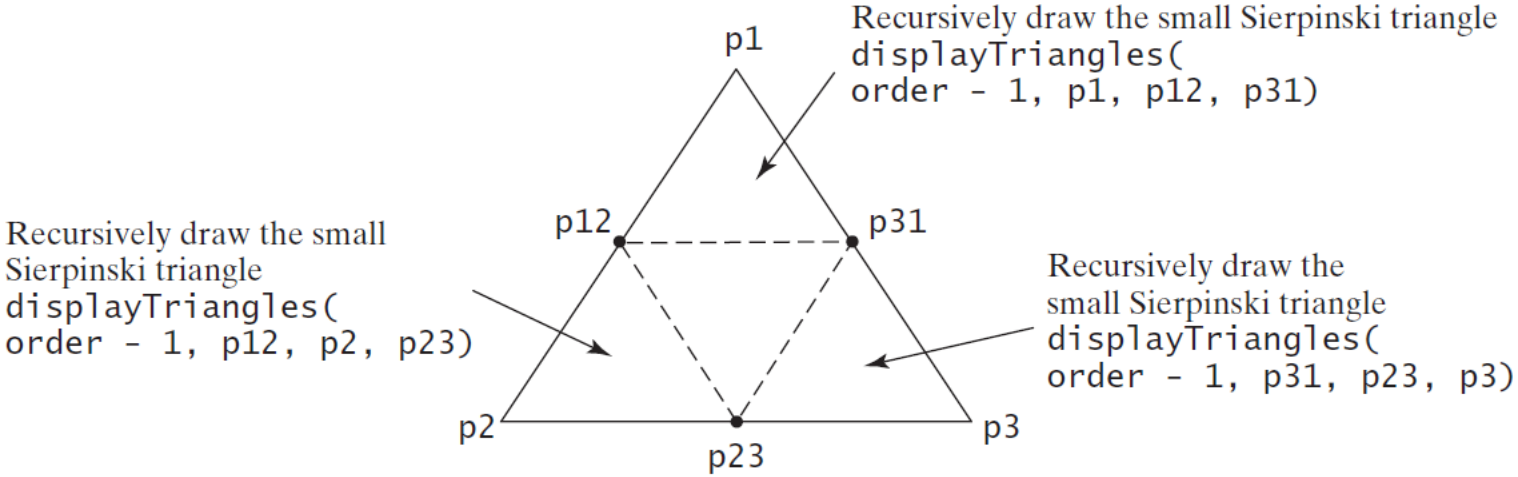
1. It begins with an equilateral triangle, which is considered to be the Sierpinski fractal of order (or level) 0, as shown in Figure (a).
2. Connect the midpoints of the sides of the triangle of order 0 to create a Sierpinski triangle of order 1, as shown in Figure (b).
3. Leave the center triangle intact. Connect the midpoints of the sides of the three other triangles to create a Sierpinski of order 2, as shown in Figure (c).
4. You can repeat the same process recursively to create a Sierpinski triangle of order 3, 4, ..., and so on, as shown in Figure (d).



Sierpinski Triangle Solution

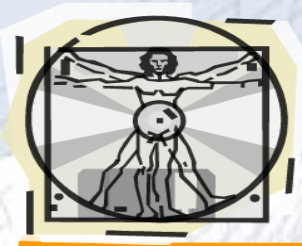


(a)



(b)

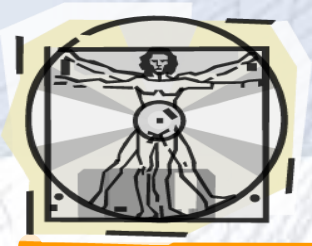
SierpinskiTriangle **Run**



Recursion vs. Iteration

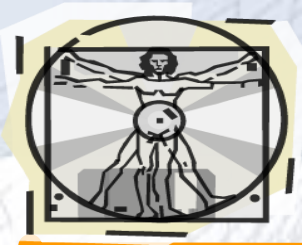
Recursion is an alternative form of program control. It is essentially repetition without a loop.

Recursion bears substantial overhead. Each time the program calls a method, the system must assign space for all of the method's local variables and parameters. This can consume considerable memory and requires extra time to manage the additional space.



Advantages of Using Recursion

Recursion is good for solving the problems that are inherently recursive.



Tail Recursion

A recursive method is said to be *tail recursive* if there are no pending operations to be performed on return from a recursive call.

Non-tail recursive — **ComputeFactorial**

Tail recursive — **ComputeFactorialTailRecursion**