

# LibBi

User Guide

User Reference

Developer Guide

Version 1.4.3

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# 1 User Guide

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## 1.1 Introduction

LibBi is used for Bayesian inference over state-space models, including simulation, filtering and smoothing for state estimation, and optimisation and sampling for parameter estimation.

LibBi supports state-space models of the form:

$$\underbrace{p(\mathbf{y}_{1:T}, \mathbf{x}_{0:T}, \boldsymbol{\theta})}_{\text{joint}} = \underbrace{\underbrace{p(\boldsymbol{\theta})}_{\text{parameter}} \underbrace{p(\mathbf{x}_0|\boldsymbol{\theta})}_{\text{initial}} \left( \prod_{t=1}^T \underbrace{p(\mathbf{x}_t|\mathbf{x}_{t-1}, \boldsymbol{\theta})}_{\text{transition}} \right)}_{\text{prior}} \underbrace{\left( \prod_{t=1}^T \underbrace{p(\mathbf{y}_t|\mathbf{x}_t, \boldsymbol{\theta})}_{\text{observation}} \right)}_{\text{likelihood}}. \quad (1.1)$$

where  $t = 1, \dots, T$  indexes time,  $\mathbf{y}_{1:T}$  are observations,  $\mathbf{x}_{1:T}$  are state variables, and  $\boldsymbol{\theta}$  are parameters.

The state-space model in (1.1) consists of four conditional probability densities:

- the *parameter* model, specifying the prior density over parameters,
- the *initial* value model, specifying the prior density over the initial value of state variables, conditioned on the parameters,
- the *transition* model, specifying the transition density, conditioned on the parameters and previous state,
- the *observation* model, specifying the observation density, conditioned on the parameters and current state.

→ See Models

Each of these is explicitly specified using the LibBi modelling language.

A brief example will help to set the scene. Consider the following Lotka-Volterra-like predator-prey model between zooplankton (predator,  $Z$ ) and phytoplankton (prey,  $P$ ):

$$\begin{aligned} \frac{dP}{dt} &= \alpha_t P - cPZ \\ \frac{dZ}{dt} &= ecPZ - m_l Z - m_q Z^2. \end{aligned}$$

Here,  $t$  is time (in days), with prescribed constants  $c = .25$ ,  $e = .3$ ,  $m_l = .1$  and  $m_q = .1$ . The stochastic growth term,  $\alpha_t$ , is updated in discrete time by drawing  $\alpha_t \sim \mathcal{N}(\mu, \sigma)$  daily. Parameters to be estimated are  $\mu$  and  $\sigma$ , and  $P$  is observed, with noise, at daily intervals.

The model above might be specified in the LibBi modelling language as follows:

```

/**
 * Lotka-Volterra-like phytoplankton-zooplankton (PZ) model.
 */
model PZ {
  const c = 0.25    // zooplankton clearance rate
  const e = 0.3     // zooplankton growth efficiency
  const m_l = 0.1   // zooplankton linear mortality
  const m_q = 0.1   // zooplankton quadratic mortality

  param mu, sigma  // mean and std. dev. of phytoplankton growth
  state P, Z       // phytoplankton, zooplankton
  noise alpha      // stochastic phytoplankton growth rate
  obs P_obs       // observations of phytoplankton

  sub parameter {
    mu ~ uniform(0.0, 1.0)
    sigma ~ uniform(0.0, 0.5)
  }

  sub initial {
    P ~ log_normal(log(2.0), 0.2)
    Z ~ log_normal(log(2.0), 0.1)
  }

  sub transition {
    alpha ~ normal(mu, sigma)
    ode {
      dP/dt = alpha*P - c*P*Z
      dZ/dt = e*c*P*Z - m_l*Z - m_q*Z*Z
    }
  }

  sub observation {
    P_obs ~ log_normal(log(P), 0.2)
  }
}

```

This would be saved in a file named PZ.bi. Various tasks can now be performed with the LibBi command-line interface, the simplest of which is just sampling from the prior distribution of the model:

```

libbi sample --target prior \
  --model-file PZ.bi \
  --nsamples 128 \
  --end-time 365 \
  --noutputs 365 \
  --output-file results/prior.nc

```

This command will sample 128 trajectories of the model (--nsamples 128), each of 365 days (--end-time 365), outputting the results every day (--noutputs 365) to the NetCDF file results/prior.nc.

Tip →

On the first occasion that a command is run, LibBi generates and compiles code for you behind the scenes. This takes some time, depending on the complexity of the model. The second time the command is run there is no such overhead, and execution time is noticeably shorter. Changes to some command-line options may also trigger a recompile.

To play with this example further, download the PZ package from [www.libbi.org](http://www.libbi.org). Inspect and run the `run.sh` script to get started.

The command-line interface provides numerous other functionality, including filtering and smoothing the model with respect to data, and optimising or sampling its parameters. The `help` command is particularly useful, and can be used to access the contents of the User Reference portion of this manual from the command line.

## 1.2 Getting started

There is a standard file and directory structure for a LibBi project. Using it for your own projects ensures that they will be easy to share and distribute as a LibBi package. To set up the standard structure, create an empty directory somewhere, and from within that directory run:

```
libbi package --create --name Name
```

replacing *Name* with the name of your project.

Tip →

By convention, names always begin with an uppercase letter, and all new words also begin with an uppercase letter, as in CamelCase. See the Style guide for more such conventions.

Each of the files that are created contains some placeholder content that is intended to be modified. The `META.yml` file can be completed immediately with the name of the package, the name of its author and a brief description. This and other files are detailed with the package command.

This early stage is also the ideal time to think about version control. LibBi developers use Git for version control, and you may like to do the same for your project. A new repository can be initialised in the same directory with:

```
git init
```

Then add all of the initial files to the repository and make the first commit:

```
git add *  
git commit -m 'Added initial files'
```

The state of the repository at each commit may be restored at any stage, allowing old versions of files to be maintained without polluting the working directory.

A complete introduction to Git is beyond the scope of this document. See [www.git-scm.com](http://www.git-scm.com) for more information. The documentation for the package command also gives some advice on what to include, and what not to include, in a version control repository.

The following command can be run at any time to validate that a project still conforms to the standard structure:

```
libbi package --validate
```

Finally, the following command can be used to build a package for distribution:



```
libbi package --build
```

This creates a `*.tar.gz` file in the current directory containing the project files.

## 1.3 Models

Models are specified in the LibBi modelling language. A model specification is put in a file with an extension of `*.bi`. Each such file contains only a single model specification.

A specification always starts with an outer `model` statement that declares and names the model. It then proceeds with declarations of constants, dimensions and variables, followed by four *top-level blocks* – `parameter`, `initial`, `transition` and `observation` – that describe the factors of the state-space model.

A suitable template is:

```
model Name {  
    // declare constants...  
    // declare dimensions...  
    // declare variables...  
  
    sub parameter {  
        // specify the parameter model...  
    }  
  
    sub initial {  
        // specify the initial condition model...  
    }  
  
    sub transition {  
        // specify the transition model...  
    }  
  
    sub observation {  
        // specify the observation model...  
    }  
}
```

Note that the contents of the `model` statement and each top-level block are contained in curly braces (`{...}`), in typical C-style. Comments are also C-style, an inline comment being wrapped by `/*` and `*/`, and the double-slash (`//`) denoting an end-of-line comment. Lines may optionally end with a semicolon.

### 1.3.1 Constants

→ See also `const`

Constants are named and immutable scalar values. They are declared using:

```
const name = constant_expression
```

Often *constant\_expression* is simply a literal value, but in general it may be any constant scalar expression (see Expressions).

### 1.3.2 Inlines

→ See also `inline`

Inlines are named scalar expressions. They are declared using:

```
inline name = expression
```

Any use of the inline *name* in subsequent expressions is precisely equivalent to wrapping *expression* in parentheses and replacing *name* with it. Inlines may be recursively nested.

### 1.3.3 Dimensions

→ See also `dim`

Dimensions are used to construct vector, matrix and higher-dimensional variables. Often, but not necessarily, they have a spatial interpretation; for example a large 3-dimensional spatial model may declare dimensions *x*, *y* and *z*. Dimensions are declared using:

```
dim name(size)
```

Tip →

Because LibBi is primarily meant for state-space models, the time dimension is special and not declared explicitly. Consider that, for continuous-time models, time is not readily represented by a dimension of a finite size.

### 1.3.4 Variables

→ See also `input`, `param`, `state`, `noise` and `obs`

Variables are named and mutable scalar, vector, matrix or higher-dimensional objects.

A simple scalar variable is declared using:

```
type name
```

where *type* is one of:

- `input` : for a variable with values that may or may not change over time, but that are prescribed according to input from a file,
- `param` : for a latent variable that does not change over time,
- `state` : for a latent variable that changes over time,
- `noise` : for a latent noise term that changes over time,
- `obs` : for an observed variable that changes over time.

For example, for the equation of the state-space model in the Introduction, the parameter  $\theta$  may be declared using:

```
param theta
```

the state variable *x* using:

```
state x
```

and the observed variable *y* using:

```
obs y
```

A vector, matrix or higher-dimensional variable is declared by listing the names of the Dimensions over which it extends, separated by commas, in square brackets after the variable name. For example:

```
dim m(50)
dim n(20)
...
state x[m,n]
```

declares a state variable  $x$ , which is a matrix of size  $50 \times 20$ .

The declaration of a variable can also include various arguments that control, for example, whether or not it is included in output files. These are specified in parentheses after the variable declaration, for example:

```
state x[m,n] (has_output = 0)
```

### 1.3.5 Actions

Within each top-level block, a probability density is specified using *actions*. Simple actions take one of three forms:

```
x ~ expression
x <- expression
dx/dt = expression
```

where  $x$  is some variable, referred to as the *target*. Named actions, available only for the first two forms, look like:

```
x ~ name(arguments, ...)
x <- name(arguments, ...)
```

The first form, with the  $\sim$  operator, indicates that the (random) variable  $x$  is distributed according to the action given on the right. Such actions are usually named, and correspond to parametric probability distributions (e.g. gaussian, gamma and uniform). If the action is not named, the expression on the right is assumed to be a probability density function.

The second form, with the  $<-$  operator, indicates that the (conditionally deterministic) variable  $x$  should be assigned the result of the action given on the right. Such actions might be simple scalar, vector or matrix expressions.

The third form, with the  $=$  operator, is for expressing ordinary differential equations.

Individual elements of a vector, matrix or higher-dimensional variable may be targeted using square brackets. Consider the following action giving the ordinary differential equation for a Lorenz '96 model:

```
dx[i]/dt = x[i - 1]*(x[i + 1] - x[i - 2]) - x[i] + F
```

If  $x$  is a vector of size  $N$ , the action should be interpreted as “for each  $i \in \{0, \dots, N-1\}$ , use the following ordinary differential equation”. The name  $i$  is an *index*, considered declared on the left with local scope to the action, so that it may be used on the right. The name  $i$  is arbitrary, but must not match the name of a constant, inline or variable. It may match the name of a dimension, and indeed matching the name of the dimension with which the index is associated can be sensible.

Elements of higher-dimensional variables may be targeted using multiple indices separated by commas. For example, consider computing a Euclidean distance matrix  $D$ :

```
D[i,j] <- sqrt(pow(x[i] - x[j], 2) - pow(y[i] - y[j], 2))
```

If  $D$  is a matrix of size  $M \times N$ , the action should be interpreted as “for each  $i \in \{0, \dots, M - 1\}$  and each  $j \in \{0, \dots, N - 1\} \dots$ ”.

Indices may be restricted with a *range*:

```
D[i=0:3,j=2:3] <- sqrt(pow(x[i] - x[j], 2) - pow(y[i] - y[j], 2))
```

Now action should be interpreted as “for each  $i \in \{0, \dots, 3\}$  and each  $j \in \{2, 3\} \dots$ ”.

Indexed actions such as this are useful for more complicated transformations that either cannot be expressed in matrix form, or that are contrived when expressed as such.

Arguments to actions may be given with either *positional* or *named* forms, or a mix of the two. Positional arguments are interpreted by the order given. For example

```
x ~ gaussian(0.0, 2.0)
```

means that  $x$  is distributed according to the *gaussian* action with, by the definition of that action, mean 0.0 and standard deviation 2.0. The *gaussian* action also happens to support named arguments, so the following is equivalent:

```
x ~ gaussian(mean = 0.0, std = 2.0)
```

The order of named arguments is unimportant, so the following is also equivalent:

```
x ~ gaussian(std = 2.0, mean = 0.0)
```

Positional and named arguments may be mixed, but all positional arguments must appear before all named arguments. Thus, this is valid:

```
x ~ gaussian(0.0, std = 2.0)
```

but these are not:

```
x ~ gaussian(mean = 0.0, 2.0)
x ~ gaussian(std = 2.0, 0.0)
```

This avoids ambiguity in the syntax.

The documentation for an action lists its arguments, and whether it may be used in named or positional form, or both.

### 1.3.6 Blocks

Some actions are more complicated and can, or must, be wrapped in *blocks*. Good examples are systems of ordinary differential equations. Two or more equations must be grouped to form the complete system. The grouping is achieved with the *ode* block.

Consider the following Lotka-Volterra-type transition model:

```

sub transition {
  sub ode {
    dP/dt = alpha*P - c*P*Z
    dZ/dt = e*c*P*Z - m_l*Z
  }
}

```

Here, the ode block combines the two actions into the one system of ordinary differential equations.

Like actions, blocks can take arguments. The ode block has parameters to select and configure the algorithm used to numerically integrate the differential equations forward through time. If the defaults are inadequate, these might be set as follows:

```

sub transition {
  sub ode(atoler = 1.0e-3, rtoler = 1.0e-3, alg = 'dopri5') {
    dP/dt = alpha*P - c*P*Z
    dZ/dt = e*c*P*Z - m_l*Z
  }
}

```

As for actions, both positional and named forms of arguments are supported.

### 1.3.7 Expressions

LibBi supports expressions over scalars, vectors, matrices and higher-dimensional variables:

- A *scalar* is a literal, constant or variable that is not declared over any dimensions.
- A *vector* is a variable declared over exactly one dimension.
- A *matrix* is a variable declared over exactly two dimensions.
- Variables declared over three or more dimensions are not given a special name.

Note that, by these definitions, a variable declared over a single dimension of size one is considered a vector, not a scalar. Any variable declared over more than one dimension, where all of those dimensions have size one, is likewise not considered a scalar. The reason for this is that the particular boundary conditions of those dimensions may convey different behaviour to that of a scalar.

Special classes of expression are:

- A *constant expression* is one that can be evaluated at compile time. It must be scalar, and may refer to literals, constants and inlines that expand to other constant expressions only.
- A *static expression* is one that does not depend on time. It may refer to literals, constants, variables of type `param`, and inlines that expand to other static expressions only.
- A *common expression* is one that does not depend on the state of a particular trajectory. It may refer to literals, constants, variables of type `param` or `input`, and inlines that expand to other common expressions only.

Note from these definitions that a constant expression is a static expression, and a static expression is a common expression.

Individual elements of multidimensional variables may be indexed with e.g. `x[i]` or `A[i, j]`. Ranges may also be referenced by giving the first and last indices of the range, separated by a colon, e.g. `x[0:3]` or `A[0:3, 2:3]`.

The following operators are supported in expressions:

Scalar/vector/matrix arithmetic operators:	<code>+ - * / % **</code>
Element-wise vector/matrix operators:	<code>.* ./ .% .**</code>
Comparison operators:	<code>== != &lt; &lt;= &gt; &gt;=</code>
Logical operators:	<code>&amp;&amp;   </code>
Ternary operators:	<code>?:</code>

The following functions are supported in expressions:

<code>abs acos acosh asin asinh atan atan2 atanh ceil cos cosh erf</code>
<code>erfc exp floor gamma lgamma log max min mod pow round sin</code>
<code>sinh sqrt tan tanh</code>

### 1.3.8 Built-in variables

The following built-in variables are supported:

<code>t_now</code>	the current time,
<code>t_last_input</code>	the last time at which an input variable was updated,
<code>t_next_obs</code>	the next time at which an obs variable will be updated.

## 1.4 Command-line interface

Methods are applied to models via the command-line interface of LibBi. This is invoked using:

```
libbi command options ...
```

where *command* is any one of the following:

- `filter`: for filtering problems using the model and observations,
- `optimise`: for parameter optimisation problems using the model and observations,
- `sample`: for parameter and state sampling problems using the model and observations,
- `package`: for creating projects and building packages for distribution,
- `help`: for accessing online help,
- `draw`: to visualise a model (useful for debugging and development),
- `rewrite`: to inspect the internal representation of a model (useful for debugging and development),

and available *options* depend on the command.

Options may be specified in a configuration file or on the command line itself. To use a configuration file, give the name of the file on the command line, preceded by `@`, e.g.

```
libbi command @command.conf
```

More than one config file may be specified, each preceded by @. An option given on the command line will override an option of the same name given in the configuration file.

A config file simply contains a list of command-line options just as they would be given on the command line itself. For readability, the command-line options may be spread over any number of lines, and end-of-line comments, preceded by #, may appear. The contents of one config file may be nested in another by using the @file.conf syntax within a file. This can be useful to avoid redundancy. For example, the `sample` command inherits all the options of `filter`. In this case it may be useful to write a `filter.conf` file that is nested within a `sample.conf` file, so that options need not be repeated.

## 1.5 Output files

LibBi uses NetCDF files for input and output. A NetCDF file consists of *dimensions*, along with *variables* extending across them. This is very similar to LibBi models. Where use of these overloaded names may cause confusion, the specific names *NetCDF variable/dimension* and *model variable/dimension* are used throughout this section and the next. A NetCDF file may also contain scalar *attributes* that provide meta information.

The *schema* of a NetCDF file is its structure with respect to the dimensions, variables and attributes that it contains. Among the dimensions will be those that correspond to model dimensions. Likewise for variables. Some additional dimensions and variables will provide critical method-specific information or diagnostics.

Tip →

To quickly inspect the structure and contents of a NetCDF file, use the `ncdump` utility, included as standard in NetCDF distributions:

```
ncdump file.nc | less
```

LibBi output files have several schemas. The schema of an output file depends on the command that produces it, and the options given to that command. The schemas are related by strong conventions and inherited structure.

This section outlines each schema in turn. The syntax `x[m,n]` is used to refer to a NetCDF variable named `x` that extends across the NetCDF dimensions named `m` and `n`.

Tip →

If using OctBi or RBi for collating and visualising the output of LibBi, it may be unnecessary to understand these schema, as they are encapsulated by the higher-level functions of these packages.

### 1.5.1 Simulation schema

This schema is used by the `sample` command when the `target` option is set to `prior`, `joint` or `prediction`. It consists of dimensions:

- `nr` indexing time,
- `np` indexing trajectories, and

- for each dimension  $n$  in the model, a dimension  $n$ .

And variables:

- `time[nr]` giving the output times,
- for each param variable  $\theta$  in the model, defined over dimensions  $m, \dots, n$ , a variable  $\theta[m, \dots, n]$ , and
- for each state and noise variable  $x$  in the model, defined over dimensions  $m, \dots, n$ , a variable  $x[nr, m, \dots, n, np]$ .

### 1.5.2 Particle filter schema

This schema is used by the `filter` command when a particle filter, besides the adaptive particle filter, is chosen. It extends the simulation schema, with the following changes:

- the `np` dimension is now interpreted as indexing *particles* not *trajectories*.

The following variables are added:

- `logweight[nr, np]` giving the log-weights vector at each time, and
- `ancestor[nr, np]` giving the ancestry vector at each time.

Tip →

To draw a whole trajectory out of a file of the particle filter schema, begin at the last time, select a particle, and use the `ancestor` vector at each time to trace that particle's ancestry back through time. One cannot simply take a row from the matrix of a state variable to obtain a complete trajectory, as with the simulation schema.

### 1.5.3 Simulation “flexi” schema

This schema is currently not used directly, but the particle filter “flexi” schema below extends it. The complication here is that the number of particles at each time can vary. The schema consists of the following dimensions:

- `nr` indexing time,
- `nrp` indexing both time and trajectories (an unlimited dimension), and
- for each dimension  $n$  in the model, a dimension  $n$ .

And variables:

- `time[nr]` giving the output times,
- `start[nr]` giving, for each time, the starting index along the `nrp` dimension for particles associated with that time,
- `len[nr]` giving, for each time, the number of particles at that time,
- for each param variable  $\theta$  in the model, defined over dimensions  $m, \dots, n$ , a variable  $\theta[m, \dots, n]$ , and
- for each state and noise variable  $x$  in the model, defined over dimensions  $m, \dots, n$ , a variable  $x[m, \dots, n, nrp]$ .

Tip →



With the “flexi” schema, to read the contents of a variable at some time index  $t$ , read `len[t]` entries along the `nrp` dimension, beginning at index `start[t]`.

#### 1.5.4 Particle filter “flexi” schema

This schema is used by the `filter` command when an adaptive particle filter is chosen. It extends the simulation “flexi” schema. The following variables are added:

- `logweight[nrp]` giving the log-weights vector at each time, and
- `ancestor[nrp]` giving the ancestry vector at each time.

#### 1.5.5 Kalman filter schema

This schema is used by the `filter` command when a Kalman filter is chosen. It extends the simulation schema, with the following changes:

- the `np` dimension is always of size 1, and
- variables that correspond to state and noise variables in the model are now interpreted as containing the mean of the filter density at each time.

The following dimensions are added:

- `nxcol` indexing columns of matrices, and
- `nxrow` indexing rows of matrices.

The following variables are added:

- `U_[nr,nxcol,nxrow]` containing the upper-triangular Cholesky factor of the covariance matrix of the filter density at each time.
- For each noise and state variable  $x$ , a variable `index.x` giving the index of the first row (and column) in `S_` pertaining to this variable, the first such index being zero.

#### 1.5.6 Optimisation schema

This schema is used by the `optimise` command. It consists of dimensions:

- `np` indexing iterations of the optimiser (an unlimited dimension).

And variables:

- for each param variable  $\theta$  in the model, defined over dimensions  $m, \dots, n$ , a variable `theta[m, ..., n, np]`,
- `optimiser.value[np]` giving the value of the function being optimised at each iteration, and
- `optimiser.size[np]` giving the value of the convergence criterion at each iteration.

#### 1.5.7 PMCMC schema

This schema is used by the `sample` command when a PMCMC method is chosen. It extends the simulation schema, with the following changes:

- the `np` dimension indexes samples, not trajectories, and

- for each param variable *theta* in the model, defined over dimensions  $m, \dots, n$ , there is a variable *theta* [ $m, \dots, n, np$ ] instead of *theta* [ $m, \dots, n$ ] (i.e. param variables are defined over the np dimension also).

The following variables are added:

- `loglikelihood[np]` giving the log-likelihood estimate  $\hat{p}(\mathbf{y}_{1:T}|\theta^p)$  for each sample  $\theta^p$ , and
- `logprior[np]` giving the log-prior density  $p(\theta^p)$  of each sample  $\theta^p$ .

### 1.5.8 SMC<sup>2</sup> schema

This schema is used by the `sample` command when an SMC<sup>2</sup> method is chosen. It extends the PMCMC schema, with the following additional variables:

- `logweight[np]` giving the log-weights vector of parameter samples.
- `logevidence[nr]` giving the incremental log-evidence of the model at each time.

## 1.6 Input files

Input files take three forms:

- initialisation files, containing the initial values of state variables,
- input files, containing the values of input variables, possibly changing across time, and
- observation files, containing the observed values of obs variables.

All of these use the same schema. That schema is quite flexible, allowing for the representation of both dense and sparse input. Sparsity may be in time or space. Sparsity in time is, for example, having a discrete-time model with a scalar obs variable that is not necessarily observed at all times. Sparsity in space is, for example, having a discrete-time model with a vector obs variable, for which not all of the elements are necessarily observed at all times.

Each model variable is associated with a NetCDF variable of the same name. Additional NetCDF variables may exist. Model variables which cannot be matched to a NetCDF variable of the same name do not receive input from the file. For input variables, this means that they will remain uninitialised. For spatially dense input, each such NetCDF variable may be defined along the following dimensions, in the order given:

1. Optionally, a dimension named `ns`, used to index multiple experiments set up in the same file. If not given for a variable, that variable is assumed to be the same for all experiments.
2. Optionally, a time dimension (see below).
3. Optionally, a number of dimensions with names that match the model dimensions along which the model variable is defined, ordered accordingly,

4. Optionally, a dimension named `np`, used to index multiple trajectories (instantiations, samples, particles) of a variable. If not given for a variable, the value of that variable is assumed to be the same for all trajectories. Variables of type `param`, `input` and `obs` may not use an `np` dimension, as by their nature they are meant to be in common across all trajectories.

For spatially sparse input, see Coordinate variables below.

Tip →

The Simulation schema is in fact a special case of the input schema, so that the `sample` command can often be used quite sensibly as input to another run. For example, a simulated data set can be generated with:

```
libbi sample \  
  --target joint \  
  --nsamples 1 \  
  --output-file data/obs.nc \  
  ...
```

The `data/obs.nc` file can then be used as an observation file, sampling from the posterior distribution conditioned on the simulated trajectory as data:

```
libbi sample \  
  --target posterior \  
  --obs-file data/obs.nc \  
  --output-file results/posterior.nc \  
  ...
```

Then, indeed, the output of that might be fed into a new sample forward in time:

```
libbi sample \  
  --target prediction \  
  --init-file results/posterior.nc \  
  --output-file results/prediction.nc \  
  ....
```

### 1.6.1 Time variables

*Time variables* are used to index time in a file. Each NetCDF variable with a name beginning with “time” is assumed to be a time variable. Each such variable may be defined along the following dimensions, in the order given:

1. Optionally, the `ns` dimension.
2. An arbitrarily named dimension.

The latter dimension becomes a *time dimension*. The time variable gives the time associated with each index of that time dimension, a sequence which must be monotonically non-decreasing. NetCDF variables that correspond to model variables, and that are defined along the same time dimension, become associated with the time variable. The time dimension thus enumerates both the times at which these variables change, and the values that they are to assume at those times. A variable may only be associated with one time dimension, and `param` variables may not be associated with one at all. If a variable is not defined across a time dimension, it is assumed to have the same value at all times.

Time variables and time dimensions are interpreted slightly differently for each of the input file types:

1. For an initialisation file, the starting time (given by the `--start-time` command-line option, see `filter`) is looked-up in each time variable, and the corresponding record in each associated variable is used for its initialisation.
2. For input files, a time variable gives the times at which each associated variable changes in value. Each variable maintains its new value until the time of the next change.
3. For observation files, a time variable gives the times at which each associated variable is observed. The value of each variable is interpreted as being its value at that precise instant in time.

Example →

#### Representing a scalar input

Assume that we have an obs variable named `y`, and we wish to construct an observation file containing our data set, which consists of observations of `y` at various times. A valid NetCDF schema would be:

- a dimension named `nr`, to be our time dimension,
- a variable named `time_y`, defined along the dimension `nr`, to be our time variable, and
- a variable named `y`, defined along the dimension `nr`, to contain the observations.

We would then fill the variable `time_y` with the observation times of our data set, and `y` with the actual observations. It may look something like this:

<code>time_y[nr]</code>	<code>y[nr]</code>
0.0	6.75
1.0	4.56
2.0	9.45
5.5	4.23
6.0	7.12
9.5	5.23

Example →

### Representing a vector input, densely

Assume that we have an obs variable named `y`, which is a vector defined across a dimension of size three called `n`. We wish to construct an observation file containing a our data set, which consists of observations of `y` at various times, where at each time all three elements of `y` are observed. A valid NetCDF schema would be:

- a dimension named `nr`, to be our time dimension,
- a variable named `time_y`, defined along the dimension `nr`, to be our time variable,
- a dimension named `n`, and
- a variable named `y`, defined along the dimensions `nr` and `n`, to contain the observations.

We would then fill the variable `time_y` with the observation times of our data set, and `y` with the actual observations. It may look something like this:

<code>time_y[nr]</code>	<code>y[nr,n]</code>		
0.0	6.75	3.34	3.45
1.0	4.56	4.54	1.34
2.0	9.45	3.43	1.65
5.5	4.23	8.65	4.64
6.0	7.12	4.56	3.53
9.5	5.23	3.45	3.24

## 1.6.2 Coordinate variables

*Coordinate variables* are used for spatially sparse input. Each variable with a name beginning with “coord” is assumed to be a coordinate variable. Each such variable may be defined along the following dimensions, in the order given:

1. Optionally, the `ns` dimension.
2. A coordinate dimension (see below).
3. Optionally, some arbitrary dimension.

The second dimension, the *coordinate dimension*, may be a time dimension as well. NetCDF variables that correspond to model variables, and that are defined along the same coordinate dimension, become associated with the coordinate variable. The coordinate variable is used to indicate which elements of these variables are active. The last dimension, if any, should have a length equal to the number of dimensions across which these variables are defined. So, for example, if these variables are matrices, the last dimension should have a length of two. If the variables are vectors, so that they have only one dimension, the coordinate variable need not have this last dimension.

If a variable specified across one or more dimensions in the model cannot be associated with a coordinate variable, then it is assumed to be represented densely.

Example →

### Representing a vector input, sparsely

Assume that we have an obs variable named `y`, which is a vector defined across a dimension of size three called `n`. We wish to construct an observation file containing our data set, which consists of observations of `y` at various times, where at each time only a subset of the elements of `y` are observed. A valid NetCDF schema would be:

- a dimension named `nr`, to be both our time and coordinate dimension,
- a variable named `time_y`, defined along the dimension `nr`, to be our time variable,
- a variable named `coord_y`, defined along the dimension `nr`, to be our coordinate variable,
- a variable named `y`, defined along the dimension `nr`, to contain the observations.

We would then fill the variable `time_y` with the observation times of our data set, `coord_y` with the coordinate of each observation, and `y` with the observations themselves. It may look something like this:

<code>time_y[nr]</code>	<code>coord_y[nr]</code>	<code>y[nr]</code>
0.0	0	6.75
0.0	1	3.34
1.0	0	4.56
1.0	1	4.54
1.0	2	1.34
2.0	1	3.43
5.5	3	4.64
6.0	0	4.23
6.0	2	3.53
9.5	1	3.45

Note that each unique value in `time_y` is repeated for as many coordinates as are active at that time. Also note that, if `y` had  $m > 1$  dimensions, the `coord_y` variable would be defined along some additional, arbitrarily named dimension of size  $m$  in the NetCDF file, so that the values of `coord_y` in the above table would be vectors.

### 1.6.3 Sampling models with input

The precise way in which input files and the model specification interact is best demonstrated in the steps taken to sample a model's prior distribution. Computing densities is similar. The *initialisation file* referred to in the proceeding steps is that given by the `--init-file` command-line option, and the *input file* that given by `--input-file`.

1. Any input variables in the input file that are not associated with a time variable are initialised by reading from the file.
2. The parameter top-level block is sampled.
3. Any param variables in the initialisation file are overwritten by reading from the file.
4. The initial top-level block is sampled.
5. Any state variables in the initialisation file are overwritten by reading from the file.
6. The transition top-level block is sampled forward through time. Sampling stops at each time that an input variable is to change, according to the input file, at which point the input variable is updated and sampling of the transition block continues.

Note two important points in this procedure:

- An input variable in the input file that is not associated with a time variable is initialised before anything else, whereas an input variable that is associated with a time variable is not initialised until simulation begins, even if the first entry of that variable indicates an update at time zero. This has implications as to which input variables are, or are not, initialised at the time that the parameter block is sampled.
- While the parameter and initial blocks are always sampled, the samples may be later overwritten from the initialisation file. Thus, the initialisation file need not contain a complete set of variables, although behaviour is more intuitive if it does. This behaviour also ensures pseudorandom reproducibility regardless of the presence, or content, of the initialisation file.

## 1.7 Getting it all working

This section contains some general advice on the statistical methods employed by LibBi and the tuning that might be required to make the most of them. It concentrates on the *particle marginal Metropolis-Hastings* (PMMH) sampler, used by default by the `sample` command when sampling from the posterior distribution.

PMMH is of the family of *particle Markov chain Monte Carlo* (PMCMC) methods (Andrieu et al., 2010), which in turn belong to the family of Markov chain Monte Carlo (MCMC). A complete introduction to PMMH is beyond the scope of this manual. Murray (2013) provides an introduction of the method and its implementation in LibBi.

When using MCMC methods it is common to perform some short pilot runs to tune the parameters of the method in order to improve its efficiency, before performing a final run. In PMMH, the parameters to be tuned are the proposal distribution, and the number of particles in the particle filter, or sequential Monte Carlo (Doucet et al., 2001), component of the method.

When running PMMH in LibBi, diagnostics are output that can be used to guide tuning. Here is an example:

22:	-116.129	-16.984	7.25272	beats	-121.853	-17.6397	7.49326	accept=0.217391
23:	-116.129	-16.984	5.63203	beats	-119.772	-18.0891	6.07209	accept=0.208333
24:	-116.129	-16.984	6.89478	beats	-121.268	-19.3354	8.25723	accept=0.2
25:	-116.129	-16.984	0.643236	beats	-136.661	-21.5339	6.44331	accept=0.192308
26:	-116.129	-16.984	3.58096	beats	-128.692	-20.3304	6.3502	accept=0.185185

The numerical columns provide, in order:

1. the iteration number,
2. the log-likelihood of the current state of the chain,
3. the prior log-density of the current state of the chain,
4. the proposal log-density of the current state of the chain, conditioned on the other state,
5. the log-likelihood of the other state of the chain (the previous state if the most recent proposal was accepted, the last proposed state if the most recent proposal was rejected),
6. the prior log-density of the other state of the chain,

7. the proposal log-density of the other state of the chain, conditioned on the current state, and
8. the acceptance rate of the chain so far.

The last of these is the most important for tuning.

For a standard Metropolis-Hastings, a reasonable guide is to aim at an acceptance rate of 0.5 for a single parameter, down to 0.23 for five or more parameters (Gelman et al., 1994). This includes the case where a Kalman filter is being used rather than a particle filter (by using the `--filter kalman` option to `sample`). In such cases the only tuning to perform is that of the proposal distribution. The proposal distribution is given in the `proposal_parameter` block of the model specification. If this block is not specified, the `parameter` block is used instead, and this may make for a poor proposal distribution, especially when there are many observations. Increasing the width of the proposal distribution will decrease the acceptance rate. Decreasing the width of the proposal distribution will increase the acceptance rate.

Tip →

Higher acceptance rates are not necessarily better. They may simply be a result of the chain exploring the posterior distribution very slowly.

PMMH has the added complication of using a particle filter to estimate the likelihood, rather than a Kalman filter to compute it exactly (although note that the Kalman filter works only for linear and Gaussian models). It is necessary to set the number of particles in the particle filter. More particles decreases the variance in the likelihood estimator and so increases the acceptance rate, but also increases the computational cost. Because the likelihood is estimated and not computed exactly, the optimal acceptance rate will be lower than for standard Metropolis-Hastings. Anecdotally, 0.1–0.15 seems reasonable.

The tradeoff between proposal size and number of particles is still under study, e.g. Doucet et al. (2013). The following procedure is suggested.

1. Start with an empty `proposal_parameter` block. Set the simulation time (`--end-time`) to the time of the first observation, and the number of particles (`--nparticles`) to a modest amount. When running PMMH, it is then the case that the same state of the chain, its starting state in fact, will be proposed repeatedly, and the acceptance rate will depend entirely on the variance of the likelihood estimator. One hopes to see a high acceptance rate here, say 0.5 or more. Increase the number of particles until this is achieved. Note that the random number seed can be fixed (`--seed`) if you wish.
2. Steadily extend the simulation time to include a few more observations on each attempt, and increase the number of particles as needed to maintain the high acceptance rate. The number of particles will typically need to scale linearly with the simulation time. Consult the Performance guide to improve execution times and further increase the number of particles if necessary.
3. If a suitable configuration is achieved for the full data set, or a workable subset, the proposal distribution can be considered. You may find it useful to add one parameter at a time to the proposal distribution, working towards an overall acceptance rate of 0.1–0.15.



4. If this fails to find a working combination with a healthy acceptance rate, consider the initialisation of the chain. By default, LibBi simply draws a sample from the parameter model to initialise the chain. If this is in an area where the variance in the likelihood estimator is high, the chain may mix untenably slowly for any sensible number of particles. It has been observed empirically that the variance in the likelihood estimator is heteroskedastic, and tends to increase with distance from the maximum likelihood (Murray et al., 2013). So initialising the chain closer to the maximum likelihood may allow it to mix well with a reasonable number of particles. Prior knowledge, optimisation of parameters (perhaps with the `optimise` command), or exploration of the data set may inform the initialisation. The initialisation can be given in the initialisation file (`--init-file`).

Tip →

It can also be the case that, in steadily extending the simulation time (`--end-time`), the acceptance rate suddenly drops at a particular time. This indicates that the particle filter degenerates at this point. Improving the initialisation of the chain is the best strategy in this case, although increasing the number of particles may help in mild cases.

Be aware that LibBi uses methods that are still being actively developed, and applied to larger and more complex models. It may be the case that your model or data set exceeds the current capabilities of the software. In such cases the only option is to consider a smaller or simpler model, or a subsample of the available data.

## 1.8 Performance guide

One of the aims of LibBi is to alleviate the user from performance considerations as much as possible. Nevertheless, there is some scope to influence performance, particularly by ensuring that appropriate hardware resources are used, and by limiting I/O where possible.

### 1.8.1 Precomputing

LibBi will:

- precompute constant subexpressions, and
- precompute static subexpressions in the transition and observation models.

Reducing redundant or repetitious expressions is thus unnecessary where these are constant or static. For example, taking the square-root of a variance parameter need not be of concern:

```
param sigma2
...
sub transition {
  epsilon ~ gaussian(mu, sqrt(sigma2))
  ...
}
```

Here, `sqrt(sigma2)` is a static expression that will be extracted and precomputed outside of the transition block.

Tip →

Use the `rewrite` command to inspect precisely which expressions have been extracted for precomputation.

## 1.8.2 I/O

The following I/O options are worth considering to reduce the size of output files and so the time spent writing to them:

- When declaring a variable, use a `has_output = 0` argument to omit it from output files if it will not be of interest.
- Enable output files by using the `--enable-single` command-line option. This will reduce write size by up to a half. Note, however, that all computations are then performed in single precision too, which may have significant numerical implications.

## 1.8.3 Configuration

The following configuration options are worth considering:

- Internally, LibBi uses extensive assertion checking to catch programming and code generation errors. These assertion checks are enabled by default, improving robustness at the expense of performance. It is recommended that they remain enabled during model development and small-scale testing, but that they are disabled for final production runs. They can be disabled by adding the `--disable-assert` command-line option.
- Experiment with the `--enable-cuda` command-line option to make use of a CUDA-enabled GPU. This should usually improve performance, as long as a sufficient number of model trajectories are to be simulated (typically upwards of 1024). If CUDA is being used, also try the `--enable-gpu-cache` command-line option if running PMCMC or SMC<sup>2</sup>. This caches particle histories in GPU rather than main memory. As GPU memory is usually much more limited than main memory, and this may result in its exhaustion, the option is disabled by default.
- Experiment with the `--enable-sse` and `--enable-avx` command-line options to make use of CPU SSE and AVX SIMD instructions. In single precision, these can provide up to a four-fold (SSE) or eight-fold (AVX) speed-up, and in double precision a two-fold (SSE) or four-fold (AVX) speed-up. These are only supported on x86 CPU architectures, however, and AVX in particular only on the most recent of these.
- Experiment with the `--nthreads` command-line option to set the number of CPU threads. Typically there are depreciating gains as the number of threads is increased, and beyond the number of physical CPU cores performance will degrade significantly. For CPUs with hyperthreading enabled, it is recommended that the number of threads is set to no more than the number of physical CPU cores. This may be half the default number of threads.
- Experiment with using single precision floating point operations by using the `--enable-single` command-line option. This can offer significant performance improvements (especially when used in conjunction with the `--enable-cuda` and `--enable-sse` options), but care should be taken to ensure that numerical error remains

tolerable. The use of single precision will also reduce memory consumption by up to a half.

- Use optimised versions of libraries, especially the BLAS and LAPACK libraries.
- Use the Intel C++ compiler if available. Anecdotally, this tends to produce code that runs faster than gcc. The configure script should automatically detect the Intel C++ compiler, and use it if available. To use the Intel Math Kernel Library as well, which is not automatically detected, use the `--enable-mkl` command-line option.

## 1.9 Style guide

The following conventions are used for LibBi model files:

- Model names are CamelCase, the first letter always capitalised.
- Action and block names are all lowercase, with multiple words separated by underscores.
- Dimension and variable names should be consistent, where possible, with their counterparts in a description of the model as it might appear in a scientific paper. For example, single upper-case letters for the names of matrix variables are appropriate, and standard symbols (rather than descriptive names) are encouraged. Greek letters should be written out in full, the first letter capitalised for the uppercase version (e.g. gamma and Gamma).
- Comments should be used liberally, with descriptions provided for all dimensions and variables in particular. Consider including units as part of the description, where relevant.
- Names ending in an underscore are intended for internal use only. They are not expected to be seen in a model file.
- Indent using two spaces, and do not use tabs.

Finally, use the package command to set up the standard files and directory structure for a LibBi project. This will make your model and its associated files easy to distribute, and your results easy to reproduce.

---

## 2 User Reference

---

### 2.1 Models

#### 2.1.1 `model`

Declare a model.

Synopsis

```
model Name {  
  ...  
}
```

Description

A `model` statement declares and names a model, and encloses declarations of the constants, dimensions, variables, inlines and top-level blocks that specify that model.

The following named top-level blocks are supported, and should usually be provided:

- `parameter`, specifying the prior density over parameters,
- `initial`, specifying the prior density over initial conditions,
- `transition`, specifying the transition density, and
- `observation`, specifying the observation density.

The following named top-level blocks are supported, and may optionally be provided:

- `proposal_parameter`, specifying a proposal density over parameters,
- `proposal_initial`, specifying a proposal density over initial conditions,
- `lookahead_transition`, specifying a lookahead density to accompany the transition density, and
- `lookahead_observation`, specifying a lookahead density to accompany the observation density.

#### 2.1.2 `dim`

Declare a dimension.

Synopsis

```
dim name(100, 'cyclic')  
dim name(size = 100, boundary = 'cyclic')
```

## Description

A `dim` statement declares a dimension with a given size and boundary condition.

A dimension may be declared anywhere in a model specification. Its scope is restricted to the block in which it is declared. A dimension must be declared before any variables that extend along it are declared.

## Arguments

`size` : (position 0, mandatory)

Length of the dimension.

`boundary` : (position 1, default 'none')

Boundary condition of the dimension. Valid values are:

'none' : No boundary condition.

'cyclic' : Cyclic boundary condition; all indexing is taken modulo the size of the dimension.

'extended' : Extended boundary condition; indexes outside the range of the dimension access the edge elements, as if these are extended indefinitely.

### 2.1.3 `input`, `noise`, `obs`, `param` and `state`

Declare an input, noise, observed, parameter or state variable.

## Synopsis

```
state x                // scalar variable
state x[i]             // vector variable
state X[i,j]           // matrix variable
state X[i,j,k]         // higher-dimensional variable
state X[i,j](has_output = 0) // omit from output files
state x, y, z          // multiple variables
```

## Description

Declares a variable of the given type, extending along the dimensions listed between the square brackets.

A variable may be declared anywhere in a model specification. Its scope is restricted to the block in which it is declared. Dimensions along which a variable extends must be declared prior to the declaration of the variable, using the `dim` statement.

## Arguments

`has_input` : (default 1)

Include variable when doing input from a file?

`has_output` : (default 1)

Include variable when doing output to a file?

`input_name` : (default the same as the name of the variable)

Name to use for the variable in input files.

`output_name` : (default the same as the name of the variable)  
Name to use for the variable in output files.

### 2.1.4 `const`

Declare a constant.

#### Synopsis

```
const name = constant_expression
```

#### Description

A `const` statement declares a constant, the value of which is evaluated using the given constant expression. The constant may then be used, by name, in other expressions.

A constant may be declared anywhere in a model specification. Its scope is restricted to the block in which it is declared.

### 2.1.5 `inline`

Declare an inline.

#### Synopsis

```
inline name = expression
...
x <- 2*name // equivalent to x <- 2*(expression)
```

#### Description

An `inline` statement declares an inline, the value of which is an expression that will be substituted in place of any occurrence of the inline's name in other expressions. The inline may be used in any expression where it will not violate the constraints on that expression (e.g. an inline expression that refers to a state variable may not be used within a constant expression).

An inline expression may be declared anywhere in a model specification. Its scope is restricted to the block in which it is declared.

## 2.2 Actions

### 2.2.1 `betabin`

Beta-binomial distribution.

#### Synopsis

```
x ~ betabin()
x ~ betabin(10, 1.0, 2.0)
x ~ betabin(n=10, alpha = 1.0, beta = 2.0)
```

#### Description

A `betabin` action specifies that a variable is distributed according to a Beta-binomial distribution with the given number of trials `n`, and `alpha` and `beta` parameters. Note that the implementation will evaluate densities for any (not necessarily integer) `x`. It is left to the user to ensure consistency (e.g., using this only with integer observations).

## Parameters

- n** : (position 0, default 1)  
Number of trials
- alpha** : (position 1, default 1.0)  
First shape parameter of the distribution.
- beta** : (position 2, default 1.0)  
Second shape parameter of the distribution.

### 2.2.2 beta

Beta distribution.

## Synopsis

```
x ~ beta()  
x ~ beta(1.0, 1.0)  
x ~ beta(alpha = 1.0, beta = 1.0)
```

## Description

A beta action specifies that a variable is beta distributed according to the given alpha and beta parameters.

## Parameters

- alpha** : (position 0, default 1.0)  
First shape parameter of the distribution.
- beta** : (position 1, default 1.0)  
Second shape parameter of the distribution.

### 2.2.3 binomial

Binomial distribution.

## Synopsis

```
x ~ binomial()  
x ~ binomial(1, 0.5)  
x ~ binomial(size = 1, prob = 0.5)
```

## Description

A binomial action specifies that a variable is distributed according to a binomial distribution with the given size and prob parameters. Note that the implementation will evaluate densities for any (not necessarily integer)  $x$ . It is left to the user to ensure consistency (e.g., using this only with integer observations).

## Parameters

**size :** (position 0, default 1)  
Mean parameter of the distribution.

**prob :** (position 1, default 0.5)  
Shape parameter of the distribution.

### 2.2.4 cholesky

Cholesky factorisation.

## Synopsis

```
U <- cholesky(A)
L <- cholesky(A, 'L')
```

## Description

A `cholesky` action performs a Cholesky factorisation of a symmetric, positive definite matrix, returning either the lower- or upper-triangular factor, with the remainder of the matrix set to zero.

## Parameters

**A :** (position 0, mandatory)  
The symmetric, positive definite matrix to factorise.

**uplo :** (position 1, default 'U')  
'U' for the upper-triangular factor, 'L' for the lower-triangular factor. As A must be symmetric, this also indicates which triangle of A is read; other elements are ignored.

### 2.2.5 exclusive\_scan

Exclusive scan primitive (also called prefix sum or cumulative sum).

## Synopsis

```
X <- exclusive_scan(x)
```

## Description

An `exclusive_scan` action computes into each element  $i$  of  $X$ , the sum of the first  $i - 1$  elements of  $x$ .

## Parameters

**x :** (position 0, mandatory)  
The vector over which to scan.

### 2.2.6 exponential

Exponential distribution.

## Synopsis



```
x ~ exponential()  
x ~ exponential(1)  
x ~ exponential(lambda = 1)
```

#### Description

A `exponential` action specifies that a variable is distributed according to an exponential distribution with the given rate `lambda`

#### Parameters

`lambda` : (position 0, default 1)  
Rate

### 2.2.7 `gamma`

Gamma distribution.

#### Synopsis

```
x ~ gamma()  
x ~ gamma(2.0, 5.0)  
x ~ gamma(shape = 2.0, scale = 5.0)
```

#### Description

A `gamma` action specifies that a variable is gamma distributed according to the given shape and scale parameters.

#### Parameters

`shape` : (position 0, default 1.0)  
Shape parameter of the distribution.  
`scale` : (position 1, default 1.0)  
Scale parameter of the distribution.

### 2.2.8 `gaussian`

Gaussian distribution.

#### Synopsis

```
x ~ gaussian()  
x ~ gaussian(0.0, 1.0)  
x ~ gaussian(mean = 0.0, std = 1.0)
```

#### Description

A `gaussian` action specifies that a variable is Gaussian distributed according to the given mean and std parameters.

#### Parameters

`mean` : (position 0, default 0.0)  
Mean.  
`std` : (position 1, default 1.0)  
Standard deviation.

### 2.2.9 inclusive\_scan

Inclusive scan primitive (also called prefix sum or cumulative sum).

#### Synopsis

```
X <- inclusive_scan(x)
```

#### Description

An `inclusive_scan` action computes into each element  $i$  of  $X$ , the sum of the first  $i$  elements of  $x$ .

#### Parameters

$x$ : (position 0, mandatory)  
The vector over which to scan.

### 2.2.10 inverse\_gamma

Inverse gamma distribution.

#### Synopsis

```
x ~ inverse_gamma()  
x ~ inverse_gamma(2.0, 1.0/5.0)  
x ~ inverse_gamma(shape = 2.0, scale = 1.0/5.0)
```

#### Description

An `inverse_gamma` action specifies that a variable is inverse-gamma distributed according to the given shape and scale parameters.

#### Parameters

$shape$ : (position 0, default 1.0)  
Shape parameter of the distribution.  
 $scale$ : (position 1, default 1.0)  
Scale parameter of the distribution.

### 2.2.11 log\_gaussian

Log-Gaussian distribution.

#### Synopsis

```
x ~ log_gaussian()  
x ~ log_gaussian(0.0, 1.0)  
x ~ log_gaussian(mean = 0.0, std = 1.0)
```

#### Description

A `log_gaussian` action specifies that the logarithm of a variable is Gaussian distributed according to the given mean and std parameters.

## Parameters

**mean :** (position 0, default 0.0)  
Mean of the log-transformed variable.

**std :** (position 1, default 1.0)  
Standard deviation of the log-transformed variable.

### 2.2.12 log\_normal

Log-normal distribution, synonym of log\_gaussian.

→ See also log\_gaussian

### 2.2.13 negbin

Negative binomial distribution.

## Synopsis

```
x ~ negbin()  
x ~ negbin(1.0, 2.0)  
x ~ negbin(mean = 1.0, shape = 2.0)
```

## Description

A negbin action specifies that a variable is distributed according to a negative binomial distribution with the given mean and shape parameters (yielding variance  $\text{mean} + \text{mean}^2/\text{shape}$ ). Note that the implementation will evaluate densities for any (not necessarily integer)  $x$ . It is left to the user to ensure consistency (e.g., using this only with integer observations).

## Parameters

**mean :** (position 0, default 1.0)  
Mean parameter of the distribution.

**shape :** (position 1, default 1.0)  
Shape parameter of the distribution.

### 2.2.14 normal

Normal distribution, synonym of gaussian.

→ See also gaussian

### 2.2.15 pdf

Arbitrary probability density function.

## Synopsis

```
x ~ expression  
x ~ pdf(pdf = expression, max_pdf = expression)  
x ~ pdf(pdf = log_expression, log = 1)
```

## Description

A pdf action specifies that a variable is distributed according to some arbitrary probability density function. It need not be used explicitly unless a maximum probability density function needs to be supplied with it: any expression using the `~` operator without naming an action is evaluated using pdf.

## Parameters

pdf : (position 0)

An expression giving the probability density function.

max\_pdf : (position 1, default inf)

An expression giving the maximum of the probability density function.

log : (default 0)

Is the expression given the log probability density function?

### 2.2.16 poisson

Poisson distribution.

## Synopsis

```
x ~ poisson()  
x ~ poisson(1.0)  
x ~ poisson(rate = 2.0)
```

## Description

A poisson action specifies that a variable is Poisson distributed according to the given rate parameter. Note that the implementation will evaluate densities for any (not necessarily integer)  $x$ . It is left to the user to ensure consistency (e.g., using this only with integer observations).

## Parameters

rate : (position 0, default 1.0)

Rate parameter of the distribution.

### 2.2.17 transpose

Transpose a matrix.

## Synopsis

```
B <- transpose(A)
```

## Description

A transpose action performs a matrix transpose.

## Parameters

A : (position 0, mandatory)

The matrix.

### 2.2.18 truncated\_gaussian

Truncated Gaussian distribution.

#### Synopsis

```
x ~ truncated_gaussian(0.0, 1.0, -2.0, 2.0)
x ~ truncated_gaussian(0.0, 1.0, lower = -2.0, upper = 2.0)
x ~ truncated_gaussian(0.0, 1.0, upper = 2.0)
```

#### Description

A `truncated_gaussian` action specifies that a variable is distributed according to a Gaussian distribution with a closed lower and/or upper bound.

For a one-sided truncation, simply omit the relevant lower or upper argument.

The current implementation uses a naive rejection sampling with the full Gaussian distribution used as a proposal. The rejection rate is simply the area under the Gaussian curve between lower and upper. If this is significantly less than one, the rejection rate will be high, and performance slow.

#### Parameters

`mean` : (position 0, default 0.0)  
Mean.

`std` : (position 1, default 1.0)  
Standard deviation.

`lower` : (position 2)  
Lower bound.

`upper` : (position 3)  
Upper bound.

### 2.2.19 truncated\_normal

Truncated normal distribution, synonym of `truncated_gaussian`.

→ See also  
`truncated_gaussian`

### 2.2.20 uniform

Uniform distribution.

#### Synopsis

```
x ~ uniform()
x ~ uniform(0.0, 1.0)
x ~ uniform(lower = 0.0, upper = 1.0)
```

#### Description

A `uniform` action specifies that a variable is uniformly distributed on a finite and left-closed interval given by the bounds lower and upper.

## Parameters

`lower` : (position 0, default 0.0)  
Lower bound on the interval.

`upper` : (position 1, default 1.0)  
Upper bound on the interval.

### 2.2.21 wiener

Wiener process.

## Synopsis

```
dW ~ wiener()
```

## Description

A `wiener` action specifies that a variable is an increment of a Wiener process: Gaussian distributed with mean zero and variance  $t_j - t_i$ , where  $t_i$  is the starting time, and  $t_j$  the ending time, of the current time interval

## 2.3 Blocks

### 2.3.1 bridge

The bridge potential.

## Synopsis

```
sub bridge {  
  ...  
}
```

## Description

Actions in the `bridge` block may reference variables of any type, but may only target variables of type `noise` and `state`. References to `obs` variables provide their next value. Use of the built-in variables `t_now` and `t_next_obs` will be useful.

### 2.3.2 initial

The prior distribution over the initial values of state variables.

## Synopsis

```
sub initial {  
  ...  
}
```

## Description

Actions in the `initial` block may only refer to variables of type `param`, `input` and `state`. They may only target variables of type `state`.

### 2.3.3 lookahead\_observation

A likelihood function for lookahead operations.

## Synopsis

```
sub lookahead_observation {  
  ...  
}
```

## Description

This may be a deterministic, computationally cheaper or perhaps inflated version of the likelihood function. It is used by the auxiliary particle filter.

Actions in the `lookahead_observation` block may only refer to variables of type `param`, `input` and `state`. They may only target variables of type `obs`.

### 2.3.4 lookahead\_transition

A transition distribution for lookahead operations.

## Synopsis

```
sub lookahead_transition {  
  ...  
}
```

## Description

This may be a deterministic, computationally cheaper or perhaps inflated version of the transition distribution. It is used by the auxiliary particle filter.

Actions in the `lookahead_transition` block may reference variables of any type except `obs`, but may only target variables of type `noise` and `state`.

### 2.3.5 observation

The likelihood function.

## Synopsis

```
sub observation {  
  ...  
}
```

## Description

Actions in the `observation` block may only refer to variables of type `param`, `input` and `state`. They may only target variables of type `obs`.

### 2.3.6 ode

System of ordinary differential equations.

## Synopsis

```
ode(alg = 'RK4(3)', h = 1.0, atoler = 1.0e-3, rtoler = 1.0e-3) {  
  dx/dt = ...  
  dy/dt = ...  
  ...  
}
```

```
ode('RK4(3)', 1.0, 1.0e-3, 1.0e-3) {
  dx/dt = ...
  dy/dt = ...
  ...
}
```

## Description

An ode block is used to group multiple ordinary differential equations into one system, and configure the numerical integrator used to simulate them.

An ode block may not contain nested blocks, and may only contain ordinary differential equation actions.

## Parameters

**alg**: (position 0, default 'RK4(3)')

The numerical integrator to use. Valid values are:

'RK4': The classic order 4 Runge-Kutta with fixed step size.

'RK5(4)': An order 5(4) Dormand-Prince with adaptive step size.

'RK4(3)': An order 4(3) low-storage Runge-Kutta with adaptive step size.

**h**: (position 1, default 1.0)

For a fixed step size, the step size to use. For an adaptive step size, the suggested initial step size to use.

**atoler**: (position 2, default 1.0e-3)

The absolute error tolerance for adaptive step size control.

**rtoler**: (position 3, default 1.0e-3)

The relative error tolerance for adaptive step size control.

### 2.3.7 parameter

The prior distribution over parameters.

## Synopsis

```
sub parameter {
  ...
}
```

## Description

Actions in the parameter block may only refer to variables of type input and param. They may only target variables of type param.

### 2.3.8 proposal\_initial

A proposal distribution over the initial values of state variables.

## Synopsis



```

sub proposal_initial {
  x ~ gaussian(x, 1.0)    // local proposal
  x ~ gaussian(0.0, 1.0) // independent proposal
}

```

## Description

This may be a local or independent proposal distribution, used by the `sample` command when the `--with-transform-initial-to-param` option is used.

Actions in the `proposal_initial` block may only refer to variables of type `param`, `input` and `state`. They may only target variables of type `state`.

### 2.3.9 proposal\_parameter

A proposal distribution over parameters.

## Synopsis

```

sub proposal_parameter {
  theta ~ gaussian(theta, 1.0) // local proposal
  theta ~ gaussian(0.0, 1.0)  // independent proposal
}

```

## Description

This may be a local or independent proposal distribution, used by the `sample` command.

Actions in the `proposal_parameter` block may only refer to variables of type `input` and `param`. They may only target variables of type `param`.

### 2.3.10 transition

The transition distribution.

## Synopsis

```

sub transition(delta = 1.0) {
  ...
}

```

## Description

Actions in the `transition` block may reference variables of any type except `obs`, but may only target variables of type `noise` and `state`.

## Parameters

`delta`: (position 0, default 1.0)

The time step for discrete-time components of the transition. Must be a constant expression.

## 2.4 Commands

### 2.4.1 Build options

Options that start with `--enable-` may be negated by instead starting them with `--disable-`.

<code>--build-dir :</code>	(default current <b>directory</b> )
	Directory in which to build and compile the model; this will be done in a subdirectory
<code>--dry-parse :</code>	(default off)
	Do not parse model file. Implies <code>--dry-gen</code> .
<code>--dry-gen :</code>	(default off)
	Do not generate code.
<code>--dry-build :</code>	(default off)
	Do not build.
<code>--force :</code>	(default off)
	Force all build steps to be performed, even when determined not to be required.
<code>--enable-warnings :</code>	(default off)
	Enable compiler warnings.
<code>--enable-assert :</code>	(default on)
	Enable assertion checking. This is recommended for test runs, but not for production runs.
<code>--enable-extra-debug :</code>	(default off)
	Enable extra debugging options in compilation. This is recommended along with <code>--with-gdb</code> , <code>--with-lldb</code> or <code>--with-cuda-gdb</code> when debugging.
<code>--enable-diagnostics n :</code>	(default 0)
	Enable diagnostic output <code>n</code> to standard error.
<code>--enable-diagnostics2 :</code>	(default off)
	Enable another type of diagnostic.
<code>--enable-single :</code>	(default off)
	Use single-precision floating point.
<code>--enable-openmp :</code>	(default on)
	Use OpenMP multithreading.
<code>--enable-cuda :</code>	(default off)
	Enable CUDA code for graphics processing units (GPU).
<code>--enable-cuda-fast-math :</code>	(default off)
	Enable fast-but-inaccurate instead of slow-but-accurate versions of functions <code>sin</code> and <code>exp</code> in CUDA.

`--enable-gpu-cache` : (default off)  
 For particle filters, enable ancestry caching in GPU memory. GPU memory is typically much more limited than main memory. If sufficient GPU memory is available this may give some performance improvement.

`--cuda-arch` : (default sm\_30)  
 For particle filters, enable ancestry caching in GPU memory. GPU memory is typically much more limited than main memory. If sufficient GPU memory is available this may give some performance improvement.

`--enable-sse` : (default off)  
 Enable SSE code.

`--enable-avx` : (default off)  
 Enable AVX code.

`--enable-mpi` : (default off)  
 Enable MPI code.

`--enable-vampir` : (default off)  
 Enable Vampir profiling.

`--enable-gperftools` : (default off)  
 Enable gperftools profiling.

## 2.4.2 Run options

Options that start with `--with-` may be negated by instead starting them with `--without-`.

`--dry-run` : Do not run.

`--seed` : (default automatic)  
 Pseudorandom number generator seed.

`--nthreads N` : (default 0)  
 Run with *N* threads. If zero, the number of threads used is the default for OpenMP on the platform.

`--with-gdb` : (default off)  
 Run within the gdb debugger.

`--with-lldb` : (default off)  
 Run within the lldb debugger.

`--with-valgrind` : (default off)  
 Run within valgrind.

`--with-cuda-gdb` : (default off)  
 Run within the cuda-gdb debugger.

`--with-cuda-memcheck` : (default off)  
 Run within cuda-memcheck.

`--gperftools-file`: (default automatic)  
Output file to use under `--enable-gperftools`. The default is *command.prof*.

`--mpi-np`: Number of processes under `--enable-mpi`, corresponding to the `-np` option to `mpirun`.

`--mpi-npernode`: Number of processes per node under `--enable-mpi`. Corresponds to the `-npernode` option to `mpirun`.

`--mpi-hostfile`: Host file under `--enable-mpi`, corresponding to the `-hostfile` option to `mpirun`.

`--role`: (default client)  
When a client-server architecture is used under MPI, the role of the process; either *client* or *server*.

`--server-file`: (default *port\_name*)  
When a client-server architecture is used under MPI, the file containing server connection information. A server process will write to this file, a client process will read from it.

### 2.4.3 Common options

The options in this section are common across all client programs.

#### Input/output options

`--model-file`: File containing the model specification.

`--init-file`: File from which to initialise parameters and initial conditions.

`--input-file`: File from which to read inputs.

`--obs-file`: File from which to read observations.

`--output-file`: (default automatic)  
File to which to write output. The default is *results/command.nc*.

`--init-ns`: (default 0)  
Index along the *ns* dimension of `--init-file` to use.

`--init-np`: (default -1)  
Index along the *np* dimension of `--init-file` to use. -1 indicates that, rather than initialising all state variables identically, the *np* dimension is used to initialise them all differently. The size of the *np* dimension must be at least the number of samples.

`--input-ns`: (default 0)  
Index along the *ns* dimension of `--input-file` to use.

`--input-np`: (default -1)  
Index along the *np* dimension of `--input-file` to use. -1 indicates that, when state variables are read from the input file, rather than initialising all state variables identically, the *np* dimension is used to initialise them all differently. The size of the *np* dimension must be at least the number of samples.

`--obs-ns` : (default 0)  
Index along the ns dimension of `--obs-file` to use.

`--obs-np` : (default 0)  
Index along the np dimension of `--obs-file` to use.

## Model transformations

`--with-transform-extended` : (default automatic)  
Transform the model for use with the extended Kalman filter. This includes symbolically deriving Jacobian expressions.

`--with-transform-param-to-state` : (default automatic)  
Treat parameters as state variables. This is useful for joint state and parameter estimation using filters.

`--with-transform-obs-to-state` : (default automatic)  
Treat observed variables as state variables. This is useful for producing simulated data sets from a model.

`--with-transform-initial-to-param` : (default off)  
Augment the parameters of the model with the initial values of state variables. This is useful when sampling from the posterior distribution. Treating initial values as parameters means that they are sampled using local Metropolis-Hastings moves rather than by importance sampling, which may be beneficial if a good importance proposal cannot be devised.

### 2.4.4 draw

Draw a model as a directed graph.

#### Synopsis

```
libbi draw --model-file Model.bi > Model.dot
dot -Tpdf -o Model.pdf Model.dot
```

#### Description

The `draw` command takes a model specification and outputs a directed graph to visualise the model. It is useful for validation and debugging purposes. The output is a dot script that can be processed by the dot program to create a figure.

### 2.4.5 filter

Filtering tasks.

#### Synopsis

```
libbi filter ...
```

## Options

The `filter` command permits the following options:

- `--start-time`: (default 0.0)  
Start time.
- `--end-time`: (default 0.0)  
End time.
- `--noutputs`: (default 0)  
Number of dense output times. The state is always output at time `--end-time` and at all observation times in `--obs-file`. This argument gives the number of additional, equispaced times at which to output. With `--end-time`  $T$  and `--noutputs`  $K$ , then for each  $k$  in  $0, \dots, K-1$ , the state will be output at time  $T*k/K$ .
- `--with-output-at-obs`: (default 1)  
Output at observation times in addition to dense output times.
- `--filter`: (default bootstrap)  
The type of filter to use; one of:
- `bootstrap`: Bootstrap particle filter,
  - `lookahead`: Auxiliary particle filter with lookahead. The lookahead operates by advancing each particle according to the `\blockref{lookahead_transition}` top-level block, and weighting according to the `\blockref{lookahead_observation}` top-level block. If the former is not defined, the `\blockref{transition}` top-level block will be used instead. If the latter is not defined, the `\blockref{observation}` top-level block will be used instead.
  - `bridge`: Particle filter with intermediate bridging weights, as described in Del Moral & Murray (2014). Bridging weights are assigned according to the `\blockref{bridge}` top-level block.
  - `kalman`: Extended Kalman filter. Jacobian terms are determined by symbolic manipulations. There are some limitations to these manipulations at present, so that some models cannot be handled. An error message will be given in these cases.  
  
Setting `--filter kalman` automatically enables the `--with-transform-extended` option.

**Particle filter-specific options** The following additional options are available when `--filter` gives a particle filter type:

- `--nparticles`: (default 1)  
Number of particles to use.
- `--ess-rel`: (default 0.5)  
Threshold for effective sample size (ESS) resampling trigger. Particles will only be resampled if ESS is below this proportion of `--nparticles`. To always resample, use `--ess-rel 1`. To never resample, use `--ess-rel 0`.
- `--resampler`: (default systematic)  
The type of resampler to use; one of:

`stratified`: for a stratified resampler (Kitagawa 1996),  
`systematic`: for a systematic (or 'deterministic stratified') resampler (Kitagawa 1996),  
`multinomial`: for a multinomial resampler,  
`metropolis`: for a Metropolis resampler (Murray 2011),  
`rejection`: for a rejection resampler (Murray, Lee & Jacob 2013), or

**Metropolis resampler-specific options** The following additional options are available when `--resampler` is set to `metropolis`.

`-C`: (default 0)  
 Number of steps to take.

**Bridge particle filter-specific options** The following additional options are available when `--filter` is set to `bridge`:

`--nbridges`: (default 0)  
 Number of dense bridge times. This argument gives the number of equispaced times at which to assign bridge weights, and potentially resample. With `--end-time`  $T$  and `--nbridges`  $K$ , then for each  $k$  in  $0, \dots, K-1$ , bridge weighting will occur at time  $T*k/K$ .

**Adaptive particle filter-specific options** The following additional options are available when `--filter` is set to 'adaptive':

`--stopper`: (default deterministic)  
 The stopping criterion to use; one of:

`deterministic`: for a fixed number of particles,  
`sumofweights`: for a sum of weights,  
`miness`: for a minimum effective sample size (ESS),  
`stddev`: for a minimum standard deviation,  
`var`: for a minimum variance.

`--stopper-threshold`: (default 128)  
 Threshold value for stopping criterion.

`--stopper-max`: (default 32768)  
 Maximum number of particles at any time, regardless of the stopping criterion.

`--stopper-block`: (default 128)  
 Number of particles per block.

## 2.4.6 help

Look up online help for an action, block or command.

## Synopsis

```
libbi help name
```

```
libbi help name --action
```

```
libbi help name --block
```

```
libbi help name --command
```

## Description

The `help` command is used to access documentation from the command line. This documentation is the same as that provided in the user reference.

*name* gives the name of any action, block or command. The documentation for the respective action, block or command is presented. Ambiguities (e.g. actions and blocks with the same name) are resolved via a prompt, or by using any of the `--action`, `--block` or `--command` options.

## Options

The following options are supported:

- `--action`: explicitly search for an action.
- `--block`: explicitly search for a block.
- `--command`: explicitly search for a command.

### 2.4.7 optimise

Optimisation of the parameters of a model.

## Synopsis

```
libbi optimise ...
```

```
libbi optimize ...
```

Alternative spellings are supported.

## Options

The `optimise` command inherits all options from `\clientref{filter}`, and permits the following additional options:

`--target`: (default `likelihood`)

Optimisation target; one of:

`likelihood`: Maximum likelihood estimation.

`posterior`: Maximum *a posteriori* estimation.

`--optimiser` or `--optimizer`: (default `nm`)

The optimisation method to use; one of:

`nm`: Nelder-Mead simplex method.



### Nelder-mead simplex method-specific options

- `--simplex-size-real`: (default 0.1)  
Size of initial simplex relative to starting point of each variable.
- `--stop-size`: (default 1.0-e4)  
Size-based stopping criterion.
- `--stop-steps`: (default 100)  
Maximum number of steps to take.

### 2.4.8 optimize

Optimization of the parameters of a model.

See also

\clientref{optimise}

### 2.4.9 package

Create and validate projects, build packages for distribution.

Synopsis

```
libbi package --create
```

```
libbi package --validate
```

```
libbi package --build
```

```
libbi package --webify
```

Description

LibBi prescribes a standard structure for a project's model, configuration, data and other files, and a standard format for packaging these. The former assists with collaboration and reproducible research, the latter with distribution.

The package command provides facilities for creating a new LibBi project with the standard file and directory structure, and bundling such a project into a package for distribution.

**A standard project** A standard project contains the following files:

- `*.bi`: Model files.
- `init.sh`: A shell script that may be run to create any derived files. A common task is to call `libbi sample --target joint ...` to simulate a data set for testing purposes.
- `run.sh`: A shell script that may be run to reproduce the results of the project. Common tasks are to call `libbi sample --target prior ...` to sample from the prior distribution, and `libbi sample --target posterior ...` to sample from the posterior distribution. While a user may not necessarily run this script directly, it should at least give them an idea of what the project is set up to do, and what

commands they might expect to work. After the README.md file, this would typically be the second file that a new user looks at.

- `*.conf` : Configuration files. Common files are `prior.conf`, `posterior.conf`, `filter.conf` and `optimise.conf`, containing command-line options for typical commands.
- `config.conf` : A particular configuration file where, by convention, a user can set any platform-specific build and run options (such as `--enable-cuda` and `--nthreads`). Any LibBi commands in the `init.sh` and `run.sh` scripts should include this configuration file to bring in the user's own options (e.g. `libbi sample @config.conf ...`).
- `data/` : Directory containing data files that are passed to LibBi using the `--init-file`, `--input-file` and `--obs-file` command-line options.
- `results/` : Directory containing results files created from LibBi using the `--output-file` command-line option.

**A standard package** The following additional files are used for the packaging of a project:

- `MANIFEST` : A list of files, one per line, to be included in the package.
- `LICENSE` : The license governing distribution of the package.
- `META.yml` : Meta data of the package. It should be formatted in YAML, giving the name, author, version and description of the package. See that produced by `libbi package --create` as a guide.
- `README.md` : A description of the package. This would typically be the first file that a new user looks at. It should be formatted in Markdown.
- `VERSION.md` : The version history of the package. It should be formatted in Markdown.

Packages should be given a three-figure version number of the form `x.y.z`, where `x` is the version number, `y` the major revision number and `z` the minor revision number. The version number would typically be incremented after an overhaul of the package, the major revision number after the addition of new functionality, and the minor revision number after bug fixes or corrections. When a number is incremented, those numbers on the right should be reset to zero. The first version number of a working package should be `1.0.0`. If a package is incomplete, only partially working or being tested, the version number may be zero.

A project may contain any additional files, and these may be listed in the MANIFEST file for distribution. Commonly included are Octave, MATLAB or R scripts for collating and plotting results, for example.

A standard package is simply a gzipped TAR archive with a file name of `Name-Version.tar.gz`. Extracting the archive produces a directory with a name of `Name-Version`, within which are all of those files listed in the MANIFEST file of the project.

**Version control** Under version control, all project files with the exception of the following would be included:

- Any files in the `results/` directory. These can be large, and at any rate are derived files that a user should be able to reproduce for themselves, perhaps with the `run.sh` script.
- The `results/` directory itself. This is always created automatically when used in the `--output-file` command-line option, and so its inclusion is unnecessary. Moreover, it is common to create a `results` symlink to another directory where output files should be written, particularly in a cluster environment where various network file systems are available. Inclusion of the `results/` directory in a version control system becomes a nuisance in such cases.

These files would also not typically be included in the package MANIFEST.

## Options

The following options are supported:

- `--create` : Set up the current working directory as a LibBi project. This creates all the standard files for a LibBi package with placeholder contents. It will prompt to overwrite existing files.
- `--validate` : Validate the current working directory as a LibBi project.
- `--build` : Validate the current working directory as a LibBi project and build the package. This produces a `Model-x.y.z.tar.gz` file in the current working directory for distribution.
- `--webify` : Create a file for publishing the package on a Jekyll website (such as [www.libbi.org](http://www.libbi.org)). This produces a `Model.md` file in the current working directory.

### Package creation-specific options

- `--name` : (default 'Untitled')  
Name of the package.

## 2.4.10 `rewrite`

Output internal model representation after applying transformations and optimisations.

### Synopsis

```
libbi rewrite --model-file Model.bi
```

### Description

The `rewrite` command takes a model specification and outputs a new specification that shows the internal transformations and optimisations applied by LibBi. It is useful for validation and debugging purposes. The new specification is written to `stdout`.

## 2.4.11 `sample`

Sample the prior, joint or posterior distribution.

### Synopsis

```
libbi sample --target prior ...
```

```
libbi sample --target joint ...
```

```
libbi sample --target posterior ...
```

## Options

The `sample` command inherits all options from `filter`, and permits the following additional options:

`--target`: (default `posterior`)

The target distribution to sample; one of:

`prior`: To sample from the prior distribution.

`joint`: To sample from the joint distribution. This is equivalent to `--target prior --with-transform-obs-to-state`.

`posterior`: To sample from the posterior distribution. Use `--obs-file` to provide observations.

`prediction`: To sample forward in time from a given initial state. Use `--init-file` to set the initial state, and this initial state determines the interpretation of the prediction. For example, the `--init-file` may be the output file of a previous sampling of the posterior distribution, in which case the result is a posterior prediction.

`--sampler`: (default `mh`)

The type of sampler to use for `--target posterior`; one of:

`mh` or (deprecated) `pmmh`: Marginal Metropolis-Hastings (Andrieu, Doucet & Holenstein, 2010).

`sir` or (deprecated) `smc2`: Marginal sequential importance resampling (Chopin, Jacob & Papaspiliopoulos, 2013).

`sis`: Marginal sequential importance sampling (SIS).

For MH, the proposal works according to the `\blockref{proposal_parameter}` top-level block in the model. If this is not defined, independent draws are taken from the `\blockref{parameter}` top-level block instead. If `--with-transform-initial-to-param` is used, the `\blockref{proposal_initial}` top-level block is used to make Metropolis-Hastings proposals over initial conditions also. If this is not defined, independent draws are taken from the `\blockref{initial}` top-level block instead.

For SIR, the same blocks are used as proposals for rejuvenation steps, unless one of the adaptation strategies below is enabled.

`--nsamples`: (default 1)

Number of samples to draw.

### SIR-specific options

`--nmoves`: (default 1)

Number of move steps to perform after each resample.

`--tmoves` : (default 0.0)  
Overall real time, in seconds, to allocate to move steps (Murray et al. 2016). If positive, `--nmoves` and `--sample-ess-rel` are ignored. Resampling is performed after each step, and the number of moves subsequently made becomes a random variable dependent on `--tmoves`.

`--sample-resampler` : (default systematic)  
The type of resampler to use on parameter particles, see `--resampler` for options.

`--sample-ess-rel` : (default 0.5)  
Threshold for effective sample size (ESS) resampling trigger. Parameter particles will only be resampled if ESS is below this proportion of `--nsamples`. To always resample, use `--sample-ess-rel 1`. To never resample, use `--sample-ess-rel 0`.

`--sample-stopper` : (default deterministic)  
The stopping criterion to use for parameter samples while adapting, see `--stopper` for options.

`--sample-stopper-threshold` : Threshold value for stopping criterion.

`--sample-stopper-max` : Maximum number of samples at any time, regardless of the stopping criterion.

`--adapter` : (default none)  
Adaptation strategy for rejuvenation proposals:

- `none` : No adaptation.
- `local` : Local proposal adaptation.
- `global` : Global proposal adaptation.

`--adapter-scale` : (default 0.25)  
When local proposal adaptation is used, the scaling factor of the local proposal standard deviation relative to the global sample standard deviation.

`--adapter-ess-rel` : (default 0.25)  
Threshold for effective sample size (ESS) adaptation trigger. Adaptation will only be performed if ESS is above this proportion of `--nsamples`. To always adapt, use `--adapter-ess-rel 1`. If adaptation is not performed, the `proposal_parameter` top-level block is used for rejuvenation proposals instead.

---

## 3 Developer Guide

---

### 3.1 Introduction

LibBi consists of two major components:

- A *library* that provides classes and functions for simulation, filtering, smoothing, optimising and sampling state-space models, as well as auxiliary functionality such as memory management, I/O, numerical computing etc. It is written in C++ using a generic programming paradigm.
- A *code generator* that parses the LibBi modelling language, constructs and optimises an internal model representation, and generates C++ code for compilation against the library. It also includes the command-line interface through which the user interacts with LibBi. It is written in Perl using an object-oriented paradigm.

Related projects might also be considered components. These include *OctBi* for GNU Octave and *RBi* for R. Both allow the querying, collation and visualisation of results output by LibBi.

Developing LibBi involves adding or modifying functionality in one or more of these components. A typical exercise is adding block and action types to the code generator to introduce new features, such as support for additional probability density functions. This may also involve implementing new functionality in the library that these blocks and actions can use. Another common task is adding a new inference method to the library, then adding or modifying the code generator to produce the client code to call it.

### 3.2 Setting up a development environment

When developing LibBi, it is recommended that you do not install it in the usual manner as a system-wide Perl module. The recommended approach is to work from a local clone of the LibBi Git repository. This way modifications take effect immediately without additional installation steps, and changes are easily committed back to the repository.

#### 3.2.1 Obtaining the source code

LibBi is hosted on GitHub at <https://github.com/libbi/LibBi>. It is recommended that you sign up for an account on GitHub and fork the repository there into your own account. You can then clone your forked repository to your local machine with:

```
git clone https://github.com/username/LibBi.git
```

You should immediately add the original LibBi repository upstream:

```
cd LibBi
git remote add upstream https://github.com/libbi/LibBi.git
```

after which the following command will pull in changes committed to the main repository by other developers:

```
git fetch upstream
```

To contribute your work back to the main LibBi repository, you can use the *Pull Request* feature of GitHub.

Similar instructions apply to working with OctBi and RBi, which are held in separate repositories under <https://github.com/libbi>.

When running the command `libbi`, you will want to make sure that you are calling the version in the Git repository that you have just created. The easiest way to do this is to add the `LibBi/script` directory to your `$PATH` variable. LibBi will find the other files itself (it uses the base directory of the `libbi` script to determine the location of other files).

### 3.2.2 Using Eclipse

LibBi is set up to use the Eclipse IDE (<http://www.eclipse.org>). From a vanilla install of the Eclipse IDE for C/C++, a number of additional plugins are supported, and recommended. These are, along with the URLs of their update sites:

- Eclox (Doxygen integration) <http://download.gna.org/eclox/update>,
- EPIC (Perl integration) <http://e-p-i-c.sf.net/updates/testing>, and
- Perl Template Toolkit Editor <http://perleclipse.com/TTEditor/UpdateSite>.

All of these plugins may be installed via the `Help > Install New Software...` menu item in Eclipse, in each case entering the update site URL given above.

The EGit plugin is bundled with all later versions of Eclipse and can be used to obtain LibBi from GitHub. Use the `File > Import...` menu item, then `Git > Projects from Git, URI`, and enter the URL of your forked repository on GitHub. Follow the prompts from there.

A custom code style is available in the `eclipse/custom_code_style.xml` file, and custom code templates in `eclipse/custom_code_templates.xml`. These can be installed via the `Window > Preferences` dialog, using the `Import...` buttons under `C/C++ > Code Style > Formatter` and `C/C++ > Code Style > Code Templates`, respectively. With these installed, `Ctrl + Shift + F` in an editor window automatically applies the prescribed style.

### 3.3 Building documentation

This user manual can be built, from the base directory of LibBi, using the command:

```
perl MakeDocs.PL
```

You will need  $\text{\LaTeX}$  installed. The manual will be built in PDF format at `docs/pdf/index.pdf`. Documentation of all actions, blocks and clients is extracted from their respective Perl modules as part of this process.

The Perl components of LibBi are documented using POD (“Plain Old Documentation”), as is conventional for Perl modules. The easiest way to inspect this is on the command line, using, e.g.

```
perldoc lib/Bi/Model.pm
```

A PDF or HTML build of the Perl module documentation is not yet available.

The C++ components of LibBi are documented using Doxygen. HTML documentation can be built by running `doxygen` in the base directory of LibBi, then opening the `docs/dev/html/index.html` file in a browser.

## 3.4 Building releases

LibBi is packaged as a Perl module. The steps for producing a new release are:

1. Update `VERSION.md`.
2. Update `MANIFEST` by removing the current file, running `perl MakeManifest.PL` and inspecting the results. Extraneous files that should not be distributed in a release are eliminated via regular expressions in the `MANIFEST.SKIP` file.
3. Run `perl Makefile.PL` followed by `make dist`.

On GitHub, the last step is eliminated in favour of tagging the repository with version numbers and having archives automatically built by the service.

## 3.5 Developing the code generator

The code generator component is implemented in Perl using an object-oriented paradigm, with extensive use of the Perl Template Toolkit (TT) for producing C++ source and other files.

### 3.5.1 Actions and blocks

A user of LibBi is exposed only to those blocks and actions which they explicitly write in their model specification file. Developers must be aware that beyond these *explicit* blocks, additional *implicit* blocks are inserted according to the actions specified by the user. In particular, each action has a preferred parent block type. If the user, in writing their model specification, does not explicitly place an action within its preferred block type, the action will be implicitly wrapped in it anyway.

For example, take the model specification:

```
sub transition {  
  x ~ uniform(x - 1.0, x + 1.0)  
  y ~ gaussian(y, 1.0)  
}
```

The `uniform` and `gaussian` actions prefer different parent block types. These additional block types will be inserted, to give the equivalent of the user having specified:



```

sub transition {
  sub uniform_ {
    x ~ uniform(x - 1.0, x + 1.0)
  }
  sub gaussian_ {
    y ~ gaussian(y, 1.0)
  }
}

```

For the developer, then, actions and blocks are always written in tandem. Note the underscore suffix on `uniform_` and `gaussian_` is a convention that marks these as blocks for internal use only – the user is not intended to use them explicitly (see the style guide).

It is worth familiarising yourself with this behaviour, and other transformations made to a model, by using the `rewrite` command.

To add a block:

1. Choose a name for the block.
2. Create a Perl module `Bi::Block::name` in the file `lib/Bi/Block/name.pm`. Note that Perl module “CamelCase” naming conventions should be ignored in favour of LibBi block naming conventions here.
3. Create a new TT template `share/tt/cpp/block/name.hpp.tt`. When rendered, the template will be passed a single variable named `block`, which is an object of the class created in the previous step.

As implementation details of the Perl module and template are subject to change, it is highly recommended that you copy and modify an existing, similar block type, as the basis for your new block type.

To add an action:

1. Choose a name for the action.
2. Create a Perl module `Bi::Action::name` in the file `lib/Bi/Action/name.pm`. Note that Perl module “CamelCase” naming conventions should be ignored in favour of LibBi action naming conventions here.
3. If necessary, create a new TT template `share/tt/cpp/action/name.hpp.tt`. During rendering, the template will be passed a single variable named `action`, which is an object of the class created in the previous step. This template is not required; the template of the containing block may generate all the necessary code.

### 3.5.2 Clients

To add a client:

1. Consider whether an existing client can be modified or a new client should be added. For an existing client, edit the corresponding Perl module `lib/Bi/Client/name.pm`. For a new client, choose a name and create a new Perl module `lib/Bi/Client/name.pm`. Again, LibBi client naming conventions should be favoured over Perl module naming conventions.
2. Consider whether an existing template can be modified or a new template should be created. For an existing template, edit the corresponding template in `share/tt/cpp/client/template.cpp.tt`. For a new template:

- (a) Create files `share/tt/cpp/client/template.cpp.tt` and `share/tt/cpp/client/...`. Note that the second typically only needs to `#include` the former, the `*.cu` file extension is merely needed for a CUDA-enabled compile.
- (b) Modify the Perl module to select this template for use where appropriate.
- (c) Add the template name to the list given at the top of `share/tt/build/Makefile` to ensure that a build rule is generated for it.

### 3.5.3 Designing an extension

Complex C++ code should form part of the library, rather than being included in TT templates. The C++ code generated by templates should typically be limited to arranging for a few function calls into the library, where most of the implementation is done.

When writing templates for client programs, consider that it is advantageous for the user that they can change command-line options without the C++ code changing, so as to avoid a recompile. This can be achieved by writing runtime checks on command-line options in C++ rather than code generation-time checks in TT templates. This need not be taken to the extreme, however: clear and simple template code is preferred over convoluted C++!

### 3.5.4 Documenting an extension

Reference documentation for actions, blocks and clients is written in the Perl module created for them, in standard Perl POD. Likewise, parameters for actions and blocks, and command-line options for clients, are enumerated in these. The idea of this is to keep implementation and documentation together.

POD documentation in these modules is automatically converted to  $\text{\LaTeX}$  and incorporated in the User Reference section of this manual.

### 3.5.5 Developing the language

The LibBi modelling language is specified in the files `share/bi.lex` (for the lexer) and `share/bi.y` (for the parser). The latter file makes extensive callbacks to the `Bi::Parser` module. After either of the two files are modified, the automatically-generated `Parse::Bi` module must be rebuilt by running `perl MakeParser.PL`.

### 3.5.6 Style guide

Further to the style guide for users, the following additional recommendations pertain to developers:

- Action, block, client and argument names are all lowercase, with multiple words separated by `'_'` (the underscore). Uppercase may be used in exceptional cases where this convention becomes contrived. A good example is matrix arguments, which might naturally be named with uppercase letters.
- Actions, blocks and arguments that are not meant to be used explicitly should be suffixed with a single underscore.

## 3.6 Developing the library

The library component is implemented in C++ using a generic programming paradigm. While classes and objects are used extensively, it is not object-oriented *per se*. In particular:

- Class inheritance is used for convenience of implementation, not necessarily to represent “is a” relationships.
- Polymorphism is seldom, if ever, employed.
- Static dispatch is favoured strongly over dynamic dispatch for reasons related to performance. Virtual functions are not used.

### 3.6.1 Header files

Header files are given two extensions in LibBi:

1. \*.hpp headers may be safely included in any C++ (e.g. \*.cpp) or CUDA (\*.cu) source files,
2. \*.cuh headers may only be safely included in CUDA (\*.cu) source files. They include CUDA C extensions.

Note that not all \*.hpp headers can be safely included in \*.cu files either, due to CUDA compiler limitations, particularly those headers that further include Boost headers which make extensive use of templates. Efforts have been made to quarantine such incidences from the CUDA compiler, and support is improving, but mileage may vary.

### 3.6.2 Pseudorandom reproducibility

It is important to maintain reproducibility of results under the same random number seed, typically passed using the `--seed` option on the command line. Real time impacts should be considered, such as favouring static scheduling over dynamic scheduling for OpenMP thread blocks. Consider the following:

```
Random rng;

#pragma omp parallel for
for (int i = 0; i < N; ++i) {
    x = rng.uniform();
    ...
}
```

Random maintains a separate pseudorandom number generator for each OpenMP thread. If dynamically scheduled, the above loop gives no guarantees as to the number of variates drawn from each generator, so that reproducibility of results for a given seed is not guaranteed. Static scheduling should be enforced in this case to ensure reproducibility:

```
Random rng;

#pragma omp parallel for schedule(static)
for (int i = 0; i < N; ++i) {
    x = rng.uniform();
    ...
}
```

Tip →

To avoid this particular case, static scheduling is enabled by default. You should not override this with dynamic scheduling in an OpenMP loop.

A more subtle consideration is the conditional generation of variates. Consider the following code, evaluating a Metropolis-Hastings acceptance criterion:

```
alpha = (l1*p1*q2)/(l2*p2*q1);
if (alpha >= 1.0 || rng.uniform() < alpha) {
    accept();
} else {
    reject();
}
```

Here, `rng.uniform()` is called only when `alpha >= 1.0` (given operator short-circuiting). We might prefer, however, that across multiple runs with the same seed, the pseudorandom number generator is always in the same state for the  $n$ th iteration, regardless of the acceptance criteria across the preceding iterations. Moving the variate generation outside the conditional will fix this:

```
alpha = (l1*p1*q2)/(l2*p2*q1);
u = rng.uniform();
if (alpha >= 1.0 || u < alpha) {
    accept();
} else {
    reject();
}
```

### 3.6.3 Shallow copy, deep assignment

Typically, classes in LibBi follow a “shallow copy, deep assignment” idiom. The idiom is most apparent in the case of vector and matrix types. A copy of a vector or matrix object is shallow – the new object merely contains a pointer to the memory buffer underlying the existing object. An assignment of a vector or matrix is deep, however – the contents of the memory buffer underlying the existing object is copied into that of the new object.

Often the default copy constructor is sufficient to achieve a shallow copy, while an override of the default assignment operator may be necessary to achieve deep assignment.

*Generic* copy constructors and assignment operators are also common. These are templated overloads of the default versions which accept, as arguments, objects of some class other than the containing class. When a generic copy constructor or assignment operator is used, the default copy constructor or assignment operator should always be overridden also.

### 3.6.4 Coding conventions

The following names are used for template parameters (where  $n$  is an integer):

- $B$  for the model type,
- $T_n$  for scalar types,
- $V_n$  for vector types,

- Mn for matrix types,
- Qn for pdf types,
- S for type lists,
- L for location (host or device),
- CL for location of a cache,
- PX for parents type,
- CX for coordinates type,
- OX for output type.

### 3.6.5 Style guide

In brief, the style of C++ code added to the library should conform to what is already there. If using Eclipse, the custom code style file (see Using Eclipse) is a good start.

C++ code written in the library should conform to the “Kernighan and Ritchie” style, indenting with two spaces and never tabs.

---

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