Defining Emergence: Learning from Flock Behavior

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The idea of emergence originates from the fact that global effects emerge from local interactions producing a collective coherent behavior. A particular instance of emergence is illustrated by a flocking model of interacting "boids" encompassing two antagonistic conducts—consensus and frustration—giving rise to highly complex, unpredictable, coherent behavior. The cohesive motion arising from consensus can be described in terms of three ordered dynamic phases. Once frustration is included in the model, local phases for specific groups of flockmates, and transitions among them, replace the global ordered phases. Following the evolution of boids in a single group, we discovered that the boids in this group will alternate among the three phases. When we compare two uncorrelated groups, the second group shows a similar behavior to the first one, but with a different sequence of phases. Besides the visual observation of our animations with marked boids, the result is evident plotting the local order parameters. Rather than adopting one of the consensus ordered phases, the flock motion resembles more an entangled dynamic sequence of phase transitions involving each group of flockmates. © 2015 Wiley Periodicals, Inc. Complexity 21: 69–78, 2016

Key Words: emergence; self-organization; collective motion

1. INTRODUCTION

The term emergence refers to the existence or formation of collective behaviors. The concept originates from the fact that global effects emerge from local interactions. Features of a system are considered to be emergent when they result from interactions among individual parts that do not show these properties. Attempts to provide a concise definition of emergence have been made. For example, Prokopenko et al. [1] and Gershenson and Fernandez [2] based on information theory, established a relation between terms such as self-organization, emergence, and complexity. Pernu and Annila [3] analyzed emergence using the principle of least action, while Halley and Winkler [4], made a classification of emergence based on self-organization.

When we try to understand collective behavior, emergence is also present and it refers to the way in which joint properties develop from the separate parts, or how

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behavior at a larger scale results from the detailed behavior at a smaller scale. The extraordinary coordination observed in large groups of moving animals is a striking form of emergent and complex behavior. In a flock, birds move together in a coherent fashion in all directions, and even spread out into smaller groups while at the same time creating amazing shapes. This biological system is able to develop a collective motion that seems to go beyond the concept of a physical phase transition.

Driven by the idea of dynamic emergence, we introduced a computational two-dimensional (2D) model of interacting boids based on two antagonistic concepts—consensus and frustration—which give rise to highly complex, unpredictable, coherent behavior [5]. In our model, each boid moves at a fixed speed creating an open system of self-propelled boids [6], and aligns itself in the average direction of a fixed number of flockmates, regardless of the physical distance between the individual and those particular partners. We refer to this alignment rule as **consensus**. The coherent motion arising from consensus can be described in terms of three ordered phases. The first phase exhibits an obvious alignment with all the members of the group moving in the same direction. There are two novel rotational phases (clockwise and counterclockwise) where the individual loop-like motion is phase-locked among all the different boids. A model-based solely on consensus is interesting in its own right. It lacks, however, the versatility observed in true flocking behavior. The cohesive emergence appears when we admit the second condition, also responsible for constraining the motion. Adequate variability can be elicited by introducing a boundary or circular basin of attraction. We refer to this reflecting condition as **frustration**, inasmuch as it antagonizes the motion induced by the consensus. The global phases disappear and in their place, a startlingly rich variety of coherent motion materializes showing an outstanding similarity to the flocking phenomena observed in nature. There is, however, a remnant of the ordered phases in this emergent motion, and this is what we describe in this article. To uncover it, we have to consider specific groups of flockmates rather than the entire flock. We found out that, if we follow the time evolution of one single group, the boids in this group will alternate among the three phases found in the consensus-only model. In other words, they remain in one ordered phase for a short time and then roll to a different phase uninterruptedly. Conversely, if we compare two uncorrelated groups, the second group will present a similar behavior to the first one, but with a different sequence of phases and different time intervals spent on each phase. These two facts combined with the coherence of the whole flock gives rise to the impressive emergent behavior of our model. Besides the visual observation of our animations with marked boids, the result is evident in the plots of the local order parameters of one and two groups as a function of time. It is in this sense that we say that emergence goes beyond the usual ordered phases and their transitions. It looks more like an entangled dynamic sequence of phase transitions involving each group. Our aim in this article is to dissect the movement of different flockmate groups and elucidate the evolution of the emergent phenomenon.

1.1. Boid Interactions

Many researchers have been intrigued by the formation of flocks for a long time [7–10]. Continuum (high density members) [11–13], Lagrangian (set of differential equations for each individual), and discrete [14,15], both theoretical and computational models have been proposed. More recently, direct observational studies [16,17] show more specific features of the flock motion. So far, we have learned that a collective motion may arise as a consequence of simple interaction rules among members of a group. For example, Reynolds [7] developed a computational model of a flock of “boids” introducing specific rules such as collision avoidance, velocity matching, and flock centering. He was able to reproduce a collective behavior of the flock. This model was later modified by Vicsek et al. [6] who also observed self-organization by introducing a single interaction rule, namely that each boid follows the average direction of its nearest neighbors, supplemented by a noise term. The idea of nearest-neighbor interaction, or radial interaction, is inspired by physical systems. However, in biology and other fields, this may not be the appropriate rule. Observations by Ballerini et al. [16] challenge this interpretation. They actually tracked three-dimensional positions of thousands of birds using a stereo photography technique. Based on these data, they concluded that birds adopt a topological interaction, that is, each member of the flock coordinates with a small number of individuals (typically six or seven) no matter how far they are with respect to each other. They hypothesized (recently confirmed by Camperi et al. [18]) that this type of interaction makes for a stronger cohesion among the flockmates and that it may be the result of natural evolutionary processes of survival origin. Other studies considering topological interactions are Ginelli and Chate [19]. They adopted Vicsek model but added metric-free interactions based on Voronoi shells about each boid to define the number of interacting neighbors. They systematically showed the effects on the collective motion that this topological rule originates. In a similar way, Dossetti et al. [20] also chose a nearest-neighbor topological interaction and added a correlated angular noise to the direction of propulsion together with a harmonic attraction toward the center of motion of the flock. They obtained a bistable region where the system may perform transitions between global oscillatory and translational

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phases. More recently, a study where interactions among birds are modeled as a mixture of topological and metric interactions has been reported by Niizato and Gunji [21].

The onset of a self-organized collective motion is what is usually understood as an emergent phenomenon. However, the richness of shapes observed in a real flock of birds is not entirely encompassed by this term. A more appropriate concept is that of "dynamic emergence" implying the onset of time-dependent patterns produced by the flock. The search for, and characterization of, dynamic emergence is the main purpose in this article.

Our simulations are based on a computational 2D model of interacting boids consisting of two stages [5]. We start off with a random initial array of boids, oriented in random directions on a 2D surface. Each boid moves at a fixed speed and aligns itself in the average direction of a fixed number of flockmates. In the consensus stage, we assume periodic boundary conditions. The results can be classified as three ordered phases: an alignment phase with the boids in the flock moving in the same direction, and two rotational phases, one clockwise and the other one counterclockwise, where each boid's loop-like motion is phase-locked to the rest of the boids in the flock. These three ordered phases are characterized by two order parameters, and we consider them as global in the sense that all the boids participate.

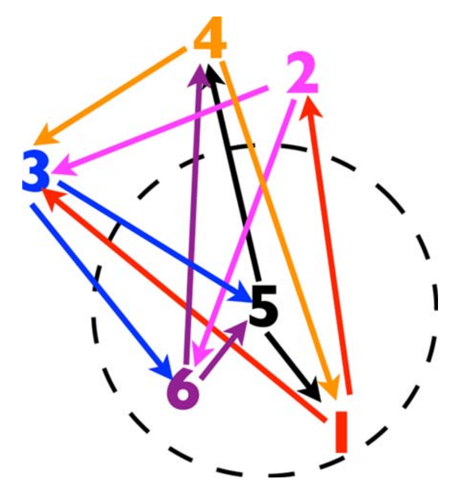
Cohesive emergence appears adding the frustration stage. Instead of imposing periodic boundary conditions, we enforce a boundary or circular basin of attraction antagonizing the consensus rule. A successful yet simple choice of reflection is to force each individual boid to make a "U-turn" at the boundary. A specular reflection boundary condition for the motion of many particles aligning each other by a metric rule was also introduced in Ref. [22]. However, the latter model does not seem to create dynamic emergence. It only seems to create stable structures like a vortex due to the presence of a specular wall.

A remarkable feature of the consensus-frustration model is the tendency of the boids to concentrate in groups, rather than become dispersed. The global phases vanish and in their place, local ordered phases appear for specific groups of flockmates rather than the entire flock. We thus introduce "local" parameters for each group of flockmates. Typically, if the flock consists of 500 boids and each group of flockmates has 25 boids, we concentrate on twenty groups and their respective order parameters.

2. MODEL

Our flock model is based on the premise that simple algorithmic rules can lead to emergent behavior, if they are appropriately chosen. We model a set of N boids moving with constant speed v_0 in a 2D space of side l, and governed by only two rules. We call these rules consensus and frustration. Consensus is the rule establishing that each boid orients its velocity vector as the average of its n topological flockmates velocity orientations. Our topological interaction is nonreciprocal in the sense that boid i sees boid j but does not necessarily vice versa. To clarify this concept, we show a diagram in Figure 1. Mathematically, consensus can be simply expressed as

FIGURE 1



Members of a flock aligning by means of a topological interaction. Note, for example, that bird $i=5$ follows $i=\{1,4\}$ rather than its nearest neighbors $i=\{1,6\}$.

$$\overline{\mathbf{v}}_i=\frac{\sum_{k=1}^{n}\mathbf{v}_k}{\left|\sum_{k=1}^{n}\mathbf{v}_k\right|}, \tag{1}$$

where $\overline{\mathbf{v}}_i$ is the instantaneous orientation velocity vector for boid i and $\mathbf{v}_k$ is the velocity vector for boid k. Note that $\overline{\mathbf{v}}_i$ is normalized to 1. Once we established Eq. (1), we evolve each bird in time according to the following discrete map [23]: if $\mathbf{r}_i(t)$ is the position of bird i at step t, its position at $\mathbf{r}_i(t+\Delta t)$ is given by

$$\mathbf{r}_i(t+\Delta t)=\mathbf{r}_i(t)+\overline{\mathbf{v}}_i\Delta t\, v_0. \tag{2}$$

In our simulations, the discrete step Δt is simply chosen to be one. By applying Eq. (2) to a flock with N members and with only the consensus rule [Eqs. (1) and (2)], this flock self-organizes and quickly reaches a steady state. However, it does not have the versatility of motion seen in a real flock: in other words, dynamic emergence is not yet present. We hence introduce the frustration. This rule frustrates the direction of motion of each boid, that is, the boid is forced to make a U-turn at the boundary and then to continue its way thus drawing the rest of its flockmates

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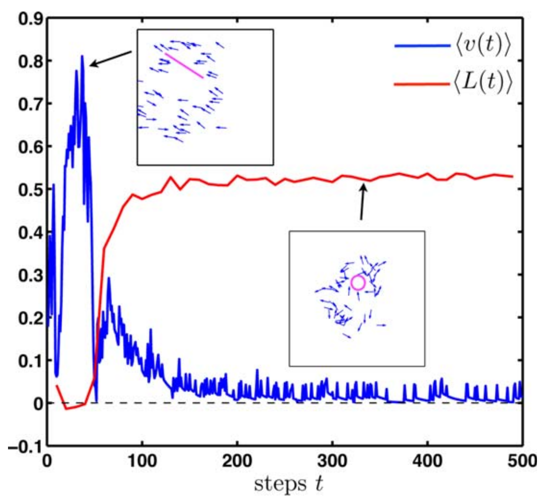
FIGURE 2



Phase transition for a flock with $N=50$ members with $v_0=0.15$, and $n=10$ topological neighbors. Snapshots for the aligned phase (illustrated by the pink straight line) and the rotational phase (illustrated by the pink circle) are also shown.

through the consensus rule. The details of the frustration rule are presented in Appendix A.

3. THE GLOBAL ORDER PARAMETERS

To characterize the flock behavior, we define two instantaneous order parameters, namely, one parameter measuring alignment and another order parameter measuring rotational motion of the flock. Alignment can also be seen as a measure of translational motion of the entire flock. We thus introduce the instantaneous alignment order parameter $\langle v\rangle$ defined as

$$\langle v\rangle=\frac{1}{Nv_0}|\sum_{i=1}^{N}\mathbf{v}_i|, \tag{3}$$

where v_0 is the norm of the velocity vector $\mathbf{v}_i$ of boid i, and N is the total number of individuals forming the flock. We can easily note that $\langle v\rangle$ tends to 0 as the flock performs a random motion, as well as when the flock performs rotational phase motion. Using this parameter alone, it is not possible to distinguish between random or rotational motion.

To quantify the two rotational phases, we define the order parameter $\langle L\rangle$ as follows

$$\langle L\rangle=\frac{1}{N}\sum_{i=1}^{N}\frac{1}{z}\sum_{k=1}^{z}\frac{\mathbf{v}_i(k)\wedge\mathbf{v}_i(k+1)}{v_0^2}, \tag{4}$$

once again, $N, \mathbf{v}_i, v_0$ represent, respectively, the total number of individuals in the flock, the velocity vector of boid i, and its norm. Here, z is an integer representing a given number of time steps such that it is possible to capture an entire rotation of an individual. Note that the wedge product is defined between two consecutive vectors, and then averaged over the entire flock and over the z steps. Thus, if the velocity vector does not change direction, the parameter $\langle L\rangle$ will vanish, a scenario occurring when the flock is aligned. However, if the individual boids in the flock rotate, then, $\langle L\rangle$ will be different than 0. Moreover, it can be shown that on average, $\langle L\rangle>0$ and $\langle L\rangle<0$ for a clockwise and counterclockwise rotation, respectively.

3.1. Phase Transition

Under particular circumstances depending on the size of the flock, the size of each group, and the initial random conditions, we have encountered a phase transition in the frustration environment. For a small flock, we found an instance in which the boids rapidly evolved to the aligned phase and with increasing fluctuations changed to one of the rotational phases, where they stayed for the remaining time of the simulation. Figure 2 shows the behavior of a flock with $N=50$, each of them interacting with $n=10$ topological neighbors, and with a flying speed $v_0=0.15$ for all the members. This scenario presents a phase transition. Its evolution starts with a random location and a random velocity direction of the individuals on a 2D square space of sides $l=10$. As they interact, the simulation shows a very pronounced alignment corresponding to an aligned metastable phase. After a few bounces off the boundary, all the members of the flock start to rotate indefinitely: thus, the flock has entered into a stable rotational phase. This behavior is completely captured by our two order parameters. Figure 2 shows that $\langle v\rangle$ is close to 1, that is, the flock is aligned, while at the same time $\langle L\rangle$ is almost 0, indicating no rotation in the flock. Then, $\langle v\rangle$ tends to 0 while $\langle L\rangle$ quickly reaches a constant value. This implies that the flock is no longer aligned but it is instead performing a stable rotation, as $\langle L\rangle$ remains constant from there on. This phase transition occurred without the addition of any external noise to the system. It only occurred as a consequence of an interplay between frustration and consensus.

4. DYNAMIC EMERGENCE

Our next numerical experiment consists of a flock with $N=500$ individuals each of them interacting with $n=10$ topological neighbors but with a flying speed $v_0=0.2$ for all members. The evolution of the flock starts with a random location and a random velocity direction for each member on a 2D square space of sides $l=10$. After $t>150$ steps, remarkable nontrivial structures emerged, and displayed in Figure 3. To quantify the emergent phenomenon, we track the evolution of the flock with the usual

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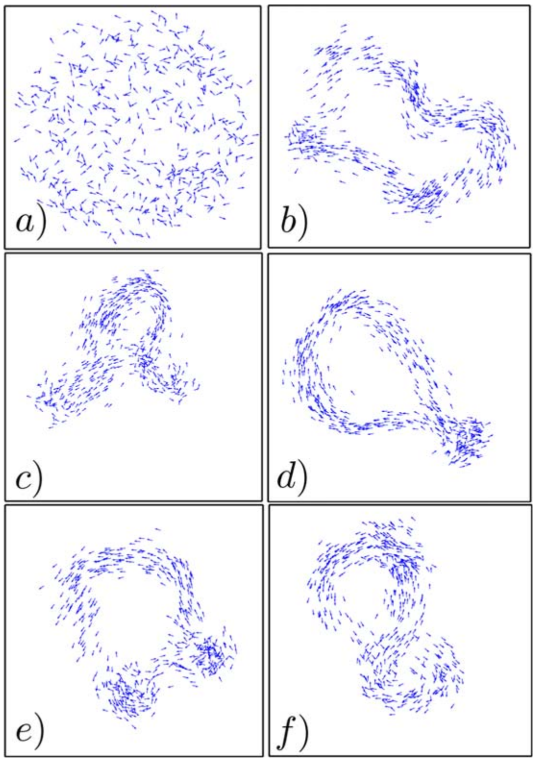
FIGURE 3



Origin of emergence and its evolution in a flock with $N=500$ members with v_0=0.2, $l=10$, and $n=10$ topological neighbors: (a), a nearly random distribution but a slight cohesion has appeared in the flock for $t=40$ steps; (b), snapshot of the flock at $t=620$ steps: emergence has started; (c), flock at $t=820$ steps; (d), flock at $t=1090$ steps; (d), flock at $t=1821$ steps; (e), flock at $t=2100$ steps; and (f), flock at $t=2300$ steps.

order parameters $\langle v \rangle$ and $\langle L \rangle$. Given that the frustrating element is present, both parameters are practically 0 as the flock evolves; hence, no further information can be gained. To overcome this, we divide the flock into N/n groups (50 in the present case), each group with n topological neighbors, and track these groups separately. Concomitantly, we introduce the jth group order parameters $\langle v_j \rangle$ and $\langle L_j \rangle$ defined, respectively, as

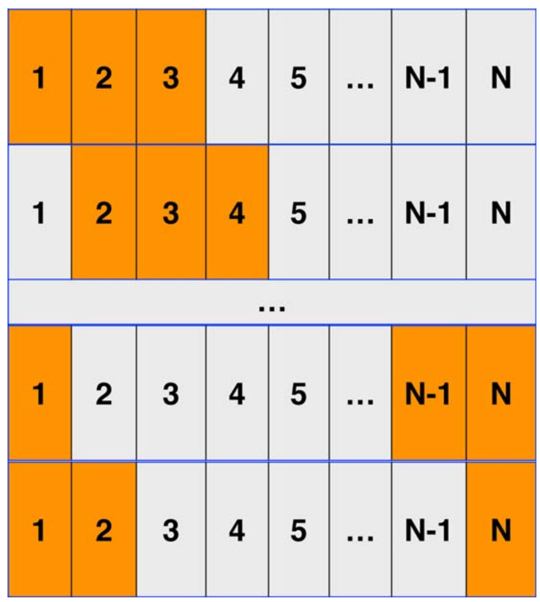
$$\langle v_j \rangle = \frac{1}{n v_0} \Big| \sum_{i=1}^{n} \mathbf{v}_i \Big|, \tag{5}$$

quantifying the partial alignment corresponding to the jth group only, and the rotation order parameter

$$\langle L_j \rangle = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{z} \sum_{k=1}^{z} \frac{\mathbf{v}_i(k) \wedge \mathbf{v}_i(k+1)}{v_0^2}, \tag{6}$$

indicating whether the jth topological group is performing rotational motion, either in a clockwise $\langle L_j \rangle > 0$, or counterclockwise $\langle L_j \rangle < 0$ direction. To describe how each jth topological group is built, we show in Figure 4, a diagram where rows contain N boids. Each row corresponds to different examples a topological group is built. For example, if boid 1 is assigned to follow two flockmates, then it will follow boids 2 and 3, which all together will make a topological group with three members. The alignment parameter $\langle v_j \rangle$ tends to 0 if the jth group has a random velocity direction as well as if the group performs rotational motion. In contrast, $\langle L_j \rangle$ tends to 1 if the jth group is rotating, but it tends to 0 if the group is aligned. Figure 5(a) shows the dynamics of the rotational order parameter $\langle L \rangle$ for the complete flock and for two specific groups ($\langle L_1 \rangle$ and $\langle L_2 \rangle$), each of them with 10 members. Emergence in the flock clearly occurs for $t > 150$ steps and it is observed to continue for $t > 2500$. Two remarkable facts can be noticed from this figure. First, rotation in both clockwise and counterclockwise direction among the jth topological neighbors is present, in spite of the fact that, on average, the global $\langle L \rangle$ is 0. Second, the rotational direction among these groups is not synchronized, that is, while the group $j=1$ rotates clockwise, the group $j=2$ might travel in the counterclockwise direction. Emergence is occurring while three dynamical phases are simultaneously

FIGURE 4



Method for building the jth topological group. Each row corresponds to different examples a topological group is built. For example, if boid 1 is assigned to follow two flockmates, then it will follow boids 2 and 3, which all together will make a topological group with three members. Boids 2, 3..., N follow the same rule as shown in the figure.

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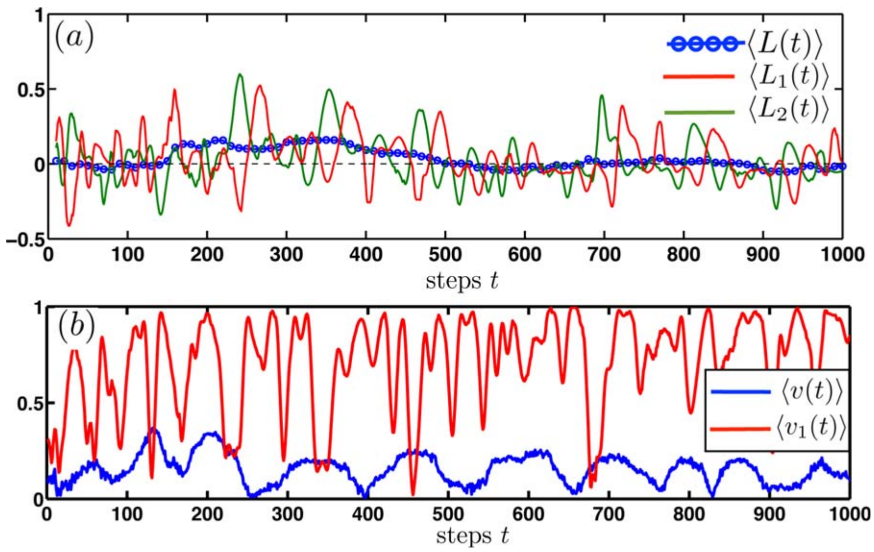
FIGURE 5

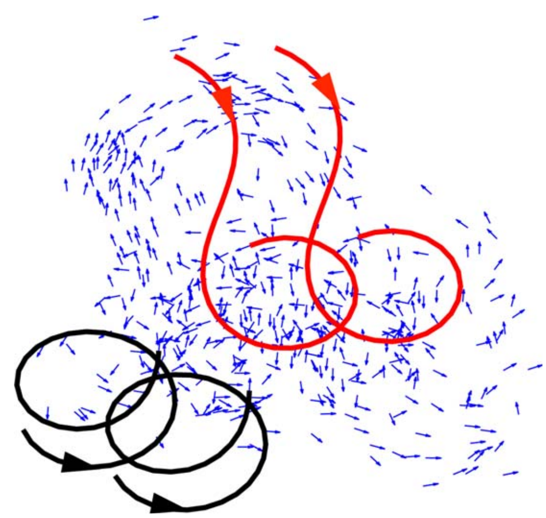


Rotational and alignment order parameters for an entire flock and two topological groups with $N=500$, $v_0=0.2$, $l=10$, and $n=10$ topological neighbors: (a), evolution of the rotational order parameters $\langle L\rangle$ for the entire flock, and $\langle L_1\rangle$ and $\langle L_2\rangle$ for two uncorrelated topological groups; (b), evolution of the alignment order parameters $\langle v\rangle$ for the entire flock, and for the same two topological groups $\langle v_1\rangle$ and $\langle v_2\rangle$.

present in the flock, namely, the aligned, clockwise, and counterclockwise rotational phases. Due to the interaction among the flockmates, these three phases are entangled, thus generating amazing time varying patterns interpreted as emergence. Figure 5(b) shows the global alignment order parameter $\langle v\rangle$ for the entire flock and for one of the topological groups $\langle v_1\rangle$. It can be observed that $\langle v\rangle$ is oscillating. Figure 6 represents the path marked boids follow during a small interval of the simulation: it clearly expands the three phases. Combining the information from $\langle L_1\rangle$ provided by Figure 5(a), and the information from $\langle v_1\rangle$ suggests that the group $j=1$, rotates clockwise and counterclockwise in a coherent internal fashion. It is possible to verify this behavior during the simulations. They indeed showed the topological group performing a combination of the three types of motion on single boid paths (see Fig. 6). These intermingled phases among the different flockmate groups appear to our eyes as cohesive, complicated, and amazing forms the flock performs as a whole.

The onset of time-dependent patterns in our system (or dynamic emergence) is what makes our research different from previous studies. We have seen that particles aligning based on metric [6,22,24] and topological [18–21] interactions only give rise to stable structures like vortices or to time-dependent global alternating patterns (oscillatory and translational phases) [20]. Even with a different interaction rule like in Touma et al. [25] where particles are coupled by an exponential potential, their system is able to self-organize and generate once again stable structures like rings, droplets, and polarized droplets. A similar study was performed by Levine et al. [26], where using a short range repulsion and a long range attractive force, found for a collection of interacting particles only stable structures. Our “flock” is very flexible and very asymmetric

FIGURE 6



Segments of paths followed by four randomly chosen boids (each pair from the same group) during 40 steps displaying the different phases.

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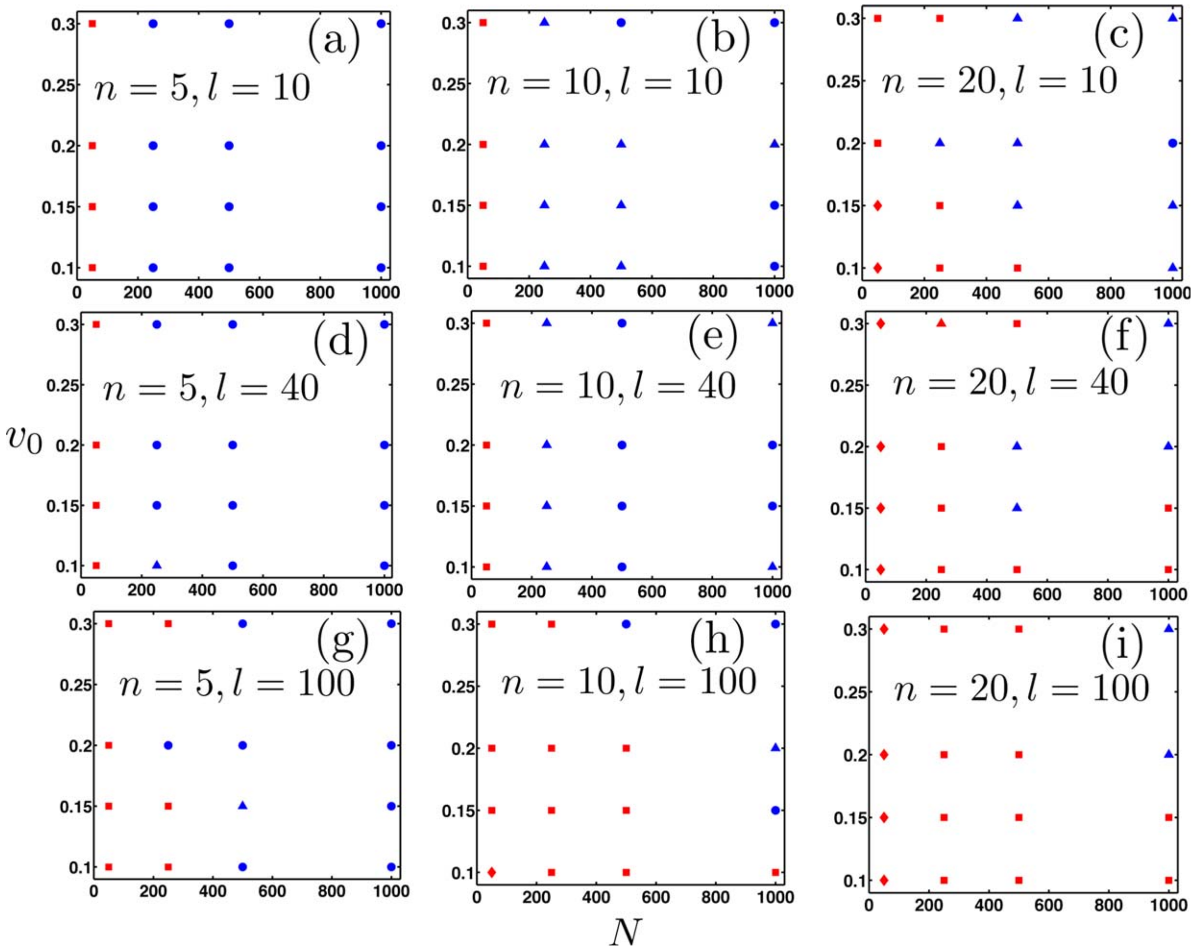
FIGURE 7



Parameter space for a flock during an interval of 10,000 steps. Red rhombi represent that the flock reached a global aligned phase without passing through dynamical emergence. Red squares represent that the flock reached a global rotational phase without passing through dynamical emergence. Blue triangles represent that the flock reached a global rotational phase after dynamical emergence was present (metastable state). Blue circles represent that during all the simulation time, dynamical emergence appeared in the flock.

in contrast to the rigid and symmetric patterns (rings, droplets, vortex) reported in previous works.

5. PARAMETER SPACE

Once we have elucidated a way of defining dynamic emergence, we turn into an analysis of the flock parameter space to see under what conditions dynamical emergence is favorable to occur or to be suppressed. Our parameter space is conformed by N, the number of individuals in the flock, n, the number of boids to align with, v_0, the speed of each boid, and l, the size of the computational space. Thus, we sampled this space for the following sets: $N=\{50, 250, 500, 1000\}$, $n=\{5, 10, 20\}$, $v_0= \{0.1, 0.15, 0.2, 0.3\}$, and $l=\{10, 40, 100\}$. The results are shown in Figure 7 where each point corresponds to the evolution of the flock after 10,000 steps. Red rhombi represent that the flock reached a global aligned phase without passing through dynamical emergence. Red squares represent that the flock reached a global rotational phase without passing through dynamical emergence. Blue triangles represent that the flock reached a metastable state, that is, the flock self-organized, then it developed dynamic emergence for some time, and finally, the flock ended up in one of the rotational phases (see next section for further detail). Blue circles represent the most interesting case, where dynamical emergence appeared in the flock and never reached a global phase. Several observations can be made from Figure 7. For example, for all the three different simulation spaces, it is clear that a global aligned phase appears for large n combined with a low

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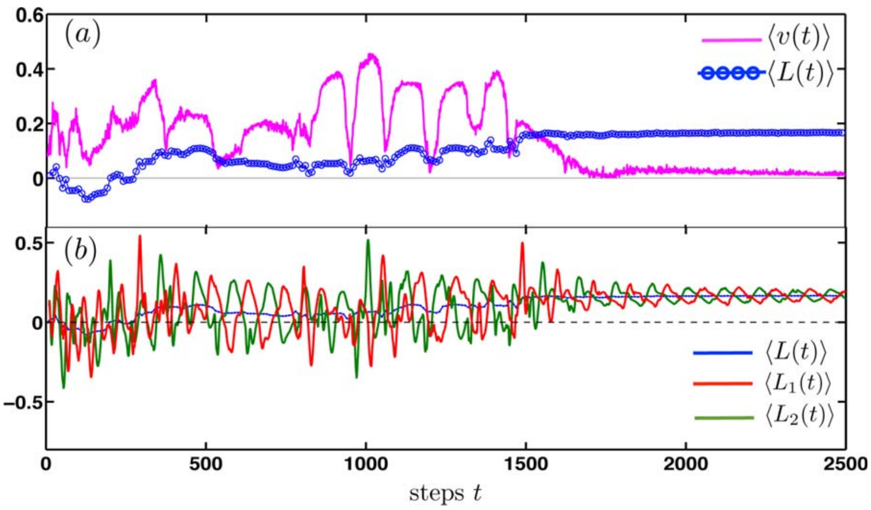
FIGURE 8



Rotational and alignment order parameters for a flock with with $N=500$ members with v_0=0.15, $l=10$, and $n=10$ topological neighbors: (a), evolution of the rotational $\langle L\rangle$ and alignment $\langle v\rangle$ order parameters for the entire flock. After $t>1500$ $\langle L\rangle$ tends to a constant value while $\langle v\rangle$ tends to 0; (b), evolution of the rotational order parameters $\langle L\rangle$ for the entire flock, and for two topological groups $\langle L_1\rangle$ and $\langle L_2\rangle$. For $t>1500$, $\langle L_1\rangle$ and $\langle L_2\rangle$ are positive.

number of boids, which is reasonable, since as the number of topological neighbors increase, the information regarding the direction of each boid is favored, thus reaching a global aligned phase. From the latter observation, one may say that a rotational phase seems to be suppressed when the flock has few individuals combined with a large number of topologic neighbors. See Figure 7(c,f,i). Figure 7 also indicates that for the velocities studied ($v_0=\{0.1, 0.15, 0.2, 0.3\}$), a global rotational phase is more likely to occur for a combination of three ingredientes: a high number of topological neighbors, high number of boids, and a large simulation space. For other conditions, the flock may reach a global aligned phase or to generate dynamic emergence.

Another important observation is that an optimal number of topological neighbors for the generation of dynamic emergence seems to exist: independently of the size of the simulation space. This value seems to be around $n=5$, which coincides with the reported number of individuals that a real bird seems to interact with [16]. The effect of the frustration rule on the appearance of dynamical emergence can be noticed in Figure 7 by looking at it in the following order: Figure 7(a,d,g), Figure 7(b,e,h), and Figure 7(c,f,i). One clearly sees that for speeds $v_0=\{0.1, 0.15, 0.2, 0.3\}$, frustration is an important factor as the simulation space increases, in other words, as the frustration rule is less an less frequent, dynamical emergence is lost. Thus, we see that the frustration rule or the presence of boundaries for the latter speeds, promotes the origin of emergence. If the flock at these speeds is not frustrated, dynamic emergence is less and less frequent.

To contrast the effect of our frustration rule, we also performed simulations with our consensus rule plus an attraction force to the center of mass of the flock. The details can be seen in Appendix B. Briefly, attraction to the center of mass only generated stable structures similar to previous works [25,26], and not time-dependent patterns as the ones observed by a real flock.

6. ILLUSTRATING METASTABILITY

We call a metastable state when a flock showed dynamic emergence for some time, only to end up in one of the global phases. However, due to the cohesiveness acquired during the metastable part, the final dynamic pattern resembles more an emergent behavior than the pure consensus phase, in spite of the phase locking among the whole flock. To illustrate metastability, we performed a simulation experiment which consists of a flock with $N=500$ boids interacting with $n=10$ flockmates, and with a flying speed v_0=0.15 for all members. The incipient emergence stops around $t>1500$. This case is very illustrative as it shows that the absence of a dynamical phase (in this case the aligned phase) seems to prevent full emergence.

Figure 8(a) shows the behavior of the corresponding order parameters $\langle v\rangle$ and $\langle L\rangle$ for the entire flock. The

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evolution of the local rotational order parameters $\langle L_1 \rangle$ and $\langle L_2 \rangle$ for two groups in the flock, each one with $n=10$ members, is plotted in Figure 8(b). For $t>1500$, the order parameter $\langle v \rangle$ tends to 0, which means the entire flock does no longer have an aligned phase which also implies the flock is not translating any more. Additionally, $\langle L \rangle$ is seen to acquire a positive constant value, implying that the flock has achieved a stable clockwise rotational phase. The rotational order parameters $\langle L_1 \rangle$ and $\langle L_2 \rangle$ also indicate that for $t>1500$, the topological groups are only rotating in a clockwise direction.

7. CONCLUDING REMARKS

The main conclusion from this article is that to understand emergent behavior, we have to go beyond the idea of global phases affecting the whole system. For the present model, this is achieved by dividing the system into meaningful subsystems and looking for (a) dynamic phase transitions within each subsystem and (b) correlation among the different phases in the different subsystems. If this correlation occurs in a smooth manner from group to group, the resulting dynamics displays coherent, cohesive, versatile, and harmonic features meriting the title of Emergence. Although we are still far from characterizing emergence in its full grandeur, we regard this vision as a first step, a stepping stone in the right direction.

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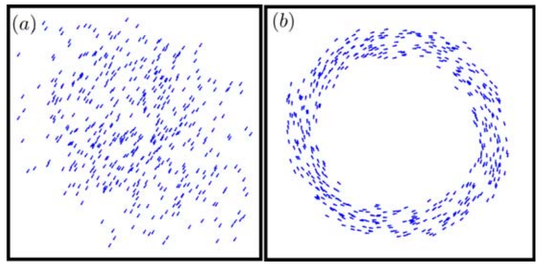
APPENDIX A

Frustration in Our Model

By only applying the consensus rule [Eqs. (1) and (2)] to a flock with N members, this flock is seen to organize and quickly reach a steady state. However, it does not have the versatility of motion seen in a real flock, in other words, dynamical Emergence is not present. Therefore, it is necessary to introduce an additional rule which we identify as a frustration term. This rule frustrates the direction of motion of the boids, that is, a boid chosen randomly is forced to make a U-turn (in one time step) and then to continue its way. To achieve this randomness, we choose a smooth function $F(R)$ (where R is a normalized distance to the center) with origin at the center $\mathbf{c}=[l/2, l/2]$ of the simulation space. Then, we measure the distance $|\mathbf{R}_i|$ of boid i with respect to $\mathbf{c}$, and substitute $R=2|\mathbf{R}_i|/l$ into our chosen function namely, $F(R)=R^6$ in this particular case. Finally, a random number λ is generated for boid i. If this number is less than or equal to $F(R)$, then boid i makes a U-turn. Note that in this way, any boid reaching $R=1$ will always be frustrated as $\lambda \in [0,1]$ and the boids will be "corralled" in a circle of radius $l/2$ about the center of the simulation space. Although this circle represents a boundary, we might consider it as a "soft" boundary given that boids are usually reflected before reaching the edge of the circle.

FIGURE B1



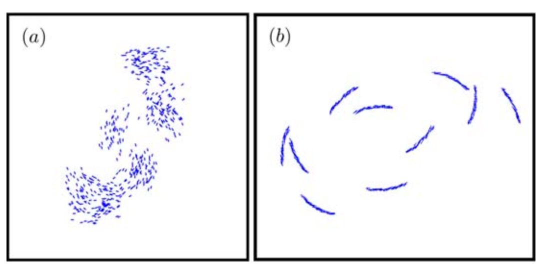
Aligned and rotational global phases for a flock with $N=500$ with $v_0=0.05$, $n=50$ and $k=0.01$. (a) Aligned phase for the flock at $t=200$. This pattern tends to reduce in size as time evolves due to the attraction to the center of mass; (b) Rotational phase at $t=300$. This ring tends to reduce its width as time evolves.

APPENDIX B

Attraction to the Center of Mass

As an alternative rule, we also studied another sort of frustration namely, attraction to the center of mass. We introduced a spring-like force toward the center of mass of the flock and we analyzed two different cases. The first case

FIGURE B2



Groups formed in a flock with $N=500$, $v_0=0.05$, $n=50$, and $k=0.01$. (a) Formation of rotating groups in the flock at $t=250$; (b) The same flock at $t=500$. Notice that the width of the groups has been reduced due to the attraction of each group to its own center of mass.

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consists in incorporating our consensus rule given by Eqs. (1) and (2) plus an attraction to the global center of mass of the flock. In this case, we observe that the flock develops aligned and rotational phases depending on the initial random positions. These phases are shown in Figure B1. The parameters used were $N=500$ with $v_0=0.05$, $n=50$, and $k=0.01$ representing Hooke's constant. Figure B1(a) shows an aligned phase where, due to the attraction to its center, the flock tends to reduce its size as time progresses. The stable vortex formed in Figure B1(b) with the new rule, has been also observed by others, and even with a different interaction rule [25,26]. We thus conclude that imposing an attraction to the center of mass does not provide the versatility that our computational flock develops when it is subject to the original frustration rule. In the present case, the flock is still very rigid and symmetric hence it is not able to generate dynamic emergence. A second attempt to generate dynamic emergence in the flock consisted in imposing our consensus rule to a flock with N members subdivided into N/n groups. This time, the attraction rule was directed to the center of mass of each group, inspired by Reynold's flock centering [7]. We observed that the flock developed groups in a similar way as the patterns obtained by Dossetti et al. [20] and Touma et al. [25]. Once again the generated patterns were very rigid and dynamic emergence was not attained. Figure B2 shows the groups that appeared in the simulation at $t=250$ [Fig. B2(a)] and $t=500$ [Fig. B2(b)] with $N=500$, $v_0=0.05$, $n=50$, and $k=0.01$. This figure indicates that as the time evolves, the groups formed reduce their size. Note that these groups rotate tangentially around a circular path which, due to the consensus rule is modified as time progresses. The formed groups move again in a rigid way that prevents any sort of dynamic emergence.

REFERENCES

1. Prokopenko, M.; Boschetti, F.; Ryan, A.J. An information-theoretic primer on complexity, self-organization, and emergence. Complexity 2009, 15, 11–28.
2. Gershenson, C.; Fernandez, N. Complexity and information: Measuring emergence, self-organization, and homeostasis at multiple scales. Complexity 2012, 18, 29–44.
3. Pernu, T.K.; Annila, A. Natural emergence. Complexity 2012, 17, 44–47.
4. Halley, J.D.; Winkler, D.A. Classification of emergence and its relation to self-organization. Complexity 2008, 13, 10–15.
5. Berrondo, M.; Krueger, W. The role of consensus and frustration in the emergence of flocking behavior. UASAL J 2008, 85, 219–232.
6. Vicsek, T.; Czirok, A.; Ben-Jacob, E.; Cohen, I.; Shochet, O. Novel type of phase transition in a system of self-driven particles. Phys Rev Lett 1995, 75, 1226–1229.
7. Reynolds, C.W. Flocks, herds, and schools: A distributed behavioral model. Comput Graph 1987, 21, 25–34.
8. Parrish, J.K.; Edelstein-Keshet, L. Complexity, pattern, and evolutionary trade-offs in animal aggregation. Science 1999, 284, 99–101.
9. Couzin, I.D. Collective minds. Nature 2007, 445, 715.
10. Berdahl, A.; Torney, C.J.; Ioannou, C.C.; Faria, J.J.; Couzin, I.D. Emergent sensing of complex environments by mobile animal groups. Science 2013, 339, 574–576.
11. Mishra, S.; Tunstrom, K.; Couzin, I.D. Collective dynamics of self-propelled particles with variable speed. Phys Rev E 2012, 86, 011901.
12. Toner, J.; Tu, Y. Long-range order in a two-dimensional dynamical xy model: How birds fly together. Phys Rev Lett 1995, 75, 4326–4329.
13. Toner, J.; Tu, Y. Flocks, herds, and schools: A quantitative theory of flocking. Phys Rev E 1998, 58, 4828–4858.
14. Chate, H.; Ginelli, F.; Gregoire, G.; Raynaud, F. Collective motion of self-propelled particles interacting without cohesion. Phys Rev E 2008. 77, 046113.
15. Gregoire, G.; Chate, H. Onset of collective and cohesive motion. Phys Rev Lett 2004, 92, 025702.
16. Ballerini, M.; Cabibbo, N.; Candelier, R.; Cavagna, A.; Cisbani, E.; Giardina, I.; Lecomte, V.; Orlandi, A.; Parisi, G.; Procaccini, A.; Viale, M.; Zdravkovic, V. Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study. Proc Natl Acad Sci 2008, 105, 1232–1235.
17. Bialek, W.; Cavagna, A.; Giardina, I.; Mora, T,; Silvestrib, E.; Viale, M.; Walczak, A.M. Statistical mechanics for natural flocks of birds. Proc Natl Acad Sci 2012, 109, 1–6.
18. Camperi, M.; Cavagna, A.; Giardina, I.; Parisi, G.; Silvestri, E. Spatially balanced topological interaction grants optimal cohesion in flocking models. Interface Focus 2012, 2, 715–725.
19. Ginelli, F.; Chaté, H. Relevance of metric-free interactions in flocking phenomena. Phys Rev Lett 2010, 105, 168103.
20. Dossetti, V.; Sevilla, F.J.; Kenkre, V.M. Phase transitions induced by complex nonlinear noise in a system of self-propelled agents. Phys Rev E 2009, 79, 051115.
21. Niizato, T.; Gunji, Y.-P. Metric-topological interaction model of collective behavior. Ecol Modell 2011, 222, 3041–3049.
22. Czirok, A.; Vicsek, T. Collective behavior of interacting self-propelled particles. Phys A: Stat Mech Appl 2000, 281, 17–29.
23. May, R.M. Simple mathematical models with very complicated dynamics. Nature 1976, 261, 459–467.
24. Huepe, C.; Aldana, M. Intermittency and clustering in a system of self-driven particles. Phys Rev Lett 2004, 92, 168701.
25. Touma, J.R.; Shreim, A.; Klushin, L.I. Self-organization in two-dimensional swarms. Phys Rev E 2010, 81, 066106.
26. Levine, H.; Rappel, W.-J.; Cohen, I. Self-organization in systems of self-propelled particles. Phys Rev E 2000, 63, 017101.

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