Longest Path Problem

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Outline:

1. Recap
2. Heuristic and Metaheuristic
3. Ant colony
4. Genetic Algorithm
5. GA in longest path problem
6. Simulated Annealing
7. Particle Swarm Optimization
8. References

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- Today we will propose meta-heuristics for our problem.
- But before we proceed any further, let's just shed some light to our previous discussion.

What is a Longest Path Problem?

Optimization Version

Given a weighted graph G, find a simple path in this graph which has the maximum weight.

But Longest Path problem is NP-Complete

- From our previous discussion, we can safely state that Longest Path problem is **NP-Complete** which makes it both **NP** and **NP-Hard**.
- So, unless **P=NP**, there is no polynomial time algorithm which gives exact solution of Longest Path problem.
- But solving a **NP-hard optimization problem** like ours optimally takes a toll on running time. So we are going to relax the criterion of getting an optimal solution.
- We have tried to find an algorithm which solves the aforementioned problem approximately in polynomial time. These were algorithms that sacrificed correctness for faster running times.
- This week we will try to find a metaheuristic algorithm which solves the aforementioned problem in polynomial time. This brings us to this week's content : **Heuristic and Metaheuristic Algorithms**.

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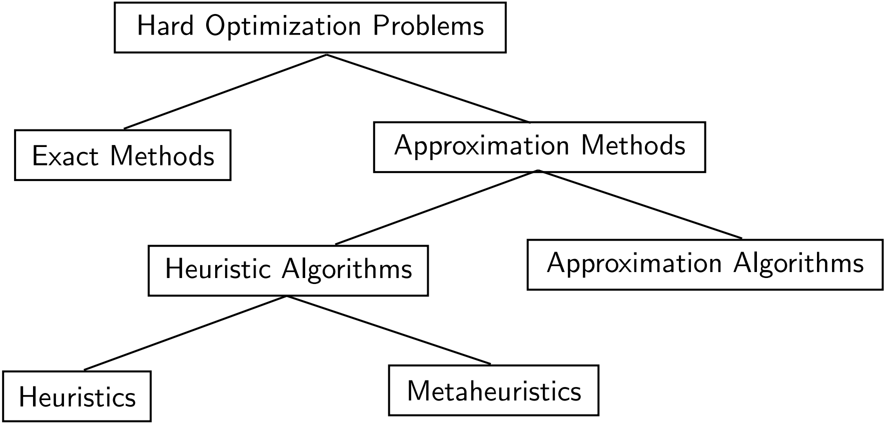
Hard Optimization Problems



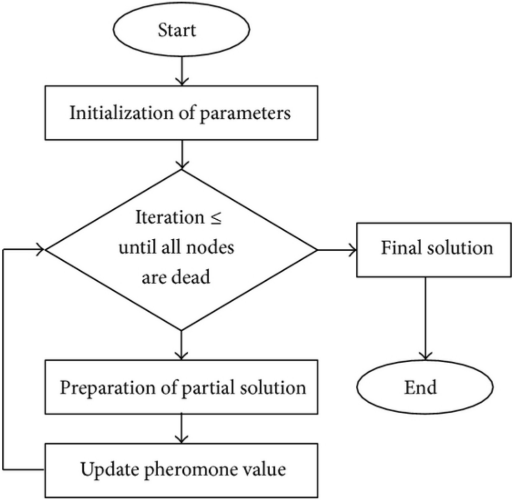
Heuristic and Metaheuristic

Heuristic	Metaheuristic
Heuristics are methods of exploration that exploit certain aspects of a problem and apply only to it.	A metaheuristic is general exploration method that applies to many problems in the same way and is often stochastic.
Heuristics are often problem-dependent.	Metaheuristics are problem-independent techniques that can be applied to a broad range of problems.
Do not guarantee to find optimal solution.	Do not guarantee to find optimal solution.

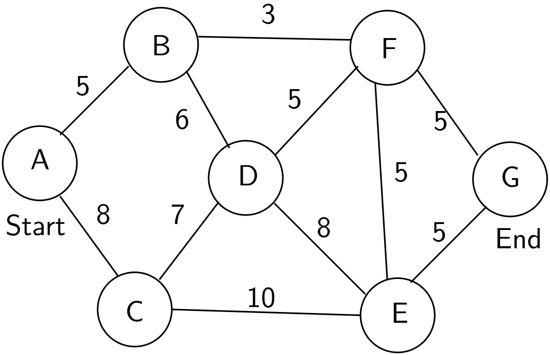
Metaheuristic Problems

- Metaheuristic algorithms
  - Trajectory Based
    - Simulated Annealing
    - Local Search
    - Random Search and Random Walk
    - Tabu Search
    - Hill Climbing
  - Population Based
    - Ant Colony Optimization
    - Particle Swarm Optimization
    - Evolutionay Algorithms
    - Bacterial Foraging
    - Artificial Bee Colony

Ant Colony Flow Chart



Simple Graph



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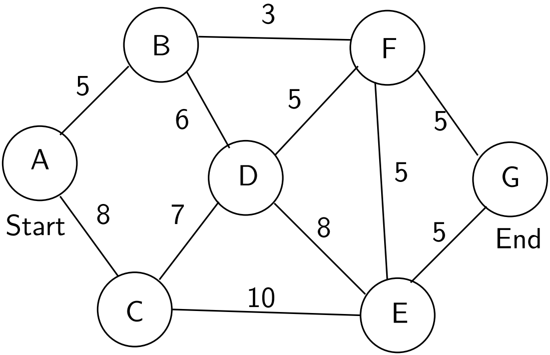
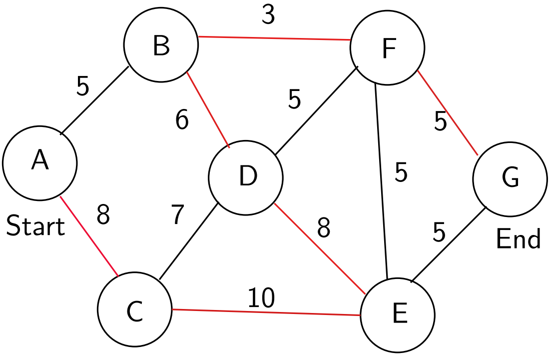


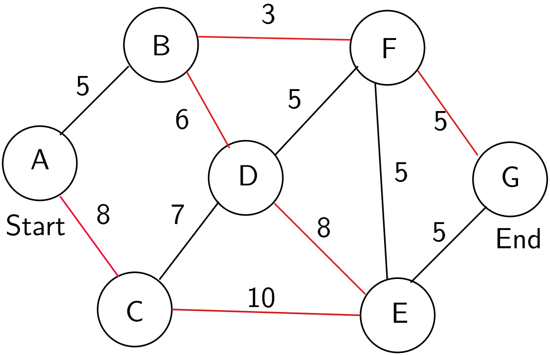
- This is a weighted undirected graph.
- We want to find the longest path between A and G.

Finding longest path



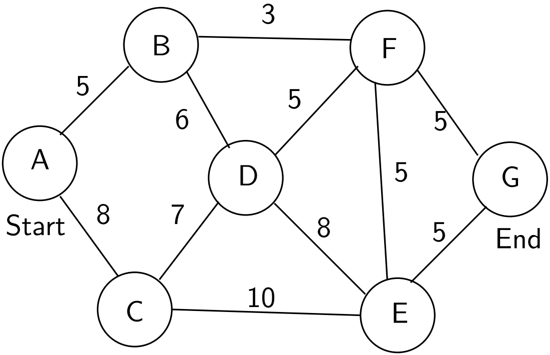
- Longest path here is A-> C-> E-> D-> B-> F-> G and path length is 40.

Finding longest path



- Longest path here is A-> C-> E-> D-> B-> F-> G and path length is 40.
- Can we exactly find this path using ant colony ?

Starting with Ant colony



- We initialize each edge with base pheromone.

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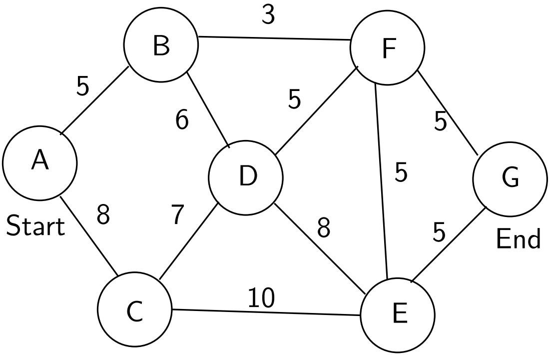
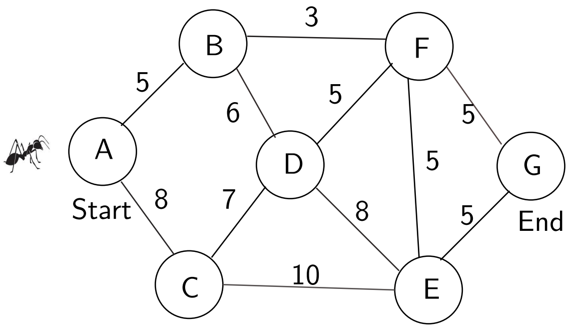


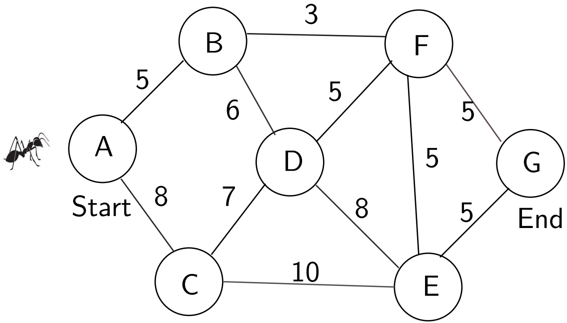
- We initialize each edge with base pheromone.
- Each edge has probability equation of format $pr = ph * \alpha + weight * \beta$ where α and β are constants given as parameters.

Starting with Ant colony



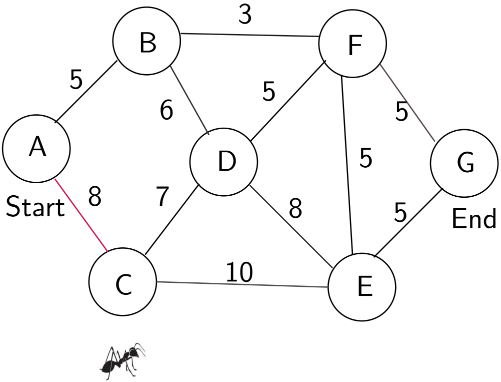
- Our very first ant at starting node.

Starting with Ant colony



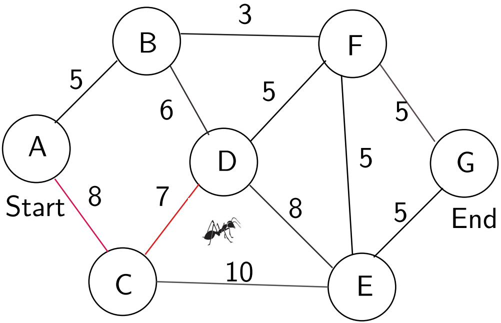
- Our very first ant at starting node.
- Which path will it go?

Starting with Ant colony



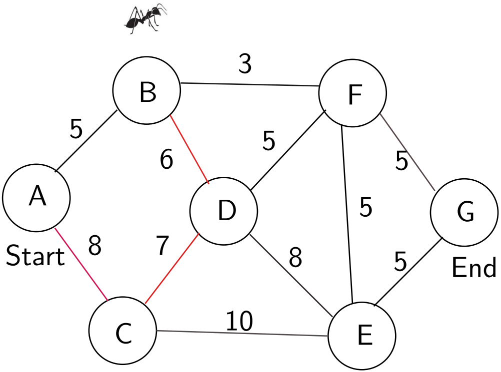
- Our Ant goes along the path with longest probability here and ends up at C.

Starting with Ant colony



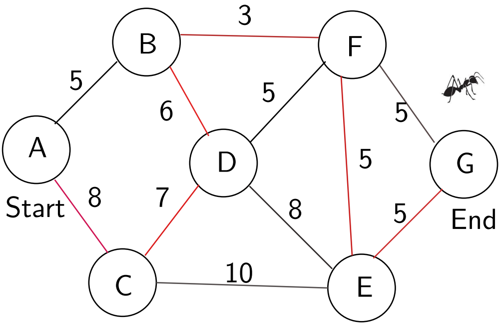
- Our Ant goes along the path CD and ends up at D.

Starting with Ant colony



- Our Ant goes along the path DB and ends up at B.

Ending with Ant colony



- All our ants are at finish point.

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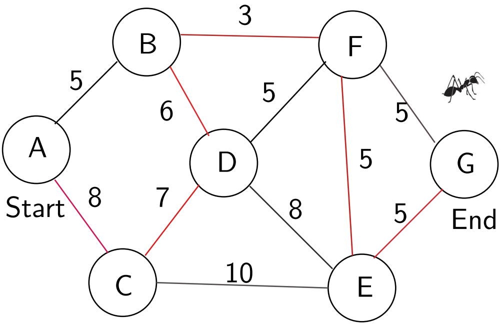

- All our ants are at finish point.
- See their paths and record the longest of them.

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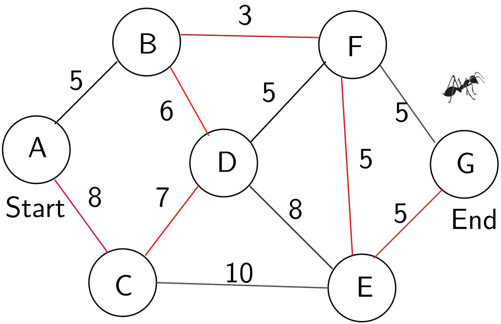
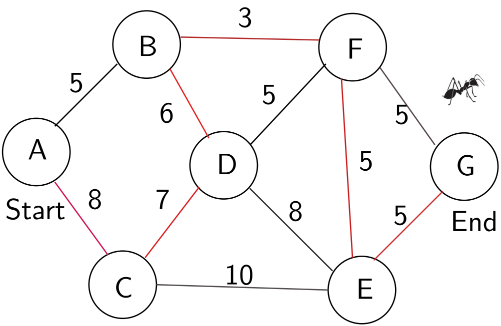


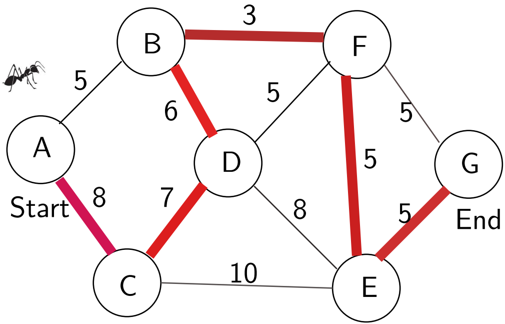
- All our ants are at finish point.
- See their paths and record the longest of them.
- Suppose the longest they have travelled is through A-> C-> D-> B-> F-> E-> G and path length is 35.

Ending with Ant colony



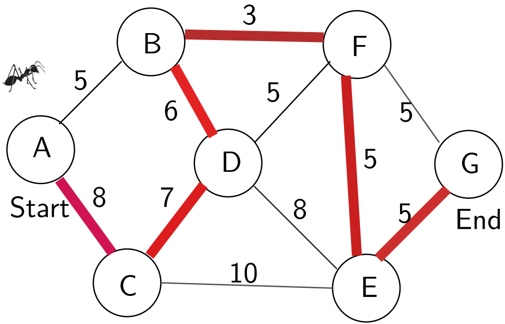
- All our ants are at finish point.
- See their paths and record the longest of them.
- Suppose the longest they have travelled is through A-> C-> D-> B-> F-> E-> G and path length is 35.
- Now lets increase pheromone along this path.

Ending with Ant colony



- With the pheromone increased along the local longest path,iteration 1 is finished.

Ending with Ant colony



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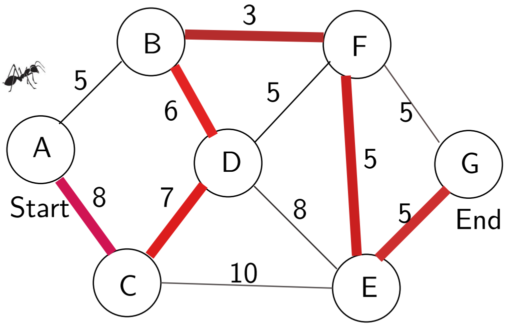

- With the pheromone increased along the local longest path,iteration 1 is finished.
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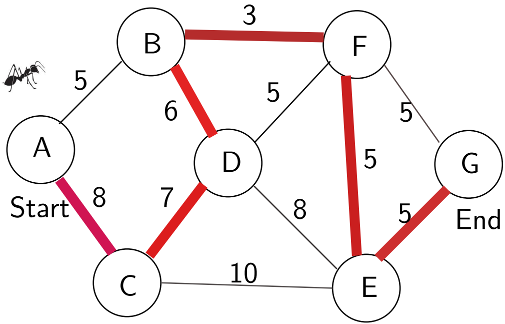

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- iteration 2 starts again at Node A.
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- After each iteration, we compare the local longest path with global and update the global path.

Final words on Ant Colony

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Final words on Ant Colony

- We will also need to evaporate pheromone after certain times.
- After our iterations have finished,we will end up with the global longest path.
- Important fact is the algorithm is polynomial as only two loops.Iteration number and ant number.Building path can be done through BFS or DFS.
- It might not be the actual longest path but chances are it is relatively close due to its design and how probability works.

Genetic Algorthim

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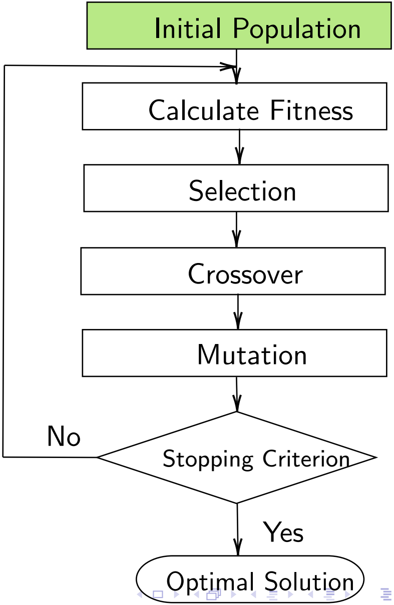
- This algorithm reflects the process of natural selection where the fittest individuals are selected for reproduction in order to produce offspring of the next generation.

Introduction to Genetic Algorithm

Initial Population: The process begins with a set of individuals which is called a Population. Each individual is a solution to the problem we want to solve.

Fitness Calculation: The fitness function determines how fit an individual is (the ability of an individual to compete with other individuals)

Selection: The idea of selection phase is to select the fittest individuals and let them pass their genes to the next generation.



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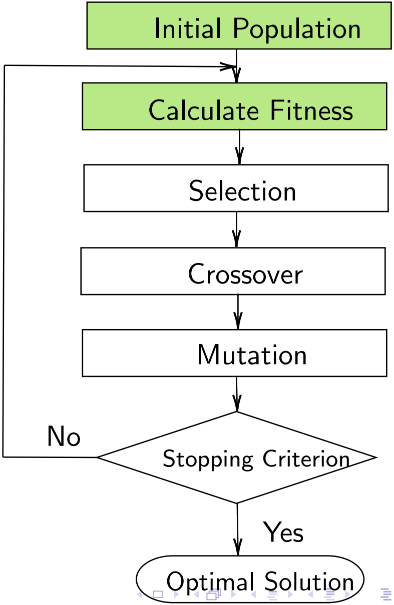

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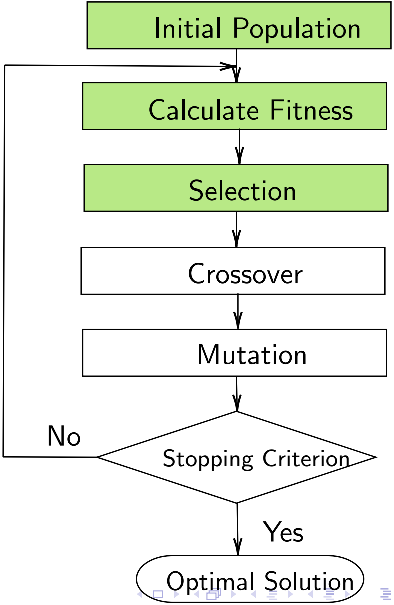

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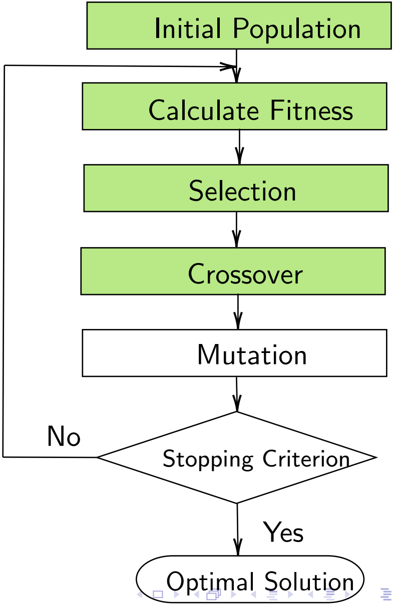
Crossover: For each pair of parents to be mated, a crossover point is chosen at random from within the genes.

Mutation: In certain new offspring formed, some of their genes can be subjected to a mutation with a low random probability.

Termination: The algorithm terminates if the population has converged (does not produce offspring which are significantly different from the previous generation).



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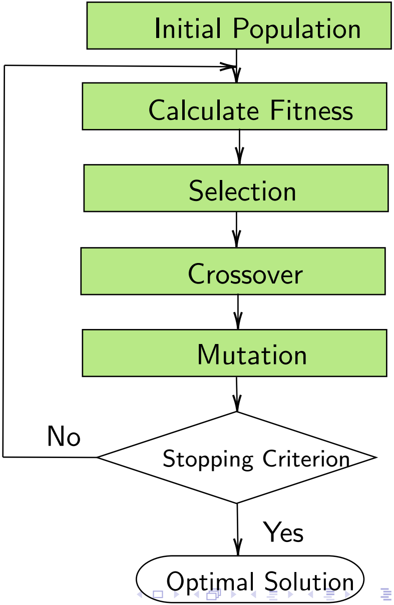

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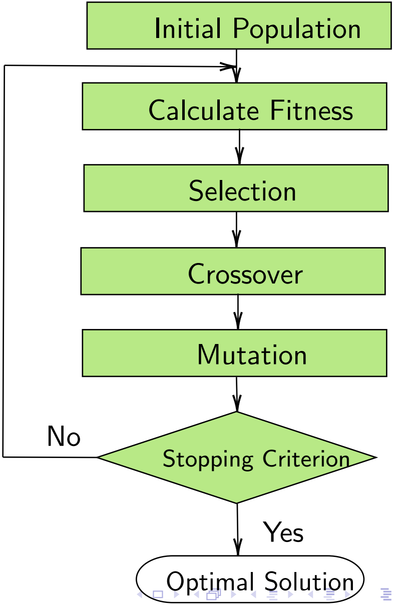

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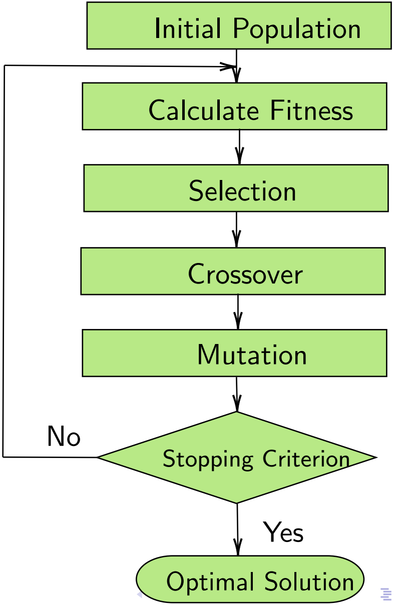

Genetic Algorithm in Longest Path Problem

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- The first three approaches (**GANP,GAIP,GABPP**) proposed are based on **crossover** mechanisms, in which two parents create a set of offspring that share their genetic material.
- The last approach (**GAMM**) is based on a **mutation** operator, in which each individual creates two offspring by perturbation of their genetic material in places specified according to the overall state of the system.

Applying GA to LPP: encoding

A question though

How will we encode path as genetic materials i.e.chromosomes?

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- Each path is represented as an ordered array of vertices with variable length.

GA: Generating initial population

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- Since GAs strongly rely on the genetic material of the initial solution population, it is important to guarantee initial solutions with good quality.
- In this case, this corresponds to long and diverse initial paths.

Generating initial population using Random method

- First select a random vertex of the graph

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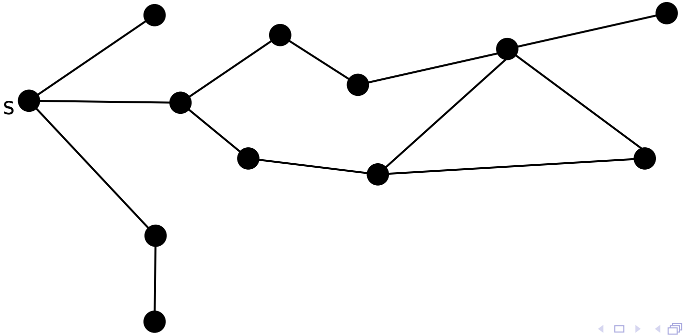
- compute paths by choosing neighbors of the current vertex at random, as long as they were not already included in the path.
- finish computing the path when there were no available neighbors left.

Generating initial population using Random method: Visualization

- Let us select a random starting vertex. We call it s.



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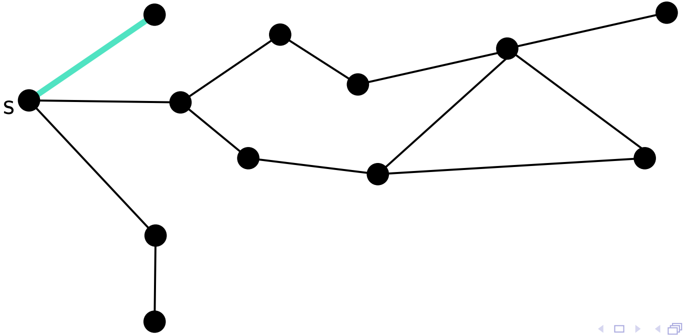
- Now if we keep adding vertices to the path, maybe we will get a path like this.



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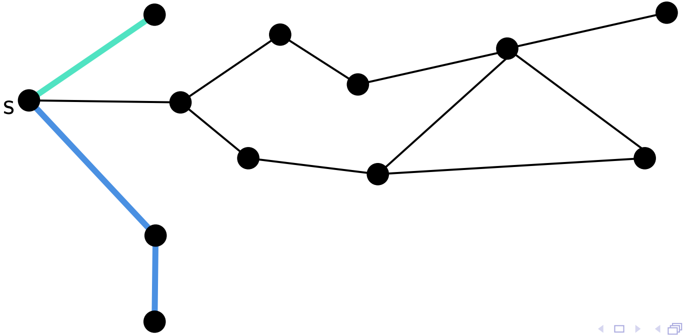

Generating initial population using Random method: Visualization

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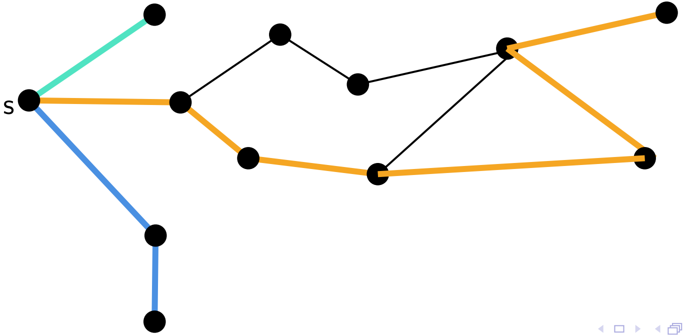
- Now if we keep adding vertices to the path, maybe we will get a path like this.
- or this (a bit longer).
- If we get really lucky, we might end up with this.



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- This proved to be an inefficient method, since most of the paths computed would be very short, mostly because degree one vertices, i.e. vertices with only one neighbor would be selected rather rapidly.
- Could we do it a bit more intelligently?

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Generating initial population using Intelligent method

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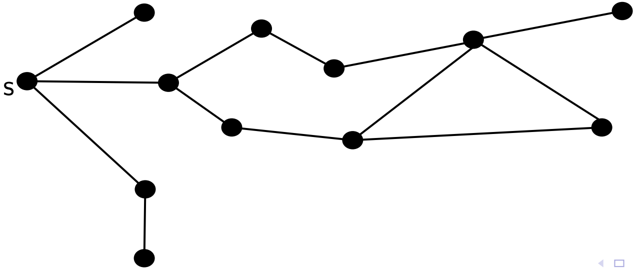
- The next vertices are selected with a probability according to their degree. For example, if two neighbors of a given vertex have degrees a and b, the first one would be selected with a probability of $a/(a+b)$ and the second one with $b/(a+b)$.
- If we have reached a vertex with unavailable neighbors, instead of stopping the method, we analyze whether the degree of the first vertex is greater than one and keep computing the path to the opposite direction until reaching a finishing point.

Generating initial population using Random method: Visualization

- Let us select a random starting vertex. We call it s.



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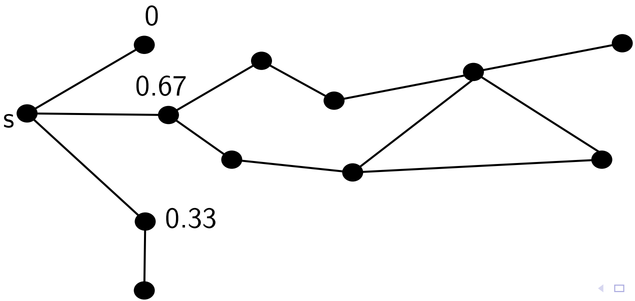

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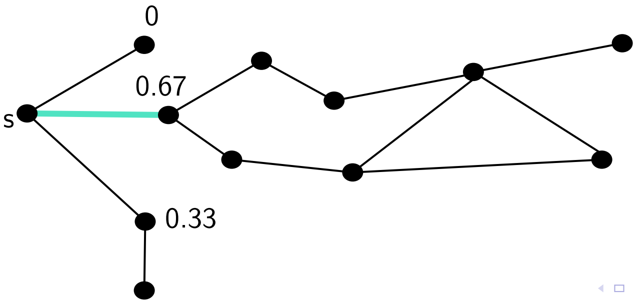

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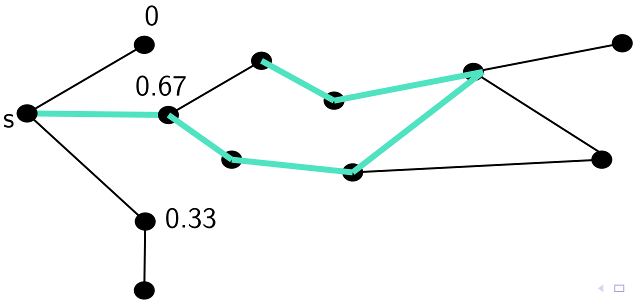

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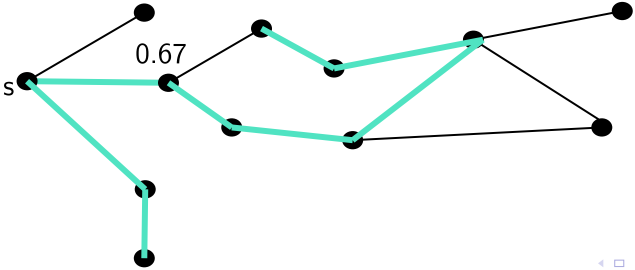
- As of now we can see there are no unavailable neighbours left so we go backwards from the starting vertex s and extend the path.



GA using non-intersecting paths (**GANP**)

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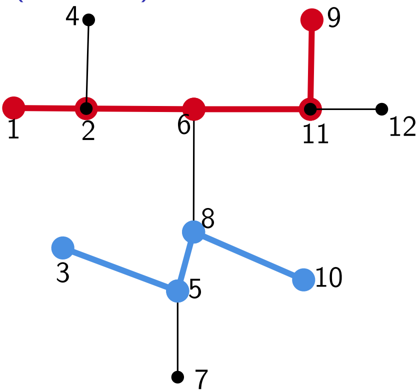

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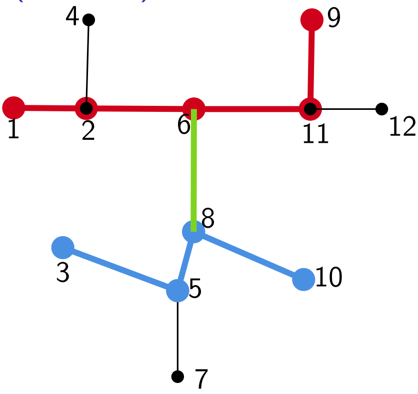

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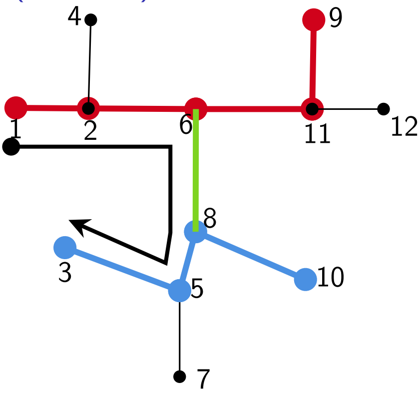

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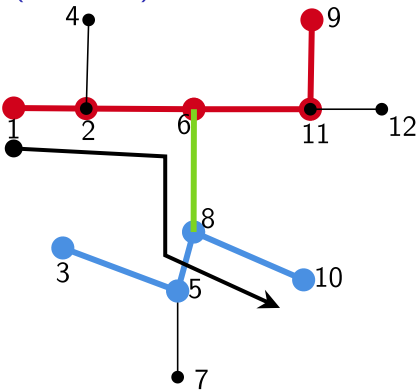

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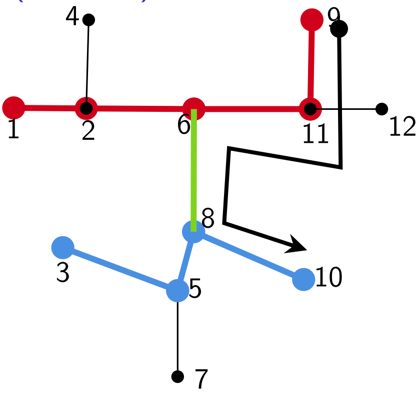

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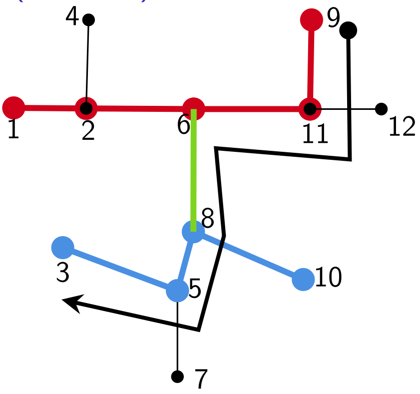

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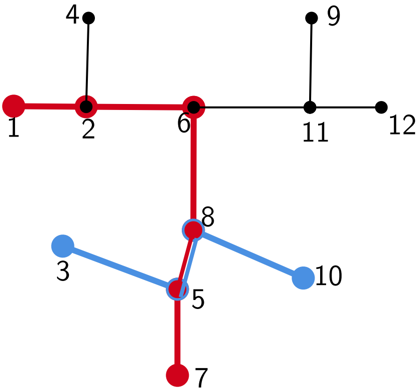

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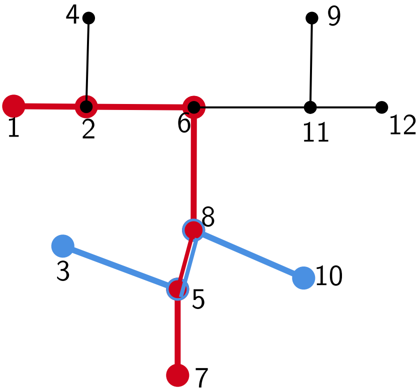

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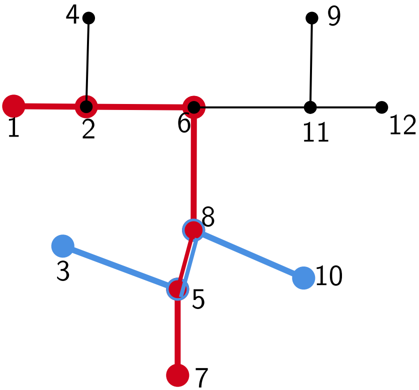

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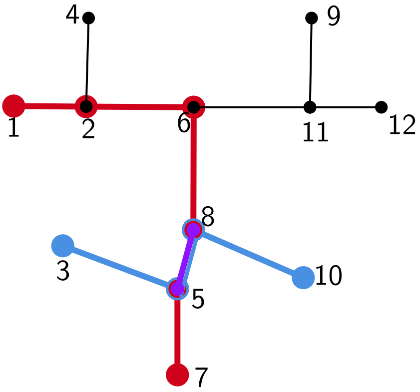
- Here junction is shown with two intersecting paths (1-2-6-8-5-7) and (3-5-8-10) with common edge (5,8).



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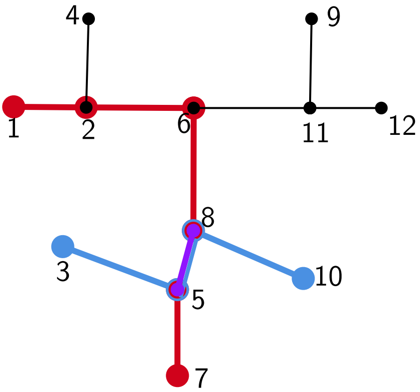

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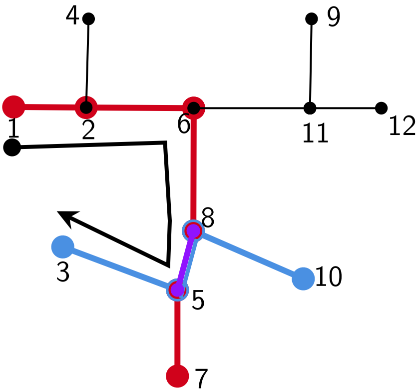

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- Two offspring, which also incorporate the junction, can be generated:
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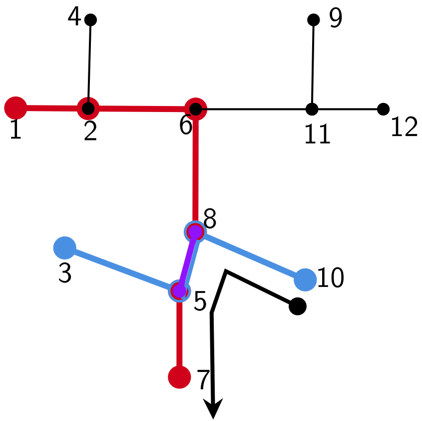

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- as the process goes on, it is expected to run faster in the beginning and slower along time, due to the detection of longer paths over time, which increases the probability of having intersecting paths and consequently more crossovers are required.

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- Rest steps are like previous algorithms.

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- A variable is initialized to measure the perturbation pressure, which is related to the rate of solution improvement obtained.
- Two offspring are generated per path consisting in two mutated solutions that result from the perturbation applied in the original path and in a flipped version of the original path,

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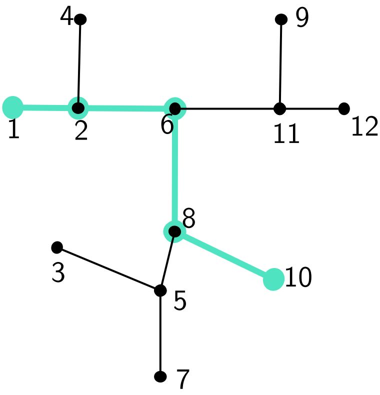
- A path and its flipped version are considered. In this case [1-2-6-8-10] and [10-8-6-2-1] are taken.



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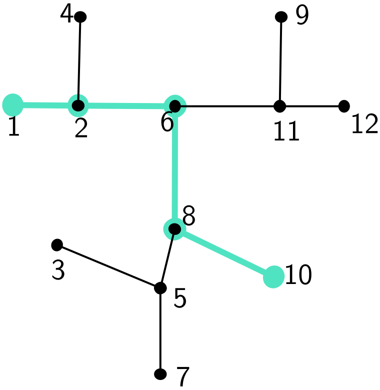

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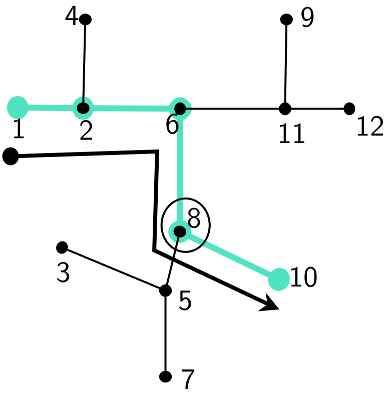

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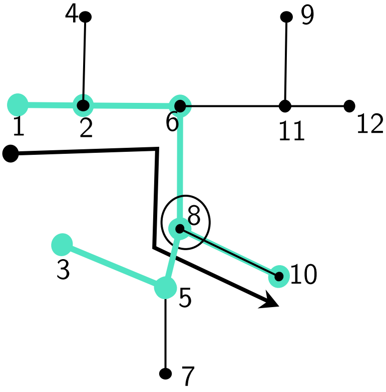
- And new path is explored from vertex 8 and we get an offspring [1-2-6-8-5-3].



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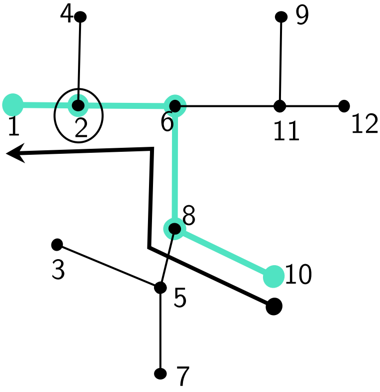

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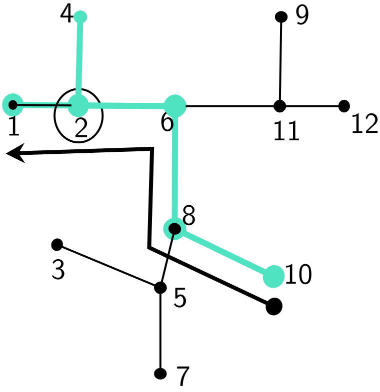
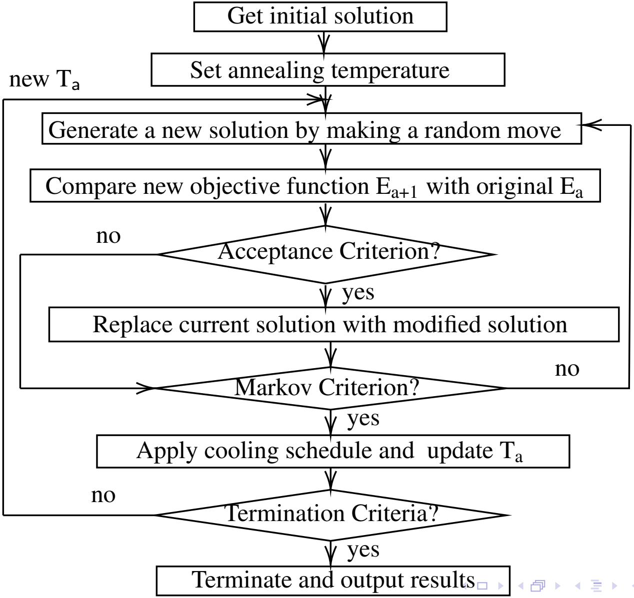
- And new path is explored from vertex 2 and we get an offspring [10-8-6-2-4].



Simulated Annealing



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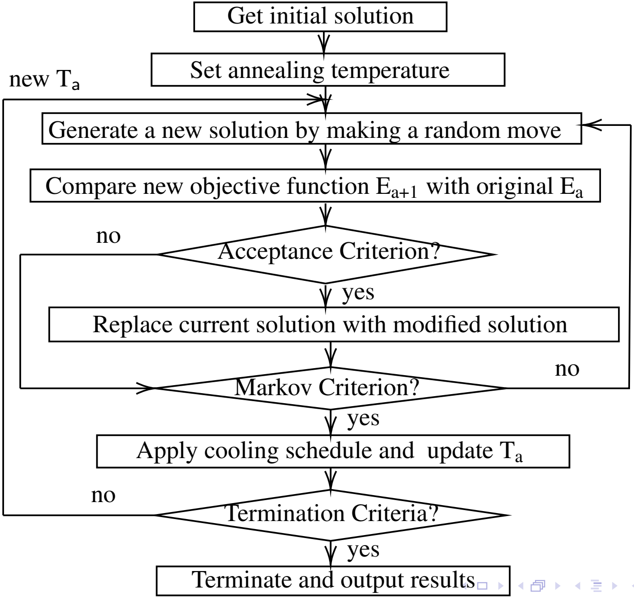

Solving LP using SA

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- We store and update the current best solution accordingly.

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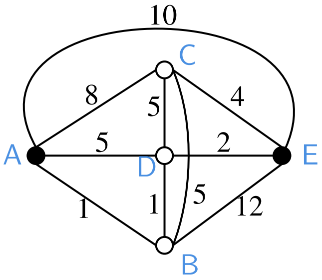

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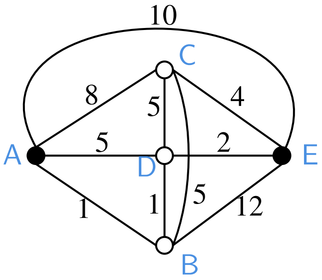

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- Let the initial temperature be 20 and the final temperature be 10.
- In each temperature, we choose 4 different paths.

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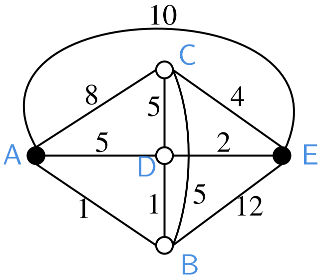

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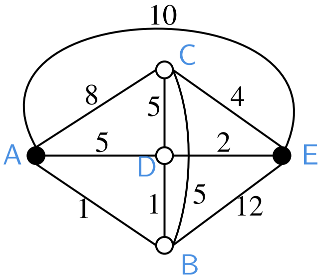

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- We decrease the temperature by 1 after each step.

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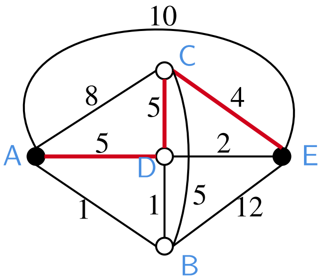
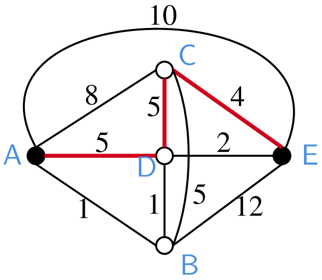


- Initially chosen random path, P $=A$-D-C-E.
- Length of P = 14

Solving LP using SA



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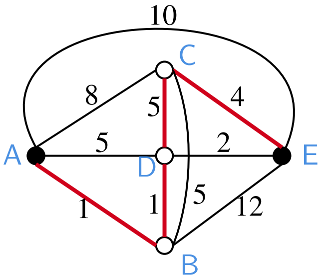

- P =A-D-C-E, length = 14
- $temp$ = 20, iteration 1:
- S =A-B-D-C-E, length = 11.

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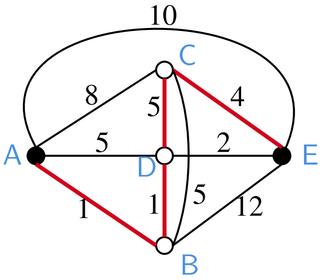

- P =A-D-C-E, length = 14
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- $d = 11 - 14 = -3$.

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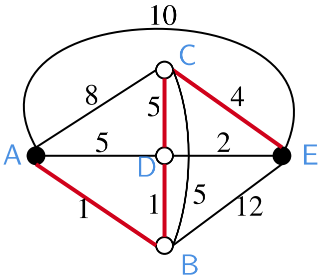
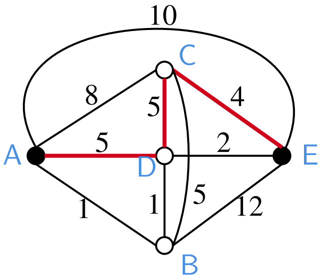


- $P =A\text{-}D\text{-}C\text{-}E$, length $= 14$
- $temp = 20$, iteration 1:
- $S =A\text{-}B\text{-}D\text{-}C\text{-}E$, length $= 11$.
- $d = 11 - 14 = -3$.
- Length of S is less than length of P.
- Random probability $= 0.90$, $e^{\frac{-3}{20}} = 0.86 < 0.90$.

Solving LP using SA



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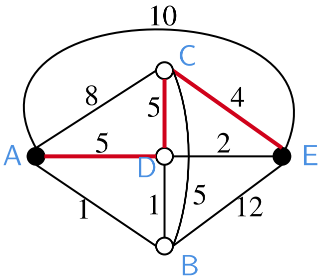
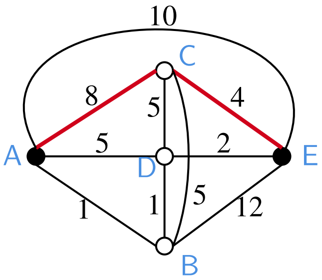
- $temp$ = 20, iteration 1:
- S =A-B-D-C-E, length = 11.
- $d = 11 - 14 = -3$.
- Length of S is less than length of P.
- Random probability = 0.90, $e^{\frac{-3}{20}} = 0.86 < 0.90$.
- Decision:P =A-D-C-E.

Solving LP using SA



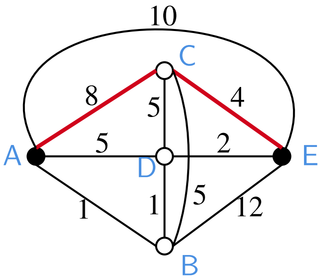
- P =A-D-C-E, length = 14
- $temp$ = 20, iteration 2:

Solving LP using SA



- P =A-D-C-E, length $= 14$
- $temp = 20$, iteration 2:
- S =A-C-E, length $= 12$.

Solving LP using SA



- P =A-D-C-E, length = 14
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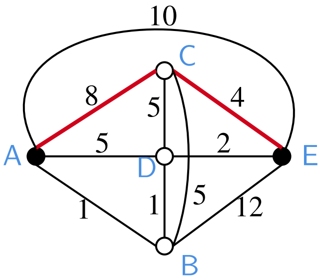
- S =A-C-E, length = 12.
- d = 12 − 14 = −2.

Solving LP using SA



- P =A-D-C-E, length = 14
- $temp = 20$, iteration 2:
- S =A-C-E, length = 12.
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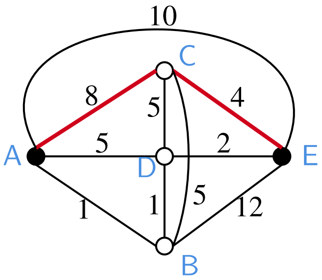
- Length of S is less than length of P.
- Random probability = 0.83, $e^{\frac{-2}{20}} = 0.90 > 0.83$.

Solving LP using SA



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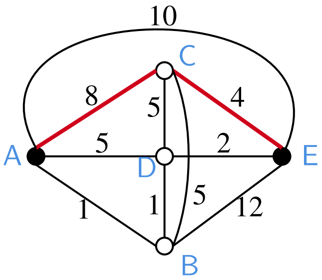

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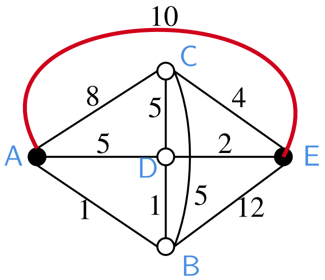
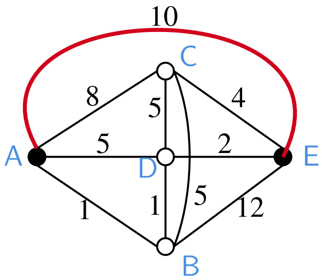
- $temp$ = 20, iteration 3:

Solving LP using SA



- P =A-C-E, length = 12
- $temp$ = 20, iteration 3:
- S =A-E, length = 10.

Solving LP using SA



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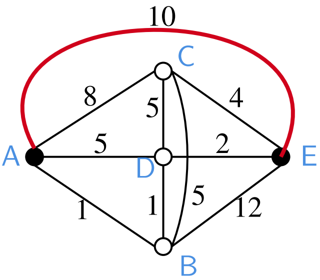
- $temp$ = 20, iteration 3:
- S =A-E, length = 10.
- d = 10 − 12 = −2.

Solving LP using SA



- $P = A\text{-}C\text{-}E$, length $= 12$
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- $S = A\text{-}E$, length $= 10$.
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- Random probability $= 0.68$, $e^{\frac{-2}{20}} = 0.90 > 0.68$.

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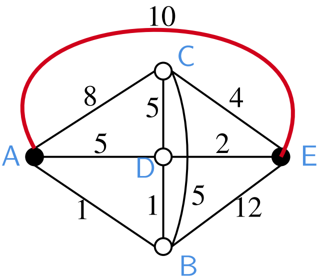

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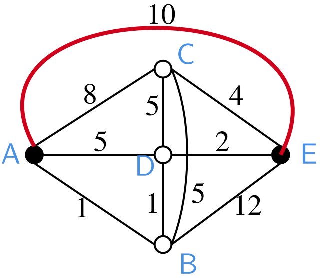

- P =A-E, length = 10
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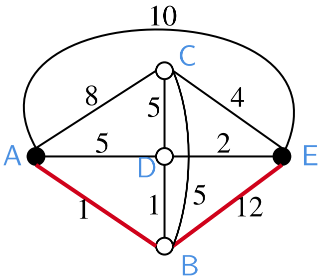
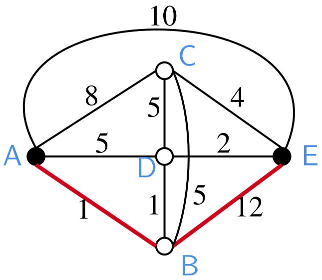
Solving LP using SA



- P =A-E, length = 10
- $temp$ = 20, iteration 4:
- S =A-B-E, length = 13.

Solving LP using SA



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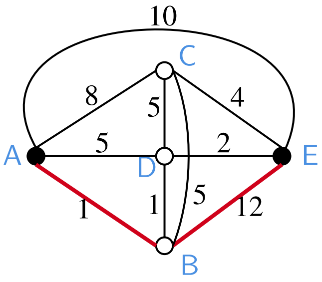

- P =A-E, length $= 10$
- $temp = 20$, iteration 4:
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- Length of S is greater than length of P.
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PSO - Introduction

- A **Stochastic Optimization** technique related to **Swarming Theory** such as :
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    2. Fish Schooling

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  2. Fish Schooling
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- Does not use gradient of the problem/function being optimized.
- Does not require function to be differentiable.

Natural Metaphor

- A flock of birds (or a school of fish) searches for food.
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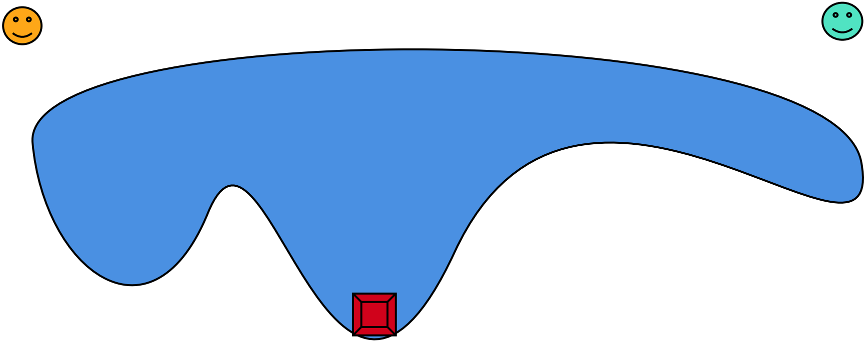
- Want to **efficiently** find the food source.
- Explore food personally **and** communicate with the team.
- Follow the bird **nearest** to the food.

A Simplified Example

Treasure (at the bottom), *Two smily boats*, *Teamwork*, **Get Rich**.



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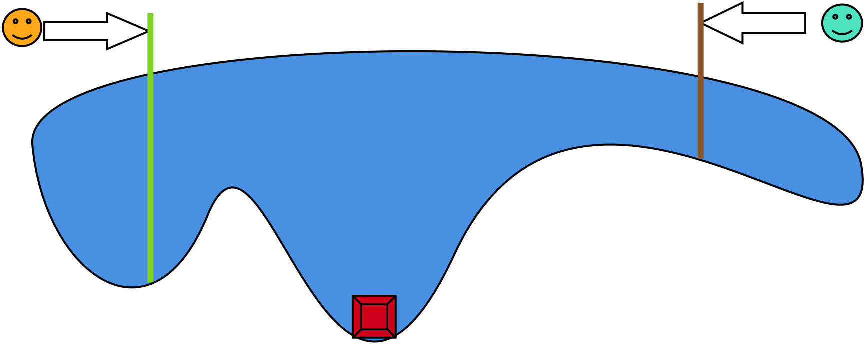

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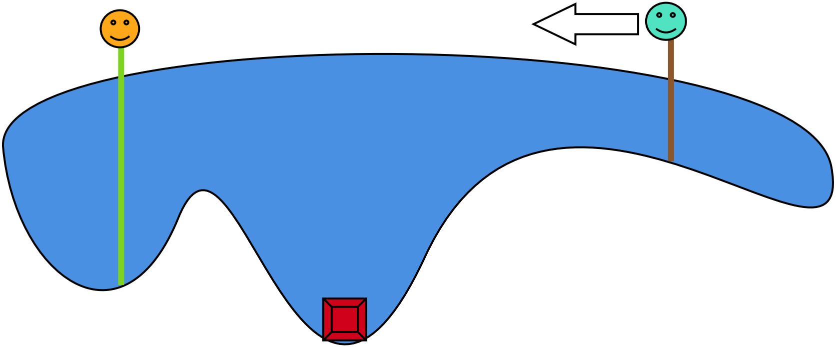

Cyan will move towards Orange

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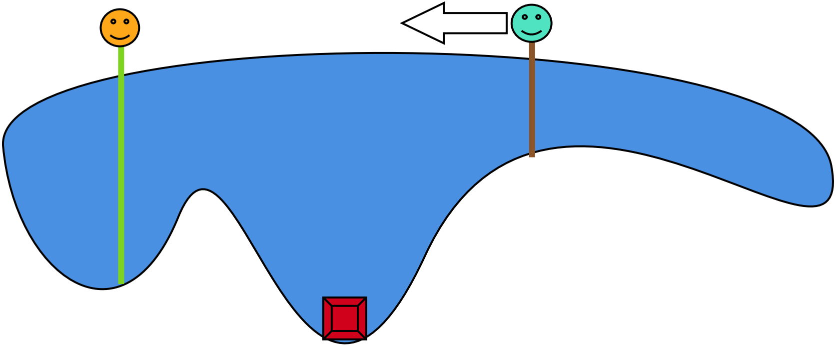

Cyan will move towards Orange AGAIN !!

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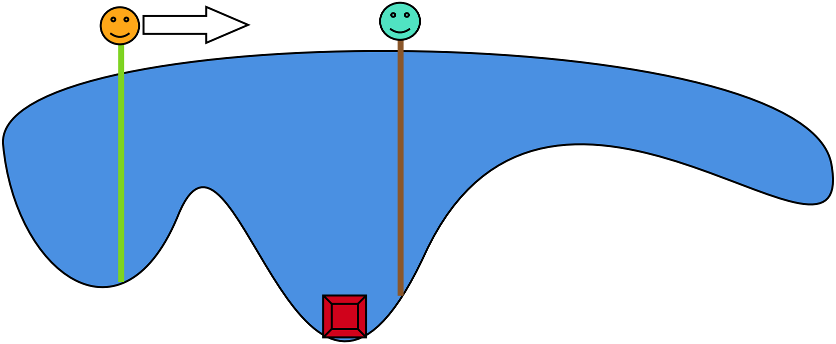

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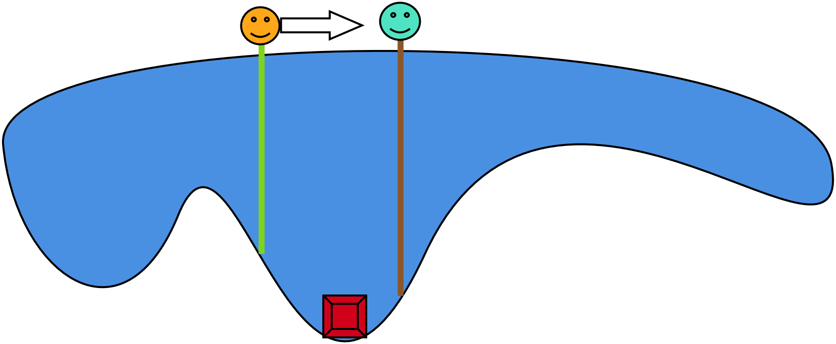

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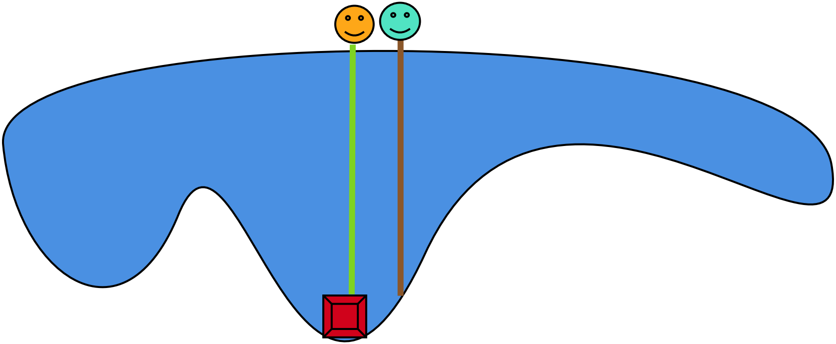

Treasure was the friendships we made along the way

Mathematical Formulation of PSO

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- On **every** iteration, position and velocity gets updated.

Position and Velocity Updates

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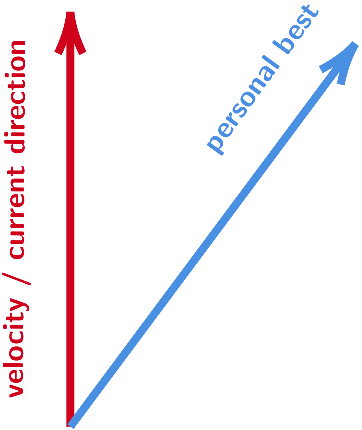

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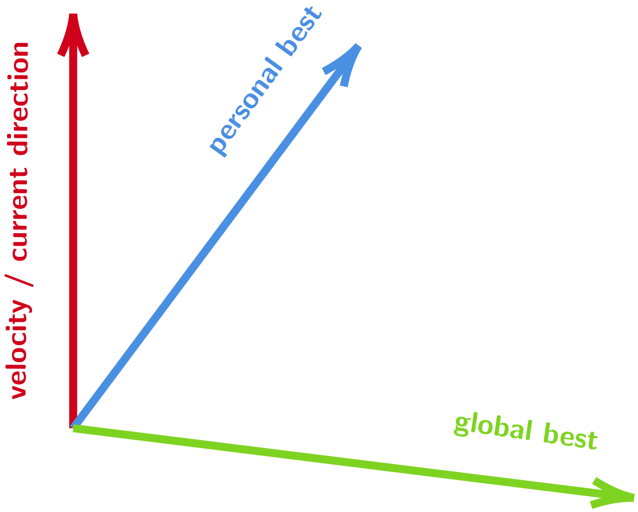

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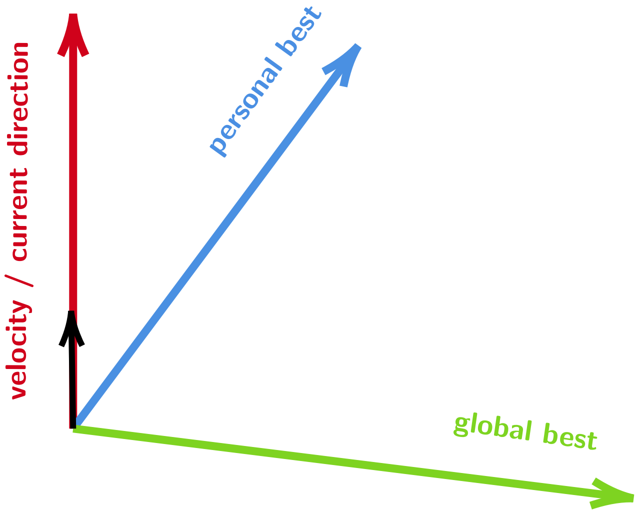

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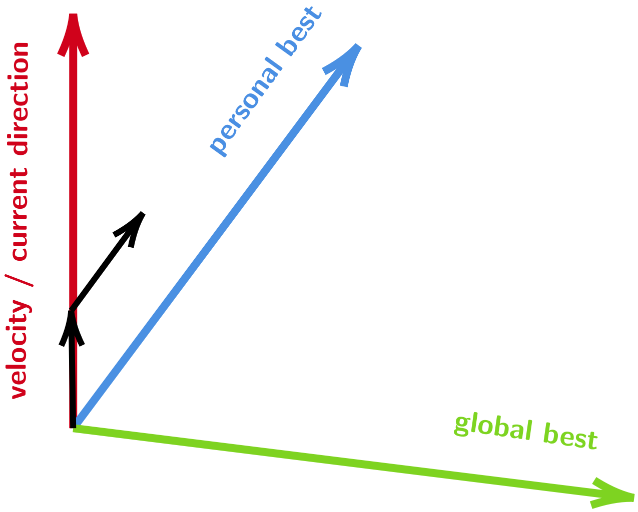

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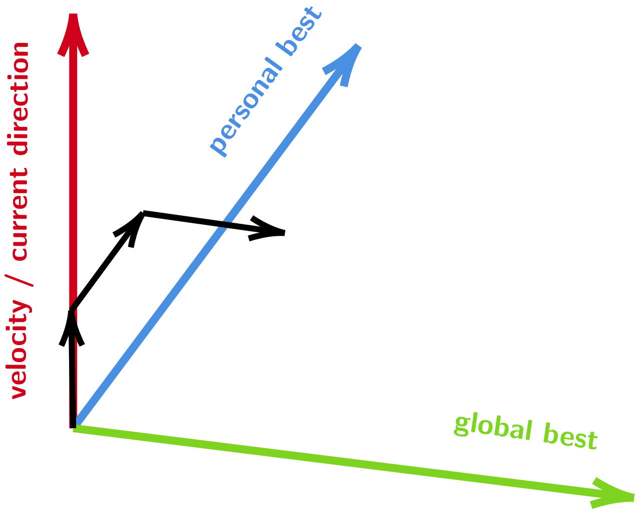

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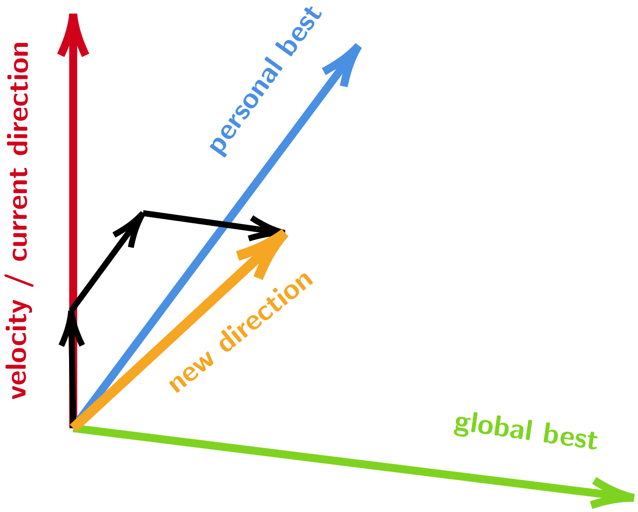

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- Update position, $x_i^{t+1} = x_i^t + v_i^{t+1}$

Flow Chart

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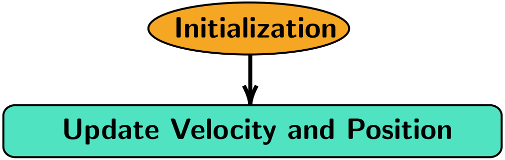
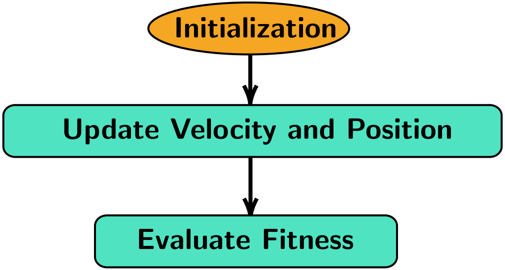
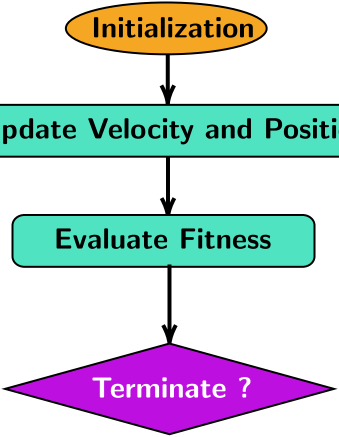


Flow Chart


Initialization
Update Velocity and Position
Evaluate Fitness

Flow Chart


Initialization
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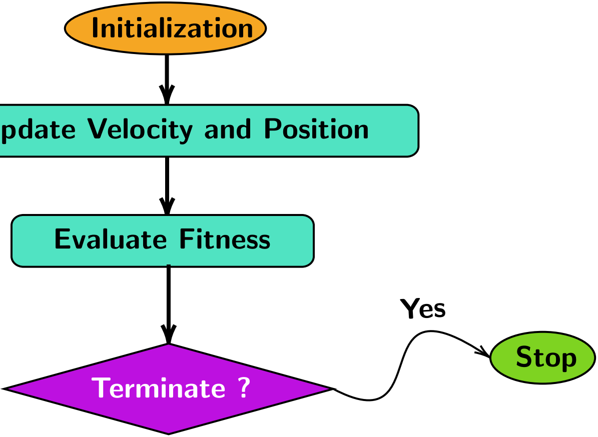
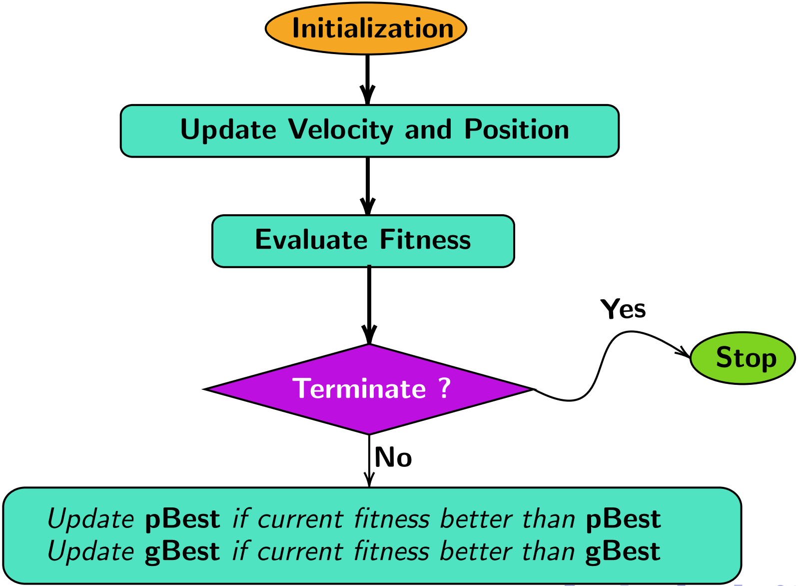
Terminate ?

Flow Chart


Initialization
Update Velocity and Position
Evaluate Fitness
Terminate ?
Yes
Stop

Flow Chart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Yes
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Update **pBest** *if current fitness better than* **pBest**
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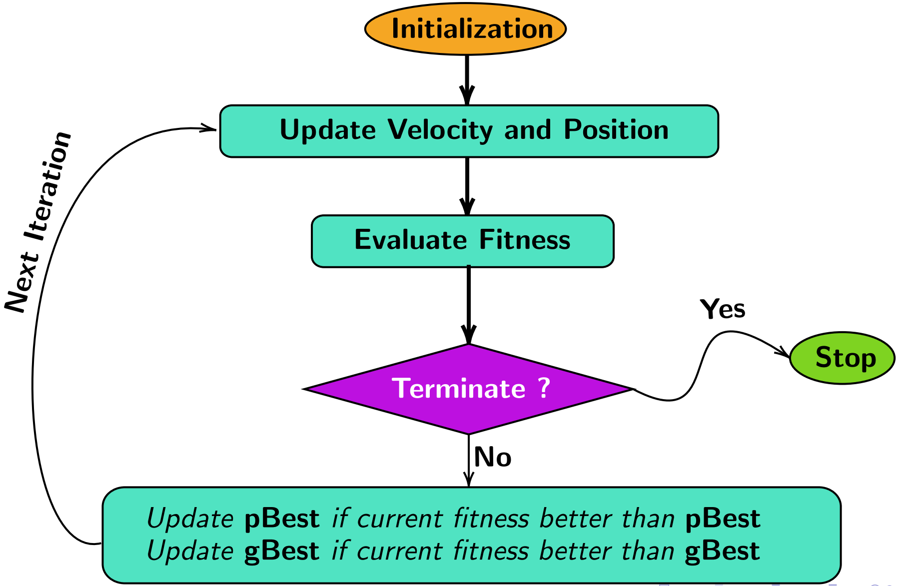
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Next Iteration

Fitting PSO to Longest Path

- We will take inspiration from the way PSO is *applied* for Shortest Path, proposed by **Mohemmad et al. (2008)** [2] and for TSP, proposed by **Wang et al. (2003)** [3].

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- Two strategies to encode **path** as **particles**.
  1. **Direct** Encoding
     Considers an array of node IDs as each particle.

Node Idx	1	2	3	4	5

  2. **Indirect** (Priority-based) Encoding
     Considers a dynamic priority array indexed by node IDs.

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Priority	574	651	670	1000	517

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- The higher the value of this fitness function, the better candidate solution is proposed.

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- Repeat until termination criterion is satisfied.
    1. Maximum number of iterations
    2. Minimum value of Longest Path to obtain
    3. Other custom conditions

Approach using Priority-Based Encoding (Indirect Encoding)

- For convenience, we will assume a starting node and ending node is given eg. **start** at node index **4**, **end** at node index **1**.

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- For convenience, we will assume a starting node and ending node is given eg. **start** at node index **4**, **end** at node index **1**.
- Initially, random values are assigned as priorities for each **particle** i.e. array of node indices with priority values.

Node Idx	1	2	3	4	5
Priority	574	651	670	1000	517

Approach using Priority-Based Encoding (Indirect Encoding)

- For convenience, we will assume a starting node and ending node is given eg. **start** at node index **4**, **end** at node index **1**.
- Initially, random values are assigned as priorities for each **particle** i.e. array of node indices with priority values.

Node Idx	1	2	3	4	5
Priority	574	651	670	1000	517

- Maintain a descending order for priority.

Node Idx	**1**	2	3	**4**	5
Priority	574	651	670	1000	517
Descending order	4	3	2	1	5

Indirect Encoding - Per Iteration Work

- Keep the candidate solution as the path starting from **4** and ending at **1** by using the descending order of priority values. This can be kept in dynamic list to save memory.

Node Idx in Path	4	3	2	1

Indirect Encoding - Per Iteration Work

- Keep the candidate solution as the path starting from **4** and ending at **1** by using the descending order of priority values. This can be kept in dynamic list to save memory.

Node Idx in Path	4	3	2	1

- Calculate F_i i.e. the **fitness value** for this candidate solution path.

Indirect Encoding - Per Iteration Work

- Keep the candidate solution as the path starting from **4** and ending at **1** by using the descending order of priority values. This can be kept in dynamic list to save memory.

Node Idx in Path	4	3	2	1

- Calculate F_i i.e. the **fitness value** for this candidate solution path.
- **If** $F_i > Fitness(pBest_i)$ **then** update $pBest_i = F_i$.
- **If** $F_i > Fitness(gBest)$ **then** update $gBest = F_i$.

Indirect Encoding - Exploration Step

- For exploration, pseudo-random numbers are added to the priority array values for all nodes **except** for the source node.
  Maximum limit of variation << max(priority array)

Indirect Encoding - Exploration Step

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  Maximum limit of variation << max(priority array)
- Example:

Node Idx	**1**	2	3	**4**	5
Priority^{old}	574	651	670	1000	517
Variation	+50	+80	-30	+0	+155
Priority^{new}	+624	+731	+640	1000	+672
Descending order	5	2	4	1	3
New possible path idx	**4**	2	5	3	**1**

Indirect Encoding - Swap with Personal Best

- To swap with personal best (for updating with respect to $pBest$), find common nodes between $pBest$ and current path to update the current path.

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- Suppose $pBest$ is as follows:

Node Idx	4	3	1	7	2

- Current path (before swapping):

Node Idx	4	3	7	9	1	5	8	2

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- Current path (before swapping):

Node Idx	4	3	7	9	1	5	8	2

- Move **7** one step towards target index.
  Target index is $^{4}/_{5}{}^{th}$ from the initial position.
- Current path (after swapping):

Before Swap	4	3	7	9	1	5	8	2
After Swap	4	3	**9**	**7**	1	5	8	2

Indirect Encoding - Replacing with Global Best

- To replace with global best (for updating with respect to $gBest$), we first segment and then replace.
- For segmenting, we keep lower half of current path, and replace upper half of $gBest$.
- For replacing, we choose a node in $gBest$ but **not** in current path, and proportionally replace that node.

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- Suppose $gBest$ is as follows:

Node Idx	4	10	6	3	2

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- Suppose $gBest$ is as follows:

Node Idx	4	10	6	3	2

- Segmenting with $gBest$:

Before Swap	4	8	9	5	1	6	7	2
After Swap	4	8	9	5	1	6	**3**	**2**

Indirect Encoding - Replacing with Global Best

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- Suppose $gBest$ is as follows:

Node Idx	4	10	6	3	2

- Segmenting with $gBest$:

Before Swap	4	8	9	5	1	6	7	2
After Swap	4	8	9	5	1	6	**3**	**2**

- Replacing with $gBest$ (10 in $gBest$ is ${}^{2}/_{5}{}^{th}$ away from the initial position)

Before Replacement	4	8	9	5	1	6	3	2
After Replacement	4	8	**10**	5	1	6	3	2

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