

TSP

$$m[S, c_i] = \min \left\{ m[S \setminus \{c_i\}, c_j \in S \setminus \{c_i\}] + d(c_j, c_i) \right\}$$

$c_i$  hoche src ;  $c_i$  tak shesh  
Min. len. er path

S er shob use kore,  $c_i$  tak shesh hoy

Longest Path can't have fixed  $c_i$ /src.

TSP  $\Rightarrow$  input: given Graph  $G$  will be complete.

~~Graph~~  
 $d(c_i, c_j)$

$c_1 \Rightarrow$  vertices

$c_2 \Rightarrow$

incomplete edges will have (+INF)

output: cycle ...

cost ... minimum

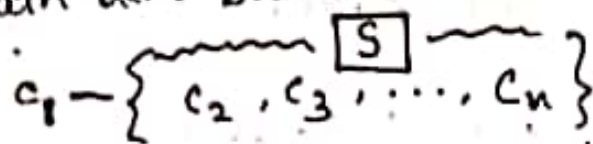
$O(n! * n)$

TSP

• 'n' length er ekta path ban kore ... loop back.

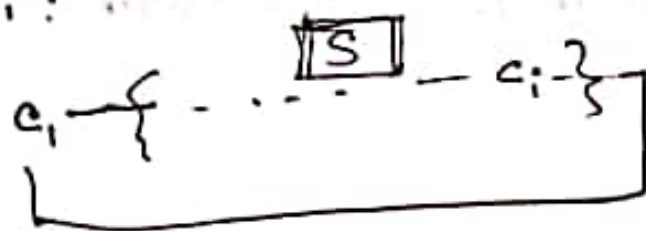
Arbitrary vertex fixed source "S"/ $c_i$

Path len. baraithe thakbo.



$c_i$  shunu, , kai giye shesh hobe ota th. jani na  
loopback kore jabe

~~XXXXXXXXXX~~  
S:  $c_i$  theke shuru kore ...  $c_i$  is the last vertex  
 $c_i$  : ...  $c_i$  lies in S



OPT/m  $\Rightarrow$  array  
 $m[S, c_i]$   $\rightarrow$  set diye indexing i.e. hashmap

$c_i$  er immediate agar vertex,  
ota min. len. then oita theke  $c_i$  tch jabo

$$m[S, c_i] = \min_{\substack{\text{all possible} \\ c_j}} \left\{ m[S - \{c_i\}, c_j] + d(c_j, c_i) \right\}$$

where  $c_j \in S - \{c_i\}$

Extra jinish all  $(c_i - c_1)$  abar  
 jaita hobe

~~Q8~~ TSP  $\leq$  LP oita diyekura jabe ?

$\pi$ : array/list; permutation of 1 to n.

$f(\pi) = c_{\pi_1}, c_{\pi_2}, \dots, c_{\pi_n}$

source janina;  $c_i$  teh shesh

$OPT[S, c_i] :-$  'S' er jotogula use kore;  
 $c_i$  teh gire shesh koro.

$V \Rightarrow$  all vertices set.

$OPT[V, c_i] \Rightarrow P$

Bam dik theke OPT

$OPT[c_i, S] :$   $c_i$  theke shure  
 { uses max no. of vertices in S  
 on max total weight of  
 vertices in S

i.e.  $c_i$    
 Largest path in S

$c_i$  belongs in S  
 $c_i$  starting vertex  
 of S.

NOT necessarily all  
 vertices in S.



$$\odot \text{OPT}[c_i, S] = \max_{\substack{\text{for all} \\ c_j \in S - \{c_i\}}} \left\{ w(c_i, c_j) + \text{OPT}[c_j, S - \{c_i\}] \right\}$$

( $c_j$  is neighbor of  $c_i$ )

If  $(c_i, c_j) \notin E$ , then  $w(c_i, c_j) = -\infty$   
 (-INF)  
 since maximization problem.

Final sol<sup>n</sup>

$$\odot \max_{c_i \in V} \{ \text{OPT}[c_i, V] \}$$

Base case:

$$\text{OPT}[c_i, c_i] = 0$$

Final sol<sup>n</sup>

$$\max_{c_i \in V} \{ \text{OPT}[c_i, V] \}$$

Time  $\rightarrow O(n^2 \cdot 2^n)$

Space  $\Omega(2^n)$

$$2^n * 2^n = 2^{2n} = 4^n$$

Space:  $\Omega(2^n)$

Time:  $O^*(2^n)$

we want polynomial space  $\Omega^*(1)$ ; but time  $O(4^n)$

S ke bhagbe dit equal set -

$C_1, C_2, C_3, C_4$

$C_1, C_2$

$C_3, C_4$

Book: Exact Algorithms  
Chapter: 10

$$\sum_k \binom{n}{k} k! = \sum_k \frac{n!}{(n-k)!}$$