# ECS 132 - Homework #5

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*r Sys.Date()*

## Contents

# Problem 1

Suppose the network flow size is a normal random variable with parameter $\mu = 71$ GBytes and $\sigma = 2.5$ GBytes. 1. What percentage of the flows are greater than 72 GBytes? 2. Suppose flows that are greater than 72 GBytes are clssified as large flows. What percentage of the large flows are greater than 77 GBytes?

## 1.1

$$
\begin{aligned}
P(X > 77) &= 1 - \phi(\frac{77-71}{2.5}) \\
&= 1 - \phi(2.4) \\
&= 1 - 0.9918 \\
&= 0.0082
\end{aligned}
$$

## 1.2

$$
\begin{aligned}
P(Y > 72) &= 1 - \phi(\frac{72-71}{2.5}) \\
&= 1 - \phi(0.4) \\
&= 1 - 0.6554 \\
&= 0.3446 \\
P(Y > 77|Y > 72) &= \frac{P(Y > 72|Y > 77) * P(Y > 77)}{P(Y > 72)} \\
&= \frac{1 * (0.0082)}{0.3446} \\
&= 0.238
\end{aligned}
$$

# Problem 2

Jobs arriving to a computer have been found to require CPU time that can be modelled by an exponentialdistribution with parameter 1/140 per millisecond. The CPU scheduling discipline is quantum-oriented sothat jobs not completing within a quantum of 100 milliseconds will be routed back to the tail of the queueof waiting jobs. 1. Find the probability that an arriving job will be forced to wait for a second quantum. 2. Of the 800 jobs coming in during a day, how many are expected to finish within the first quantum.

Let $X$ represent the time taken for a task, and $Y$ the number of tasks waiting for the second quantum.

## 2.1

$$
\begin{aligned}
P(X > 100) &= 1 - e^{-\lambda * 100} \\
&= 1 - e^{-\frac{1}{140} * 100} \\
&= 0.5105
\end{aligned}
$$

## 2.2

$$
\begin{aligned}
E[Y] &= 800 * 0.5105 \\
&= 408
\end{aligned}
$$

# Problem 3

Two core in a CPU serve jobs from a single queue. The time it takes one core to serve a randomly selectedjob is distributed as $Expo(\frac{1}{5})$so that his average service time is 5 minutes. The other, more powerful core's service times are distributed as $Expo(\frac{1}{4})$, averaging 4 minutes. Cores work independently. Both cores are busy and a new job arrives in the queue. 1. What is the probability it will be longer than 5 minutes before the new job started to be served? 2. Write a R code to simulate the problem and verify the result in Part 1.

Let $X$ represent the 1st CPU, and let $Y$ represent the 2nd CPU. Let $Z$ represent the time before a new process is serviced from the queue.

## 3.1

$$
\begin{aligned}
P(Z > 5) &= P(min(X, Y) > 5) \\
&= P(X > 5) * P(Y > 5) \\
&= e^{-\frac{1}{5}*5)} *^{-\frac{1}{4}*5)} \\
&= e^{-\frac{9}{5}} \\
&= 0.1653
\end{aligned}
$$

## 3.2

```
trails <- 10000
cpu1Prob <- rexp(trails,1/5)
cpu2Prob <- rexp(trails,1/4)

count <- 0

for(i in 1:trails){
  if(cpu1Prob[i] > 5 && cpu2Prob[i] >= 5){
    count = count + 1
  }
}

(count/trails)
```

```
## [1] 0.106
```

# Problem 4

A parallel system hasnindependent components, each with lifetime distributed as $Expo(\lambda)$. 1. Show that the cumulative distribution of the lifetime of this system, denoted by $W$, is $F_W(t) = (1 - e^{-\lambda t})^n$. 2. For $n = 5$ and $\lambda = \frac{1}{4}$ find the probability $P(W > 5)$.

## 4.1

The CDF for an exponential distribution is given as $1 - e^{-\lambda t}$. Since there are $n$ number of componnents which with the same CDF , therefore the CD for this problem is $(1 - e^{-\lambda t})^n$.

### 4.2

$$
\begin{aligned}
P(W > 5) &= (1 - e^{-\lambda t})^n \\
&= (1 - e^{-\frac{1}{4}*5})^5 \\
&= 0.1849
\end{aligned}
$$

## Problem 5

This question deals with the Poisson process and is formulated in terms of a subway station. It could aswell be formulated in terms of a networking problem (keep in mind a router with input interfaces which arebeing served by the switch fabric in a random process).

Two one-way subway lines, the A train line and the B train line, intersect at a transfer station, A trains and B trains arrive at the station according to independently operating Poisson processes with rates $\lambda_A = 3$ trains/hr and $\lambda_B = 6$ trains/hr. We assume that passenger boarding and un-boarding occurs almost instantaneously. At a random time, Bart, a prospective passenger, arrives at the station, awaiting an A train.

[Note that the superposition of two Poisson processes with rates $\lambda_1$ and $\lambda_2$ is also a Poisson process withrate $\lambda_1 + \lambda_2$]

1. What is the probability that the station handles exactly 9 trains during any given hour?
2. If an observer counts the number of trains that the station handles each hour, starting at 8:00 A.M. onTuesday, what is the expected number of hours until he or she will first count exactly 9 trains duringan hour that commences "on the hour"? (e.g., 9: 00 A.M., 10: 00 A.M., 2: 00 P.M.)

### 5.1

$$
\begin{aligned}
\lambda_A + \lambda_B &= 9 \\
P(N(t) = x) &= e^{-rt}\frac{(rt)^x}{x!} \\
&= e^{-9}\frac{9^9}{9!} \\
&= 0.1318
\end{aligned}
$$

### 5.2

The probablity of the station handling exactly 9 trains in the hour is 0.1318. Therefore, there are $N-1$ failures before the $N^{th}$ hour where it is 9 trains exactly. This becomes a geometrically distributed random variable which succeeeds or fails. There are $N-1$ failures with the $N^{th}$ being a success. The expected success formula for this is $E[X] = \frac{1}{p}$.

$$
\begin{aligned}
P(success) &= 0.1318 \\
P(failure) &= 1 - 0.1318 \\
&= 0.8682 \\
E[X] &= \frac{1}{P(success)} \\
&= 7.593
\end{aligned}
$$

## Classmate Collaborators

Inclulde the names and email IDs of everyone you collaborated with. You are free to discuss with your peers but everyone's work must be individual.

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