ECS 132 - Homework #6

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r Sys.Date()

Contents

Problem 1

Based from the proof from class, and that $Q_i = 1 - P_i$

1.1

$$\begin{aligned} Q_i &= 1 - P_i \\ &= 1 - \frac{1 - (\frac{q}{p})^i}{1 - (\frac{q}{p})^N} \\ &= \frac{1 - (\frac{q}{p})^N - 1 + (\frac{q}{p})^i}{1 - (\frac{q}{p})^N} \\ &= \frac{-(\frac{q}{p})^N + (\frac{q}{p})^i}{1 - (\frac{q}{p})^N} \end{aligned}$$

1.2

$$\begin{aligned} Q_i &= \frac{-(\frac{q}{p})^N + (\frac{q}{p})^i}{1 - (\frac{q}{p})^N} \\ &= \lim_{\frac{p}{q} \to 1} \frac{-(\frac{q}{p})^N + (\frac{q}{p})^i}{1 - (\frac{q}{p})^N} \\ &= \frac{-N + i}{-N} \\ &= \frac{N - i}{N} \end{aligned}$$

Problem 2

$$\begin{aligned} P(i) &= \frac{1}{3}P(i+1) + \frac{2}{3}P(i-1) \\ P(1) &= \frac{1}{3}P(2) + \frac{2}{3}P(0) \\ &= \frac{1}{3}P(2) + 0 \\ P(2) &= 3P(1) \\ P(3) &= 7P(1) \\ P(i) &= (2^i - 1)P(1) \\ P(i+2) &= 1 = (2^{i+2} - 1)P(1) \\ P(1) &= \frac{1}{2^{i+2} - 1} = \frac{1}{3}P(2) \\ P(2) &= \frac{3}{2^{i+2} - 1} \\ &= \frac{2^i - 1}{2^{i+2} - 1} \\ P(i) &= \frac{2^i - 1}{2^{i+2} - 1} < \frac{1}{4} \quad \forall i \geq 0 \end{aligned}$$

Problem 3

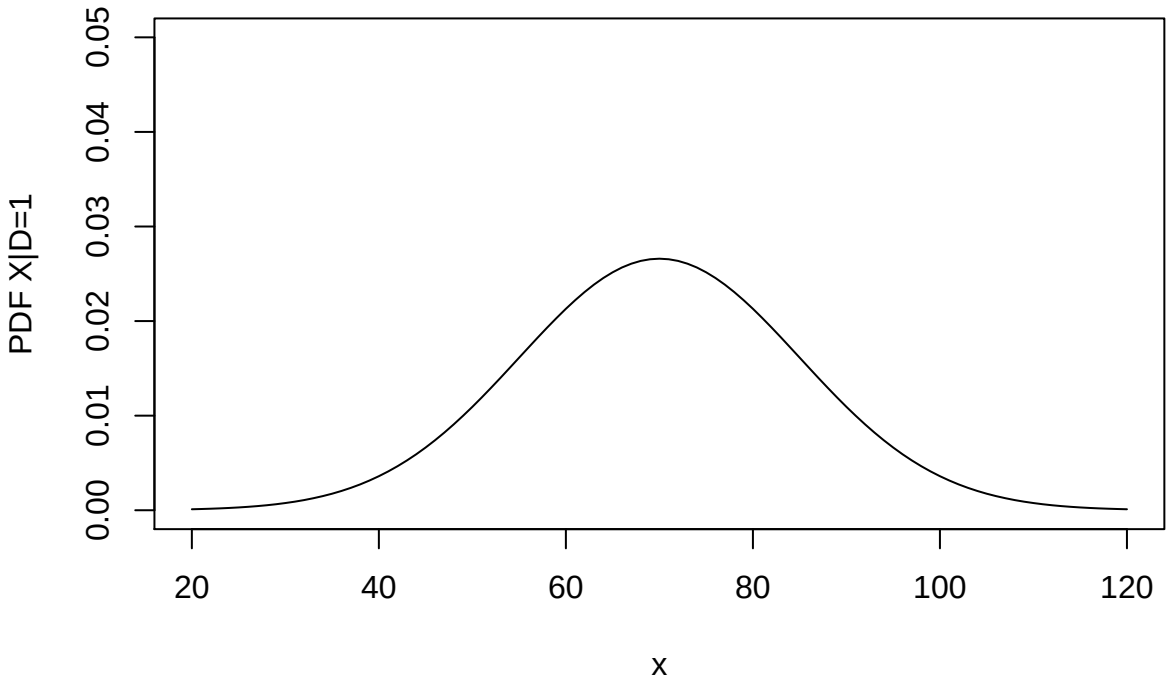
D = 0 (Truth) | D = 1 (Lie)
T = 0 (Pass) | T = 1 (Fail)
P(T = 0| D = 1) = .1
P(T = 1| D = 0) = .2

$$Sensitivity(n) = 1 - P(T = 1|D = 0) = 0.8$$
$$Specificity(\theta) = 1 - P(T = 0|D = 1) = 0.9$$

Problem 4

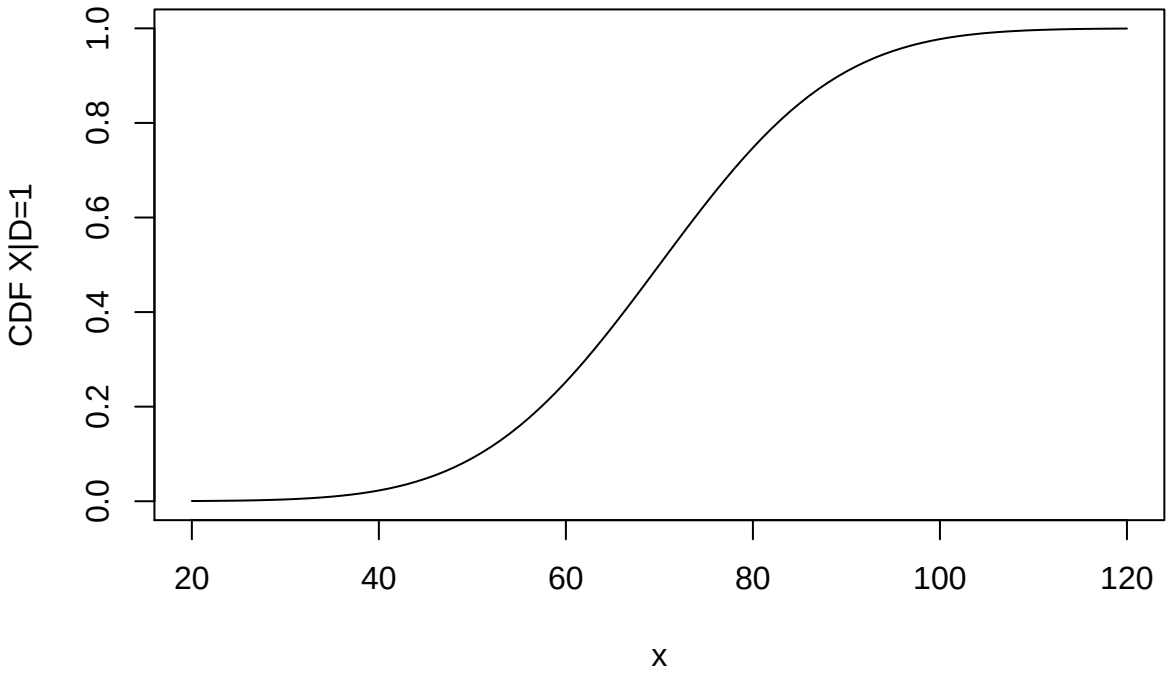
1. Plot the pdf and the cdf of $X|(D = 1)???Norm(70, 15)$ for values 20 to 120.

```
par(mfrow=c(1,1))
x =seq(20,120)
hx =dnorm(x,70,15)
plot(x, hx,xlim =c(20,120),ylim =c(0,0.05),type='l',ylab="PDF X|D=1",xlab="x")
```



2. Let $x*$ denote the cutoff value such that if $X > x*$ then the test is positive test otherwise it is considered be negative. Plot the ROC curve for values of $x*$ ranging from $52 < x \leq 65$.

```
par(mfrow=c(1,1))
x =seq(20,120)
hx =pnorm(x,70,15)
plot(x, hx,xlim =c(20,120),ylim =c(0,1),type='l',ylab="CDF X|D=1",xlab="x")
```



3. If False Positive Rate and False Negative Rates are equally bad, determine the value of $x*$.

Classmate Collaborators

Includle the names and email IDs of everyone you collaborated with. You are free to discuss with your peers but everyone's work must be individual.

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