ECS 132 - Homework #1

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Contents

Problem 1

Consider the Manhattan grid network shown in Figure 1. Suppose that starting at the point labeled A, you can go one step up or one step to the right at each move. This procedure is continued until the point labelled B is reached. How many different paths are there from A to B that go through point C?

4 possible paths choices from A to C: { (RR), (RU), (UR), (UU) }, R-right, U-up.
2 choices per path. So $\binom{4}{2}$
3 possible path choices from C to B: { (RR), (RU), (UR) }, R-right, U-up.
2 choices per path. So $\binom{3}{2}$
Total possible choices $\binom{4}{2} * \binom{3}{2} = 18$ total paths.

Problem 2

Four (4) fair dice are rolled. What is the probability that the total is 21? Possible dice rolls that sum to 21

6	6	6	3
6	6	5	4
6	5	5	5

Assuming order does matter, there are ${}_4P_1$ permutations of the first and last possibility, and ${}_2P_4$ possibilities of 2nd possibility. So 20 total possibilities in total. Total number of possibilities: 6^4.
Probability of the sum being 21: $\frac{20}{1296} = 0.01543 = 1.543\%$.

Problem 3

Suppose your MP3 player contains 100 songs. 10 of those songs are by your favorite group (say U2). Suppose the shuffle feature is used to play the songs in random order.

Part 3.1

What is the probability that the first U2 song heard is the 5th song played? The first 4 songs start at .9 probablity then decrease by .01 per song. The 5th song has a .1 probability of being U2. Therefore: $(0.9)(0.89)(0.88)(0.87)(0.1) = 0.06132$

Part 3.2

What is the probability that one of the first five songs played is a U2 song? $\frac{90}{100} * \frac{89}{99} * \frac{88}{98} * \frac{87}{97} * \frac{10}{96} = 0.06717$
Number of permutations: 5. Total probablity: $0.06132 * 5 = .3066$

Problem 4

A server has 5 incoming packet flows (flows are distinct and uniquely identified by a 5-tuple flow-id). Packets from each flow are stored in a circular buffer (in the Linux kernel this is called the Ring Buffer). Suppose there are three packets from each flow (packets have sequence numbers and hence are distinguishable) which are randomly assigned to locations in the circular buffern of size 15. What is the probability that no three packets from the same flow are in adjacent locations in the buffer?

Let E_i, i= 1,2,3, . . . ,10, be the event that packets from the ith flow are placed in adjacent locations in the buffer. The probability the question asks for, denoted by A: $A = P(\bigcup_{i=1}^{5} E_i)$

There are 3^x ways to organize packets from the same flow that are adjacent. Due to this being a circular buffer, there are $(15-1)!$ ways to pick an order. Therefore $(15-x-1)!$, where x is the number of flows which have adjacent packets.

$$
\begin{aligned}
P(\bigcup_{i=1}^{5} E_i) &= \sum_{x=1}^{5}(-1^{x+1})\binom{5}{x}3^x\frac{(15-x-1)!}{(15-1)!} \\
P(\bigcup_{i=1}^{5} E_i) &= \binom{5}{1}3^1\frac{(15-1-1)!}{(15-1)!} \\
&- \binom{5}{2}3^2\frac{(15-2-1)!}{(15-1)!} \\
&+ \binom{5}{3}3^3\frac{(15-3-1)!}{(15-1)!} \\
&- \binom{5}{4}3^4\frac{(15-4-1)!}{(15-1)!} \\
&+ \binom{5}{5}3^5\frac{(15-5-1)!}{(15-1)!} \\
&= 0.6847 \\
1 - P(\bigcup_{i=1}^{5} E_i) &= 0.3153
\end{aligned}
$$

Problem 5

Part 5.1

Suppose there are 100 memory chips in a box, of which 90 are good and 10 are bad. We withdraw five of the 100 chips at random to upgrade a computer. We want to find the probability that all five (5) chips are good, $P(AllGood)$. $P(AllGood) = \binom{90}{5}/\binom{100}{5}$ $P(AllGood) = 0.5835$

Part 5.2

```
choose(90,5)/choose(100,5)
```

```
## [1] 0.5837524
```

Part 5.3

```
# strategy: We will consider numbers 1 - 90 as good and 91 - 100 are defective
# set.seed(1237)
```

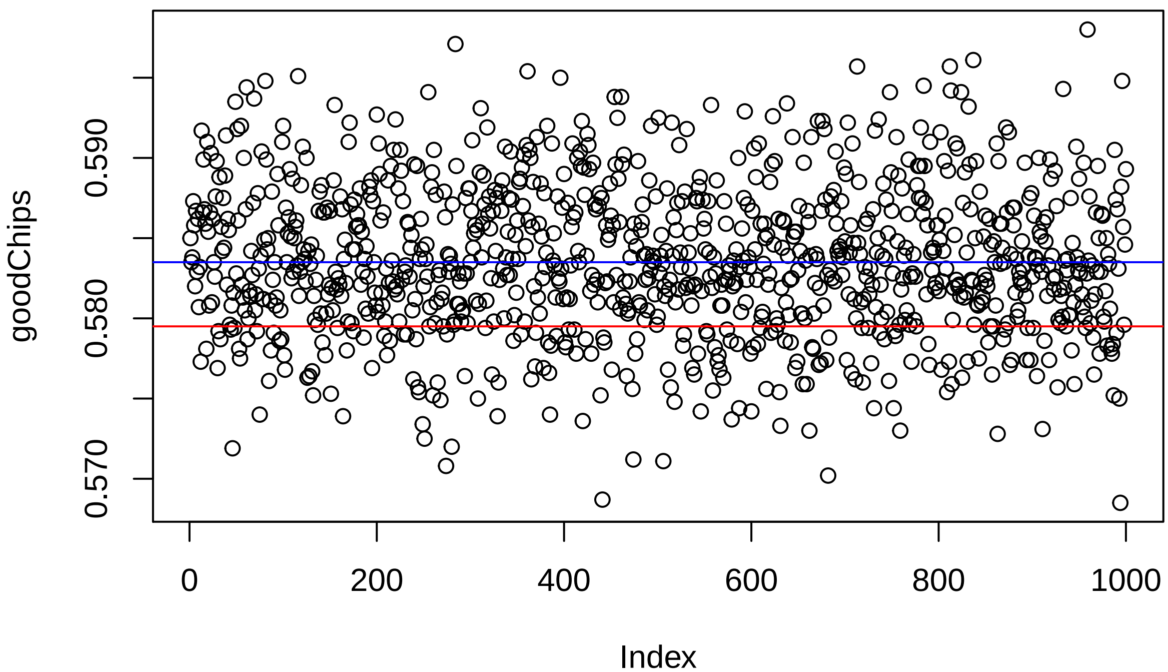
```
m = 10000 # number of samples to simulate
good = numeric(m) # initialize for use in loop
for (i in 1:m)
{
  pick = sample(1:100, 5) # vector of 5 items from ith experiment
  good[i] = sum(pick <= 90) # number good in ith experiment
}
mean(good == 5)
```

```
## [1] 0.5795
```

Close average to the calculated average from part 2.

Part 5.4

Modify the code to determine the probability of all good, P(All Good), for the first 1000 samples and plot it.

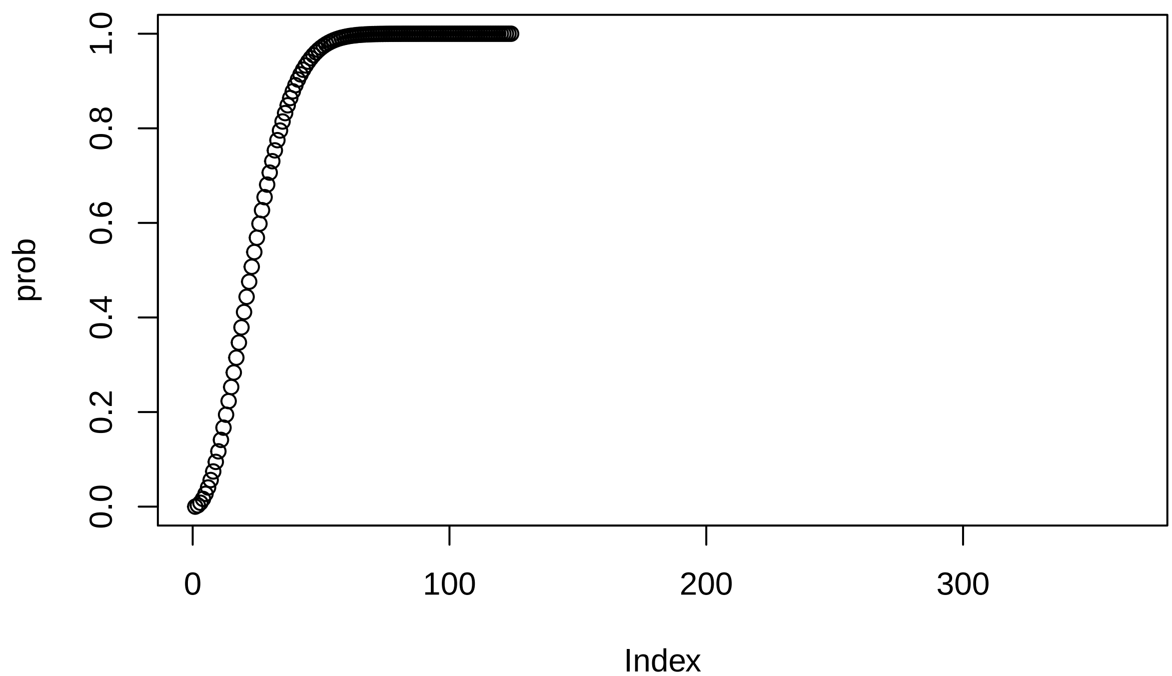


Red line is the experimental average, Blue line is the theoretical average.

Problem 6

Consider a small change in the problem definition of the Birthday paradox. Suppose you walk into a room with k people. What is the probability that there will be atleast 1 person with the same birthday as yours.

Write a R code to plot the probability as a function of k.



Problem 7

Martian birthdays. In his science fiction trilogy on the human colonization of Mars, Kim Stanley Robinson arranges the 669 Martian days of the Martian year into 24 months with distinct names [Rob96]. Imagine a time when the Martian population consists entirely of people born on Mars and that birthdays in the Martian born population are uniformly distributed across the year. Write an R-code to determine the value of n such that the probability of a match is just greater than 0.75.

```
prob = 0
people = 0;
while(prob < 0.75){
  people = people + 1
  prob = 1-(factorial(people)*(choose(669,people))/669^people)
}
prob
```

```
## [1] 0.7644274
```

```
people
```

```
## [1] 44
```

Classmate Collaborators

Inclulde the names and email IDs of everyone you collaborated with. You are free to discuss with your peers but everyone's work must be individual.

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