PURBANCHAL UNIVERSITY

2023

Bachelor in Information Technology (B.I.T.)/Second Semester/*Final*

Time: 03:00 hrs. Full Marks: 80/Pass Marks: 32

BIT151HS: Mathematics-II *(New Course)*

Candidates are required to give their answers in their own words as far as practicable.

Figure in the margin indicate full marks.

Group A

Answer ALL questions. **10×2=20**

1. Evaluate the double integral: 24

$$\int_0^3 \int_1^2 xy(x+y)\,dx\,dy$$

2. Evaluate the triple integral: 64/3

$$\int_0^4 \int_0^z \int_0^y (x+1)\,dx\,dy\,dz$$

3. Define essential singularity and removable singularity.

4. Determine the order and degree of the differential equation:

$$\sqrt{\frac{d^2y}{dx^2}} = \frac{dy}{dx}.$$ 2|1

5. Solve: $(1+x)dy + (1-y)dx = 0$. y-cx=c-1

6. Find the general solution of the differential equation:
$y'' + 4y' + 8y = 0$

7. Define add and even functions with examples.

8. If f(z) is differentiable at z_0 then show that f(z) is continuous at $z = z_0$.

9. Express the function $f(z) = z^3$ in the form of $f(z) = u(x,y) + iv(x,y)$.

Contd. ...

10. Determine the order of the pole of the function: $f(z)=\frac{\sinh z}{z^5}$.

Group B

Answer EIGHT questions. 8×5=40

11. Find the area in the XY-plane bounded by the leminiscate $r^2=4\sin 2\theta$.
12. Solve: $xdy-ydx+a(x^2+y^2)dx=0$.
13. Find the volume bounded by the sphere $x^2+y^2+z^2=a^2$ using triple integral..
14. Solve:

$$\frac{d^2y}{dx^2}-13\frac{dy}{dx}+12y=e^{-2x}$$

15. Find the Fourier series for the function defined by:

$$f(x)=\begin{cases}1 & \text{for } 0\le x<\pi\\ 0 & \text{for } \pi\le x<2\pi\end{cases}$$

16. Expand $f(x)=x$ as cosine series in th interval $0\le x\le\pi$ and hence show that:

$$1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$$

17. Verify Cauchy-Riemann equation for the function:

$$f(z)=e^x(\cos y+i\sin y)$$

18. Expand $f(z)=\frac{1}{z}$ by Taylor's series about the point z=1.
19. Show that:

$$f(z)=\frac{1-e^{2z}}{z^3}$$

has a pole of order 2 at z = 0.

20. Find the residue of: $f(z)=\frac{3z-4}{z(z-1)}$.

Contd. ...

Group C

Answer FOUR questions. 4×5=20

21. Find the orthogonal trajectories of the family of curves given by $y=kx^2;\ k\neq 0$
22. Find the Fourier integral of the function:

$$f(x) = 1 \text{ for } |x|<1$$
$$= 0 \text{ for } |x|>1$$

23. Find the Fourier transform of:

$$f(x)=\begin{cases}1-x^2, & \text{for } -1<x<1\\ 0, & \text{otherwise}\end{cases}$$

24. Find the analytic function whose real part is $u=e^x(x\cos y+y\sin y)$.
25. Obtain the Laurent series of the function:

$f(z)=\frac{\sin z}{z^6}$ and hence show that $\int_C \frac{\sin z}{z^6}dz=\frac{1}{60}\pi i$

where C is the circle $|z|=2$ taken in the counter-clockwise sense.

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PURBANCHAL UNIVERSITY

2023

Bachelor in Information Technology (B.I.T.)/Second Semester/*Back*

Time: 03:00 hrs. Full Marks: 80/Pass Marks: 32

BIT102SH/BIT121HS: Mathematics-II *(Old Course)*

Candidates are required to give their answers in their own words as far as practicable.

Figure in the margin indicate full marks.

Group A

Answer ALL questions. **10×2=20**

1. Solve (1+x)ydx +(1+y)xdy = 0.
2. State Laurent's series for a function f(z).
3. Solve: $\frac{dy^2}{dx^2}+5\frac{dy}{dx}=0$.
4. Find the laplace transform of $t\sin at$.
5. State Taylor's Theorem.
6. Define odd and even function.
7. Define exact differential equation.
8. Express the function f(z) =z^3 in the form u(x,y) + iv (x,y).
9. Define the harmonic function.
10. Show that $f(z)=\frac{1-e^{2z}}{2^3}$ has a pole of order 2 at z=0.

Group B

Answer EIGHT questions. **8×5=40**

11. Solve the differential equation $\sin 2x\frac{dy}{dx}-y=Tanx$.
12. Solve the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$.
13. Find the general and singular solution of y= px + ap (l-p).

Contd. ...

14. Find the inverse Laplace transforms of $\frac{1}{s^2 - 5s + 6}$.

15. Find the analytic function whose real part is
$u(x, y) = e^x(x\cos y - y\sin y)$.

16. Find the Laurent's series for $f(z) = \frac{1}{(1-z)(z+2)}$ valid for the domain $1 < |z| < 2$.

17. Obtain the Fourier series of $f(x) = x^2, \quad 0 < x < 2\pi$.

18. A metal ball is heated to a temperature of 100°C and at time t=0 it is placed in water which is maintained at 40°C. If the temperature of ball is reduced to 60°C in 4 minutes, find the time at which temperature of ball is 50°C.

19. Solve: $zp + yq = x$

Group C

Answer FOUR questions. **4×5=20**

20. Solve the differential equation by Laplace transform method:

$$y'' + y' - 2y = t, \; y(0) = 1, \; y'(0) = 0$$

21. Find the Fourier cosine integral of $f(x) = \begin{matrix} x^2 & for & 0 < x < a \\ 0 & for & x > a \end{matrix}$

22. Solve the partial differential equation by the method of separation of variables $\frac{du}{dt} = C^2 \frac{d^2u}{dx^2}$.

23. Find the poles and residue of the function: $f(z) = \frac{4-3z}{z(z-1)(z-2)}$.

24. Find the orthogonal trajectories of the hyperbola $xy = c$

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BIT151HS: Mathematics-II *(New Course)*

Candidates are required to give their answers in their own words as far as practicable.

Figure in the margin indicate full marks.

Group A

Answer TWO questions. **2×10=20**

1. Evaluate the double integral:

$$\int_0^2 \int_1^2 (x^2 + y^2)dx\, dy$$

2. Define Taylor's theorem and Laurent's series.
3. Determine the order and degree of the differential equation:

$$\sqrt{\frac{d^3y}{dx^3}} = \frac{dy}{dx}$$

4. Solve: $(x+1)dy+(y-1)dx=0$.
5. Find the general solution of the differential equation: $y'' + 4y' - 5y = 0$.
6. Define odd and even function with examples.
7. If $f(z)$ is differentiable at z_0, then show that $f(z)$ is continuous at $z = z_0$.
8. Express the function $f(z) = z^2$ in the form of $f(z) = u(x,y) + iv(x,y)$.
9. Define isolated singularity with example.
10. Show that the function.

$$f(z) = \frac{z^2 - 2z + 5}{z - 2}$$

has a simple pole at $z = 2$.

Contd. ...

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Group B

Answer EIGHT questions. **8×5=40**

11. Find by double integration, the area which lies inside the cardoid $r = a(1+\cos\theta)$ and outside the circle r = a.

12. Solve:

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

13. Find the orthogonal trajectories of the family of curves given by

$$y = kx^2;\ \ k \neq 0$$

14. Solve:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2x + x^2$$

15. Find the Fourier series for the function defined by:

$$f(x) = \begin{cases} 1 & \text{for } 0 \le x \le \pi \\ -1 & \text{for } \pi \le x < 2\pi \end{cases}$$

16. Expand f(x) = x as a cosine series in the interval $0 \le x \le \pi$ and hence show that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

17. State and prove Cauchy-Riemann equation.

18. Expand $f(z) = \frac{1}{z}$ by Taylor's series about the point z=1.

19. Show that:

$$f(z) = \frac{1 - e^{2z}}{z^3}$$

has a pole of order 2 at z = 0.

20. Find the residue of:

$$f(z) = \frac{3z - 4}{z(z-1)(z-2)}$$

Contd. ...

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Group C

Answer FOUR questions. 4×5=20

21. Find the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$ using triple integral.

22. Solve:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{3x} + \cos 5x$$

23. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 - x^2, & \text{for } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

24. Show that $u=e^x \cos y$ is harmonic and find an analytic function.

25. Obtain the Laurent series of the function

$$f(z) = \frac{\sin z}{z^6}$$

and hence show that

$$\int_C \frac{\sin z}{z^6} dz = \frac{1}{60}\pi i$$

where C is the circle $|z| = 2$ taken in the counter-clockwise sense.

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PURBANCHAL UNIVERSITY
2022

Bachelor in Information Technology (B.I.T.)/Second Semester/*Back*

Time: 03:00 hrs. Full Marks: 80/Pass Marks: 32

BIT102SH/BIT121HS: Mathematics-II *(Old Course)*

Candidates are required to give their answers in their own words as far as practicable.

Figure in the margin indicate full marks.

Group A

Answer ALL questions. **10×2=20**

1. State the order and degree of the given differential equation,

$$\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

2. State Taylor's series for a function f(z).

3. Find the complementary function for the given differential equation y"+9=-tanx.

4. Find the inverse Laplace Transform of: $\int_0^t x^2 e^x dx$..

5. Find the Laplace Transform of : t^2e^{2t}.

6. If $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ represent the Fourier expressions of a function f(x) of period 2π. Write the expression for a_n and b_n.

7. Express e^z in the form $u(x, y) + iv(x, y)$.

e^{x+iy}

$e^x . e^{iy}$

8. State Laurent's series of a function f(z).

9. Show that the function: $f(x) = \frac{z^2 - 2z + 5}{z - 2}$ has a simple pole at z = 2.

10. Obtain a partial differential equation by eliminating arbitrary constant : z = axy + b.

Contd. ...

Group B

Answer EIGHT questions. 2 **8×5=40**

11. Solve the differential equation $x^2dy + xydx + 2\sqrt{1-x^2y^2}\,dx = 0$.

12. Solve the differential equation by operator method: $(D^2 - 4D + 4)y = x^3e^{2X}$.

13. Find the general and singular solution of $y = px + {}^{a}/_{p}$.

14. Use the convolution to find the inverse Laplace transforms of:

$$\frac{2s}{(s^2+4)(s^2+9)}$$

15. Show that: $u = e^x(x\cos y - y\sin y)$ is a harmonic function and find corresponding analytic function.

16. Find the residue of:

$$f(x) = \frac{3z-4}{z(z-5)(z-3)}.$$

17. Expand $f(x)=x$ as a half range sine series in $0 \le x \le 3$.

18. In a circuit with resistance R, inductance L, electromotive E, The current I, satisfies the differential equation $L\frac{di}{dt} + Ri = E$, Taking L and R as constant and E = wt, solve this equation subject to the initial condition that i=0 at t = 0.

19. Solve the partial differential equation by the method of separation of variables:

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial y^2} = 0.$$

$u = XY$

$\ddot{X}Y + 4X\ddot{Y} = 0$

Group C

Answer FOUR questions. **4×5=20**

20. Solve the given differential equation by the Laplace transformation method:

$$y'' + 4y' + 3y = e^{-t}, y(0) = y'(0) = 0$$

21. Find the orthogonal trajectories of the family of curves given by $y = k_1x^2$.

22. Suppose that an object is heated to 300^0 F and allowed to $0°$ cool in a room whose air temperature is 80^0 F. If after 10 minutes the temperature of object is $250°$ F what will be its temperature after 20 minutes?

23. Find fourier cosine integral of $f(x) = e^{-kx}$, $x>0$, $k>0$ and hence show that $\int_0^\infty \frac{\cos wx}{k^2 + w^2} dw = \frac{\Pi}{2k} e^{-kx}, x > 0, k > 0$.

24. Find the Laurent series expansion if $f(z) = \frac{1}{(1-z)(z+2)}$ in the domain $1 < |z| < 2$.

PURBANCHAL UNIVERSITY
2021

Bachelor in Information Technology (B.I.T.)/Second Semester/*Final*

Time: 03:00 hrs. Full Marks: 80/Pass Marks: 32

BIT102SH: Mathematics-II *(New Course)*

Candidates are required to give their answers in their own words as far as practicable.

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Group A

Answer TWO questions. **2×10=20**

1. Find the solution of the differential equation $(x+1)dy+(y-1)dx=0$.

2. Find the inverse Laplace transform of $\frac{1}{s^2+3s+2}$.

3. Express f(z)=log z in the form u(x,y)+i v(x,y).

4. Find the Fourier expansion of the function in the interval $0 \le x \le 2\pi, f(x)=2x$.

5. Find the general solution of the partial differential equation ap+bq = c.

6. Evaluate $\int_C f(z)dz, when f(z)=\frac{1}{z-a}, c$ is the circle with centre at a and radius r.

7. Calculate the residue of (Z)= $f(z)=\frac{1}{z+\frac{1}{z}}$.

8. Find the Laplace transform of $e^{3t}\cos 2t$.

9. Find the general solution of the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y.=0$.

10. Define fourier cosine and sine integral of f(x).

Contd. ...

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Group B

Answer EIGHT questions. **8×5=40**

11. Solve the differential equation: $\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$.

12. Solve the second order differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \sin x$.

13. Solve $xdy - ydx = \sqrt[x]{x^2 + y^2 dx}$.

14. Find the Laplace transform of t e^{-t} cost.

15. Find the inverse Laplace transform of $\frac{2s+3}{(s-1)(s-2)(s-3)}$.

16. Expand the function $f(x) = x^2, 0 \le x \le \pi$ in a Fourier cosine series and deduce that $\sum_{n=1}^{\infty} \frac{1}{x^2} = \frac{\pi^2}{6}$.

17. Verify Cauchy Riemann equations for the following function $e^x(\cos y + i \sin y)$.

18. Obtain the Laurent Series for $f(z) = \frac{1}{(1-z)(z+2)}$ in the domain $1 < |z| < 2$.

19. Solve the partial differential equation $p^2 + qy - z = 0$.

Group C

Answer TWO questions. **2×10=20**

20. Solve the differential equation by the method of Laplace transform

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t,\ y(0) = 0,\ y'(0) = 1$$

21(a) Find an analytic function f(z) = u+iv, if u = e^xsiny.

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(b) Find the fourier sine integral of the function

$$f(x) \begin{cases} = x^2 & for \quad 0 < x < b \\ = 0 & for \quad x > b \end{cases}$$

22. Obtain the general solution of wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. using variable separation method.

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PURBANCHAL UNIVERSITY
2019

Bachelor in Information Technology (B.I.T.)/Second Semester/*Final*

Time: 03:00 hrs. Full Marks: 80/Pass Marks: 32

BIT102SH: Mathematics-II *(New Course)*

Candidates are required to give their answers in their own words as far as practicable.

Figure in the margin indicate full marks.

Group A

Answer ALL questions. **10×2=20**

1. Solve $\frac{dy}{dx} = 3x^2$.

2. Solve $p^2 - 3p + 2 = 0$.

3. Find the Laplace transform of $\frac{\cos 2t}{t}$.

4. Find the residue of $\frac{z}{z^2 - 5z + 6}$ at $z = 3$.

5. Express in Taylor's series of cosz.

6. Define the fourier series ~~of the given funcyion~~.

7. State Cauchy Riemann equation for a analytic function f(z).

8. Test whether (2x - 3y) dx - 3xdy = 0 is exact or not.

9. Write one dimensional heat and wave equation.

10. Define Laplace transform.

Group B

Answer all questions. **8×5=40**

11. Solve $(D-4)^2 y = e^{4x}$.

12. Solve $\frac{dy}{dx} + \frac{2x}{x^2+2} y = \frac{1}{x}$.

Contd. ...

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13. Find the laplace transform of $\left(\frac{1-\cos t}{t}\right)$.

14. Find the inverse laplace transform of $\frac{s+1}{(s+2)(s^2+2)}$.

15. Express $f(x)=x-x^2; \pi \leq x \leq \pi$. in fourier series.

16. Show that by fourier sine integral of $e^{-x}\cos x$ is $\int_0^\infty \frac{w^3 \sin wx}{w^4+4} dw$

$= \pi/2\, e^{-x} \cos x$.

17. Express $f(z)=\frac{1}{z^2-3z+2}$ in Laurents series in the region

$1 \leq |z| \leq 2$.

18. Show that the function $u(x,y)=3x^2y+x^2-y^3-y^2$ is harmonic function. Find its harmonic conjugate.

Group C

Answer all questions. **2×10=20**

19. Solve using Laplace Transform

$$2y''+5y'+2y=e^{-2t}; y(0)=1; y'(0)=1.$$

20. A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperate and temperature initially is

$$f(x)=x; 0 \leq x \leq 50$$
$$=100-x; 50 \leq x \leq 100$$

Find the temperature $u(x,t)$ at anytime.

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