

Question 1 Provide a short answer to the following questions.

(a) What is the benefit to using a binary tree over other data structures?

(b) Give an example of when you would use a graph?

(c) What is a clique?

(d) What is one quality that *all* trees exhibit, that graphs, in general, do not?

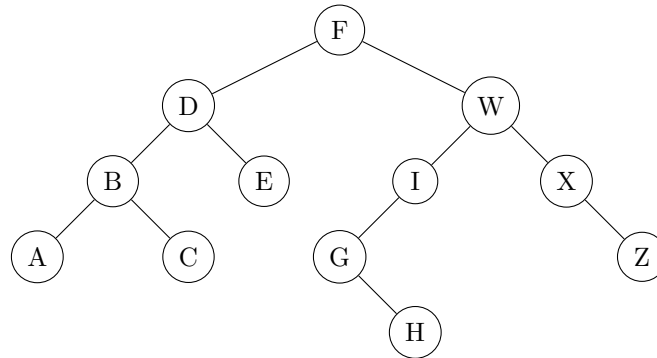
(e) What is the balance property all *AVL Trees* seek to maintain?

Question 2 Build a proper *AVL Tree* given the following inputs. Show all steps. Include – at each step – the balance factor of each node.

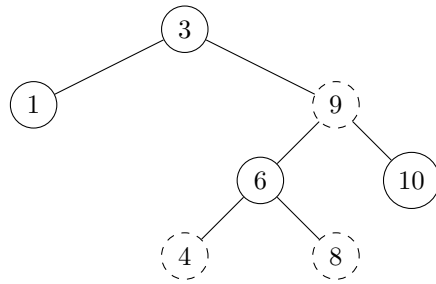
3, 2, 1, 4, 5, 6

Question 3 Provide all of the listed traversals for the following binary tree. Be sure to label them.

- Pre-order traversal.
- In-order traversal.
- Breadth-first traversal.



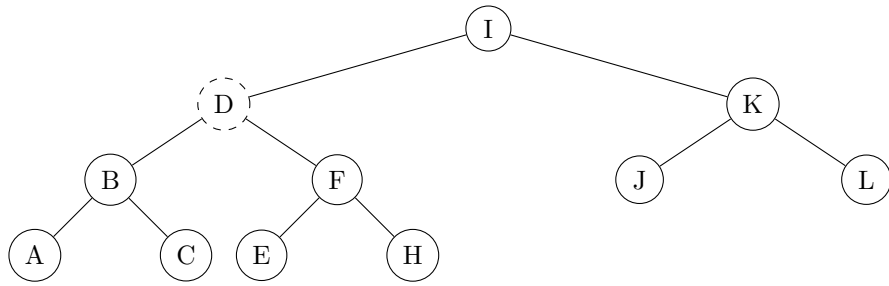
Question 4 Insert the value 5 into the following *Red-Black Tree*. Denote red nodes with a dashed outline, black nodes with a solid circle, and double-black nodes with a double-solid circle. Show all steps.



Question 5 Build a proper *2-3-4 Tree* from the following input. Show all steps, including rotations.

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insert 1,3,4,6  
delete 3  
insert 2, 7, 8, 9  
delete 6
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Question 6 Delete the I node from the following *Red-Black Tree*. Denote red nodes with a dashed outline, black nodes with a solid circle, and double-black nodes with a double-solid circle. Show all steps.



Question 7 Given the following adjacency matrix, draw the weighted, undirected graph with $V = \{v_0, v_1, v_2, v_3, v_4, v_5\}$.

$$\begin{bmatrix} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 4 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

NO ILLUSTRATIONS

Question 8 Given the following graph $G = (V, E)$, list the vertices that form a connected component with v_3 .

$$V = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$$

$$E = \{\{v_0, v_1\}, \{v_1, v_3\}, \{v_0, v_3\}, \{v_3, v_4\}, \{v_4, v_6\}, \{v_2, v_5\}, \{v_5, v_7\}, \{v_5, v_8\}, \{v_7, v_8\}, \{v_7, v_9\}, \{v_8, v_9\}\}$$

NO ILLUSTRATIONS

Question 9 Use Kruskal's Algorithm to calculate the minimum spanning forest of the following graph $G = (V, E, w)$. Show all steps. List all vertices in a particular spanning tree, and give its final cost.

$$V = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$$

E	w
v_1, v_2	1
v_1, v_4	2.5
v_2, v_3	1.5
v_5, v_0	7
v_0, v_7	0.2
v_6, v_3	8.4
v_8, v_9	2.6

NO ILLUSTRATIONS

Question 10 Given the graph $G = (V, E, w)$, below, find the shortest path between v_2 and v_6 .

$$V = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$$

E	w
v_0, v_1	0.5
v_0, v_3	1.2
v_0, v_4	0.3
v_1, v_2	1.9
v_1, v_3	2.2
v_1, v_5	1.3
v_2, v_3	4.7
v_2, v_7	9.1
v_4, v_6	2.7
v_5, v_6	3.1
v_6, v_7	2.8