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PART I: Analysis of Normal Code

QUESTION#1

Choose the best matching order of growth of the running time

- ```

public static int f1(int N) {
 int x = 0;
 for (int i = 0; i < N; i++)
 x++;
 return x;
}

```

(A) N (B) X (C) NX
- ```

public static int f2(int N, int R) {
    int x = 0;
    for (int i = 0; i < R; i++)
        x += f1(i);
    return x;
}

```

(A) 2^R (B) R^2 (C) RN
- ```

public static int f4(int N, int R) {
 int x = 0;
 for (int i = 0; i < N; i++)
 for (int j = 1; j <= R; j += j)
 x++;
 return x;
}

```

(A) RN (B)  $N \log(R)$  (C)  $N^R$

| Question | Answer |
|----------|--------|
| 1        |        |
| 2        |        |
| 3        |        |
| 4        |        |
| 5        |        |

### QUESTION#2

For each group of functions, sort the functions in increasing order of asymptotic (big-O) complexity:

| Group1                                                                                            | Group2                                                                                              | Group3                                                                                                        |
|---------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| $f_1(n) = n^{0.999999} \log n$<br>$f_2(n) = 10000000n$<br>$f_3(n) = 1.000001^n$<br>$f_4(n) = n^2$ | $f_1(n) = 2^{1000000}$<br>$f_2(n) = 2^{100000n}$<br>$f_3(n) = \binom{n}{2}$<br>$f_4(n) = n\sqrt{n}$ | $f_1(n) = n^{\sqrt{n}}$<br>$f_2(n) = 2^n$<br>$f_3(n) = n^{10} \cdot 2^{n/2}$<br>$f_4(n) = \sum_{i=1}^n (i+1)$ |

1 -> 2 -> 4 -> 3



### Group1

**Solution:** The correct order of these functions is  $f_1(n)$ ,  $f_2(n)$ ,  $f_4(n)$ ,  $f_3(n)$ . To see why  $f_1(n)$  grows asymptotically slower than  $f_2(n)$ , recall that for any  $c > 0$ ,  $\log n$  is  $O(n^c)$ . Therefore we have:

$$f_1(n) = n^{0.999999} \log n = O(n^{0.999999} \cdot n^{0.000001}) = O(n) = O(f_2(n))$$

The function  $f_2(n)$  is linear, while the function  $f_4(n)$  is quadratic, so  $f_2(n)$  is  $O(f_4(n))$ . Finally, we know that  $f_3(n)$  is exponential, which grows much faster than quadratic, so  $f_4(n)$  is  $O(f_3(n))$ .

### Group2

**Solution:** The correct order of these functions is  $f_1(n)$ ,  $f_4(n)$ ,  $f_3(n)$ ,  $f_2(n)$ . The variable  $n$  never appears in the formula for  $f_1(n)$ , so despite the multiple exponentials,  $f_1(n)$  is constant. Hence, it is asymptotically smaller than  $f_4(n)$ , which does grow with  $n$ . We may rewrite the formula for  $f_4(n)$  to be  $f_4(n) = n\sqrt{n} = n^{1.5}$ . The value of  $f_3(n) = \binom{n}{2}$  is given by the formula  $n(n-1)/2$ , which is  $\Theta(n^2)$ . Hence,  $f_4(n) = n^{1.5} = O(n^2) = O(f_3(n))$ . Finally,  $f_2(n)$  is exponential, while  $f_3(n)$  is quadratic, meaning that  $f_3(n)$  is  $O(f_2(n))$ .

## PART II: Analysis of Recursive Code

### QUESTION#1

|                                                           |                                                 |
|-----------------------------------------------------------|-------------------------------------------------|
| What's the exact <b>complexity</b> of the following code? |                                                 |
| <b>Fun2</b> (A, S, E)                                     | <input checked="" type="radio"/> a. $\Theta(N)$ |
| <b>IF</b> (S == E) <b>return</b> A[S]                     | b. $\Theta(N^3)$                                |
| S1 = (S + E) / 3                                          | c. $\Theta((\log(N))^3)$                        |
| S2 = 2 × (S + E) / 3                                      | d. $\Theta(\log(N))$                            |
| R1 = <b>Fun2</b> (A, S, S1)                               | e. $\Theta(3 \times \log(N))$                   |
| R2 = <b>Fun2</b> (A, S1+1, S2)                            | f. $\Theta(1)$                                  |
| R3 = <b>Fun2</b> (A, S2+1, E)                             |                                                 |
| <b>Return</b> Max (R1, R2, R3)                            |                                                 |

### QUESTION#2

For the following “Main” function:

|                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Main</b> (A, N)<br>I = N<br>Sum = 0<br><b>For</b> (J = 0; J < I; J += N/2)<br><b>While</b> (I > 1)<br>Sum = Sum + 1<br>I = I - 1<br><b>EndWhile</b><br>I = N<br><b>EndFor</b><br><b>Return</b> 5 × <b>Fun</b> (N, 4)<br><b>End Main</b> | <b>Fun</b> (Z, L)<br><b>If</b> (Z == 2)<br><b>Return</b> 0<br><b>For</b> I = 1 to L<br><b>If</b> (random(1,1000) % 2 == 0)<br>X += I × <b>Fun</b> ( $\frac{Z}{L}$ , L)<br><b>Else</b><br>X += (I + 1) × <b>Fun</b> ( $\frac{Z}{2 \times L}$ , L)<br><b>EndFor</b><br><b>End Fun</b><br>//random(1,1000) : generates a random integer<br>between 1 & 1000 in O(1) |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

1. What's the complexity of the non-recursive part of code?  $O(n)$
2. What's the recurrence equation (T(N)) that represents the LOWER BOUND of the entire code?  $\text{theta}(n^{0.6667})$ ,  $T(N) = 4T(N/8) + \text{theta}(1)$ ;
3. What's the LOWER BOUND complexity of entire code?  $\text{omega}(n)$
4. What's the recurrence equation (T(N)) that represents the UPPER BOUND of the entire code?  $T(N) = 4T(N/4) + \text{theta}(1)$ ; -->  $\text{theta}(n)$
5. What's the UPPER BOUND complexity of entire code?  $O(n)$

so, as  $\text{omega} == \text{Big O}$   
then,  $\text{theta}$  of entire code = n;



## PART III: Asymptotic Notations

### QUESTION#1

|                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|----------------|-----------|----------------------------------|----------------|----------------|------------------|------------------------|
| <p>Chose the notation that <u><b>BEST REPRESENTS</b></u> the <b>WORST</b> case?</p> <p><b>Fun2 (N)</b></p> <pre>Sorted = true I = 1 While (I &lt; N AND Sorted == true)     If (A[I] &gt; A[I + 1])         Sorted = false     End If     I = I + 1 End while Print Sorted</pre> | <table><tr><td>a. <math>O(1)</math></td></tr><tr><td>b. <math>\Theta(1)</math></td></tr><tr><td>c. <math>O(N)</math></td></tr><tr><td><b>d. <math>\Omega(N)</math></b></td></tr><tr><td>e. <math>\Omega(1)</math></td></tr><tr><td>f. <math>\Omega(N)</math></td></tr><tr><td>g. <math>\Theta(N-1)</math></td></tr><tr><td>h. None of the choices</td></tr></table> | a. $O(1)$ | b. $\Theta(1)$ | c. $O(N)$ | <b>d. <math>\Omega(N)</math></b> | e. $\Omega(1)$ | f. $\Omega(N)$ | g. $\Theta(N-1)$ | h. None of the choices |
| a. $O(1)$                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |
| b. $\Theta(1)$                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |
| c. $O(N)$                                                                                                                                                                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |
| <b>d. <math>\Omega(N)</math></b>                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |
| e. $\Omega(1)$                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |
| f. $\Omega(N)$                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |
| g. $\Theta(N-1)$                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |
| h. None of the choices                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                     |           |                |           |                                  |                |                |                  |                        |

### QUESTION#2

|                                                                                                                                                   |                     |                 |                                   |                                              |
|---------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|-----------------|-----------------------------------|----------------------------------------------|
| Let $f(N) = 10N \times \log_2(N)$ and $g(N) = N$ , what's the possible value(s) of $N_0$ that makes $f(N) = \Omega(g(N))$ for constant $C = 20$ ? |                     |                 |                                   |                                              |
| a. $1 \leq N_0 \leq 32$                                                                                                                           | b. $0 < N_0 \leq 2$ | c. $N_0 \geq 2$ | <b>d. <math>N_0 \geq 4</math></b> | e. No values exist. The relation is no valid |

### QUESTION#3

|                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                |                |                       |                |                   |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|----------------|-----------------------|----------------|-------------------|
| <p>Given the following orders (<b>log</b> is base 2):</p> <p>1) <math>\sqrt{e^{\log(N)} + N^3}</math>, 2) <math>1/\sqrt{\log(N)}</math>, 3) <math>N</math>, 4) <math>N^3</math>, 5) <math>4^{\log(N)}</math>, 6) <math>N \log(N)</math></p> <p>Arrange them in <b>increasing order of growth rate</b> (with <math>g(n)</math> following <math>f(n)</math> in your list if and only if <math>f(n) = O(g(n))</math>). Chose the correct order?</p> |                |                |                       |                |                   |
| a. 2,3,6,5,1,4                                                                                                                                                                                                                                                                                                                                                                                                                                   | b. 2,3,1,6,5,4 | c. 2,6,3,1,5,4 | <b>d. 2,3,6,1,5,4</b> | e. 4,5,1,3,6,2 | f. None of choice |

## PART IV: Recurrence Equations

### QUESTION#1

Using the RECURSION TREE method, Answer the following questions:

$$T(N) = T(N/5) + T(7N/10) + \Theta(N); T(1) = 1$$

| Question                                                 | Answer          |
|----------------------------------------------------------|-----------------|
| 1. What's the EXACT total number of levels in this tree? | $\lg_{10/7}(n)$ |
| 2. What's the complexity of each level?                  | $(9/10)^i * n$  |
| 3. What's the complexity of LAST level?                  | 1               |
| 4. What's the UPPER BOUND order of this tree?            | $O(n)$          |

### QUESTION#2

Given  $T(N) = 64 T(N / 16) + (22/7) N \sqrt{N}$ , using Master Method: what's the correct value of  $\epsilon$

- ☐  $0 < \epsilon \leq 1.5$
- ☐  $0 < \epsilon \leq 0.5$
- ☐  $\epsilon = 1.6$
- ☐ No possible  $\epsilon$
- ☒ It's case2: no  $\epsilon$  is required

### QUESTION#3

1. Select the correct asymptotic complexity of an algorithm with runtime  $T(n; n)$  where:

$$\begin{aligned}
 T(x, c) &= \Theta(x) && \text{for } c \leq 2, \\
 T(c, y) &= \Theta(y) && \text{for } c \leq 2, \text{ and} \\
 T(x, y) &= \Theta(x + y) + T(x/2, y/2).
 \end{aligned}$$

1.  $\Theta(\log n)$ .
2.  $\Theta(n)$ .
3.  $\Theta(n \log n)$ .
4.  $\Theta(n \log^2 n)$ .
5.  $\Theta(n^2)$ .
6.  $\Theta(2^n)$ .

**Solution:** The correct answer is  $\Theta(n)$ . To see why, we rewrite the recurrence relation to avoid  $\Theta$  notation as follows:

$$T(x, y) = c(x + y) + T(x/2, y/2).$$

We may then begin to replace  $T(x/2, y/2)$  with the recursive formula containing it:

$$T(x, y) = c(x + y) + c\left(\frac{x + y}{2}\right) + c\left(\frac{x + y}{4}\right) + c\left(\frac{x + y}{8}\right) + \dots$$

This geometric sequence is bounded from above by  $2c(x + y)$ , and is obviously bounded from below by  $c(x + y)$ . Therefore,  $T(x, y)$  is  $\Theta(x + y)$ , and so  $T(n, n)$  is  $\Theta(n)$ .

2. Select the correct asymptotic complexity of an algorithm with runtime  $T(n; n)$  where:

$$\begin{aligned} T(x, c) &= \Theta(x) && \text{for } c \leq 2, \\ T(c, y) &= \Theta(y) && \text{for } c \leq 2, \text{ and} \\ T(x, y) &= \Theta(x) + T(x, y/2). \end{aligned}$$

1.  $\Theta(\log n)$ .
2.  $\Theta(n)$ .
3.  $\Theta(n \log n)$ .
4.  $\Theta(n \log^2 n)$ .
5.  $\Theta(n^2)$ .
6.  $\Theta(2^n)$ .



**Solution:** The correct answer is  $\Theta(n \log n)$ . To see why, we rewrite the recurrence relation to avoid  $\Theta$  notation as follows:

$$T(x, y) = cx + T(x, y/2).$$

We may then begin to replace  $T(x, y/2)$  with the recursive formula containing it:

$$T(x, y) = \underbrace{cx + cx + cx + \dots + cx}_{\Theta(\log y) \text{ times}}.$$

As a result,  $T(x, y)$  is  $\Theta(x \log y)$ . When we substitute  $n$  for  $x$  and  $y$ , we get that  $T(n, n)$  is  $\Theta(n \log n)$ .

3. Select the correct asymptotic complexity of an algorithm with runtime  $T(n; n)$  where:

$$\begin{aligned} T(x, c) &= \Theta(x) && \text{for } c \leq 2, \\ T(x, y) &= \Theta(x) + S(x, y/2), \\ S(c, y) &= \Theta(y) && \text{for } c \leq 2, \text{ and} \\ S(x, y) &= \Theta(y) + T(x/2, y). \end{aligned}$$

1.  $\Theta(\log n)$ .
2.  $\Theta(n)$ .
3.  $\Theta(n \log n)$ .
4.  $\Theta(n \log^2 n)$ .
5.  $\Theta(n^2)$ .
6.  $\Theta(2^n)$ .