Summary of TLA$^+$

Module-Level Constructs

──── MODULE M ────

Begins the module or submodule named M.

EXTENDS $M_1, \ldots, M_n$

Incorporates the declarations, definitions, assumptions, and theorems from the modules named $M_1, \ldots, M_n$ into the current module.

CONSTANTS $C_1, \ldots, C_n$ [(1)]

Declares the C_j to be constant parameters (rigid variables). Each C_j is either an identifier or has the form $C(\_, \ldots, \_)$, the latter form indicating that C is an operator with the indicated number of arguments.

VARIABLES $x_1, \ldots, x_n$ [(1)]

Declares the x_j to be variables (parameters that are flexible variables).

ASSUME P

Asserts P as an assumption.

$F(x_1, \ldots, x_n) \;\triangleq\; exp$

Defines F to be the operator such that $F(e_1, \ldots, e_n)$ equals *exp* with each identifier x_k replaced by e_k. (For $n = 0$, it is written $F \triangleq exp$.)

$f[x \in S] \;\triangleq\; exp$ [(2)]

Defines f to be the function with domain S such that $f[x] = exp$ for all x in S. (The symbol f may occur in *exp*, allowing a recursive definition.)

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(1) The terminal S in the keyword is optional.

(2) $x \in S$ may be replaced by a comma-separated list of items $v \in S$, where v is either a comma-separated list or a tuple of identifiers.

INSTANCE M WITH $p_1 \leftarrow e_1, \ldots, p_m \leftarrow e_m$

For each defined operator F of module M, this defines F to be the operator whose definition is obtained from the definition of F in M by replacing each declared constant or variable p_j of M with e_j. (If $m = 0$, the WITH is omitted.)

$N(x_1, \ldots, x_n) \;\triangleq\;$ INSTANCE M WITH $p_1 \leftarrow e_1, \ldots, p_m \leftarrow e_m$

For each defined operator F of module M, this defines $N(d_1, \ldots, d_n)!F$ to be the operator whose definition is obtained from the definition of F by replacing each declared constant or variable p_j of M with e_j, and then replacing each identifier x_k with d_k. (If $m = 0$, the WITH is omitted.)

THEOREM P

Asserts that P can be proved from the definitions and assumptions of the current module.

LOCAL *def*

Makes the definition(s) of *def* (which may be a definition or an INSTANCE statement) local to the current module, thereby not obtained when extending or instantiating the module.

Ends the current module or submodule.

The Constant Operators

Logic

$\land \quad \lor \quad \neg \quad \Rightarrow \quad \equiv$

TRUE   FALSE   BOOLEAN   [the set {TRUE, FALSE}]

$\forall x \in S : p$ (1) $\exists x \in S : p$ (1)

CHOOSE $x \in S : p$ [An x in S satisfying p]

Sets

$= \quad \neq \quad \in \quad \notin \quad \cup \quad \cap \quad \subseteq \quad \setminus$ [set difference]

$\{e_1, \ldots, e_n\}$	[Set consisting of elements e_i]
$\{x \in S : p\}$ (2)	[Set of elements x in S satisfying p]
$\{e : x \in S\}$ (1)	[Set of elements e such that x in S]
SUBSET S	[Set of subsets of S]
UNION S	[Union of all elements of S]

Functions

$f[e]$	[Function application]
DOMAIN f	[Domain of function f]
$[x \in S \mapsto e]$ (1)	[Function f such that $f[x] = e$ for $x \in S$]
$[S \rightarrow T]$	[Set of functions f with $f[x] \in T$ for $x \in S$]
$[f$ EXCEPT $![e_1] = e_2]$ (3)	[Function $\widehat{f}$ equal to f except $\widehat{f}[e_1] = e_2$]

Records

$e.h$	[The h-field of record e]
$[h_1 \mapsto e_1, \ldots, h_n \mapsto e_n]$	[The record whose h_i field is e_i]
$[h_1 : S_1, \ldots, h_n : S_n]$	[Set of all records with h_i field in S_i]
$[r$ EXCEPT $!.h = e]$ (3)	[Record $\widehat{r}$ equal to r except $\widehat{r}.h = e$]

Tuples

$e[i]$	[The i^{th} component of tuple e]
$\langle e_1, \ldots, e_n \rangle$	[The n-tuple whose i^{th} component is e_i]
$S_1 \times \ldots \times S_n$	[The set of all n-tuples with i^{th} component in S_i]

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(1) $x \in S$ may be replaced by a comma-separated list of items $v \in S$, where v is either a comma-separated list or a tuple of identifiers.

(2) x may be an identifier or tuple of identifiers.

(3) $![e_1]$ or $!.h$ may be replaced by a comma separated list of items $!a_1 \cdots a_n$, where each a_i is $[e_i]$ or $.h_i$.

Miscellaneous Constructs

IF p THEN e_1 ELSE e_2	[e_1 if p true, else e_2]
CASE $p_1 \rightarrow e_1 \;\square\; \ldots \;\square\; p_n \rightarrow e_n$	[Some e_i such that p_i true]
CASE $p_1 \rightarrow e_1 \;\square\; \ldots \;\square\; p_n \rightarrow e_n \;\square$ OTHER $\rightarrow e$	[Some e_i such that p_i true, or e if all p_i are false]

LET $d_1 \triangleq e_1 \;\ldots\; d_n \triangleq e_n$ IN e [e in the context of the definitions]

$$\begin{array}{l} \land\; p_1 \\ \quad\vdots \\ \land\; p_n \end{array} \quad \text{[the conjunction } p_1 \land \ldots \land p_n\text{]} \qquad \begin{array}{l} \lor\; p_1 \\ \quad\vdots \\ \lor\; p_n \end{array} \quad \text{[the disjunction } p_1 \lor \ldots \lor p_n\text{]}$$

Action Operators

e'	[The value of e in the final state of a step]
$[A]_e$	[$A \lor (e' = e)$]
$\langle A \rangle_e$	[$A \land (e' \neq e)$] 要求e'会变化
ENABLED A	[An A step is possible]
UNCHANGED e	[$e' = e$]
$A \cdot B$	[Composition of actions]

Temporal Operators

$\square F$	[F is always true]
$\diamond F$	[F is eventually true]
$\mathrm{WF}_e(A)$	[Weak fairness for action A]
$\mathrm{SF}_e(A)$	[Strong fairness for action A]
$F \leadsto G$	[F leads to G]

User-Definable Operator Symbols

Infix Operators

$+$ (1)	$-$ (1)	$*$ (1)	$/$ (2)	$\circ$ (3)	$++$
$\div$ (1)	$\%$ (1)	$\hat{}$ (1,4)	$..$ (1)	$\dots$	$--$
$\oplus$ (5)	$\ominus$ (5)	$\otimes$	$\oslash$	$\odot$	$**$
$<$ (1)	$>$ (1)	$\leq$ (1)	$\geq$ (1)	$\sqcap$	$//$
$\prec$	$\succ$	$\preceq$	$\succeq$	$\sqcup$	$\hat{}\hat{}$
$\ll$	$\gg$	$<:$	$:>$ (6)	$\&$	$\&\&$
$\sqsubset$	$\sqsupset$	$\sqsubseteq$ (5)	$\sqsupseteq$	$\mid$	$\%\%$
$\subset$	$\supset$		$\supseteq$	$\star$	$@@$ (6)
$\vdash$	$\dashv$	$\models$	$=\!\mid$	$\bullet$	$\#\#$
$\sim$	$\simeq$	$\approx$	$\cong$	$\$$	$\$\$$
$\bigcirc$	$::=$	$\asymp$	$\doteq$	$??$	$!!$
$\propto$	$\wr$	$\uplus$			

Postfix Operators (7)

$\hat{}+$ $\hat{}*$ $\hat{}\#$

(1) Defined by the *Naturals*, *Integers*, and *Reals* modules.
(2) Defined by the *Reals* module.
(3) Defined by the *Sequences* module.
(4) $x\hat{}y$ is printed as x^y.
(5) Defined by the *Bags* module.
(6) Defined by the *TLC* module.
(7) $e\hat{}+$ is printed as e^+, and similarly for $\hat{}*$ and $\hat{}\#$.

Precedence Ranges of Operators

The relative precedence of two operators is unspecified if their ranges overlap. Left-associative operators are indicated by (a).

Prefix Operators

$\neg$	4–4	$\Box$	4–15	UNION	8–8
ENABLED	4–15	$\Diamond$	4–15	DOMAIN	9–9
UNCHANGED	4–15	SUBSET	8–8	$-$	12–12

Infix Operators

$\Rightarrow$	1–1	$\leq$	5–5	$<:$	7–7	$\ominus$	11–11 (a)
$\stackrel{+}{\rhd}$	2–2	$\ll$	5–5	$\backslash$	8–8	$-$	11–11 (a)
$\equiv$	2–2	$\prec$	5–5	$\cap$	8–8 (a)	$--$	11–11 (a)
$\leadsto$	2–2	$\preceq$	5–5	$\cup$	8–8 (a)	$\&$	13–13 (a)
$\land$	3–3 (a)	$\propto$	5–5	$..$	9–9	$\&\&$	13–13 (a)
$\lor$	3–3 (a)	$\sim$	5–5	$\ldots$	9–9	$\odot$	13–13 (a)
$\neq$	5–5	$\simeq$	5–5	$!!$	9–13	$\oslash$	13–13
$\dashv$	5–5	$\sqsubset$	5–5	$\#\#$	9–13 (a)	$\otimes$	13–13 (a)
$::=$	5–5	$\sqsubseteq$	5–5	$\$$	9–13 (a)	$*$	13–13 (a)
$:=$	5–5	$\sqsupset$	5–5	$\$\$$	9–13 (a)	$**$	13–13 (a)
$<$	5–5	$\sqsupseteq$	5–5	$??$	9–13 (a)	$/$	13–13
$=$	5–5	$\subset$	5–5	$\sqcap$	9–13 (a)	$//$	13–13
$=\mid$	5–5	$\subseteq$	5–5	$\sqcup$	9–13 (a)	$\bigcirc$	13–13 (a)
$>$	5–5	$\succ$	5–5	$\uplus$	9–13 (a)	$\bullet$	13–13 (a)
$\approx$	5–5	$\succeq$	5–5	$\wr$	9–14	$\div$	13–13
$\asymp$	5–5	$\supset$	5–5	$\oplus$	10–10 (a)	$\circ$	13–13 (a)
$\cong$	5–5	$\supseteq$	5–5	$+$	10–10 (a)	$\star$	13–13 (a)
$\doteq$	5–5	$\vdash$	5–5	$++$	10–10 (a)	$\hat{}$	14–14
$\geq$	5–5	$\models$	5–5	$\%$	10–11	$\hat{}\hat{}$	14–14
$\gg$	5–5	$\cdot$ [1]	5–14 (a)	$\%\%$	10–11 (a)	. [2]	17–17 (a)
$\in$	5–5	$@@$	6–6 (a)	$\vert$	10–11 (a)		
$\notin$	5–5	$:>$	7–7	$\Vert$	10–11 (a)		

Postfix Operators

`^+`	15–15	`^*`	15–15	`^#`	15–15	$'$	15–15

(1) Action composition (`\cdot`).
(2) Record field (period).

Operators Defined in Standard Modules.

Modules *Naturals*, *Integers*, *Reals*

$+$	$-$ (1)	$*$	$/$ (2)	$\hat{}$ (3)	$..$	Nat	$Real$ (2)
$\div$	$\%$	$\leq$	$\geq$	$<$	$>$	Int (4)	$Infinity$ (2)

(1) Only infix $-$ is defined in *Naturals*.
(2) Defined only in *Reals* module.
(3) Exponentiation.
(4) Not defined in *Naturals* module.

Module *Sequences*

$\circ$	$Head$	$SelectSeq$	$SubSeq$
$Append$	Len	Seq	$Tail$

Module *FiniteSets*

$IsFiniteSet$ $\quad$ $Cardinality$

Module *Bags*

$\oplus$	$BagIn$	$CopiesIn$	$SubBag$
$\ominus$	$BagOfAll$	$EmptyBag$	
$\sqsubseteq$	$BagToSet$	$IsABag$	
$BagCardinality$	$BagUnion$	$SetToBag$	

Module *RealTime*

$RTBound$ $\quad$ $RTnow$ $\quad$ now (declared to be a variable)

Module *TLC*

$:>$ $\quad$ $@@$ $\quad$ $Print$ $\quad$ $Assert$ $\quad$ $JavaTime$ $\quad$ $Permutations$
$SortSeq$

ASCII Representation of Typeset Symbols

<table>
<tr><td>$\land$</td><td>/\ or \land</td><td>$\lor$</td><td>\/ or \lor</td><td>$\Rightarrow$</td><td>=></td></tr>
<tr><td>$\lnot$</td><td>~ or \lnot or \neg</td><td>$\equiv$</td><td><=> or \equiv</td><td>$\triangleq$</td><td>==</td></tr>
<tr><td>$\in$</td><td>\in</td><td>$\notin$</td><td>\notin</td><td>$\neq$</td><td># or /=</td></tr>
<tr><td>$\langle$</td><td><<</td><td>$\rangle$</td><td>>></td><td>$\Box$</td><td>[]</td></tr>
<tr><td>$<$</td><td><</td><td>$>$</td><td>></td><td>$\Diamond$</td><td><></td></tr>
<tr><td>$\leq$</td><td>\leq or =< or <=</td><td>$\geq$</td><td>\geq or >=</td><td>$\leadsto$</td><td>~></td></tr>
<tr><td>$\ll$</td><td>\ll</td><td>$\gg$</td><td>\gg</td><td>$\stackrel{+}{\rhd}$</td><td>-+-></td></tr>
<tr><td>$\prec$</td><td>\prec</td><td>$\succ$</td><td>\succ</td><td>$\mapsto$</td><td>|-></td></tr>
<tr><td>$\preceq$</td><td>\preceq</td><td>$\succeq$</td><td>\succeq</td><td>$\div$</td><td>\div</td></tr>
<tr><td>$\subseteq$</td><td>\subseteq</td><td>$\supseteq$</td><td>\supseteq</td><td>$\cdot$</td><td>\cdot</td></tr>
<tr><td>$\subset$</td><td>\subset</td><td>$\supset$</td><td>\supset</td><td>$\circ$</td><td>\o or \circ</td></tr>
<tr><td>$\sqsubset$</td><td>\sqsubset</td><td>$\sqsupset$</td><td>\sqsupset</td><td>$\bullet$</td><td>\bullet</td></tr>
<tr><td>$\sqsubseteq$</td><td>\sqsubseteq</td><td>$\sqsupseteq$</td><td>\sqsupseteq</td><td>$\star$</td><td>\star</td></tr>
<tr><td>$\vdash$</td><td>|-</td><td>$\dashv$</td><td>-|</td><td>$\bigcirc$</td><td>\bigcirc</td></tr>
<tr><td>$\models$</td><td>|=</td><td>$=\!\!|$</td><td>=|</td><td>$\sim$</td><td>\sim</td></tr>
<tr><td>$\rightarrow$</td><td>-></td><td>$\leftarrow$</td><td><-</td><td>$\simeq$</td><td>\simeq</td></tr>
<tr><td>$\cap$</td><td>\cap or \intersect</td><td>$\cup$</td><td>\cup or \union</td><td>$\asymp$</td><td>\asymp</td></tr>
<tr><td>$\sqcap$</td><td>\sqcap</td><td>$\sqcup$</td><td>\sqcup</td><td>$\approx$</td><td>\approx</td></tr>
<tr><td>$\oplus$</td><td>(+) or \oplus</td><td>$\uplus$</td><td>\uplus</td><td>$\cong$</td><td>\cong</td></tr>
<tr><td>$\ominus$</td><td>(-) or \ominus</td><td>$\times$</td><td>\X or \times</td><td>$\doteq$</td><td>\doteq</td></tr>
<tr><td>$\odot$</td><td>(.) or \odot</td><td>$\wr$</td><td>\wr</td><td>x^y</td><td>x^y [2]</td></tr>
<tr><td>$\otimes$</td><td>(\X) or \otimes</td><td>$\propto$</td><td>\propto</td><td>x^+</td><td>x^+ [2]</td></tr>
<tr><td>$\oslash$</td><td>(/) or \oslash</td><td>“s”</td><td>"s" [1]</td><td>x^*</td><td>x^* [2]</td></tr>
<tr><td>$\exists$</td><td>\E</td><td>$\forall$</td><td>\A</td><td>$x^\#$</td><td>x^# [2]</td></tr>
<tr><td>$\boldsymbol{\exists}$</td><td>\EE</td><td>$\boldsymbol{\forall}$</td><td>\AA</td><td>$'$</td><td>'</td></tr>
<tr><td>$]_v$</td><td>]_v</td><td>$\rangle_v$</td><td>>>_v</td><td></td><td></td></tr>
<tr><td>WF_v</td><td>WF_v</td><td>SF_v</td><td>SF_v</td><td></td><td></td></tr>
<tr><td></td><td>-------- [3]</td><td></td><td>-------- [3]</td><td></td><td></td></tr>
<tr><td></td><td>-------- [3]</td><td></td><td>======== [3]</td><td></td><td></td></tr>
</table>

(1) s is a sequence of characters.

(2) x and y are any expressions.

(3) a sequence of four or more - or = characters.