Maximisation Problem

2058 (10)

$Max(Z) = Rs\,16x + Rs\,40y$

s.t. $3x + 5y \le 30$

$x + 2y \le 12$

$4x + 3y \le 36$

where $x, y \ge 0$

Let OX and OY be the two axes intersecting at origin O. Any point (x, y) satisfying the condition $x \ge 0$, $y \ge 0$ lies on the first quadrant.

The corresponding equations of the given constraints are

$3x + 5y = 30$ ·····(i)

$x + 2y = 12$ ·····(ii)

$4x + 3y = 36$ ·····(iii)

27

Now, let us find out extreme points of each equation

In eqn (i), put $x = 0$, $y = 6$ i.e. (0,6)

Put $y = 0$, $x = 10$ i.e. (10,0)

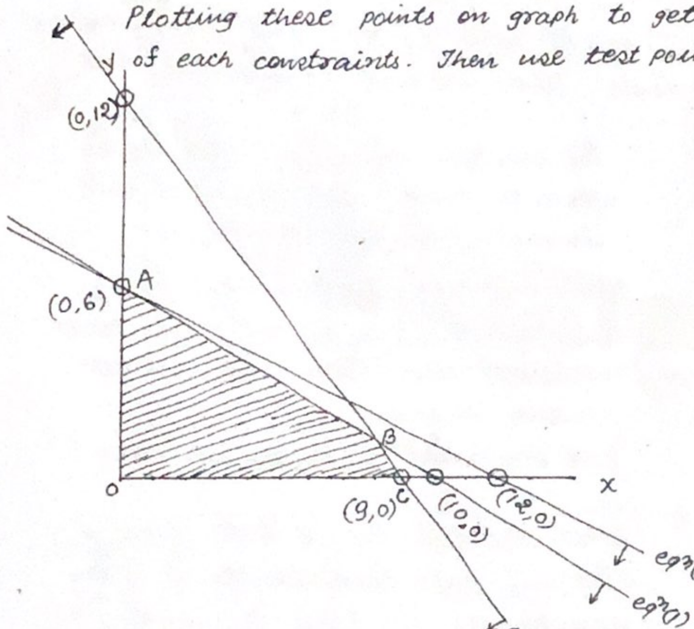
In eqn (ii), Put $x = 0$, $y = 6$ i.e. (0,6)

Put $y = 0$, $x = 12$ i.e. (12,0)

In eqn (iii), Put $x = 0$ $y = 12$ i.e. (0,12)

Put $y = 0$ $x = 9$ i.e. (9,0)

Plotting these points on graph to get the corresponding boundary lines of each constraints. Then use test point and get the feasible region.



OABC is the feasible solution. But coordinates of B is unknown. To find it, solve eqns (i) & (iii).

$3x + 5y = 30$ —(i) } ×4

$4x + 3y = 36$ —(iii) } ×3

$\cancel{12x} + 20y = 120$

$-\cancel{12x} + 9y = -108$

$11y = 12$

$y = 12/11$

$3x = 30 - 5y$

$3x = 30 - 5 \times \frac{12}{11}$

$x = \frac{270}{11 \times 3} = \frac{90}{11}$

i.e. $B(90/11, 12/11)$

Vertices	x	y	Z = Rs16x + Rs40y
O(0,0)	0	0	$Z = 16\times0 + 40\times0 = 0$
A (0,6)	0	6	$Z = 16\times0 + 40\times6 = 240$ ★
$B(\frac{90}{11}, \frac{12}{11})$	$\frac{90}{11}$	$\frac{12}{11}$	$Z = 16\times\frac{90}{11} + 40\times\frac{12}{11} = 174.54$
C (9,0)	9.	0	$Z = 16\times9 + 40\times0 = 144$

Decision: Maximum profit is Rs 240 which is at A(0,6).

∴ Max profit = Rs 240 when $x = 0$, $y = 6$

* Minimization Problem

$Min(C) = 2x_1 + 8x_2$

s.t. $x_1 \le 20$

$x_2 \ge 14$

$5x_1 + 10x_2 = 150$

$(x_1, x_2 \ge 0)$

Let Ox_1 and Ox_2 be the two axes intersecting at origin $O(0,0)$. Any point (x_1, x_2) satisfying the condition $x_1 \ge 0$, $x_2 \ge 0$ lies on the 1st quadrant only. The corresponding equations of the given constraints are

$x_1 = 20$ ...... (I)

$x_2 = 14$ ...... (II)

$5x_1 + 10x_2 = 150$ ... (III)

28

Now, let us find out extreme points of each equation

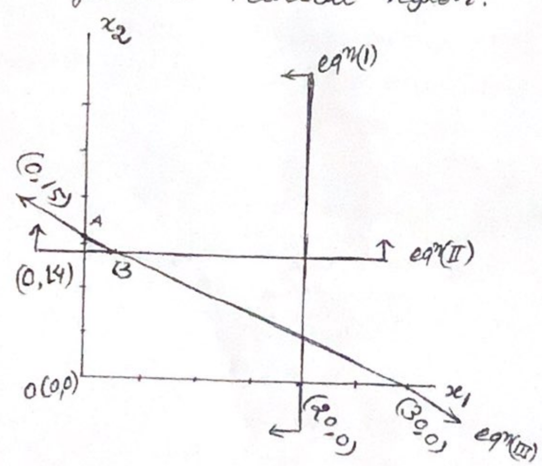
eqn(I) represents the st. line parallel to x_2-axis with distance of 20 units from the origin

eqn(II) represents the st. line parallel to x_1-axis with distance of 14 units from the origin.

In eqn(III), Put $x_1 = 0$, $x_2 = 15$ i.e. $(0,15)$

Put $x_2 = 0$, $x_1 = 30$ i.e. $(30,0)$

Plot these points along with (I) and (II) eqns to get the corresponding boundry lines of each constraints. Then we use test point and get the Feasible region.



Put $x_1=0$, $x_2=0$ in $x_1 \le 20$, then $0 \le 20$ which is true. plane region of this inequality contain the origin

Again put $x_1=0$, $x_2=0$ in $x_2 \ge 14$, then $0 \ge 14$ which is not true. Plane region of this inequality does not contain origin.

3rd constraint is purely equation.

Line segment AB is the Feasible region. But co-ordinates of B is not known. To Find it, solve eqn (II) and (III).

$x_2 = 14$ — (II)

$5x_1 + 10x_2 = 150$ — (III)

Put (II) in (III), we get $x_1 = \frac{10}{5} = 2$

$\therefore B(2,14)$

Evaluation of obj Function

vertices	obj Func. $Min(C) = 2x_1 + 8x_2$	Remarks
A(0,15)	$C = 2\times0 + 8\times15 = 120$	
B(2,14)	$C = 2\times2 + 8\times14 = 116$	Min(C)=116

$Min(C) = 116$ when $x_1 = 2$ and $x_2 = 14$

Minimisation Problem

$Min(Z) = 120x + 60y$

s.t. $3x+y \geq 15$, $x+5y \geq 20$, $3x+2y \geq 24$

$(x, y \geq 0)$

Let Ox and Oy be the two axes intersecting at origin O. Any point (x,y) satisfying the condition $x \geq 0, y \geq 0$ lies on the 1st quadrant only.

The corresponding equations of the given constraints are

$3x+y = 15$ --- (i)

$x+5y = 20$ -- (ii)

$3x+2y = 24$ --- (iii)

29

Now, let us find out extreme points of each equation.

In eqⁿ (i), put $x=0$, $y=15$ i.e. $(0,15)$

Put $y=0$, $x=5$ i.e. $(5,0)$

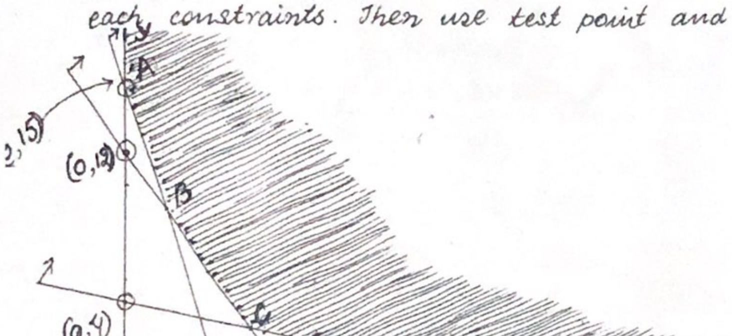
In eqⁿ (ii), Put $x=0$, $y=4$ i.e. $(0,4)$

Put $y=0$, $x=20$ i.e. $(20,0)$

In eqⁿ (iii), Put $x=0$, $y=12$ i.e. $(0,12)$

Put $y=0$ $x=8$ i.e. $(8,0)$

Plotting these points on graph to get the corresponding boundry lines of each constraints. Then use test point and get the feasible region.



ABCD is the feasible region. But co-ordinates B and C are unknown. To find B, solve eqⁿ (i) and (iii) and To find C, solve eqⁿ (ii) and (iii)

$3x + y = 15$ — (i)

$3x + 2y = 24$ —(iii)

$-y = -9 \quad \therefore y = 9$

and $x = (15-9)/3 = 2$

$\therefore B(2,9)$

$x + 5y = 20$ — (ii) $\times 3$

$3x + 2y = 24$ — (iii) $\times 1$

$3x + 15y = 60$

$3x + 2y = 24$

$13y = 36 \quad \therefore y = \frac{36}{13}$

and $x = 20 - 5 \times \frac{36}{13} = \frac{80}{13}$

$\therefore C = \left(\frac{80}{13}, \frac{36}{13}\right)$

vertices	x	y	$Z = 120x + 60y$
A (0,15)	0	15	$Z = 120\times0 + 60\times15 = 900$
B (2,9)	2	9	$Z = 120\times2 + 60\times9 = 780$
$C\left(\frac{80}{13}, \frac{36}{13}\right)$	$\frac{80}{13}$	$\frac{36}{13}$	$Z = 120\times\frac{80}{13} + 60\times\frac{36}{13} = 904.6$
D (20,0)	20	0	$Z = 120\times20 + 60\times0 = 2400$

Decision: Minimum value of Z = Rs 780 which occurs at vertex B(2,9).

$\therefore$ minimum cost = Rs 780 when $x = 2$ and $y = 9$.

$Z = 2x_1 + 5x_2$

s.t. $4x_2 - 3x_1 \leq 12$, $x_1 + x_2 \geq 2$, $x_1 \leq 4$

$(x_1, x_2 \geq 0)$

Let Ox_1 and Ox_2 be the two axes intersecting at origin O. Any point (x_1, x_2) satisfying the condition $x_1 \geq 0$, $x_2 \geq 0$ lies on the 1st quadrant only.

The corresponding equations of the given constraints are

$4x_2 - 3x_1 = 12$ —(i)

$x_1 + x_2 = 2$ —(ii)

$x_1 = 4$ —(iii)

30

Now, let us find out extreme points of each equation

In eqn (i), Put $x_1 = 0$, $x_2 = 3$ i.e (0, 3)

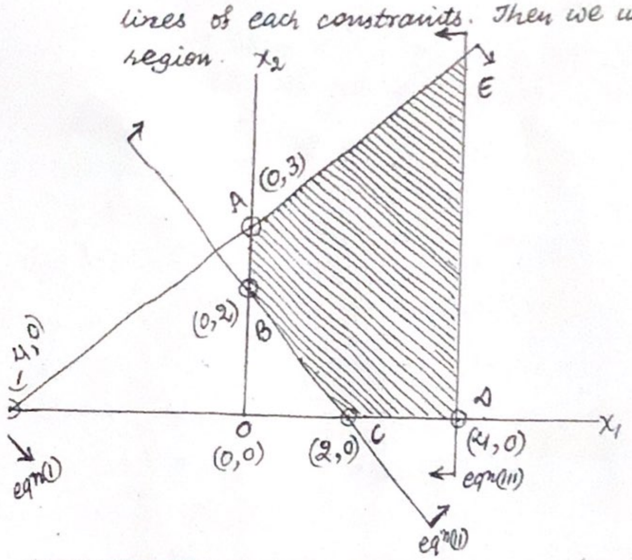
Put $x_2 = 0$, $x_1 = -4$ ie (-4, 0)

In eqn (ii), Put $x_1 = 0$, $x_2 = 2$ ie (0, 2)

Put $x_2 = 0$, $x_1 = 2$ ie (2, 0)

eqn (iii) represents the st. line parallel to x_2-axis with distance of 4 units from the origin

Plotting these points along with 3rd eqn to get the corresponding boundary lines of each constraints. Then we use test point and get the feasible region.



ABCDE is the feasible region. But co-ordinates of E is unknown. To find it, solve eqns (i) and (iii).

$4x_2 - 3x_1 = 12$ – (i)

$x_1 = 4$ — (iii)

Put (iii) in (i), we get

$x_2 = (12 + 3\times4)/4 = 6$

$\therefore$ E(4, 6)

vertices	x_1	x_2	$Z = 2x_1 + 5x_2$
A (0,3)	0	3	$Z = 2\times0 + 5\times3 = 15$
B (0,2)	0	2	$Z = 2\times0 + 5\times2 = 10$
C (2,0)	2	0	$Z = 2\times2 + 5\times0 = 4$
D (4,0)	4	0	$Z = 2\times4 + 5\times0 = 8$
E (4,6)	4	6	$Z = 2\times4 + 5\times6 = 38$

Decision:

i) Maximum value of $Z = 38$ at the vertex E (4,6) i.e. $x_1 = 4$, $x_2 = 6$

ii) Minimum value of $Z = 4$ at the vertex C (2,0) i.e. $x_1 = 2$, $x_2 = 0$

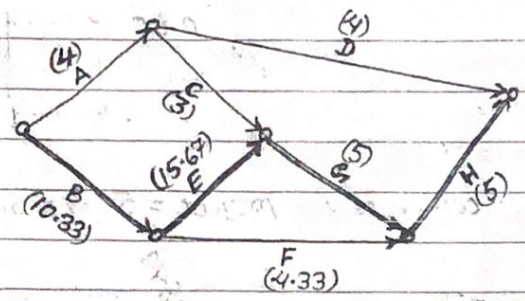
Network Analysis

(1) A small project composed of 8 activities and their time estimates in days are as follows:

Activity	A	B	C	D	E	F	G	H
Predecessor	None	None	A	A	B	B	CE	GF
optimistic(t_o)	1	5	3	1	8	2	5	2
most likely(t_m)	4	10	3	4	15	4	5	5
Pessimistic(t_p)	7	17	3	7	26	8	5	8

(a) Draw the project network and identify all the paths through it.
(b) Find the expected duration, s.d. and variance for each activity
(c) Calculate the s.d. and variance of the project
(d) Find the probability that project will be completed within 41 days
(e) Find the probability that project will be completed 5 days before the expected project completion time
(f) Find the probability that it would take 4 days more than the expected project completion time.
(g) Find the project completion time which will have 95% confidence.

35



Network diagram

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$\delta = \frac{t_p - t_o}{6}$$

$$\delta^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Calculation of Expected time, s.d. and variance of each activity

Activities	t_o	t_m	t_p	Expected time (t_e)	s.d.(δ)	variance(δ^2)
A	1	4	7	4	1	1
B	5	10	17	10.33	2	4
C	3	3	3	3	0	0
D	1	4	7	4	1	1
E	8	15	26	15.67	3	9
F	2	4	8	4.33	1	1
G	5	5	5	5	0	0
H	2	5	8	5	1	1

Calculation of total time taken by all Possible Paths

Possible Paths	Total time taken
A-D	4+4 = 8 days
A-C-G-H	4+3+5+5 = 17 days
B-E-G-H	10.33 + 15.67 + 5 + 5 = 36 days ★
B-F-H	10.33 + 4.33 + 5 = 19.67 days

BEGH is the longest time taking Path. So BEGH is the critical Path. Therefore expected project length = 36 days

S.d. of critical Path/Project, $\sigma_{cp} = \sqrt{\sigma_B^2 + \sigma_E^2 + \sigma_G^2 + \sigma_H^2}$

$= \sqrt{4+9+0+1} = 3.7416$ days

Variance of critical Path/project, $\sigma_{cp}^2 = \sigma_B^2 + \sigma_E^2 + \sigma_G^2 + \sigma_H^2$

Now, we have $= 4+9+0+1 = 14$

T_E = Expected Project completion time = 36 days

T_S = Scheduled time to complete the project

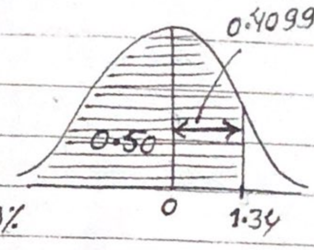
σ_{cp} = S.d. of critical Path/project = 3.7416 days

36

According to Normal distribution, we have

$$\boxed{Z = \frac{T_S - T_E}{\sigma_{cp}} = \frac{T_S - 36}{3.7416}}$$

(d) $P(T_S \leq 41) = ?$



when $T_S = 41$ days, $Z = \frac{41-36}{3.7416} = 1.34$

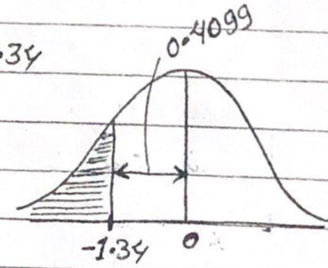
$\therefore P(T_S \leq 41) = P(Z \leq 1.34)$

$= 0.50 + 0.4099 = 0.9099 = 90.99\%$

(e) T_S = scheduled time to complete the project $= 36-5 = 31$ days

$P(T_S \leq 31) = ?$

when $T_S = 31$ days, $Z = \frac{31-36}{3.7416} = -1.34$



$\therefore P(T_S \leq 31) = P(Z \leq -1.34)$

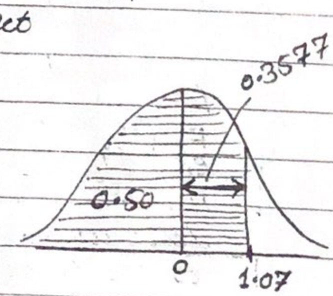
$= 0.50 - 0.4099$

$= 0.0901 = 9.01\%$

(f) T_S = Scheduled time to complete the project

$= 36 + 4 = 40$ days

$P(T_S \leq 40) = ?$



when $T_S = 40$ days, $Z = \frac{40-36}{3.7416} = 1.07$

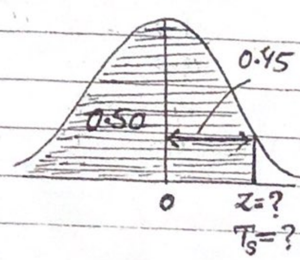
$\therefore P(T_S \leq 40) = P(Z \leq 1.07)$

$= 0.50 + 0.3577$

$= 0.8577 = 85.77\%$

(g) Scheduled completion time for the probability of completion to be 95% = ?

For P = 95%, Area under Normal curve on right side of perpendicular 0.45 corresponds to $Z = 1.65$



$Z = \frac{T_S - 36}{3.7416}$

$1.65 = \frac{T_S - 36}{3.7416}$, $\therefore T_S = 42.17$ days

(2) Following table lists the activity of a project along with their time estimates in days.

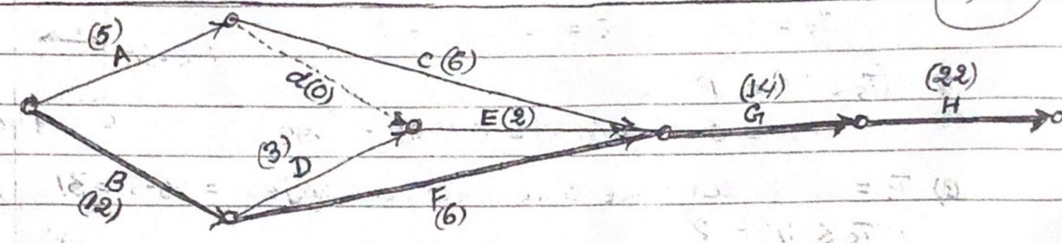
Activity	A	B	C	D	E	F	G	H
Predecessors	—	—	A	B	DA	B	CEF	G
Most likely (t_m)	5	12	5	3	2	6	14	20
Optimistic (t_o)	4	8	4	1	2	4	10	18
Pessimistic (t_p)	6	16	12	5	2	8	18	34

The scheduled completion date for this project is 60 days. Draw network diagram and find:

(a) Probability that project will be finished within scheduled date

(b) Probability that project will be completed 4 days prior to the expected time.

(c) What should be the scheduled completion time for the probability of completion to be 90%. 37



Network diagram

calculation of Expected time and Variance of each activities

Activity	t_m	t_o	t_p	Expected time $t_e = \frac{4t_m + t_o + t_p}{6}$	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A	5	4	6	5	1/9
B	12	8	16	12	16/9 •
C	5	4	12	6	16/9
D	3	1	5	3	4/9
E	2	2	2	2	0
F	6	4	8	6	4/9 •
G	14	10	18	14	16/9 •
H	20	18	34	22	64/9 •

Calculation of total time taken by all possible Paths

Possible Paths	Total time taken
A-C-G-H	5+6+14+22 = 47 days
A-d-E-G-H	5+0+2+14+22 = 43 days
B-D-E-G-H	12+3+2+14+22 = 53 days
B-F-G-H	12+6+14+22 = 54 days ★

BFGH is the longest time taking Path. So BFGH is the critical Path. Therefore expected project completion time = 54 days

S.d. of critical Path/Project, $\sigma_{cp} = \sqrt{\sigma_B^2 + \sigma_F^2 + \sigma_G^2 + \sigma_H^2}$

$$= \sqrt{16/9 + 4/9 + 16/9 + 64/9}$$

$$= \sqrt{\frac{100}{9}} = \frac{10}{3} = 3.33 \text{ days}$$

Now, we have

T_E = Expected Project completion time = 54 days

σ_{cp} = S.d. of critical Path/project = 3.33 days

T_S = Scheduled time to complete the project

38

According to Normal distribution, we have

$$\boxed{Z = \frac{T_S - T_E}{\sigma_{cp}} = \frac{T_S - 54}{3.33}}$$

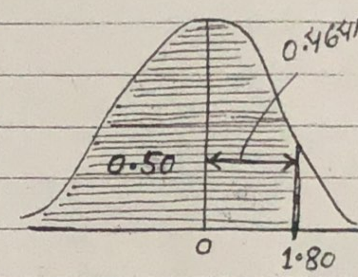
(a) $P(T_S \leq 60) = ?$

when $T_S = 60$, $Z = \frac{60-54}{3.33} = 1.80$

$\therefore P(T_S \leq 60) = P(Z \leq 1.80)$

$= 0.50 + 0.4641$

$= 0.9641 = 96.41\%$



(b) T_S = Scheduled time to complete the project = 54 − 4 = 50 days

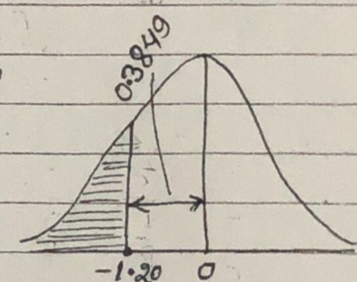
$P(T_S \leq 50) = ?$

when $T_S = 50$, $Z = \frac{50-54}{3.33} = -1.20$

$\therefore P(T_S \leq 50) = P(Z \leq -1.20)$

$= 0.50 - 0.3849$

$= 0.1151 = 11.51\%$

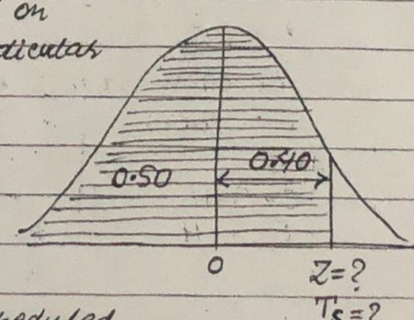


(c) Scheduled completion time for the probability of completion to be 90% = ?

For $P = 90\%$, Area under normal on ~~right side of perpendicular~~ 0.40 corresponds to $Z = 1.28$

$Z = \frac{T_S - 54}{3.33}$

$1.28 = \frac{T_S - 54}{3.33}$ $\therefore T_S = 58.26$ days



Hence for 90% probability, the scheduled completion time should be 58.26 days.