FIRST MULTICOLOUR EDITION

A TEXTBOOK OF

Machine Design

(S.I. UNITS)

[A Textbook for the Students of B.E. / B.Tech., U.P.S.C. (Engg. Services); Section 'B' of A.M.I.E. (I)]

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Top

Preface to the Fourteenth Edition

We feel satisfied in presenting the new edition of this popular treatise. The favourable and warm reception which the previous editions and reprints of this book have enjoyed all over India and abroad, is a matter of great satisfaction for us.

The present multicolour edition has been thoroughly revised and brought up-to-date. Multicolour pictures have been added to enhance the content value and to give the students an idea of what he will be dealing in reality, and to bridge the gap between theory and practice. This book has already been included in the 'Suggested Reading' for the A.M.I.E. (India) examinations. The mistakes which had crept in, have been eliminated. We wish to express our sincere thanks to numerous professors and students, both at home and abroad, for sending their valuable suggestions and recommending the book to their students and friends. We hope, that they will continue to patronise this book in the future also.

Our grateful thanks are due to the Editorial staff of S. Chand & Company Ltd., especially to Mr. E.J. Jawahardatham and Mr. Rupesh Gupta, for their help in conversion of the book into multicolour edition and Mr. Pradeep Kr. Joshi for Designing & Layouting of this book.

Any errors, omissions and suggestions, for the improvement of this volume brought to our notice, will be thankfully acknowledged and incorporated in the next edition.

R.S. KHURMI
J.K. GUPTA

Top

Preface to the First Edition

We take an opportunity to present this standard treatise entitled as 'A TEXTBOOK OF MACHINE DESIGN' to the students of Degree, Diploma and A.M.I.E. (India) classes in M.K.S. and S.I. units. The objective of this book is to present the subject matter in a most concise, compact, to the point and lucid manner.

While writing the book, we have continuously kept in mind the examination requirement of the students preparing for U.P.S.C. (Engg. Services) and A.M.I.E. (India) examinations. In order to make this volume more useful for them, complete solutions of their examination papers upto 1977 have also been included. Every care has been taken to make this treatise as self-explanatory as possible. The subject matter has been amply illustrated by incorporating a good number of solved, unsolved and well graded examples of almost every variety. Most of these examples are taken from the recent examination papers of Indian and foreign universities as well as professional examining bodies, to make the students familiar with the type of questions, usually, set in their examinations. At the end of each chapter, a few exercises have been added for the students to solve them independently. Answers to these problems have been provided, but it is too much to hope that these are entirely free from errors. In short, it is earnestly hoped that the book will earn appreciation of all the teachers and students alike.

Although every care has been taken to check mistakes and misprints, yet it is difficult to claim perfection. Any errors, omissions and suggestions for the improvement of this treatise, brought to our notice, will be thankfully acknowledged and incorporated in the next edition.

R.S. KHURMI
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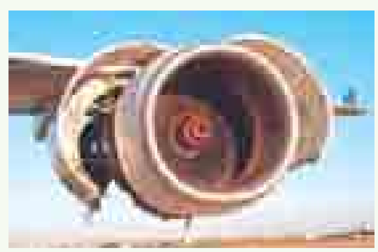
Top

CONTENTS



Top

Top

Top

Top

Top

Top

Top

CHAPTER

1

Introduction

1.1 Definition

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

1.2 Classifications of Machine Design

The machine design may be classified as follows :

1. *Adaptive design.* In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. *Development design.* This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. *New design.* This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows :

(*a*) ***Rational design.*** This type of design depends upon mathematical formulae of principle of mechanics.

(*b*) ***Empirical design.*** This type of design depends upon empirical formulae based on the practice and past experience.

(*c*) ***Industrial design.*** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(*d*) ***Optimum design.*** It is the best design for the given objective function under the specified constraints. It may be achieved by minimising the undesirable effects.

(*e*) ***System design.*** It is the design of any complex mechanical system like a motor car.

(*f*) ***Element design.*** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(*g*) ***Computer aided design.*** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

1.3 General Considerations in Machine Design

Following are the general considerations in designing a machine component :

1. *Type of load and stresses caused by the load.* The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed in chapters 4 and 5.

2. *Motion of the parts or kinematics of the machine.* The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required. The motion of the parts may be :

(*a*) Rectilinear motion which includes unidirectional and reciprocating motions.

(*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic.

(*c*) Constant velocity.

(*d*) Constant or variable acceleration.

3. *Selection of materials.* It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are : strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed in chapter 2.

Contents

4. *Form and size of the parts.* The form and size are based on judgement. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. *Frictional resistance and lubrication.* There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. *Convenient and economical features.* In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take-up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible.

The economical operation of a machine which is to be used for production, or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. *Use of standard parts.* The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order.

The standard or stock parts should be used whenever possible ; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

Design considerations play important role in the successful production of machines.

8. *Safety of operation.* Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. *Workshop facilities.* A design engineer should be familiar with the limitations of his employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. *Number of machines to be manufactured.* The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue

Top

Contents

expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. *Cost of construction.* The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of hand made samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions, should be to reduce the manufacturing cost to the minimum.

Car assembly line.

12. *Assembling.* Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

1.4 General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows :

1. *Recognition of need.* First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. *Synthesis (Mechanisms).* Select the possible mechanism or group of mechanisms which will give the desired motion.

3. *Analysis of forces.* Find the forces acting on each member of the machine and the energy transmitted by each member.

4. *Material selection.* Select the material best suited for each member of the machine.

5. *Design of elements (Size and Stresses).* Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. *Modification.* Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

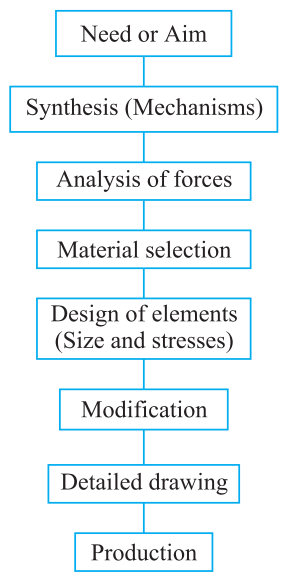


Fig. 1.1. General procedure in Machine Design.

7. *Detailed drawing.* Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. *Production.* The component, as per the drawing, is manufactured in the workshop.

The flow chart for the general procedure in machine design is shown in Fig. 1.1.

Top

Note : When there are number of components in the market having the same qualities of efficiency, durability and cost, then the customer will naturally attract towards the most appealing product. The aesthetic and ergonomics are very important features which gives grace and lustre to product and dominates the market.

1.5 Fundamental Units

The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called ***fundamental units.***

1.6 Derived Units

Some units are expressed in terms of other units, which are derived from fundamental units, are known as ***derived units*** *e.g.* the unit of area, velocity, acceleration, pressure, etc.

1.7 System of Units

There are only four systems of units, which are commonly used and universally recognised. These are known as :

1. C.G.S. units, 2. F.P.S. units, 3. M.K.S. units, and 4. S.I. units.

Since the present course of studies are conducted in S.I. system of units, therefore, we shall discuss this system of unit only.

1.8 S.I. Units (International System of Units)

The 11th General Conference* of Weights and Measures have recommended a unified and systematically constituted system of fundamental and derived units for international use. This system is now being used in many countries. In India, the standards of Weights and Measures Act 1956 (vide which we switched over to M.K.S. units) has been revised to recognise all the S.I. units in industry and commerce.

In this system of units, there are seven fundamental units and two supplementary units, which cover the entire field of science and engineering. These units are shown in Table 1.1

Table 1.1. Fundamental and supplementary units.

S.No.	*Physical quantity*	*Unit*
	Fundamental units	
1.	Length (*l*)	Metre (m)
2.	Mass (*m*)	Kilogram (kg)
3.	Time (*t*)	Second (s)
4.	Temperature (*T*)	Kelvin (K)
5.	Electric current (*I*)	Ampere (A)
6.	Luminous intensity(*Iv*)	Candela (cd)
7.	Amount of substance (*n*)	Mole (mol)
	Supplementary units	
1.	Plane angle (α, β, θ, ϕ)	Radian (rad)
2.	Solid angle (Ω)	Steradian (sr)

* It is known as General Conference of Weights and Measures (G.C.W.M). It is an international organisation of which most of the advanced and developing countries (including India) are members. The conference has been entrusted with the task of prescribing definitions for various units of weights and measures, which are the very basics of science and technology today.

The derived units, which will be commonly used in this book, are given in Table 1.2.

Table 1.2. Derived units.

S.No.	Quantity	Symbol	Units
1.	Linear velocity	V	m/s
2.	Linear acceleration	a	m/s^2
3.	Angular velocity	ω	rad/s
4.	Angular acceleration	α	rad/s^2
5.	Mass density	ρ	kg/m^3
6.	Force, Weight	F, W	N ; 1N = 1kg-m/s^2
7.	Pressure	P	N/m^2
8.	Work, Energy, Enthalpy	W, E, H	J ; 1J = 1N-m
9.	Power	P	W ; 1W = 1J/s
10.	Absolute or dynamic viscosity	μ	N-s/m^2
11.	Kinematic viscosity	v	m^2/s
12.	Frequency	f	Hz ; 1Hz = 1cycle/s
13.	Gas constant	R	J/kg K
14.	Thermal conductance	h	W/m^2 K
15.	Thermal conductivity	k	W/m K
16.	Specific heat	c	J/kg K
17.	Molar mass or Molecular mass	M	kg/mol

1.9 Metre

The metre is defined as the length equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels 2 p_{10} and 5 d_5 of the Krypton– 86 atom.

1.10 Kilogram

The kilogram is defined as the mass of international prototype (standard block of platinum-iridium alloy) of the kilogram, kept at the International Bureau of Weights and Measures at Sevres near Paris.

1.11 Second

The second is defined as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium – 133 atom.

1.12 Presentation of Units and their Values

The frequent changes in the present day life are facilitated by an international body known as International Standard Organisation (ISO) which makes recommendations regarding international standard procedures. The implementation of lSO recommendations, in a country, is assisted by its organisation appointed for the purpose. In India, Bureau of Indian Standards (BIS), has been created for this purpose. We have already discussed that the fundamental units in S.I. units for length, mass and time is metre, kilogram and second respectively. But in actual practice, it is not necessary to express all lengths in metres, all masses in kilograms and all times in seconds. We shall, sometimes, use the convenient units, which are multiples or divisions of our basic units in tens. As a typical example, although the metre is the unit of length, yet a smaller length of one-thousandth of a metre proves to be more convenient unit, especially in the dimensioning of drawings. Such convenient units are formed by using a prefix in the basic units to indicate the multiplier. The full list of these prefixes is given in the following table :

Table 1.3. Prefixes used in basic units.

Factor by which the unit is multiplied	*Standard form*	*Prefix*	*Abbreviation*
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1000	10^{3}	kilo	k
100	10^{2}	hecto*	h
10	10^{1}	deca*	da
0.1	10^{-1}	deci*	d
0.01	10^{-2}	centi*	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p

1.13 Rules for S.I. Units

The eleventh General Conference of Weights and Measures recommended only the fundamental and derived units of S.I. units. But it did not elaborate the rules for the usage of the units. Later on many scientists and engineers held a number of meetings for the style and usage of S.I. units. Some of the decisions of the meeting are :

1. For numbers having five or more digits, the digits should be placed in groups of three separated by spaces (instead of commas)** counting both to the left and right of the decimal point.
2. In a four*** digit number, the space is not required unless the four digit number is used in a column of numbers with five or more digits.
3. A dash is to be used to separate units that are multiplied together. For example, newton × metre is written as N-m. It should not be confused with mN, which stands for milli newton.
4. Plurals are never used with symbols. For example, metre or metres are written as m.
5. All symbols are written in small letters except the symbol derived from the proper names. For example, N for newton and W for watt.
6. The units with names of the scientists should not start with capital letter when written in full. For example, 90 newton and not 90 Newton.

At the time of writing this book, the authors sought the advice of various international authorities, regarding the use of units and their values. Keeping in view the international reputation of the authors, as well as international popularity of their books, it was decided to present **** units and

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* These prefixes are generally becoming obsolete, probably due to possible confusion. Moreover it is becoming a conventional practice to use only those power of ten which conform to 10^{3x}, where x is a positive or negative whole number.

** In certain countries, comma is still used as the decimal mark

*** In certain countries, a space is used even in a four digit number.

**** In some of the question papers of the universities and other examining bodies standard values are not used. The authors have tried to avoid such questions in the text of the book. However, at certain places the questions with sub-standard values have to be included, keeping in view the merits of the question from the reader's angle.

Contents

their values as per recommendations of ISO and BIS. It was decided to use :

4500	not	4 500	or	4,500
75 890 000	not	75890000	or	7,58,90,000
0.012 55	not	0.01255	or	.01255
30×10^6	not	3,00,00,000	or	3×10^7

The above mentioned figures are meant for numerical values only. Now let us discuss about the units. We know that the fundamental units in S.I. system of units for length, mass and time are metre, kilogram and second respectively. While expressing these quantities, we find it time consuming to write the units such as metres, kilograms and seconds, in full, every time we use them. As a result of this, we find it quite convenient to use some standard abbreviations :

We shall use :

m	for metre or metres
km	for kilometre or kilometres
kg	for kilogram or kilograms
t	for tonne or tonnes
s	for second or seconds
min	for minute or minutes
N-m	for netwon × metres (*e.g.* work done)
kN-m	for kilonewton × metres
rev	for revolution or revolutions
rad	for radian or radians

1.14 Mass and Weight

Sometimes much confusion and misunderstanding is created, while using the various systems of units in the measurements of force and mass. This happens because of the lack of clear understanding of the difference between the mass and weight. The following definitions of mass and weight should be clearly understood :

Mass. It is the amount of matter contained in a given body and does not vary with the change in its position on the earth's surface. The mass of a body is measured by direct comparison with a standard mass by using a lever balance.

Weight. It is the amount of pull, which the earth exerts upon a given body. Since the pull varies with the distance of the body from the centre of the earth, therefore, the weight of the body will vary with its position on the earth's surface (say latitude and elevation). It is thus obvious, that the weight is a force.

The pointer of this spring gauge shows the tension in the hook as the brick is pulled along.

Top

The earth's pull in metric units at sea level and 45° latitude has been adopted as one force unit and named as one kilogram of force. Thus, it is a definite amount of force. But, unfortunately, has the same name as the unit of mass.

The weight of a body is measured by the use of a spring balance, which indicates the varying tension in the spring as the body is moved from place to place.

Note : The confusion in the units of mass and weight is eliminated to a great extent, in S.I units . In this system, the mass is taken in kg and the weight in newtons. The relation between mass (m) and weight (W) of a body is

$$W = m.g \quad \text{or} \quad m = W / g$$

where W is in newtons, m in kg and g is the acceleration due to gravity in m/s^2.

1.15 Inertia

It is that property of a matter, by virtue of which a body cannot move of itself nor change the motion imparted to it.

1.16 Laws of Motion

Newton has formulated three laws of motion, which are the basic postulates or assumptions on which the whole system of dynamics is based. Like other scientific laws, these are also justified as the results, so obtained, agree with the actual observations. Following are the three laws of motion :

1. *Newton's First Law of Motion.* It states, "*Every body continues in its state of rest or of uniform motion in a straight line, unless acted upon by some external force*". This is also known as *Law of Inertia.*

2. *Newton's Second Law of Motion.* It states, "*The rate of change of momentum is directly proportional to the impressed force and takes place in the same direction in which the force acts*".

3. *Newton's Third Law of Motion.* It states, "*To every action, there is always an equal and opposite reaction*".

1.17 Force

It is an important factor in the field of Engineering science, which may be defined as **an agent, which produces or tends to produce, destroy or tends to destroy motion.**

According to Newton's Second Law of Motion, the applied force or impressed force is directly proportional to the rate of change of momentum. We know that

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

Let m = Mass of the body,

u = Initial velocity of the body,

v = Final velocity of the body,

a = Constant acceleration, and

t = Time required to change velocity from u to v.

$\therefore$ Change of momentum $= mv - mu$

and rate of change of momentum

$$= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = m.a \qquad \ldots \left(\because \frac{v-u}{t} = a\right)$$

or Force, $F \propto ma$ or $F = k\,m\,a$

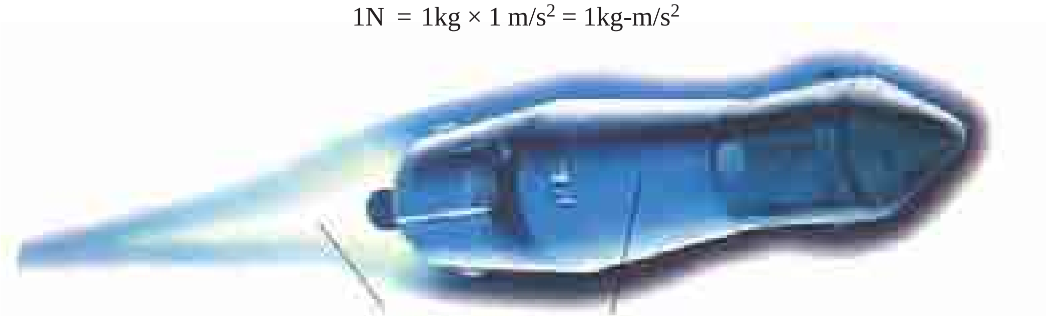
where k is a constant of proportionality.

For the sake of convenience, the unit of force adopted is such that it produces a unit acceleration to a body of unit mass.

$$\therefore \quad F = m.a = \text{Mass} \times \text{Acceleration}$$

In S.I. system of units, the unit of force is called **newton** (briefly written as N). A ***newton may be defined as the force, while acting upon a mass of one kg, produces an acceleration of 1 m/s² in the direction in which it acts.*** Thus

$$1\text{N} = 1\text{kg} \times 1\ \text{m/s}^2 = 1\text{kg-m/s}^2$$



Far away from Earth's gravity and its frictional forces, a spacecraft shows Newton's three laws of motion at work.

1.18 Absolute and Gravitational Units of Force

We have already discussed, that when a body of mass 1 kg is moving with an acceleration of 1 m/s^2, the force acting on the body is one newton (briefly written as 1 N). Therefore, when the same body is moving with an acceleration of 9.81 m/s^2, the force acting on the body is 9.81N. But we denote 1 kg mass, attracted towards the earth with an acceleration of 9.81 m/s^2 as 1 kilogram force (briefly written as kgf) or 1 kilogram weight (briefly written as kg-wt). It is thus obvious that

$$1\text{kgf} = 1\text{kg} \times 9.81\ \text{m/s}^2 = 9.81\ \text{kg-m/s}^2 = 9.81\ \text{N} \quad \ldots (\because 1\text{N} = 1\text{kg-m/s}^2)$$

The above unit of force *i.e.* kilogram force (kgf) is called ***gravitational*** or ***engineer's unit of force,*** whereas netwon is the ***absolute*** or ***scientific*** or ***S.I. unit of force.*** It is thus obvious, that the gravitational units are '*g*' times the unit of force in the absolute or S. I. units.

It will be interesting to know that ***the mass of a body in absolute units is numerically equal to the weight of the same body in gravitational units.***

For example, consider a body whose mass, m = 100 kg.

∴ The force, with which it will be attracted towards the centre of the earth,

$$F = m.a = m.g = 100 \times 9.81 = 981\ \text{N}$$

Now, as per definition, we know that the weight of a body is the force, by which it is attracted towards the centre of the earth.

∴ Weight of the body,

$$W = 981\ \text{N} = \frac{981}{9.81} = 100\ \text{kgf} \quad \ldots (\because 1\ \text{kgf} = 9.81\ \text{N})$$

In brief, the weight of a body of mass m kg at a place where gravitational acceleration is 'g' m/s^2 is $m.g$ newtons.

1.19 Moment of Force

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required, and the line of action of the force. Mathematically,

$$\text{Moment of a force} = F \times l$$

where F = Force acting on the body, and

l = Perpendicular distance of the point and the line of action of the force (F) as shown in Fig. 1.2.

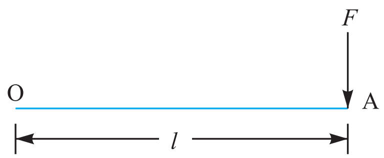


Fig. 1.2. Moment of a force.

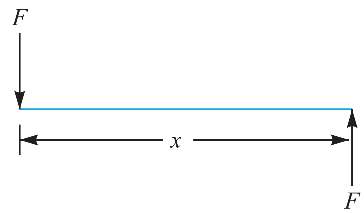


Fig. 1.3. Couple.

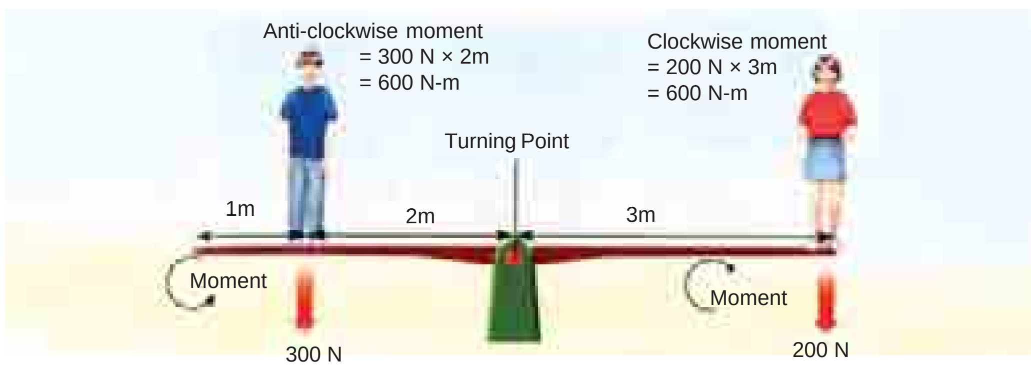
1.20 Couple

The two equal and opposite parallel forces, whose lines of action are different form a couple, as shown in Fig. 1.3.

The perpendicular distance (x) between the lines of action of two equal and opposite parallel forces is known as ***arm of the couple.*** The magnitude of the couple (*i.e.* moment of a couple) is the product of one of the forces and the arm of the couple. Mathematically,

$$\text{Moment of a couple} = F \times x$$

A little consideration will show, that a couple does not produce any translatory motion (*i.e.* motion in a straight line). But, a couple produces a motion of rotation of the body on which it acts.



A see saw is balanced when the clockwise moment equals the anti-clockwise moment. The boy's weight is 300 newtons (300 N) and he stands 2 metres (2 m) from the pivot. He causes the anti-clockwise moment of 600 newton-metres (N-m). The girl is lighter (200 N) but she stands further from the pivot (3m). She causes a clockwise moment of 600 N-m, so the seesaw is balanced.

1.21 Mass Density

The mass density of the material is the mass per unit volume. The following table shows the mass densities of some common materials used in practice.

Table 1.4. Mass density of commonly used materials.

Material	*Mass density (kg/m^3)*	*Material*	*Mass density (kg/m^3)*
Cast iron	7250	Zinc	7200
Wrought iron	7780	Lead	11 400
Steel	7850	Tin	7400
Brass	8450	Aluminium	2700
Copper	8900	Nickel	8900
Cobalt	8850	Monel metal	8600
Bronze	8730	Molybdenum	10 200
Tungsten	19 300	Vanadium	6000

1.22 Mass Moment of Inertia

It has been established since long that a rigid body is composed of small particles. If the mass of every particle of a body is multiplied by the square of its perpendicular distance from a fixed line, then the sum of these quantities (for the whole body) is known as ***mass moment of inertia*** of the body. It is denoted by I.

Consider a body of total mass m. Let it be composed of small particles of masses m_1, m_2, m_3, m_4, etc. If k_1, k_2, k_3, k_4, etc., are the distances from a fixed line, as shown in Fig. 1.4, then the mass moment of inertia of the whole body is given by

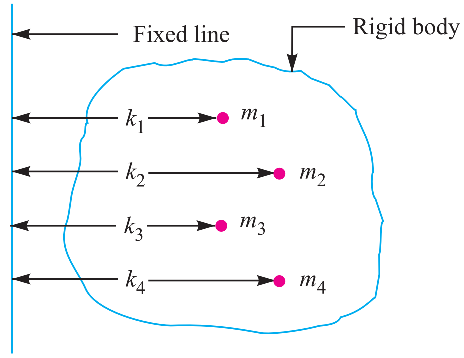


Fig. 1.4. Mass moment of inertia.

$$I = m_1 (k_1)^2 + m_2 (k_2)^2 + m_3 (k_3)^2 + m_4 (k_4)^2 + ...$$

If the total mass of a body may be assumed to concentrate at one point (known as centre of mass or centre of gravity), at a distance k from the given axis, such that

$$mk^2 = m_1 (k_1)^2 + m_2 (k_2)^2 + m_3 (k_3)^2 + m_4 (k_4)^2 + ...$$

then $$I = m\,k^2$$

The distance k is called the ***radius of gyration.*** It may be defined ***as the distance, from a given reference, where the whole mass of body is assumed to be concentrated to give the same value of I.***

The unit of mass moment of inertia in S.I. units is kg-m^2.

Notes : 1. If the moment of inertia of body about an axis through its centre of gravity is known, then the moment of inertia about any other parallel axis may be obtained by using a parallel axis theorem *i.e.* moment of inertia about a parallel axis,

$$I_p = I_G + mh^2$$

where I_G = Moment of inertia of a body about an axis through its centre of gravity, and

h = Distance between two parallel axes.

2. The following are the values of I for simple cases :

(*a*) The moment of inertia of a thin disc of radius r, about an axis through its centre of gravity and perpendicular to the plane of the disc is,

$$I = mr^2/2 = 0.5\,mr^2$$

and moment of inertia about a diameter,

$$I = mr^2/4 = 0.25\,mr^2$$

(*b*) The moment of inertia of a thin rod of length l, about an axis through its centre of gravity and perpendicular to its length,

$$I_G = ml^2/12$$

and moment of inertia about a parallel axis through one end of a rod,

$$I_P = ml^2/3$$

3. The moment of inertia of a solid cylinder of radius r and length l,about the longitudinal axis or polar axis

$$= mr^2/2 = 0.5\,mr^2$$

and moment of inertia through its centre perpendicular to the longitudinal axis

$$= m\left(\frac{r^2}{4} + \frac{l^2}{12}\right)$$

1.23 Angular Momentum

It is the product of the mass moment of inertia and the angular velocity of the body. Mathematically,

$$\text{Angular momentum} = I.\omega$$

where I = Mass moment of inertia, and

ω = Angular velocity of the body.

1.24 Torque

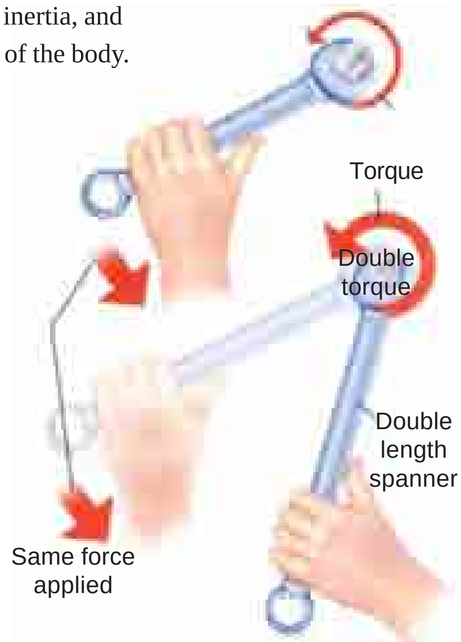
It may be defined as the product of force and the perpendicular distance of its line of action from the given point or axis. A little consideration will show that the torque is equivalent to a couple acting upon a body.

The Newton's second law of motion when applied to rotating bodies states, the ***torque is directly proportional to the rate of change of angular momentum.*** Mathematically,

$$\text{Torque, } T \propto \frac{d(I\omega)}{dt}$$

Since I is constant, therefore,

$$T = I \times \frac{d\omega}{dt} = I.\alpha \qquad \ldots \left[\because \frac{d\omega}{dt} = \text{Angular acceleration } (\alpha)\right]$$



Same force applied at double the length, doubles the torque.

1.25 Work

Whenever a force acts on a body and the body undergoes a displacement in the direction of the force, then work is said to be done. For example, if a force F acting on a body causes a displacement x of the body in the direction of the force, then

$$\text{Work done} = \text{Force} \times \text{Displacement} = F \times x$$

If the force varies linearly from zero to a maximum value of F, then

$$\text{Work done} = \frac{0 + F}{2} \times x = \frac{F}{2} \times x$$

When a couple or torque (T) acting on a body causes the angular displacement (θ) about an axis perpendicular to the plane of the couple, then

$$\text{Work done} = \text{Torque} \times \text{Angular displacement} = T.\theta$$

The unit of work depends upon the units of force and displacement. In S. I. system of units, the practical unit of work is N-m. It is the work done by a force of 1 newton, when it displaces a body through 1 metre. The work of 1 N-m is known as joule (briefly written as J), such that 1 N-m = 1 J.

Note : While writing the unit of work, it is a general practice to put the units of force first followed by the units of displacement (*e.g.* N-m).

1.26 Power

It may be defined as the rate of doing work or work done per unit time. Mathematically,

$$\text{Power, } P = \frac{\text{Work done}}{\text{Time taken}}$$

In S.I system of units, the unit of power is watt (briefly written as W) which is equal to 1 J/s or 1N-m/s. Thus, the power developed by a force of F (in newtons) moving with a velocity v m/s is $F.v$ watt. Generally, a bigger unit of power called kilowatt (briefly written as kW) is used which is equal to 1000 W

Notes : 1. If T is the torque transmitted in N-m or J and ω is angular speed in rad/s, then

$$\text{Power, } P = T.\omega = T \times 2\,\pi\,N/60 \text{ watts} \qquad \ldots (\because\ \omega = 2\,\pi\,N/60)$$

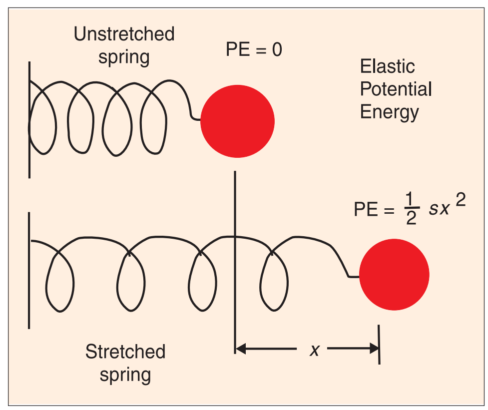
where N is the speed in r.p.m.

2. The ratio of the power output to power input is known as ***efficiency*** of a machine. It is always less than unity and is represented as percentage. It is denoted by a Greek letter eta (η). Mathematically,

$$\text{Efficiency, } \eta = \frac{\text{Power output}}{\text{Power input}}$$

1.27 Energy

It may be defined as the capacity to do work. The energy exists in many forms *e.g.* mechanical, electrical, chemical, heat, light, etc. But we are mainly concerned with mechanical energy.



The mechanical energy is equal to the work done on a body in altering either its position or its velocity. The following three types of mechanical energies are important from the subject point of view :

1. *Potential energy.* It is the energy possessed by a body, for doing work, by virtue of its position. For example, a body raised to some height above the ground level possesses potential energy, because it can do some work by falling on earth's surface.

Let W = Weight of the body,

m = Mass of the body, and

h = Distance through which the body falls.

∴ Potential energy,

$$\text{P.E.} = W.\,h = m.g.h$$

It may be noted that

(a) When W is in newtons and h in metres, then potential energy will be in N-m.

(b) When m is in kg and h in metres, then the potential energy will also be in N-m as discussed below :

We know that potential energy

$$= m.g.h = \text{kg} \times \frac{\text{m}}{\text{s}^2} \times \text{m} = \text{N-m} \qquad \ldots \left(\because\ 1\text{N} = \frac{1\text{ kg-m}}{\text{s}^2}\right)$$

2. *Strain energy.* It is the potential energy stored by an elastic body when deformed. A compressed spring possesses this type of energy, because it can do some work in recovering its original shape. Thus, if a compressed spring of stiffness (s) N per unit deformation (*i.e.* extension or compression) is deformed through a distance x by a weight W, then

$$\text{Strain energy} = \text{Work done} = \frac{1}{2}\,W.x = \frac{1}{2}\,s.x^2 \qquad \ldots (\because\ W = s.x)$$

Contents

In case of a torsional spring of stiffness (q) N-m per unit angular deformation when twisted through an angle θ radians, then

$$\text{Strain energy} = \text{Work done} = \frac{1}{2}\, q.\theta^2$$

3. ***Kinetic energy.*** It is the energy possessed by a body, for doing work, by virtue of its mass and velocity of motion. If a body of mass m attains a velocity v from rest in time t, under the influence of a force F and moves a distance s, then

$$\text{Work done} = F.s = m.a.s \qquad ...(\because\ F = m.a)$$

∴ Kinetic energy of the body or the kinetic energy of translation,

$$\text{K.E.} = m.a.s = m \times a \times {}^{*}\frac{v^2}{2a} = \frac{1}{2}\, mv^2$$

It may be noted that when m is in kg and v in m/s, then kinetic energy will be in N-m as discussed below :

We know that kinetic energy,

$$\text{K.E.} = \frac{1}{2}\, m\, v^2 = \text{kg} \times \frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg - m}}{\text{s}^2} \times \text{m} = \text{N-m} \ ...\left(\because 1\text{N} = \frac{1\ \text{kg-m}}{\text{s}^2}\right)$$

Notes : 1. When a body of mass moment of inertia I (about a given axis) is rotated about that axis, with an angular velocity ω, then it possesses some kinetic energy. In this case,

$$\text{Kinetic energy of rotation} = \frac{1}{2}\, I.\omega^2$$

2. When a body has both linear and angular motions, *e.g.* wheels of a moving car, then the total kinetic energy of the body is equal to the sum of linear and angular kinetic energies.

$$\therefore \qquad \text{Total kinetic energy} = \frac{1}{2}\, m.v^2 + \frac{1}{2}\, I.\omega^2$$

3. The energy can neither be created nor destroyed, though it can be transformed from one form into any of the forms, in which energy can exist. This statement is known as ***'Law of Conservation of Energy'*.**

4. The loss of energy in any one form is always accompanied by an equivalent increase in another form. When work is done on a rigid body, the work is converted into kinetic or potential energy or is used in overcoming friction. If the body is elastic, some of the work will also be stored as strain energy.

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* We know that $v^2 - u^2 = 2\, a.s$

Since the body starts from rest (*i.e.* $u = 0$), therefore,

$$v^2 = 2\, a.s \quad \text{or} \quad s = v^2 / 2a$$

Top

C H A P T E R

2

Engineering Materials and their Properties

2.1 Introduction

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. In this chapter, we shall discuss the commonly used engineering materials and their properties in Machine Design.

2.2 Classification of Engineering Materials

The engineering materials are mainly classified as :

1. Metals and their alloys, such as iron, steel, copper, aluminium, etc.
2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as :

(*a*) Ferrous metals, and (*b*) Non-ferrous metals.

The *ferrous metals* are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminium, brass, tin, zinc, etc.

2.3 Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problem for the designer. The best material is one which serve the desired objective at the minimum cost. The following factors should be considered while selecting the material :

1. Availability of the materials,
2. Suitability of the materials for the working conditions in service, and
3. The cost of the materials.

A filament of bulb needs a material like tungsten which can withstand high temperatures without undergoing deformation.

The important properties, which determine the utility of the material are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

Valuable Metals

2.4 Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

* The word 'ferrous' is derived from a latin word 'ferrum' which means iron.

Contents

Table 2.1. Physical properties of metals.

Metal	*Density* *(kg/m³)*	*Melting point* *(°C)*	*Thermal conductivity* *(W/m°C)*	*Coefficient of linear expansion at 20°C (μm/m/°C)*
Aluminium	2700	660	220	23.0
Brass	8450	950	130	16.7
Bronze	8730	1040	67	17.3
Cast iron	7250	1300	54.5	9.0
Copper	8900	1083	393.5	16.7
Lead	11 400	327	33.5	29.1
Monel metal	8600	1350	25.2	14.0
Nickel	8900	1453	63.2	12.8
Silver	10 500	960	420	18.9
Steel	7850	1510	50.2	11.1
Tin	7400	232	67	21.4
Tungsten	19 300	3410	201	4.5
Zinc	7200	419	113	33.0
Cobalt	8850	1490	69.2	12.4
Molybdenum	10 200	2650	13	4.8
Vanadium	6000	1750	—	7.75

2.5 Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. ***Strength.*** It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called *stress.

2. ***Stiffness.*** It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. ***Elasticity.*** It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. ***Plasticity.*** It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. ***Ductility.*** It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

Note : The ductility of a material is commonly measured by means of percentage elongation and percentage reduction in area in a tensile test. (Refer Chapter 4, Art. 4.11).

* For further details, refer Chapter 4 on Simple Stresses in Machine Parts.

Top

6. *Brittleness.* It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads, snap off without giving any sensible elongation. Cast iron is a brittle material.

7. *Malleability.* It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. *Toughness.* It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. *Machinability.* It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. *Resilience.* It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. *Creep.* When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called ***creep***. This property is considered in designing internal combustion engines, boilers and turbines.

12. *Fatigue.* When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ****fatigue***. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

Brinell Tester : *Hardness can be defined as the resistance of a metal to attempts to deform it. This machine invented by the Swedish metallurgist Johann August Brinell (1849-1925), measure hardness precisely.*

13. *Hardness.* It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually

* For further details, refer Chapter 6 (Art. 6.3) on Variable Stresses in Machine Parts.

expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests :

(*a*) Brinell hardness test,

(*b*) Rockwell hardness test,

(*c*) Vickers hardness (also called Diamond Pyramid) test, and

(*d*) Shore scleroscope.

2.6 Ferrous Metals

We have already discussed in Art. 2.2 that the ferrous metals are those which have iron as their main constituent. The ferrous metals commonly used in engineering practice are cast iron, wrought iron, steels and alloy steels. The principal raw material for all ferrous metals is pig iron which is obtained by smelting iron ore with coke and limestone, in the blast furnace. The principal iron ores with their metallic contents are shown in the following table :

Table 2.2. Principal iron ores.

Iron ore	*Chemical formula*	*Colour*	*Iron content (%)*
Magnetite	Fe_2O_3	Black	72
Haematite	Fe_3O_4	Red	70
Limonite	$FeCO_3$	Brown	60–65
Siderite	Fe_2O_3 (H_2O)	Brown	48

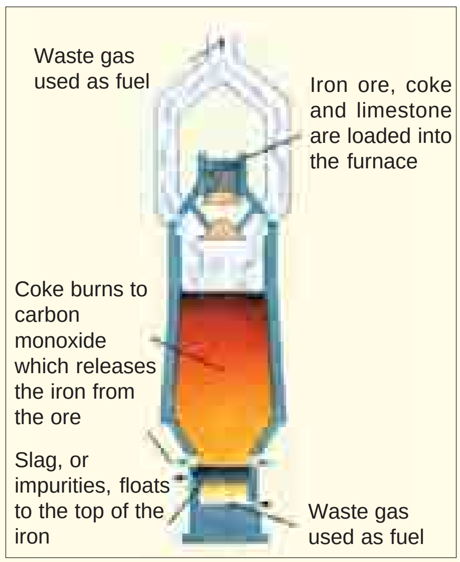
2.7 Cast Iron

The cast iron is obtained by re-melting pig iron with coke and limestone in a furnace known as cupola. It is primarily an alloy of iron and carbon. The carbon contents in cast iron varies from 1.7 per cent to 4.5 per cent. It also contains small amounts of silicon, manganese, phosphorous and sulphur. The carbon in a cast iron is present in either of the following two forms:

1. Free carbon or graphite, and **2.** Combined carbon or cementite.

Since the cast iron is a brittle material, therefore, it cannot be used in those parts of machines which are subjected to shocks. The properties of cast iron which make it a valuable material for engineering purposes are its low cost, good casting characteristics, high compressive strength, wear resistance and excellent machinability. The compressive strength of cast iron is much greater than the tensile strength. Following are the values of ultimate strength of cast iron :

Tensile strength	=	100 to 200 MPa*
Compressive strength	=	400 to 1000 MPa
Shear strength	=	120 MPa



Smelting : *Ores consist of non-metallic elements like oxygen or sulphur combined with the wanted metal. Iron is separated from the oxygen in its ore heating it with carbon monoxide derived from coke (a form of carbon made from coal). Limestone is added to keep impurities liquid so that the iron can separate from them.*

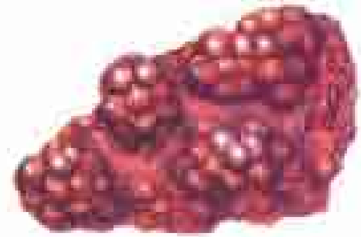
* 1MPa = 1MN/m^2 = 1 × 10^6 N/m^2 = 1 N/mm^2

2.8 Types of Cast Iron

The various types of cast iron in use are discussed as follows :

1. *Grey cast iron.* It is an ordinary commercial iron having the following compositions :

Carbon = 3 to 3.5%; Silicon = 1 to 2.75%; Manganese = 0.40 to 1.0%; Phosphorous = 0.15 to 1% ; Sulphur = 0.02 to 0.15% ; and the remaining is iron.



Haematite is an ore of iron. It often forms kidney-shaped lumps, These give the ore its nickname of kidney ore.

The grey colour is due to the fact that the carbon is present in the form of *free graphite. It has a low tensile strength, high compressive strength and no ductility. It can be easily machined. A very good property of grey cast iron is that the free graphite in its structure acts as a lubricant. Due to this reason, it is very suitable for those parts where sliding action is desired. The grey iron castings are widely used for machine tool bodies, automotive cylinder blocks, heads, housings, fly-wheels, pipes and pipe fittings and agricultural implements.

Table 2.3. Grey iron castings, as per IS : 210 – 1993.

IS Designation	*Tensile strength (MPa or N/mm^2)*	*Brinell hardness number (B.H.N.)*
FG 150	150	130 to 180
FG 200	200	160 to 220
FG 220	220	180 to 220
FG 260	260	180 to 230
FG 300	300	180 to 230
FG 350	350	207 to 241
FG 400	400	207 to 270

According to Indian standard specifications (IS: 210 – 1993), the grey cast iron is designated by the alphabets 'FG' followed by a figure indicating the minimum tensile strength in MPa or N/mm^2. For example, 'FG 150' means grey cast iron with 150 MPa or N/mm^2 as minimum tensile strength. The seven recommended grades of grey cast iron with their tensile strength and Brinell hardness number (B.H.N) are given in Table 2.3.

2. *White cast iron.* The white cast iron shows a white fracture and has the following approximate compositions :

Carbon = 1.75 to 2.3% ; Silicon = 0.85 to 1.2% ; Manganese = less than 0.4% ; Phosphorus = less than 0.2% ; Sulphur = less than 0.12%, and the remaining is iron.

The white colour is due to fact that it has no graphite and whole of the carbon is in the form of carbide (known as cementite) which is the hardest constituent of iron. The white cast iron has a high tensile strength and a low compressive strength. Since it is hard, therefore, it cannot be machined with ordinary cutting tools but requires grinding as shaping process. The white cast iron may be produced by casting against metal chills or by regulating analysis. The chills are used when a hard, wear resisting surface is desired for such products as for car wheels, rolls for crushing grains and jaw crusher plates.

3. *Chilled cast iron.* It is a white cast iron produced by quick cooling of molten iron. The quick cooling is generally called chilling and the cast iron so produced is called chilled cast iron. All castings

* When filing or machining cast iron makes our hands black, then it shows that free graphite is present in cast iron.

are chilled at their outer skin by contact of the molten iron with the cool sand in the mould. But on most castings, this hardness penetrates to a very small depth (less than 1 mm). Sometimes, a casting is chilled intentionally and sometimes chilled becomes accidently to a considerable depth. The intentional chilling is carried out by putting inserts of iron or steel (chills) into the mould. When the molten metal comes into contact with the chill, its heat is readily conducted away and the hard surface is formed. Chills are used on any faces of a casting which are required to be hard to withstand wear and friction.

4. ***Mottled cast iron.*** It is a product in between grey and white cast iron in composition, colour and general properties. It is obtained in castings where certain wearing surfaces have been chilled.

5. ***Malleable cast iron.*** The malleable iron is a cast iron-carbon alloy which solidifies in the as-cast condition in a graphite free structure, *i.e.* total carbon content is present in its combined form as cementite (Fe_3C).

It is ductile and may be bent without breaking or fracturing the section. The tensile strength of the malleable cast iron is usually higher than that of grey cast iron and has excellent machining qualities. It is used for machine parts for which the steel forgings would be too expensive and in which the metal should have a fair degree of accuracy, *e.g.* hubs of wagon wheels, small fittings for railway rolling stock, brake supports, parts of agricultural machinery, pipe fittings, door hinges, locks etc.

In order to obtain a malleable iron castings, it is first cast into moulds of white cast iron. Then by a suitable heat treatment (*i.e.* annealing), the combined carbon of the white cast iron is separated into nodules of graphite. The following two methods are used for this purpose :

1. Whiteheart process, and **2.** Blackheart process.

In a ***whiteheart process,*** the white iron castings are packed in iron or steel boxes surrounded by a mixture of new and used haematite ore. The boxes are slowly heated to a temperature of 900 to 950°C and maintained at this temperature for several days. During this period, some of the carbon is oxidised out of the castings and the remaining carbon is dispersed in small specks throughout the structure. The heating process is followed by the cooling process which takes several more days. The result of this heat treatment is a casting which is tough and will stand heat treatment without fracture.

In a ***blackheart process***, the castings used contain less carbon and sulphur. They are packed in a neutral substance like sand and the reduction of sulphur helps to accelerate the process. The castings are heated to a temperature of 850 to 900°C and maintained at that temperature for 3 to 4 days. The carbon in this process transforms into globules, unlike whiteheart process. The castings produced by this process are more malleable.

Notes : (*a*) According to Indian standard specifications (*IS : 14329 – 1995), the malleable cast iron may be either whiteheart, blackheart or pearlitic, according to the chemical composition, temperature and time cycle of annealing process.

(*b*) The ***whiteheart malleable cast iron*** obtained after annealing in a decarburizing atmosphere have a silvery-grey fracture with a heart dark grey to black. The microstructure developed in a section depends upon the size of the section. In castings of small sections, it is mainly ferritic with certain amount of pearlite. In large sections, microstructure varies from the surface to the core as follows :

Core and intermediate zone : Pearlite + ferrite + temper carbon

Surface zone : Ferrite.

The microstructure shall not contain flake graphite.

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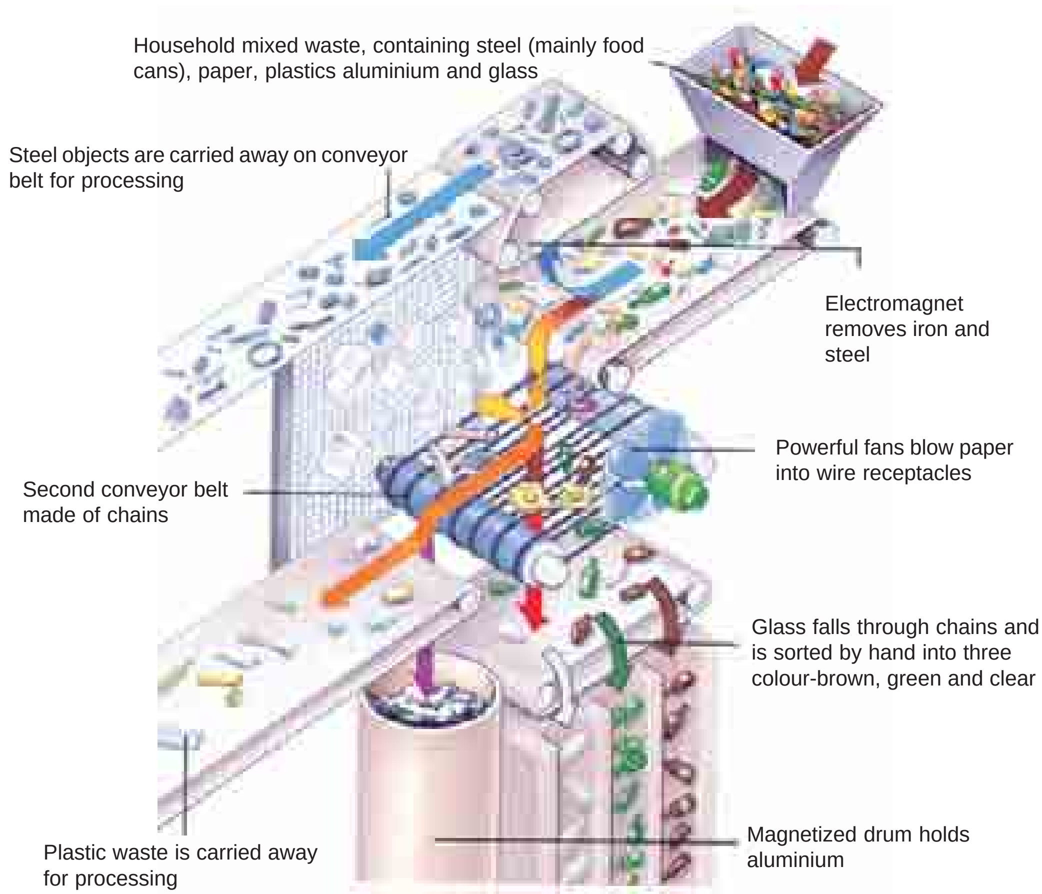
* This standard (IS : 14329-1995) supersedes the previous three standards, *i.e.*

(*a*) IS : 2107–1977 for white heart malleable iron casting,

(*b*) IS : 2108–1977 for black heart malleable iron casting, and

(*c*) IS : 2640–1977 for pearlitic malleable iron casting.

Contents



In a modern materials recovery plant, mixed waste (but no organic matter) is passed along a conveyor belt and sorted into reusable materials-steel, aluminium, paper, glass. Such recycling plants are expensive, but will become essential as vital resources become scarce.

Note : This picture is given as additional information and is not a direct example of the current chapter.

(*c*) The ***blackheart malleable cast iron*** obtained after annealing in an inert atmosphere have a black fracture. The microstructure developed in the castings has a matrix essentially of ferrite with temper carbon and shall not contain flake graphite.

(*d*) The ***pearlitic malleable cast iron*** obtained after heat-treatment have a homogeneous matrix essentially of pearlite or other transformation products of austenite. The graphite is present in the form of temper carbon nodules. The microstructure shall not contain flake graphite.

(*e*) According to IS: 14329 – 1995, the whiteheart, blackheart and pearlitic malleable cast irons are designated by the alphabets WM, BM and PM respectively. These designations are followed by a figure indicating the minimum tensile strength in MPa or N/mm^2. For example 'WM 350' denotes whiteheart malleable cast iron with 350 MPa as minimum tensile strength. The following are the different grades of malleable cast iron :

Whiteheart malleable cast iron — WM 350 and WM 400

Blackheart malleable cast iron — BM 300 ; BM 320 and BM 350

Pearlitic malleable cast iron — PM 450 ; PM 500 ; PM 550 ; PM 600 and PM 700

6. ***Nodular or spheroidal graphite cast iron.*** The nodular or spheroidal graphite cast iron is also called ***ductile cast iron*** or ***high strength cast iron***. This type of cast iron is obtained by adding small amounts of magnesium (0.1 to 0.8%) to the molten grey iron. The addition of magnesium

Top

Contents

causes the *graphite to take form of small nodules or spheroids instead of the normal angular flakes. It has high fluidity, castability, tensile strength, toughness, wear resistance, pressure tightness, weldability and machinability. It is generally used for castings requiring shock and impact resistance along with good machinability, such as hydraulic cylinders, cylinder heads, rolls for rolling mill and centrifugally cast products.

According to Indian standard specification (IS : 1865-1991), the nodular or spheroidal graphite cast iron is designated by the alphabets 'SG' followed by the figures indicating the minimum tensile strength in MPa or N/mm^2 and the percentage elongation. For example, SG 400/15 means spheroidal graphite cast iron with 400 MPa as minimum tensile strength and 15 percent elongation. The Indian standard (IS : 1865 – 1991) recommends nine grades of spheroidal graphite cast iron based on mechanical properties measured on separately-cast test samples and six grades based on mechanical properties measured on cast-on sample as given in the Table 2.4.

The letter A after the designation of the grade indicates that the properties are obtained on cast-on test samples to distinguish them from those obtained on separately-cast test samples.

Table 2.4. Recommended grades of spheroidal graphite cast iron as per IS : 1865–1991.

Grade	*Minimum tensile strength (MPa)*	*Minimum percentage elongation*	*Brinell hardness number (BHN)*	*Predominant constituent of matrix*
SG 900/2	900	2	280 – 360	Bainite or tempered martensite
SG 800/2	800	2	245 – 335	Pearlite or tempered structure
SG 700/2	700	2	225 – 305	Pearlite
SG 600/3	600	3	190 – 270	Ferrite + Pearlite
SG 500/7	500	7	160 – 240	Ferrite + Pearlite
SG 450/10	450	10	160 – 210	Ferrite
SG 400/15	400	15	130 – 180	Ferrite
SG 400/18	400	18	130 – 180	Ferrite
SG 350/22	350	22	≤ 150	Ferrite
SG 700/2A	700	2	220 – 320	Pearlite
SG 600/3A	600	2	180 – 270	Pearlite + Ferrite
SG 500/7A	450	7	170 – 240	Pearlite + Ferrite
SG 400/15A	390	15	130 – 180	Ferrite
SG 400/18A	390	15	130 – 180	Ferrite
SG 350/22A	330	18	≤ 150	Ferrite

2.9 Alloy Cast Iron

The cast irons as discussed in Art. 2.8 contain small percentages of other constituents like silicon, manganese, sulphur and phosphorus. These cast irons may be called as **plain cast irons.** The alloy cast iron is produced by adding alloying elements like nickel, chromium, molybdenum, copper and manganese in sufficient quantities. These alloying elements give more strength and result in improvement of properties. The alloy cast iron has special properties like increased strength, high wear resistance, corrosion resistance or heat resistance. The alloy cast irons are extensively used for

* The graphite flakes in cast iron act as discontinuities in the matrix and thus lower its mechanical properties. The sharp corners of the flakes also act as stress raisers. The weakening effect of the graphite can be reduced by changing its form from a flake to a spheroidal form.

Top

gears, automobile parts like cylinders, pistons, piston rings, crank cases, crankshafts, camshafts, sprockets, wheels, pulleys, brake drums and shoes, parts of crushing and grinding machinery etc.

2.10 Effect of Impurities on Cast Iron

We have discussed in the previous articles that the cast iron contains small percentages of silicon, sulphur, manganese and phosphorous. The effect of these impurities on the cast iron are as follows:

1. ***Silicon.*** It may be present in cast iron upto 4%. It provides the formation of free graphite which makes the iron soft and easily machinable. It also produces sound castings free from blow-holes, because of its high affinity for oxygen.

2. ***Sulphur.*** It makes the cast iron hard and brittle. Since too much sulphur gives unsound casting, therefore, it should be kept well below 0.1% for most foundry purposes.

3. ***Manganese.*** It makes the cast iron white and hard. It is often kept below 0.75%. It helps to exert a controlling influence over the harmful effect of sulphur.

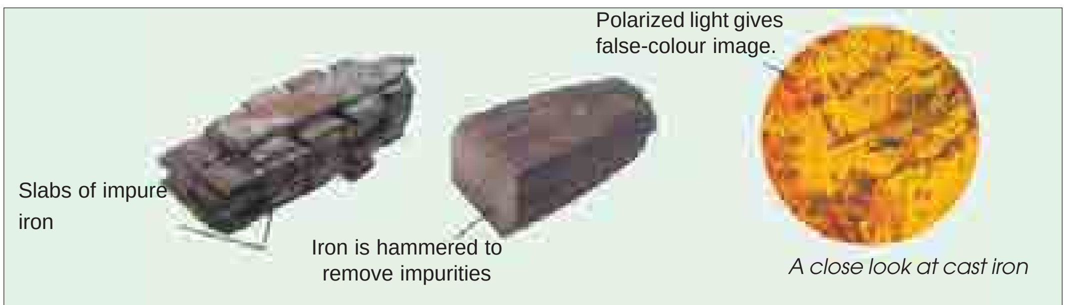
4. ***Phosphorus.*** It aids fusibility and fluidity in cast iron, but induces brittleness. It is rarely allowed to exceed 1%. Phosphoric irons are useful for casting of intricate design and for many light engineering castings when cheapness is essential.

Phosphorus is a non-metallic element. It must be stored underwater (above), since it catches fire when exposed to air, forming a compound.

2.11 Wrought Iron

It is the purest iron which contains at least 99.5% iron but may contain upto 99.9% iron. The typical composition of a wrought iron is

Carbon = 0.020%, Silicon = 0.120%, Sulphur = 0.018%, Phosphorus = 0.020%, Slag = 0.070%, and the remaining is iron.

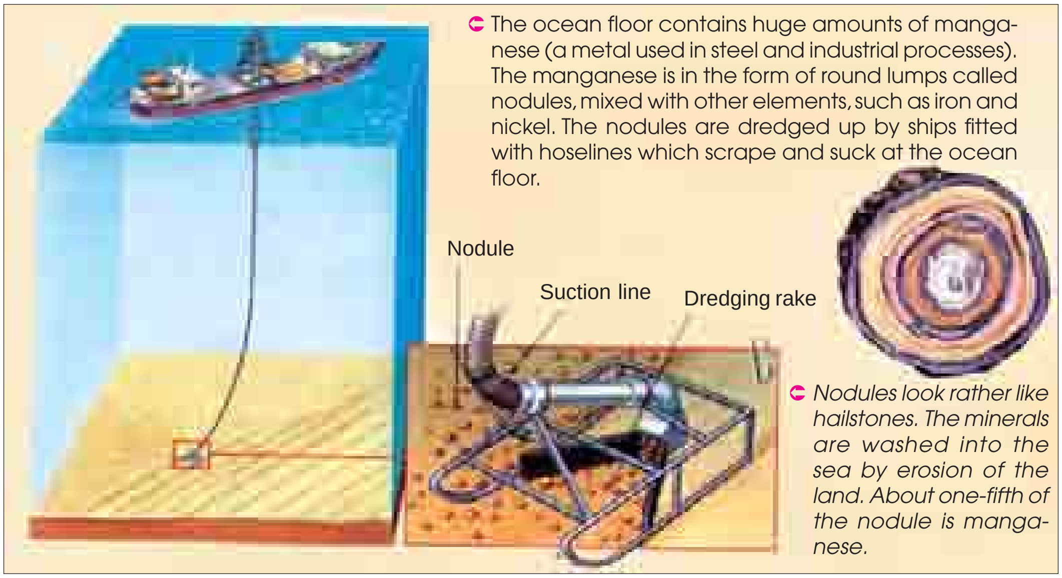


Wrought Iron

The wrought iron is produced from pig iron by remelting it in the puddling furnace of reverberatory type. The molten metal free from impurities is removed from the furnace as a pasty mass of iron and slag. The balls of this pasty mass, each about 45 to 65 kg are formed. These balls are then mechanically worked both to squeeze out the slag and to form it into some commercial shape.

The wrought iron is a tough, malleable and ductile material. It cannot stand sudden and excessive shocks. Its ultimate tensile strength is 250 MPa to 500 MPa and the ultimate compressive strength is 300 MPa.

It can be easily forged or welded. It is used for chains, crane hooks, railway couplings, water and steam pipes.



Note : This picture is given as additional information and is not a direct example of the current chapter.

2.12 Steel

It is an alloy of iron and carbon, with carbon content up to a maximum of 1.5%. The carbon occurs in the form of iron carbide, because of its ability to increase the hardness and strength of the steel. Other elements *e.g.* silicon, sulphur, phosphorus and manganese are also present to greater or lesser amount to impart certain desired properties to it. Most of the steel produced now-a-days is ***plain carbon steel*** or simply ***carbon steel.*** A carbon steel is defined as a steel which has its properties mainly due to its carbon content and does not contain more than 0.5% of silicon and 1.5% of manganese. The plain carbon steels varying from 0.06% carbon to 1.5% carbon are divided into the following types depending upon the carbon content.

1. Dead mild steel — up to 0.15% carbon
2. Low carbon or mild steel — 0.15% to 0.45% carbon
3. Medium carbon steel — 0.45% to 0.8% carbon
4. High carbon steel — 0.8% to 1.5% carbon

According to Indian standard *[IS : 1762 (Part-I)–1974], a new system of designating the steel is recommended. According to this standard, steels are designated on the following two basis :

(*a*) On the basis of mechanical properties, and (*b*) On the basis of chemical composition.

We shall now discuss, in detail, the designation of steel on the above two basis, in the following pages.

2.13 Steels Designated on the Basis of Mechanical Properties

These steels are carbon and low alloy steels where the main criterion in the selection and inspection of steel is the tensile strength or yield stress. According to Indian standard **IS: 1570 (Part–I)-1978 (Reaffirmed 1993), these steels are designated by a symbol 'Fe' or 'Fe E' depending on whether

* This standard was reaffirmed in 1993 and covers the code designation of wrought steel based on letter symbols.

** The Indian standard IS : 1570-1978 (Reaffirmed 1993) on wrought steels for general engineering purposes has been revised on the basis of experience gained in the production and use of steels. This standard is now available in seven parts.

the steel has been specified on the basis of minimum tensile strength or yield strength, followed by the figure indicating the minimum tensile strength or yield stress in N/mm^2. For example 'Fe 290' means a steel having minimum tensile strength of 290 N/mm^2 and 'Fe E 220' means a steel having yield strength of 220 N/mm^2.

Table 2.5 shows the tensile and yield properties of standard steels with their uses according to IS : 1570 (Part I)-1978 (Reaffirmed 1993).

Table 2.5. Indian standard designation of steel according to IS : 1570 (Part I)-1978 (Reaffirmed 1993).

Indian standard designation	*Tensile strength (Minimum) N/mm^2*	*Yield stress (Minimum) N/mm^2*	*Minimum percentage elongation*	*Uses as per IS : 1871 (Part I)–1987 (Reaffirmed 1993)*
Fe 290 Fe E 220	290 290	170 220	27 27	It is used for plain drawn or enamelled parts, tubes for oil well casing, steam, water and air passage, cycle, motor cycle and automobile tubes, rivet bars and wire.
Fe 310 Fe E 230 Fe 330 Fe E 250	310 310 330 330	180 230 200 250	26 26 26 26	These steels are used for locomotive carriages and car structures, screw stock and other general engineering purposes.
Fe 360 Fe E 270	360 360	220 270	25 25	It is used for chemical pressure vessels and other general engineering purposes.
Fe 410 Fe E 310	410 410	250 310	23 23	It is used for bridges and building construction, railway rolling stock, screw spikes, oil well casing, tube piles, and other general engineering purposes.
Fe 490 Fe E 370	490 490	290 370	21 21	It is used for mines, forgings for marine engines, sheet piling and machine parts.
Fe 540 Fe E 400	540 540	320 400	20 20	It is used for locomotive, carriage, wagon and tramway axles, arches for mines, bolts, seamless and welded tubes.
Fe 620 Fe E 460	620 620	380 460	15 15	It is used for tramway axles and seamless tubes.
Fe 690 Fe E 520	690 690	410 520	12 12	It is used for locomotive, carriage and wagon wheels and tyres, arches for mines, seamless oil well casing and drill tubes, and machine parts for heavy loading.
Fe 770 Fe E 580	770 770	460 580	10 10	It is used for locomotive, carriage and wagon wheels and tyres, and machine parts for heavy loading.
Fe 870 Fe E 650	870 870	520 650	8 8	It is used for locomotive, carriage and wagon wheels and tyres.

Contents

Notes : 1. The steels from grades Fe 290 to Fe 490 are general structural steels and are available in the form of bars, sections, tubes, plates, sheets and strips.

2. The steels of grades Fe 540 and Fe 620 are medium tensile structural steels.

3. The steels of grades Fe 690, Fe 770 and Fe 870 are high tensile steels.

2.14 Steels Designated on the Basis of Chemical Composition

According to Indian standard, IS : 1570 (Part II/Sec I)-1979 (Reaffirmed 1991), the carbon steels are designated in the following order :

(*a*) Figure indicating 100 times the average percentage of carbon content,

(*b*) Letter 'C', and

(*c*) Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.

For example 20C8 means a carbon steel containing 0.15 to 0.25 per cent (0.2 per cent on an average) carbon and 0.60 to 0.90 per cent (0.75 per cent rounded off to 0.8 per cent on an average) manganese.

Table 2.6 shows the Indian standard designation of carbon steel with composition and their uses.

Table 2.6. Indian standard designation of carbon steel according to IS : 1570 (Part II/Sec 1) – 1979 (Reaffirmed 1991).

Indian standard designation	*Composition in percentages*		*Uses as per IS : 1871 (Part II)–1987 (Reaffirmed 1993)*
	Carbon (C)	*Manganese (Mn)*	
4C2	0.08 Max.	0.40 Max.	It is a dead soft steel generally used in electrical industry.
5C4 7C4 10C4	0.10 Max. 0.12 Max. 0.15 Max.	0.50 Max. 0.50 Max. 0.30 – 0.60	These steels are used where cold formability is the primary requirement. In the rimming quality, they are used as sheet, strip, rod and wire especially where excellent surface finish or good drawing qualities are required, such as automobile body, and fender stock, hoods, lamps, oil pans and a multiple of deep drawn and formed products. They are also used for cold heading wire and rivets and low carbon wire products. The killed steel is used for forging and heat treating applications.
10C4 14C6	0.15 Max. 0.10 – 0.18	0.30 – 0.60 0.40 – 0.70	The case hardening steels are used for making camshafts, cams, light duty gears, worms, gudgeon pins, spindles, pawls, ratchets, chain wheels, tappets, etc.
15C4	0.20 Max.	0.30 – 0.60	It is used for lightly stressed parts. The material, although easily machinable, is not designed specifically for rapid cutting, but is suitable where cold web, such as bending and riveting may be necessary.

Top

Contents

Indian standard designation	*Composition in percentages*		*Uses as per IS : 1871 (Part II)–1987 (Reaffirmed 1993)*
	Carbon (C)	*Manganese (Mn)*	
15C8	0.10 – 0.20	0.60 – 0.90	These steels are general purposes steels used for low stressed components.
20C8	0.15 – 0.25	0.60 – 0.90	
25C4	0.20 – 0.30	0.30 – 0.60	
25C8	0.20 – 0.30	0.60 – 0.90	
30C8	0.25 – 0.35	0.60 – 0.90	It is used for making cold formed parts such as shift and brake levers. After suitable case hardening or hardening and tempering, this steel is used for making sprockets, tie rods, shaft fork and rear hub, 2 and 3 wheeler scooter parts such as sprocket, lever, hubs for forks, cams, rocket arms and bushes. Tubes for aircraft, automobile, bicycle and furniture are also made of this steel.
35C4	0.30 – 0.40	0.30 – 0.60	It is used for low stressed parts, automobile tubes and fasteners.
35C8	0.30 – 0.40	0.60 – 0.90	It is used for low stressed parts in machine structures, cycle and motor cycle tubes, fish plates for rails and fasteners.
40C8	0.35 – 0.45	0.60 – 0.90	It is used for crankshafts, shafts, spindles, push rods, automobile axle beams, connecting rods, studs, bolts, lightly stressed gears, chain parts, umbrella ribs, washers, etc.
45C8	0.40 – 0.50	0.60 – 0.90	It is used for spindles of machine tools, bigger gears, bolts, lead screws, feed rods, shafts and rocks.
50C4	0.45 – 0.55	0.30 – 0.60	It is used for keys, crankshafts, cylinders and machine parts requiring moderate wear resistance. In surface hardened condition, it is also suitable for large pitch worms and gears.
50C12	0.45 – 0.55	1.1 – 1.50	It is a rail steel. It is also used for making spike bolts, gear shafts, rocking levers and cylinder liners.
55C4	0.50 – 0.60	0.30 – 0.60	These steels are used for making gears, coil springs, cylinders, cams, keys, crankshafts, sprockets and machine parts requiring moderate wear resistance for which toughness is not of primary importance. It is also used for cycle and industrial chains, spring, can opener, umbrella ribs, parts of camera and typewriter.
55C8	0.50 – 0.60	0.60 – 0.90	
60C4	0.55 – 0.65	0.30 – 0.60	It is used for making clutch springs, hardened screws and nuts, machine tool spindles, couplings, crankshafts, axles and pinions.
65C9	0.60 – 0.70	0.50 – 0.80	It is a high tensile structural steel used for making locomotive carriage and wagon tyres. It is also used for engine valve springs, small washers and thin stamped parts.

Top

Contents

Indian standard designation	*Composition in percentages*		*Uses as per IS : 1871 (Part II)–1987 (Reaffirmed 1993)*
	Carbon (C)	*Manganese (Mn)*	
70C6	0.65 – 0.75	0.50 – 0.80	It is used for making baffle springs, shock absorbers, springs for seat cushions for road vehicles. It is also used for making rail tyres, unhardened gears and worms, washers, wood working saw, textile and jute machinery parts and clutch plates, etc.
75C6	0.70 – 0.80	0.50 – 0.80	It is used for making light flat springs formed from annealed stock. Because of good wear properties when properly heat treated, it is used for making shear blades, rack teeth, scrappers and cutlivators' shovels.
80C6 85C6	0.75 – 0.85 0.80 – 0.90	0.50 – 0.80 0.50 – 0.80	These steels are used for making flat and coil springs for automobile and railway vehicles. It is also used for girder rails. The valve spring wire and music wire are special applications of steel 85 C6. After suitable heat treatment, these steels are also used for making scraper blades, discs and spring tooth harrows. It is also used for clutch parts, wood working saw, band saw and textile and jute machinery parts.
98C6 113C6	0.90 – 1.05 1.05 – 1.20	0.50 – 0.80 0.50 – 0.80	These steels in the oil hardened and tempered condition are used for coil or spiral springs. It is also used for pen nibs, volute spring, spring cutlery, knitting needle and hacksaw blades.

2.15 Effect of Impurities on Steel

The following are the effects of impurities like silicon, sulphur, manganese and phosphorus on steel.

1. *Silicon.* The amount of silicon in the finished steel usually ranges from 0.05 to 0.30%. Silicon is added in low carbon steels to prevent them from becoming porous. It removes the gases and oxides, prevent blow holes and thereby makes the steel tougher and harder.

2. *Sulphur.* It occurs in steel either as iron sulphide or manganese sulphide. Iron sulphide because of its low melting point produces red shortness, whereas manganese sulphide does not effect so much. Therefore, manganese sulphide is less objectionable in steel than iron sulphide.

3. *Manganese.* It serves as a valuable deoxidising and purifying agent in steel. Manganese also combines with sulphur and thereby decreases the harmful effect of this element remaining in the steel. When used in ordinary low carbon steels, manganese makes the metal ductile and of good bending qualities. In high speed steels, it is used to toughen the metal and to increase its critical temperature.

4. *Phosphorus.* It makes the steel brittle. It also produces cold shortness in steel. In low carbon steels, it raises the yield point and improves the resistance to atmospheric corrosion. The sum of carbon and phosphorus usually does not exceed 0.25%.

Top

Contents

2.16 Free Cutting Steels

The free cutting steels contain sulphur and phosphorus. These steels have higher sulphur content than other carbon steels. In general, the carbon content of such steels vary from 0.1 to 0.45 per cent and sulphur from 0.08 to 0.3 per cent. These steels are used where rapid machining is the prime requirement. It may be noted that the presence of sulphur and phosphorus causes long chips in machining to be easily broken and thus prevent clogging of machines. Now a days, lead is used from 0.05 to 0.2 per cent instead of sulphur, because lead also greatly improves the machinability of steel without the loss of toughness.

According to Indian standard, IS : 1570 (Part III)-1979 (Reaffirmed 1993), carbon and carbon manganese free cutting steels are designated in the following order :

1. Figure indicating 100 times the average percentage of carbon,
2. Letter 'C',
3. Figure indicating 10 times the average percentage of manganese, and
4. Symbol 'S' followed by the figure indicating the 100 times the average content of sulphur. If instead of sulphur, lead (Pb) is added to make the steel free cutting, then symbol 'Pb' may be used.

Table 2.7 shows the composition and uses of carbon and carbon-manganese free cutting steels, as per IS : 1570 (Part III)-1979 (Reaffirmed 1993).

2.17 Alloy Steel

An alloy steel may be defined as a steel to which elements other than carbon are added in sufficient amount to produce an improvement in properties. The alloying is done for specific purposes to increase wearing resistance, corrosion resistance and to improve electrical and magnetic properties, which cannot be obtained in plain carbon steels. The chief alloying elements used in steel are nickel, chromium, molybdenum, cobalt, vanadium, manganese, silicon and tungsten. Each of these elements confer certain qualities upon the steel to which it is added. These elements may be used separately or in combination to produce the desired characteristic in steel. Following are the effects of alloying elements on steel:

1. ***Nickel***. It increases the strength and toughness of the steel. These steels contain 2 to 5% nickel and from 0.1 to 0.5% carbon. In this range, nickel contributes great strength and hardness with high elastic limit, good ductility and good resistance to corrosion. An alloy containing 25% nickel possesses maximum toughness and offers the greatest resistance to rusting, corrosion and burning at high temperature. It has proved to be of advantage in the manufacture of boiler tubes, valves for use with superheated steam, valves for I.C. engines and spark plugs for petrol engines. A nickel steel alloy containing 36% of nickel is known as ***invar***. It has nearly zero coefficient of expansion. So it is in great demand for measuring instruments and standards of lengths for everyday use.

2. ***Chromium.*** It is used in steels as an alloying element to combine hardness with high strength and high elastic limit. It also imparts corrosion-resisting properties to steel. The most common chrome steels contains from 0.5 to 2% chromium and 0.1 to 1.5% carbon. The chrome steel is used for balls, rollers and races for bearings. A ***nickel chrome steel*** containing 3.25% nickel, 1.5% chromium and 0.25% carbon is much used for armour plates. Chrome nickel steel is extensively used for motor car crankshafts, axles and gears requiring great strength and hardness.

3. ***Tungsten.*** It prohibits grain growth, increases the depth of hardening of quenched steel and confers the property of remaining hard even when heated to red colour. It is usually used in conjuction with other elements. Steel containing 3 to 18% tungsten and 0.2 to 1.5% carbon is used for cutting tools. The principal uses of tungsten steels are for cutting tools, dies, valves, taps and permanent magnets.

Top

Contents

Table 2.7. Indian standard designation of carbon and carbon-manganese free cutting steels according to IS:1570 (Part III) – 1979 (Reaffirmed 1993).

Indian standard designation	*Composition in percentages*					*Uses as per IS : 1871 (Part III)–1987 (Reaffirmed 1993)*
	Carbon (C)	*Silicon (Si)*	*Manganese (Mn)*	*Sulphur (S)*	*Phosphorus (P) Max*	
10C8S10	0.15 Max.	0.05 – 0.30	0.60 – 0.90	0.08 – 0.13	0.06	It is used for small parts to be cyanided or carbonitrided.
14C14S14	0.10 – 0.18	0.05 – 0.30	1.20 – 1.50	0.1 – 0.18	0.06	It is used for parts where good machinability and finish are important.
25C12S14	0.20 – 0.30	0.25 Max.	1.00 – 1.50	0.10 – 0.18	0.06	It is used for bolts, studs and other heat treated parts of small section. It is suitable in either cold drawn, normalised or heat treated condition for moderately stressed parts requiring more strength than mild steel.
40C10S18	0.35 – 0.45	0.25 Max.	0.80 – 1.20	0.14 – 0.22	0.06	It is used for heat treated bolts, engine shafts, connecting rods, miscellaneous gun carriage, and small arms parts not subjected to high stresses and severe wear.
11C10S25	0.08 – 0.15	0.10 Max.	0.80 – 1.20	0.20 – 0.30	0.06	It is used for lightly stressed components not subjected to shock (nuts, studs, etc.) and suitable for production on automatic lathes. It is not recommended for general case hardening work but should be used when ease of machining is the deciding factor.
40C15S12	0.35 – 0.45	0.25 Max.	1.30 – 1.70	0.08 – 0.15	0.06	It is used for heat treated axles, shafts, small crankshafts and other vehicle parts. It is not recommended for forgings in which transverse properties are important.

Top

4. *Vanadium.* It aids in obtaining a fine grain structure in tool steel. The addition of a very small amount of vanadium (less than 0.2%) produces a marked increase in tensile strength and elastic limit in low and medium carbon steels without a loss of ductility. The ***chrome-vanadium steel*** containing about 0.5 to 1.5% chromium, 0.15 to 0.3% vanadium and 0.13 to 1.1% carbon have extremely good tensile strength, elastic limit, endurance limit and ductility. These steels are frequently used for parts such as springs, shafts, gears, pins and many drop forged parts.

This is a fan blade from a jumbo jet engine. On take-off, the stress on the metal is immense, so to prevent the fan from flying apart, the blades must be both light and very strong. Titanium, though expensive, is the only suitable metal.

5. *Manganese.* It improves the strength of the steel in both the hot rolled and heat treated condition. The manganese alloy steels containing over 1.5% manganese with a carbon range of 0.40 to 0.55% are used extensively in gears, axles, shafts and other parts where high strength combined with fair ductility is required. The principal uses of manganese steel is in machinery parts subjected to severe wear. These steels are all cast and ground to finish.

6. *Silicon.* The silicon steels behave like nickel steels. These steels have a high elastic limit as compared to ordinary carbon steel. Silicon steels containing from 1 to 2% silicon and 0.1 to 0.4% carbon and other alloying elements are used for electrical machinery, valves in I.C. engines, springs and corrosion resisting materials.

7. *Cobalt.* It gives red hardness by retention of hard carbides at high temperatures. It tends to decarburise steel during heat-treatment. It increases hardness and strength and also residual magnetism and coercive magnetic force in steel for magnets.

8. *Molybdenum.* A very small quantity (0.15 to 0.30%) of molybdenum is generally used with chromium and manganese (0.5 to 0.8%) to make molybdenum steel. These steels possess extra tensile strength and are used for air-plane fuselage and automobile parts. It can replace tungsten in high speed steels.

2.18 Indian Standard Designation of Low and Medium Alloy Steels

According to Indian standard, IS : 1762 (Part I)-1974 (Reaffirmed 1993), low and medium alloy steels shall be designated in the following order :

1. Figure indicating 100 times the average percentage carbon.
2. Chemical symbol for alloying elements each followed by the figure for its average percentage content multiplied by a factor as given below :

Element	*Multiplying factor*
Cr, Co, Ni, Mn, Si and W	4
Al, Be, V, Pb, Cu, Nb, Ti, Ta, Zr and Mo	10
P, S and N	100

For example 40 Cr 4 Mo 2 means alloy steel having average 0.4% carbon, 1% chromium and 0.25% molybdenum.

Notes : 1. The figure after multiplying shall be rounded off to the nearest integer.

2. Symbol 'Mn' for manganese shall be included in case manganese content is equal to or greater than 1 per cent.

3. The chemical symbols and their figures shall be listed in the designation in the order of decreasing content.

Table 2.8 shows the composition and uses of some low and medium alloy steels according to Indian standard IS : 1570-1961 (Reaffirmed 1993).

Contents

Table 2.8. Composition and uses of alloy steels according to IS : 1570-1961 (Reaffirmed 1993).

Indian standard designation	*Composition in percentages*						*Uses as per IS : 1871–1965*
	Carbon (C)	*Silicon (Si)*	*Manganese (Mn)*	*Nickel (Ni)*	*Chromium (Cr)*	*Molybdenum (Mo)*	
11Mn2	0.16 Max.	0.10 – 0.35	1.30 – 1.70	–	–	–	It is a notch ductile steel for general purposes. It is also used in making filler rods, colliery cage suspension gear tub, mine car draw gear, couplings and rope sockets.
20Mn2	0.16 – 0.24	0.10 – 0.35	1.30 – 1.70	–	–	–	These are used for welded structures, crankshafts, steering levers, shafting spindles, etc.
27Mn2	0.22 – 0.32	0.10 – 0.35	1.30 – 1.70	–	–	–	
37Mn2	0.32 – 0.42	0.10 – 0.35	1.30 – 1.70	–	–	–	It is used for making axles, shafts, crankshafts, connecting rods, etc.
47Mn2	0.42 – 0.52	0.10 – 0.35	1.30 – 1.70	–	–	–	It is used for tram rails and similar other structural purposes.
40Cr1	0.35 – 0.45	0.10 – 0.35	0.60 – 0.09	–	0.90 – 1.20	–	It is used for making gears, connecting rods, stub axles, steering arms, wear resistant plates for earth moving and concrete handling equipment, etc.
50Cr1	0.45 – 0.55	0.10 – 0.35	0.60 – .90	–	0.90 – 1.20	–	It is spring steel. It is used in a helical automobile front suspension springs.
35Mn2Mo28	0.30 – 0.40	0.10 – 0.35	1.30 – 1.80	–	–	0.20 – 0.35	These are used for making general engineering components such as crankshafts, bolts, wheel studs, axle shafts, levers and connecting rods.
35Mn2Mo45	0.30 – 0.40	0.10 – 0.35	1.30 – 1.80	–	–	0.35 – 0.55	
40Cr1Mo28	0.35 – 0.45	0.10 – 0.35	0.50 – 0.80	–	0.90 – 1.20	0.20 – 0.35	It is used for making axle shafts, crankshafts, connecting rods, gears, high tensile bolts and studs, propeller shaft joints, etc.

Contd..

Top

Contents

Indian standard designation	Composition in percentages						Uses as per IS : 1871–1965
	Carbon (C)	Silicon (Si)	Manganese (Mn)	Nickel (Ni)	Chromium (Cr)	Molybdenum (Mo)	
15Cr3Mo55 25Cr3Mo55	0.10 – 0.20 0.20 – 0.30	0.10 – 0.35 0.10 – 0.35	0.40 – 0.70 0.40 – 0.70	0.30 Max. 0.30 Max.	2.90 – 3.40 2.90 – 3.40	0.45 – 0.65 0.45 – 0.65	These are used for components requiring medium to high tensile properties. In the nitrided condition, it is used for crank-shafts, cylinder liners for aero and automobile engines, gears, and machine parts requiring high surface hardness and wear resistance.
40Ni3	0.35 – 0.45	0.10 – 0.35	0.50 – 0.80	3.20 – 3.60	0.30 Max.	–	It is used for parts requiring excessively high toughness. In particular, it is used for components working at low temperatures (in refrigerators, compressors, locomotives and aircraft) and for heavy forgings, turbine blades, severely stressed screws, bolts and nuts.
30Ni4Crl	0.26 – 0.34	0.10 – 0.35	0.40 – 0.70	3.90 – 4.30	1.10 – 1.40	–	It is used for highly stressed gears and other components requiring high tensile strength of the order of 16 N/mm^2 and where minimum distortion in heat treatment is essential.
35NilCr60	0.30 – 0.40	0.10 – 0.35	0.60 – 0.90	1.00 – 1.50	0.45 – 0.75	–	It is used in the construction of aircraft and heavy vehicles for crankshafts, connecting rods, gear shafts, chain parts, clutches, flexible shafts for plenary gears, camshafts, etc.
40Ni2CrlMo28	0.35 – 0.45	0.10 – 0.35	0.40 – 70	1.25 – 1.75	0.90 – 1.30	0.20 – 0.35	It is used for high strength machine parts collets, spindles, screws, high tensile bolts and studs, gears, pinions, axle shafts, tappets, crankshafts, connecting rods, boring bars, arbours, etc.

Top

2.19 Stainless Steel

It is defined as that steel which when correctly heat treated and finished, resists oxidation and corrosive attack from most corrosive media. The different types of stainless steels are discussed below :

1. *Martensitic stainless steel.* The chromium steels containing 12 to 14 per cent chromium and 0.12 to 0.35 per cent carbon are the first stainless steels developed. Since these steels possess martensitic structure, therefore, they are called ***martensitic stainless steels.*** These steels are magnetic and may be hardened by suitable heat treatment and the hardness obtainable depends upon the carbon content. These steels can be easily welded and machined. When formability, softness, etc. are required in fabrication, steel having 0.12 per cent maximum carbon is often used in soft condition. With increasing carbon, it is possible by hardening and tempering to obtain tensile strength in the range of 600 to 900 N/mm^2, combined with reasonable toughness and ductility. In this condition, these steels find many useful general applications where mild corrosion resistance is required. Also, with the higher carbon range in the hardened and lightly tempered condition, tensile strength of about 1600 N/mm^2 may be developed with lowered ductility.

Stainless steel was invented in 1913 by British metallurgist Harry Brearley (1871-1948). He made a steel containing 13 per cent chromium. The new alloy proved to be highly resistant to corrosion: chromium reacts with oxygen in the air to form a tough, protective film which renews itself if the metal is scratched.

These steels may be used where the corrosion conditions are not too severe, such as for hydraulic, steam and oil pumps, valves and other engineering components. However, these steels are not suitable for shafts and parts working in contact with non-ferrous metals (*i.e.* brass, bronze or gun metal bearings) and with graphite packings, because electrolytic corrosion is likely to occur. After hardening and light tempering, these steels develop good cutting properties. Therefore, they are used for cutlery, springs, surgical and dental instruments.

Note: The presence of chromium provides good resistance to scaling upto a temperature of about 750°C, but it is not suitable where mechanical strength in the temperature range of 600 to 750°C is required. In fact, creep resistance of these steels at this temperature is not superior to that of mild steel. But at temperature below 600°C, the strength of these steels is better than that of carbon steels and upto 480°C is even better than that of austenitic steels.

2. *Ferritic stainless steel.* The steels containing greater amount of chromium (from 16 to 18 per cent) and about 0.12 per cent carbon are called ***ferritic stainless steels.*** These steels have better corrosion resistant property than martensitic stainless steels. But, such steels have little capacity for hardening by heat treatment. However, in the softened condition, they possess good ductility and are mainly used as sheet or strip for cold forming and pressing operations for purposes where moderate corrosion resistance is required. They may be cold worked or hot worked. They are ferro-magnetic, usually undergo excessive grain growth during prolonged exposure to elevated temperatures, and may develop brittleness after electric arc resistance or gas welding. These steels have lower strength at elevated temperatures than martensitic steels. However, resistance to scaling and corrosion at elevated temperatures are usually better. The machinability is good and they show no tendency to intercrystalline corrosion.

Note: When nickel from 1.5 to 2.5 per cent is added to 16 to 18 per cent chromium steel, it not only makes more resistant to corrosion than martensitic steel but also makes it hardenable by heat treatment. Such a steel has good resistance to electrolytic corrosion when in contact with non-ferrous metals and graphite packings. Thus it is widely used for pump shafts, spindles and valves as well as for many other fittings where a good combination of mechanical and corrosion properties are required.

3. *Austenitic stainless steel.* The steel containing high content of both chromium and nickel are called ***austenitic stainless steels.*** There are many variations in chemical composition of these steels, but the most widely used steel contain 18 per cent chromium and 8 per cent nickel with carbon content as low as possible. Such a steel is commonly known as ***18/8 steel.*** These steels cannot be hardened by quenching, in fact they are softened by rapid cooling from about 1000°C. They are non-magnetic and possess greatest resistance to corrosion and good mechanical properties at elevated temperature.

These steels are very tough and can be forged and rolled but offer great difficulty in machining. They can be easily welded, but after welding, it is susceptible to corrosive attack in an area adjacent to the weld. This susceptibility to corrosion (called intercrystalline corrosion or weld decay) may be removed by softening after welding by heating to about 1100°C and cooling rapidly. These steels are used in the manufacture of pump shafts, rail road car frames and sheathing, screws, nuts and bolts and small springs. Since 18/8 steel provide excellent resistance to attack by many chemicals, therefore, it is extensively used in chemical, food, paper making and dyeing industries.

Note : When increased corrosion resistance properties are required, for some purposes, then molybdenum from 2 to 3 per cent may be added.

2.20 Heat Resisting Steels

The steels which can resist creep and oxidation at high temperatures and retain sufficient strength are called ***heat resisting steels.*** A number of heat resisting steels have been developed as discussed below :

1. *Low alloy steels.* These steels contain 0.5 per cent molybdenum. The main application of these steels are for superheater tubes and pipes in steam plants, where service temperatures are in the range of 400°C to 500°C.

2. *Valve steels.* The chromium-silicon steels such as ***silchrome*** (0.4% C, 8% Cr, 3.5% Si) and ***Volmax*** (0.5% C, 8% Cr, 3.5% Si, 0.5% Mo) are used for automobile valves. They possess good resistance to scaling at dull red heat, although their strength at elevated temperatures is relatively low. For aeroplane engines and marine diesel engine valves, 13/13/3 nickel-chromium-tungsten valve steel is usually used.

3. *Plain chromium steel.* The plain chromium steel consists of

(*a*) Martensitic chromium steel with 12–13% Cr, and

(*b*) Ferritic chromium steels with 18–30% Cr.

These steels are very good for oxidation resistance at high temperatures as compared to their strength which is not high at such conditions. The maximum operating temperature for martensitic steels is about 750°C, whereas for ferritic steels it is about 1000 – 1150°C.

4. *Austenitic chromium-nickel steels.* These steels have good mechanical properties at high temperatures with good scaling resistance. These alloys contain a minimum of 18 per cent chromium and 8 per cent nickel stabilised with titanium or niobium. Other carbide forming elements such as molybdenum or tungsten may also be added in order to improve creep strength. Such alloys are suitable for use upto 1100°C and are used for gas turbine discs and blades.

Table 2.9. Indian standard designation of high alloy steels (stainless steel and heat resisting steels) according to IS : 1570 (Part V)-1985 (Reaffirmed 1991).

Indian standard designation	*Composition in percentages*						*Uses as per IS : 1871–1965*
	Carbon (C)	*Silicon (Si)*	*Manganese (Mn)*	*Nickel (Ni)*	*Chromium . (Cr)*	*Molybdenum (Mo)*	
30Cr13	0.26 – 0.35	1.0 Max.	1.0 Max.	1.0 Max.	12.0 – 14.0	–	It is used for structural parts with high strength and kitchen utensils.
15Cr16Ni2	0.10 – 0.20	1.0 Max.	1.0 Max.	1.5 – 3.0	15.0 – 18.0	–	It is used for aircraft fittings, wind shield wiper arms, bolting materials, paper machinery etc.
07Cr18Ni9	0.12 Max.	1.0 Max.	2.0 Max.	8.0 – 10.0	17.0 – 19.0	–	It is used for aircraft fire walls and cawlings, radar and microwaves antennae, jewellery, household novelties, automotive wheel covers, refrigerator trays, kitchen utensils, railway passenger car bodies, ice making equipment, tubular furniture, screen door and storm window frames, electric switch parts, flexible couplings etc.
04Cr17Ni12 Mo2	0.08 Max.	1.0 Max.	2.0 Max.	10.5 – 14.0	16.0 – 18.5	2.0 – 3.0	It is used for high temperature chemical handling equipment for rayon, rubber and marine industries, photographic developing equipment, pulp handling equipment, steam jacketed kettles, coke plant equipment, food processing equipment, edible oil storage tanks.
45Cr9Si4	0.40 – 0.50	3.25 – 3.75	0.30 – 0.60	0.05 Max.	7.50 – 9.50	–	It is used for heat resisting outlet valves in oil engines, lorries and cars.
80Cr20Si2 Nil	0.75 – 0.85	1.75 – 2.25	0.20 – 0.60	1.20 – 1.70	19.0 – 21.0	–	It is used for highly stressed outlet valves in high speed carburetors and heavy oil engines.

Contents

Top

2.21 Indian Standard Designation of High Alloy Steels (Stainless Steel and Heat Resisting Steel)

According to Indian standard, IS : 1762 (Part I)-1974 (Reaffirmed 1993), the high alloy steels (*i.e.* stainless steel and heat resisting steel) are designated in the following order:

1. Letter 'X'.
2. Figure indicating 100 times the percentage of carbon content.
3. Chemical symbol for alloying elements each followed by a figure for its average percentage content rounded off to the nearest integer.
4. Chemical symbol to indicate specially added element to allow the desired properties.

For example, X 10 Cr 18 Ni 9 means alloy steel with average carbon 0.10 per cent, chromium 18 per cent and nickel 9 per cent.

Table 2.9 shows the composition and uses of some types of the stainless steels and heat resisting steels according to Indian standard IS : 1570 (Part V)-1985 (Reaffirmed 1991).

2.22 High Speed Tool Steels

These steels are used for cutting metals at a much higher cutting speed than ordinary carbon tool steels. The carbon steel cutting tools do not retain their sharp cutting edges under heavier loads and higher speeds. This is due to the fact that at high speeds, sufficient heat may be developed during the cutting operation and causes the temperature of the cutting edge of the tool to reach a red heat. This temperature would soften the carbon tool steel and thus the tool will not work efficiently for a longer period. The high speed steels have the valuable property of retaining their hardness even when heated to red heat. Most of the high speed steels contain tungsten as the chief alloying element, but other elements like cobalt, chromium, vanadium, etc. may be present in some proportion. Following are the different types of high speed steels:

Gold is found mixed with quartz rock, deep underground. Most metals occur in their ores as compounds. Gold is so unreactive that it occurs naturally as pure metal.

1. ***18-4-1 High speed steel.*** This steel, on an average, contains 18 per cent tungsten, 4 per cent chromium and 1 per cent vanadium. It is considered to be one of the best of all purpose tool steels. It is widely used for drills, lathe, planer and shaper tools, milling cutters, reamers, broaches, threading dies, punches, etc.

2. ***Molybdenum high speed steel.*** This steel, on an average, contains 6 per cent tungsten, 6 per cent molybdenum, 4 per cent chromium and 2 per cent vanadium. It has excellent toughness and cutting ability. The molybdenum high speed steels are better and cheaper than other types of steels. It is particularly used for drilling and tapping operations.

3. ***Super high speed steel.*** This steel is also called ***cobalt high speed steel*** because cobalt is added from 2 to 15 per cent, in order to increase the cutting efficiency especially at high temperatures. This steel, on an average, contains 20 per cent tungsten, 4 per cent chromium, 2 per cent vanadium and 12 per cent cobalt. Since the cost of this steel is more, therefore, it is principally used for heavy cutting operations which impose high pressure and temperatures on the tool.

2.23 Indian Standard Designation of High Speed Tool Steel

According to Indian standard, IS : 1762 (Part I)-1974 (Reaffirmed 1993), the high speed tool steels are designated in the following order :

1. Letter 'XT'.
2. Figure indicating 100 times the percentage of carbon content.
3. Chemical symbol for alloying elements each followed by the figure for its average percentage content rounded off to the nearest integer, and
4. Chemical symbol to indicate specially added element to attain the desired properties.

For example, XT 75 W 18 Cr 4 V 1 means a tool steel with average carbon content 0.75 per cent, tungsten 18 per cent, chromium 4 per cent and vanadium 1 per cent.

Table 2.10 shows the composition of high speed tool steels as per Indian standard, IS : 7291-1981 (Reaffirmed 1993).

2.24 Spring Steels

The most suitable material for springs are those which can store up the maximum amount of work or energy in a given weight or volume of spring material, without permanent deformation. These steels should have a high elastic limit as well as high deflection value. The spring steel, for aircraft and automobile purposes should possess maximum strength against fatigue effects and shocks. The steels most commonly used for making springs are as follows:

1. ***High carbon steels.*** These steels contain 0.6 to 1.1 per cent carbon, 0.2 to 0.5 per cent silicon and 0.6 to 1 per cent manganese. These steels are heated to 780 – 850°C according to the composition and quenched in oil or water. It is then tempered at 200 – 500°C to suit the particular application. These steels are used for laminated springs for locomotives, carriages, wagons, and for heavy road vehicles. The higher carbon content oil hardening steels are used for volute, spiral and conical springs and for certain types of petrol engine inlet valve springs.

2. ***Chrome-vanadium steels.*** These are high quality spring steels and contain 0.45 to 0.55 per cent carbon, 0.9 to 1.2 per cent chromium, 0.15 to 0.20 per cent vanadium, 0.3 to 0.5 per cent silicon and 0.5 to 0.8 per cent manganese. These steels have high elastic limit, resistance to fatigue and impact stresses. Moreover, these steels can be machined without difficulty and can be given a smooth surface free from tool marks. These are hardened by oil quenching at 850 – 870°C and tempered at 470 – 510°C for vehicle and other spring purposes. These steels are used for motor car laminated and coil springs for suspension purposes, automobile and aircraft engine valve springs.

Sodium is in Group I. Although it is a metal, it is so soft that a knife can cut easily through a piece. Sodium is stored in oil to stop air or moisture reacting with it.

3. ***Silicon-manganese steels.*** These steels contain 1.8 to 2.0 per cent silicon, 0.5 to 0.6 per cent carbon and 0.8 to 1 per cent manganese. These steels have high fatigue strength, resistance and toughness. These are hardened by quenching in oil at 850 – 900°C and tempered at 475 – 525°C. These are the usual standard quality modern spring materials and are much used for many engineering purposes.

Contents

Table 2.10. Indian standard designation of high speed tool steel according to IS : 7291-1981 (Reaffirmed 1993).

Indian standard designation	*Chemical composition in percentages*								*Brinell hardness in annealed condition (HB)Max.*
	Carbon (C)	*Silicon (Si)*	*Manganese (Mn)*	*Chromium (Cr)*	*Molybdenum (Mo)*	*Vanadium (V)*	*Tungsten (W)*	*Cobalt (Co)*	
XT 72 W 18 Cr 4 V 1	0.65 – 0.80	0.15 – 0.40	0.20 – 0.40	3.75 – 4.50	–	1.00 – 1.25	17.50 – 19.0	–	255
XT 75 W 18 Co 5 Cr 4 Mo V 1	0.70 – 0.80	0.15 – 0.40	0.20 – 0.40	3.75 – 4.50	0.40 – 1.00	1.00 – 1.25	17.50 – 19.0	4.50 – 5.50	269
XT 80 W 20 Co 12 Cr 4 V 2 Mo 1	0.75 – 0.85	0.15 – 0.40	0.20 – 0.40	4.00 – 4.75	0.40 – 1.00	1.25 – 1.75	19.50 – 21.0	11.00 – 12.50	302
XT 125 W Co 10 Cr Mo 4 V 3	1.20 – 1.30	0.15 – 0.40	0.20 – 0.40	3.75 – 4.75	3.00 – 4.00	2.80 – 3.50	8.80 – 10.70	8.80 – 10.70	269
XT 87 W 6 Mo 5 Cr 4 V 2	0.82 – 0.92	0.15 – 0.40	0.15 – 0.40	3.75 – 4.75	4.75 – 5.50	1.75 – 2.05	5.75 – 6.75	–	248
XT 90 W 6 Co Mo 5 Cr 4 V 2	0.85 – 0.95	0.15 – 0.40	0.20 – 0.40	3.75 – 4.75	4.75 – 5.50	1.70 – 2.20	5.75 – 6.75	4.75 – 5.25	269
XT 110 Mo 10 Co 8 Cr 4 W 2	1.05 – 1.15	0.15 – 0.40	0.15 – 0.40	3.50 – 4.50	9.0 – 10.0	0.95 – 1.35	1.15 – 1.85	7.75 – 8.75	269

Notes:
1. For all steels, sulphur (*S*) and phosphorus (*P*) is 0.030 per cent Max.
2. If sulphur is added to give free machining properties, then it shall be between 0.09 and 0.15 per cent.

Top

2.25 Heat Treatment of Steels

The term heat treatment may be defined as an operation or a combination of operations, involving the heating and cooling of a metal or an alloy in the solid state for the purpose of obtaining certain desirable conditions or properties without change in chemical composition. The aim of heat treatment is to achieve one or more of the following objects :

1. To increase the hardness of metals.
2. To relieve the stresses set up in the material after hot or cold working.
3. To improve machinability.
4. To soften the metal.
5. To modify the structure of the material to improve its electrical and magnetic properties.
6. To change the grain size.
7. To increase the qualities of a metal to provide better resistance to heat, corrosion and wear.

Following are the various heat treatment processes commonly employed in engineering practice:

1. ***Normalising.*** The main objects of normalising are :

1. To refine the grain structure of the steel to improve machinability, tensile strength and structure of weld.
2. To remove strains caused by cold working processes like hammering, rolling, bending, etc., which makes the metal brittle and unreliable.
3. To remove dislocations caused in the internal structure of the steel due to hot working.
4. To improve certain mechanical and electrical properties.

The process of normalising consists of heating the steel from 30 to 50°C above its upper critical temperature (for hypoeutectoid steels) or Acm line (for hypereutectoid steels). It is held at this temperature for about fifteen minutes and then allowed to cool down in still air.

This process provides a homogeneous structure consisting of ferrite and pearlite for hypoeutectoid steels, and pearlite and cementite for hypereutectoid steels. The homogeneous structure provides a higher yield point, ultimate tensile strength and impact strength with lower ductility to steels. The process of normalising is frequently applied to castings and forgings, etc. The alloy steels may also be normalised but they should be held for two hours at a specified temperature and then cooling in the furnace.

Notes : (*a*) The upper critical temperature for a steel depends upon its carbon content. It is 900°C for pure iron, 860°C for steels with 2.2% carbon, 723°C for steel with 0.8% carbon and 1130°C for steel with 1.8% carbon.

(*b*) Steel containing 0.8% carbon is known as ***eutectoid steel,*** steel containing less than 0.8% carbon is called ***hypoeutectoid steel*** and steel containing above 0.8% carbon is called ***hypereutectoid steel.***

2. ***Annealing.*** The main objects of annealing are :

1. To soften the steel so that it may be easily machined or cold worked.
2. To refine the grain size and structure to improve mechanical properties like strength and ductility.
3. To relieve internal stresses which may have been caused by hot or cold working or by unequal contraction in casting.
4. To alter electrical, magnetic or other physical properties.
5. To remove gases trapped in the metal during initial casting.

The annealing process is of the following two types :

(*a*) ***Full annealing.*** The purpose of full annealing is to soften the metal to refine the grain structure, to relieve the stresses and to remove trapped gases in the metal. The process consists of

Contents

(*i*) heating the steel from 30 to 50°C above the upper critical temperature for hypoeutectoid steel and by the same temperature above the lower critical temperature *i.e.* 723°C for hypereutectoid steels.

(*ii*) holding it at this temperature for sometime to enable the internal changes to take place. The time allowed is approximately 3 to 4 minutes for each millimetre of thickness of the largest section, and

(*iii*) cooling slowly in the furnace. The rate of cooling varies from 30 to 200°C per hour depending upon the composition of steel.

In order to avoid decarburisation of the steel during annealing, the steel is packed in a cast iron box containing a mixture of cast iron borings, charcoal, lime, sand or ground mica. The box along with its contents is allowed to cool slowly in the furnace after proper heating has been completed.

The following table shows the approximate temperatures for annealing depending upon the carbon contents in steel.

Table 2.11. Annealing temperatures.

S.No.	*Carbon content, per cent*	*Annealing temperature, °C*
1.	Less than 0.12 (Dead mild steel)	875 – 925
2.	0.12 to 0.45 (Mild steel)	840 – 970
3.	0.45 to 0.50 (Medium carbon steel)	815 – 840
4.	0.50 to 0.80 (Medium carbon steel)	780 – 810
5.	0.80 to 1.50 (High carbon or tool steel)	760 – 780

(*b*) *Process annealing.* The process annealing is used for relieving the internal stresses previously set up in the metal and for increasing the machinability of the steel. In this process, steel is heated to a temperature below or close to the lower critical temperature, held at this temperature for sometime and then cooled slowly. This causes complete recrystallisation in steels which have been severely cold worked and a new grain structure is formed. The process annealing is commonly used in the sheet and wire industries.

3. *Spheroidising.* It is another form of annealing in which cementite in the granular form is produced in the structure of steel. This is usually applied to high carbon tool steels which are difficult to machine. The operation consists of heating the steel to a temperature slightly above the lower critical temperature (730 to 770°C). It is held at this temperature for some time and then cooled slowly to a temperature of 600°C. The rate of cooling is from 25 to 30°C per hour.

The spheroidising improves the machinability of steels, but lowers the hardness and tensile strength. These steels have better elongation properties than the normally annealed steel.

4. *Hardening.* The main objects of hardening are :

1. To increase the hardness of the metal so that it can resist wear.

2. To enable it to cut other metals *i.e.* to make it suitable for cutting tools.

Clay can be hardened by heat. Bricks and ceramic items are made by firing soft clay objects in a kiln.

Top

The process of hardening consists of

(*a*) heating the metal to a temperature from 30 to 50°C above the upper critical point for hypoeutectoid steels and by the same temperature above the lower critical point for hypereutectoid steels.

(*b*) keeping the metal at this temperature for a considerable time, depending upon its thickness.

(*c*) quenching (cooling suddenly) in a suitable cooling medium like water, oil or brine.

It may be noted that the low carbon steels cannot be hardened appreciably, because of the presence of ferrite which is soft and is not changed by the treatment. As the carbon content goes on increasing, the possible obtainable hardness also increases.

Notes : 1. The greater the rate of quenching, the harder is the resulting structure of steel.

2. For hardening alloy steels and high speed steels, they are heated from 1100°C to 1300°C followed by cooling in a current of air.

5. *Tempering.* The steel hardened by rapid quenching is very hard and brittle. It also contains internal stresses which are severe and unequally distributed to cause cracks or even rupture of hardened steel. The tempering (also known as ***drawing***) is, therefore, done for the following reasons :

1. To reduce brittleness of the hardened steel and thus to increase ductility.
2. To remove the internal stresses caused by rapid cooling of steel.
3. To make steel tough to resist shock and fatigue.

The tempering process consists of reheating the hardened steel to some temperature below the lower critical temperature, followed by any desired rate of cooling. The exact tempering temperature depends upon the purpose for which the article or tool is to be used.

6. *Surface hardening or case hardening.* In many engineering applications, it is desirable that a steel being used should have a hardened surface to resist wear and tear. At the same time, it should have soft and tough interior or core so that it is able to absorb any shocks, etc. This is achieved by hardening the surface layers of the article while the rest of it is left as such. This type of treatment is applied to gears, ball bearings, railway wheels, etc.

Following are the various *surface or case hardening processes by means of which the surface layer is hardened:

1. Carburising, **2.** Cyaniding, **3.** Nitriding, **4.** Induction hardening, and **5.** Flame hardening.

2.26 Non-ferrous Metals

We have already discussed that the non-ferrous metals are those which contain a metal other than iron as their chief constituent. The non-ferrous metals are usually employed in industry due to the following characteristics :

1. Ease of fabrication (casting, rolling, forging, welding and machining),
2. Resistance to corrosion,
3. Electrical and thermal conductivity, and
4. Weight.

The various non-ferrous metals used in engineering practice are aluminium, copper, lead, tin, zinc, nickel, etc. and their alloys. We shall now discuss these non-ferrous metals and their alloys in detail, in the following pages.

2.27 Aluminium

It is white metal produced by electrical processes from its oxide (alumina), which is prepared from a clayey mineral called ***bauxite.*** It is a light metal having specific gravity 2.7 and melting point 658°C. The tensile strength of the metal varies from 90 MPa to 150 MPa.

* For complete details, please refer authors' popular book **'A Text Book of Workshop Technology'.**

In its pure state, the metal would be weak and soft for most purposes, but when mixed with small amounts of other alloys, it becomes hard and rigid. So, it may be blanked, formed, drawn, turned, cast, forged and die cast. Its good electrical conductivity is an important property and is widely used for overhead cables. The high resistance to corrosion and its non-toxicity makes it a useful metal for cooking utensils under ordinary condition and thin foils are used for wrapping food items. It is extensively used in aircraft and automobile components where saving of weight is an advantage.

2.28 Aluminium Alloys

The aluminium may be alloyed with one or more other elements like copper, magnesium, manganese, silicon and nickel. The addition of small quantities of alloying elements converts the soft and weak metal into hard and strong metal, while still retaining its light weight. The main aluminium alloys are discussed below:

1. ***Duralumin.*** It is an important and interesting wrought alloy. Its composition is as follows:

Copper = 3.5 – 4.5%; Manganese = 0.4 – 0.7%; Magnesium = 0.4 – 0.7%, and the remainder is aluminium.

This alloy possesses maximum tensile strength (upto 400 MPa) after heat treatment and age hardening. After working, if the metal is allowed to age for 3 or 4 days, it will be hardened. This phenomenon is known as ***age hardening.***

It is widely used in wrought conditions for forging, stamping, bars, sheets, tubes and rivets. It can be worked in hot condition at a temperature of 500°C. However, after forging and annealing, it can also be cold worked. Due to its high strength and light weight, this alloy may be used in automobile and aircraft components. It is also used in manufacturing connecting rods, bars, rivets, pulleys, etc.

2. ***Y-alloy.*** It is also called copper-aluminium alloy. The addition of copper to pure aluminium increases its strength and machinability. The composition of this alloy is as follows :

Copper = 3.5 – 4.5%; Manganese = 1.2 – 1.7%; Nickel = 1.8 – 2.3%; Silicon, Magnesium, Iron = 0.6% each; and the remainder is aluminium.

This alloy is heat treated and age hardened like duralumin. The ageing process is carried out at room temperature for about five days.

It is mainly used for cast purposes, but it can also be used for forged components like duralumin. Since *Y*-alloy has better strength (than duralumin) at high temperature, therefore, it is much used in aircraft engines for cylinder heads and pistons.

3. ***Magnalium.*** It is made by melting the aluminium with 2 to 10% magnesium in a vacuum and then cooling it in a vacuum or under a pressure of 100 to 200 atmospheres. It also contains about 1.75% copper. Due to its light weight and good mechanical properties, it is mainly used for aircraft and automobile components.

4. ***Hindalium.*** It is an alloy of aluminium and magnesium with a small quantity of chromium. It is the trade name of aluminium alloy produced by Hindustan Aluminium Corporation Ltd, Renukoot (U.P.). It is produced as a rolled product in 16 gauge, mainly for anodized utensil manufacture.

2.29 Copper

It is one of the most widely used non-ferrous metals in industry. It is a soft, malleable and ductile material with a reddish-brown appearance. Its specific gravity is 8.9 and melting point is 1083°C. The tensile strength varies from 150 MPa to 400 MPa under different conditions. It is a good conductor of electricity. It is largely used in making electric cables and wires for electric machinery and appliances, in electrotyping and electroplating, in making coins and household utensils.

It may be cast, forged, rolled and drawn into wires. It is non-corrosive under ordinary conditions and resists weather very effectively. Copper in the form of tubes is used widely in mechanical engineering. It is also used for making ammunitions. It is used for making useful alloys with tin, zinc, nickel and aluminium.

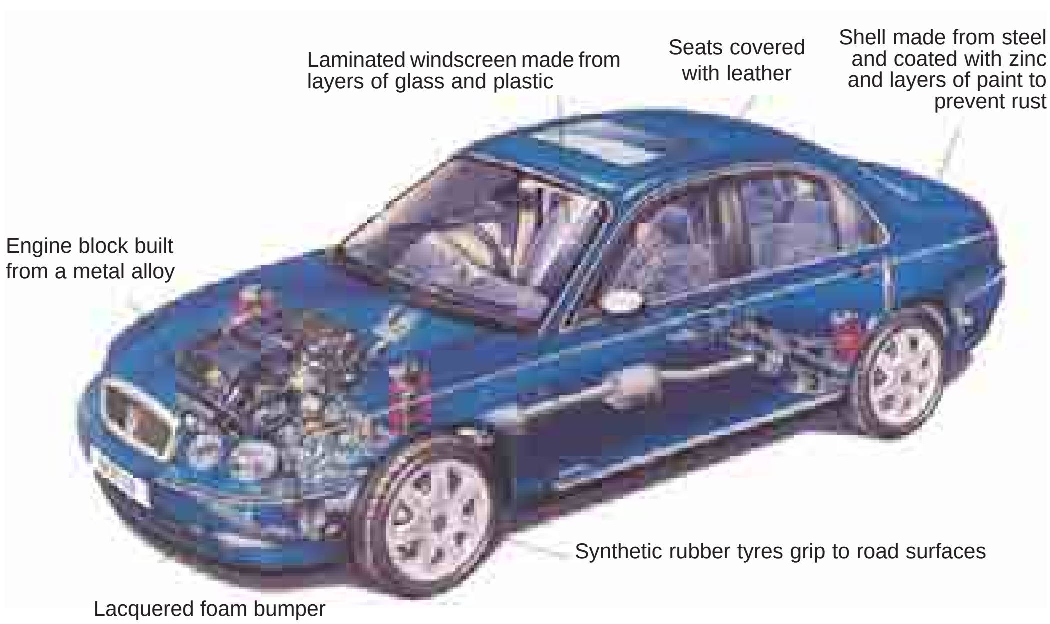
2.30 Copper Alloys

The copper alloys are broadly classified into the following two groups :

1. *Copper-zinc alloys* (*Brass*). The most widely used copper-zinc alloy is brass. There are various types of brasses, depending upon the proportions of copper and zinc. This is fundamentally a binary alloy of copper with zinc each 50%. By adding small quantities of other elements, the properties of brass may be greatly changed. For example, the addition of lead (1 to 2%) improves the machining quality of brass. It has a greater strength than that of copper, but have a lower thermal and electrical conductivity. Brasses are very resistant to atmospheric corrosion and can be easily soldered. They can be easily fabricated by processes like spinning and can also be electroplated with metals like nickel and chromium. The following table shows the composition of various types of brasses according to Indian standards.

Malachite is an ore of copper. Its dramatic bands of dark green make it popular in jewellery.

Electrical cables often consist of fine strands of copper wire woven together and encased in a plastic sleeve.



Materials are used to build a modern car.

Contents

Table 2.12. Composition and uses of brasses.

Indian standard designation	*Composition in percentages*	*Uses*
Cartridge brass	Copper = 70 Zinc = 30	It is a cold working brass used for cold rolled sheets, wire drawing, deep drawing, pressing and tube manufacture.
Yellow brass (Muntz metal)	Copper = 60 Zinc = 40	It is suitable for hot working by rolling, extrusion and stamping.
Leaded brass	Copper = 62.5 Zinc = 36 Lead = 1.5	
Admiralty brass	Copper = 70 Zinc = 29 Tin = 1	These are used for plates, tubes, etc.
Naval brass	Copper = 59 Zinc = 40 Tin = 1	It is used for marine castings.
Nickel brass (German silver or Nickel silver)	Copper = 60 – 45 Zinc = 35 – 20 Nickel = 5 – 35	It is used for valves, plumbing fittings, automobile fitting, type writer parts and musical instruments.

2. ***Copper-tin alloys (Bronze).*** The alloys of copper and tin are usually termed as bronzes. The useful range of composition is 75 to 95% copper and 5 to 25% tin. The metal is comparatively hard, resists surface wear and can be shaped or rolled into wires, rods and sheets very easily. In corrosion resistant properties, bronzes are superior to brasses. Some of the common types of bronzes are as follows:

(*a*) ***Phosphor bronze.*** When bronze contains phosphorus, it is called phosphor bronze. Phosphorus increases the strength, ductility and soundness of castings. The tensile strength of this alloy when cast varies from 215 MPa to 280 MPa but increases upto 2300 MPa when rolled or drawn. This alloy possesses good wearing qualities and high elasticity. The metal is resistant to salt water corrosion. The composition of the metal varies according to whether it is to be forged, wrought or made into castings. A common type of phosphor bronze has the following composition according to Indian standards :

Copper = 87–90%, Tin = 9–10%, and Phosphorus = 0.1–3%.

It is used for bearings, worm wheels, gears, nuts for machine lead screws, pump parts, linings and for many other purposes. It is also suitable for making springs.

(*b*) ***Silicon bronze.*** It contains 96% copper, 3% silicon and 1% manganese or zinc. It has good general corrosion resistance of copper combined with higher strength. It can be cast, rolled, stamped, forged and pressed either hot or cold and it can be welded by all the usual methods.

It is widely used for boilers, tanks, stoves or where high strength and good corrosion resistance is required.

(*c*) ***Beryllium bronze.*** It is a copper base alloy containing about 97.75% copper and 2.25% beryllium. It has high yield point, high fatigue limit and excellent cold and hot corrosion resistance. It is particularly suitable material for springs, heavy duty electrical switches, cams and bushings. Since the wear resistance of beryllium copper is five times that of phosphor bronze, therefore, it may be used as a bearing metal in place of phosphor bronze.

Top

It has a film forming and a soft lubricating property, which makes it more suitable as a bearing metal.

(*d*) ***Manganese bronze.*** It is an alloy of copper, zinc and little percentage of manganese. The usual composition of this bronze is as follows:

Copper = 60%, Zinc = 35%, and Manganese = 5%

This metal is highly resistant to corrosion. It is harder and stronger than phosphor bronze. It is generally used for bushes, plungers, feed pumps, rods etc. Worm gears are frequently made from this bronze.

(*e*) ***Aluminium bronze.*** It is an alloy of copper and aluminium. The aluminium bronze with 6–8% aluminium has valuable cold working properties. The maximum tensile strength of this alloy is 450 MPa with 11% of aluminium. They are most suitable for making components exposed to severe corrosion conditions. When iron is added to these bronzes, the mechanical properties are improved by refining the grain size and improving the ductility.

Aluminium bronzes are widely used for making gears, propellers, condenser bolts, pump components, tubes, air pumps, slide valves and bushings, etc. Cams and rollers are also made from this alloy. The 6% aluminium alloy has a fine gold colour which is used for imitation jewellery and decorative purposes.

2.31 Gun Metal

It is an alloy of copper, tin and zinc. It usually contains 88% copper, 10% tin and 2% zinc. This metal is also known as ***Admiralty gun metal.*** The zinc is added to clean the metal and to increase its fluidity.

It is not suitable for being worked in the cold state but may be forged when at about 600°C. The metal is very strong and resistant to corrosion by water and atmosphere. Originally, it was made for casting guns. It is extensively used for casting boiler fittings, bushes, bearings, glands, etc.

2.32 Lead

It is a bluish grey metal having specific gravity 11.36 and melting point 326°C. It is so soft that it can be cut with a knife. It has no tenacity. It is extensively used for making solders, as a lining for acid tanks, cisterns, water pipes, and as coating for electrical cables.

The lead base alloys are employed where a cheap and corrosion resistant material is required. An alloy containing 83% lead, 15% antimony, 1.5% tin and 0.5% copper is used for large bearings subjected to light service.

2.33 Tin

It is brightly shining white metal. It is soft, malleable and ductile. It can be rolled into very thin sheets. It is used for making important alloys, fine solder, as a protective coating for iron and steel sheets and for making tin foil used as moisture proof packing.

A tin base alloy containing 88% tin, 8% antimony and 4% copper is called ***babbit metal.*** It is a soft material with a low coefficient of friction and has little strength. It is the most common bearing metal used with cast iron boxes where the bearings are subjected to high pressure and load.

Note : Those alloys in which lead and tin are predominating are designated as ***white metal bearing alloys.*** This alloy is used for lining bearings subjected to high speeds like the bearings of aero-engines.

2.34 Bearing Metals

The following are the widely used bearing metals :

1. Copper-base alloys, **2.** Lead-base alloys, **3.** Tin-base alloys, and **4.** Cadmium-base alloys

The **copper base alloys** are the most important bearing alloys. These alloys are harder and stronger than the white metals (lead base and tin base alloys) and are used for bearings subjected to heavy pressures. These include brasses and bronzes which are discussed in Art 2.30. The **lead base** and **tin base alloys** are discussed in Art. 2.32 and 2.33. The **cadmium base alloys** contain 95% cadmium and 5% silver. It is used for medium loaded bearings subjected to high temperature.

The selection of a particular type of bearing metal depends upon the conditions under which it is to be used. It involves factors relating to bearing pressures, rubbing speeds, temperatures, lubrication, etc. A bearing material should have the following properties:

1. It should have low coefficient of friction.
2. It should have good wearing qualities.
3. It should have ability to withstand bearing pressures.
4. It should have ability to operate satisfactorily with suitable lubrication means at the maximum rubbing speeds.
5. It should have a sufficient melting point.
6. It should have high thermal conductivity.
7. It should have good casting qualities.
8. It should have minimum shrinkage after casting.
9. It should have non-corrosive properties.
10. It should be economical in cost.

2.35 Zinc Base Alloys

The most of the die castings are produced from zinc base alloys. These alloys can be casted easily with a good finish at fairly low temperatures. They have also considerable strength and are low in cost. The usual alloying elements for zinc are aluminium, copper and magnesium and they are all held in close limits.

The composition of two standard die casting zinc alloys are as follows :

1. Aluminium 4.1%, copper 0.1%, magnesium 0.04% and the remainder is zinc.
2. Aluminium 4.1%, copper 1%, magnesium 0.04% and the remainder is zinc.

Aluminium improves the mechanical properties and also reduces the tendency of zinc to dissolve iron. Copper increases the tensile strength, hardness and ductility. Magnesium has the beneficial effect of making the castings permanently stable. These alloys are widely used in the automotive industry and for other high production markets such as washing machines, oil burners, refrigerators, radios, photographs, television, business machines, etc.

This copper statue, believed to be the world's oldest metal sculpture, is an image of Egyptian pharaoh Pepi I. This old kingdom pharaoh reigned from 2289 to 2244 BC.

2.36 Nickel Base Alloys

The nickel base alloys are widely used in engineering industry on account of their high mechanical strength properties, corrosion resistance, etc. The most important nickel base alloys are discussed below:

1. ***Monel metal.*** It is an important alloy of nickel and copper. It contains 68% nickel, 29% copper and 3% other constituents like iron, manganese, silicon and carbon. Its specific gravity is 8.87 and melting point 1360°C. It has a tensile strength from 390 MPa to 460 MPa. It resembles nickel in appearance and is strong, ductile and tough. It is superior to brass and bronze in corrosion resisting properties. It is used for making propellers, pump fittings, condenser tubes, steam turbine blades, sea water exposed parts, tanks and chemical and food handling plants.

Contents

2. ***Inconel.*** It consists of 80% nickel, 14% chromium, and 6% iron. Its specific gravity is 8.55 and melting point 1395°C. This alloy has excellent mechanical properties at ordinary and elevated temperatures. It can be cast, rolled and cold drawn. It is used for making springs which have to withstand high temperatures and are exposed to corrosive action. It is also used for exhaust manifolds of aircraft engines.

3. ***Nichrome.*** It consists of 65% nickel, 15% chromium and 20% iron. It has high heat and oxidation resistance. It is used in making electrical resistance wire for electric furnaces and heating elements.

4. ***Nimonic.*** It consists of 80% nickel and 20% chromium. It has high strength and ability to operate under intermittent heating and cooling conditions. It is widely used in gas turbine engines.

2.37 Non-metallic Materials

The non-metallic materials are used in engineering practice due to their low density, low cost, flexibility, resistant to heat and electricity. Though there are many non-metallic materials, yet the following are important from the subject point of view.

1. ***Plastics.*** The plastics are synthetic materials which are moulded into shape under pressure with or without the application of heat. These can also be cast, rolled, extruded, laminated and machined. Following are the two types of plastics :

(*a*) Thermosetting plastics, and

(*b*) Thermoplastic.

The ***thermosetting plastics*** are those which are formed into shape under heat and pressure and results in a permanently hard product. The heat first softens the material, but as additional heat and pressure is applied, it becomes hard by a chemical change known as phenol-formaldehyde (Bakelite), phenol-furfural (Durite), urea-formaldehyde (Plaskon), etc.

Reinforced plastic with fibreglass makes the material to withstand high compressive as well as tensile stresses.

The ***thermoplastic*** materials do not become hard with the application of heat and pressure and no chemical change occurs. They remain soft at elevated temperatures until they are hardened by cooling. These can be remelted repeatedly by successive application of heat. Some of the common thermoplastics are cellulose nitrate (Celluloid), polythene, polyvinyl acetate, polyvinyl chloride (P.V.C.), etc.

The plastics are extremely resistant to corrosion and have a high dimensional stability. They are mostly used in the manufacture of aeroplane and automobile parts. They are also used for making safety glasses, laminated gears, pulleys, self-lubricating bearing, etc. due to their resilience and strength.

2. ***Rubber.*** It is one of the most important natural plastics. It resists abrasion, heat, strong alkalis and fairly strong acids. Soft rubber is used for electrical insulations. It is also used for power transmission belting, being applied to woven cotton or cotton cords as a base. The hard rubber is used for piping and as lining for pickling tanks.

3. ***Leather.*** It is very flexible and can withstand considerable wear under suitable conditions. It is extensively used for power transmission belting and as a packing or as washers.

4. ***Ferrodo.*** It is a trade name given to asbestos lined with lead oxide. It is generally used as a friction lining for clutches and brakes.

Top

QUESTIONS

1. How do you classify materials for engineering use?
2. What are the factors to be considered for the selection of materials for the design of machine elements? Discuss.
3. Enumerate the most commonly used engineering materials and state at least one important property and one application of each.
4. Why are metals in their pure form unsuitable for industrial use?
5. Define 'mechanical property' of an engineering material. State any six mechanical properties, give their definitions and one example of the material possessing the properties.
6. Define the following properties of a material :
   (*i*) Ductility, (*ii*) Toughness, (*iii*) Hardness, and (*iv*) Creep.
7. Distinguish clearly amongst cast iron, wrought iron and steel regarding their constituents and properties.
8. How cast iron is obtained? Classify and explain different types of cast irons.
9. How is grey cast iron designated in Indian standards?
10. Discuss the effect of silicon, manganese, sulphur and phosphorus on cast iron.
11. Define plain carbon steel. How it is designated according to Indian standards?
12. Define alloy steel. Discuss the effects of nickel, chromium and manganese on steel.
13. What are the common materials used in Mechanical Engineering Design? How can the properties of steel be improved?
14. State the alloying elements added to steel to get alloy steels and the effect they produce. Give at least one example of each.
15. Give the composition of 35 Mn 2 Mo 45 steel. List its main uses.
16. Write short notes on free cutting steel, and stainless steel.
17. Select suitable material for the following cases, indicating the reason;
    1. A shaft subjected to variable torsional and bending load ; 2. Spring used in a spring loaded safety valve; 3. Nut of a heavy duty screw jack; and 4. Low speed line-shaft coupling.
18. Select suitable materials for the following parts stating the special property which makes it most suitable for use in manufacturing:
    1. Turbine blade, 2. Bush bearing, 3. Dies, 4. Carburetor body, 5. Keys (used for fastening), 6. Cams, 7. Heavy duty machine tool beds, 8. Ball bearing, 9. Automobile cylinder block, 10. Helical springs.
19. Suggest suitable materials for the following parts stating the special property which makes it more suitable for use in manufacturing:
    1. Diesel engine crankshaft ; 2. Automobile tyres ; 3. Roller bearings ; 4. High pressure steam pipes ; 5. Stay bar of boilers ; 6. Worm and worm gear ; 7. Dies; 8. Tramway axle ; 9. Cam follower ; 10. Hydraulic brake piston.
20. Write short notes on high speed tool steel and spring steel.
21. Explain the following heat treatment processes:
    1. Normalising; 2. Hardening; and 3. Tempering.
22. Write short note on the type of bearing metals.
23. Discuss the important non-metallic materials of construction used in engineering practice.

OBJECTIVE TYPE QUESTIONS

1. Which of the following material has the maximum ductility?
   (*a*) Mild steel (*b*) Copper
   (*c*) Zinc (*d*) Aluminium
2. According to Indian standard specifications, a grey cast iron designated by 'FG 200' means that the
   (*a*) carbon content is 2%
   (*b*) maximum compressive strength is 200 N/mm^2
   (*c*) minimum tensile strength is 200 N/mm^2
   (*d*) maximum shear strength is 200 N/mm^2

3. Steel containing upto 0.15% carbon is known as
(*a*) mild steel (*b*) dead mild steel
(*c*) medium carbon steel (*d*) high carbon steel

4. According to Indian standard specifications, a plain carbon steel designated by 40C8 means that
(*a*) carbon content is 0.04 per cent and manganese is 0.08 per cent
(*b*) carbon content is 0.4 per cent and manganese is 0.8 per cent
(*c*) carbon content is 0.35 to 0.45 per cent and manganese is 0.60 to 0.90 per cent
(*d*) carbon content is 0.60 to 0.80 per cent and manganese is 0.8 to 1.2 per cent

5. The material commonly used for machine tool bodies is
(*a*) mild steel (*b*) aluminium
(*c*) brass (*d*) cast iron

6. The material commonly used for crane hooks is
(*a*) cast iron (*b*) wrought iron
(*c*) mild steel (*d*) aluminium

7. Shock resistance of steel is increased by adding
(*a*) nickel (*b*) chromium
(*c*) nickel and chromium (*d*) sulphur, lead and phosphorus

8. The steel widely used for motor car crankshafts is
(*a*) nickel steel (*b*) chrome steel
(*c*) nickel-chrome steel (*d*) silicon steel

9. A steel with 0.8 per cent carbon is known as
(*a*) eutectoid steel (*b*) hypereutectoid steel
(*c*) hypoeutectoid steel (*d*) none of these

10. 18/8 steel contains
(*a*) 18 per cent nickel and 8 per cent chromium
(*b*) 18 per cent chromium and 8 per cent nickel
(*c*) 18 per cent nickel and 8 per cent vanadium
(*d*) 18 per cent vanadium and 8 per cent nickel

11. Ball bearing are usually made from
(*a*) low carbon steel (*b*) high carbon steel
(*c*) medium carbon steel (*d*) high speed steel

12. The process which improves the machinability of steels, but lower the hardness and tensile strength is
(*a*) normalising (*b*) full annealing
(*c*) process annealing (*d*) spheroidising

13. The metal suitable for bearings subjected to heavy loads is
(*a*) silicon bronze (*b*) white metal
(*c*) monel metal (*d*) phosphor bronze

14. The metal suitable for bearings subjected to light loads is
(*a*) silicon bronze (*b*) white metal
(*c*) monel metal (*d*) phosphor bronze

15. Thermoplastic materials are those materials which
(*a*) are formed into shape under heat and pressure and results in a permanently hard product
(*b*) do not become hard with the application of heat and pressure and no chemical change occurs
(*c*) are flexible and can withstand considerable wear under suitable conditions
(*d*) are used as a friction lining for clutches and brakes

ANSWERS

1. (*a*)	**2.** (*c*)	**3.** (*b*)	**4.** (*c*)	**5.** (*d*)
6. (*b*)	**7.** (*c*)	**8.** (*b*)	**9.** (*a*)	**10.** (*b*)
11. (*c*)	**12.** (*d*)	**13.** (*b*)	**14.** (*d*)	**15.** (*b*)

CHAPTER 3

Manufacturing Considerations in Machine Design

3.1 Introduction

In the previous chapter, we have only discussed about the composition, properties and uses of various materials used in Mechanical Engineering. We shall now discuss in this chapter a few of the manufacturing processes, limits and fits, etc.

3.2 Manufacturing Processes

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

1. ***Primary shaping processes.*** The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

2. ***Machining processes.*** The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbing, etc.

3. ***Surface finishing processes.*** The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, superfinishing, sheradizing, etc.

4. ***Joining processes.*** The processes used for joining machine components are known as joining processes. The common operations used for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering, etc.

5. ***Processes effecting change in properties.*** These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

To discuss in detail all these processes is beyond the scope of this book, but a few of them which are important from the subject point of view will be discussed in the following pages.

3.3 Casting

It is one of the most important manufacturing process used in Mechanical Engineering. The castings are obtained by remelting of ingots* in a cupola or some other foundry furnace and then pouring this molten metal into metal or sand moulds. The various important casting processes are as follows:

1. ***Sand mould casting.*** The casting produced by pouring molten metal in sand mould is called sand mould casting. It is particularly used for parts of larger sizes.

2. ***Permanent mould casting.*** The casting produced by pouring molten metal in a metallic mould is called permanent mould casting. It is used for casting aluminium pistons, electric iron parts, cooking utensils, gears, etc. The permanent mould castings have the following advantages:

1. Shaping the Sand : A wooden pattern cut to the shape of one half of the casting is positioned in an iron box and surrounded by tightly packed moist sand.

2. Ready for the Metal : After the wooden patterns have been removed, the two halves of the mould are clamped together. Molten iron is poured into opening called the runner.

* Most of the metals used in industry are obtained from ores. These ores are subjected to suitable reducing or refining process which gives the metal in a molten form. This molten metal is poured into moulds to give commercial castings, called **ingots.**

(a) It has more favourable fine grained structure.

(b) The dimensions may be obtained with close tolerances.

(c) The holes up to 6.35 mm diameter may be easily cast with metal cores.

3. ***Slush casting.*** It is a special application of permanent metal mould casting. This method is used for production of hollow castings without the use of cores.

4. ***Die casting.*** The casting produced by forcing molten metal under pressure into a permanent metal mould (known as die) is called die casting. A die is usually made in two halves and when closed it forms a cavity similar to the casting desired. One half of the die that remains stationary is known as ***cover die*** and the other movable half is called ***ejector die.*** The die casting method is mostly used for castings of non-ferrous metals of comparatively low fusion temperature. This process is cheaper and quicker than permanent or sand mould casting. Most of the automobile parts like fuel pump, carburettor bodies, horn, heaters, wipers, brackets, steering wheels, hubs and crank cases are made with this process. Following are the advantages and disadvantages of die casting :

Aluminium die casting component

Advantages

(a) The production rate is high, ranging up to 700 castings per hour.

(b) It gives better surface smoothness.

(c) The dimensions may be obtained within tolerances.

(d) The die retains its trueness and life for longer periods. For example, the life of a die for zinc base castings is upto one million castings, for copper base alloys upto 75 000 castings and for aluminium base alloys upto 500 000 castings.

Sand Casting

Investment Casting

(*e*) It requires less floor area for equivalent production by other casting methods.

(*f*) By die casting, thin and complex shapes can be easily produced.

(*g*) The holes up to 0.8 mm can be cast.

Disadvantages

(*a*) The die casting units are costly.

(*b*) Only non-ferrous alloys are casted more economically.

(*c*) It requires special skill for maintenance and operation of a die casting machine.

5. ***Centrifugal casting.*** The casting produced by a process in which molten metal is poured and allowed to solidify while the mould is kept revolving, is known as centrifugal casting. The metal thus poured is subjected to centrifugal force due to which it flows in the mould cavities. This results in the production of high density castings with promoted directional solidification. The examples of centrifugal castings are pipes, cylinder liners and sleeves, rolls, bushes, bearings, gears, flywheels, gun barrels, piston rings, brake drums, etc.

3.4 Casting Design

An engineer must know how to design the castings so that they can effectively and efficiently render the desired service and can be produced easily and economically. In order to design a casting, the following factors must be taken into consideration :

1. The function to be performed by the casting,
2. Soundness of the casting,
3. Strength of the casting,
4. Ease in its production,
5. Consideration for safety, and
6. Economy in production.

In order to meet these requirements, a design engineer should have a thorough knowledge of production methods including pattern making, moulding, core making, melting and pouring, etc. The best designs will be achieved only when one is able to make a proper selection out of the various available methods. However, a few rules for designing castings are given below to serve as a guide:

1. The sharp corners and frequent use of fillets should be avoided in order to avoid concentration of stresses.
2. All sections in a casting should be designed of uniform thickness, as far as possible. If, however, variation is unavoidable, it should be done gradually.
3. An abrupt change of an extremely thick section into a very thin section should always be avoided.
4. The casting should be designed as simple as possible, but with a good appearance.
5. Large flat surfaces on the casting should be avoided because it is difficult to obtain true surfaces on large castings.
6. In designing a casting, the various allowances must be provided in making a pattern.
7. The ability to withstand contraction stresses of some members of the casting may be improved by providing the curved shapes *e.g.*, the arms of pulleys and wheels.
8. The stiffening members such as webs and ribs used on a casting should be minimum possible in number, as they may give rise to various defects like hot tears and shrinkage, etc.
9. The casting should be designed in such a way that it will require a simpler pattern and its moulding is easier.
10. In order to design cores for casting, due consideration should be given to provide them adequate support in the mould.

Contents

11. The deep and narrow pockets in the casting should invariably be avoided to reduce cleaning costs.
12. The use of metal inserts in the casting should be kept minimum.
13. The markings such as names or numbers, etc., should never be provided on vertical surfaces because they provide a hindrance in the withdrawl of pattern.
14. A tolerance of ± 1.6 mm on small castings (below 300 mm) should be provided. In case more dimensional accuracy is desired, a tolerance of ± 0.8 mm may be provided.

3.5 Forging

It is the process of heating a metal to a desired temperature in order to acquire sufficient plasticity, followed by operations like hammering, bending and pressing, etc. to give it a desired shape. The various forging processes are :

1. Smith forging or hand forging
2. Power forging,
3. Machine forging or upset forging, and
4. Drop forging or stamping

The **smith** or **hand forging** is done by means of hand tools and it is usually employed for small jobs. When the forging is done by means of power hammers, it is then known as ***power forging***. It is used for medium size and large articles requiring very heavy blows. The **machine forging** is done by means of forging machines. The **drop forging** is carried out with the help of drop hammers and is particularly suitable for mass production of identical parts. The forging process has the following advantages :

1. It refines the structure of the metal.
2. It renders the metal stronger by setting the direction of grains.
3. It effects considerable saving in time, labour and material as compared to the production of a similar item by cutting from a solid stock and then shaping it.
4. The reasonable degree of accuracy may be obtained by forging.
5. The forgings may be welded.

It may be noted that wrought iron and various types of steels and steel alloys are the common raw material for forging work. Low carbon steels respond better to forging work than the high carbon steels. The common non-ferrous metals and alloys used in forging work are brass, bronze, copper, aluminium and magnesium alloys. The following table shows the temperature ranges for forging some common metals.

Table 3.1. Temperature ranges for forging.

Material	*Forging temperature (°C)*	*Material*	*Forging temperature (°C)*
Wrought iron	900 – 1300	Stainless steel	940 – 1180
Mild steel	750 – 1300	Aluminium and magnesium alloys	350 – 500
Medium carbon steel	750 – 1250		
High carbon and alloy steel	800 – 1150	Copper, brass and bronze	600 – 950

Top

3.6 Forging Design

In designing a forging, the following points should always be considered.

1. The forged components should ultimately be able to achieve a radial flow of grains or fibres.
2. The forgings which are likely to carry flash, such as drop and press forgings, should preferably have the parting line in such a way that the same will divide them in two equal halves.
3. The parting line of a forging should lie, as far as possible, in one plane.
4. Sufficient draft on surfaces should be provided to facilitate easy removal of forgings from dies.
5. The sharp corners should always be avoided in order to prevent concentration of stress and to facilitate ease in forging.
6. The pockets and recesses in forgings should be minimum in order to avoid increased die wear.
7. The ribs should not be high and thin.
8. Too thin sections should be avoided to facilitate easy flow of metal.

3.7 Mechanical Working of Metals

The mechanical working of metals is defined as an intentional deformation of metals plastically under the action of externally applied forces.

The mechanical working of metal is described as hot working and cold working depending upon whether the metal is worked above or below the recrystallisation temperature. The metal is subjected to mechanical working for the following purposes :

1. To reduce the original block or ingot into desired shapes,
2. To refine grain size, and
3. To control the direction of flow lines.

3.8 Hot Working

The working of metals above the *recrystallisation temperature is called ***hot working***. This temperature should not be too high to reach the solidus temperature, otherwise the metal will burn and become unsuitable for use. The hot working of metals has the following advantages and disadvantages :

Advantages

1. The porosity of the metal is largely eliminated.
2. The grain structure of the metal is refined.
3. The impurities like slag are squeezed into fibres and distributed throughout the metal.
4. The mechanical properties such as toughness, ductility, percentage elongation, percentage reduction in area, and resistance to shock and vibration are improved due to the refinement of grains.

Disadvantages

1. It requires expensive tools.
2. It produces poor surface finish, due to the rapid oxidation and scale formation on the metal surface.
3. Due to the poor surface finish, close tolerance cannot be maintained.

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* The temperature at which the new grains are formed in the metal is known as **recrystallisation temperature.**

Contents

3.9 Hot Working Processes

The various *hot working processes are described as below :

1. *Hot rolling.* The hot rolling process is the most rapid method of converting large sections into desired shapes. It consists of passing the hot ingot through two rolls rotating in opposite directions at the same speed. The space between the rolls is adjusted to conform to the desired thickness of the rolled section. The rolls, thus, squeeze the passing ingot to reduce its cross-section and increase its length. The forming of bars, plates, sheets, rails, angles, I-beam and other structural sections are made by hot rolling.

Hot Rolling : When steel is heated until it glows bright red, it becomes soft enough to form into elabrate shapes.

2. *Hot forging.* It consists of heating the metal to plastic state and then the pressure is applied to form it into desired shapes and sizes. The pressure applied in this is not continuous as for hot rolling, but intermittent. The pressure may be applied by hand hammers, power hammers or by forging machines.

3. *Hot spinning.* It consists of heating the metal to forging temperature and then forming it into the desired shape on a spinning lathe. The parts of circular cross-section which are symmetrical about the axis of rotation, are made by this process.

4. *Hot extrusion.* It consists of pressing a metal inside a chamber to force it out by high pressure through an orifice which is shaped to provide the desired form of the finished part. Most commercial metals and their alloys such as steel, copper, aluminium and nickel are directly extruded at elevated temperatures. The rods, tubes, structural shapes, flooring strips and lead covered cables, etc., are the typical products of extrusion.

5. *Hot drawing or cupping.* It is mostly used for the production of thick walled seamless tubes and cylinders. It is usually performed in two stages. The first stage consists of drawing a cup out of a hot circular plate with the help of a die and punch. The second stage consists of reheating the drawn cup and drawing it further to the desired length having the required wall thickness. The second drawing operation is performed through a number of dies, which are arranged in a descending order of their diameters, so that the reduction of wall thickness is gradual in various stages.

Cold Rolled Steel : Many modern products are made from easily shaped sheet metal.

6. *Hot piercing.* This process is used for the manufacture of seamless tubes. In its operation, the heated cylindrical billets of steel are passed between two conical shaped rolls operating in the same direction. A mandrel is provided between these rolls which assist in piercing and controls the size of the hole, as the billet is forced over it.

* For complete details, please refer to Authors' popular book **'A Text Book of Workshop Technology'.**

Top

3.10 Cold Working

The working of metals below their recrystallisation temperature is known as ***cold working.*** Most of the cold working processes are performed at room temperature. The cold working distorts the grain structure and does not provide an appreciable reduction in size. It requires much higher pressures than hot working. The extent to which a metal can be cold worked depends upon its ductility. The higher the ductility of the metal, the more it can be cold worked. During cold working, severe stresses known as residual stresses are set up. Since the presence of these stresses is undesirable, therefore, a suitable heat treatment may be employed to neutralise the effect of these stresses. The cold working is usually used as finishing operation, following the shaping of the metal by hot working. It also increases tensile strength, yield strength and hardness of steel but lowers its ductility. The increase in hardness due to cold working is called ***work-hardening.***

In general, cold working produces the following effects :

1. The stresses are set up in the metal which remain in the metal, unless they are removed by subsequent heat treatment.
2. A distortion of the grain structure is created.
3. The strength and hardness of the metal are increased with a corresponding loss in ductility.
4. The recrystalline temperature for steel is increased.
5. The surface finish is improved.
6. The close dimensional tolerance can be maintained.

3.11 Cold Working Processes

The various cold working processes are discussed below:

1. ***Cold rolling.*** It is generally employed for bars of all shapes, rods, sheets and strips, in order to provide a smooth and bright surface finish. It is also used to finish the hot rolled components to close tolerances and improve their toughness and hardness. The hot rolled articles are first immersed in an acid to remove the scale and washed in water, and then dried. This process of cleaning the articles is known as ***pickling.*** These cleaned articles are then passed through rolling mills. The rolling mills are similar to that used in hot rolling.

Gallium arsenide (GaAs) *is now being manufactured as an alternative to silicon for microchips. This combination of elements is a semiconductor like silicon, but is electronically faster and therefore better for microprocessors.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

2. ***Cold forging.*** The cold forging is also called ***swaging.*** During this method of cold working, the metal is allowed to flow in some pre-determined shape according to the design of dies, by a compressive force or impact. It is widely used in forming ductile metals. Following are the three, commonly used cold forging processes :

(*a*) ***Sizing.*** It is the simplest form of cold forging. It is the operation of slightly compressing a forging, casting or steel assembly to obtain close tolerance and a flat surface. The metal is confined only in a vertical direction.

(*b*) ***Cold heading.*** This process is extensively used for making bolts, rivets and other similar headed parts. This is usually done on a cold header machine. Since the cold header is made from unheated material, therefore, the equipment must be able to withstand the high pressures that develop. The rod is fed to the machine where it is cut off and moved into the header die. The operation may be either single or double and upon completion, the part is ejected from the dies.

After making the bolt head, the threads are produced on a thread rolling machine. This is also a cold working process. The process consists of pressing the blank between two rotating rolls which have the thread form cut in their surface.

(*c*) ***Rotary swaging.*** This method is used for reducing the diameter of round bars and tubes by rotating dies which open and close rapidly on the work. The end of rod is tapered or reduced in size by a combination of pressure and impact.

3. ***Cold spinning.*** The process of cold spinning is similar to hot spinning except that the metal is worked at room temperature. The process of cold spinning is best suited for aluminium and other soft metals. The commonly used spun articles out of aluminum and its alloys are processing kettles, cooking utensils, liquid containers, and light reflectors, etc.

4. ***Cold extrusion.*** The principle of cold extrusion is exactly similar to hot extrusion. The most common cold extrusion process is ***impact extrusion.*** The operation of cold extrusion is performed with the help of a punch and die. The work material is placed in position into a die and struck from top

Making microchips demands extreme control over chemical components. The layers of conducting and insulating materials that are laid down on the surface of a silicon chip may be only a few atoms thick yet must perform to the highest specifications. Great care has to be taken in their manufacture (right), and each chip is checked by test probes to ensure it performs correctly.

Note : This picture is given as additional information and is not a direct example of the current chapter.

by a punch operating at high pressure and speed. The metal flows up along the surface of the punch forming a cup-shaped component. When the punch moves up, compressed air is used to separate the component from the punch. The thickness of the side wall is determined by the amount of clearance between the punch and die. The process of impact extrusion is limited to soft and ductile materials such as lead, tin, aluminium, zinc and some of their alloys. The various items of daily use such as tubes for shaving creams and tooth pastes and such other thin walled products are made by impact extrusion.

5. ***Cold drawing.*** It is generally employed for bars, rods, wires, etc. The important cold drawing processes are as follows:

(*a*) ***Bar or rod drawing.*** In bar drawing, the hot drawn bars or rods from the mills are first pickled, washed and coated to prevent oxidation. A draw bench, is employed for cold drawing. One end of the bar is reduced in diameter by the swaging operation to permit it to enter a drawing die. This end of bar is inserted through the die and gripped by the jaws of the carriage fastened to the chain of the draw bench. The length of bars which can be drawn is limited by the maximum travel of the carriage, which may be from 15 metres to 30 metres. A high surface finish and dimensional accuracy is obtained by cold drawing. The products may be used directly without requiring any machining.

(*b*) ***Wire drawing.*** In wire drawing, the rolled bars from the mills are first pickled, washed and coated to prevent oxidation. They are then passed through several dies of decreasing diameter to provide the desired reduction in size. The dies are usually made of carbide materials.

(*c*) ***Tube drawing.*** The tube drawing is similar to bar drawing and in most cases it is accomplished with the use of a draw bench.

6. ***Cold bending.*** The bars, wires, tubes, structural shapes and sheet metal may be bent to many shapes in cold condition through dies. A little consideration will show that when the metal is bend beyond the elastic limit, the inside of the bend will be under compression while the outside will be under tension. The stretching of the metal on the outside makes the stock thinner. Usually, a flat strip of metal is bend by ***roll forming***. The materials commonly used for roll forming are carbon steel, stainless steel, bronze, copper, brass, zinc and aluminium. Some of its products are metal windows, screen frame parts, bicycle wheel rims, trolley rails, etc. Most of the tubing is now-a-days are roll formed in cold conditions and then welded by resistance welding.

7. ***Cold peening.*** This process is used to improve the fatigue resistance of the metal by setting up compressive stresses in its surface. This is done by blasting or hurling a rain of small shot at high velocity against the surface to be peened. The shot peening is done by air blast or by some mechanical means. As the shot strikes, small indentations are produced, causing a slight plastic flow of the surface metal to a depth of a few hundreds of a centimetre. This stretching of the outer fibres is resisted by those underneath, which tend to return them to their original length, thus producing an outer layer having a compressive stress while those below are in tension. In addition, the surface is slightly hardened and strengthened by the cold working operation.

3.12 Interchangeability

The term interchangeability is normally employed for the mass production of indentical items within the prescribed limits of sizes. A little consideration will show that in order to maintain the sizes of the part within a close degree of accuracy, a lot of time is required. But even then there will be small variations. If the variations are within certain limits, all parts of equivalent size will be equally fit for operating in machines and mechanisms. Therefore, certain variations are recognised and allowed in the sizes of the mating parts to give the required fitting. This facilitates to select at random from a

Contents

large number of parts for an assembly and results in a considerable saving in the cost of production. In order to control the size of finished part, with due allowance for error, for interchangeable parts is called ***limit system.***

It may be noted that when an assembly is made of two parts, the part which enters into the other, is known as ***enveloped surface*** (or **shaft** for cylindrical part) and the other in which one enters is called ***enveloping surface*** (or **hole** for cylindrical part).

Notes: 1. The term ***shaft*** refers not only to the diameter of a circular shaft, but it is also used to designate any external dimension of a part.

2. The term ***hole*** refers not only to the diameter of a circular hole, but it is also used to designate any internal dimension of a part.

3.13 Important Terms used in Limit System

The following terms used in limit system (or interchangeable system) are important from the subject point of view:

1. ***Nominal size.*** It is the size of a part specified in the drawing as a matter of convenience.

2. ***Basic size.*** It is the size of a part to which all limits of variation (*i.e.* tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often the same.

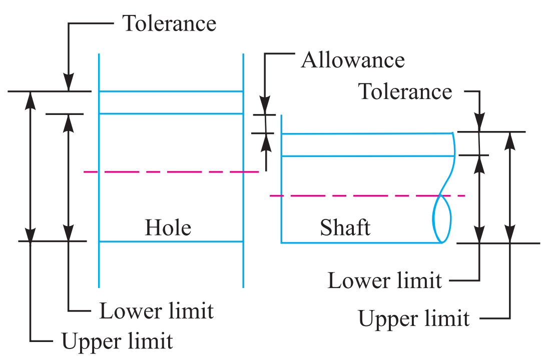


Fig. 3.1. Limits of sizes.

3. ***Actual size.*** It is the actual measured dimension of the part. The difference between the basic size and the actual size should not exceed a certain limit, otherwise it will interfere with the interchangeability of the mating parts.

4. ***Limits of sizes.*** There are two extreme permissible sizes for a dimension of the part as shown in Fig. 3.1. The largest permissible size for a dimension of the part is called ***upper*** or ***high*** or ***maximum limit,*** whereas the smallest size of the part is known as ***lower*** or ***minimum limit.***

5. ***Allowance.*** It is the difference between the basic dimensions of the mating parts. The allowance may be ***positive*** or ***negative***. When the shaft size is less than the hole size, then the allowance is ***positive*** and when the shaft size is greater than the hole size, then the allowance is ***negative***.

6. ***Tolerance.*** It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be ***unilateral*** or ***bilateral***. When all the tolerance is allowed on one side of the nominal size, *e.g.* $20^{+0.000}_{-0.004}$, then it is said to be ***unilateral system of tolerance***. The unilateral system is mostly used in industries as it permits changing the tolerance value while still retaining the same allowance or type of fit.

(*a*) Unilateral tolerance. (*b*) Bilateral tolerance.

Fig. 3.2. Method of assigning tolerances.

Top

Contents

When the tolerance is allowed on both sides of the nominal size, *e.g.* $20^{+0.002}_{-0.002}$, then it is said to be ***bilateral system of tolerance.*** In this case + 0.002 is the upper limit and – 0.002 is the lower limit.

The method of assigning unilateral and bilateral tolerance is shown in Fig. 3.2 (*a*) and (*b*) respectively.

7. ***Tolerance zone.*** It is the zone between the maximum and minimum limit size, as shown in Fig. 3.3.

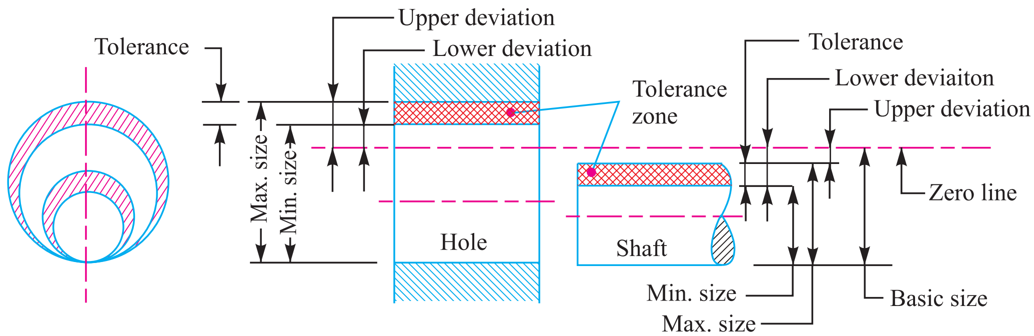


Fig. 3.3. Tolerance zone.

8. ***Zero line.*** It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero line respectively.

9. ***Upper deviation.*** It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol *ES* (Ecart Superior) and of a shaft, it is represented by *es*.

10. ***Lower deviation.*** It is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol *EI* (Ecart Inferior) and of a shaft, it is represented by *ei*.

11. ***Actual deviation.*** It is the algebraic difference between an actual size and the corresponding basic size.

12. ***Mean deviation.*** It is the arithmetical mean between the upper and lower deviations.

13. ***Fundamental deviation.*** It is one of the two deviations which is conventionally chosen to define the position of the tolerance zone in relation to zero line, as shown in Fig. 3.4.

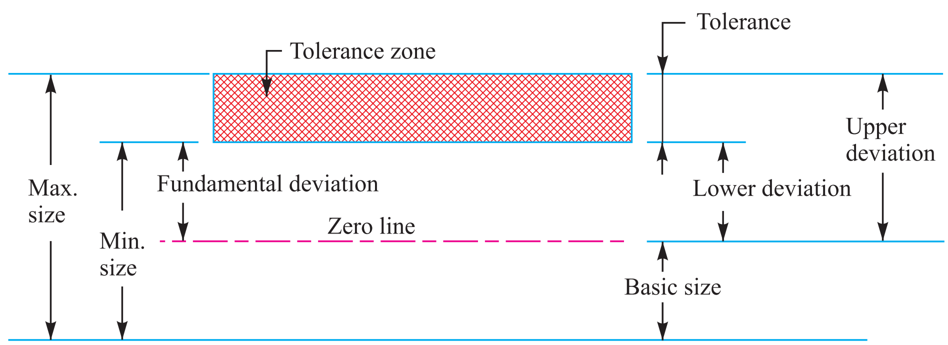


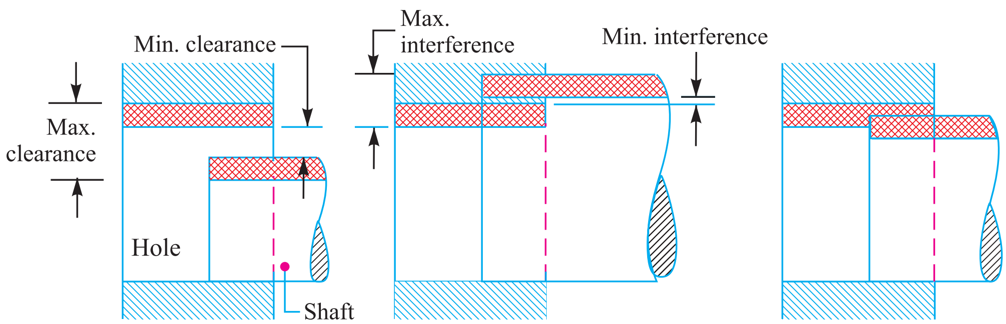
Fig. 3.4. Fundamental deviation.

Top

3.14 Fits

The degree of tightness or looseness between the two mating parts is known as a *fit* of the parts. The nature of fit is characterised by the presence and size of clearance and interference.

The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly as shown in Fig. 3.5 (*a*). In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be ***positive.***



(*a*) Clearance fit. (*b*) Interference fit. (*c*) Transition fit.

Fig. 3.5. Types of fits.

The ***interference*** is the amount by which the actual size of a shaft is larger than the actual finished size of the mating hole in an assembly as shown in Fig. 3.5 (*b*). In other words, the interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must be ***negative.***

3.15 Types of Fits

According to Indian standards, the fits are classified into the following three groups :

1. *Clearance fit.* In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, as shown in Fig. 3.5 (*a*). It may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft.

In a clearance fit, the difference between the minimum size of the hole and the maximum size of the shaft is known as ***minimum clearance*** whereas the difference between the maximum size of the hole and minimum size of the shaft is called ***maximum clearance*** as shown in Fig. 3.5 (*a*).

A Jet Engine : *In a jet engine, fuel is mixed with air, compressed, burnt, and exhausted in one smooth, continuous process. There are no pistons shuttling back and forth to slow it down.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

Contents

The clearance fits may be slide fit, easy sliding fit, running fit, slack running fit and loose running fit.

2. ***Interference fit.*** In this type of fit, the size limits for the mating parts are so selected that interference between them always occur, as shown in Fig. 3.5 (*b*). It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft.

In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as ***minimum interference,*** whereas the difference between the minimum size of the hole and the maximum size of the shaft is called ***maximum interference,*** as shown in Fig. 3.5 (*b*).

The interference fits may be shrink fit, heavy drive fit and light drive fit.

3. ***Transition fit.*** In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts, as shown in Fig. 3.5 (*c*). It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap.

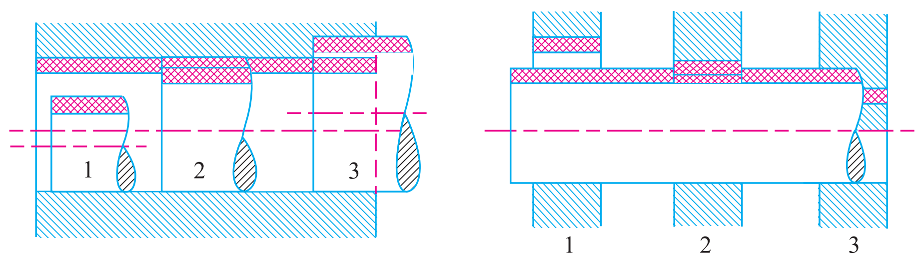
The transition fits may be force fit, tight fit and push fit.

3.16 Basis of Limit System

The following are two bases of limit system:

1. ***Hole basis system.*** When the hole is kept as a constant member (*i.e.* when the lower deviation of the hole is zero) and different fits are obtained by varying the shaft size, as shown in Fig. 3.6 (*a*), then the limit system is said to be on a hole basis.

2. ***Shaft basis system.*** When the shaft is kept as a constant member (*i.e.* when the upper deviation of the shaft is zero) and different fits are obtained by varying the hole size, as shown in Fig. 3.6 (*b*), then the limit system is said to be on a shaft basis.

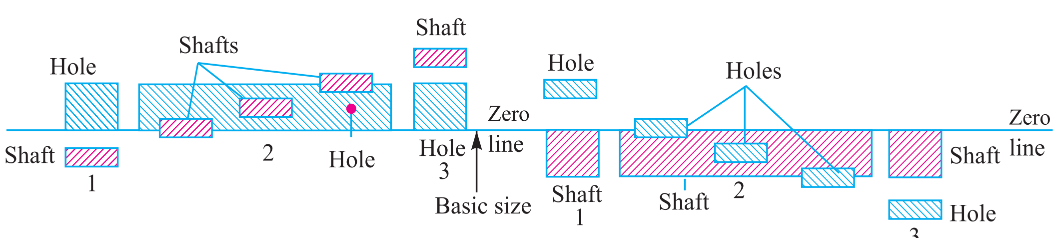


1. Clearance fit. 2. Transition fit. 3. Interference fit.

(*a*) Hole basis system. (*b*) Shaft basis system.

Fig. 3.6. Bases of limit system.

The hole basis and shaft basis system may also be shown as in Fig. 3.7, with respect to the zero line.



1. Clearance fit. 2. Transition fit. 3. Interference fit.

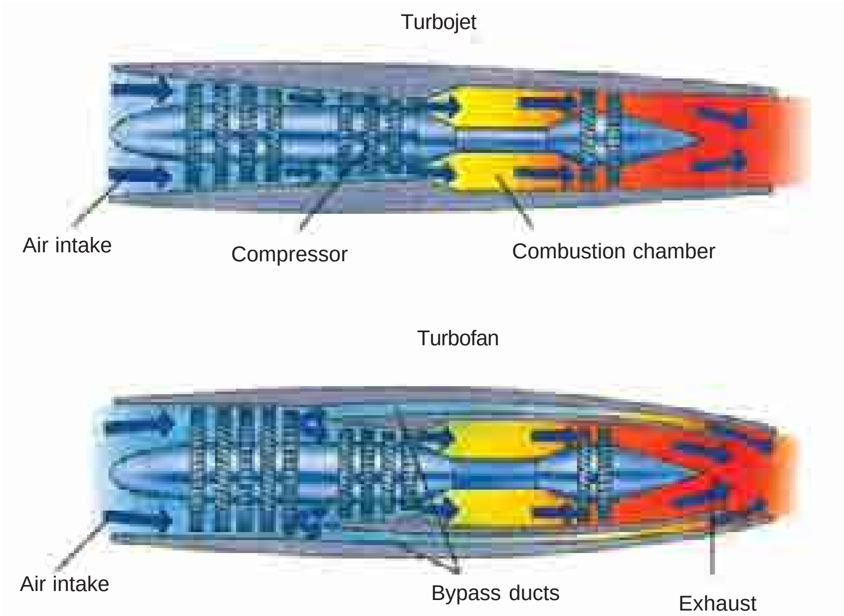
(*a*) Hole basis system. (*b*) Shaft basis system.

Fig. 3.7. Bases of limit system.

Top

Contents

It may be noted that from the manufacturing point of view, a hole basis system is always preferred. This is because the holes are usually produced and finished by standard tooling like drill, reamers, etc., whose size is not adjustable easily. On the other hand, the size of the shaft (which is to go into the hole) can be easily adjusted and is obtained by turning or grinding operations.



Turbofan engines are quieter and more efficient than simple turbojet engines. Turbofans drive air around the combustion engine as well as through it.

Note : This picture is given as additional information and is not a direct example of the current chapter.

3.17 Indian Standard System of Limits and Fits

According to Indian standard [IS : 919 (Part I)-1993], the system of limits and fits comprises 18 grades of fundamental tolerances *i.e.* grades of accuracy of manufacture and 25 types of fundamental deviations indicated by letter symbols for both holes and shafts (capital letter *A* to *ZC* for holes and small letters *a* to *zc* for shafts) in diameter steps ranging from 1 to 500 mm. A unilateral hole basis system is recommended but if necessary a unilateral or bilateral shaft basis system may also be used. The 18 tolerance grades are designated as IT 01, IT 0 and IT 1 to IT 16. These are called ***standard tolerances***. The standard tolerances for grades IT 5 to IT 7 are determined in terms of standard tolerance unit (i) in microns, where

i (microns) = $0.45 \sqrt[3]{D} + 0.001 D$, where D is the size or geometric mean diameter in mm.

The following table shows the relative magnitude for grades between IT 5 and IT 16.

Table 3.2. Relative magnitude of tolerance grades.

Tolerance grade	IT 5	IT 6	IT 7	IT 8	IT 9	IT 10	IT 11	IT 12	IT 13	IT 14	IT 15	IT 16
Magnitude	7 i	10 i	16 i	25 i	40 i	64 i	100 i	160 i	250 i	400 i	640 i	1000 i

Top

The values of standard tolerances corresponding to grades IT 01, IT 0 and IT 1 are as given below:

For IT 01, i (microns) $= 0.3 + 0.008\,D$,

For IT 0, i (microns) $= 0.5 + 0.012\,D$, and

For IT 1, i (microns) $= 0.8 + 0.020\,D$,

where D is the size or geometric mean diameter in mm.

The tolerance values of grades IT 2 to IT 4 are scaled approximately geometrically between IT 1 and IT 5. The fundamental tolerances of grades IT 01, IT 0 and IT 1 to IT 16 for diameter steps ranging from 1 to 500 mm are given in Table 3.3. The manufacturing processes capable of producing the particular IT grades of work are shown in Table 3.4.

The alphabetical representation of fundamental deviations for basic shaft and basic hole system is shown in Fig. 3.8.

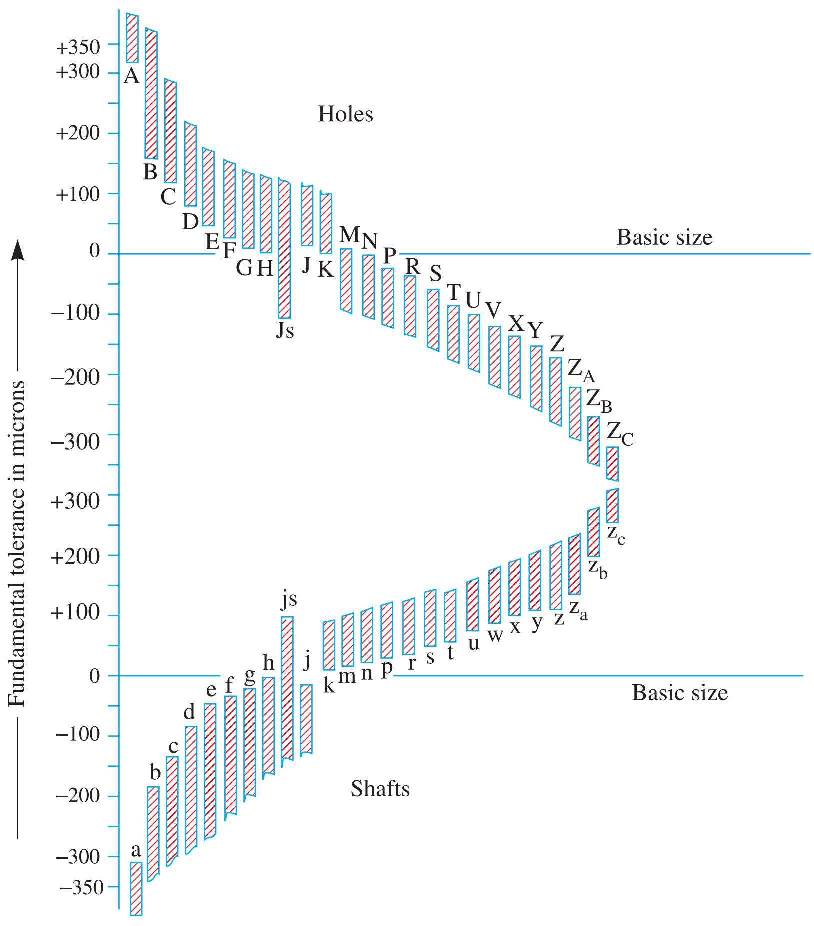


Fig. 3.8. Fundamental deviations for shafts and holes.

Contents

Table 3.3. Fundamental tolerances of grades IT01, IT0 and IT1 to IT16, according to IS : 919 (Part I) – 1993.

Basic size (Diameter steps) in mm		*Standard tolerance grades, in micron (1 micron = 0.001 mm)*																	
		IT01	IT0	IT1	IT2	IT3	IT4	IT5	IT6	IT7	IT8	IT9	IT10	IT11	IT12	IT13	IT14	IT15	IT16
Over To and inc.	1 3	0.3	0.5	0.8	1.2	2	3	4	6	10	14	25	40	60	100	140	250	400	600
Over To and inc.	3 6	0.4	0.6	1	1.5	2.5	4	5	8	12	18	30	48	75	120	180	300	480	750
Over To and inc.	6 10	0.4	0.6	1	1.5	2.5	4	6	9	15	22	36	58	90	150	220	360	580	900
Over To and inc.	10 18	0.5	0.8	1.2	2	3	5	8	11	18	27	43	70	110	180	270	430	700	1100
Over To and inc.	18 30	0.6	1	1.5	2.5	4	6	9	13	21	33	52	84	130	210	330	520	840	1300
Over To and inc.	30 50	0.6	1	1.5	2.5	4	7	11	16	25	39	62	100	160	250	390	620	1000	1600
Over To and inc.	50 80	0.8	1.2	2	3	5	8	13	19	30	46	74	120	190	300	460	740	1200	1900
Over To and inc.	80 120	1	1.5	2.5	4	6	10	15	22	35	54	87	140	220	350	540	870	1400	2200
Over To and inc.	120 180	1.2	2	3.5	5	8	12	18	25	40	63	100	160	250	400	630	1000	1600	2500
Over To and inc.	180 250	2	3	4.5	7	10	14	20	29	46	72	115	185	290	460	720	1150	1850	2900
Over To and inc.	250 315	2.5	4	6	8	12	16	23	32	52	81	130	210	320	520	810	1300	2100	3200
Over To and inc.	315 400	3	5	7	9	13	18	25	36	57	89	140	230	360	570	890	1400	2300	3800
Over To and inc.	400 500	4	6	8	10	15	20	27	40	63	97	155	250	400	630	970	1550	2500	4000

Top

Table 3.4. Manufacturing processes and IT grades produced.

S.No.	Manufacturing process	IT grade produced	S.No.	Manufacturing process	IT grade produced
1.	Lapping	4 and 5	9.	Extrusion	8 to 10
2.	Honing	4 and 5	10	Boring	8 to 13
3.	Cylindrical grinding	5 to 7	11.	Milling	10 to 13
4.	Surface grinding	5 to 8	12.	Planing and shaping	10 to 13
5.	Broaching	5 to 8	13.	Drilling	10 to 13
6.	Reaming	6 to 10	14.	Die casting	12 to 14
7.	Turning	7 to 13	15.	Sand casting	14 to 16
8.	Hot rolling	8 to 10	16.	Forging	14 to 16

For hole, *H* stands for a dimension whose lower deviation refers to the basic size. The hole *H* for which the lower deviation is zero is called a ***basic hole.*** Similarly, for shafts, *h* stands for a dimension whose upper deviation refers to the basic size. The shaft *h* for which the upper deviation is zero is called a ***basic shaft.***

This view along the deck of a liquefied natural gas (LNG) carrier shows the tops of its large, insulated steel tanks. The tanks contain liquefied gas at-162°C.

A fit is designated by its basic size followed by symbols representing the limits of each of its two components, the hole being quoted first. For example, 100 *H*6/*g*5 means basic size is 100 mm and the tolerance grade for the hole is 6 and for the shaft is 5. Some of the fits commonly used in engineering practice, for holes and shafts are shown in Tables 3.5 and 3.6 respectively according to IS : 2709 – 1982 (Reaffirmed 1993).

Contents

Table 3.5. Commonly used fits for holes according to IS : 2709 – 1982 (Reaffirmed 1993).

Type of fit	*Class of shaft*	*With holes* *H6*	*H7*	*H8*	*H11*	*Remarks and uses*
Clearance fit	*a*	—	—	—	*a*11	Large clearance fit and widely used.
	b	—	—	—	*b*11	
	c	—	*c*8	**c* 9	*c* 11	Slack running fit.
	d	—	*d*8	**d* 8 *d* 9, *d*10	*d* 11	Loose running fit—used for plummer block bearings and loose pulleys.
	e	*e*7	*e*8	**e* 8-*e* 9	—	Easy running fit—used for properly lubricated bearings requiring appreciable clearance. In the finer grades, it may be used on large electric motor and turbogenerator bearings according to the working condition.
	f	**f* 6	*f* 7	**f* 8	—	Normal running fit—widely used for grease lubricated or oil lubricated bearings where no substantial temperature differences are encountered—Typical applications are gear box shaft bearings and the bearings of small electric motors, pumps, etc.
	g	**g* 5	**g* 6	*g* 7	—	Close running fit or sliding fit—Also fine spigot and location fit—used for bearings for accurate link work and for piston and slide valves.
	h	**h* 5	**h* 6	**h* 7–*h* 8	**h*11	Precision sliding fit. Also fine spigot and location fit—widely used for non-running parts.
Transition fit	*j*	**j*5	**j*6	**j*7	—	Push fit for very accurate location with easy assembly and dismantling—Typical applications are coupling, spigots and recesses, gear rings clamped to steel hubs, etc.
	k	**k* 5	**k* 6	*k* 7	—	True transition fit (light keying fit)—used for keyed shaft, non-running locked pins, etc.
	m	**m* 5	**m* 6	*m* 7	—	Medium keying fit.
	n	*n* 5	**n* 6	*n*7	—	Heavy keying fit—used for tight assembly of mating parts.

* Second preference fits.

Top

Type of fit	*Class of shaft*	*With holes* H6	H7	H8	H11	*Remarks and uses*
Interference fit	*p*	*p*5	**p*6	—	—	Light press fit with easy dismantling for non-ferrous parts. Standard press fit with easy dismantling for ferrous and non-ferrous parts assembly.
	r	*r*5	**r*6	—	—	Medium drive fit with easy dismantling for ferrous parts assembly. Light drive fit with easy dismantling for non-ferrous parts assembly.
	s	*s*5	**s*6	*s*7	—	Heavy drive fit on ferrous parts for permanent or semi-permanent assembly. Standard press fit for non-ferrous parts.
	t	*t*5	*t*6	**t*7	—	Force fit on ferrous parts for permanent assembly.
	u	*u*5	*u*6	**u*7	—	Heavy force fit or shrink fit.
	v, *x*	—	—	—	—	Very large interference fits — not recommended for use
	y, *z*	—	—	—	—	

Table 3.6. Commonly used fits for shafts according to IS : 2709 – 1982 (Reaffirmed 1993).

Type of fit	*Class of hole*	*With shafts* **h*5	*h*6	*h*7	**h*8	*h*9	*h*11	*Remarks and uses*
Clearance fit	*A*	—	—	—	—	—	*A*11	Large clearance fit and widely used.
	B	—	—	—	—	—	*B*11	
	C	—	—	—	—	—	*C*11	Slack running fit.
	D	—	**D*9	—	*D*10	*D*10	**D*11	Loose running fit.
	E	—	**E*8	—	*E*8*	*E*9	—	Easy running fit.
	F	—	**F*7	—	*F*8	**F*8	—	Normal running fit.
	G	**G*6	*G*7	—	—	—	—	Close running fit or sliding fit, also spigot and location fit.
	H	**H*6	*H*7	*H*8	*H*8	*H*8, *H*9	*H*11	Precision sliding fit. Also fine spigot and location fit.
	Js	**Js*6	*Js*7	**Js*8	—	—	—	Push fit for very accurate location with easy assembly and disassembly.

* Second preference fits.

Contents

Type of fit	*Class of hole*	*With shafts*						*Remarks and uses*
		*h5	h6	h7	*h8	h9	h11	
Transition fit	K	*K6	K7	*K8	—	—	—	Light keying fit (true transition) for keyed shafts, non-running locked pins, etc.
	M	*M6	*M7	*M8	—	—	—	Medium keying fit.
	N	*N6	N7	*N8	—	—	—	Heavy keying fit (for tight assembly of mating surfaces).
Interference fit	P	*P6	P7	—	—	—	—	Light press fit with easy dismantling for non-ferrous parts. Standard press fit with easy dismantling for ferrous and non-ferrous parts assembly.
	R	*R6	R7	—	—	—	—	Medium drive fit with easy dismantling for ferrous parts assembly. Light drive fit with easy dismantling for non-ferrous parts assembly.
	S	*S6	S7	—	—	—	—	Heavy drive fit for ferrous parts permanent or semi- permanent assembly, standard press fit for non-ferrous parts.
	T	*T6	T7	—	—	—	—	Force fit on ferrous parts for permanent assembly.

3.18 Calculation of Fundamental Deviation for Shafts

We have already discussed that for holes, the upper deviation is denoted by *ES* and the lower deviation by *EI*. Similarly for shafts, the upper deviation is represented by *es* and the lower deviation by *ei*. According to Indian standards, for each letter symbol, the magnitude and sign for one of the two deviations (*i.e.* either upper or lower deviation), which is known as fundamental deviation, have been determined by means of formulae given in Table 3.7. The other deviation may be calculated by using the absolute value of the standard tolerance (*IT*) from the following relation:

$$ei = es - IT \qquad \text{or} \qquad es = ei + IT$$

It may be noted for shafts *a* to *h*, the upper deviations (*es*) are considered whereas for shafts *j* to *Zc*, the lower deviation (*ei*) is to be considered.

Computer simulation of stresses on a jet engine blades.

* Second preference fits.

Top

Contents

The fundamental deviation for Indian standard shafts for diameter steps from 1 to 200 mm may be taken directly from Table 3.10 (page 76).

Table 3.7. Formulae for fundamental shaft deviations.

Upper deviation (es)		*Lower deviation (ei)*	
Shaft designation	*In microns (for D in mm)*	*Shaft designation*	*In microns (for D in mm)*
a	$= -(265 + 1.3\,D)$ for $D \le 120$	*J* 5 to *j* 8	No formula
	$= -3.5\,D$ for $D > 120$	*k* 4 to *k* 7	$= +0.6\sqrt[3]{D}$
		k for grades ≤ 3 and ≤ 8	$= 0$
b	$= -(140 + 0.85\,D)$ for $D \le 160$	*m*	$= +(IT\,7 - IT\,6)$
	$= -1.8\,D$	*n*	$= +5\,(D)^{0.34}$
	for $D > 160$	*p*	$= +IT\,7 + 0$ to 5
c	$= -52\,(D)^{0.2}$ for $D \le 40$	*r*	= Geometric mean of values of *ei* for shaft *p* and *s*
	$= -(95 + 0.8\,D)$ for $D > 40$	*s*	$= +(IT\,8 + 1$ to $4)$ for $D \le 50$; $= +(IT\,7 + 0.4\,D)$ for $D > 50$
d	$= -16\,(D)^{0.44}$	*t*	$= +(IT\,7 + 0.63\,D)$
e	$= -11\,(D)^{0.41}$	*u*	$= +(IT\,7 + D)$
f	$= -5.5\,(D)^{0.41}$	*v*	$= +(IT\,7 + 1.25\,D)$
		x	$= +(IT\,7 + 1.6\,D)$
g	$= -2.5\,(D)^{0.34}$	*y*	$= +(IT\,7 + 2\,D)$
		z	$= +(IT\,7 + 2.5\,D)$
h	$= 0$	*za*	$= +(IT\,8 + 3.15\,D)$
		zb	$= +(IT\,9 + 4\,D)$
		zc	$= +(IT\,10 + 5\,D)$

For *js*, the two deviations are equal to $\pm IT/2$.

3.19 Calculation of Fundamental Deviation for Holes

The fundamental deviation for holes for those of the corresponding shafts, are derived by using the rule as given in Table 3.8.

Top

Table 3.8. Rules for fundamental deviation for holes.

All deviation except those below			*General rule* Hole limits are identical with the shaft limits of the same symbol (letter and grade) but disposed on the other side of the zero line. *EI* = Upper deviation *es* of the shaft of the same letter symbol but of opposite sign.
For sizes above 3 mm	*N*	9 and coarser grades	*ES* = 0
	J, K, M and *N* *P* to *ZC*	Upto grade 8 inclusive upto grade 7 inclusive	*Special rule* *ES* = Lower deviation *ei* of the shaft of the same letter symbol but one grade finer and of opposite sign increased by the difference between the tolerances of the two grades in question.

The fundamental deviation for Indian standard holes for diameter steps from 1 to 200 mm may be taken directly from the following table.

Table 3.9. Indian standard 'H' Hole
Limits for H5 to H13 over the range 1 to 200 mm as per IS : 919 (Part II) -1993.

Diameter steps in mm		*Deviations in micron (1 micron = 0.001 mm)*									
		H5	H6	H7	H8	H9	H10	H11	H12	H13	H5 – H13
Over	To	High +	High +	High +	High +	High +	High +	High +	High +	High +	Low
1	3	5	7	9	14	25	40	60	90	140	0
3	6	5	8	12	18	30	48	75	120	180	0
6	10	6	9	15	22	36	58	90	150	220	0
10 14	14 18	8	11	18	27	43	70	110	180	270	0
18 24	24 30	9	13	21	33	52	84	130	210	330	0
30 40	40 50	11	16	25	39	62	100	160	250	460	0
50 65	65 80	13	19	30	46	74	120	190	300	390	0
80 100	100 120	15	22	35	54	87	140	220	350	540	0
120 140	140 160	18	25	40	63	100	160	250	400	630	0
160 180	180 200	20	29	46	72	115	185	290	460	720	0

Contents

Table 3.10. Indian standard shafts for common use as per IS : 919 (Part II)–1993.

Shaft	*Limit*	*Values of deviations in microns for diameter steps 1 to 200 mm (1 micron = 0.001 mm)*																
		1 to 3	3 to 6	6 to 10	10 to 14	14 to 18	18 to 24	24 to 30	30 to 40	40 to 50	50 to 65	65 to 80	80 to 100	100 to 120	120 to 140	140 to 160	160 to 180	180 to 200
s6	High+	22	37	32	39	39	48	48	59	59	72	78	93	101	117	125	133	151
s7	High+	24	31	38	46	46	56	66	68	68	83	89	106	114	132	140	148	168
s6 & *s7*	Low+	15	19	23	28	28	35	35	43	43	53	59	71	79	92	100	108	122
p9	High+	16	20	24	29	29	35	35	42	42	51	51	59	59	68	68	68	79
	Low+	9	12	15	18	18	22	22	26	26	32	32	37	37	43	43	43	50
k6	High+	—	—	10	12	12	15	15	18	18	21	21	25	25	28	28	28	33
k7	High+	—	—	16	19	19	23	23	27	27	32	32	38	38	43	43	43	50
k6 & *k7*	Low+	—	—	1	1	1	2	2	2	2	2	2	3	3	3	3	3	4
	High+	7	9	10	12	12	13	13	15	15	18	18	20	20	22	22	22	25
	Low–	2	3	5	6	6	8	8	10	10	12	12	15	15	18	18	18	21
h6	High–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Top

Shaft	Limit	Values of deviations in microns for diameter steps 1 to 200 mm (1 micron = 0.001 mm)																
		1 to 3	3 to 6	6 to 10	10 to 14	14 to 18	18 to 24	24 to 30	30 to 40	40 to 50	50 to 65	65 to 80	80 to 100	100 to 120	120 to 140	140 to 160	160 to 180	180 to 200
*h*7	High–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
*h*6	Low–	7	8	9	11	11	13	13	16	16	19	19	22	22	25	25	26	29
&		&	&	&	&	&	&	&	&	&	&	&	&	&	&	&	&	&
*h*7		9	12	15	18	18	21	21	25	25	30	30	35	35	40	40	40	46
g 6	High–	3	4	5	6	6	7	7	9	9	10	10	12	12	14	14	14	15
	Low–	10	12	14	17	17	20	20	25	25	29	29	34	34	39	39	39	44
f 7	High–	7	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50
f 8	High–	7	10	13	16	16	20	20	25	25	30	30	36	36	43	43	43	50
f 7	Low–	16	22	28	34	34	41	41	50	50	60	60	71	71	83	83	83	96
&		&	&	&	&	&	&	&	&	&	&	&	&	&	&	&	&	&
f 8		21	28	35	43	43	53	53	64	64	76	76	90	90	106	106	106	122
e 8	High–	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100
e 9	High–	14	20	25	32	32	40	40	50	50	60	60	72	72	85	85	85	100
e 8	Low–	28	38	47	59	58	73	73	89	89	106	106	126	126	148	148	148	172
&		&	&	&	&	&	&	&	&	&	&	&	&	&	&	&	&	&
e 9		39	50	61	75	75	92	92	112	112	134	134	158	158	185	185	185	215
d 9	High–	20	30	40	50	50	65	65	80	80	100	100	120	120	145	145	145	170
	Low–	45	60	76	93	93	117	117	142	142	174	174	207	207	245	245	245	285
c 9	High–	60	70	80	95	95	110	110	120	130	140	150	170	180	200	210	230	240
	Low–	85	100	116	138	138	162	162	182	192	214	224	257	267	300	310	330	355
b 9	High–	140	140	150	150	150	160	160	170	180	190	200	220	240	260	280	310	340
	Low–	165	170	186	193	193	212	212	232	242	264	274	307	327	360	380	410	545

Contents

Example 3.1. *The dimensions of the mating parts, according to basic hole system, are given as follows :*

Hole : 25.00 mm	*Shaft : 24.97 mm*
25.02 mm	*24.95 mm*

Find the hole tolerance, shaft tolerance and allowance.

Solution. Given : Lower limit of hole = 25 mm ; Upper limit of hole = 25.02 mm ; Upper limit of shaft = 24.97 mm ; Lower limit of shaft = 24.95 mm

Hole tolerance

We know that hole tolerance

= Upper limit of hole – Lower limit of hole

= 25.02 – 25 = 0.02 mm **Ans.**

Shaft tolerance

We know that shaft tolerance

= Upper limit of shaft – Lower limit of shaft

= 24.97 – 24.95 = 0.02 mm **Ans.**

Allowance

We know that allowance

= Lower limit of hole – Upper limit of shaft

= 25.00 – 24.97 = 0.03 mm **Ans.**

Example 3.2. *Calculate the tolerances, fundamental deviations and limits of sizes for the shaft designated as 40 H8 / f7.*

Solution. Given: Shaft designation = 40 *H*8 / *f* 7

The shaft designation 40 *H*8 / *f* 7 means that the basic size is 40 mm and the tolerance grade for the hole is 8 (*i.e.* *I T* 8) and for the shaft is 7 (*i.e.* *I T* 7).

Tolerances

Since 40 mm lies in the diameter steps of 30 to 50 mm, therefore the geometric mean diameter,

$$D = \sqrt{30 \times 50} = 38.73 \text{ mm}$$

We know that standard tolerance unit,

$$i = 0.45 \sqrt[3]{D} + 0.001\,D$$

$$= 0.45 \sqrt[3]{38.73} + 0.001 \times 38.73$$

$$= 0.45 \times 3.38 + 0.03873 = 1.559\,73 \quad \text{or} \quad 1.56 \text{ microns}$$

$$= 1.56 \times 0.001 = 0.001\,56 \text{ mm} \qquad \text{...}(\because 1 \text{ micron} = 0.001 \text{ mm})$$

From Table 3.2, we find that standard tolerance for the hole of grade 8 (*I T* 8)

= 25 *i* = 25 × 0.001 56 = 0.039 mm **Ans.**

and standard tolerance for the shaft of grade 7 (*I T* 7)

= 16 *i* = 16 × 0.001 56 = 0.025 mm **Ans.**

Note : The value of *I T* 8 and *I T* 7 may be directly seen from Table 3.3.

Top

Contents

Fundamental deviation

We know that fundamental deviation (lower deviation) for hole *H*,

$$EI = 0$$

From Table 3.7, we find that fundamental deviation (upper deviation) for shaft *f*,

$$es = -5.5\,(D)^{0.41}$$
$$= -5.5\,(38.73)^{0.41} = -24.63 \text{ or } -25 \text{ microns}$$
$$= -25 \times 0.001 = -0.025 \text{ mm } \textbf{Ans.}$$

∴ Fundamental deviation (lower deviation) for shaft *f*,

$$ei = es - IT = -0.025 - 0.025 = -0.050 \text{ mm } \textbf{Ans.}$$

The –ve sign indicates that fundamental deviation lies below the zero line.

Limits of sizes

We know that lower limit for hole

= Basic size = 40 mm **Ans.**

Upper limit for hole = Lower limit for hole + Tolerance for hole

= 40 + 0.039 = 40.039 mm **Ans.**

Upper limit for shaft = Lower limit for hole or Basic size – Fundamental deviation (upper deviation) ...(∵ Shaft *f* lies below the zero line)

= 40 – 0.025 = 39.975 mm **Ans.**

and lower limit for shaft = Upper limit for shaft – Tolerance for shaft

= 39.975 – 0.025 = 39.95 mm **Ans.**

Example 3.3. *Give the dimensions for the hole and shaft for the following:*

(*a*) *A 12 mm electric motor sleeve bearing;*

(*b*) *A medium force fit on a 200 mm shaft; and*

(*c*) *A 50 mm sleeve bearing on the elevating mechanism of a road grader.*

Solution.

(*a*) *Dimensions for the hole and shaft for a 12 mm electric motor sleeve bearing*

From Table 3.5, we find that for an electric motor sleeve bearing, a shaft *e* 8 should be used with *H* 8 hole.

Since 12 mm size lies in the diameter steps of 10 to 18 mm, therefore the geometric mean diameter,

$$D = \sqrt{10 \times 18} = 13.4 \text{ mm}$$

We know that standard tolerance unit,

$$i = 0.45\sqrt[3]{D} + 0.001\,D$$
$$= 0.45\sqrt[3]{13.4} + 0.001 \times 13.4 = 1.07 + 0.0134 = 1.0834 \text{ microns}$$

∴ *Standard tolerance for shaft and hole of grade 8 (*IT* 8)

= 25 *i* ...(From Table 3.2)

= 25 × 1.0834 = 27 microns

= 27 × 0.001 = 0.027 mm ...(∵ 1 micron = 0.001 mm)

From Table 3.7, we find that upper deviation for shaft '*e*',

$$es = -11(D)^{0.41} = -11\,(13.4)^{0.41} = -32 \text{ microns}$$
$$= -32 \times 0.001 = -0.032 \text{ mm}$$

* The tolerance values may be taken directly from Table 3.3.

Top

We know that lower deviation for shaft 'e',

$$ei = es - IT = -0.032 - 0.027 = -0.059 \text{ mm}$$

∴ Dimensions for the hole (H 8)

$$= 12^{+0.027}_{+0.000} \quad \textbf{Ans.}$$

and dimension for the shaft (e 8)

$$= 12^{-0.032}_{-0.059} \quad \textbf{Ans.}$$

(b) ***Dimensions for the hole and shaft for a medium force fit on a 200 mm shaft***

From Table 3.5, we find that shaft r 6 with hole H 7 gives the desired fit.

Since 200 mm lies in the diameter steps of 180 mm to 250 mm, therefore the geometric mean diameter,

$$D = \sqrt{180 \times 250} = 212 \text{ mm}$$

We know that standard tolerance unit,

$$i = 0.45 \sqrt[3]{D} + 0.001\, D$$

$$= 0.45 \sqrt[3]{212} + 0.001 \times 212 = 2.68 + 0.212 = 2.892 \text{ microns}$$

∴ Standard tolerance for the shaft of grade 6 (IT6) from Table 3.2

$$= 10\, i = 10 \times 2.892 = 28.92 \text{ microns}$$

$$= 28.92 \times 0.001 = 0.02892 \text{ or } 0.029 \text{ mm}$$

and standard tolerance for the hole of grade 7 (IT 7)

$$= 16\, i = 16 \times 2.892 = 46 \text{ microns}$$

$$= 46 \times 0.001 = 0.046 \text{ mm}$$

We know that lower deviation for shaft 'r' from Table 3.7

$$ei = \frac{1}{2}\left[(IT\,7 + 0.4\,D) + (IT\,7 + 0 \text{ to } 5)\right]$$

$$= \frac{1}{2}\left[(46 + 0.4 \times 212) + (46 + 3)\right] = 90 \text{ microns}$$

$$= 90 \times 0.001 = 0.09 \text{ mm}$$

and upper deviation for the shaft r,

$$es = ei + IT = 0.09 + 0.029 = 0.119 \text{ mm}$$

∴ Dimension for the hole H 7

$$= 200^{+0.046}_{+0.00} \quad \textbf{Ans.}$$

and dimension for the shaft r 6

$$= 200^{+0.119}_{+0.09} \quad \textbf{Ans.}$$

(c) ***Dimensions for the hole and shaft for a 50 mm sleeve bearing on the elevating mechanism of a road grader***

From Table 3.5, we find that for a sleeve bearing, a loose running fit will be suitable and a shaft d 9 should be used with hole H 8.

Since 50 mm size lies in the diameter steps of 30 to 50 mm or 50 to 80 mm, therefore the geometric mean diameter,

$$D = \sqrt{30 \times 50} = 38.73 \text{ mm}$$

Contents

We know that standard tolerance unit,

$$i = 0.45\sqrt[3]{D} + 0.001\,D$$
$$= 0.45\sqrt[3]{38.73} + 0.001 \times 38.73$$
$$= 1.522 + 0.03873 = 1.56073 \quad \text{or} \quad 1.56 \text{ microns}$$

∴ Standard tolerance for the shaft of grade 9 (*IT* 9) from Table 3.2

$$= 40\,i = 40 \times 1.56 = 62.4 \text{ microns}$$
$$= 62.4 \times 0.001 = 0.0624 \quad \text{or} \quad 0.062 \text{ mm}$$

and standard tolerance for the hole of grade 8 (*IT* 8)

$$= 25\,i = 25 \times 1.56 = 39 \text{ microns}$$
$$= 39 \times 0.001 = 0.039 \text{ mm}$$

We know that upper deviation for the shaft *d*, from Table 3.7

$$es = -16\,(D)^{0.44} = -16\,(38.73)^{0.44} = -80 \text{ microns}$$
$$= -80 \times 0.001 = -0.08 \text{ mm}$$

and lower deviation for the shaft *d*,

$$ei = es - IT = -0.08 - 0.062 = -0.142 \text{ mm}$$

∴ Dimension for the hole *H* 8

$$= 50^{+0.039}_{+0.000} \quad \textbf{Ans.}$$

and dimension for the shaft *d* 9

$$= 50^{-0.08}_{-0.142} \quad \textbf{Ans.}$$

Example 3.4. *A journal of nominal or basic size of 75 mm runs in a bearing with close running fit. Find the limits of shaft and bearing. What is the maximum and minimum clearance?*

Solution. Given: Nominal or basic size = 75 mm

From Table 3.5, we find that the close running fit is represented by *H* 8/*g* 7, *i.e.* a shaft *g* 7 should be used with *H* 8 hole.

Since 75 mm lies in the diameter steps of 50 to 80 mm, therefore the geometric mean diameter,

$$D = \sqrt{50 \times 80} = 63 \text{ mm}$$

We know that standard tolerance unit,

$$i = 0.45\sqrt[3]{D} + 0.001\,D = 0.45\sqrt[3]{63} + 0.001 \times 63$$
$$= 1.79 + 0.063 = 1.853 \text{ micron}$$
$$= 1.853 \times 0.001 = 0.001\,853 \text{ mm}$$

∴ Standard tolerance for hole '*H*' of grade 8 (*IT* 8)

$$= 25\,i = 25 \times 0.001\,853 = 0.046 \text{ mm}$$

and standard tolerance for shaft '*g*' of grade 7 (*IT* 7)

$$= 16\,i = 16 \times 0.001\,853 = 0.03 \text{ mm}$$

From Table 3.7, we find that upper deviation for shaft *g*,

$$es = -2.5\,(D)^{0.34} = -2.5\,(63)^{0.34} = -10 \text{ micron}$$
$$= -10 \times 0.001 = -0.01 \text{ mm}$$

Top

Contents

∴ Lower deviation for shaft g,

$$ei = es - IT = -0.01 - 0.03 = -0.04 \text{ mm}$$

We know that lower limit for hole

= Basic size = 75 mm

Upper limit for hole = Lower limit for hole + Tolerance for hole

= 75 + 0.046 = 75.046 mm

Upper limit for shaft = Lower limit for hole – Upper deviation for shaft

...(∵ Shaft g lies below zero line)

= 75 – 0.01 = 74.99 mm

and lower limit for shaft = Upper limit for shaft – Tolerance for shaft

= 74.99 – 0.03 = 74.96 mm

We know that maximum clearance

= Upper limit for hole – Lower limit for shaft

= 75.046 – 74.96 = 0.086 mm **Ans.**

and minimum clearance = Lower limit for hole – Upper limit for shaft

= 75 – 74.99 = 0.01 mm **Ans.**

3.20 Surface Roughness and its Measurement

A little consideration will show that surfaces produced by different machining operations (*e.g.* turning, milling, shaping, planing, grinding and superfinishing) are of different characteristics. They show marked variations when compared with each other. The variation is judged by the degree of smoothness. A surface produced by superfinishing is the smoothest, while that by planing is the roughest. In the assembly of two mating parts, it becomes absolutely necessary to describe the surface finish in quantitative terms which is measure of micro-irregularities of the surface and expressed in microns. In order to prevent stress concentrations and proper functioning, it may be necessary to avoid or to have certain surface roughness.

There are many ways of expressing the surface roughness numerically, but the following two methods are commonly used :

1. Centre line average method (briefly known as CLA method), and
2. Root mean square method (briefly known as RMS method).

The ***centre line average method*** is defined as the average value of the ordinates between the surface and the mean line, measured on both sides of it. According to Indian standards, the surface finish is measured in terms of 'CLA' value and it is denoted by *Ra*.

Landing Gear : *When an aircraft comes in to land, it has to lose a lot of energy in a very short time. the landing gear deals with this and prevents disaster. First, mechanical or liquid springs absorb energy rapidly by being compressed. As the springs relax, this energy will be released again, but in a slow controlled manner in a damper-the second energy absorber. Finally, the tyres absorb energy, getting hot in the process.*

Top

$$\text{CLA value or } Ra \text{ (in microns)} = \frac{y_1 + y_2 + y_3 + \ldots y_n}{n}$$

where, $y_1, y_2, \ldots y_n$ are the ordinates measured on both sides of the mean line and n are the number of ordinates.

The ***root mean square method*** is defined as the square root of the arithmetic mean of the squares of the ordinates. Mathematically,

$$\text{R.M.S. value (in microns)} = \sqrt{\frac{y_1^2 + y_2^2 + y_3^2 + \ldots y_n^2}{n}}$$

According to Indian standards, following symbols are used to denote the various degrees of surface roughness :

Symbol	Surface roughness (*Ra*) in microns
∇	8 to 25
∇ ∇	1.6 to 8
∇ ∇ ∇	0.025 to 1.6
∇ ∇ ∇ ∇	Less than 0.025

The following table shows the range of surface roughness that can be produced by various manufacturing processes.

Table 3.11. Range of surface roughness.

S.No.	*Manufacturing process*	*Surface roughness in microns*	*S.No.*	*Manufacturing process*	*Surface roughness in microns*
1.	Lapping	0.012 to 0.016	9	Extrusion	0.16 to 5
2.	Honing	0.025 to 0.40	10.	Boring	0.40 to 6.3
3.	Cylindrical grinding	0.063 to 5	11.	Milling	0.32 to 25
4.	Surface grinding	0.063 to 5	12.	Planing and shaping	1.6 to 25
5.	Broaching	0.40 to 3.2	13.	Drilling	1.6 to 20
6.	Reaming	0.40 to 3.2	14.	Sand casting	5 to 50
7.	Turning	0.32 to 25	15.	Die casting	0.80 to 3.20
8.	Hot rolling	2.5 to 50	16.	Forging	1.60 to 2.5

3.21 Preferred Numbers

When a machine is to be made in several sizes with different powers or capacities, it is necessary to decide what capacities will cover a certain range efficiently with minimum number of sizes. It has been shown by experience that a certain range can be covered efficiently when it follows a geometrical progression with a constant ratio. The preferred numbers are the conventionally rounded off values derived from geometric series including the integral powers of 10 and having as common ratio of the following factors:

$$\sqrt[5]{10},\ \sqrt[10]{10},\ \sqrt[20]{10} \text{ and } \sqrt[40]{10}$$

These ratios are approximately equal to 1.58, 1.26, 1.12 and 1.06. The series of preferred numbers are designated as *R5, R10, R20 and R40 respectively. These four series are called ***basic series.*** The other series called ***derived series*** may be obtained by simply multiplying or dividing the basic sizes by 10, 100, etc. The preferred numbers in the series R5 are 1, 1.6, 2.5, 4.0 and 6.3. Table 3.12 shows basic series of preferred numbers according to IS : 1076 (Part I) – 1985 (Reaffirmed 1990).

* The symbol R is used as a tribute to Captain Charles Renard, the first man to use preferred numbers.

Contents

Notes : 1. The standard sizes (in mm) for wrought metal products are shown in Table 3.13 according to IS : 1136 – 1990. The standard G.P. series used correspond to R10, R20 and R40.

2. The hoisting capacities (in tonnes) of cranes are in R10 series, while the hydraulic cylinder diameters are in R40 series and hydraulic cylinder capacities are in R5 series.

3. The basic thickness of sheet metals and diameter of wires are based on R10, R20 and R40 series. Wire diameter of helical springs are in R20 series.

Table 3.12. Preferred numbers of the basic series, according to IS : 1076 (Part I)–1985 (Reaffirmed 1990).

Basic series	*Preferred numbers*
R5	1.00, 1.60, 2.50, 4.00, 6.30, 10.00
R10	1.00, 1.25, 1.60, 2.00, 2.50, 3.15, 4.00, 5.00, 6.30, 8.00, 10.00
R20	1.00, 1.12, 1.25, 1.40, 1.60, 1.80, 2.00, 2.24, 2.50, 2.80, 3.15, 3.55, 4.00, 4.50, 5.00, 5.60, 6.30, 7.10, 8.00, 9.00, 10.00
R40	1.00, 1.06, 1.12, 1.18, 1.25, 1.32, 1.40, 1.50, 1.60, 1.70, 1.80, 1.90, 2.00, 2.12, 2.24, 2.36, 2.50, 2.65, 2.80, 3.00, 3.15, 3.35, 3.55, 3.75, 4.00, 4.25, 4.50, 4.75, 5.00, 5.30, 5.60, 6.00, 6.30, 6.70, 7.10, 7.50, 8.00, 8.50, 9.00, 9.50, 10.00

Table 3.13. Preferred sizes for wrought metal products according to IS : 1136 – 1990.

Size range	*Preferred sizes (mm)*
0.01 – 0.10 mm	0.02, 0.025, 0.030, 0.04, 0.05, 0.06, 0.08 and 0.10
0.10 – 1 mm	0.10, 0.11, 0.12, 0.14, 0.16, 0.18, 0.20, 0.22, 0.25, 0.28, 0.30, 0.32, 0.35, 0.36, 0.40, 0.45, 0.50, 0.55, 0.60, 0.63, 0.70, 0.80, 0.90 and 1
1 – 10 mm	1, 1.1, 1.2, 1.4, 1.5, 1.6, 1.8, 2.22, 2.5, 2.8, 3, 3.2, 3.5, 3.6, 4, 4.5, 5, 5.5, 5.6, 6, 6.3, 7, 8, 9 and 10
10 – 100 mm	10 to 25 (in steps of 1 mm), 28, 30, 32, 34, 35, 36, 38, 40, 42, 44, 45, 46, 48,50, 52, 53, 55, 56, 58, 60, 62, 63, 65, 67, 68, 70, 71, 72, 75, 78, 80, 82, 85, 88, 90, 92, 95, 98 and 100
100 – 1000 mm	100 to 200 (in steps of 5 mm), 200 to 310 (in steps of 10 mm), 315, 320, 330, 340, 350, 355, 360, 370, 375, 380 to 500 (in steps of 10 mm), 520, 530,550, 560, 580, 600, 630, 650, 670, 700, 710 and 750 – 1000 (in steps of 50 mm)
1000 – 10 000 mm	1000, 1100, 1200, 1250, 1400, 1500, 1600, 1800, 2000, 2200, 2500, 2800, 3000, 3200, 3500, 3600, 4000, 4500, 5000, 5500, 5600, 6000, 6300, 7000, 7100, 8000, 9000 and 10 000

EXERCISES

1. A journal of basic size of 75 mm rotates in a bearing. The tolerance for both the shaft and bearing is 0.075 mm and the required allowance is 0.10 mm. Find the dimensions of the shaft and the bearing bore. **[Ans. For shaft : 74.90 mm, 74.825 mm ; For hole : 75.075 mm, 75 mm]**

2. A medium force fit on a 75 mm shaft requires a hole tolerance and shaft tolerance each equal to 0.225 mm and average interference of 0.0375 mm. Find the hole and shaft dimensions.

[Ans. 75 mm, 75.225 mm ; 75.2625 mm, 75.4875 mm]

Top

Contents

3. Calculate the tolerances, fundamental deviations and limits of size for hole and shaft in the following cases of fits :

(*a*) 25 H 8 / d 9; and (*b*) 60 H 7 / m 6

[Ans. (*a*) 0.033 mm, 0.052 mm; 0, – 0.064 mm, – 0.116 mm; 25 mm, 25.033 mm, 24.936 mm, 24.884 mm (*b*) 0.03 mm, 0.019 mm; 0.011 mm, – 0.008 mm; 60 mm, 60.03 mm, 59.989 mm, 59.97 mm]

4. Find the extreme diameters of shaft and hole for a transition fit $H7/n6$, if the nominal or basic diameter is 12 mm. What is the value of clearance and interference?

[Ans. 12.023 mm, 12.018 mm; 0.006 mm, – 0.023 mm]

5. A gear has to be shrunk on a shaft of basic size 120 mm. An interference fit $H7/u6$ is being selected. Determine the minimum and maximum diameter of the shaft and interference.

[Ans. 120.144 mm, 120.166 mm; 0.109 mm, 0.166 mm]

QUESTIONS

1. Enumerate the various manufacturing methods of machine parts which a designer should know.

2. Explain briefly the different casting processes.

3. Write a brief note on the design of castings?

4. State and illustrate two principal design rules for casting design.

5. List the main advantages of forged components.

6. What are the salient features used in the design of forgings? Explain.

7. What do you understand by ‘hot working’ and ‘cold working’ processes? Explain with examples.

8. State the advantages and disadvantages of hot working of metals. Discuss any two hot working processes.

9. What do you understand by cold working of metals? Describe briefly the various cold working processes.

10. What are fits and tolerances? How are they designated?

11. What do you understand by the nominal size and basic size?

12. Write short notes on the following :

(*a*) Interchangeability; (*b*) Tolerance; (*c*) Allowance; and (*d*) Fits.

13. What is the difference in the type of assembly generally used in running fits and interference fits?

14. State briefly unilateral system of tolerances covering the points of definition, application and advantages over the bilateral system.

15. What is meant by ‘hole basis system’ and ‘shaft basis system’? Which one is preferred and why?

16. Discuss the Indian standard system of limits and fits.

17. What are the commonly used fits according to Indian standards?

18. What do you understand by preferred numbers? Explain fully.

OBJECTIVE TYPE QUESTIONS

1. The castings produced by forcing molten metal under pressure into a permanent metal mould is known as

(*a*) permanent mould casting (*b*) slush casting

(*c*) die casting (*d*) centrifugal casting

2. The metal is subjected to mechanical working for

(*a*) refining grain size (*b*) reducing original block into desired shape

(*c*) controlling the direction of flow lines (*d*) all of these

Top

Contents

3. The temperature at which the new grains are formed in the metal is called
   (*a*) lower critical temperature (*b*) upper critical temperature
   (*c*) eutectic temperature (*d*) recrystallisation temperature
4. The hot working of metals is carried out
   (*a*) at the recrystallisation temperature (*b*) below the recrystallisation temperature
   (*c*) above the recrystallisation temperature (*d*) at any temperature
5. During hot working of metals
   (*a*) porosity of the metal is largely eliminated
   (*b*) grain structure of the metal is refined
   (*c*) mechanical properties are improved due to refinement of grains
   (*d*) all of the above
6. The parts of circular cross-section which are symmetrical about the axis of rotation are made by
   (*a*) hot forging (*b*) hot spinning
   (*c*) hot extrusion (*d*) hot drawing
7. The cold working of metals is carried out .... the recrystallisation temperature.
   (*a*) above (*b*) below
8. The process extensively used for making bolts and nuts is
   (*a*) hot piercing (*b*) extrusion
   (*c*) cold peening (*d*) cold heading
9. In a unilateral system of tolerance, the tolerance is allowed on
   (*a*) one side of the actual size (*b*) one side of the nominal size
   (*c*) both sides of the actual size (*d*) both sides of the nominal size
10. The algebraic difference between the maximum limit and the basic size is called
    (*a*) actual deviation (*b*) upper deviation
    (*c*) lower deviation (*d*) fundamental deviation
11. A basic shaft is one whose
    (*a*) lower deviation is zero (*b*) upper deviation is zero
    (*c*) lower and upper deviations are zero (*d*) none of these
12. A basic hole is one whose
    (*a*) lower deviation is zero (*b*) upper deviation is zero
    (*c*) lower and upper deviations are zero (*d*) none of these
13. According to Indian standard specifications, 100 $H\,6\,/\,g\,5$ means that the
    (*a*) actual size is 100 mm
    (*b*) basic size is 100 mm
    (*c*) difference between the actual size and basic size is 100 mm
    (*d*) none of the above
14. According to Indian standards, total number of tolerance grades are
    (*a*) 8 (*b*) 12
    (*c*) 18 (*d*) 20
15. According to Indian standard specification, 100 $H6/g5$ means that
    (*a*) tolerance grade for the hole is 6 and for the shaft is 5
    (*b*) tolerance grade for the shaft is 6 and for the hole is 5
    (*c*) tolerance grade for the shaft is 4 to 8 and for the hole is 3 to 7
    (*d*) tolerance grade for the hole is 4 to 8 and for the shaft is 3 to 7

ANSWERS

1. (*c*)	**2.** (*d*)	**3.** (*d*)	**4.** (*c*)	**5.** (*d*)
6. (*b*)	**7.** (*b*)	**8.** (*d*)	**9.** (*b*)	**10.** (*b*)
11. (*b*)	**12.** (*a*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)

Top

Contents

CHAPTER

4

Simple Stresses in Machine Parts

1. Introduction.
2. Load.
3. Stress.
4. Strain.
5. Tensile Stress and Strain.
6. Compressive Stress and Strain.
7. Young's Modulus or Modulus of Elasticity.
8. Shear Stress and Strain
9. Shear Modulus or Modulus of Rigidity.
10. Bearing Stress.
11. Stress-Strain Diagram.
12. Working Stress.
13. Factor of Safety.
14. Selection of Factor of Safety.
15. Stresses in Composite Bars.
16. Stresses due to Change in Temperature—Thermal Stresses.
17. Linear and Lateral Strain.
18. Poisson's Ratio.
19. Volumetric Strain.
20. Bulk Modulus.
21. Relation between Bulk Modulus and Young's Modulus.
22. Relation between Young's Modulus and Modulus of Rigidity.
23. Impact Stress.
24. Resilience.

4.1 Introduction

In engineering practice, the machine parts are subjected to various forces which may be due to either one or more of the following:

1. Energy transmitted,
2. Weight of machine,
3. Frictional resistances,
4. Inertia of reciprocating parts,
5. Change of temperature, and
6. Lack of balance of moving parts.

The different forces acting on a machine part produces various types of stresses, which will be discussed in this chapter.

4.2 Load

It is defined as **any external force acting upon a machine part.** The following four types of the load are important from the subject point of view:

Top

Contents

1. *Dead or steady load.* A load is said to be a dead or steady load, when it does not change in magnitude or direction.

2. *Live or variable load.* A load is said to be a live or variable load, when it changes continually.

3. *Suddenly applied or shock loads*. A load is said to be a suddenly applied or shock load, when it is suddenly applied or removed.

4. *Impact load.* A load is said to be an impact load, when it is applied with some initial velocity.

Note: A machine part resists a dead load more easily than a live load and a live load more easily than a shock load.

4.3 Stress

When some external system of forces or loads act on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as ***unit stress*** or simply a ***stress***. It is denoted by a Greek letter sigma (σ). Mathematically,

$$\text{Stress, } \sigma = P/A$$

where P = Force or load acting on a body, and

A = Cross-sectional area of the body.

In S.I. units, the stress is usually expressed in Pascal (Pa) such that 1 Pa = 1 N/m^2. In actual practice, we use bigger units of stress *i.e.* megapascal (MPa) and gigapascal (GPa), such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

and

$$1 \text{ GPa} = 1 \times 10^9 \text{ N/m}^2 = 1 \text{ kN/mm}^2$$

4.4 Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as ***unit strain*** or simply a ***strain.*** It is denoted by a Greek letter epsilon (ε). Mathematically,

$$\text{Strain, } \varepsilon = \delta l / l \quad \text{or} \quad \delta l = \varepsilon . l$$

where δl = Change in length of the body, and

l = Original length of the body.

4.5 Tensile Stress and Strain

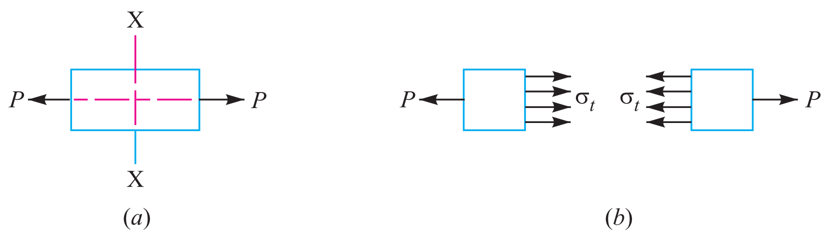


Fig. 4.1. Tensile stress and strain.

When a body is subjected to two equal and opposite axial pulls P (also called tensile load) as shown in Fig. 4.1 (*a*), then the stress induced at any section of the body is known as ***tensile stress*** as shown in Fig. 4.1 (*b*). A little consideration will show that due to the tensile load, there will be a decrease in cross-sectional area and an increase in length of the body. The ratio of the increase in length to the original length is known as ***tensile strain.***

Top

Let P = Axial tensile force acting on the body,

A = Cross-sectional area of the body,

l = Original length, and

δl = Increase in length.

∴ Tensile stress, $\sigma_t = P/A$

and tensile strain, $\varepsilon_t = \delta l / l$

4.6 Compressive Stress and Strain

When a body is subjected to two equal and opposite axial pushes P (also called compressive load) as shown in Fig. 4.2 (*a*), then the stress induced at any section of the body is known as ***compressive stress*** as shown in Fig. 4.2 (*b*). A little consideration will show that due to the compressive load, there will be an increase in cross-sectional area and a decrease in length of the body. The ratio of the decrease in length to the original length is known as ***compressive strain.***

Shock absorber of a motorcycle absorbs stresses.

Note : This picture is given as additional information and is not a direct example of the current chapter.

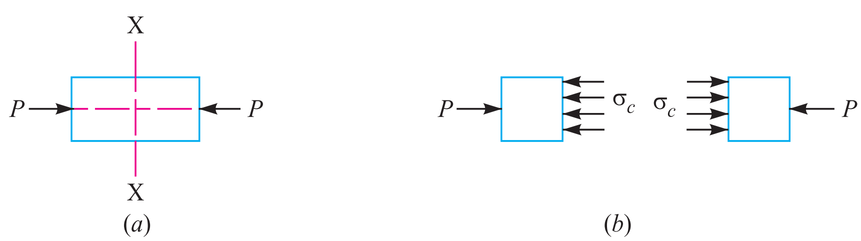


Fig. 4.2. Compressive stress and strain.

Let P = Axial compressive force acting on the body,

A = Cross-sectional area of the body,

l = Original length, and

δl = Decrease in length.

∴ Compressive stress, $\sigma_c = P/A$

and compressive strain, $\varepsilon_c = \delta l / l$

Note : In case of tension or compression, the area involved is at right angles to the external force applied.

4.7 Young's Modulus or Modulus of Elasticity

Hooke's law* states that when a material is loaded within elastic limit, the stress is directly proportional to strain, *i.e.*

$$\sigma \propto \varepsilon \qquad \text{or} \qquad \sigma = E.\varepsilon$$

$$\therefore \qquad E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$$

* It is named after Robert Hooke, who first established it by experiments in 1678.

where E is a constant of proportionality known as ***Young's modulus*** or ***modulus of elasticity.*** In S.I. units, it is usually expressed in GPa *i.e.* GN/m^2 or kN/mm^2. It may be noted that Hooke's law holds good for tension as well as compression.

The following table shows the values of modulus of elasticity or Young's modulus (E) for the materials commonly used in engineering practice.

Table 4.1. Values of E for the commonly used engineering materials.

Material	*Modulus of elasticity (E) in GPa i.e. GN/m^2 or kN/mm^2*
Steel and Nickel	200 to 220
Wrought iron	190 to 200
Cast iron	100 to 160
Copper	90 to 110
Brass	80 to 90
Aluminium	60 to 80
Timber	10

Example 4.1. *A coil chain of a crane required to carry a maximum load of 50 kN, is shown in Fig. 4.3.*

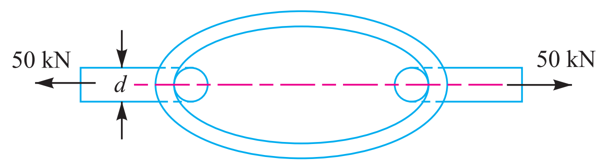


Fig. 4.3

Find the diameter of the link stock, if the permissible tensile stress in the link material is not to exceed 75 MPa.

Solution. Given : $P = 50$ kN $= 50 \times 10^3$ N ; $\sigma_t = 75$ MPa $= 75$ N/mm^2

Let $\quad d$ = Diameter of the link stock in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$$

We know that the maximum load (P),

$$50 \times 10^3 = \sigma_t \,.\, A = 75 \times 0.7854\, d^2 = 58.9\, d^2$$

$$\therefore \quad d^2 = 50 \times 10^3 / 58.9 = 850 \quad \text{or} \quad d = 29.13 \text{ say } 30 \text{ mm}$$ **Ans.**

Example 4.2. *A cast iron link, as shown in Fig. 4.4, is required to transmit a steady tensile load of 45 kN. Find the tensile stress induced in the link material at sections A-A and B-B.*

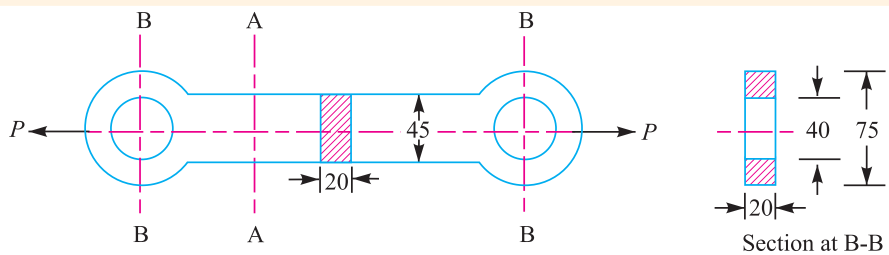


Fig. 4.4. All dimensions in mm.

Solution. Given : $P = 45$ kN $= 45 \times 10^3$ N

Tensile stress induced at section A-A

We know that the cross-sectional area of link at section *A-A*,

$$A_1 = 45 \times 20 = 900 \text{ mm}^2$$

∴ Tensile stress induced at section *A-A*,

$$\sigma_{t1} = \frac{P}{A_1} = \frac{45 \times 10^3}{900} = 50 \text{ N/mm}^2 = 50 \text{ MPa} \quad \textbf{Ans.}$$

Tensile stress induced at section B-B

We know that the cross-sectional area of link at section B-B,

$$A_2 = 20\,(75 - 40) = 700 \text{ mm}^2$$

∴ Tensile stress induced at section *B-B*,

$$\sigma_{t2} = \frac{P}{A_2} = \frac{45 \times 10^3}{700} = 64.3 \text{ N/mm}^2 = 64.3 \text{ MPa} \quad \textbf{Ans.}$$

Example 4.3. *A hydraulic press exerts a total load of 3.5 MN. This load is carried by two steel rods, supporting the upper head of the press. If the safe stress is 85 MPa and E = 210 kN/mm², find : 1. diameter of the rods, and 2. extension in each rod in a length of 2.5 m.*

Solution. Given : $P = 3.5$ MN $= 3.5 \times 10^6$ N ; $\sigma_t = 85$ MPa $= 85$ N/mm² ; $E = 210$ kN/mm² $= 210 \times 10^3$ N/mm² ; $l = 2.5$ m $= 2.5 \times 10^3$ mm

1. Diameter of the rods

Let d = Diameter of the rods in mm.

∴ Area, $A = \dfrac{\pi}{4} \times d^2 = 0.7854\, d^2$

Since the load P is carried by two rods, therefore load carried by each rod,

$$P_1 = \frac{P}{2} = \frac{3.5 \times 10^6}{2} = 1.75 \times 10^6 \text{ N}$$

We know that load carried by each rod (P_1),

$$1.75 \times 10^6 = \sigma_t \,.\, A = 85 \times 0.7854\, d^2 = 66.76\, d^2$$

$$\therefore \quad d^2 = 1.75 \times 10^6 / 66.76 = 26\,213 \quad \text{or} \quad d = 162 \text{ mm} \quad \textbf{Ans.}$$

2. Extension in each rod

Let δl = Extension in each rod.

We know that Young's modulus (E),

$$210 \times 10^3 = \frac{P_1 \times l}{A \times \delta l} = \frac{\sigma_t \times l}{\delta l} = \frac{85 \times 2.5 \times 10^3}{\delta l} = \frac{212.5 \times 10^3}{\delta l} \quad \ldots \left(\because \frac{P_1}{A} = \sigma_t \right)$$

$$\therefore \quad \delta l = 212.5 \times 10^3 / (210 \times 10^3) = 1.012 \text{ mm} \quad \textbf{Ans.}$$

Example 4.4. *A rectangular base plate is fixed at each of its four corners by a 20 mm diameter bolt and nut as shown in Fig. 4.5. The plate rests on washers of 22 mm internal diameter and 50 mm external diameter. Copper washers which are placed between the nut and the plate are of 22 mm internal diameter and 44 mm external diameter.*

If the base plate carries a load of 120 kN (including self-weight, which is equally distributed on the four corners), calculate the stress on the lower washers before the nuts are tightened.

What could be the stress in the upper and lower washers, when the nuts are tightened so as to produce a tension of 5 kN on each bolt?

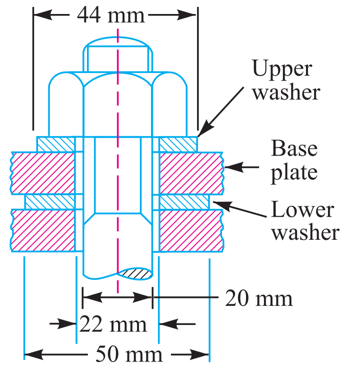


Fig. 4.5

Solution. Given : d = 20 mm ; d_1 = 22 mm ; d_2 = 50 mm ; d_3 = 22 mm ; d_4 = 44 mm ; P_1 = 120 kN ; P_2 = 5 kN

Stress on the lower washers before the nuts are tightened

We know that area of lower washers,

$$A_1 = \frac{\pi}{4}\left[(d_2)^2 - (d_1)^2\right] = \frac{\pi}{4}\left[(50)^2 - (22)^2\right] = 1583 \text{ mm}^2$$

and area of upper washers,

$$A_2 = \frac{\pi}{4}\left[(d_4)^2 - (d_3)^2\right] = \frac{\pi}{4}\left[(44)^2 - (22)^2\right] = 1140 \text{ mm}^2$$

Since the load of 120 kN on the four washers is equally distributed, therefore load on each lower washer before the nuts are tightened,

$$P_1 = \frac{120}{4} = 30 \text{ kN} = 30\ 000 \text{ N}$$

We know that stress on the lower washers before the nuts are tightened,

$$\sigma_{c1} = \frac{P_1}{A_1} = \frac{30\ 000}{1583} = 18.95 \text{ N/mm}^2 = 18.95 \text{ MPa} \quad \textbf{Ans.}$$

Stress on the upper washers when the nuts are tightened

Tension on each bolt when the nut is tightened,

$$P_2 = 5 \text{ kN} = 5000 \text{ N}$$

∴ Stress on the upper washers when the nut is tightened,

$$\sigma_{c2} = \frac{P_2}{A_2} = \frac{5000}{1140} = 4.38 \text{ N/mm}^2 = 4.38 \text{ MPa} \quad \textbf{Ans.}$$

Stress on the lower washers when the nuts are tightened

We know that the stress on the lower washers when the nuts are tightened,

$$\sigma_{c3} = \frac{P_1 + P_2}{A_1} = \frac{30\ 000 + 5000}{1583} = 22.11 \text{ N/mm}^2 = 22.11 \text{ MPa} \quad \textbf{Ans.}$$

Example 4.5. *The piston rod of a steam engine is 50 mm in diameter and 600 mm long. The diameter of the piston is 400 mm and the maximum steam pressure is 0.9 N/mm². Find the compression of the piston rod if the Young's modulus for the material of the piston rod is 210 kN/mm².*

Solution. Given : d = 50 mm ; l = 600 mm ; D = 400 mm ; p = 0.9 N/mm² ; E = 210 kN/mm² = 210 × 10³ N/mm²

Let δl = Compression of the piston rod.

We know that cross-sectional area of piston,

$$= \frac{\pi}{4} \times D^2 = \frac{\pi}{4}(400)^2 = 125\ 680 \text{ mm}^2$$

∴ Maximum load acting on the piston due to steam,

$$P = \text{Cross-sectional area of piston} \times \text{Steam pressure}$$

$$= 125\ 680 \times 0.9 = 113\ 110 \text{ N}$$

Contents

We also know that cross-sectional area of piston rod,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}\,(50)^2$$

$$= 1964 \text{ mm}^2$$

and Young's modulus (E),

$$210 \times 10^3 = \frac{P \times l}{A \times \delta l}$$

$$= \frac{113\,110 \times 600}{1964 \times \delta l} = \frac{34\,555}{\delta l}$$

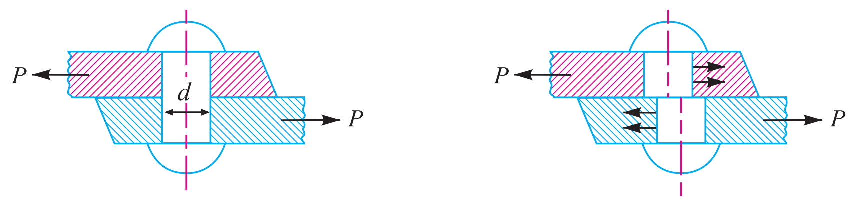
$$\therefore \quad \delta l = 34\,555 / (210 \times 10^3)$$

$$= 0.165 \text{ mm}$$ **Ans.**

This picture shows a jet engine being tested for bearing high stresses.

4.8 Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called ***shear stress.***



(*a*) (*b*)

Fig. 4.6. Single shearing of a riveted joint.

The corresponding strain is known as ***shear strain*** and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (τ) and phi (ϕ) respectively. Mathematically,

$$\text{Shear stress, } \tau = \frac{\text{Tangential force}}{\text{Resisting area}}$$

Consider a body consisting of two plates connected by a rivet as shown in Fig. 4.6 (*a*). In this case, the tangential force P tends to shear off the rivet at one cross-section as shown in Fig. 4.6 (*b*). It may be noted that when the tangential force is resisted by one cross-section of the rivet (or when shearing takes place at one cross-section of the rivet), then the rivets are said to be in ***single shear.*** In such a case, the area resisting the shear off the rivet,

$$A = \frac{\pi}{4} \times d^2$$

and shear stress on the rivet cross-section,

$$\tau = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2} = \frac{4P}{\pi\, d^2}$$

Now let us consider two plates connected by the two cover plates as shown in Fig. 4.7 (*a*). In this case, the tangential force P tends to shear off the rivet at two cross-sections as shown in Fig. 4.7 (*b*). It may be noted that when the tangential force is resisted by two cross-sections of the rivet (or

Top

when the shearing takes place at two cross-sections of the rivet), then the rivets are said to be in ***double shear.*** In such a case, the area resisting the shear off the rivet,

$$A = 2 \times \frac{\pi}{4} \times d^2 \qquad \text{... (For double shear)}$$

and shear stress on the rivet cross-section,

$$\tau = \frac{P}{A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2} = \frac{2P}{\pi d^2}$$

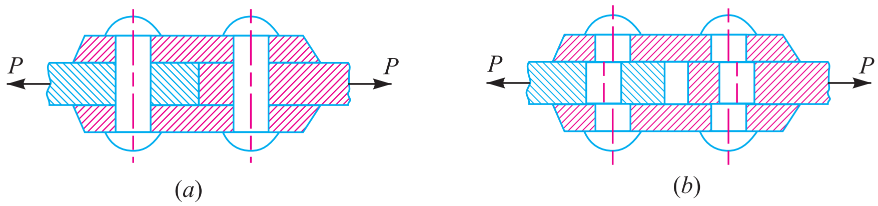


Fig. 4.7. Double shearing of a riveted joint.

Notes : 1. All lap joints and single cover butt joints are in single shear, while the butt joints with double cover plates are in double shear.

2. In case of shear, the area involved is parallel to the external force applied.

3. When the holes are to be punched or drilled in the metal plates, then the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. If a hole of diameter '*d*' is to be punched in a metal plate of thickness '*t*', then the area to be sheared,

$$A = \pi\, d \times t$$

and the maximum shear resistance of the tool or the force required to punch a hole,

$$P = A \times \tau_u = \pi\, d \times t \times \tau_u$$

where τ_u = Ultimate shear strength of the material of the plate.

4.9 Shear Modulus or Modulus of Rigidity

It has been found experimentally that within the elastic limit, the shear stress is directly proportional to shear strain. Mathematically

$$\tau \propto \phi \qquad \text{or} \qquad \tau = C \,.\, \phi \qquad \text{or} \qquad \tau / \phi = C$$

where τ = Shear stress,

ϕ = Shear strain, and

C = Constant of proportionality, known as shear modulus or modulus of rigidity. It is also denoted by *N* or *G*.

The following table shows the values of modulus of rigidity (*C*) for the materials in every day use:

Table 4.2. Values of *C* for the commonly used materials.

Material	*Modulus of rigidity (C) in GPa i.e. GN/m² or kN/mm²*
Steel	80 to 100
Wrought iron	80 to 90
Cast iron	40 to 50
Copper	30 to 50
Brass	30 to 50
Timber	10

Example 4.6. *Calculate the force required to punch a circular blank of 60 mm diameter in a plate of 5 mm thick. The ultimate shear stress of the plate is 350 N/mm².*

Solution. Given: $d = 60$ mm ; $t = 5$ mm ; $\tau_u = 350$ N/mm²

We know that area under shear,

$$A = \pi\, d \times \tau = \pi \times 60 \times 5 = 942.6 \text{ mm}^2$$

and force required to punch a hole,

$$P = A \times \tau_u = 942.6 \times 350 = 329\,910 \text{ N} = 329.91 \text{ kN} \quad \textbf{Ans.}$$

Example 4.7. *A pull of 80 kN is transmitted from a bar X to the bar Y through a pin as shown in Fig. 4.8.*

If the maximum permissible tensile stress in the bars is 100 N/mm² and the permissible shear stress in the pin is 80 N/mm², find the diameter of bars and of the pin.

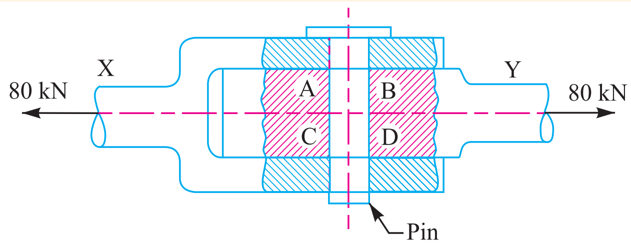


Fig. 4.8

Solution. Given : $P = 80$ kN $= 80 \times 10^3$ N; $\sigma_t = 100$ N/mm² ; $\tau = 80$ N/mm²

High force injection moulding machine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Diameter of the bars

Let D_b = Diameter of the bars in mm.

$\therefore$ Area, $A_b = \frac{\pi}{4}(D_b)^2 = 0.7854\,(D_b)^2$

We know that permissible tensile stress in the bar (σ_t),

$$100 = \frac{P}{A_b} = \frac{80 \times 10^3}{0.7854\,(D_b)^2} = \frac{101\,846}{(D_b)^2}$$

$\therefore$ $(D_b)^2 = 101\,846 / 100 = 1018.46$

or $D_b = 32$ mm **Ans.**

Diameter of the pin

Let D_p = Diameter of the pin in mm.

Since the tensile load P tends to shear off the pin at two sections *i.e.* at AB and CD, therefore the pin is in double shear.

$\therefore$ Resisting area,

$$A_p = 2 \times \frac{\pi}{4}(D_p)^2 = 1.571\,(D_p)^2$$

We know that permissible shear stress in the pin (τ),

$$80 = \frac{P}{A_p} = \frac{80 \times 10^3}{1.571\,(D_p)^2} = \frac{50.9 \times 10^3}{(D_p)^2}$$

$\therefore$ $(D_p)^2 = 50.9 \times 10^3/80 = 636.5$ or $D_p = 25.2$ mm **Ans.**

Contents

4.10 Bearing Stress

A localised compressive stress at the surface of contact between two members of a machine part, that are relatively at rest is known as ***bearing stress*** or ***crushing stress*.** The bearing stress is taken into account in the design of riveted joints, cotter joints, knuckle joints, etc. Let us consider a riveted joint subjected to a load P as shown in Fig. 4.9. In such a case, the bearing stress or crushing stress (stress at the surface of contact between the rivet and a plate),

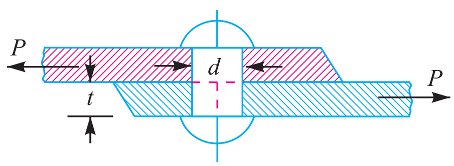
$$\sigma_b \text{ (or } \sigma_c) = \frac{P}{d.t.n}$$

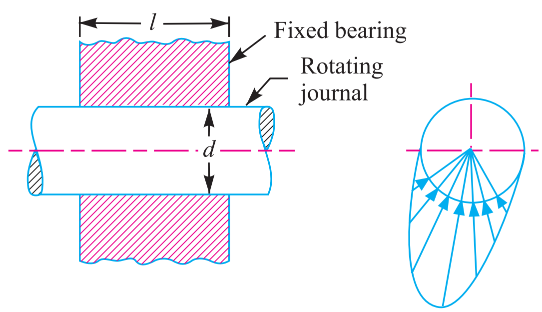
where d = Diameter of the rivet,

t = Thickness of the plate,

$d.t$ = Projected area of the rivet, and

n = Number of rivets per pitch length in bearing or crushing.





(*a*) Journal supported in a bearing.

(*b*) Distribution of bearing pressure.

Fig. 4.9. Bearing stress in a riveted joint.

Fig. 4.10. Bearing pressure in a journal supported in a bearing.

It may be noted that the local compression which exists at the surface of contact between two members of a machine part that are in relative motion, is called ***bearing pressure*** (not the bearing stress). This term is commonly used in the design of a journal supported in a bearing, pins for levers, crank pins, clutch lining, etc. Let us consider a journal rotating in a fixed bearing as shown in Fig. 4.10 (*a*). The journal exerts a bearing pressure on the curved surfaces of the brasses immediately below it. The distribution of this bearing pressure will not be uniform, but it will be in accordance with the shape of the surfaces in contact and deformation characteristics of the two materials. The distribution of bearing pressure will be similar to that as shown in Fig. 4.10 (*b*). Since the actual bearing pressure is difficult to determine, therefore the average bearing pressure is usually calculated by dividing the load to the projected area of the curved surfaces in contact. Thus, the average bearing pressure for a journal supported in a bearing is given by

$$p_b = \frac{P}{l.d}$$

where p_b = Average bearing pressure,

P = Radial load on the journal,

l = Length of the journal in contact, and

d = Diameter of the journal.

Top

Example 4.8. *Two plates 16 mm thick are joined by a double riveted lap joint as shown in Fig. 4.11. The rivets are 25 mm in diameter.*

Find the crushing stress induced between the plates and the rivet, if the maximum tensile load on the joint is 48 kN.

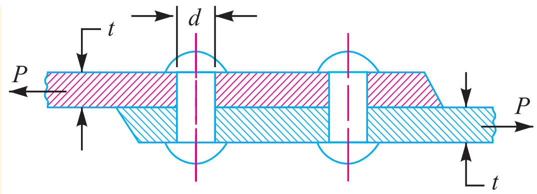


Fig. 4.11

Solution. Given : $t = 16$ mm ; $d = 25$ mm ; $P = 48$ kN $= 48 \times 10^3$ N

Since the joint is double riveted, therefore, strength of two rivets in bearing (or crushing) is taken. We know that crushing stress induced between the plates and the rivets,

$$\sigma_c = \frac{P}{d.t.n} = \frac{48 \times 10^3}{25 \times 16 \times 2} = 60 \text{ N/mm}^2 \textbf{ Ans.}$$

Example 4.9. *A journal 25 mm in diameter supported in sliding bearings has a maximum end reaction of 2500 N. Assuming an allowable bearing pressure of 5 N/mm², find the length of the sliding bearing.*

Solution. Given : $d = 25$ mm ; $P = 2500$ N ; $p_b = 5$ N/mm²

Let l = Length of the sliding bearing in mm.

We know that the projected area of the bearing,

$$A = l \times d = l \times 25 = 25\, l \text{ mm}^2$$

$\therefore$ Bearing pressure (p_b),

$$5 = \frac{P}{A} = \frac{2500}{25\, l} = \frac{100}{l} \quad \text{or} \quad l = \frac{100}{5} = 20 \text{ mm } \textbf{Ans.}$$

4.11 Stress-strain Diagram

In designing various parts of a machine, it is necessary to know how the material will function in service. For this, certain characteristics or properties of the material should be known. The mechanical properties mostly used in mechanical engineering practice are commonly determined from a standard tensile test. This test consists of gradually loading a standard specimen of a material and noting the corresponding values of load and elongation until the specimen fractures. The load is applied and measured by a testing machine. The stress is determined by dividing the load values by the original cross-sectional area of the specimen. The elongation is measured by determining the amounts that two reference points on the specimen are moved apart by the action of the machine. The original distance between the two reference points is known as ***gauge length***. The strain is determined by dividing the elongation values by the gauge length.

In addition to bearing the stresses, some machine parts are made of stainless steel to make them corrosion resistant.

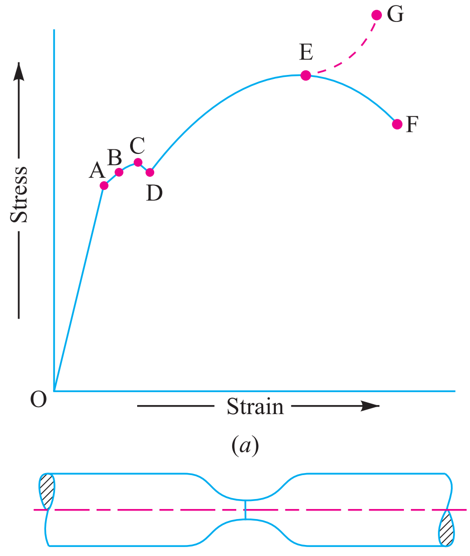
Note : This picture is given as additional information and is not a direct example of the current chapter.

The values of the stress and corresponding strain are used to draw the stress-strain diagram of the material tested. A stress-strain diagram for a mild steel under tensile test is shown in Fig. 4.12 (*a*). The various properties of the material are discussed below :

1. *Proportional limit.* We see from the diagram that from point *O* to *A* is a straight line, which represents that the stress is proportional to strain. Beyond point *A*, the curve slightly deviates from the straight line. It is thus obvious, that Hooke's law holds good up to point *A* and it is known as ***proportional limit***. It is defined as that stress at which the stress-strain curve begins to deviate from the straight line.

2. *Elastic limit.* It may be noted that even if the load is increased beyond point *A* upto the point *B*, the material will regain its shape and size when the load is removed. This means that the material has elastic properties up to the point *B*. This point is known as ***elastic limit.*** It is defined as the stress developed in the material without any permanent set.

Note: Since the above two limits are very close to each other, therefore, for all practical purposes these are taken to be equal.



(*b*) Shape of specimen after elongation.

Fig. 4.12. Stress-strain diagram for a mild steel.

3. *Yield point.* If the material is stressed beyond point *B*, the plastic stage will reach *i.e.* on the removal of the load, the material will not be able to recover its original size and shape. A little consideration will show that beyond point *B*, the strain increases at a faster rate with any increase in the stress until the point *C* is reached. At this point, the material yields before the load and there is an appreciable strain without any increase in stress. In case of mild steel, it will be seen that a small load drops to *D*, immediately after yielding commences. Hence there are two yield points *C* and *D*. The points *C* and *D* are called the ***upper*** and ***lower yield points*** respectively. The stress corresponding to yield point is known as ***yield point stress.***

4. *Ultimate stress.* At *D*, the specimen regains some strength and higher values of stresses are required for higher strains, than those between *A* and *D*. The stress (or load) goes on increasing till the

A crane used on a ship.

Note : This picture is given as additional information and is not a direct example of the current chapter.

point E is reached. The gradual increase in the strain (or length) of the specimen is followed with the uniform reduction of its cross-sectional area. The work done, during stretching the specimen, is transformed largely into heat and the specimen becomes hot. At E, the stress, which attains its maximum value is known as ***ultimate stress***. It is defined as the largest stress obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

5. ***Breaking stress.*** After the specimen has reached the ultimate stress, a neck is formed, which decreases the cross-sectional area of the specimen, as shown in Fig. 4.12 (*b*). A little consideration will show that the stress (or load) necessary to break away the specimen, is less than the maximum stress. The stress is, therefore, reduced until the specimen breaks away at point F. The stress corresponding to point F is known as ***breaking stress.***

A recovery truck with crane.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Note : The breaking stress (*i.e.* stress at F which is less than at E) appears to be somewhat misleading. As the formation of a neck takes place at E which reduces the cross-sectional area, it causes the specimen suddenly to fail at F. If for each value of the strain between E and F, the tensile load is divided by the reduced cross-sectional area at the narrowest part of the neck, then the true stress-strain curve will follow the dotted line EG. However, it is an established practice, to calculate strains on the basis of original cross-sectional area of the specimen.

6. ***Percentage reduction in area.*** It is the difference between the original cross-sectional area and cross-sectional area at the neck (*i.e.* where the fracture takes place). This difference is expressed as percentage of the original cross-sectional area.

Let A = Original cross-sectional area, and

a = Cross-sectional area at the neck.

Then reduction in area $= A - a$

and percentage reduction in area $= \dfrac{A - a}{A} \times 100$

7. ***Percentage elongation.*** It is the percentage increase in the standard gauge length (*i.e.* original length) obtained by measuring the fractured specimen after bringing the broken parts together.

Let l = Gauge length or original length, and

L = Length of specimen after fracture or final length.

$\therefore$ Elongation $= L - l$

and percentage elongation $= \dfrac{L - l}{l} \times 100$

Note : The percentage elongation gives a measure of ductility of the metal under test. The amount of local extensions depends upon the material and also on the transverse dimensions of the test piece. Since the specimens are to be made from bars, strips, sheets, wires, forgings, castings, etc., therefore it is not possible to make all specimens of one standard size. Since the dimensions of the specimen influence the result, therefore some standard means of comparison of results are necessary.

As a result of series of experiments, Barba estabilished a law that in tension, similar test pieces deform similarly and two test pieces are said to be similar if they have the same value of $\frac{l}{\sqrt{A}}$, where l is the gauge length and A is the cross-sectional area. A little consideration will show that the same material will give the same percentage elongation and percentage reduction in area.

It has been found experimentally by Unwin that the general extension (up to the maximum load) is proportional to the gauge length of the test piece and that the local extension (from maximum load to the breaking load) is proportional to the square root of the cross-sectional area. According to Unwin's formula, the increase in length,

$$\delta l = b.l + C\sqrt{A}$$

and percentage elongation

$$= \frac{\delta l}{l} \times 100$$

where l = Gauge length,

A = Cross-sectional area, and

b and C = Constants depending upon the quality of the material.

The values of b and C are determined by finding the values of δl for two test pieces of known length (l) and area (A).

Example 4.10. *A mild steel rod of 12 mm diameter was tested for tensile strength with the gauge length of 60 mm. Following observations were recorded :*

Final length = 80 mm; Final diameter = 7 mm; Yield load = 3.4 kN and Ultimate load = 6.1 kN.

Calculate : 1. yield stress, 2. ultimate tensile stress, 3. percentage reduction in area, and 4. percentage elongation.

Solution. Given : D = 12 mm ; l = 60 mm ; L = 80 mm ; d = 7 mm ; W_y = 3.4 kN = 3400 N; W_u = 6.1 kN = 6100 N

We know that original area of the rod,

$$A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4}(12)^2 = 113 \text{ mm}^2$$

and final area of the rod,

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(7)^2 = 38.5 \text{ mm}^2$$

1. *Yield stress*

We know that yield stress

$$= \frac{W_y}{A} = \frac{3400}{113} = 30.1 \text{ N/mm}^2 = 30.1 \text{ MPa} \quad \textbf{Ans.}$$

2. *Ultimate tensile stress*

We know the ultimate tensile stress

$$= \frac{W_u}{A} = \frac{6100}{113} = 54 \text{ N/mm}^2 = 54 \text{ MPa} \quad \textbf{Ans.}$$

3. *Percentage reduction in area*

We know that percentage reduction in area

$$= \frac{A - a}{A} = \frac{113 - 38.5}{113} = 0.66 \text{ or } 66\% \quad \textbf{Ans.}$$

4. Percentage elongation

We know that percentage elongation

$$= \frac{L - l}{L} = \frac{80 - 60}{80} = 0.25 \text{ or } 25\% \textbf{ Ans.}$$

4.12 Working Stress

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the ***working stress*** or ***design stress.*** It is also known as ***safe*** or ***allowable stress.***

Note : By failure it is not meant actual breaking of the material. Some machine parts are said to fail when they have plastic deformation set in them, and they no more perform their function satisfactory.

4.13 Factor of Safety

It is defined, in general, as the **ratio of the maximum stress to the working stress.** Mathematically,

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety} = \frac{\text{Yield point stress}}{\text{Working or design stress}}$$

In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\therefore \qquad \text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working or design stress}}$$

This relation may also be used for ductile materials.

Note: The above relations for factor of safety are for static loading.

4.14 Selection of Factor of Safety

The selection of a proper factor of safety to be used in designing any machine component depends upon a number of considerations, such as the material, mode of manufacture, type of stress, general service conditions and shape of the parts. Before selecting a proper factor of safety, a design engineer should consider the following points :

1. The reliability of the properties of the material and change of these properties during service ;
2. The reliability of test results and accuracy of application of these results to actual machine parts ;
3. The reliability of applied load ;
4. The certainty as to exact mode of failure ;
5. The extent of simplifying assumptions ;
6. The extent of localised stresses ;
7. The extent of initial stresses set up during manufacture ;
8. The extent of loss of life if failure occurs ; and
9. The extent of loss of property if failure occurs.

Each of the above factors must be carefully considered and evaluated. The high factor of safety results in unnecessary risk of failure. The values of factor of safety based on ultimate strength for different materials and type of load are given in the following table:

Contents

Table 4.3. Values of factor of safety.

Material	*Steady load*	*Live load*	*Shock load*
Cast iron	5 to 6	8 to 12	16 to 20
Wrought iron	4	7	10 to 15
Steel	4	8	12 to 16
Soft materials and alloys	6	9	15
Leather	9	12	15
Timber	7	10 to 15	20

4.15 Stresses in Composite Bars

A composite bar may be defined as a bar made up of two or more different materials, joined together, in such a manner that the system extends or contracts as one unit, equally, when subjected to tension or compression. In case of composite bars, the following points should be kept in view:

1. The extension or contraction of the bar being equal, the strain *i.e.* deformation per unit length is also equal.
2. The total external load on the bar is equal to the sum of the loads carried by different materials.

Consider a composite bar made up of two different materials as shown in Fig. 4.13.

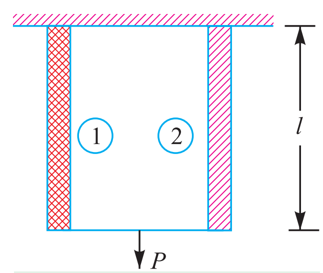


Fig. 4.13. Stresses in composite bars.

Let P_1 = Load carried by bar 1,

A_1 = Cross-sectional area of bar 1,

σ_1 = Stress produced in bar 1,

E_1 = Young's modulus of bar 1,

P_2, A_2, σ_2, E_2 = Corresponding values of bar 2,

P = Total load on the composite bar,

l = Length of the composite bar, and

δl = Elongation of the composite bar.

We know that $$P = P_1 + P_2 \qquad \text{...}(i)$$

Stress in bar 1, $$\sigma_1 = \frac{P_1}{A_1}$$

and strain in bar 1, $$\varepsilon = \frac{\sigma_1}{E_1} = \frac{P_1}{A_1.E_1}$$

∴ Elongation of bar 1,

$$\delta l_1 = \frac{P_1.l}{A_1.E_1}$$

Similarly, elongation of bar 2,

$$\delta l_2 = \frac{P_2.l}{A_2.E_2}$$

Since $$\delta l_1 = \delta l_2$$

A Material handling system

Note : This picture is given as additional information and is not a direct example of the current chapter.

Top

Contents

Therefore, $\dfrac{P_1 . l}{A_1 . E_1} = \dfrac{P_2 . l}{A_2 . E_2}$ or $P_1 = P_2 \times \dfrac{A_1 . E_1}{A_2 . E_2}$ ...(*ii*)

But $$P = P_1 + P_2 = P_2 \times \frac{A_1 . E_1}{A_2 . E_2} + P_2 = P_2 \left(\frac{A_1 . E_1}{A_2 . E_2} + 1 \right)$$

$$= P_2 \left(\frac{A_1 . E_1 + A_2 . E_2}{A_2 . E_2} \right)$$

or $$P_2 = P \times \frac{A_2 . E_2}{A_1 . E_1 + A_2 . E_2} \quad ...(iii)$$

Similarly $$P_1 = P \times \frac{A_1 . E_1}{A_1 . E_1 + A_2 . E_2} \quad ...[\text{From equation } (ii)] \quad ...(iv)$$

We know that

$$\frac{P_1 . l}{A_1 . E_1} = \frac{P_2 . l}{A_2 . E_2}$$

$\therefore$ $$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

or $$\sigma_1 = \frac{E_1}{E_2} \times \sigma_2 \quad ...(v)$$

Similarly, $$\sigma_2 = \frac{E_2}{E_1} \times \sigma_1 \quad ...(vi)$$

From the above equations, we can find out the stresses produced in the different bars. We also know that

$$P = P_1 + P_2 = \sigma_1 . A_1 + \sigma_2 . A_2$$

From this equation, we can also find out the stresses produced in different bars.

Note : The ratio E_1 / E_2 is known as ***modular ratio*** of the two materials.

Example 4.11. *A bar 3 m long is made of two bars, one of copper having E = 105 GN/m² and the other of steel having E = 210 GN/m². Each bar is 25 mm broad and 12.5 mm thick. This compound bar is stretched by a load of 50 kN. Find the increase in length of the compound bar and the stress produced in the steel and copper. The length of copper as well as of steel bar is 3 m each.*

Solution. Given : $l_c = l_s = 3$ m $= 3 \times 10^3$ mm ; $E_c = 105$ GN/m² $= 105$ kN/mm² ; $E_s = 210$ GN/m² $= 210$ kN/mm² ; $b = 25$ mm ; $t = 12.5$ mm ; $P = 50$ kN

Increase in length of the compound bar

Let δl = Increase in length of the compound bar.

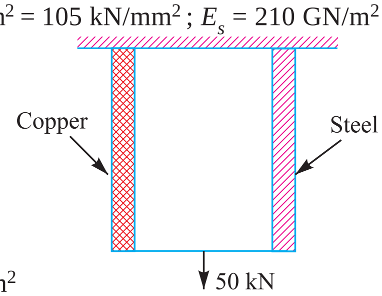


Fig. 4.14

The compound bar is shown in Fig. 4.14. We know that cross-sectional area of each bar,

$$A_c = A_s = b \times t = 25 \times 12.5 = 312.5 \text{ mm}^2$$

Top

∴ Load shared by the copper bar,

$$P_c = P \times \frac{A_c . E_c}{A_c . E_c + A_s . E_s} = P \times \frac{E_c}{E_c + E_s} \qquad \ldots (\because A_c = A_s)$$

$$= 50 \times \frac{105}{105 + 210} = 16.67 \text{ kN}$$

and load shared by the steel bar,

$$P_s = P - P_c = 50 - 16.67 = 33.33 \text{ kN}$$

Since the elongation of both the bars is equal, therefore

$$\delta l = \frac{P_c . l_c}{A_c . E_c} = \frac{P_s . l_s}{A_s . E_s} = \frac{16.67 \times 3 \times 10^3}{312.5 \times 105} = 1.52 \text{ mm} \quad \textbf{Ans.}$$

Stress produced in the steel and copper bar

We know that stress produced in the steel bar,

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{210}{105} \times \sigma_c = 2\,\sigma_c$$

and total load, $P = P_s + P_c = \sigma_s . A_s + \sigma_c . A_c$

∴ $50 = 2\,\sigma_c \times 312.5 + \sigma_c \times 312.5 = 937.5\,\sigma_c$

or $\sigma_c = 50 / 937.5 = 0.053 \text{ kN/mm}^2 = 53 \text{ N/mm}^2 = 53 \text{ MPa}$ **Ans.**

and $\sigma_s = 2\,\sigma_c = 2 \times 53 = 106 \text{ N/mm}^2 = 106 \text{ MPa}$ **Ans.**

Example 4.12. *A central steel rod 18 mm diameter passes through a copper tube 24 mm inside and 40 mm outside diameter, as shown in Fig. 4.15. It is provided with nuts and washers at each end. The nuts are tightened until a stress of 10 MPa is set up in the steel.*

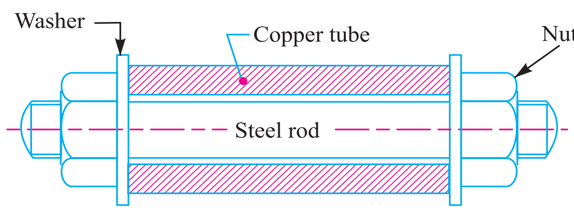


Fig. 4.15

The whole assembly is then placed in a lathe and turned along half the length of the tube removing the copper to a depth of 1.5 mm. Calculate the stress now existing in the steel. Take $E_s = 2E_c$.

Solution. Given : $d_s = 18$ mm ; $d_{c1} = 24$ mm ; $d_{c2} = 40$ mm ; $\sigma_s = 10$ MPa = 10 N/mm²

We know that cross-sectional area of steel rod,

$$A_s = \frac{\pi}{4}(d_s)^2 = \frac{\pi}{4}(18)^2 = 254.5 \text{ mm}^2$$

and cross-sectional area of copper tube,

$$A_c = \frac{\pi}{4}\left[(d_{c2})^2 - (d_{c1})^2\right] = \frac{\pi}{4}\left[(40)^2 - (24)^2\right] = 804.4 \text{ mm}^2$$

We know that when the nuts are tightened on the tube, the steel rod will be under tension and the copper tube in compression.

Let σ_c = Stress in the copper tube.

Since the tensile load on the steel rod is equal to the compressive load on the copper tube, therefore

$$\sigma_s \times A_s = \sigma_c \times A_c$$

$$10 \times 254.5 = \sigma_c \times 804.4$$

$$\therefore \quad \sigma_c = \frac{10 \times 254.5}{804.4} = 3.16 \text{ N/mm}^2$$

When the copper tube is reduced in the area for half of its length, then outside diameter of copper tube,

$$= 40 - 2 \times 1.5 = 37 \text{ mm}$$

∴ Cross-sectional area of the half length of copper tube,

$$A_{c1} = \frac{\pi}{4}(37^2 - 24^2) = 623 \text{ mm}^2$$

The cross-sectional area of the other half remains same. If A_{c2} be the area of the remainder, then

$$A_{c2} = A_c = 804.4 \text{ mm}^2$$

Let σ_{c1} = Compressive stress in the reduced section,

σ_{c2} = Compressive stress in the remainder, and

σ_{s1} = Stress in the rod after turning.

Since the load on the copper tube is equal to the load on the steel rod, therefore

$$A_{c1} \times \sigma_{c1} = A_{c2} \times \sigma_{c2} = A_s \times \sigma_{s1}$$

$$\therefore \quad \sigma_{c1} = \frac{A_s}{A_{c1}} \times \sigma_{s1} = \frac{254.5}{623} \times \sigma_{s1} = 0.41\,\sigma_{s1} \qquad \text{...}(i)$$

and

$$\sigma_{c2} = \frac{A_s}{A_{c2}} \times \sigma_{s1} = \frac{254.5}{804.4} \times \sigma_{s1} = 0.32\,\sigma_{s1} \qquad \text{...}(ii)$$

Let δl = Change in length of the steel rod before and after turning,

l = Length of the steel rod and copper tube between nuts,

δl_1 = Change in length of the reduced section (*i.e.* $l/2$) before and after turning, and

δl_2 = Change in length of the remainder section (*i.e.* $l/2$) before and after turning.

Since $\delta l = \delta l_1 + \delta l_2$

$$\therefore \quad \frac{\sigma_s - \sigma_{s1}}{E_s} \times l = \frac{\sigma_{c1} - \sigma_c}{E_c} \times \frac{l}{2} + \frac{\sigma_{c2} - \sigma_c}{E_c} \times \frac{l}{2}$$

or

$$\frac{10 - \sigma_{s1}}{2E_c} = \frac{0.41\,\sigma_{s1} - 3.16}{2E_c} + \frac{0.32\,\sigma_{s1} - 3.16}{2E_c} \qquad \text{...(Cancelling } l \text{ throughout)}$$

$$\therefore \quad \sigma_{s1} = 9.43 \text{ N/mm}^2 = 9.43 \text{ MPa}$$ **Ans.**

4.16 Stresses due to Change in Temperature—Thermal Stresses

Whenever there is some increase or decrease in the temperature of a body, it causes the body to expand or contract. A little consideration will show that if the body is allowed to expand or contract freely, with the rise or fall of the temperature, no stresses are induced in the body. But, if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are known as ***thermal stresses.***

Let l = Original length of the body,

t = Rise or fall of temperature, and

α = Coefficient of thermal expansion,

∴ Increase or decrease in length,

$$\delta l = l.\,\alpha.t$$

If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the body,

$$\varepsilon_c = \frac{\delta l}{l} = \frac{l.\alpha.t}{l} = \alpha.t$$

∴ Thermal stress, $\sigma_{th} = \varepsilon_c.E = \alpha.t.E$

Notes : 1. When a body is composed of two or different materials having different coefficient of thermal expansions, then due to the rise in temperature, the material with higher coefficient of thermal expansion will be subjected to compressive stress whereas the material with low coefficient of expansion will be subjected to tensile stress.

2. When a thin tyre is shrunk on to a wheel of diameter D, its internal diameter d is a little less than the wheel diameter. When the tyre is heated, its circumferance $\pi\, d$ will increase to $\pi\, D$. In this condition, it is slipped on to the wheel. When it cools, it wants to return to its original circumference $\pi\, d$, but the wheel if it is assumed to be rigid, prevents it from doing so.

∴ Strain, $\varepsilon = \dfrac{\pi\, D - \pi d}{\pi d} = \dfrac{D - d}{d}$

This strain is known as ***circumferential*** or ***hoop strain.***

∴ Circumferential or hoop stress,

$$\sigma = E.\varepsilon = \frac{E\,(D - d)}{d}$$

Steel tyres of a locomotive.

Example 4.13. *A thin steel tyre is shrunk on to a locomotive wheel of 1.2 m diameter. Find the internal diameter of the tyre if after shrinking on, the hoop stress in the tyre is 100 MPa. Assume E = 200 kN/mm^2. Find also the least temperature to which the tyre must be heated above that of the wheel before it could be slipped on. The coefficient of linear expansion for the tyre is 6.5×10^{-6} per °C.*

Solution. Given : D = 1.2 m = 1200 mm ; σ = 100 MPa = 100 N/mm^2 ; E = 200 kN/mm^2 = 200×10^3 N/mm^2 ; $\alpha = 6.5 \times 10^{-6}$ per °C

Contents

Internal diameter of the tyre

Let d = Internal diameter of the tyre.

We know that hoop stress (σ),

$$100 = \frac{E\,(D-d)}{d} = \frac{200 \times 10^3\,(D-d)}{d}$$

$$\therefore \quad \frac{D-d}{d} = \frac{100}{200 \times 10^3} = \frac{1}{2 \times 10^3} \quad \ldots(i)$$

$$\frac{D}{d} = 1 + \frac{1}{2 \times 10^3} = 1.0005$$

$$\therefore \quad d = \frac{D}{1.0005} = \frac{1200}{1.0005} = 1199.4 \text{ mm} = 1.1994 \text{ m} \quad \textbf{Ans.}$$

Least temperature to which the tyre must be heated

Let t = Least temperature to which the tyre must be heated.

We know that

$$\pi D = \pi d + \pi d\,.\,\alpha.t = \pi d\,(1 + \alpha.t)$$

$$\alpha.t = \frac{\pi D}{\pi d} - 1 = \frac{D-d}{d} = \frac{1}{2 \times 10^3} \quad \ldots\text{[From equation } (i)]$$

$$\therefore \quad t = \frac{1}{\alpha \times 2 \times 10^3} = \frac{1}{6.5 \times 10^{-6} \times 2 \times 10^3} = 77°\text{C} \quad \textbf{Ans.}$$

Example 4.14. *A composite bar made of aluminium and steel is held between the supports as shown in Fig. 4.16. The bars are stress free at a temperature of 37°C. What will be the stress in the two bars when the temperature is 20°C, if (a) the supports are unyielding; and (b) the supports yield and come nearer to each other by 0.10 mm?*

It can be assumed that the change of temperature is uniform all along the length of the bar. Take E_s = 210 GPa ; E_a = 74 GPa ; $\alpha_s = 11.7 \times 10^{-6}$ / °C ; and $\alpha_a = 23.4 \times 10^{-6}$ / °C.

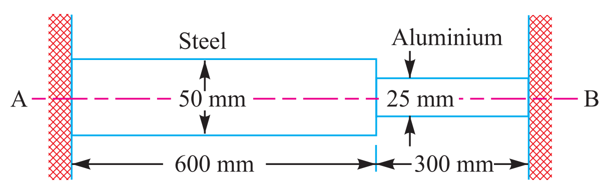


Fig. 4.16

Solution. Given : t_1 = 37°C ; t_2 = 20°C ; E_s = 210 GPa = 210 × 10^9 N/m^2 ; E_a = 74 GPa = 74 × 10^9 N/m^2 ; $\alpha_s = 11.7 \times 10^{-6}$ / °C ; $\alpha_a = 23.4 \times 10^{-6}$ / °C , d_s = 50 mm = 0.05 m ; d_a = 25 mm = 0.025 m ; l_s = 600 mm = 0.6 m ; l_a = 300 mm = 0.3 m

Let us assume that the right support at B is removed and the bar is allowed to contract freely due to the fall in temperature. We know that the fall in temperature,

$$t = t_1 - t_2 = 37 - 20 = 17°\text{C}$$

∴ Contraction in steel bar

$$= \alpha_s\,.\,l_s\,.\,t = 11.7 \times 10^{-6} \times 600 \times 17 = 0.12 \text{ mm}$$

and contraction in aluminium bar

$$= \alpha_a\,.\,l_a\,.\,t = 23.4 \times 10^{-6} \times 300 \times 17 = 0.12 \text{ mm}$$

$$\text{Total contraction} = 0.12 + 0.12 = 0.24 \text{ mm} = 0.24 \times 10^{-3} \text{ m}$$

It may be noted that even after this contraction (*i.e.* 0.24 mm) in length, the bar is still stress free as the right hand end was assumed free.

Top

Let an axial force P is applied to the right end till this end is brought in contact with the right hand support at B, as shown in Fig. 4.17.

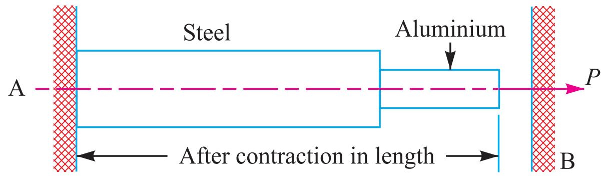


Fig. 4.17

We know that cross-sectional area of the steel bar,

$$A_s = \frac{\pi}{4}(d_s)^2 = \frac{\pi}{4}(0.05)^2 = 1.964 \times 10^{-3}\ \text{m}^2$$

and cross-sectional area of the aluminium bar,

$$A_a = \frac{\pi}{4}(d_a)^2 = \frac{\pi}{4}(0.025)^2 = 0.491 \times 10^{-3}\ \text{m}^2$$

We know that elongation of the steel bar,

$$\delta l_s = \frac{P \times l_s}{A_s \times E_s} = \frac{P \times 0.6}{1.964 \times 10^{-3} \times 210 \times 10^9} = \frac{0.6P}{412.44 \times 10^6}\ \text{m}$$

$$= 1.455 \times 10^{-9}\ P\ \text{m}$$

and elongation of the aluminium bar,

$$\delta l_a = \frac{P \times l_a}{A_a \times E_a} = \frac{P \times 0.3}{0.491 \times 10^{-3} \times 74 \times 10^9} = \frac{0.3P}{36.334 \times 10^6}\ \text{m}$$

$$= 8.257 \times 10^{-9}\ P\ \text{m}$$

∴ Total elongation, $\delta l = \delta l_s + \delta l_a$

$$= 1.455 \times 10^{-9}\ P + 8.257 \times 10^{-9} P = 9.712 \times 10^{-9}\ P\ \text{m}$$

Let σ_s = Stress in the steel bar, and

σ_a = Stress in the aluminium bar.

(a) When the supports are unyielding

When the supports are unyielding, the total contraction is equated to the total elongation,*i.e.*

$$0.24 \times 10^{-3} = 9.712 \times 10^{-9} P \quad \text{or} \quad P = 24\ 712\ \text{N}$$

∴ Stress in the steel bar,

$$\sigma_s = P/A_s = 24\ 712\ /\ (1.964 \times 10^{-3}) = 12\ 582 \times 10^3\ \text{N/m}^2$$

$$= 12.582\ \text{MPa}$$ **Ans.**

and stress in the aluminium bar,

$$\sigma_a = P/A_a = 24\ 712\ /\ (0.491 \times 10^{-3}) = 50\ 328 \times 10^3\ \text{N/m}^2$$

$$= 50.328\ \text{MPa}$$ **Ans.**

(b) When the supports yield by 0.1 mm

When the supports yield and come nearer to each other by 0.10 mm, the net contraction in length

$$= 0.24 - 0.1 = 0.14\ \text{mm} = 0.14 \times 10^{-3}\ \text{m}$$

Equating this net contraction to the total elongation, we have

$$0.14 \times 10^{-3} = 9.712 \times 10^{-9} P \quad \text{or} \quad P = 14\,415 \text{ N}$$

∴ Stress in the steel bar,

$$\sigma_s = P/A_s = 14\,415 / (1.964 \times 10^{-3}) = 7340 \times 10^3 \text{ N/m}^2$$
$$= 7.34 \text{ MPa} \quad \textbf{Ans.}$$

and stress in the aluminium bar,

$$\sigma_a = P/A_a = 14\,415 / (0.491 \times 10^{-3}) = 29\,360 \times 10^3 \text{ N/m}^2$$
$$= 29.36 \text{ MPa} \quad \textbf{Ans.}$$

Example 4.15. *A copper bar 50 mm in diameter is placed within a steel tube 75 mm external diameter and 50 mm internal diameter of exactly the same length. The two pieces are rigidly fixed together by two pins 18 mm in diameter, one at each end passing through the bar and tube. Calculate the stress induced in the copper bar, steel tube and pins if the temperature of the combination is raised by 50°C. Take E_s = 210 GN/m^2 ; E_c = 105 GN/m^2 ; $\alpha_s = 11.5 \times 10^{-6}$/°C and $\alpha_c = 17 \times 10^{-6}$/°C.*

Solution. Given: d_c = 50 mm ; d_{se} = 75 mm ; d_{si} = 50 mm ; d_p = 18 mm = 0.018 m ; t = 50°C; E_s = 210 GN/m^2 = 210 × 10^9 N/m^2 ; E_c = 105 GN/m^2 = 105 × 10^9 N/m^2 ; $\alpha_s = 11.5 \times 10^{-6}$/°C ; $\alpha_c = 17 \times 10^{-6}$/°C

The copper bar in a steel tube is shown in Fig. 4.18.

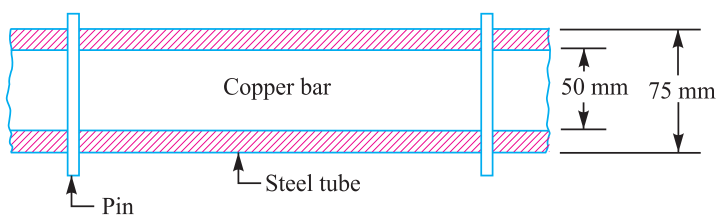


Fig. 4.18

We know that cross-sectional area of the copper bar,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(50)^2 = 1964 \text{ mm}^2 = 1964 \times 10^{-6} \text{ m}^2$$

and cross-sectional area of the steel tube,

$$A_s = \frac{\pi}{4}\left[(d_{se})^2 - (d_{si})^2\right] = \frac{\pi}{4}\left[(75)^2 - (50)^2\right] = 2455 \text{ mm}^2$$
$$= 2455 \times 10^{-6} \text{ m}^2$$

Let l = Length of the copper bar and steel tube.

We know that free expansion of copper bar

$$= \alpha_c \,.\, l \,.\, t = 17 \times 10^{-6} \times l \times 50 = 850 \times 10^{-6}\, l$$

and free expansion of steel tube

$$= \alpha_s \,.\, l \,.\, t = 11.5 \times 10^{-6} \times l \times 50 = 575 \times 10^{-6}\, l$$

∴ Difference in free expansion

$$= 850 \times 10^{-6}\, l - 575 \times 10^{-6}\, l = 275 \times 10^{-6}\, l \qquad \text{...}(i)$$

Since the free expansion of the copper bar is more than the free expansion of the steel tube, therefore the copper bar is subjected to a *compressive stress, while the steel tube is subjected to a tensile stress.

Let a compressive force P newton on the copper bar opposes the extra expansion of the copper bar and an equal tensile force P on the steel tube pulls the steel tube so that the net effect of reduction in length of copper bar and the increase in length of steel tube equalises the difference in free expansion of the two.

Main wheels on the undercarraige of an airliner. Air plane landing gears and wheels need to bear high stresses and shocks.

Note : This picture is given as additional information and is not a direct example of the current chapter.

∴ Reduction in length of copper bar due to force P

$$= \frac{P.l}{A_c.E_c}$$

$$= \frac{P.l}{1964 \times 10^{-6} \times 105 \times 10^{9}} = \frac{P.l}{206.22 \times 10^{6}} \text{ m}$$

and increase in length of steel bar due to force P

$$= \frac{P.l}{A_s.E_s} = \frac{P.l}{2455 \times 10^{-6} \times 210 \times 10^{9}} = \frac{P.l}{515.55 \times 10^{6}} \text{ m}$$

$$\therefore \text{Net effect in length} = \frac{P.l}{206.22 \times 10^{6}} + \frac{P.l}{515.55 \times 10^{6}}$$

$$= 4.85 \times 10^{-9} \, P.l + 1.94 \times 10^{-9} \, P.l = 6.79 \times 10^{-9} \, P.l$$

Equating this net effect in length to the difference in free expansion, we have

$$6.79 \times 10^{-9} \, P.l = 275 \times 10^{-6} \, l \quad \text{or} \quad P = 40\,500 \text{ N}$$

Stress induced in the copper bar, steel tube and pins

We know that stress induced in the copper bar,

$$\sigma_c = P / A_c = 40\,500 / (1964 \times 10^{-6}) = 20.62 \times 10^{6} \text{ N/m}^2 = 20.62 \text{ MPa } \textbf{Ans.}$$

Stress induced in the steel tube,

$$\sigma_s = P / A_s = 40\,500 / (2455 \times 10^{-6}) = 16.5 \times 10^{6} \text{ N / m}^2 = 16.5 \text{ MPa } \textbf{Ans.}$$

* In other words, we can also say that since the coefficient of thermal expansion for copper (α_c) is more than the coefficient of thermal expansion for steel (α_s), therefore the copper bar will be subjected to compressive stress and the steel tube will be subjected to tensile stress.

and shear stress induced in the pins,

$$\tau_p = \frac{P}{2\,A_p} = \frac{40\,500}{2 \times \frac{\pi}{4}(0.018)^2} = 79.57 \times 10^6 \text{ N/m}^2 = 79.57 \text{ MPa} \quad \textbf{Ans.}$$

...($\because$ The pin is in double shear)

4.17 Linear and Lateral Strain

Consider a circular bar of diameter *d* and length *l*, subjected to a tensile force *P* as shown in Fig. 4.19 (*a*).

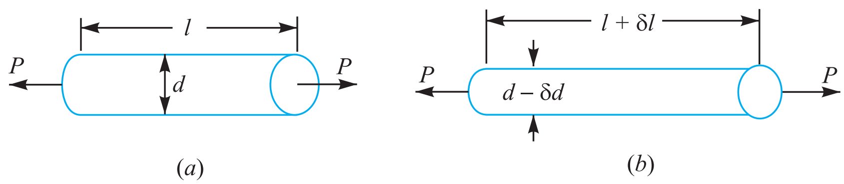


Fig. 4.19. Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δl and the diameter decreases by an amount δd, as shown in Fig. 4.19 (*b*). Similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as ***linear strain*** and an opposite kind of strain in every direction, at right angles to it, is known as ***lateral strain.***

4.18 Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{Lateral strain}}{\text{Linear strain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice.

Table 4.4. Values of Poisson’s ratio for commonly used materials.

S.No.	*Material*	*Poisson’s ratio (1/m or* μ)
1	Steel	0.25 to 0.33
2	Cast iron	0.23 to 0.27
3	Copper	0.31 to 0.34
4	Brass	0.32 to 0.42
5	Aluminium	0.32 to 0.36
6	Concrete	0.08 to 0.18
7	Rubber	0.45 to 0.50

Top

Contents

4.19 Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

where δV = Change in volume, and V = Original volume.

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right); \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where ε_x, ε_y and ε_z are the strains in the directions *x*-axis, *y*-axis and *z*-axis respectively.

4.20 Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by *K*. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$

4.21 Relation Between Bulk Modulus and Young's Modulus

The bulk modulus (*K*) and Young's modulus (*E*) are related by the following relation,

$$K = \frac{m.E}{3\,(m - 2)} = \frac{E}{3\,(1 - 2\mu)}$$

4.22 Relation Between Young's Modulus and Modulus of Rigidity

The Young's modulus (*E*) and modulus of rigidity (*G*) are related by the following relation,

$$G = \frac{m.E}{2\,(m + 1)} = \frac{E}{2\,(1 + \mu)}$$

Example 4.16. *A mild steel rod supports a tensile load of 50 kN. If the stress in the rod is limited to 100 MPa, find the size of the rod when the cross-section is 1. circular, 2. square, and 3. rectangular with width = 3 × thickness.*

Solution. Given : P = 50 kN = 50 × 10^3 N ; σ_t = 100 MPa = 100 N/mm^2

1. Size of the rod when it is circular

Let d = Diameter of the rod in mm.

$\therefore$ Area, $$A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$$

We know that tensile load (*P*),

$$50 \times 10^3 = \sigma_t \times A = 100 \times 0.7854\, d^2 = 78.54\, d^2$$

$\therefore$ $d^2 = 50 \times 10^3 / 78.54 = 636.6$ or $d = 25.23$ mm **Ans.**

Top

Contents

2. Size of the rod when it is square

Let x = Each side of the square rod in mm.

∴ Area, $A = x \times x = x^2$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times x^2$$

∴ $x^2 = 50 \times 10^3/100 = 500$ or $x = 22.4$ mm **Ans.**

3. Size of the rod when it is rectangular

Let t = Thickness of the rod in mm, and

b = Width of the rod in mm = $3\,t$ ...(Given)

∴ Area, $A = b \times t = 3t \times t = 3\,t^2$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times 3\,t^2 = 300\,t^2$$

∴ $t^2 = 50 \times 10^3 / 300 = 166.7$ or $t = 12.9$ mm **Ans.**

and $b = 3t = 3 \times 12.9 = 38.7$ mm **Ans.**

Example 4.17. *A steel bar 2.4 m long and 30 mm square is elongated by a load of 500 kN. If poisson's ratio is 0.25, find the increase in volume. Take E = 0.2 × 10⁶ N/mm².*

Solution. Given : l = 2.4 m = 2400 mm ; $A = 30 \times 30 = 900$ mm² ; P = 500 kN = 500 × 10³ N ; $1/m = 0.25$; $E = 0.2 \times 10^6$ N/mm²

Let δV = Increase in volume.

We know that volume of the rod,

$$V = \text{Area} \times \text{length} = 900 \times 2400 = 2160 \times 10^3 \text{ mm}^3$$

and Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\varepsilon}$$

$$\therefore \quad \varepsilon = \frac{P}{A.E} = \frac{500 \times 10^3}{900 \times 0.2 \times 10^6} = 2.8 \times 10^{-3}$$

We know that volumetric strain,

$$\frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right) = 2.8 \times 10^{-3}\ (1 - 2 \times 0.25) = 1.4 \times 10^{3}$$

$$\therefore \quad \delta V = V \times 1.4 \times 10^{-3} = 2160 \times 10^3 \times 1.4 \times 10^{-3} = 3024 \text{ mm}^3 \textbf{ Ans.}$$

4.23 Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as ***impact stress.***

Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end, as shown in Fig. 4.20.

Let A = Cross-sectional area of the bar,

E = Young's modulus of the material of the bar,

l = Length of the bar,

δl = Deformation of the bar,

P = Force at which the deflection δl is produced,

σ_i = Stress induced in the bar due to the application of impact load, and

h = Height through which the load falls.

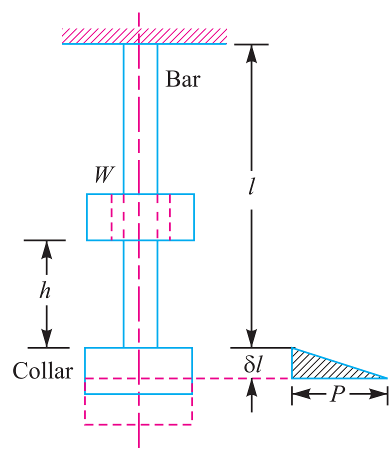


Fig. 4.20. Impact stress.

Top

We know that energy gained by the system in the form of strain energy

$$= \frac{1}{2} \times P \times \delta l$$

and potential energy lost by the weight

$$= W (h + \delta l)$$

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W \ (h + \delta l)$$

$$\frac{1}{2} \sigma_i \times A \times \frac{\sigma_i \times l}{E} = W \left(h + \frac{\sigma_i \times l}{E} \right) \qquad \ldots \left[\because P = \sigma_i \times A, \text{ and } \delta l = \frac{\sigma_i \times l}{E} \right]$$

$$\therefore \qquad \frac{Al}{2E} (\sigma_i)^2 - \frac{Wl}{E} (\sigma_i) - Wh = 0$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A} \left(1 + \sqrt{1 + \frac{2h\ A\ E}{W\ l}} \right) \qquad \ldots \text{[Taking +ve sign for maximum value]}$$

Note : When $h = 0$, then $\sigma_i = 2W/A$. This means that the stress in the bar when the load in applied suddenly is double of the stress induced due to gradually applied load.

Example 4.18. *An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm² in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take E = 200 kN/mm².*

Solution. Given : h = 10 mm ; l = 3 m = 3000 mm ; A = 600 mm^2 ; δl = 2 mm ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2

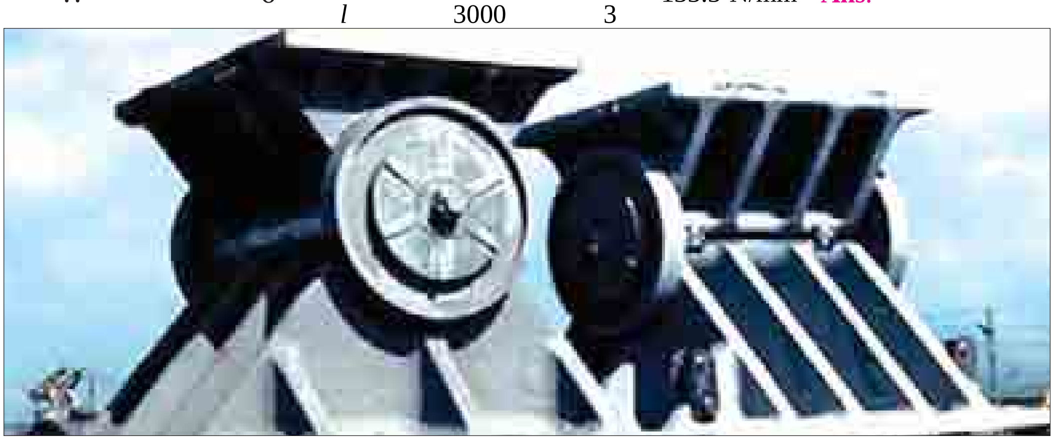
Stress in the bar

Let σ = Stress in the bar.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma . l}{\delta l}$$

$$\therefore \qquad \sigma = \frac{E . \delta l}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3 \text{ N/mm}^2 \quad \textbf{Ans.}$$



These bridge shoes are made to bear high compressive stresses.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Value of the unknown weight

Let W = Value of the unknown weight.

We know that
$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3W} = 1 + \sqrt{1 + \frac{800\ 000}{W}}$$

$$\frac{80\ 000}{W} - 1 = \sqrt{1 + \frac{800\ 000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160\,000}{W} = 1 + \frac{800\,000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80 \quad \text{or} \quad \frac{6400 \times 10^2}{W} = 96$$

∴ $W = 6400 \times 10^2 / 96 = 6666.7$ N **Ans.**

4.24 Resilience

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as ***strain energy***. The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as ***resilience*** and the maximum energy which can be stored in a body up to the elastic limit is called ***proof resilience***. The proof resilience per unit volume of a material is known as ***modulus of resilience***. It is an important property of a material and gives capacity of the material to bear impact or shocks. Mathematically, strain energy stored in a body due to tensile or compressive load or resilience,
$$U = \frac{\sigma^2 \times V}{2E}$$

and Modulus of resilience $= \dfrac{\sigma^2}{2E}$

where σ = Tensile or compressive stress,

V = Volume of the body, and

E = Young's modulus of the material of the body.

Notes : 1. When a body is subjected to a shear load, then modulus of resilience (shear)
$$= \frac{\tau^2}{2C}$$

where τ = Shear stress, and

C = Modulus of rigidity.

2. When the body is subjected to torsion, then modulus of resilience
$$= \frac{\tau^2}{4C}$$

Example 4.19. *A wrought iron bar 50 mm in diameter and 2.5 m long transmits a shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take E = 200 GN/m².*

Solution. Given : d = 50 mm ; l = 2.5 m = 2500 mm ; U = 100 N-m = 100 × 10^3 N-mm ; E = 200 GN/m² = 200 × 10^3 N/mm²

Maximum instantaneous stress

Let σ = Maximum instantaneous stress.

We know that volume of the bar,

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4} (50)^2 \times 2500 = 4.9 \times 10^6 \text{ mm}^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \frac{\sigma^2 \times V}{2E} = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25\, \sigma^2$$

$\therefore$ $\sigma^2 = 100 \times 10^3 / 12.25 = 8163$ or $\sigma = 90.3$ N/mm² **Ans.**

Elongation produced

Let δl = Elongation produced.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\delta l / l}$$

$$\therefore \quad \delta l = \frac{\sigma \times l}{E} = \frac{90.3 \times 2500}{200 \times 10^3} = 1.13 \text{ mm} \quad \textbf{Ans.}$$

A double-decker train.

EXERCISES

1. A reciprocating steam engine connecting rod is subjected to a maximum load of 65 kN. Find the diameter of the connecting rod at its thinnest part, if the permissible tensile stress is 35 N/mm². **[Ans. 50 mm]**

2. The maximum tension in the lower link of a Porter governor is 580 N and the maximum stress in the link is 30 N/mm². If the link is of circular cross-section, determine its diameter. **[Ans. 5 mm]**

3. A wrought iron rod is under a compressive load of 350 kN. If the permissible stress for the material is 52.5 N/mm^2, calculate the diameter of the rod. **[Ans. 95 mm]**

4. A load of 5 kN is to be raised by means of a steel wire. Find the minimum diameter required, if the stress in the wire is not to exceed 100 N/mm^2. **[Ans. 8 mm]**

5. A square tie bar 20 mm × 20 mm in section carries a load. It is attached to a bracket by means of 6 bolts. Calculate the diameter of the bolt if the maximum stress in the tie bar is 150 N/mm^2 and in the bolts is 75 N/mm^2. **[Ans. 13 mm]**

6. The diameter of a piston of the steam engine is 300 mm and the maximum steam pressure is 0.7 N/mm^2. If the maximum permissible compressive stress for the piston rod material is 40 N/mm^2, find the size of the piston rod. **[Ans. 40 mm]**

7. Two circular rods of 50 mm diameter are connected by a knuckle joint, as shown in Fig. 4.21, by a pin of 40 mm in diameter. If a pull of 120 kN acts at each end, find the tensile stress in the rod and shear stress in the pin. **[Ans. 61 N/mm^2; 48 N/mm^2]**

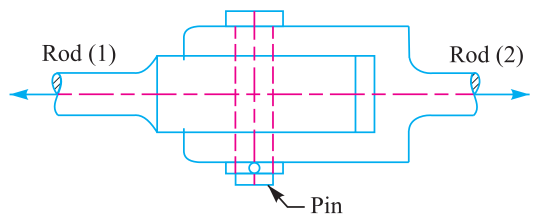


Fig. 4.21

8. Find the minimum size of a hole that can be punched in a 20 mm thick mild steel plate having an ultimate shear strength of 300 N/mm^2. The maximum permissible compressive stress in the punch material is 1200 N/mm^2. **[Ans. 20 mm]**

9. The crankpin of an engine sustains a maximum load of 35 kN due to steam pressure. If the allowable bearing pressure is 7 N/mm^2, find the dimensions of the pin. Assume the length of the pin equal to 1.2 times the diameter of the pin. **[Ans. 64.5 mm; 80 mm]**

10. The following results were obtained in a tensile test on a mild steel specimen of original diameter 20 mm and gauge length 40 mm.

Load at limit of proportionality	=	80 kN
Extension at 80 kN load	=	0.048 mm
Load at yield point	=	85 kN
Maximum load	=	150 kN

When the two parts were fitted together after being broken, the length between gauge length was found to be 55.6 mm and the diameter at the neck was 15.8 mm.

Calculate Young's modulus, yield stress, ultimate tensile stress, percentage elongation and percentage reduction in area. **[Ans. 213 kN/mm^2; 270 N/mm^2; 478 N/mm^2; 39%; 38%]**

11. A steel rod of 25 mm diameter is fitted inside a brass tube of 25 mm internal diameter and 375 mm external diameter. The projecting ends of the steel rod are provided with nuts and washers. The nuts are tightened up so as to produce a pull of 5 kN in the rod. The compound is then placed in a lathe and the brass is turned down to 4 mm thickness. Calculate the stresses in the two materials.

[Ans. 7 N/mm^2, 7.8 N/mm^2]

12. A composite bar made up of aluminium bar and steel bar, is firmly held between two unyielding supports as shown in Fig. 4.22.

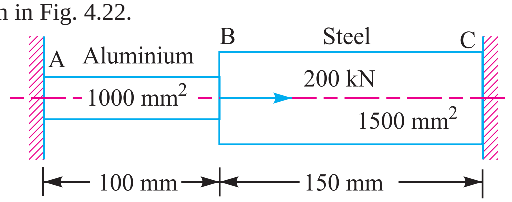


Fig. 4.22

An axial load of 200 kN is applied at *B* at 47°C. Find the stresses in each material, when the temperature is 97°C. Take E_a = 70 GPa ; E_s = 210 GPa ; $\alpha_a = 24 \times 10^{-6}$/°C and $\alpha_s = 12 \times 10^{6}$/°C.

[Ans. 60.3 MPa; 173.5 MPa]

13. A steel rod of 20 mm diameter passes centrally through a copper tube of external diameter 40 mm and internal diameter 20 mm. The tube is closed at each end with the help of rigid washers (of negligible thickness) which are screwed by the nuts. The nuts are tightened until the compressive load on the copper tube is 50 kN. Determine the stresses in the rod and the tube, when the temperature of whole assembly falls by 50°C. Take E_s = 200 GPa ; E_c = 100 GPa ; $\alpha_s = 12 \times 10^{-6}$/°C and $\alpha_c = 18 \times 10^{6}$/°C.

[Ans. 99.6 MPa; 19.8 MPa]

14. A bar of 2 m length, 20 mm breadth and 15 mm thickness is subjected to a tensile load of 30 kN. Find the final volume of the bar, if the Poisson's ratio is 0.25 and Young's modulus is 200 GN/m^2.

[Ans. 600 150 mm^3]

15. A bar of 12 mm diameter gets stretched by 3 mm under a steady load of 8 kN. What stress would be produced in the bar by a weight of 800 N, which falls through 80 mm before commencing the stretching of the rod, which is initially unstressed. Take *E* = 200 kN/mm^2.

[Ans. 170.6 N/mm^2]

QUESTIONS

1. Define the terms load , stress and strain. Discuss the various types of stresses and strain.
2. What is the difference between modulus of elasticity and modulus of rigidity?
3. Explain clearly the bearing stress developed at the area of contact between two members.
4. What useful informations are obtained from the tensile test of a ductile material?
5. What do you mean by factor of safety?
6. List the important factors that influence the magnitude of factor of safety.
7. What is meant by working stress and how it is calculated from the ultimate stress or yield stress of a material? What will be the factor of safety in each case for different types of loading?
8. Describe the procedure for finding out the stresses in a composite bar.
9. Explain the difference between linear and lateral strain.
10. Define the following :
    (*a*) Poisson's ratio, (*b*) Volumetric strain, and (*c*) Bulk modulus
11. Derive an expression for the impact stress induced due to a falling load.
12. Write short notes on :
    (*a*) Resilience (*b*) Proof resilience, and (*c*) Modulus of resilience

OBJECTIVE TYPE QUESTIONS

1. Hooke's law holds good upto
   (*a*) yield point (*b*) elastic limit
   (*c*) plastic limit (*d*) breaking point
2. The ratio of linear stress to linear strain is called
   (*a*) Modulus of elasticity (*b*) Modulus of rigidity
   (*c*) Bulk modulus (*d*) Poisson's ratio

Contents

3. The modulus of elasticity for mild steel is approximately equal to
   (*a*) 80 kN/mm^2 (*b*) 100 kN/mm^2
   (*c*) 110 kN/mm^2 (*d*) 210 kN/mm^2
4. When the material is loaded within elastic limit, then the stress is .... to strain.
   (*a*) equal (*b*) directly proportional (*c*) inversely proportional
5. When a hole of diameter '*d*' is punched in a metal of thickness `*t*', then the force required to punch a hole is equal to
   (*a*) $d.t.\tau_u$ (*b*) $\pi\, d.t.\tau_u$
   (*c*) $\frac{\pi}{4} \times d^2\ \tau_u$ (*d*) $\frac{\pi}{4} \times d^2.t.\tau_u$

   where τ_u = Ultimate shear strength of the material of the plate.
6. The ratio of the ultimate stress to the design stress is known as
   (*a*) elastic limit (*b*) strain
   (*c*) factor of safety (*d*) bulk modulus
7. The factor of safety for steel and for steady load is
   (*a*) 2 (*b*) 4
   (*c*) 6 (*d*) 8
8. An aluminium member is designed based on
   (*a*) yield stress (*b*) elastic limit stress
   (*c*) proof stress (*d*) ultimate stress
9. In a body, a thermal stress is one which arises because of the existence of
   (*a*) latent heat (*b*) temperature gradient
   (*c*) total heat (*d*) specific heat
10. A localised compressive stress at the area of contact between two members is known as
   (*a*) tensile stress (*b*) bending stress
   (*c*) bearing stress (*d*) shear stress
11. The Poisson's ratio for steel varies from
   (*a*) 0.21 to 0.25 (*b*) 0.25 to 0.33
   (*c*) 0.33 to 0.38 (*d*) 0.38 to 0.45
12. The stress in the bar when load is applied suddenly is .... as compared to the stress induced due to gradually applied load.
   (*a*) same (*b*) double
   (*c*) three times (*d*) four times
13. The energy stored in a body when strained within elastic limit is known as
   (*a*) resilience (*b*) proof resilience
   (*c*) strain energy (*d*) impact energy
14. The maximum energy that can be stored in a body due to external loading upto the elastic limit is called
   (*a*) resilience (*b*) proof resilience
   (*c*) strain energy (*d*) modulus of resilience
15. The strain energy stored in a body, when suddenly loaded, is .... the strain energy stored when same load is applied gradually.
   (*a*) equal to (*b*) one-half
   (*c*) twice (*d*) four times

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*d*)	**4.** (*b*)	**5.** (*b*)
6. (*c*)	**7.** (*b*)	**8.** (*a*)	**9.** (*b*)	**10.** (*c*)
11. (*b*)	**12.** (*b*)	**13.** (*c*)	**14.** (*b*)	**15.** (*d*)

Top

C H A P T E R

5

Torsional and Bending Stresses in Machine Parts

5.1 Introduction

Sometimes machine parts are subjected to pure torsion or bending or combination of both torsion and bending stresses. We shall now discuss these stresses in detail in the following pages.

5.2 Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion.*** The stress set up by torsion is known as ***torsional shear stress.*** It is zero at the centroidal axis and maximum at the outer surface.

Consider a shaft fixed at one end and subjected to a torque (T) at the other end as shown in Fig. 5.1. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the

Contents
Top

torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \quad \text{...}(i)$$

where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l.

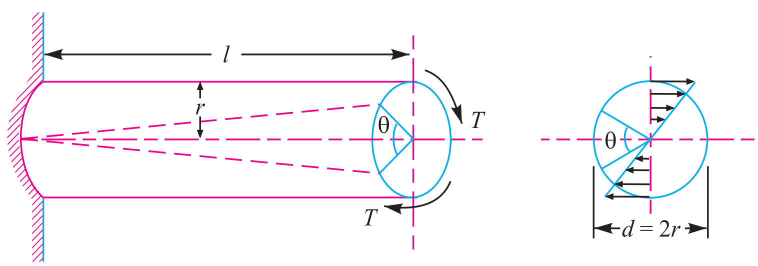


Fig. 5.1. Torsional shear stress.

The equation (*i*) is known as ***torsion equation***. It is based on the following assumptions:

1. The material of the shaft is uniform throughout.

2. The twist along the length of the shaft is uniform.

3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Notes : 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance x from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2. From equation (*i*), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (d), the polar moment of inertia,

$$J = I_{XX} + I_{YY} = \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

$$\therefore \quad T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J = \frac{\pi}{32}\,[(d_o)^4 - (d_i)^4] \text{ and } r = \frac{d_o}{2}$$

$$\therefore \qquad T = \tau \times \frac{\pi}{32}[(d_o)^4 - (d_4)^4] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

$$= \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \ldots\left(\text{Substituting, } k = \frac{d_i}{d_o}\right)$$

3. The expression ($C \times J$) is called ***torsional rigidity*** of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2\,\pi\,N\,.\,T}{60} = T\,.\,\omega \qquad \ldots\left(\because\ \omega = \frac{2\,\pi\,N}{60}\right)$$

where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Example 5.1. *A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.*

Solution. Given : P = 100 kW = 100 × 10^3 W ; N = 160 r.p.m ; T_{max} = 1.25 T_{mean} ; τ = 70 MPa = 70 N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\,\pi\,N\,.\,T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\,T_{mean}$$

$$\therefore \qquad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

A Helicopter propeller shaft has to bear torsional, tensile, as well as bending stresses.

Note : This picture is given as additional information and is not a direct example of the current chapter.

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \quad \text{or} \quad d = 81.5 \text{ mm} \quad \textbf{Ans.}$$

Example 5.2. *A steel shaft 35 mm in diameter and 1.2 m long held rigidly at one end has a hand wheel 500 mm in diameter keyed to the other end. The modulus of rigidity of steel is 80 GPa.*

1. What load applied to tangent to the rim of the wheel produce a torsional shear of 60 MPa?

2. How many degrees will the wheel turn when this load is applied?

Solution. Given : d = 35 mm or r = 17.5 mm ; l = 1.2 m = 1200 mm ; D = 500 mm or R = 250 mm ; C = 80 GPa = 80 kN/mm^2 = 80 × 10^3 N/mm^2 ; τ = 60 MPa = 60 N/mm^2

1. Load applied to the tangent to the rim of the wheel

Let W = Load applied (in newton) to tangent to the rim of the wheel.

We know that torque applied to the hand wheel,

$$T = W.R = W \times 250 = 250\, W \text{ N-mm}$$

and polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} (35)^4 = 147.34 \times 10^3 \text{ mm}^4$$

We know that $\dfrac{T}{J} = \dfrac{\tau}{r}$

$$\therefore \quad \frac{250\, W}{147.34 \times 10^3} = \frac{60}{17.5} \quad \text{or} \quad W = \frac{60 \times 147.34 \times 10^3}{17.5 \times 250} = 2020 \text{ N} \quad \textbf{Ans.}$$

2. Number of degrees which the wheel will turn when load W = 2020 N is applied

Let θ = Required number of degrees.

We know that $\dfrac{T}{J} = \dfrac{C.\theta}{l}$

$$\therefore \quad \theta = \frac{T.l}{C.J} = \frac{250 \times 2020 \times 1200}{80 \times 10^3 \times 147.34 \times 10^3} = 0.05° \quad \textbf{Ans.}$$

Example 5.3. *A shaft is transmitting 97.5 kW at 180 r.p.m. If the allowable shear stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more that 1° in a length of 3 metres. Take C = 80 GPa.*

Solution. Given : P = 97.5 kW = 97.5 × 10^3 W ; N = 180 r.p.m. ; τ = 60 MPa = 60 N/mm^2 ; θ = 1° = π / 180 = 0.0174 rad ; l = 3 m = 3000 mm ; C = 80 GPa = 80 × 10^9 N/m^2 = 80 × 10^3 N/mm^2

Let T = Torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted by the shaft (P),

$$97.5 \times 10^3 = \frac{2\,\pi\, N.T}{60} = \frac{2\pi \times 180 \times T}{60} = 18.852\, T$$

$$\therefore \quad T = 97.5 \times 10^3/18.852 = 5172 \text{ N-m} = 5172 \times 10^3 \text{ N-mm}$$

Now let us find the diameter of the shaft based on the strength and stiffness.

1. *Considering strength of the shaft*

We know that the torque transmitted (T),

$$5172 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$\therefore \quad d^3 = 5172 \times 10^3/11.78 = 439 \times 10^3 \text{ or } d = 76 \text{ mm} \quad ...(i)$$

2. *Considering stiffness of the shaft*

Polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = 0.0982\, d^4$$

We know that
$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{5172 \times 10^3}{0.0982\, d^4} = \frac{80 \times 10^3 \times 0.0174}{3000} \text{ or } \frac{52.7 \times 10^6}{d^4} = 0.464$$

$$\therefore \quad d^4 = 52.7 \times 10^6/0.464 = 113.6 \times 10^6 \text{ or } d = 103 \text{ mm} \quad ...(ii)$$

Taking larger of the two values, we shall provide d = 103 say 105 mm **Ans.**

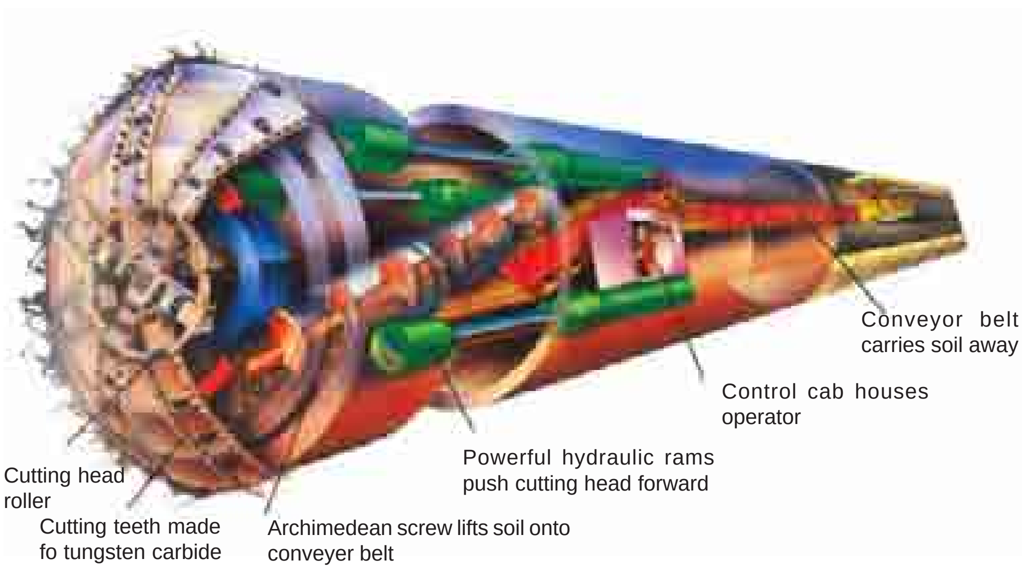
Example 5.4. *A hollow shaft is required to transmit 600 kW at 110 r.p.m., the maximum torque being 20% greater than the mean. The shear stress is not to exceed 63 MPa and twist in a length of 3 metres not to exceed 1.4 degrees. Find the external diameter of the shaft, if the internal diameter to the external diameter is 3/8. Take modulus of rigidity as 84 GPa.*

Solution. Given : P = 600 kW = 600 × 10^3 W ; N = 110 r.p.m. ; T_{max} = 1.2 T_{mean} ; τ = 63 MPa = 63 N/mm^2 ; l = 3 m = 3000 mm ; θ = 1.4 × π / 180 = 0.024 rad ; $k = d_i / d_o$ = 3/8 ; C = 84 GPa = 84 × 10^9 N/m^2 = 84 × 10^3 N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft,

d_o = External diameter of the shaft, and

d_i = Internal diameter of the shaft.



A tunnel-boring machine can cut through rock at up to one kilometre a month. Powerful hydraulic rams force the machine's cutting head fowards as the rock is cut away.

Note : This picture is given as additional information and is not a direct example of the current chapter.

We know that power transmitted by the shaft (P),

$$600 \times 10^3 = \frac{2\,\pi\,N.T_{mean}}{60} = \frac{2\pi \times 110 \times T_{mean}}{60} = 11.52\ T_{mean}$$

$$\therefore \qquad T_{mean} = 600 \times 10^3/11.52 = 52 \times 10^3 \text{ N-m} = 52 \times 10^6 \text{ N-mm}$$

and maximum torque transmitted by the shaft,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 52 \times 10^6 = 62.4 \times 10^6 \text{ N-mm}$$

Now let us find the diameter of the shaft considering strength and stiffness.

1. *Considering strength of the shaft*

We know that maximum torque transmitted by the shaft,

$$T_{max} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$62.4 \times 10^6 = \frac{\pi}{16} \times 63 \times (d_o)^3 \left[1 - \left(\frac{3}{8}\right)^4\right] = 12.12\,(d_o)^3$$

$$\therefore \qquad (d_o)^3 = 62.4 \times 10^6/12.12 = 5.15 \times 10^6 \text{ or } d_o = 172.7 \text{ mm} \qquad \text{...}(i)$$

2. *Considering stiffness of the shaft*

We know that polar moment of inertia of a hollow circular section,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{32}\,(d_o)^4 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$$

$$= \frac{\pi}{32}\,(d_o)^4\,(1 - k^4) = \frac{\pi}{32}\,(d_o)^4 \left[1 - \left(\frac{3}{8}\right)^4\right] = 0.0962\,(d_o)^4$$

We also know that

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{62.4 \times 10^6}{0.0962\,(d_o)^4} = \frac{84 \times 10^3 \times 0.024}{3000} \quad \text{or} \quad \frac{648.6 \times 10^6}{(d_o)^4} = 0.672$$

$$\therefore \qquad (d_o)^4 = 648.6 \times 10^6/0.672 = 964 \times 10^6 \text{ or } d_o = 176.2 \text{ mm} \qquad \text{...}(ii)$$

Taking larger of the two values, we shall provide

$$d_o = 176.2 \text{ say } 180 \text{ mm}$$ **Ans.**

5.3 Shafts in Series and Parallel

When two shafts of different diameters are connected together to form one shaft, it is then known as ***composite shaft.*** If the driving torque is applied at one end and the resisting torque at the other end, then the shafts are said to be connected in series as shown in Fig. 5.2 (a). In such cases, each shaft transmits the same torque and the total angle of twist is equal to the sum of the angle of twists of the two shafts.

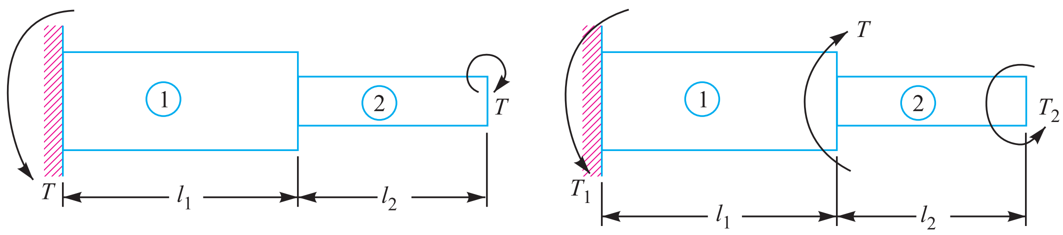
Mathematically, total angle of twist,

$$\theta = \theta_1 + \theta_2 = \frac{T.l_1}{C_1 J_1} + \frac{T.l_2}{C_2 J_2}$$

If the shafts are made of the same material, then $C_1 = C_2 = C$.

$$\therefore \qquad \theta = \frac{T.l_1}{CJ_1} + \frac{T.l_2}{CJ_2} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2}\right]$$

Contents



(*a*) Shafts in series. (*b*) Shafts in parallel.

Fig. 5.2. Shafts in series and parallel.

When the driving torque (T) is applied at the junction of the two shafts, and the resisting torques T_1 and T_2 at the other ends of the shafts, then the shafts are said to be connected in parallel, as shown in Fig. 5.2 (*b*). In such cases, the angle of twist is same for both the shafts, *i.e.*

$$\theta_1 = \theta_2$$

or
$$\frac{T_1 l_1}{C_1 J_1} = \frac{T_2 l_2}{C_2 J_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{C_1}{C_2} \times \frac{J_1}{J_2}$$

and
$$T = T_1 + T_2$$

If the shafts are made of the same material, then $C_1 = C_2$.

$$\therefore \qquad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{J_1}{J_2}$$

Example 5.5. *A steel shaft ABCD having a total length of 3.5 m consists of three lengths having different sections as follows:*

AB is hollow having outside and inside diameters of 100 mm and 62.5 mm respectively, and BC and CD are solid. BC has a diameter of 100 mm and CD has a diameter of 87.5 mm. If the angle of twist is the same for each section, determine the length of each section. Find the value of the applied torque and the total angle of twist, if the maximum shear stress in the hollow portion is 47.5 MPa and shear modulus, C = 82.5 GPa.

Solution. Given: $L = 3.5$ m ; $d_o = 100$ mm ; $d_i = 62.5$ mm ; $d_2 = 100$ mm ; $d_3 = 87.5$ mm ; $\tau = 47.5$ MPa $= 47.5$ N/mm^2 ; $C = 82.5$ GPa $= 82.5 \times 10^3$ N/mm^2

The shaft *ABCD* is shown in Fig. 5.3.

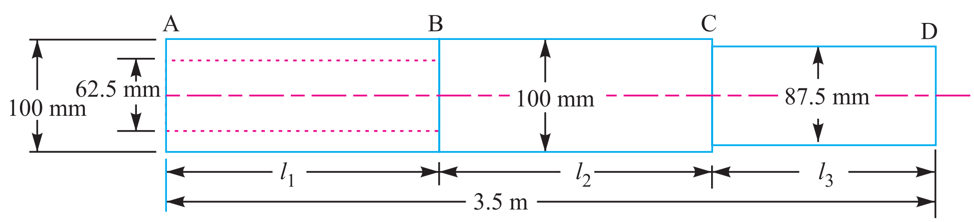


Fig. 5.3

Length of each section

Let l_1, l_2 and l_3 = Length of sections *AB*, *BC* and *CD* respectively.

We know that polar moment of inertia of the hollow shaft *AB*,

$$J_1 = \frac{\pi}{32}\,[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}\,[(100)^4 - (62.5)^4] = 8.32 \times 10^6 \text{ mm}^4$$

Polar moment of inertia of the solid shaft *BC*,

$$J_2 = \frac{\pi}{32}\,(d_2)^4 = \frac{\pi}{32}\,(100)^4 = 9.82 \times 10^6 \text{ mm}^4$$

Top

Contents

and polar moment of inertia of the solid shaft *CD*,

$$J_3 = \frac{\pi}{32}(d_3)^4 = \frac{\pi}{32}(87.5)^4 = 5.75 \times 10^6 \text{ mm}^4$$

We also know that angle of twist,

$$\theta = T \,.\, l \,/\, C \,.\, J$$

Assuming the torque T and shear modulus C to be same for all the sections, we have

Angle of twist for hollow shaft *AB*,

$$\theta_1 = T \,.\, l_1 \,/\, C \,.\, J_1$$

Similarly, angle of twist for solid shaft *BC*,

$$\theta_2 = T \,.\, l_2 \,/\, C \,.\, J_2$$

and angle of twist for solid shaft *CD*,

$$\theta_3 = T \,.\, l_3 \,/\, C \,.\, J_3$$

Since the angle of twist is same for each section, therefore

Machine part of a jet engine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$\theta_1 = \theta_2$$

$$\frac{T \,.\, l_1}{C \,.\, J_1} = \frac{T \,.\, l_2}{C \,.\, J_2} \quad \text{or} \quad \frac{l_1}{l_2} = \frac{J_1}{J_2} = \frac{8.32 \times 10^6}{9.82 \times 10^6} = 0.847 \qquad ...(i)$$

Also $\theta_1 = \theta_3$

$$\frac{T \,.\, l_1}{C \,.\, J_1} = \frac{T \,.\, l_3}{C \,.\, J_3} \quad \text{or} \quad \frac{l_1}{l_3} = \frac{J_1}{J_3} = \frac{8.32 \times 10^6}{5.75 \times 10^6} = 1.447 \qquad ...(ii)$$

We know that $l_1 + l_2 + l_3 = L = 3.5 \text{ m} = 3500 \text{ mm}$

$$l_1\left(1 + \frac{l_2}{l_1} + \frac{l_3}{l_1}\right) = 3500$$

$$l_1\left(1 + \frac{1}{0.847} + \frac{1}{1.447}\right) = 3500$$

$l_1 \times 2.8717 = 3500$ or $l_1 = 3500 / 2.8717 = 1218.8$ mm **Ans.**

From equation ***(i)***,

$l_2 = l_1 / 0.847 = 1218.8 / 0.847 = 1439$ mm **Ans.**

and from equation ***(ii)***, $l_3 = l_1 / 1.447 = 1218.8 / 1.447 = 842.2$ mm **Ans.**

Value of the applied torque

We know that the maximum shear stress in the hollow portion,

$$\tau = 47.5 \text{ MPa} = 47.5 \text{ N/mm}^2$$

For a hollow shaft, the applied torque,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times 47.5 \left[\frac{(100)^4 - (62.5)^4}{100}\right]$$

$= 7.9 \times 10^6$ N-mm = 7900 N-m **Ans.**

Total angle of twist

When the shafts are connected in series, the total angle of twist is equal to the sum of angle of twists of the individual shafts. Mathematically, the total angle of twist,

$$\theta = \theta_1 + \theta_2 + \theta_3$$

Top

Contents

$$= \frac{T \cdot l_1}{C \cdot J_1} + \frac{T \cdot l_2}{C \cdot J_2} + \frac{T \cdot l_3}{C \cdot J_3} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}\right]$$

$$= \frac{7.9 \times 10^6}{82.5 \times 10^3}\left[\frac{1218.8}{8.32 \times 10^6} + \frac{1439}{9.82 \times 10^6} + \frac{842.2}{5.75 \times 10^6}\right]$$

$$= \frac{7.9 \times 10^6}{82.5 \times 10^3 \times 10^6}\,[146.5 + 146.5 + 146.5] = 0.042 \text{ rad}$$

$= 0.042 \times 180 / \pi = 2.406°$ **Ans.**

5.4 Bending Stress in Straight Beams

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses.

Consider a straight beam subjected to a bending moment M as shown in Fig. 5.4. The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).
2. The material of the beam obeys Hooke's law.
3. The transverse sections (*i.e.* BC or GH) which were plane before bending, remain plane after bending also.
4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.
5. The Young's modulus (E) is the same in tension and compression.
6. The loads are applied in the plane of bending.

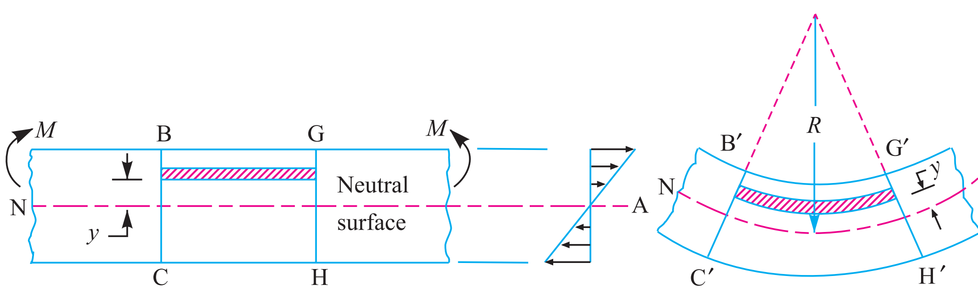


Fig. 5.4. Bending stress in straight beams.

A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called ***neutral surface***. The intersection of the neutral surface with any normal cross-section of the beam is known *as* ***neutral axis***. The stress distribution of a beam is shown in Fig. 5.4. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where M = Bending moment acting at the given section,

σ = Bending stress,

Top

I = Moment of inertia of the cross-section about the neutral axis,
y = Distance from the neutral axis to the extreme fibre,
E = Young's modulus of the material of the beam, and
R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y, *i.e.* the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I / y} = \frac{M}{Z}$$

The ratio I/y is known as ***section modulus*** and is denoted by Z.

Notes : 1. The neutral axis of a section always passes through its centroid.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fibre from the neutral axis is $y = d / 2$, where d is the diameter in case of circular section or depth in case of square or rectangular section.

3. In case of unsymmetrical sections such as L-section or T-section, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

Parts in a machine.

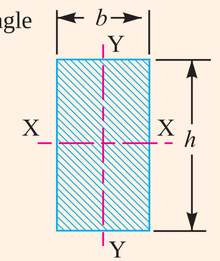
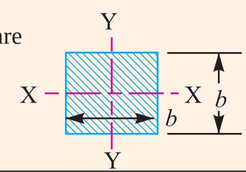
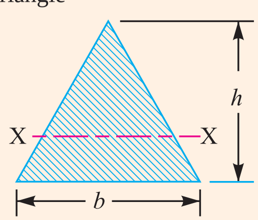
Table 5.1 (from pages 130 to 134) shows the properties of some common cross-sections.

This is the first revolver produced in a production line using interchangeable parts.

Note : This picture is given as additional information and is not a direct example of the current chapter.

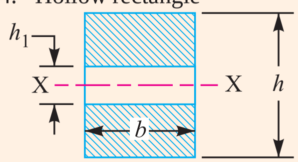
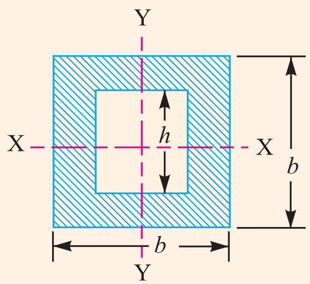
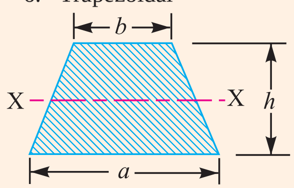
Contents

Table 5.1. Properties of commonly used cross-sections.

Section	*Area* (A)	*Moment of inertia* (I)	*Distance from the neutral axis to the extreme fibre (y)	*Section modulus* $\left[Z = \frac{I}{y}\right]$	*Radius of gyration* $\left[k = \sqrt{\frac{I}{A}}\right]$
1. Rectangle 	bh	$I_{xx} = \frac{b.h^3}{12}$ $I_{yy} = \frac{h.b^3}{12}$	$\frac{h}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{b.h^2}{6}$ $Z_{yy} = \frac{h.b^2}{6}$	$k_{xx} = 0.289\ h$ $k_{yy} = 0.289\ b$
2. Square 	b^2	$I_{xx} = I_{yy} = \frac{b^4}{12}$	$\frac{b}{2}$	$Z_{xx} = Z_{yy} = \frac{b^3}{6}$	$k_{xx} = k_{yy} = 0.289\ b$
3. Triangle 	$\frac{bh}{2}$	$I_{xx} = \frac{b.h^3}{36}$	$\frac{h}{3}$	$Z_{xx} = \frac{bh^2}{12}$	$k_{xx} = 0.2358\ h$

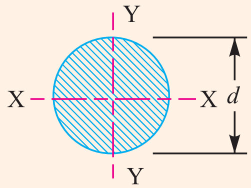
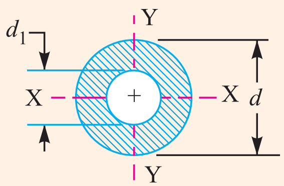
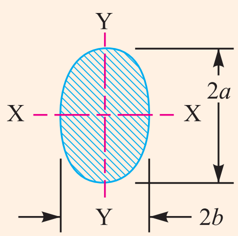
* The distances from the neutral axis to the bottom extreme fibre is taken into consideration.

Top

Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
4. Hollow rectangle 	$b\,(h - h_1)$	$I_{xx} = \frac{b}{12}(h^3 - h_1^3)$	$\frac{h}{2}$	$Z_{xx} = \frac{b}{6}\left(\frac{h^3 - h_1^3}{h}\right)$	$k_{xx} = 0.289\sqrt{\frac{h^3 - h_1^3}{h - h_1}}$
5. Hollow square 	$b^2 - h^2$	$I_{xx} = I_{yy} = \frac{b^4 - h^4}{12}$	$\frac{b}{2}$	$Z_{xx} = Z_{yy} = \frac{b^4 - h^4}{6b}$	$0.289\sqrt{b^2 + h^2}$
6. Trapezoidal 	$\frac{a + b}{2} \times h$	$I_{xx} = \frac{h^2(a^2 + 4ab + b^2)}{36\,(a + b)}$	$\frac{a + 2b}{3\,(a + b)} \times h$	$Z_{xx} = \frac{a^2 + 4ab + b^2}{12\,(a + 2b)}$	$\frac{0.236}{a+b}\sqrt{h\,(a^2 + 4ab + b^2)}$

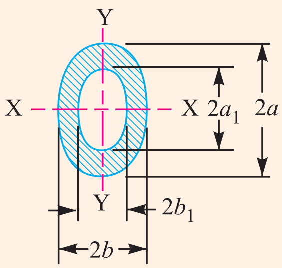
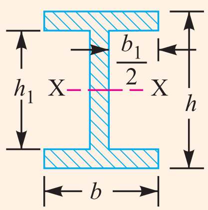
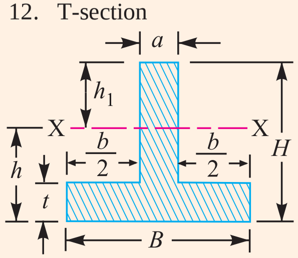
Top

Contents

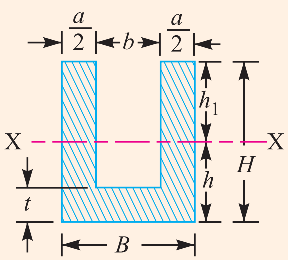
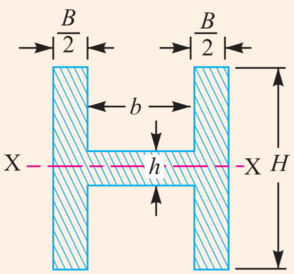
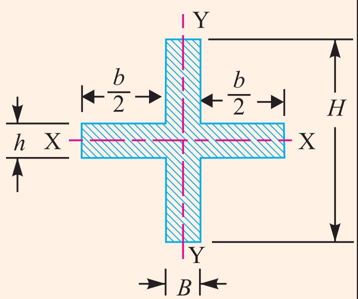
Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
7. Circle 	$\frac{\pi}{4} \times d^2$	$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$	$\frac{d}{2}$	$Z_{xx} = Z_{yy} = \frac{\pi d^3}{32}$	$k_{xx} = k_{yy} = \frac{d}{2}$
8. Hollow circle 	$\frac{\pi}{4}(d^2 - d_1^2)$	$I_{xx} = I_{yy} = \frac{\pi}{64}(d^4 - d_1^4)$	$\frac{d}{2}$	$Z_{xx} = Z_{yy} = \frac{\pi}{32}\left(\frac{d^4 - d_1^4}{d}\right)$	$k_{xx} = k_{yy} = \frac{\sqrt{d^2 + d_1^2}}{4}$
9. Elliptical 	πab	$I_{xx} = \frac{\pi}{4} \times a^3 b$ $I_{yy} = \frac{\pi}{4} \times ab^3$	a b	$Z_{xx} = \frac{\pi}{4} \times a^2 b$ $Z_{yy} = \frac{\pi}{4} \times ab^2$	$k_{xx} = 0.5a$ $k_{yy} = 0.5b$

Contents
Top

Contents

Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
10. Hollow elliptical 	$\pi\,(ab - a_1 b_1)$	$I_{xx} = \frac{\pi}{4}\,(ba^3 - b_1\,a_1^{\,3})$ $I_{yy} = \frac{\pi}{4}\,(ab^3 - a_1\,b_1^{\,3})$	a b	$Z_{xx} = \frac{\pi}{4a}\,(ba^3 - b_1\,a_1^{\,3})$ $Z_{yy} = \frac{\pi}{4b}\,(ab^3 - a_1\,b_1^{\,3})$	$k_{xx} = \frac{1}{2}\sqrt{\frac{ba^3 - b_1 a_1^{\,3}}{ab - a_1 b_1}}$ $k_{yy} = \frac{1}{2}\sqrt{\frac{ab^3 - a_1 b_1^{\,3}}{ab - a_1 b_1}}$
11. I-section 	$bh - b_1 h_1$	$I_{xx} = \frac{bh^3 - b_1 h_1^{\,3}}{12}$	$\frac{h}{2}$	$Z_{xx} = \frac{bh^3 - b_1 h_1^{\,3}}{6h}$	$k_{xx} = 0.289\sqrt{\frac{bh^3 - b_1 h_1^{\,3}}{bh - b_1 h_1}}$
12. T-section 	$Bt + (H - t)\,a$	$I_{xx} = \frac{Bh^3 - b(h-t)^3 + ah_1^{\,3}}{3}$	$h = H - h_1$ $= \frac{aH^2 + bt^2}{2(aH + bt)}$	$Z_{xx} = \frac{2I_{xx}(aH + bt)}{aH^2 + bt^2}$	$k_{xx} = \sqrt{\frac{I_{xx}}{Bt + (H - t)a}}$

Top

Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
13. Channel Section 	$Bt + (H - t)\,a$	$I_{xx} = \frac{Bh^3 - b(h-t)^3 + ah_1^3}{3}$	$h = H - h_1 = \frac{aH^2 + bt^2}{2(aH + bt)}$	$Z_{xx} = \frac{2I_{xx}(aH + bt)}{aH^2 + bt^2}$	$k_{xx} = \sqrt{\frac{I_{xx}}{Bt + (H-t)\,a}}$
14. H-Section 	$BH + bh$	$I_{xx} = \frac{BH^3 + bh^3}{12}$	$\frac{H}{2}$	$Z_{xx} = \frac{BH^3 + bh^3}{6H}$	$k_{xx} = 0.289\sqrt{\frac{BH^3 + bh^3}{BH + bh}}$
15. Cross-section 	$BH + bh$	$I_{xx} = \frac{Bh^3 + bh^3}{12}$	$\frac{H}{2}$	$Z_{xx} = \frac{BH^3 + bh^3}{6H}$	$k_{xx} = 0.289\sqrt{\frac{BH^3 + bh^3}{BH + bh}}$

Contents

Top

Contents

Example 5.6. *A pump lever rocking shaft is shown in Fig. 5.5. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100 MPa.*

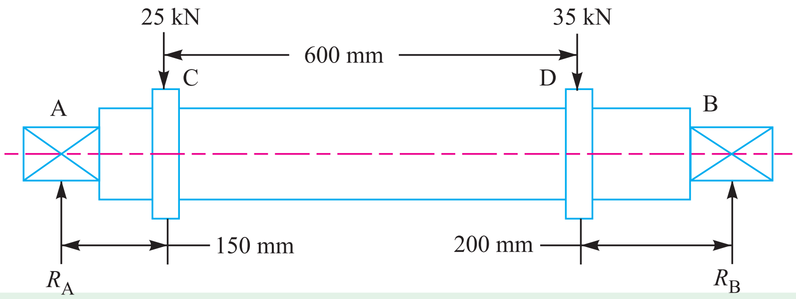


Fig. 5.5

Solution. Given : σ_b = 100 MPa = 100 N/mm^2

Let R_A and R_B = Reactions at A and B respectively.

Taking moments about A, we have

$$R_B \times 950 = 35 \times 750 + 25 \times 150 = 30\ 000$$

$$\therefore \quad R_B = 30\ 000 / 950 = 31.58 \text{ kN} = 31.58 \times 10^3 \text{ N}$$

and $$R_A = (25 + 35) - 31.58 = 28.42 \text{ kN} = 28.42 \times 10^3 \text{ N}$$

$\therefore$ Bending moment at C

$$= R_A \times 150 = 28.42 \times 10^3 \times 150 = 4.263 \times 10^6 \text{ N-mm}$$

and bending moment at $D = R_B \times 200 = 31.58 \times 10^3 \times 200 = 6.316 \times 10^6$ N-mm

We see that the maximum bending moment is at D, therefore maximum bending moment, $M = 6.316 \times 10^6$ N-mm.

Let d = Diameter of the shaft.

$\therefore$ Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

We know that bending stress (σ_b),

$$100 = \frac{M}{Z} = \frac{6.316 \times 10^6}{0.0982\, d^3} = \frac{64.32 \times 10^6}{d^3}$$

$$\therefore \quad d^3 = 64.32 \times 10^6/100 = 643.2 \times 10^3 \text{ or } d = 86.3 \text{ say } 90 \text{ mm}$$ **Ans.**

The picture shows a method where sensors are used to measure torsion

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 5.7. *An axle 1 metre long supported in bearings at its ends carries a fly wheel weighing 30 kN at the centre. If the stress (bending) is not to exceed 60 MPa, find the diameter of the axle.*

Solution. Given : L = 1 m = 1000 mm ; W = 30 kN = 30 × 10^3 N ; σ_b = 60 MPa = 60 N/mm^2

The axle with a flywheel is shown in Fig. 5.6.

Let d = Diameter of the axle in mm.

Top

Contents

∴ Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

Maximum bending moment at the centre of the axle,

$$M = \frac{W.L}{4} = \frac{30 \times 10^3 \times 1000}{4} = 7.5 \times 10^6 \text{ N-mm}$$

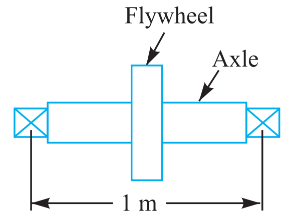


Fig. 5.6

We know that bending stress (σ_b),

$$60 = \frac{M}{Z} = \frac{7.5 \times 10^6}{0.0982\, d^3} = \frac{76.4 \times 10^6}{d^3}$$

∴ $d^3 = 76.4 \times 10^6/60 = 1.27 \times 10^6$ or $d = 108.3$ say 110 mm **Ans.**

Example 5.8. *A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.*

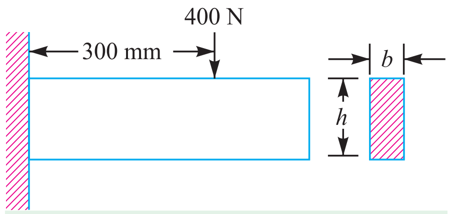


Fig. 5.7

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa = 40 N/mm^2 ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

∴ Section modulus,

$$Z = \frac{b \cdot h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3} \text{ mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

∴ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

Example 5.9. *A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.*

Solution. Given : $P = 10$ kW $= 10 \times 10^3$ W ; $N = 400$ r.p.m ; $D = 1.2$ m = 1200 mm or $R = 600$ mm ; $\sigma_b = 15$ MPa = 15 N/mm^2

Let T = Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\,\pi\,N\,.\,T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\,T$$

∴ $T = 10 \times 10^3/42 = 238$ N-m $= 238 \times 10^3$ N-mm

Top

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\ 520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm = $2 \times 2b = 4b$ ...(Given)

∴ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4} (2b)^2 \times b = \pi\, b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\ 520}{\pi\, b^3} = \frac{18\ 943}{b^3}$$

or $b^3 = 18\ 943/15 = 1263$ or $b = 10.8$ mm

∴ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**

5.5 Bending Stress in Curved Beams

We have seen in the previous article that for the straight beams, the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in case of curved beams, the neutral axis of the cross-section is shifted towards the centre of curvature of the beam causing a non-linear (hyperbolic) distribution of stress, as shown in Fig. 5.8. It may be noted that the neutral axis lies between the centroidal axis and the centre of curvature and always occurs within the curved beams. The application of curved beam principle is used in crane hooks, chain links and frames of punches, presses, planers etc.

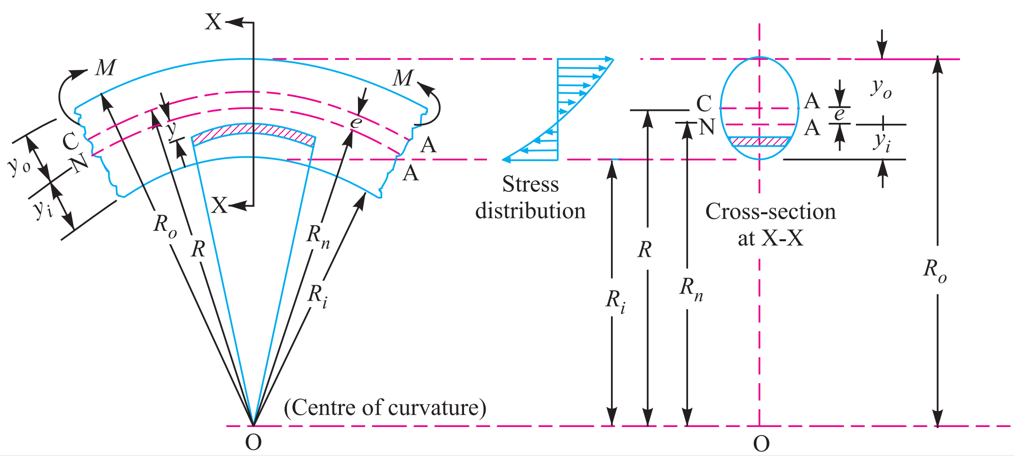


Fig. 5.8. Bending stress in a curved beam.

Consider a curved beam subjected to a bending moment M, as shown in Fig. 5.8. In finding the bending stress in curved beams, the same assumptions are used as for straight beams. The general expression for the bending stress (σ_b) in a curved beam at any fibre at a distance y from the neutral axis, is given by

$$\sigma_b = \frac{M}{A \,.\, e}\left(\frac{y}{R_n - y}\right)$$

where M = Bending moment acting at the given section about the centroidal axis,

A = Area of cross-section,

e = Distance from the centroidal axis to the neutral axis = $R - R_n$,

R = Radius of curvature of the centroidal axis,

R_n = Radius of curvature of the neutral axis, and

y = Distance from the neutral axis to the fibre under consideration. It is positive for the distances towards the centre of curvature and negative for the distances away from the centre of curvature.

Notes : 1. The bending stress in the curved beam is zero at a point other than at the centroidal axis.

2. If the section is symmetrical such as a circle, rectangle, I-beam with equal flanges, then the maximum bending stress will always occur at the inside fibre.

3. If the section is unsymmetrical, then the maximum bending stress may occur at either the inside fibre or the outside fibre. The maximum bending stress at the inside fibre is given by

$$\sigma_{bi} = \frac{M \,.\, y_i}{A \,.\, e \,.\, R_i}$$

where y_i = Distance from the neutral axis to the inside fibre = $R_n - R_i$, and

R_i = Radius of curvature of the inside fibre.

The maximum bending stress at the outside fibre is given by

$$\sigma_{bo} = \frac{M \,.\, y_o}{A \,.\, e \,.\, R_o}$$

where y_o = Distance from the neutral axis to the outside fibre = $R_o - R_n$, and

R_o = Radius of curvature of the outside fibre.

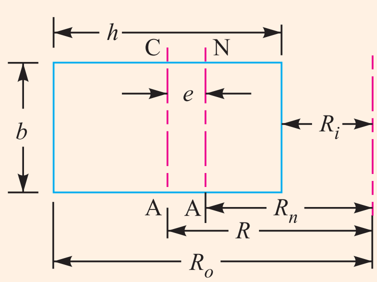
It may be noted that the bending stress at the inside fibre is ***tensile*** while the bending stress at the outside fibre is ***compressive.***

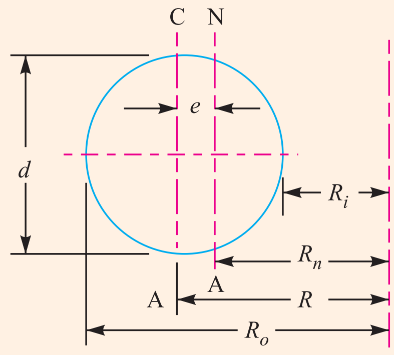
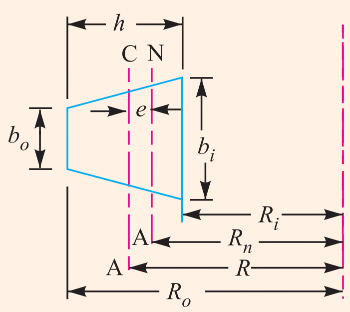
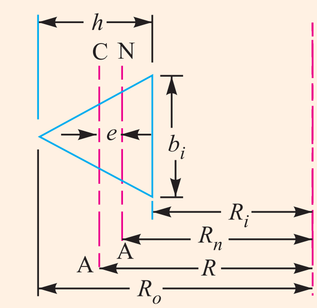
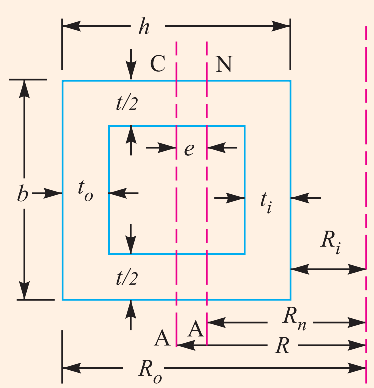
4. If the section has an axial load in addition to bending, then the axial or direct stress (σ_d) must be added algebraically to the bending stress, in order to obtain the resultant stress on the section. In other words,

Resultant stress, $\sigma = \sigma_d \pm \sigma_b$

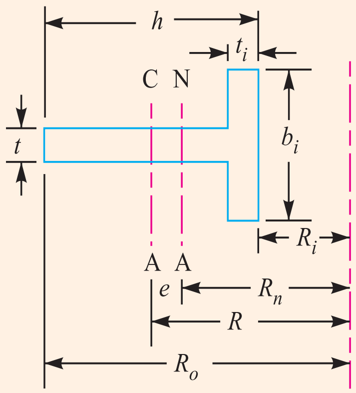
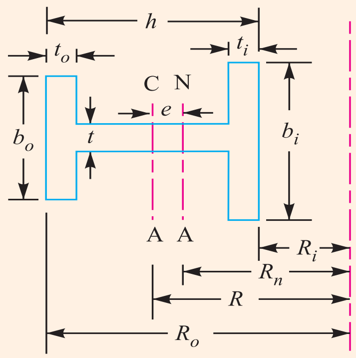
The following table shows the values of R_n and R for various commonly used cross-sections in curved beams.

Table 5.2. Values of R_n and R for various commonly used cross-section in curved beams.

Section	*Values of R_n and R*
	$R_n = \dfrac{h}{\log_e\left(\dfrac{R_o}{R_i}\right)}$ $R = R_i + \dfrac{h}{2}$

Section	*Values of R_n and R*
	$R_n = \dfrac{\left[\sqrt{R_o} + \sqrt{R_i}\right]^2}{4}$ $R = R_i + \dfrac{d}{2}$
	$R_n = \dfrac{\left(\dfrac{b_i + b_o}{2}\right)h}{\left(\dfrac{b_i\,R_o - b_o\,R_i}{h}\right)\log_e\left(\dfrac{R_o}{R_i}\right) - (b_i - b_o)}$ $R = R_i + \dfrac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)}$
	$R_n = \dfrac{\dfrac{1}{2}\,b_i \times h}{\dfrac{b_i\,R_o}{h}\log_e\left(\dfrac{R_o}{R_i}\right) - b_i}$ $R = R_i + \dfrac{h}{3}$
	$R_n = \dfrac{(b-t)(t_i + t_o) + t.h}{b\left[\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + \log_e\left(\dfrac{R_o}{R_o - t_o}\right)\right] + t.\log_e\left(\dfrac{R_o - t_o}{R_i + t_i}\right)}$ $R = R_i + \dfrac{\dfrac{1}{2}h^2.t + \dfrac{1}{2}\,t_i^{\,2}(b-t) + (b-t)\,t_o\left(h - \dfrac{1}{2}t_o\right)}{h.t + (b-t)(t_i + t_o)}$

Contents

Section	*Values of R_n and R*
	$R_n = \dfrac{t_i\,(b_i - t) + t.h}{(b_i - t)\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + t.\log_e\left(\dfrac{R_o}{R_i}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t)}{h.t + t_i\,(b_i - t)}$
	$R_n = \dfrac{t_i\,(b_i - t) + t_o(b_o - t) + t.h}{b_i\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + t\log_e\left(\dfrac{R_o - t_o}{R_i + t_i}\right) + b_o\log_e\left(\dfrac{R_o}{R_o - t_o}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t) + (b_o - t)\,t_o\,(h - \frac{1}{2}t_o)}{t_i\,(b_i - t) + t_o\,(b_o - t) + t.h}$

Example 5.10. *The frame of a punch press is shown in Fig. 5.9. Find the stresses at the inner and outer surface at section X-X of the frame, if W = 5000 N.*

Solution. Given : W = 5000 N ; b_i = 18 mm ; b_o = 6 mm ; h = 40 mm ; R_i = 25 mm ; R_o = 25 + 40 = 65 mm

We know that area of section at *X-X*,

$$A = \frac{1}{2}(18 + 6)\,40 = 480 \text{ mm}^2$$

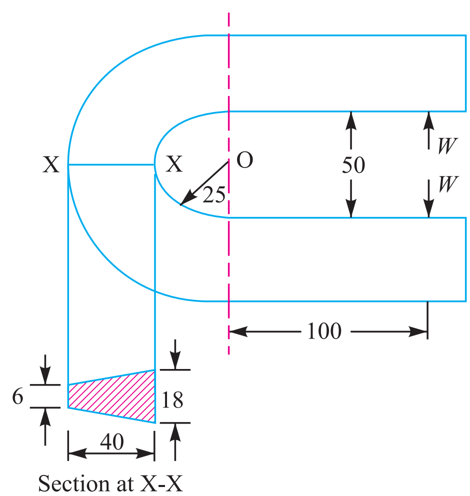
The various distances are shown in Fig. 5.10.

We know that radius of curvature of the neutral axis,

$$R_n = \frac{\left(\dfrac{b_i + b_o}{2}\right)h}{\left(\dfrac{b_i\,R_o - b_o\,R_i}{h}\right)\log_e\left(\dfrac{R_o}{R_i}\right) - (b_i - b_o)}$$

$$= \frac{\left(\dfrac{18 + 6}{2}\right)\times 40}{\left(\dfrac{18\times 65 - 6\times 25}{40}\right)\log_e\left(\dfrac{65}{25}\right) - (18 - 6)}$$

$$= \frac{480}{(25.5\times 0.9555) - 12} = 38.83 \text{ mm}$$



All dimensions in mm.

Fig. 5.9

Top

Contents

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)} = 25 + \frac{40\,(18 + 2 \times 6)}{3\,(18 + 6)} \text{ mm}$$

$$= 25 + 16.67 = 41.67 \text{ mm}$$

Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 41.67 - 38.83 = 2.84 \text{ mm}$$

and the distance between the load and centroidal axis,

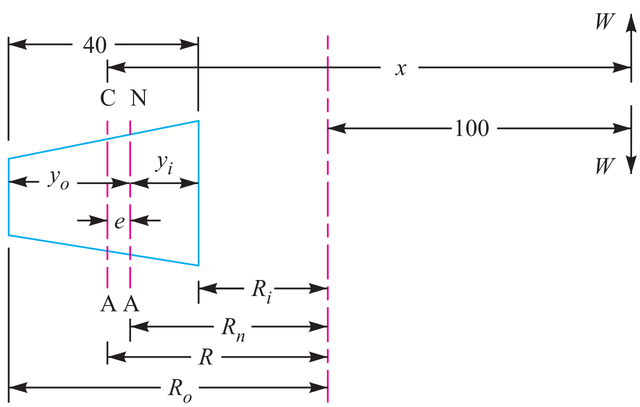
$$x = 100 + R = 100 + 41.67 = 141.67 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W.x = 5000 \times 141.67 = 708\,350 \text{ N-mm}$$

The section at *X-X* is subjected to a direct tensile load of W = 5000 N and a bending moment of M = 708 350 N-mm. We know that direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{5000}{480} = 10.42 \text{ N/mm}^2 = 10.42 \text{ MPa}$$



All dimensions in mm.

Fig. 5.10

Distance from the neutral axis to the inner surface,

$$y_i = R_n - R_i = 38.83 - 25 = 13.83 \text{ mm}$$

Distance from the neutral axis to the outer surface,

$$y_o = R_o - R_n = 65 - 38.83 = 26.17 \text{ mm}$$

We know that maximum bending stress at the inner surface,

$$\sigma_{bi} = \frac{M\,.\,y_i}{A\,.\,e\,.\,R_i} = \frac{708\,350 \times 13.83}{480 \times 2.84 \times 25} = 287.4 \text{ N/mm}^2$$

$$= 287.4 \text{ MPa (tensile)}$$

and maximum bending stress at the outer surface,

$$\sigma_{b0} = \frac{M\,.\,y_o}{A\,.\,e\,.\,R_o} = \frac{708\,350 \times 26.17}{480 \times 2.84 \times 65} = 209.2 \text{ N/mm}^2$$

$$= 209.2 \text{ MPa (compressive)}$$

Top

Contents

∴ Resultant stress on the inner surface

$$= \sigma_t + \sigma_{bi} = 10.42 + 287.4 = 297.82 \text{ MPa (tensile)} \textbf{ Ans.}$$

and resultant stress on the outer surface,

$$= \sigma_t - \sigma_{bo} = 10.42 - 209.2 = -198.78 \text{ MPa}$$

$$= 198.78 \text{ MPa (compressive)} \textbf{ Ans.}$$

A big crane hook

Example 5.11. *The crane hook carries a load of 20 kN as shown in Fig. 5.11. The section at X-X is rectangular whose horizontal side is 100 mm. Find the stresses in the inner and outer fibres at the given section.*

Solution. Given : $W = 20 \text{ kN} = 20 \times 10^3 \text{ N}$; $R_i = 50$ mm ; $R_o = 150$ mm ; $h = 100$ mm ; $b = 20$ mm

We know that area of section at *X-X*,

$$A = b.h = 20 \times 100 = 2000 \text{ mm}^2$$

The various distances are shown in Fig. 5.12.

We know that radius of curvature of the neutral axis,

$$R_n = \frac{h}{\log_e\left(\frac{R_o}{R_i}\right)} = \frac{100}{\log_e\left(\frac{150}{50}\right)} = \frac{100}{1.098} = 91.07 \text{ mm}$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h}{2} = 50 + \frac{100}{2} = 100 \text{ mm}$$

∴ Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 100 - 91.07 = 8.93 \text{ mm}$$

and distance between the load and the centroidal axis,

$$x = R = 100 \text{ mm}$$

∴ Bending moment about the centroidal axis,

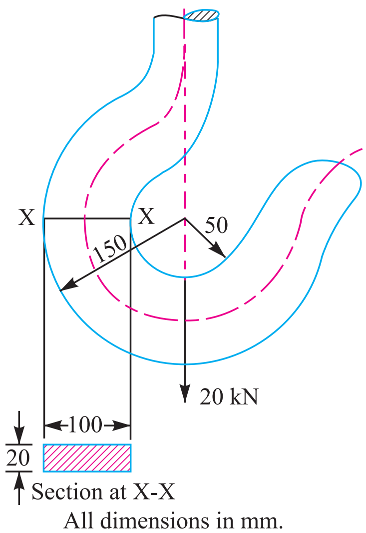
$$M = W \times x = 20 \times 10^3 \times 100 = 2 \times 10^6 \text{ N-mm}$$

Top

Contents

The section at *X-X* is subjected to a direct tensile load of $W = 20 \times 10^3$ N and a bending moment of $M = 2 \times 10^6$ N-mm. We know that direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{20 \times 10^3}{2000} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$



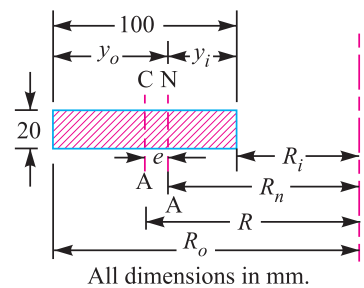


Fig. 5.11 **Fig. 5.12**

We know that the distance from the neutral axis to the inside fibre,

$$y_i = R_n - R_i = 91.07 - 50 = 41.07 \text{ mm}$$

and distance from the neutral axis to outside fibre,

$$y_o = R_o - R_n = 150 - 91.07 = 58.93 \text{ mm}$$

∴ Maximum bending stress at the inside fibre,

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i} = \frac{2 \times 10^6 \times 41.07}{2000 \times 8.93 \times 50} = 92 \text{ N/mm}^2 = 92 \text{ MPa (tensile)}$$

and maximum bending stress at the outside fibre,

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{2 \times 10^6 \times 58.93}{2000 \times 8.93 \times 150} = 44 \text{ N/mm}^2$$

$$= 44 \text{ MPa (compressive)}$$

∴ Resultant stress at the inside fibre

$$= \sigma_t + \sigma_{bi} = 10 + 92 = 102 \text{ MPa (tensile)}$$ **Ans.**

and resultant stress at the outside fibre

$$= \sigma_t - \sigma_{bo} = 10 - 44 = -34 \text{ MPa} = 34 \text{ MPa (compressive)}$$ **Ans.**

Example 5.12. *A C-clamp is subjected to a maximum load of W, as shown in Fig. 5.13. If the maximum tensile stress in the clamp is limited to 140 MPa, find the value of load W.*

Solution. Given : $\sigma_{t(max)}$ = 140 MPa = 140 N/mm^2 ; R_i = 25 mm ; R_o = 25 + 25 = 50 mm ; b_i = 19 mm ; t_i = 3 mm ; t = 3 mm ; h = 25 mm

We know that area of section at *X-X*,

$$A = 3 \times 22 + 3 \times 19 = 123 \text{ mm}^2$$

Top

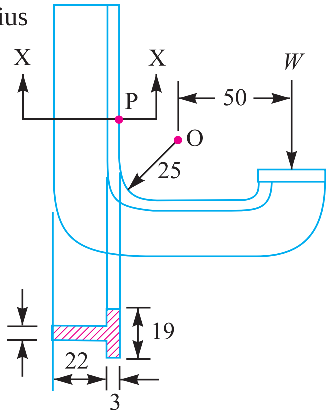
Contents

The various distances are shown in Fig. 5.14. We know that radius of curvature of the neutral axis,

$$R_n = \frac{t_i\,(b_i - t) + t\,.\,h}{(b_i - t)\log_e\left(\dfrac{R_i + t_i}{R_i}\right) + t\log_e\left(\dfrac{R_o}{R_i}\right)}$$

$$= \frac{3\,(19 - 3) + 3 \times 25}{(19 - 3)\log_e\left(\dfrac{25 + 3}{25}\right) + 3\log_e\left(\dfrac{50}{25}\right)}$$

$$= \frac{123}{16 \times 0.113 + 3 \times 0.693} = \frac{123}{3.887} = 31.64 \text{ mm}$$



Section of X-X

All dimensions in mm.

Fig. 5.13

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{\frac{1}{2}h^2\,.\,t + \frac{1}{2}t_i^2\,(b_i - t)}{h\,.\,t + t_i\,(b_i - t)}$$

$$= 25 + \frac{\frac{1}{2} \times 25^2 \times 3 + \frac{1}{2} \times 3^2\,(19 - 3)}{25 \times 3 + 3\,(19 - 3)} = 25 + \frac{937.5 + 72}{75 + 48}$$

$$= 25 + 8.2 = 33.2 \text{ mm}$$

Distance between the centroidal axis and neutral axis,

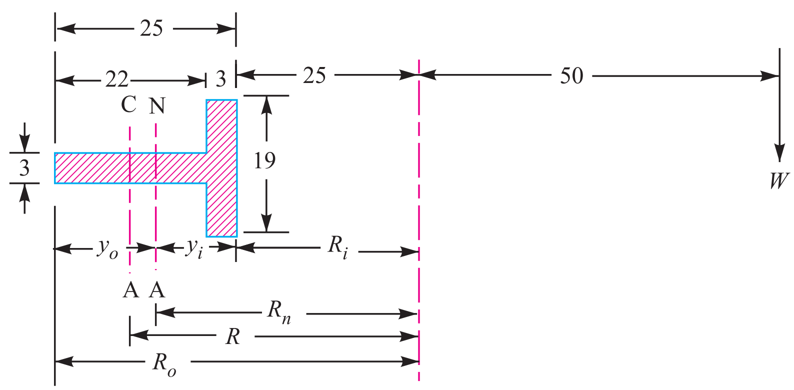
$$e = R - R_n = 33.2 - 31.64 = 1.56 \text{ mm}$$

and distance between the load W and the centroidal axis,

$$x = 50 + R = 50 + 33.2 = 83.2 \text{ mm}$$

$\therefore$ Bending moment about the centroidal axis,

$$M = W.x = W \times 83.2 = 83.2\,W \text{ N-mm}$$



All dimensions in mm.

Fig. 5.14

The section at X-X is subjected to a direct tensile load of W and a bending moment of 83.2 W. The maximum tensile stress will occur at point P (*i.e.* at the inner fibre of the section).

Distance from the neutral axis to the point P,

$$y_i = R_n - R_i = 31.64 - 25 = 6.64 \text{ mm}$$

Top

Contents

Direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{W}{123} = 0.008\,W \text{ N/mm}^2$$

and maximum bending stress at point *P*,

$$\sigma_{bi} = \frac{M\,.\,y_i}{A\,.\,e\,.\,R_i} = \frac{83.2\,W \times 6.64}{123 \times 1.56 \times 25} = 0.115\,W \text{ N/mm}^2$$

We know that the maximum tensile stress $\sigma_{t(max)}$,

$$140 = \sigma_t + \sigma_{bi} = 0.008\,W + 0.115\,W = 0.123\,W$$

$$\therefore \quad W = 140/0.123 = 1138 \text{ N} \quad \textbf{Ans.}$$

Note : We know that distance from the neutral axis to the outer fibre,

$$y_o = R_o - R_n = 50 - 31.64 = 18.36 \text{ mm}$$

$\therefore$ Maximum bending stress at the outer fibre,

$$\sigma_{bo} = \frac{M\,.\,y_o}{A\,.\,e\,.\,R_o} = \frac{83.2\,W \times 18.36}{123 \times 1.56 \times 50} = 0.16\,W$$

and maximum stress at the outer fibre,

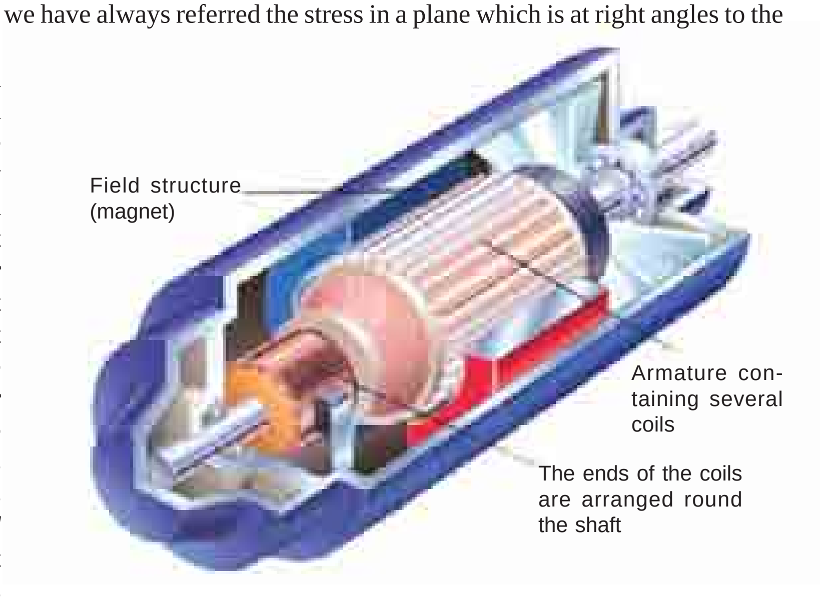
$$= \sigma_t - \sigma_{bo} = 0.008\,W - 0.16\,W = -\,0.152\,W \text{ N/mm}^2$$

$$= 0.152\,W \text{ N/mm}^2 \text{ (compressive)}$$

From above we see that stress at the outer fibre is larger in this case than at the inner fibre, but this stress at outer fibre is compressive.

5.6 Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as ***principal planes*** and the direct stresses along these planes are known as ***principal stresses.*** The planes on which the maximum shear stress act are known as planes of maximum shear.



Big electric generators undergo high torsional stresses.

Top

5.7 Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (*i.e.* direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body *ABCD* of uniform cross-sectional area and unit thickness subjected to normal stresses σ_1 and σ_2 as shown in Fig. 5.15 (*a*). In addition to these normal stresses, a shear stress τ also acts.

It has been shown in books on ***'Strength of Materials'*** that the normal stress across any oblique section such as *EF* inclined at an angle θ with the direction of σ_2, as shown in Fig. 5.15 (*a*), is given by

$$\sigma_t = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta \qquad \text{...}(i)$$

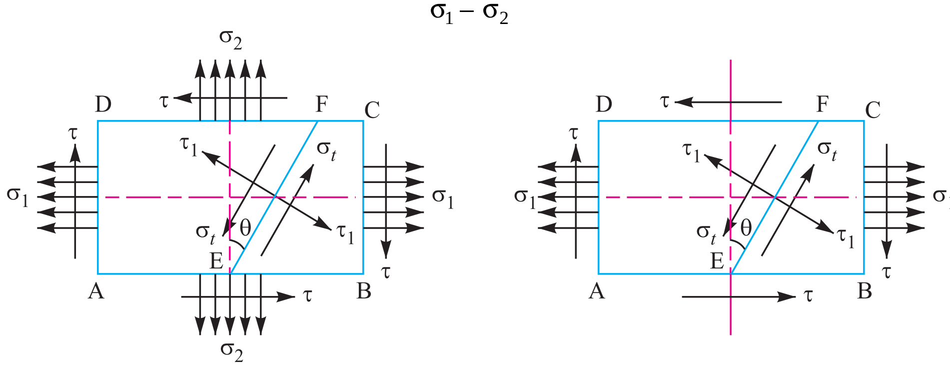
and tangential stress (*i.e.* shear stress) across the section *EF*,

$$\tau_1 = \frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta \qquad \text{...}(ii)$$

Since the planes of maximum and minimum normal stress (*i.e.* principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating $\tau_1 = 0$ in the above equation (*ii*), *i.e.*

$$\frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta = 0$$

$$\therefore \qquad \tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} \qquad \text{...}(iii)$$



(*a*) Direct stress in two mutually prependicular planes accompanied by a simple shear stress.

(*b*) Direct stress in one plane accompanied by a simple shear stress.

Fig. 5.15. Principal stresses for a member subjected to bi-axial stress.

We know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be the inclinations of these planes with the normal cross-section.

From Fig. 5.16, we find that

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\therefore \qquad \sin 2\theta_1 = +\frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

and
$$\sin 2\theta_2 = -\frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

Also
$$\cos 2\theta = \pm\frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

$$\therefore \qquad \cos 2\theta_1 = +\frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

and
$$\cos 2\theta_2 = -\frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

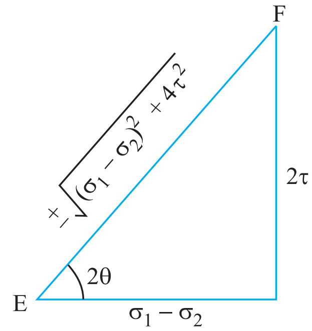


Fig. 5.16

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 2θ and cos 2θ in equation ***(i)***.

∴ Maximum principal (or normal) stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad \text{...}(iv)$$

and minimum principal (or normal) stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad \text{...}(v)$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by ***one-half the algebraic difference between the principal stresses,*** *i.e.*

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad \text{...}(vi)$$

A Boring mill.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Notes: 1. When a member is subjected to direct stress in one plane accompanied by a simple shear stress as shown in Fig. 5.15 (*b*), then the principal stresses are obtained by substituting $\sigma_2 = 0$ in equation **(*iv*), (*v*)** and **(*vi*).**

$$\therefore \quad \sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

and

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

2. In the above expression of σ_{t2}, the value of $\frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$ is more than $\frac{\sigma_1}{2}$. Therefore the nature of σ_{t2} will be opposite to that of σ_{t1}, *i.e.* if σ_{t1} is tensile then σ_{t2} will be compressive and *vice-versa*.

5.8 Application of Principal Stresses in Designing Machine Members

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained.

The results obtained in the previous article may be written as follows:

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

2. Maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

3. Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

where σ_t = Tensile stress due to direct load and bending,

σ_c = Compressive stress, and

τ = Shear stress due to torsion.

Notes : 1. When $\tau = 0$ as in the case of thin cylindrical shell subjected in internal fluid pressure, then

$$\sigma_{t\,(max)} = \sigma_t$$

2. When the shaft is subjected to an axial load (P) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). This will give the resultant tensile stress or compressive stress (σ_t or σ_c) depending upon the type of axial load (*i.e.* pull or push).

Example 5.13. *A hollow shaft of 40 mm outer diameter and 25 mm inner diameter is subjected to a twisting moment of 120 N-m, simultaneously, it is subjected to an axial thrust of 10 kN and a bending moment of 80 N-m. Calculate the maximum compressive and shear stresses.*

Solution. Given: d_o = 40 mm ; d_i = 25 mm ; T = 120 N-m = 120 × 10^3 N-mm ; P = 10 kN = 10 × 10^3 N ; M = 80 N-m = 80 × 10^3 N-mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] = \frac{\pi}{4}\left[(40)^2 - (25)^2\right] = 766 \text{ mm}^2$$

∴ Direct compressive stress due to axial thrust,

$$\sigma_o = \frac{P}{A} = \frac{10 \times 10^3}{766} = 13.05 \text{ N/mm}^2 = 13.05 \text{ MPa}$$

Section modulus of the shaft,

$$Z = \frac{\pi}{32}\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{32}\left[\frac{(40)^4 - (25)^4}{40}\right] = 5325 \text{ mm}^3$$

∴ Bending stress due to bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{80 \times 10^3}{5325} = 15.02 \text{ N/mm}^2 = 15.02 \text{ MPa (compressive)}$$

and resultant compressive stress,

$$\sigma_c = \sigma_b + \sigma_o = 15.02 + 13.05 = 28.07 \text{ N/mm}^2 = 28.07 \text{ MPa}$$

We know that twisting moment (T),

$$120 \times 10^3 = \frac{\pi}{16} \times \tau\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau\left[\frac{(40)^4 - (25)^4}{40}\right] = 10\,650\ \tau$$

$$\therefore \quad \tau = 120 \times 10^3/10\,650 = 11.27 \text{ N/mm}^2 = 11.27 \text{ MPa}$$

Maximum compressive stress

We know that maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

$$= \frac{28.07}{2} + \frac{1}{2}\left[\sqrt{(28.07)^2 + 4\,(11.27)^2}\right]$$

$$= 14.035 + 18 = 32.035 \text{ MPa } \textbf{Ans.}$$

Maximum shear stress

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(28.07)^2 + 4(11.27)^2}\right] = 18 \text{ MPa } \textbf{Ans.}$$

Example 5.14. *A shaft, as shown in Fig. 5.17, is subjected to a bending load of 3 kN, pure torque of 1000 N-m and an axial pulling force of 15 kN.*

Calculate the stresses at A and B.

Solution. Given : W = 3 kN = 3000 N ; T = 1000 N-m = 1 × 10^6 N-mm ; P = 15 kN = 15 × 10^3 N ; d = 50 mm; x = 250 mm

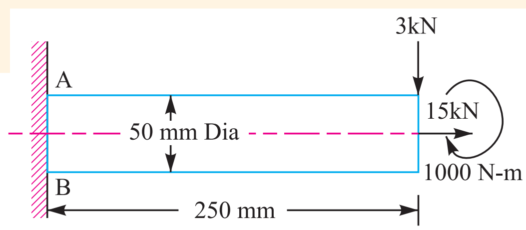


Fig. 5.17

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4}(50)^2 = 1964 \text{ mm}^2$$

∴ Tensile stress due to axial pulling at points A and B,

$$\sigma_o = \frac{P}{A} = \frac{15 \times 10^3}{1964} = 7.64 \text{ N/mm}^2 = 7.64 \text{ MPa}$$

Bending moment at points A and B,

$$M = W.x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (50)^3$$

$$= 12.27 \times 10^3 \text{ mm}^3$$

∴ Bending stress at points *A* and *B*,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3}$$

$$= 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa}$$

This bending stress is tensile at point *A* and compressive at point *B*.

∴ Resultant tensile stress at point *A*,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$
$$= 68.74 \text{ MPa}$$

and resultant compressive stress at point *B*,

$$\sigma_B = \sigma_b - \sigma_o = 61.1 - 7.64 = 53.46 \text{ MPa}$$

This picture shows a machine component inside a crane

Note : This picture is given as additional information and is not a direct example of the current chapter.

We know that the shear stress at points *A* and *B* due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\, d^3} = \frac{16 \times 1 \times 10^6}{\pi\,(50)^3} = 40.74 \text{ N/mm}^2 = 40.74 \text{ MPa} \qquad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Stresses at point A

We know that maximum principal (or normal) stress at point *A*,

$$\sigma_{A(max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right]$$

$$= \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 34.37 + 53.3 = 87.67 \text{ MPa (tensile)}$$ **Ans.**

Minimum principal (or normal) stress at point *A*,

$$\sigma_{A(min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = 34.37 - 53.3 = -18.93 \text{ MPa}$$

$$= 18.93 \text{ MPa (compressive)}$$ **Ans.**

and maximum shear stress at point *A*,

$$\tau_{A(max)} = \tfrac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 53.3 \text{ MPa}$$ **Ans.**

Stresses at point B

We know that maximum principal (or normal) stress at point *B*,

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right]$$

$$= \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 26.73 + 48.73 = 75.46 \text{ MPa (compressive)}$$ **Ans.**

Top

Minimum principal (or normal) stress at point *B*,

$$\sigma_{B(min)} = \frac{\sigma_B}{2} - \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right]$$

$$= 26.73 - 48.73 = -22 \text{ MPa}$$

$$= 22 \text{ MPa (tensile)}$$ **Ans.**

and maximum shear stress at point *B*,

$$\tau_{B(max)} = \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 48.73 \text{ MPa}$$ **Ans.**

Example 5.15. *An overhang crank with pin and shaft is shown in Fig. 5.18. A tangential load of 15 kN acts on the crank pin. Determine the maximum principal stress and the maximum shear stress at the centre of the crankshaft bearing.*

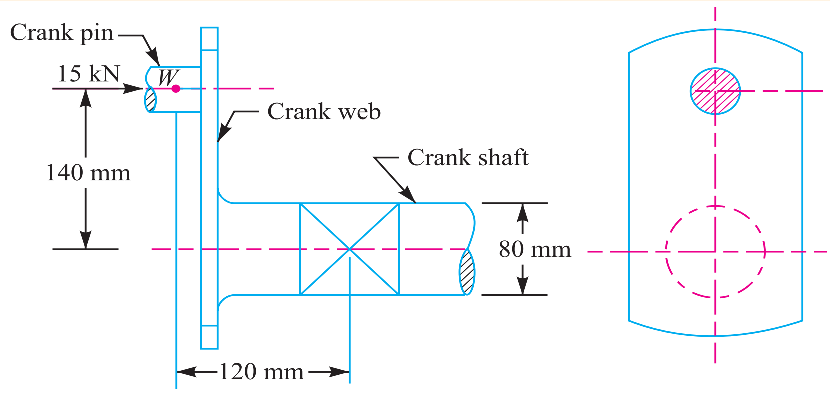


Fig. 5.18

Solution. Given : $W = 15$ kN $= 15 \times 10^3$ N ; $d = 80$ mm ; $y = 140$ mm ; $x = 120$ mm

Bending moment at the centre of the crankshaft bearing,

$$M = W \times x = 15 \times 10^3 \times 120 = 1.8 \times 10^6 \text{ N-mm}$$

and torque transmitted at the axis of the shaft,

$$T = W \times y = 15 \times 10^3 \times 140 = 2.1 \times 10^6 \text{ N-mm}$$

We know that bending stress due to the bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{32\,M}{\pi\,d^3} \qquad \ldots\left(\because Z = \frac{\pi}{32} \times d^3\right)$$

$$= \frac{32 \times 1.8 \times 10^6}{\pi\,(80)^3} = 35.8 \text{ N/mm}^2 = 35.8 \text{ MPa}$$

and shear stress due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16 \times 2.1 \times 10^6}{\pi\,(80)^3} = 20.9 \text{ N/mm}^2 = 20.9 \text{ MPa}$$

Maximum principal stress

We know that maximum principal stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

$$= \frac{35.8}{2} + \frac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right] \qquad \ldots (\text{Substituting } \sigma_t = \sigma_b)$$

$$= 17.9 + 27.5 = 45.4 \text{ MPa}$$ **Ans.**

Maximum shear stress

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right]$$

$$= 27.5 \text{ MPa } \textbf{Ans.}$$

5.9 Theories of Failure Under Static Load

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uni-axial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi-axial stress are as follows:

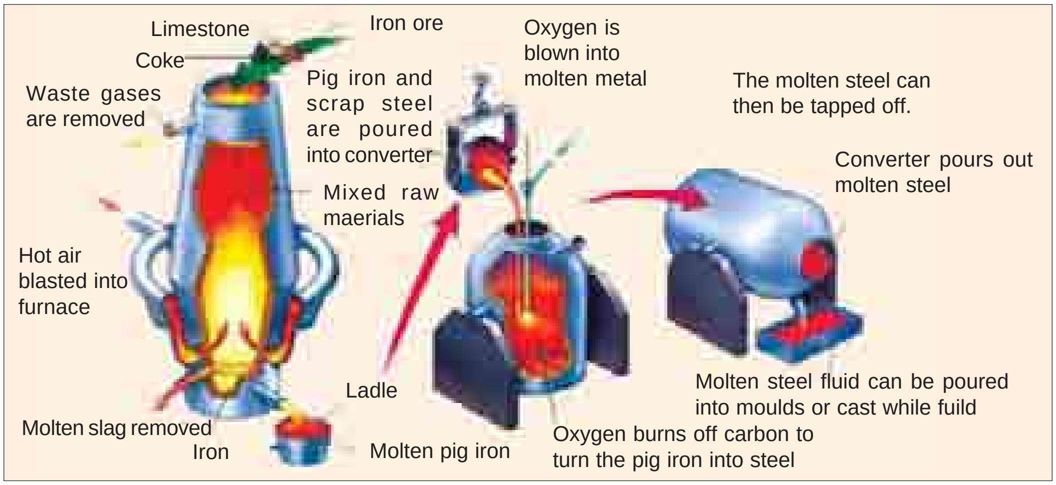
1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding *i.e.* when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

5.10 Maximum Principal or Normal Stress Theory (Rankine's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.

Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according



Pig iron is made from iron ore in a blast furnace. It is a brittle form of iron that contains 4-5 per cent carbon.

Note : This picture is given as additional information and is not a direct example of the current chapter.

to the above theory, taking factor of safety ($F.S.$) into consideration, the maximum principal or normal stress (σ_{t1}) in a bi-axial stress system is given by

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials}$$

$$= \frac{\sigma_u}{F.S.}, \text{ for brittle materials}$$

where σ_{yt} = Yield point stress in tension as determined from simple tension test, and

σ_u = Ultimate stress.

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

Note : The value of maximum principal stress (σ_{t1}) for a member subjected to bi-axial stress system may be determined as discussed in Art. 5.7.

5.11 Maximum Shear Stress Theory (Guest's or Tresca's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt}/F.S. \qquad ...(i)$$

where τ_{max} = Maximum shear stress in a bi-axial stress system,

τ_{yt} = Shear stress at yield point as determined from simple tension test, and

$F.S.$ = Factor of safety.

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation *(i)* may be written as

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.}$$

This theory is mostly used for designing members of ductile materials.

Note: The value of maximum shear stress in a bi-axial stress system (τ_{max}) may be determined as discussed in Art. 5.7.

5.12 Maximum Principal Strain Theory (Saint Venant's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (*i.e.* strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E}$$

∴ According to the above theory,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E} = \varepsilon = \frac{\sigma_{yt}}{E \times F.S.} \qquad ...(i)$$

where σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,

ε = Strain at yield point as determined from simple tension test,

$1/m$ = Poisson's ratio,

E = Young's modulus, and

$F.S.$ = Factor of safety.

From equation *(i)*, we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.S.}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

5.13 Maximum Strain Energy Theory (Haigh's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (*i.e.* strain energy at the yield point) per unit volume as determined from simple tension test.

This double-decker A 380 has a passenger capacity of 555. Its engines and parts should be robust which can bear high torsional and variable stresses.

We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

According to the above theory, $U_1 = U_2$.

$$\therefore \quad \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory may be used for ductile materials.

5.14 Maximum Distortion Energy Theory (Hencky and Von Mises Theory)

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (*i.e.* distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory is mostly used for ductile materials in place of maximum strain energy theory.

Note: The maximum distortion energy is the difference between the total strain energy and the strain energy due to uniform stress.

Contents

Example 5.16. *The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to*

1. Maximum principal stress theory; 2. Maximum shear stress theory; 3. Maximum principal strain theory; 4. Maximum strain energy theory; and 5. Maximum distortion energy theory.

Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3.

Solution. Given : P_{t1} = 10 kN ; P_s = 5 kN ; $\sigma_{t(el)}$ = 100 MPa = 100 N/mm^2 ; $1/m$ = 0.3

Let d = Diameter of the bolt in mm.

∴ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854\, d^2} = \frac{12.73}{d^2} \text{ kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854\, d^2} = \frac{6.365}{d^2} \text{ kN/mm}^2$$

1. *According to maximum principal stress theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{12.73}{2\,d^2} + \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4 + 4}\right]$$

$$= \frac{6.365}{d^2}\left[1 + \frac{1}{2}\sqrt{4+4}\right] = \frac{15.365}{d^2} \text{ kN/mm}^2 = \frac{15\,365}{d^2} \text{ N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \quad \text{or} \quad \frac{15\,365}{d^2} = 100$$

∴ d^2 = 15 365/100 = 153.65 or d = 12.4 mm **Ans.**

2. *According to maximum shear stress theory*

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right] = \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{9}{d^2} \text{ kN/mm}^2 = \frac{9000}{d^2} \text{ N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \quad \text{or} \quad \frac{9000}{d^2} = \frac{100}{2} = 50$$

∴ d^2 = 9000 / 50 = 180 or d = 13.42 mm **Ans.**

Top

3. *According to maximum principal strain theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\ 365}{d^2} \qquad \text{...(As calculated before)}$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-\,2.635}{d^2}\ \text{kN/mm}^2$$

$$= \frac{-\,2635}{d^2}\ \text{N/mm}^2$$

Front view of a jet engine. The rotors undergo high torsional and bending stresses.

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \quad \text{or} \quad \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\ 365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\ 156}{d^2} = 100$$

$d^2 = 16\ 156 / 100 = 161.56$ or $d = 12.7$ mm **Ans.**

4. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\ 365}{d^2}\right]^2 + \left[\frac{-\,2635}{d^2}\right]^2 - 2 \times \frac{15\ 365}{d^2} \times \frac{-\,2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\ 600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\ 724}{d^4} = 1$$

$\therefore$ $d^4 = 26\ 724$ or $d = 12.78$ mm **Ans.**

5. *According to maximum distortion energy theory*

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\ 365}{d^2}\right]^2 + \left[\frac{-\,2635}{d^2}\right]^2 - 2 \times \frac{15\ 365}{d^2} \times \frac{-\,2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\ 600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\ 391}{d^4} = 1$$

$\therefore$ $d^4 = 32\ 391$ or $d = 13.4$ mm **Ans.**

Contents

Example 5.17. *A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of bending moment 10 kN-m and a torsional moment 30 kN-m. Determine the diameter of the shaft using two different theories of failure, and assuming a factor of safety of 2. Take E = 210 GPa and poisson's ratio = 0.25.*

Solution. Given : σ_{yt} = 700 MPa = 700 N/mm^2 ; M = 10 kN-m = 10 × 10^6 N-mm ; T = 30 kN-m = 30 × 10^6 N-mm ; $F.S.$ = 2 ; E = 210 GPa = 210 × 10^3 N/mm^2 ; $1/m$ = 0.25

Let d = Diameter of the shaft in mm.

First of all, let us find the maximum and minimum principal stresses.

We know that section modulus of the shaft

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

∴ Bending (tensile) stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{10 \times 10^6}{0.0982\, d^3} = \frac{101.8 \times 10^6}{d^3} \text{ N/mm}^2$$

and shear stress due to torsional moment,

$$\tau = \frac{16\, T}{\pi\, d^3} = \frac{16 \times 30 \times 10^6}{\pi\, d^3} = \frac{152.8 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{101.8 \times 10^6}{2d^3} + \frac{1}{2}\left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8 \times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{161 \times 10^6}{d^3} = \frac{211.9 \times 10^6}{d^3} \text{ N/mm}^2$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{50.9 \times 10^6}{d^3} - \frac{161 \times 10^6}{d^3} = \frac{-110.1 \times 10^6}{d^3} \text{ N/mm}^2$$

Let us now find out the diameter of shaft (d) by considering the maximum shear stress theory and maximum strain energy theory.

1. According to maximum shear stress theory

We know that maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\left[\frac{211.9 \times 10^6}{d^3} + \frac{110.1 \times 10^6}{d^3}\right] = \frac{161 \times 10^6}{d^3}$$

We also know that according to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{yt}}{2\, F.S.} \quad \text{or} \quad \frac{161 \times 10^6}{d^3} = \frac{700}{2 \times 2} = 175$$

∴ d^3 = 161 × 10^6 / 175 = 920 × 10^3 or d = 97.2 mm **Ans.**

Top

Note: The value of maximum shear stress (τ_{max}) may also be obtained by using the relation,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8 \times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{1}{2} \times \frac{10^6}{d^3} \times 322 = \frac{161 \times 10^6}{d^3} \text{ N/mm}^2 \qquad \text{...(Same as before)}$$

2. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$\frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or
$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

$$\left[\frac{211.9 \times 10^6}{d^3}\right]^2 + \left[\frac{-110.1 \times 10^6}{d^3}\right]^2 - 2 \times \frac{211.9 \times 10^6}{d^3} \times \frac{-110.1 \times 10^6}{d^3} \times 0.25 = \left(\frac{700}{2}\right)^2$$

or
$$\frac{44\,902 \times 10^{12}}{d^6} + \frac{12\,122 \times 10^{12}}{d^6} + \frac{11\,665 \times 10^{12}}{d^6} = 122\,500$$

$$\frac{68\,689 \times 10^{12}}{d^6} = 122\,500$$

$\therefore$ $d^6 = 68\,689 \times 10^{12}/122\,500 = 0.5607 \times 10^{12}$ or $d = 90.8$ mm **Ans.**

Example 5.18. *A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000 N-m and a torque T. If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to 1. the maximum principal stress; 2. the maximum shear stress; and 3. the maximum distortion strain energy theory of yielding.*

Solution. Given: $d = 50$ mm ; $M = 2000$ N-m $= 2 \times 10^6$ N-mm ; $\sigma_{yt} = 200$ MPa $= 200$ N/mm^2

Let T = Maximum torque without causing yielding of the shaft, in N-mm.

1. *According to maximum principal stress theory*

We know that section modulus of the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}\,(50)^3 = 12\,273 \text{ mm}^3$$

$\therefore$ Bending stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{2 \times 10^6}{12\,273} = 163 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16\,T}{\pi\,(50)^3} = 0.0407 \times 10^{-3}\,T \text{ N/mm}^2$$

$$\ldots\left[\because T = \frac{\pi}{16} \times \tau \times d^3\right]$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} + \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

Minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} - \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\, T)^2}\right]$$

$$= 81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\, T)^2}\right]$$

$$= \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} \text{ N/mm}^2$$

We know that according to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{yt} \qquad \text{...(Taking } F.S. = 1)$$

$$\therefore \quad 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} = 200$$

$$6642.5 + 1.65 + 10^{-9}\, T^2 = (200 - 81.5)^2 = 14\,042$$

$$T^2 = \frac{14\,042 - 6642.5}{1.65 \times 10^{-9}} = 4485 \times 10^9$$

or $\quad T = 2118 \times 10^3$ N-mm = 2118 N-m **Ans.**

2. *According to maximum shear stress theory*

We know that according to maximum shear stress theory,

$$\tau_{max} = \tau_{yt} = \frac{\sigma_{yt}}{2}$$

$$\therefore \quad \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2} = \frac{200}{2} = 100$$

$$6642.5 + 1.65 \times 10^{-9}\, T^2 = (100)^2 = 10\,000$$

$$T^2 = \frac{10\,000 - 6642.5}{1.65 \times 10^{-9}} = 2035 \times 10^9$$

$\therefore \quad T = 1426 \times 10^3$ N-mm = 1426 N-m **Ans.**

3. *According to maximum distortion strain energy theory*

We know that according to maximum distortion strain energy theory

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \sigma_{t1} \times \sigma_{t2} = (\sigma_{yt})^2$$

$$\left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]^2 + \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]^2$$

$$- \left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right]\left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\, T^2}\right] = (200)^2$$

$$2\left[(81.5)^2 + 6642.5 + 1.65 \times 10^{-9}\, T^2\right] - \left[(81.5)^2 - 6642.5 + 1.65 \times 10^{-9}\, T^2\right] = (200)^2$$

$$(81.5)^2 + 3 \times 6642.5 + 3 \times 1.65 \times 10^{-9}\, T^2 = (200)^2$$

$$26\,570 + 4.95 \times 10^{-9}\, T^2 = 40\,000$$

$$T^2 = \frac{40\,000 - 26\,570}{4.95 \times 10^{-9}} = 2713 \times 10^9$$

$\therefore \quad T = 1647 \times 10^3$ N-mm = 1647 N-m **Ans.**

5.15 Eccentric Loading - Direct and Bending Stresses Combined

An external load, whose line of action is parallel but does not coincide with the centroidal axis of the machine component, is known as an ***eccentric load***. The distance between the centroidal axis of the machine component and the eccentric load is called ***eccentricity*** and is generally denoted by *e*. The examples of eccentric loading, from the subject point of view, are *C*-clamps, punching machines, brackets, offset connecting links etc.

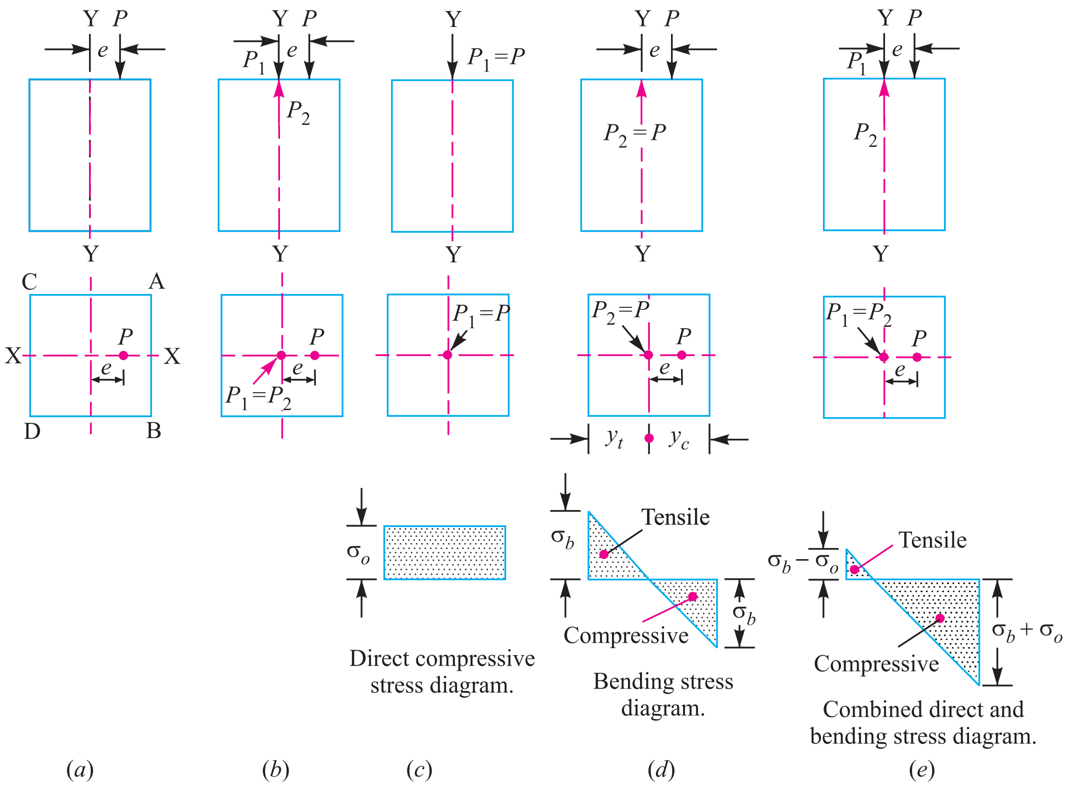


Fig. 5.19. Eccentric loading.

Consider a short prismatic bar subjected to a compressive load *P* acting at an eccentricity of *e* as shown in Fig. 5.19 (*a*).

Let us introduce two forces P_1 and P_2 along the centre line or neutral axis equal in magnitude to *P*, without altering the equilibrium of the bar as shown in Fig. 5.19 (*b*). A little consideration will show that the force P_1 will induce a direct compressive stress over the entire cross-section of the bar, as shown in Fig. 5.19 (*c*).

The magnitude of this direct compressive stress is given by

$$\sigma_o = \frac{P_1}{A} \text{ or } \frac{P}{A}, \text{ where } A \text{ is the cross-sectional area of the bar.}$$

The forces P_1 and P_2 will form a couple equal to $P \times e$ which will cause bending stress. This bending stress is compressive at the edge *AB* and tensile at the edge *CD*, as shown in Fig. 5.19 (*d*). The magnitude of bending stress at the edge *AB* is given by

$$\sigma_b = \frac{P \,.\, e \,.\, y_c}{I} \quad \text{(compressive)}$$

and bending stress at the edge *CD*,

$$\sigma_b = \frac{P \,.\, e \,.\, y_t}{I} \quad \text{(tensile)}$$

where y_c and y_t = Distances of the extreme fibres on the compressive and tensile sides, from the neutral axis respectively, and

I = Second moment of area of the section about the neutral axis *i.e.* Y-axis.

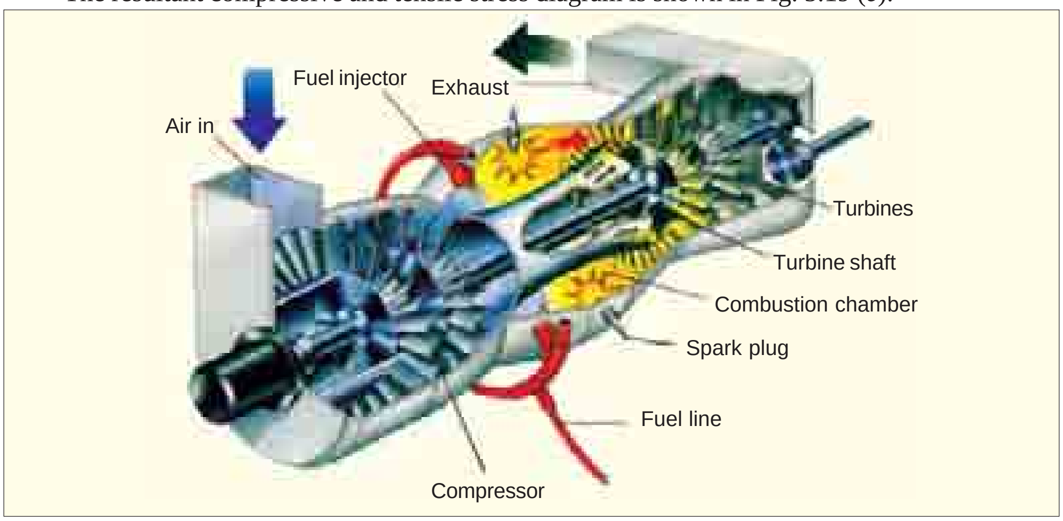
According to the principle of superposition, the maximum or the resultant compressive stress at the edge AB,

$$\sigma_c = \frac{P \,.\, e \,.\, y_c}{I} + \frac{P}{A} = {}^{*}\frac{M}{Z} + \frac{P}{A} = \sigma_b + \sigma_o$$

and the maximum or resultant tensile stress at the edge CD,

$$\sigma_t = \frac{P \,.\, e \,.\, y_t}{I} - \frac{P}{A} = \frac{M}{Z} - \frac{P}{A} = \sigma_b - \sigma_o$$

The resultant compressive and tensile stress diagram is shown in Fig. 5.19 (*e*).



In a gas-turbine system, a compressor forces air into a combustion chamber. There, it mixes with fuel. The mixture is ignited by a spark. Hot gases are produced when the fuel burns. They expand and drive a series of fan blades called a turbine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Notes: 1. When the member is subjected to a tensile load, then the above equations may be used by interchanging the subscripts c and t.

2. When the direct stress σ_o is greater than or equal to bending stress σ_b, then the compressive stress shall be present all over the cross-section.

3. When the direct stress σ_o is less than the bending stress σ_b, then the tensile stress will occur in the left hand portion of the cross-section and compressive stress on the right hand portion of the cross-section. In Fig. 5.19, the stress diagrams are drawn by taking σ_o less than σ_b.

In case the eccentric load acts with eccentricity about two axes, as shown in Fig. 5.20, then the total stress at the extreme fibre

$$= \frac{P}{A} \pm \frac{P \,.\, e_x \,.\, x}{I_{XX}} \pm \frac{P \,.\, e_y \,.\, y}{I_{YY}}$$

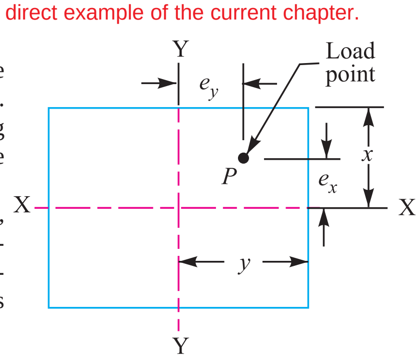


Fig. 5.20. Eccentric load with eccentricity about two axes.

* We know that bending moment, $M = P.e$ and section modulus, $Z = \dfrac{I}{y} = \dfrac{I}{y_c \text{ or } y_t}$

$\therefore$ Bending stress, $\sigma_b = M / Z$

Example 5.19. *A rectangular strut is 150 mm wide and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness as shown in Fig. 5.21. Find the maximum and minimum intensities of stress in the section.*

Solution. Given : b = 150 mm ; d = 120 mm ; P = 180 kN = 180×10^3 N ; e = 10 mm

We know that cross-sectional area of the strut,

$$A = b.d = 150 \times 120 = 18 \times 10^3 \text{ mm}^2$$

∴ Direct compressive stress,

$$\sigma_o = \frac{P}{A} = \frac{180 \times 10^3}{18 \times 10^3} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$

Section modulus for the strut,

$$Z = \frac{I_{YY}}{y} = \frac{d \cdot b^3/12}{b/2} = \frac{d \cdot b^2}{6} = \frac{120\,(150)^2}{6} = 450 \times 10^3 \text{ mm}^3$$

Bending moment, $M = P.e = 180 \times 10^3 \times 10 = 1.8 \times 10^6$ N-mm

∴ Bending stress, $\sigma_b = \dfrac{M}{Z} = \dfrac{1.8 \times 10^6}{450 \times 10^3} = 4 \text{ N/mm}^2 = 4$ MPa

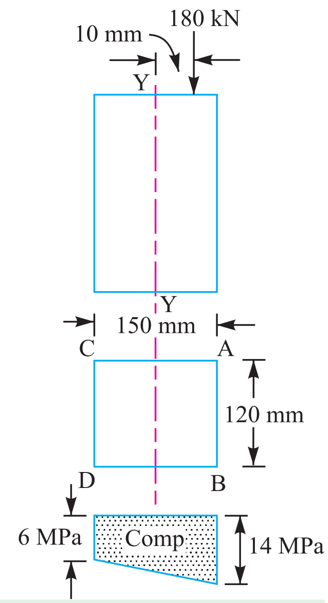


Fig. 5.21

Since σ_o is greater than σ_b, therefore the entire cross-section of the strut will be subjected to compressive stress. The maximum intensity of compressive stress will be at the edge *AB* and minimum at the edge *CD*.

∴ Maximum intensity of compressive stress at the edge *AB*

$$= \sigma_o + \sigma_b = 10 + 4 = 14 \text{ MPa } \textbf{Ans.}$$

and minimum intensity of compressive stress at the edge *CD*

$$= \sigma_o - \sigma_b = 10 - 4 = 6 \text{ MPa } \textbf{Ans.}$$

Example 5.20. *A hollow circular column of external diameter 250 mm and internal diameter 200 mm, carries a projecting bracket on which a load of 20 kN rests, as shown in Fig. 5.22. The centre of the load from the centre of the column is 500 mm. Find the stresses at the sides of the column.*

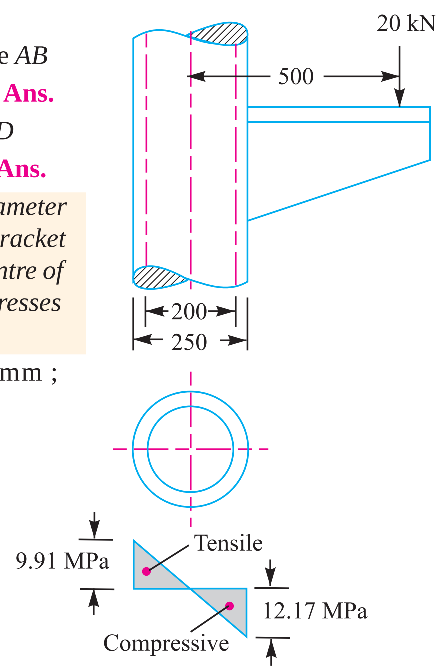


Fig. 5.22

Solution. Given : D = 250 mm ; d = 200 mm ; P = 20 kN = 20×10^3 N ; e = 500 mm

We know that cross-sectional area of column,

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(250)^2 - (200)^2] = 17\ 674 \text{ mm}^2$$

∴ Direct compressive stress,

$$\sigma_o = \frac{P}{A} = \frac{20 \times 10^3}{17\ 674} = 1.13 \text{ N/mm}^2 = 1.13 \text{ MPa}$$

Section modulus for the column,

$$Z = \frac{I}{y} = \frac{\frac{\pi}{64}\left[D^4 - d^4\right]}{D/2} = \frac{\frac{\pi}{64}\left[(250)^4 - (200)^4\right]}{250/2}$$

$$= 905.8 \times 10^3 \text{ mm}^3$$

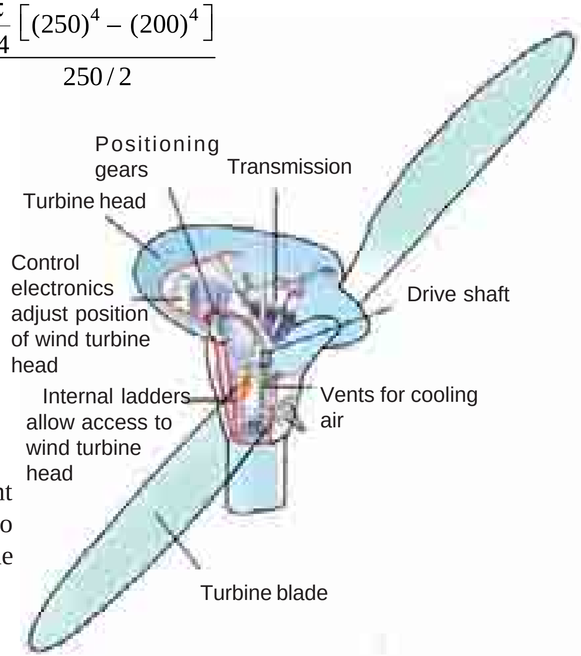
Bending moment,

$$M = P.e$$
$$= 20 \times 10^3 \times 500$$
$$= 10 \times 10^6 \text{ N-mm}$$

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{10 \times 10^6}{905.8 \times 10^3}$$
$$= 11.04 \text{ N/mm}^2$$
$$= 11.04 \text{ MPa}$$

Since σ_o is less than σ_b, therefore right hand side of the column will be subjected to compressive stress and the left hand side of the column will be subjected to tensile stress.



Wind turbine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

∴ Maximum compressive stress,

$$\sigma_c = \sigma_b + \sigma_o = 11.04 + 1.13$$
$$= 12.17 \text{ MPa } \textbf{Ans.}$$

and maximum tensile stress,

$$\sigma_t = \sigma_b - \sigma_o = 11.04 - 1.13 = 9.91 \text{ MPa } \textbf{Ans.}$$

Example 5.21. *A masonry pier of width 4 m and thickness 3 m, supports a load of 30 kN as shown in Fig. 5.23. Find the stresses developed at each corner of the pier.*

Solution. Given: $b = 4$ m ; $d = 3$ m ; $P = 30$ kN ; $e_x = 0.5$ m ; $e_y = 1$ m

We know that cross-sectional area of the pier,

$$A = b \times d = 4 \times 3 = 12 \text{ m}^2$$

Moment of inertia of the pier about X-axis,

$$I_{XX} = \frac{b \cdot d^3}{12} = \frac{4 \times 3^3}{12} = 9 \text{ m}^4$$

and moment of inertia of the pier about Y-axis,

$$I_{YY} = \frac{d \cdot b^3}{12} = \frac{3 \times 4^3}{12} = 16 \text{ m}^4$$

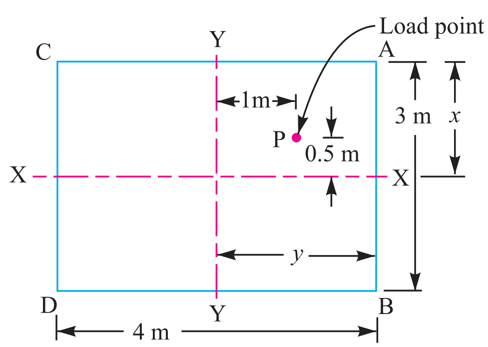


Fig. 5.23

Distance between X-axis and the corners A and B,

$$x = 3/2 = 1.5 \text{ m}$$

Distance between Y-axis and the corners A and C,

$$y = 4/2 = 2 \text{ m}$$

We know that stress at corner A,

$$\sigma_A = \frac{P}{A} + \frac{P \cdot e_x \cdot x}{I_{XX}} + \frac{P \cdot e_y \cdot y}{I_{YY}} \qquad \text{... [}\because \text{ At } A\text{, both } x \text{ and } y \text{ are +ve]}$$

Contents

$$= \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 + 2.5 + 3.75 = 8.75 \text{ kN/m}^2 \textbf{ Ans.}$$

Similarly stress at corner B,

$$\sigma_B = \frac{P}{A} + \frac{P \cdot e_x \cdot x}{I_{XX}} - \frac{P \cdot e_y \cdot y}{I_{YY}} \quad \text{... [}\because \text{ At } B, x \text{ is +ve and } y \text{ is –ve]}$$

$$= \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 + 2.5 - 3.75 = 1.25 \text{ kN/m}^2 \textbf{ Ans.}$$

Stress at corner C,

$$\sigma_C = \frac{P}{A} - \frac{P \cdot e_x \cdot x}{I_{XX}} + \frac{P \cdot e_y \cdot y}{I_{YY}} \quad \text{... [At } C, x \text{ is –ve and } y \text{ is +ve]}$$

$$= \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 - 2.5 + 3.75 = 3.75 \text{ kN/m}^2 \textbf{ Ans.}$$

and stress at corner D,

$$\sigma_D = \frac{P}{A} - \frac{P \cdot e_x \cdot x}{I_{XX}} - \frac{P \cdot e_y \cdot y}{I_{YY}} \quad \text{... [At } D, \text{ both } x \text{ and } y \text{ are – ve]}$$

$$= \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 - 2.5 - 3.75 = -3.75 \text{ kN/m}^2 = 3.75 \text{ kN/m}^2 \text{ (tensile) } \textbf{Ans.}$$

Example 5.22. *A mild steel link, as shown in Fig. 5.24 by full lines, transmits a pull of 80 kN. Find the dimensions b and t if b = 3t. Assume the permissible tensile stress as 70 MPa. If the original link is replaced by an unsymmetrical one, as shown by dotted lines in Fig. 5.24, having the same thickness t, find the depth b_1, using the same permissible stress as before.*

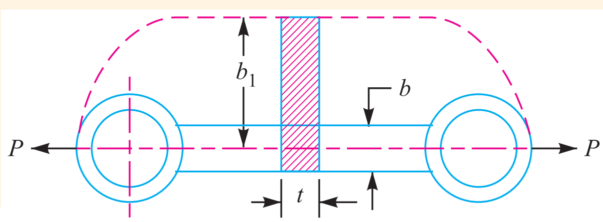


Fig. 5.24

Solution. Given : P = 80 kN = 80×10^3 N ; σ_t = 70 MPa = 70 N/mm^2

When the link is in the position shown by full lines in Fig. 5.24, the area of cross-section,

$$A = b \times t = 3\,t \times t = 3\,t^2 \quad \text{...(}\because\ b = 3t\text{)}$$

We know that tensile load (P),

$$80 \times 10^3 = \sigma_t \times A = 70 \times 3\,t^2 = 210\,t^2$$

$\therefore$ $t^2 = 80 \times 10^3 / 210 = 381$ or $t = 19.5$ say 20 mm **Ans.**

and $b = 3\,t = 3 \times 20 = 60$ mm **Ans.**

When the link is in the position shown by dotted lines, it will be subjected to direct stress as well as bending stress. We know that area of cross-section,

$$A_1 = b_1 \times t$$

Top

Contents

∴ Direct tensile stress,

$$\sigma_o = \frac{P}{A} = \frac{P}{b_1 \times t}$$

and bending stress,
$$\sigma_b = \frac{M}{Z} = \frac{P\,.\,e}{Z} = \frac{6\,P\,.\,e}{t\,(b_1)^2} \qquad \ldots \left(\because Z = \frac{t\,(b_1)^2}{6}\right)$$

∴ Total stress due to eccentric loading

$$= \sigma_b + \sigma_o = \frac{6\,P\,.\,e}{t\,(b_1)^2} + \frac{P}{b_1 \times t} = \frac{P}{t\,.\,b_1}\left(\frac{6\,e}{b_1} + 1\right)$$

Since the permissible tensile stress is the same as 70 N/mm^2, therefore

$$70 = \frac{80 \times 10^3}{20\,b_1}\left(\frac{6 \times b_1}{b_1 \times 2} + 1\right) = \frac{16 \times 10^3}{b_1} \qquad \ldots \left(\because \text{Eccentricity, } e = \frac{b_1}{2}\right)$$

∴ $b_1 = 16 \times 10^3 / 70 = 228.6$ say 230 mm **Ans.**

Example 5.23. *A cast-iron link, as shown in Fig. 5.25, is to carry a load of 20 kN. If the tensile and compressive stresses in the link are not to exceed 25 MPa and 80 MPa respectively, obtain the dimensions of the cross-section of the link at the middle of its length.*

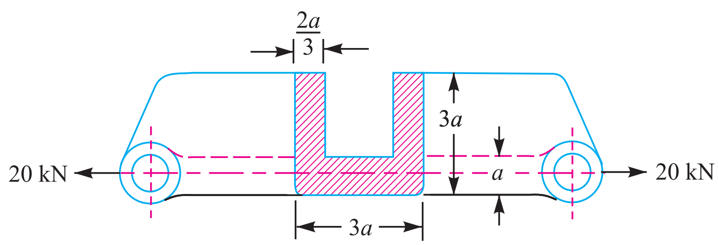


Fig. 5.25

Solution. Given : P = 20 kN = 20 × 10^3 N ; $\sigma_{t(max)}$ = 25 MPa = 25 N/mm^2 ; $\sigma_{c(max)}$ = 80 MPa = 80 N/mm^2

Since the link is subjected to eccentric loading, therefore there will be direct tensile stress as well as bending stress. The bending stress at the bottom of the link is tensile and in the upper portion is compressive.

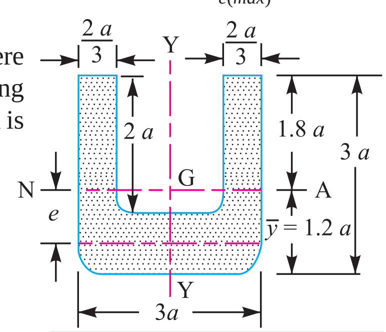


Fig. 5.26

We know that cross-sectional area of the link,

$$A = 3a \times a + 2 \times \frac{2a}{3} \times 2a$$
$$= 5.67\,a^2 \text{ mm}^2$$

∴ Direct tensile stress,

$$\sigma_o = \frac{P}{A} = \frac{20 \times 10^3}{5.67\,a^2} = \frac{3530}{a^2} \text{ N/mm}^2$$

Now let us find the position of centre of gravity (or neutral axis) in order to find the bending stresses.

Let $\bar{y}$ = Distance of neutral axis (N.A.) from the bottom of the link as shown in Fig. 5.26.

$$\therefore \quad \bar{y} = \frac{3a^2 \times \dfrac{a}{2} + 2 \times \dfrac{4a^2}{3} \times 2a}{5.67\,a^2} = 1.2\,a \text{ mm}$$

Top

Moment of inertia about N.A.,

$$I = \left[\frac{3a \times a^3}{12} + 3a^2 (1.2a - 0.5a)^2\right] + 2\left[\frac{\frac{2}{3} a \times (2a)^3}{12} + \frac{4a^2}{3} (2a - 1.2a)^2\right]$$

$$= (0.25\ a^4 + 1.47\ a^4) + 2\ (0.44a^4 + 0.85\ a^4) = 4.3\ a^4 \text{ mm}^4$$

Distance of N.A. from the bottom of the link,

$$y_t = \bar{y} = 1.2\ a \text{ mm}$$

Distance of N.A. from the top of the link,

$$y_c = 3\ a - 1.2\ a = 1.8\ a \text{ mm}$$

Eccentricity of the load (*i.e.* distance of N.A. from the point of application of the load),

$$e = 1.2\ a - 0.5\ a = 0.7\ a \text{ mm}$$

We know that bending moment exerted on the section,

$$M = P.e = 20 \times 10^3 \times 0.7\ a = 14 \times 10^3\ a \text{ N-mm}$$

∴ Tensile stress in the bottom of the link,

$$\sigma_t = \frac{M}{Z_t} = \frac{M}{I / y_t} = \frac{M\ .\ y_t}{I} = \frac{14 \times 10^3\ a \times 1.2\ a}{4.3\ a^4} = \frac{3907}{a^2}$$

and compressive stress in the top of the link,

$$\sigma_c = \frac{M}{Z_c} = \frac{M}{I / y_c} = \frac{M\ .\ y_c}{I} = \frac{14 \times 10^3\ a \times 1.8\ a}{4.3\ a^4} = \frac{5860}{a^2}$$

We know that maximum tensile stress [$\sigma_{t\ (max)}$],

$$25 = \sigma_t + \sigma_c = \frac{3907}{a^2} + \frac{5860}{a^2} = \frac{9767}{a^2}$$

∴ $a^2 = 9767 / 25 = 390.7$ or $a = 19.76$ mm ...(*i*)

and maximum compressive stress [$\sigma_{c(max)}$],

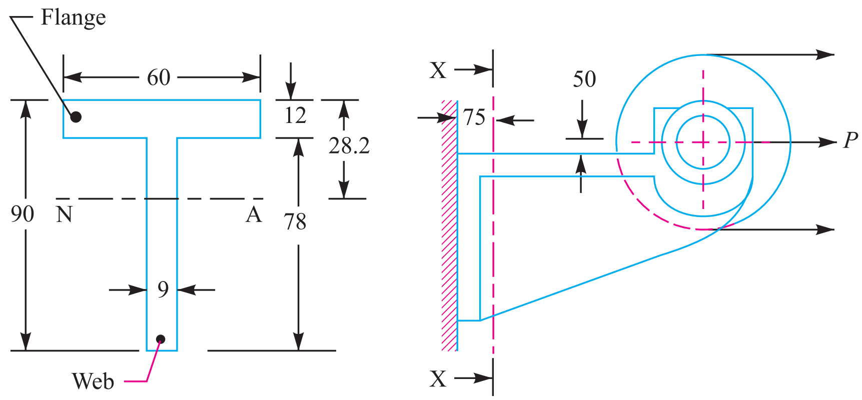
$$80 = \sigma_c - \sigma_0 = \frac{5860}{a^2} - \frac{3530}{a^2} = \frac{2330}{a^2}$$

∴ $a^2 = 2330 / 80 = 29.12$ or $a = 5.4$ mm ...(*ii*)

We shall take the larger of the two values, *i.e.*

$$a = 19.76 \text{ mm}$$ **Ans.**

Example 5.24. *A horizontal pull P = 5 kN is exerted by the belting on one of the cast iron wall brackets which carry a factory shafting. At a point 75 mm from the wall, the bracket has a T-section as shown in Fig. 5.27. Calculate the maximum stresses in the flange and web of the bracket due to the pull.*



All dimensions in mm.

Fig. 5.27

Solution. Given : Horizontal pull, P = 5 kN = 5000 N

Since the section is subjected to eccentric loading, therefore there will be direct tensile stress as well as bending stress. The bending stress at the flange is tensile and in the web is compressive.

We know that cross-sectional area of the section,

$$A = 60 \times 12 + (90 - 12)9 = 720 + 702 = 1422 \text{ mm}^2$$

$\therefore$ Direct tensile stress, $\sigma_0 = \dfrac{P}{A} = \dfrac{5000}{1422} = 3.51 \text{ N/mm}^2 = 3.51 \text{ MPa}$

Now let us find the position of neutral axis in order to determine the bending stresses. The neutral axis passes through the centre of gravity of the section.

Let $\bar{y}$ = Distance of centre of gravity (*i.e.* neutral axis) from top of the flange.

$$\therefore \quad \bar{y} = \frac{60 \times 12 \times \frac{12}{2} + 78 \times 9 \left(12 + \frac{78}{2}\right)}{720 + 702} = 28.2 \text{ mm}$$

Moment of inertia of the section about N.A.,

$$I = \left[\frac{60\,(12)^3}{12} + 720\,(28.2 - 6)^2\right] + \left[\frac{9\,(78)^3}{12} + 702\,(51 - 28.2)^2\right]$$

$$= (8640 + 354\,845) + (355\,914 + 364\,928) = 1\,084\,327 \text{ mm}^4$$

This picture shows a reconnoissance helicopter of air force. Its dark complexion absorbs light that falls on its surface. The flat and sharp edges deflect radar waves and they do not return back to the radar. These factors make it difficult to detect the helicopter.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Distance of N.A. from the top of the flange,

$$y_t = \bar{y} = 28.2 \text{ mm}$$

Distance of N.A. from the bottom of the web,

$$y_c = 90 - 28.2 = 61.8 \text{ mm}$$

Distance of N.A. from the point of application of the load (*i.e.* eccentricity of the load),

$$e = 50 + 28.2 = 78.2 \text{ mm}$$

We know that bending moment exerted on the section,

$$M = P \times e = 5000 \times 78.2 = 391 \times 10^3 \text{ N-mm}$$

∴ Tensile stress in the flange,

$$\sigma_t = \frac{M}{Z_t} = \frac{M}{I / y_t} = \frac{M \cdot y_t}{I} = \frac{391 \times 10^3 \times 28.2}{1\,084\,327} = 10.17 \text{ N/mm}^2$$
$$= 10.17 \text{ MPa}$$

and compressive stress in the web,

$$\sigma_c = \frac{M}{Z_c} = \frac{M}{I / y_c} = \frac{M \cdot y_c}{I} = \frac{391 \times 10^3 \times 61.8}{1\,084\,327} = 22.28 \text{ N/mm}^2$$
$$= 22.28 \text{ MPa}$$

We know that maximum tensile stress in the flange,

$$\sigma_{t(max)} = \sigma_b + \sigma_o = \sigma_t + \sigma_o = 10.17 + 3.51 = 13.68 \text{ MPa}$$ **Ans.**

and maximum compressive stress in the flange,

$$\sigma_{c(max)} = \sigma_b - \sigma_o = \sigma_c - \sigma_o = 22.28 - 3.51 = 18.77 \text{ MPa}$$ **Ans.**

Example 5.25. *A mild steel bracket as shown in Fig. 5.28, is subjected to a pull of 6000 N acting at 45° to its horizontal axis. The bracket has a rectangular section whose depth is twice the thickness. Find the cross-sectional dimensions of the bracket, if the permissible stress in the material of the bracket is limited to 60 MPa.*

Solution. Given : $P = 6000$ N ; $\theta = 45°$; $\sigma = 60$ MPa $= 60$ N/mm²

Let t = Thickness of the section in mm, and

b = Depth or width of the section $= 2\,t$ ...(Given)

We know that area of cross-section,

$$A = b \times t = 2\,t \times t = 2\,t^2 \text{ mm}^2$$

and section modulus,

$$Z = \frac{t \times b^2}{6} = \frac{t\,(2t)^2}{6} = \frac{4\,t^3}{6} \text{ mm}^3$$

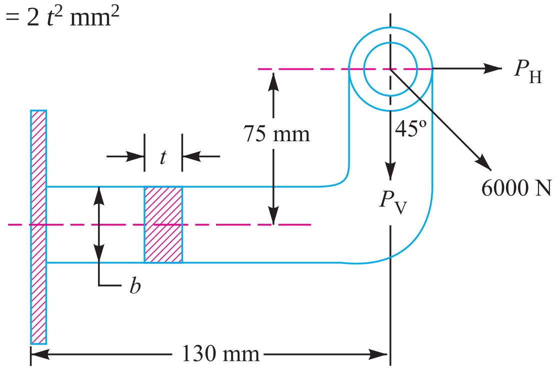


Fig. 5.28

Horizontal component of the load,

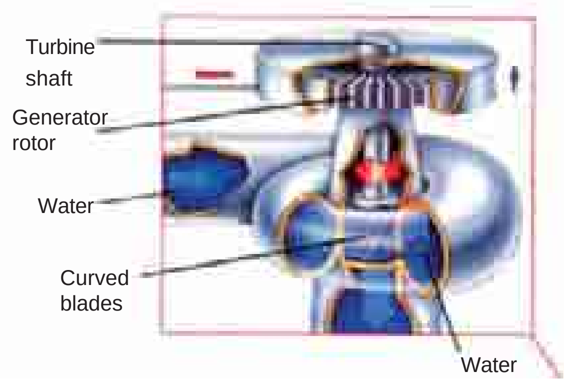
$$P_H = 6000 \cos 45° = 6000 \times 0.707 = 4242 \text{ N}$$

∴ Bending moment due to horizontal component of the load,

$$M_H = P_H \times 75 = 4242 \times 75 = 318\,150 \text{ N-mm}$$

A little consideration will show that the bending moment due to the horizontal component of the load induces tensile stress on the upper surface of the bracket and compressive stress on the lower surface of the bracket.

∴ Maximum bending stress on the upper surface due to horizontal component,



Schematic of a hydel turbine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$\sigma_{bH} = \frac{M_H}{Z} = \frac{318\ 150 \times 6}{4\ t^3}$$

$$= \frac{477\ 225}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

Vertical component of the load,

$$P_V = 6000 \sin 45° = 6000 \times 0.707 = 4242 \text{ N}$$

∴ Direct stress due to vertical component,

$$\sigma_{oV} = \frac{P_V}{A} = \frac{4242}{2t^2} = \frac{2121}{t^2} \text{ N/mm}^2 \text{ (tensile)}$$

Bending moment due to vertical component of the load,

$$M_V = P_V \times 130 = 4242 \times 130 = 551\ 460 \text{ N-mm}$$

This bending moment induces tensile stress on the upper surface and compressive stress on the lower surface of the bracket.

∴ Maximum bending stress on the upper surface due to vertical component,

$$\sigma_{bV} = \frac{M_V}{Z} = \frac{551\ 460 \times 6}{4\ t^3} = \frac{827\ 190}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

and total tensile stress on the upper surface of the bracket,

$$\sigma = \frac{477\ 225}{t^3} + \frac{2121}{t^2} + \frac{827\ 190}{t^3} = \frac{1\ 304\ 415}{t^3} + \frac{2121}{t^2}$$

Since the permissible stress (σ) is 60 N/mm^2, therefore

$$\frac{1\ 304\ 415}{t^3} + \frac{2121}{t^2} = 60 \quad \text{or} \quad \frac{21\ 740}{t^3} + \frac{35.4}{t^2} = 1$$

∴ $t = 28.4$ mm **Ans.** ... (By hit and trial)

and $b = 2t = 2 \times 28.4 = 56.8$ mm **Ans.**

Example 5.26. *A C-clamp as shown in Fig. 5.29, carries a load P = 25 kN. The cross-section of the clamp at X-X is rectangular having width equal to twice thickness. Assuming that the clamp is made of steel casting with an allowable stress of 100 MPa, find its dimensions. Also determine the stresses at sections Y-Y and Z-Z.*

Solution. Given : $P = 25$ kN $= 25 \times 10^3$ N ; $\sigma_{t(max)} = 100$ MPa $= 100$ N/mm^2

Dimensions at X-X

Let t = Thickness of the section at X-X in mm, and

b = Width of the section at X-X in mm = $2t$ ...(Given)

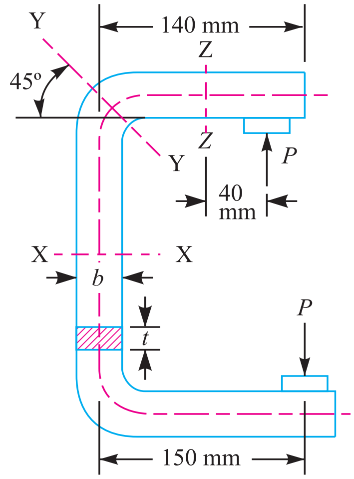


Fig. 5.29

We know that cross-sectional area at *X-X*,

$$A = b \times t = 2\,t \times t = 2\,t^2 \text{ mm}^2$$

∴ Direct tensile stress at *X-X*,

$$\sigma_o = \frac{P}{A} = \frac{25 \times 10^3}{2\,t^2} = \frac{12.5 \times 10^3}{t^3} \text{ N/mm}^2$$

Bending moment at *X-X* due to the load *P*,

$$M = P \times e = 25 \times 10^3 \times 140 = 3.5 \times 10^6 \text{ N-mm}$$

Section modulus, $$Z = \frac{t\,.\,b^2}{6} = \frac{t\,(2t)^2}{6} = \frac{4\,t^3}{6} \text{ mm}^3 \quad ...(\because b = 2t)$$

∴ Bending stress at *X-X*,

$$\sigma_b = \frac{M}{Z} = \frac{3.5 \times 10^6 \times 6}{4\,t^3} = \frac{5.25 \times 10^6}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

We know that the maximum tensile stress $[\sigma_{t\,(max)}]$,

$$100 = \sigma_o + \sigma_b = \frac{12.5 \times 10^3}{t^2} + \frac{5.25 \times 10^6}{t^3}$$

or $$\frac{125}{t^2} + \frac{52.5 \times 10^3}{t^3} - 1 = 0$$

∴ $t = 38.5$ mm **Ans.** ...(By hit and trial)

and $b = 2\,t = 2 \times 38.5 = 77$ mm **Ans.**

Stresses at section Y-Y

Since the cross-section of frame is uniform throughout, therefore cross-sectional area of the frame at section *Y-Y*,

$$A = b \sec 45° \times t = 77 \times 1.414 \times 38.5 = 4192 \text{ mm}^2$$

Component of the load perpendicular to the section

$$= P \cos 45° = 25 \times 10^3 \times 0.707 = 17\,675 \text{ N}$$

This component of the load produces uniform tensile stress over the section.

∴ Uniform tensile stress over the section,

$$\sigma = 17\,675 / 4192 = 4.2 \text{ N/mm}^2 = 4.2 \text{ MPa}$$

Component of the load parallel to the section

$$= P \sin 45° = 25 \times 10^3 \times 0.707 = 17\,675 \text{ N}$$

This component of the load produces uniform shear stress over the section.

∴ Uniform shear stress over the section,

$$\tau = 17\,675 / 4192 = 4.2 \text{ N/mm}^2 = 4.2 \text{ MPa}$$

We know that section modulus,

$$Z = \frac{t\,(b \sec 45°)^2}{6} = \frac{38.5\,(77 \times 1.414)^2}{6} = 76 \times 10^3 \text{ mm}^3$$

Bending moment due to load (P) over the section Y-Y,

$$M = 25 \times 10^3 \times 140 = 3.5 \times 10^6 \text{ N-mm}$$

∴ Bending stress over the section,

$$\sigma_b = \frac{M}{Z} = \frac{3.5 \times 10^6}{76 \times 10^3} = 46 \text{ N/mm}^2 = 46 \text{ MPa}$$

Due to bending, maximum tensile stress at the inner corner and the maximum compressive stress at the outer corner is produced.

∴ Maximum tensile stress at the inner corner,

$$\sigma_t = \sigma_b + \sigma_o = 46 + 4.2 = 50.2 \text{ MPa}$$

and maximum compressive stress at the outer corner,

$$\sigma_c = \sigma_b - \sigma_o = 46 - 4.2 = 41.8 \text{ MPa}$$

Since the shear stress acts at right angles to the tensile and compressive stresses, therefore maximum principal stress (tensile) on the section Y-Y at the inner corner

$$= \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{50.2}{2} + \frac{1}{2}\left[\sqrt{(50.2)^2 + 4 \times (4.2)^2}\right] \text{ MPa}$$

$= 25.1 + 25.4 = 50.5$ MPa **Ans.**

and maximum principal stress (compressive) on section Y-Y at outer corner

$$= \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right] = \frac{41.8}{2} + \frac{1}{2}\left[\sqrt{(41.8)^2 + 4 \times (4.2)^2}\right] \text{ MPa}$$

$= 20.9 + 21.3 = 42.2$ MPa **Ans.**

Maximum shear stress $= \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(50.2)^2 + 4 \times (4.2)^2}\right] = 25.4$ MPa **Ans.**

Stresses at section Z-Z

We know that bending moment at section Z-Z,

$$= 25 \times 10^3 \times 40 = 1 \times 10^6 \text{ N-mm}$$

and section modulus,

$$Z = \frac{t\,.\,b^2}{6} = \frac{38.5\,(77)^2}{6} = 38 \times 10^3 \text{ mm}^3$$

∴ Bending stress at section Z-Z,

$$\sigma_b = \frac{M}{Z} = \frac{1 \times 10^6}{38 \times 10^3} = 26.3 \text{ N/mm}^2 = 26.3 \text{ MPa}$$ **Ans.**

The bending stress is tensile at the inner edge and compressive at the outer edge. The magnitude of both these stresses is 26.3 MPa. At the neutral axis, there is only transverse shear stress. The shear stress at the inner and outer edges will be zero.

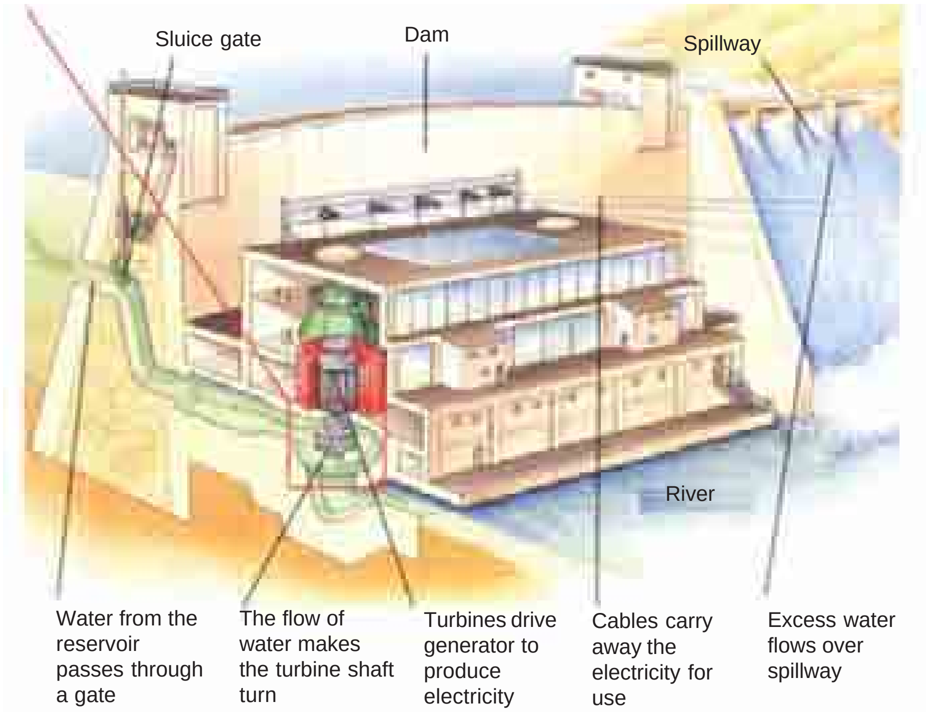
We know that *maximum transverse shear stress,

$$\tau_{max} = 1.5 \times \text{Average shear stress} = 1.5 \times \frac{P}{b.t} = 1.5 \times \frac{25 \times 10^3}{77 \times 38.5}$$

$= 12.65$ N/mm^2 $= 12.65$ MPa **Ans.**

* Refer Art. 5.16

Contents

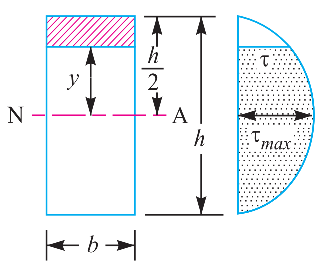


General layout of a hydroelectric plant.

Note : This picture is given as additional information and is not a direct example of the current chapter.

5.16 Shear Stresses in Beams

In the previous article, we have assumed that no shear force is acting on the section. But, in actual practice, when a beam is loaded, the shear force at a section always comes into play along with the bending moment. It has been observed that the effect of the shear stress, as compared to the bending stress, is quite negligible and is of not much importance. But, sometimes, the shear stress at a section is of much importance in the design. It may be noted that the shear stress in a beam is not uniformly distributed over the cross-section but varies from zero at the outer fibres to a maximum at the neutral surface as shown in Fig. 5.30 and Fig. 5.31.



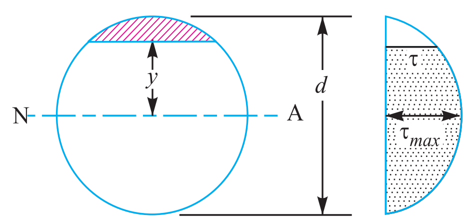


Fig. 5.30. Shear stress in a rectangular beam. **Fig. 5.31.** Shear stress in a circular beam.

The shear stress at any section acts in a plane at right angle to the plane of the bending stress and its value is given by

$$\tau = \frac{F}{I\,.\,b} \times A.\bar{y}$$

Top

Contents

where F = Vertical shear force acting on the section,

I = Moment of inertia of the section about the neutral axis,

b = Width of the section under consideration,

A = Area of the beam above neutral axis, and

$\bar{y}$ = Distance between the C.G. of the area and the neutral axis.

The following values of maximum shear stress for different cross-section of beams may be noted :

1. For a beam of rectangular section, as shown in Fig. 5.30, the shear stress at a distance y from neutral axis is given by

$$\tau = \frac{F}{2I}\left(\frac{h^2}{4} - y^2\right) = \frac{3F}{2b \cdot h^3}(h^2 - 4y^2) \qquad \ldots\left[\because I = \frac{b \cdot h^3}{12}\right]$$

and maximum shear stress,

$$\tau_{max} = \frac{3F}{2b \cdot h} \qquad \ldots\left(\text{Substituting } y = \frac{h}{2}\right)$$

$$= 1.5\ \tau_{(average)} \qquad \ldots\left[\because \tau_{(average)} = \frac{F}{\text{Area}} = \frac{F}{b.h}\right]$$

The distribution of stress is shown in Fig. 5.30.

2. For a beam of circular section as shown in Fig. 5.31, the shear stress at a distance y from neutral axis is given by

$$\tau = \frac{F}{3I}\left(\frac{d^2}{4} - y^2\right) = \frac{16\ F}{3\ \pi\ d^4}(d^2 - 4y^2)$$

and the maximum shear stress,

$$\tau_{max} = \frac{4F}{3 \times \frac{\pi}{4} d^2} \qquad \ldots\left(\text{Substituting } y = \frac{d}{2}\right)$$

$$= \frac{4}{3}\tau_{(average)} \qquad \ldots\left[\because \tau_{(average)} = \frac{F}{\text{Area}} = \frac{F}{\frac{\pi}{4} d^2}\right]$$

The distribution of stress is shown in Fig. 5.31.

3. For a beam of I-section as shown in Fig. 5.32, the maximum shear stress occurs at the neutral axis and is given by

$$\tau_{max} = \frac{F}{I \cdot b}\left[\frac{B}{8}(H^2 - h^2) + \frac{b.h^2}{8}\right]$$

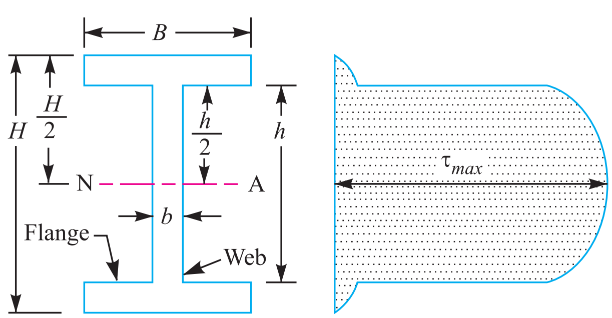


Fig. 5.32

Top

Contents

Shear stress at the joint of the web and the flange

$$= \frac{F}{8I}(H^2 - h^2)$$

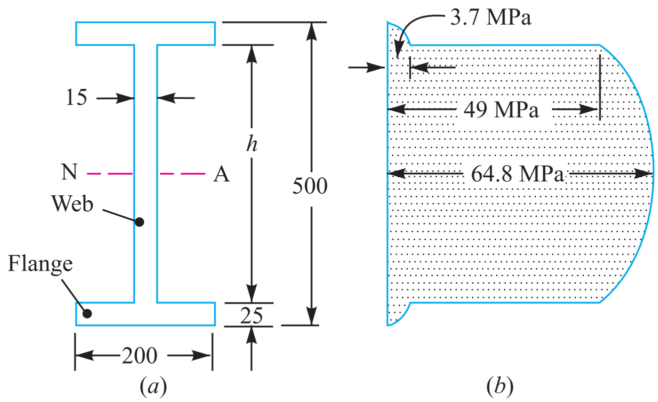
and shear stress at the junction of the top of the web and bottom of the flange

$$= \frac{F}{8I} \times \frac{B}{b}(H^2 - h^2)$$

The distribution of stress is shown in Fig. 5.32.

Example 5.27. *A beam of I-section 500 mm deep and 200 mm wide has flanges 25 mm thick and web 15 mm thick, as shown in Fig. 5.33 (a). It carries a shearing force of 400 kN. Find the maximum intensity of shear stress in the section, assuming the moment of inertia to be 645 × 10⁶ mm⁴. Also find the shear stress at the joint and at the junction of the top of the web and bottom of the flange.*

Solution. Given : H = 500 mm ; B = 200 mm ; h = 500 – 2 × 25 = 450 mm ; b = 15 mm ; F = 400 kN = 400×10^3 N ; $I = 645 \times 10^6$ mm^4



All dimensions in mm.

Fig. 5.33

Maximum intensity of shear stress

We know that maximum intensity of shear stress,

$$\tau_{max} = \frac{F}{I \cdot b}\left[\frac{B}{8}(H^2 - h^2) + \frac{b \cdot h^2}{8}\right]$$

$$= \frac{400 \times 10^3}{645 \times 10^6 \times 15}\left[\frac{200}{8}(500^2 - 450^2) + \frac{15 \times 450^2}{8}\right] \text{N/mm}^2$$

$$= 64.8 \text{ N/mm}^2 = 64.8 \text{ MPa } \textbf{Ans.}$$

The maximum intensity of shear stress occurs at neutral axis.

Note : The maximum shear stress may also be obtained by using the following relation :

$$\tau_{max} = \frac{F \cdot A \cdot \bar{y}}{I \cdot b}$$

We know that area of the section above neutral axis,

$$A = 200 \times 25 + \frac{450}{2} \times 15 = 8375 \text{ mm}^2$$

Top

Distance between the centre of gravity of the area and neutral axis,

$$\bar{y} = \frac{200 \times 25\,(225 + 12.5) + 225 \times 15 \times 112.5}{8375} = 187 \text{ mm}$$

$$\therefore \qquad \tau_{max} = \frac{400 \times 10^3 \times 8375 \times 187}{645 \times 10^6 \times 15} = 64.8 \text{ N/mm}^2 = 64.8 \text{ MPa} \textbf{ Ans.}$$

Shear stress at the joint of the web and the flange

We know that shear stress at the joint of the web and the flange

$$= \frac{F}{8I}(H^2 - h^2) = \frac{400 \times 10^3}{8 \times 645 \times 10^6}\left[(500)^2 - (450)^2\right] \text{N/mm}^2$$

$$= 3.7 \text{ N/mm}^2 = 3.7 \text{ MPa} \textbf{ Ans.}$$

Shear stress at the junction of the top of the web and bottom of the flange

We know that shear stress at junction of the top of the web and bottom of the flange

$$= \frac{F}{8I} \times \frac{B}{b}(H^2 - h^2) = \frac{400 \times 10^3}{8 \times 645 \times 10^6} \times \frac{200}{15}\left[(500)^2 - (450)^2\right] \text{N/mm}^2$$

$$= 49 \text{ N/mm}^2 = 49 \text{ MPa} \textbf{ Ans.}$$

The stress distribution is shown in Fig. 5.33 (*b*)

EXERCISES

1. A steel shaft 50 mm diameter and 500 mm long is subjected to a twisting moment of 1100 N-m, the total angle of twist being 0.6°. Find the maximum shearing stress developed in the shzaft and modulus of rigidity. **[Ans. 44.8 MPa; 85.6 kN/m²]**

2. A shaft is transmitting 100 kW at 180 r.p.m. If the allowable stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more than 1° in a length of 3 metres. Take C = 80 GPa. **[Ans. 105 mm]**

3. Design a suitable diameter for a circular shaft required to transmit 90 kW at 180 r.p.m. The shear stress in the shaft is not to exceed 70 MPa and the maximum torque exceeds the mean by 40%. Also find the angle of twist in a length of 2 metres. Take C = 90 GPa. **[Ans. 80 mm; 2.116°]**

4. Design a hollow shaft required to transmit 11.2 MW at a speed of 300 r.p.m. The maximum shear stress allowed in the shaft is 80 MPa and the ratio of the inner diameter to outer diameter is 3/4. **[Ans. 240 mm; 320 mm]**

5. Compare the weights of equal lengths of hollow shaft and solid shaft to transmit a given torque for the same maximum shear stress. The material for both the shafts is same and inside diameter is 2/3 of outside diameter in case of hollow shaft. **[Ans. 0.56]**

6. A spindle as shown in Fig. 5.34, is a part of an industrial brake and is loaded as shown. Each load P is equal to 4 kN and is applied at the mid point of its bearing. Find the diameter of the spindle, if the maximum bending stress is 120 MPa. **[Ans. 22 mm]**

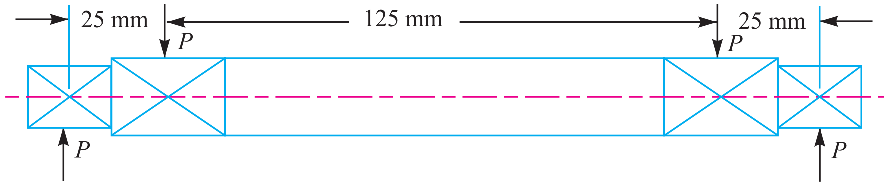


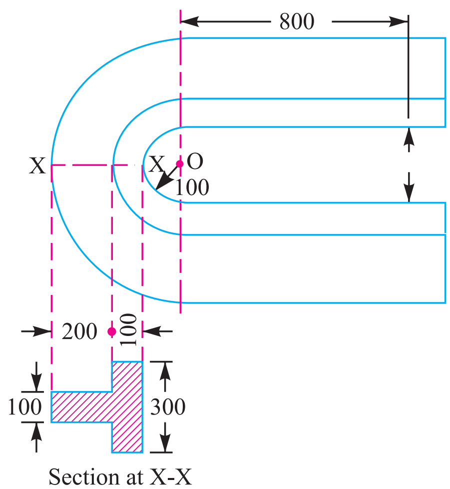
Fig. 5.34

7. A cast iron pulley transmits 20 kW at 300 r.p.m. The diameter of the pulley is 550 mm and has four straight arms of elliptical cross-section in which the major axis is twice the minor axis. Find the dimensions of the arm, if the allowable bending stress is 15 MPa. **[Ans. 60 mm; 30 mm]**

Contents

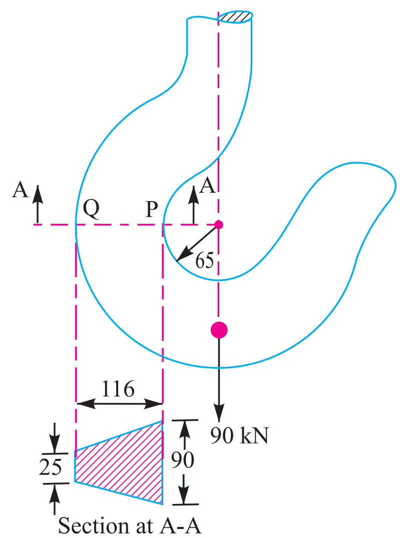
8. A shaft is supported in bearings, the distance between their centres being 1 metre. It carries a pulley in the centre and it weighs 1 kN. Find the diameter of the shaft, if the permissible bending stress for the shaft material is 40 MPa. **[Ans. 40 mm]**

9. A punch press, used for stamping sheet metal, has a punching capacity of 50 kN. The section of the frame is as shown in Fig. 5.35. Find the resultant stress at the inner and outer fibre of the section.
[Ans. 28.3 MPa (tensile); 17.7 MPa (compressive)]



All dimensions in mm.

Fig. 5.35



All dimensions in mm.

Fig. 5.36

10. A crane hook has a trapezoidal section at *A-A* as shown in Fig. 5.36. Find the maximum stress at points *P* and *Q*. **[Ans. 118 MPa (tensile); 62 MPa (compressive)]**

11. A rotating shaft of 16 mm diameter is made of plain carbon steel. It is subjected to axial load of 5000 N, a steady torque of 50 N-m and maximum bending moment of 75 N-m. Calculate the factor of safety available based on 1. Maximum normal stress theory; and 2. Maximum shear stress theory.

Assume yield strength as 400 MPa for plain carbon steel. If all other data remaining same, what maximum yield strength of shaft material would be necessary using factor of safety of 1.686 and maximum distortion energy theory of failure. Comment on the result you get.
[Ans. 1.752; 400 MPa]

12. A hand cranking lever, as shown in Fig. 5.37, is used to start a truck engine by applying a force F = 400 N. The material of the cranking lever is 30C8 for which yield strength = 320 MPa; Ultimate tensile strength = 500 MPa ; Young's modulus = 205 GPa ; Modulus of rigidity = 84 GPa and poisson's ratio = 0.3.

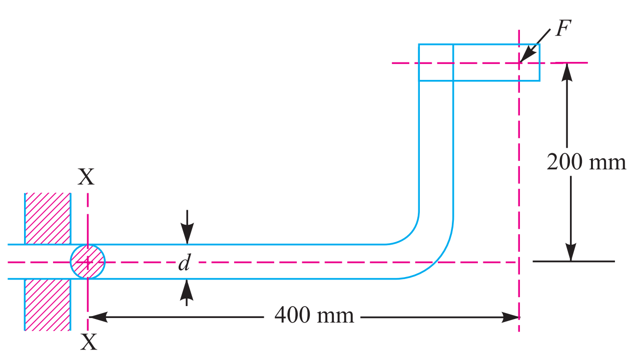


Fig. 5.37

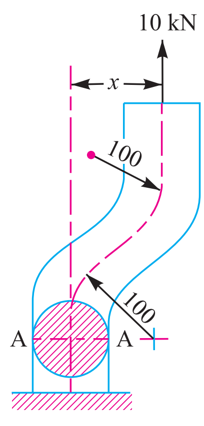
Top

Contents

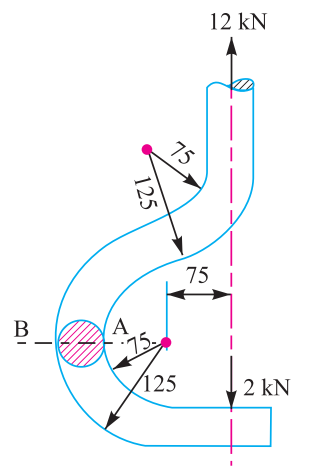
Assuming factor of safety to be 4 based on yield strength, design the diameter '*d*' of the lever at section *X-X* near the guide bush using : 1. Maximum distortion energy theory; and 2. Maximum shear stress theory. **[Ans. 28.2 mm; 28.34 mm]**

13. An offset bar is loaded as shown in Fig. 5.38. The weight of the bar may be neglected. Find the maximum offset (*i.e.*, the dimension *x*) if allowable stress in tension is limited to 70 MPa.

[Ans. 418 mm]



All dimensions in mm.

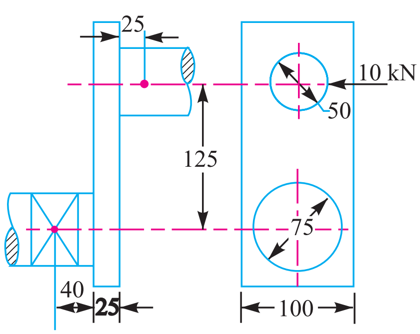


All dimensions in mm.

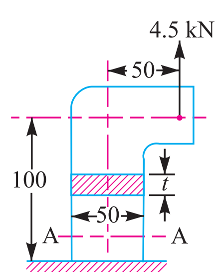
Fig. 5.38 **Fig. 5.39**

14. A crane hook made from a 50 mm diameter bar is shown in Fig. 5.39. Find the maximum tensile stress and specify its location. **[Ans. 35.72 MPa at A]**

15. An overhang crank, as shown in Fig. 5.40 carries a tangential load of 10 kN at the centre of the crankpin. Find the maximum principal stress and the maximum shear stress at the centre of the crank-shaft bearing. **[Ans. 29.45 MPa; 18.6 MPa]**



All dimensions in mm.



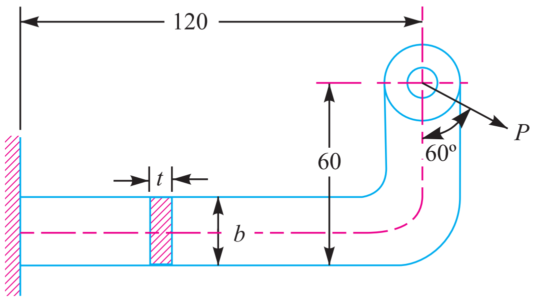
All dimensions in mm.

Fig. 5.40 **Fig. 5.41**

16. A steel bracket is subjected to a load of 4.5 kN, as shown in Fig. 5.41. Determine the required thickness of the section at A-A in order to limit the tensile stress to 70 MPa. **[Ans. 9 mm]**

Top

17. A wall bracket, as shown in Fig. 5.42, is subjected to a pull of P = 5 kN, at 60° to the vertical. The cross-section of bracket is rectangular having $b = 3t$. Determine the dimensions b and t if the stress in the material of the bracket is limited to 28 MPa. **[Ans. 75 mm; 25 mm]**



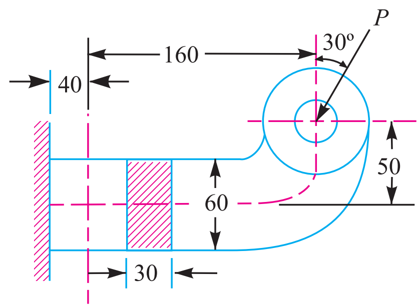


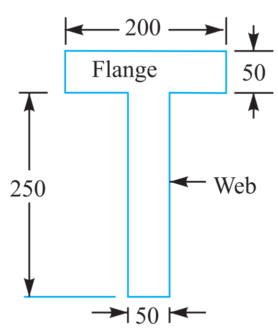
Fig. 5.42 **Fig. 5.43**

18. A bracket, as shown in Fig. 5.43, is bolted to the framework of a machine which carries a load P. The cross-section at 40 mm from the fixed end is rectangular with dimensions, 60 mm × 30 mm. If the maximum stress is limited to 70 MPa, find the value of P.

[Ans. 3000 N]

19. A T-section of a beam, as shown in Fig. 5.44, is subjected to a vertical shear force of 100 kN. Calculate the shear stress at the neutral axis and at the junction of the web and the flange. The moment of inertia at the neutral axis is 113.4×10^6 mm^4.

[Ans. 11.64 MPa; 11 MPa; 2.76 MPa]



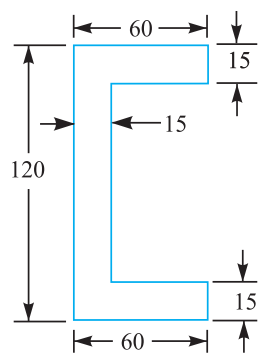


Fig. 5.44 **Fig. 5.45**

20. A beam of channel section, as shown in Fig. 5.45, is subjected to a vertical shear force of 50 kN. Find the ratio of maximum and mean shear stresses. Also draw the distribution of shear stresses.

[Ans. 2.22]

QUESTIONS

1. Derive a relation for the shear stress developed in a shaft, when it is subjected to torsion.
2. State the assumptions made in deriving a bending formula.

Contents

3. Prove the relation: $M/I = \sigma/y = E/R$

   where M = Bending moment; I = Moment of inertia; σ = Bending stress in a fibre at a distance y from the neutral axis; E = Young's modulus; and R = Radius of curvature.
4. Write the relations used for maximum stress when a machine member is subjected to tensile or compressive stresses along with shearing stresses.
5. Write short note on maximum shear stress theory *verses* maximum strain energy theory.
6. Distinguish clearly between direct stress and bending stress.
7. What is meant by eccentric loading and eccentricity?
8. Obtain a relation for the maximum and minimum stresses at the base of a symmetrical column, when it is subjected to

   (*a*) an eccentric load about one axis, and (*b*) an eccentric load about two axes.

OBJECTIVE TYPE QUESTIONS

1. When a machine member is subjected to torsion, the torsional shear stress set up in the member is
   (*a*) zero at both the centroidal axis and outer surface of the member
   (*b*) Maximum at both the centroidal axis and outer surface of the member
   (*c*) zero at the centroidal axis and maximum at the outer surface of the member
   (*d*) none of the above
2. The torsional shear stress on any cross-section normal to the axis is .... the distance from the centre of the axis.
   (*a*) directly proportional to (*b*) inversely proportional to
3. The neutral axis of a beam is subjected to
   (*a*) zero stress (*b*) maximum tensile stress
   (*c*) maximum compressive stress (*d*) maximum shear stress
4. At the neutral axis of a beam,
   (*a*) the layers are subjected to maximum bending stress
   (*b*) the layers are subjected to tension (*c*) the layers are subjected to compression
   (*d*) the layers do not undergo any strain
5. The bending stress in a curved beam is
   (*a*) zero at the centroidal axis (*b*) zero at the point other than centroidal axis
   (*c*) maximum at the neutral axis (*d*) none of the above
6. The maximum bending stress, in a curved beam having symmetrical section, always occur, at the
   (*a*) centroidal axis (*b*) neutral axis
   (*c*) inside fibre (*d*) outside fibre
7. If d = diameter of solid shaft and τ = permissible stress in shear for the shaft material, then torsional strength of shaft is written as
   (*a*) $\frac{\pi}{32} d^4 \tau$ (*b*) $d \log_e \tau$
   (*c*) $\frac{\pi}{16} d^3 \tau$ (*d*) $\frac{\pi}{32} d^3 \tau$
8. If d_i and d_o are the inner and outer diameters of a hollow shaft, then its polar moment of inertia is
   (*a*) $\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$ (*b*) $\frac{\pi}{32}\left[(d_o)^3 - (d_i)^3\right]$
   (*c*) $\frac{\pi}{32}\left[(d_o)^2 - (d_i)^2\right]$ (*d*) $\frac{\pi}{32}(d_o - d_i)$

Top

9. Two shafts under pure torsion are of identical length and identical weight and are made of same material. The shaft *A* is solid and the shaft *B* is hollow. We can say that
   (*a*) shaft *B* is better than shaft *A*
   (*b*) shaft *A* is better than shaft *B*
   (*c*) both the shafts are equally good
10. A solid shaft transmits a torque *T*. The allowable shear stress is τ. The diameter of the shaft is
   (*a*) $\sqrt[3]{\dfrac{16\,T}{\pi\,\tau}}$ (*b*) $\sqrt[3]{\dfrac{32\,T}{\pi\,\tau}}$
   (*c*) $\sqrt[3]{\dfrac{64\,T}{\pi\,\tau}}$ (*d*) $\sqrt[3]{\dfrac{16\,T}{\tau}}$
11. When a machine member is subjected to a tensile stress (σ_t) due to direct load or bending and a shear stress (τ) due to torsion, then the maximum shear stress induced in the member will be
   (*a*) $\frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$ (*b*) $\frac{1}{2}\left[\sqrt{(\sigma_t)^2 - 4\,\tau^2}\right]$
   (*c*) $\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$ (*d*) $(\sigma_t)^2 + 4\,\tau^2$
12. Rankine's theory is used for
   (*a*) brittle materials (*b*) ductile materials
   (*c*) elastic materials (*d*) plastic materials
13. Guest's theory is used for
   (*a*) brittle materials (*b*) ductile materials
   (*c*) elastic materials (*d*) plastic materials
14. At the neutral axis of a beam, the shear stress is
   (*a*) zero (*b*) maximum
   (*c*) minimum
15. The maximum shear stress developed in a beam of rectangular section is .... the average shear stress.
   (*a*) equal to (*b*) $\frac{4}{3}$ times
   (*c*) 1.5 times

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*a*)	**4.** (*d*)	**5.** (*b*)
6. (*c*)	**7.** (*c*)	**8.** (*a*)	**9.** (*a*)	**10.** (*a*)
11. (*a*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)	**15.** (*c*)

C H A P T E R

6

Variable Stresses in Machine Parts

6.1 Introduction

We have discussed, in the previous chapter, the stresses due to static loading only. But only a few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads), therefore we shall discuss, in this chapter, the variable or alternating stresses.

6.2 Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load W, as shown in Fig. 6.1. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (*i.e.* at point A) are under compressive stress and the lower fibres (*i.e.* at point B) are under tensile stress. After half a revolution, the point *B* occupies the position of point *A* and the point *A* occupies the position of point *B*. Thus the point *B* is now under compressive stress and the point *A* under tensile stress. The speed of variation of these stresses depends upon the speed of the beam.

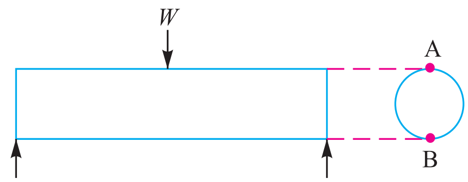


Fig. 6.1. Reversed or cyclic stresses.

From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or ***vice versa,*** are known as ***completely reversed*** or ***cyclic stresses.***

Notes: 1. The stresses which vary from a minimum value to a maximum value of the same nature, (*i.e.* tensile or compressive) are called ***fluctuating stresses.***

2. The stresses which vary from zero to a certain maximum value are called ***repeated stresses.***

3. The stresses which vary from a minimum value to a maximum value of the opposite nature (*i.e.* from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called ***alternating stresses.***

6.3 Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

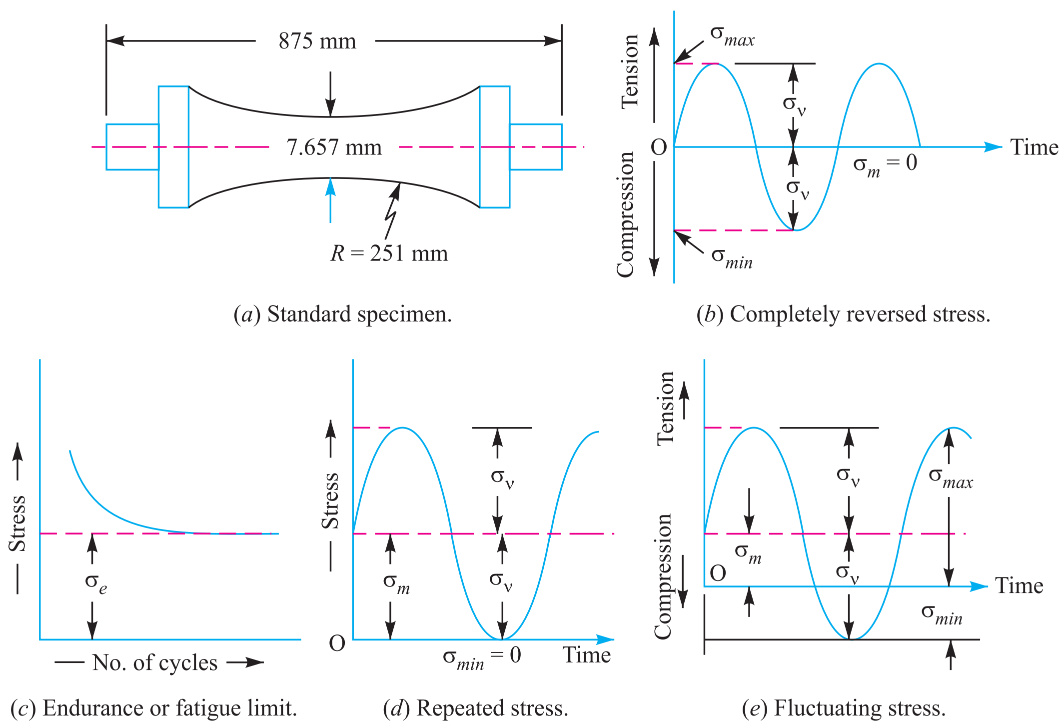


Fig. 6.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 6.2 (*a*), is rotated in a fatigue

Contents

testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. 6.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig. 6.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 6.2 (*c*), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as ***endurance*** or ***fatigue limit*** (σ_e). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).

A machine part is being turned on a Lathe.

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 6.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress.

The stress ***verses*** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig. 6.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Fig. 6.2 (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Note: For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma_{min} = 0$) in each cycle as shown in Fig. 6.2 (*d*).

$$\therefore \qquad \sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, $R = \frac{\sigma_{max}}{\sigma_{min}}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

Top

where σ'_e = Endurance limit for any stress range represented by R.

σ_e = Endurance limit for completely reversed stresses, and

R = Stress ratio.

6.4 Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Shaft drive.

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

$\therefore$ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e.K_b = \sigma_e$ ...($\because K_b = 1$)

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e.K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e.K_s$

6.5 Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. 6.3 shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.

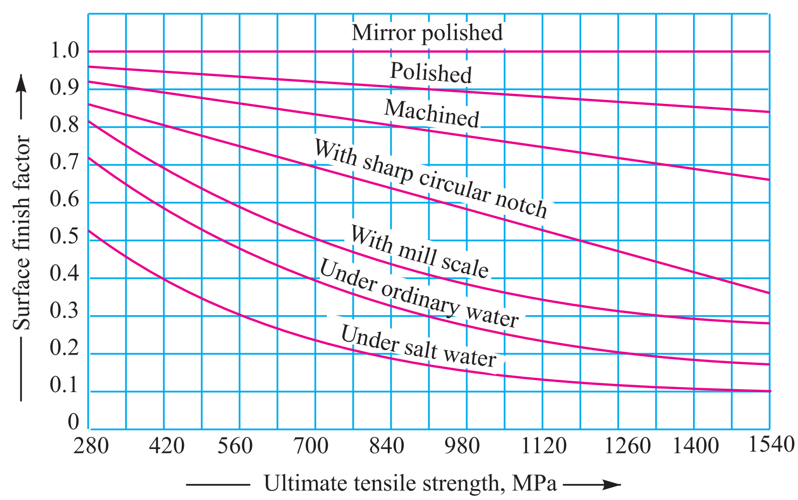


Fig. 6.3. Surface finish factor for various surface conditions.

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that

Contents

for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e.K_b.K_{sur} = \sigma_e.K_{sur} \quad \text{...}(\because K_b = 1)$$

...(For reversed bending load)

$$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur} \quad \text{...(For reversed torsional or shear load)}$$

Note : The surface finish factor for non-ferrous metals may be taken as unity.

6.6 Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig. 6.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

∴ Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$

$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$

$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

Notes: 1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.

2. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.

3. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

6.7 Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions :

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

In addition to shear, tensile, compressive and torsional stresses, temperature can add its own stress (Ref. Chapter 4)

Note : This picture is given as additional information and is not a direct example of the current chapter.

Top

Contents

6.8 Relation Between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u). Fig. 6.4 shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice :

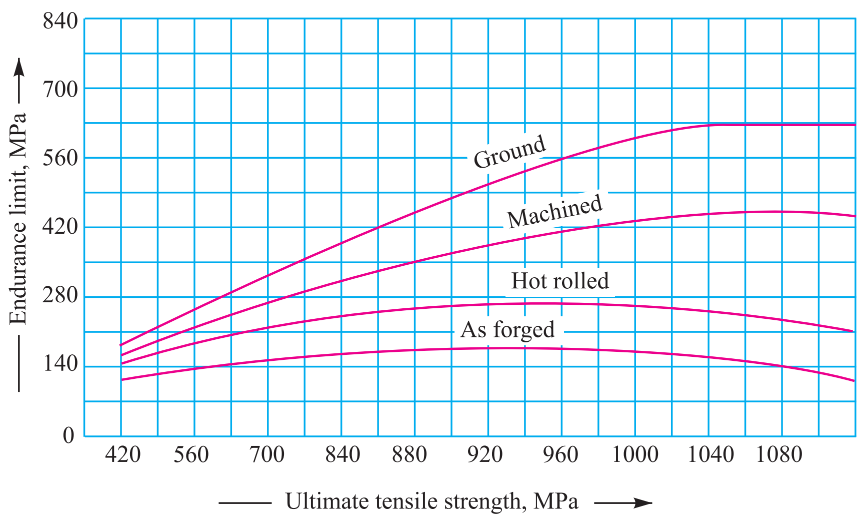


Fig. 6.4. Endurance limit of steel corresponding to ultimate tensile strength.

For steel, $\sigma_e = 0.5\ \sigma_u$;

For cast steel, $\sigma_e = 0.4\ \sigma_u$;

For cast iron, $\sigma_e = 0.35\ \sigma_u$;

For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

6.9 Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for faliure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

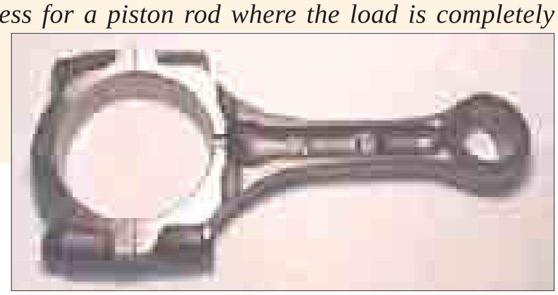
$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

Note: For steel, $\sigma_e = 0.8$ to $0.9\ \sigma_y$

where σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Example 6.1. *Determine the design stress for a piston rod where the load is completely reversed. The surface of the rod is ground and the surface finish factor is 0.9. There is no stress concentration. The load is predictable and the factor of safety is 2.*



Piston rod

Solution. Given : $K_{sur} = 0.9$; $F.S. = 2$

The piston rod is subjected to reversed axial loading. We know that for reversed axial loading, the load correction factor (K_a) is 0.8.

Top

If σ_e is the endurance limit for reversed bending load, then endurance limit for reversed axial load,

$$\sigma_{ea} = \sigma_e \times K_a \times K_{sur} = \sigma_e \times 0.8 \times 0.9 = 0.72\ \sigma_e$$

We know that design stress,

$$\sigma_d = \frac{\sigma_{ea}}{F.S.} = \frac{0.72\ \sigma_e}{2} = 0.36\ \sigma_e \textbf{ Ans.}$$

6.10 Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.*** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.

In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. 6.5. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

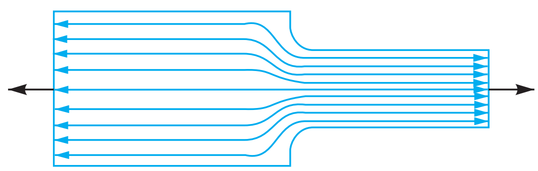


Fig. 6.5. Stress concentration.

6.11 Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

The value of K_t depends upon the material and geometry of the part.

Notes: 1. In static loading, stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration. In brittle materials, cracks may appear at these local concentrations of stress which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

2. In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness, or any sharp discontinuity in the geometrical form of the member. If the stress at any point in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

6.12 Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig. 6.6 (*a*). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

Top