6 LOWER PAIRS

Introduction

In the chapter of mechanisms and machines, basic mechanisms with their inversions were introduced. In this chapter, some more mechanisms of the lower pair category will be discussed. Lower pairs usually comprise turning (pivoted) and sliding pairs. Mechanisms with pivoted links are widely used in machines and the required movements of links are produced by using them in a variety of forms and methods. In this chapter, some of the more common mechanisms will be studied. Pantographs are used to copy the curves on reduced or enlarged scales. Some pivoted-link mechanisms are used to guide reciprocating parts either exactly or approximately in straight paths to eliminate the friction of the straight guides of the sliding pairs. However, these days, sliders are also being used to get linear motions.

An exact straight-line mechanism guides a reciprocating part in an exact straight line. On the other hand, an approximate straight-line mechanism is designed in such a way that the middle and the two extreme positions of the guided point are in a straight line and the intermediate positions deviate as little as possible from the line.

Although this chapter will be restricted to the more elementary aspects of the analysis of mechanisms, the possibilities of their use in the mechanisms and the machine of daily use can easily be glimpsed. Moreover, systematic design techniques are being developed so that these mechanisms can be used for accurate control of the processes and the machines being needed in modern technology.

6.1 PANTOGRAPH

A pantograph is a four-bar linkage used to produce paths exactly similar to the ones traced out by a point on the linkage. The paths so produced are, usually, on an enlarged or reduced scale and may be straight or curved ones.

The four links of a pantograph are arranged in such a way that a parallelogram *ABCD* is formed (Fig. 6.1). Thus, *AB* = *DC* and *BC* = *AD*. If some point *O* in one of the links is made fixed and three other points *P*, *Q* and *R* on the other three links are located in such a way that *OPQR* is a straight line, it can be shown that the points *P*, *Q* and *R* always move parallel and similar to each other over any path, straight or curved. Their motions will be proportional to their distances from the fixed point.

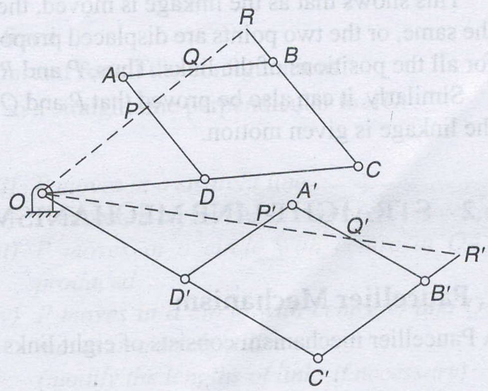


Fig. 6.1

Let O, P, Q and R lie on links CD, DA, AB and BC respectively. $ABCD$ is the initial assumed position shown in the figure.

Let the linkage be moved to another position so that A moves to A', B to B', and so on.

In ΔODP and OCR,

O, P and R lie on a straight line and thus OP and OR coincide.

$\angle DOP = \angle COR$ (common angle)

$\angle ODP = \angle OCR$ ($\because DP$ ...)

Therefore, the Δs are similar and

$$\frac{OD}{OC} = \frac{OP}{OR} = \frac{DP}{CR}$$

Now, $A'B' = AB = DC = D'C'$

And $B'C' = BC = AD = A'D'$

Therefore, $A'B'C'D'$ is again a parallelogram.

In Δs $OD'P'$ and $OC'R'$,

$$\frac{OD'}{OC'} = \frac{OD}{OC} = \frac{DP}{CR}$$

$$= \frac{D'P'}{C'R'}$$

and,

$$\angle OD'P' = \angle OC'R' \qquad (D'P' \parallel C'B' \text{ as } A'B'C'D' \text{ is a } \parallel \text{gm})$$

Thus, the $\angle$s are similar.

$\therefore$ $\angle D'OP' = \angle C'OR'$

or O, P' and R' lie on a straight line.

Now

$$\frac{OP}{OR} = \frac{OD}{OC}$$

$$= \frac{OD'}{OC'}$$

$$= \frac{OP'}{OR'} \qquad (\because \Delta s\ OD'P' \text{ and } OC'R' \text{ are similar})$$

This shows that as the linkage is moved, the ratio of the distances of P and R from the fixed point remains the same, or the two points are displaced proportional to their distances from the fixed point. This will ... for all the positions of the links. Thus, P and R will trace exactly similar paths.

Similarly, it can also be proved that P and Q trace similar paths. Thus, P, Q and R trace similar paths when the linkage is given motion.

6.2 STRAIGHT-LINE MECHANISMS

1. Paucellier Mechanism

A Paucellier mechanism consists of eight links (Fig. 6.2) such that,

$$OA = OQ; \qquad AB = AC$$

and $$BP = PC = CQ = QB$$

OA is the fixed link and OQ is a rotating link. It can be proved that as the link OQ moves around O, P moves in a straight line perpendicular to OA. All the joints are pin-jointed.

Since $BPCQ$ is a rhombus,

QP always bisects the angle BQC,

i.e.,

$$\angle 1 = \angle 2 \qquad \text{(i)}$$

in all the positions

Also, in Δs AQC and AQB,

AQ is common,

$AC = AB$

$QC = QB$

$\therefore$ Δs are congruent in all positions.

or $\angle 3 = \angle 4$ (ii)

Adding (i) and (ii),

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^o$

$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4 = 180^o$

or A, Q and P lie on a straight line.

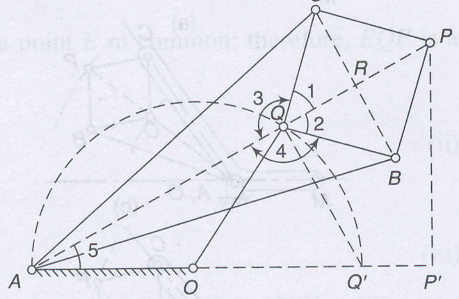


Fig. 6.2

Let PP' be the perpendicular on AO produced.

Δs AQQ' and APP' are similar because $\angle 5$ is common and $\angle AQQ' = \angle AP'P = 90^o$

$$\therefore \quad \frac{AQ}{AP'} = \frac{AQ'}{AP}$$

or $$AQ'.AP' = (AQ)(AP)$$

$$= (AR - RQ)(AR + RP)$$

$$= (AR - RQ)(AR + RQ) \qquad \ldots (RP = RQ)$$

$$= (AR)^2 - (RQ)^2$$

$$= [(AC)^2 - (CR)^2) - [(CQ)^2 - (CR)^2]$$

or $$AP' = \frac{(AC)^2 - (CQ)^2}{AQ'}$$

= constant, as AC, CQ and AQ' are always fixed

This means that the projection of P and AQ produced is constant for all the configurations.

Thus, PP' is always a normal to AO produced or P moves in a straight line perpendicular to AO.

Example 6.1 *Figure 6.3(a) shows the link MAC which oscillates on a fixed centre A. Another link OQ oscillates on the centre O. The links AB and AC are equal. Also BP = PC = CQ = QB. Locate the position of O such that,*

(i) P moves in a straight line

(ii) P moves in a circle with centre A

(iii) P moves in a circle with centre at OA produced

(iv) P moves in a circle with centre O and Q moves in a straight line

(modify the lengths of links if necessary)

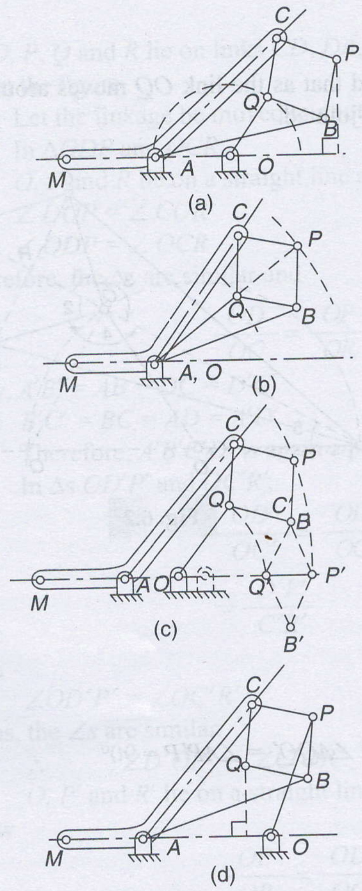


Fig. 6.3

Solution

(i) As in the Paucellier mechanism, O is lo… by drawing a straight line throug… perpendicular to the motion of P su… $AO = OQ$ [Fig. 6.3(a)].

(ii) If O is made to coincide with A, AQ w… be equal to OQ. Thus, Q and P will be… on AP. Q will rotate about A and thus P… also rotate in a circle about A with AP… radius [Fig. 6.3(b)].

(iii) From the above two cases, it can be obs… that in (i) P moves in a circle with the… at infinity on OA produced and in… moves in circle with the centre at A… if P is to move in a circle with the… in-between A and infinity on OA pro… O must lie in-between O and A or in… words OQ should be greater than OA… 6.3(c)].

(iv) The mechanism will be similar… Paucellier mechanism. P is to be… with O by a link so that P moves in a… about O and $OA = OP$. The lengths… modified in two ways [Fig. 6.3(d)].

(a) OA is increased and OA and OP ar… equal.

(b) Lengths AB and AC are reduced in su… way that $OA = OP$.

2. Hart Mechanism

A Hart mechanism consists of six links as shown in Fig. 6.4 such that

$$AB = CD; \quad AD = BC \quad \text{and } OE = OQ$$

OE is the fixed link and OQ, the rotating link. The links are arranged in such a way that $ABDC$ is a trapezium (AC parallel to BD). Pins E and Q on the links AB and AD respectively, and the point P on the link CB are located in such a way that

$$\frac{AE}{AB} = \frac{AQ}{AD} = \frac{CP}{CB} \qquad \text{(i)}$$

It can be shown that as OQ rotates about O, P moves in a line perpendicular to EO produced.

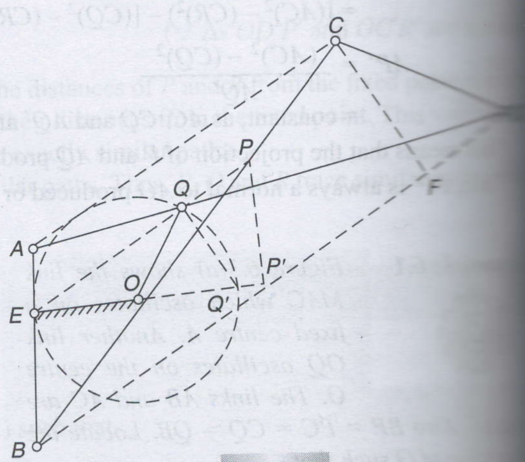


Fig. 6.4

In $\Delta ABD \ \dfrac{AE}{AB} = \dfrac{AQ}{AD}$ (Given)

Therefore, EQ is parallel to BD and thus parallel to AC.

In $\Delta ABC \ \dfrac{AE}{AB} = \dfrac{CP}{CB}$ (Given)

Therefore, EP is parallel to AC and thus parallel to BD.

Now, EQ and EP are both parallel to AC and BD and have a point E in common; therefore, EQP is a straight line.

Δs AEQ and ABD are similar ($\because EQ \parallel BD$).

$$\therefore \quad \frac{EQ}{BD} = \frac{AE}{AB} \quad \text{or } EQ = BD \times \frac{AE}{AB} \qquad \text{(ii)}$$

Δs BEP and BAC are similar ($\because EP \parallel AC$).

$$\therefore \quad \frac{EP}{AC} = \frac{BE}{BA} \quad \text{or } EP = AC \times \frac{BE}{AB} \qquad \text{(iii)}$$

Δs EQQ' and $EP'P$ are similar, because $\angle QEQ'$ or $\angle PEP'$ is common and $\angle EQQ' = \angle QP'P = 90°$.

$$\therefore \quad \frac{EQ}{EP'} = \frac{EQ'}{EP}$$

$$\text{or} \quad EQ' \times EP' = EQ \times EP$$

$$= \left(BD \times \frac{AE}{AB} \right)\left(AC \times \frac{BE}{AB} \right) \qquad \text{[from (ii) and (iii)]}$$

$$\text{or} \quad EP' = \frac{AE \times BE}{(EQ')(AB)^2}[(BD)(AC)]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF + FD)(BF - FD)\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF)^2 - (FD)^2\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[\{(BC)^2 - (CF)^2\} - \{(CD)^2 - (CF)^2\}\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BC)^2 - (CD)^2\right]$$

= constant, as all the parameters are fixed.

Thus, EP' is always constant. Therefore, the projection of P on EO produced is always the same point or P moves in a straight line perpendicular to EO.

Example 6.2 *A circle with EQ' as diameter has a point Q on its circumference. P is a point on EQ produced such that if Q turns about E, $EQ.EP$ is constant. Prove that the point P moves in a straight line perpendicular to EQ'.*

Solution Let PP' be perpendicular to EQ' produced (Fig. 6.5).

For any position of Q on the circumference of the circle with diameter EQ', Δs EQQ' and $EP'P$ are similar ($\angle QEQ'$ is common and $\angle EQQ' = \angle EP'P = 90°$).

$$\therefore \quad \frac{EQ}{EQ'} = \frac{EP'}{EP}$$

or $\quad EQ'.\,EP' = EQ.\,EP$

or $\quad EP' = \dfrac{EQ.EP}{EQ'}$

$= \text{constant, as } EQ' \text{ is fixed and } EQ.$

$EP = \text{constant}$

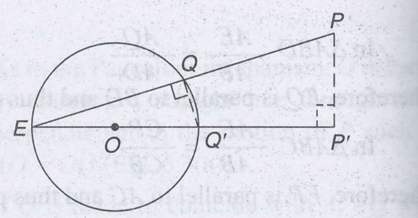


Fig. 6.5

Thus, EP' will be constant for all positions
Therefore, the location of P' is fixed which m
that P moves in a straight line perpendicula
EQ'.

3. Scott–Russel Mechanism

A Scott–Russel mechanism consists of three movable links; OQ, PS and slider S which moves along OS. OQ is the crank (Fig. 6.6). The links are connected in such a way that

$$QO = QP = QS$$

It can be proved that P moves in a straight line perpendicular to OS as the slider S moves along OS.

As $QO = QP = QS$, a circle can be drawn passing through O, P and S with PS as the diameter and Q as the centre.

Now, O lies on the circumference of the circle and PS is the diameter. Therefore, $\angle POS$ is a right angle. This is true for all the positions of S and is possible only if P moves in a straight line perpendicular to OS at O.

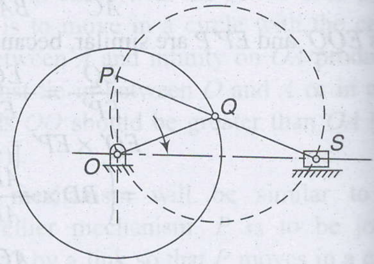


Fig. 6.6

Note that in such a mechanism, the path of P is through the joint O which is not desirable. This can be avoided if the links are proportioned in a way that QS is the mean proportional between OQ and QP, i.e.,

$$\frac{OQ}{QS} = \frac{QS}{QP}$$

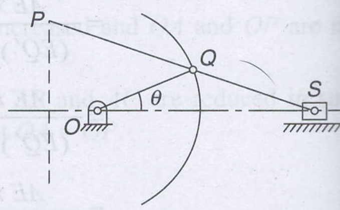


Fig. 6.7

However, in this case P will approximately traverse a straight line perpendicular to OS and that also for small movements of S or for small values of the angle θ (Fig. 6.7). A mathematical proof of this, being not simple, is omitted here. However, by drawing the mechanism in a number of positions, the fact can be verified.

Usually, this is known as the *modified Scott–Russel mechanism*.

4. Grass–Hopper Mechanism

This mechanism is a derivation of the modified Scott–Russel mechanism in which the sliding pair at S is replaced by a turning pair. This is achieved by replacing the slider with a link AS perpendicular to OS in the mean position. AS is pin-jointed at A (Fig. 6.8).

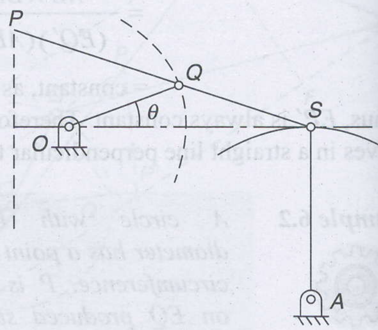


Fig. 6.8

If the length AS is large enough, S moves in an approximated straight line perpendicular to AS (or in line with OS) for small angular movements. P again will move in an approximate straight line if QS is the mean proportional between OQ and QP, i.e.,

$$\frac{OQ}{QS} = \frac{QS}{QP}$$

Example 6.3 *In a Grass–Hopper mechanism shown in Fig. 6.9, $OQ = 80$ mm, $SQ = 120$ mm and $SP = 300$ mm. Find the magnitude of the vertical force at P necessary to resist a torque of 100 N.m applied to the link OQ when it makes angles of 0°, 10° and 20° with the horizontal.*

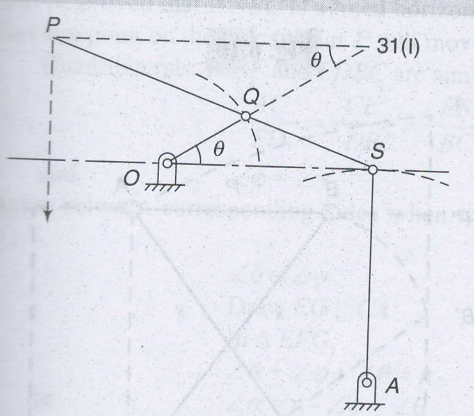


Fig. 6.9

Solution

$OQ = 80$ mm

$QS = 120$ mm

$QP = 300 - 120 = 180$ mm

$$\frac{OQ}{QS} = \frac{QS}{QP}, \text{ i.e., } \frac{80}{120} = \frac{120}{180}$$

As the condition for the dimensions of the Grass–Hopper mechanism is satisfied, P moves in an approximate straight line for small angles of OQ with the horizontal.

Now

$$F_p \times v_p = T_q \times \omega_q$$

$$F_p = \frac{T_p \omega_q}{v_p} = \frac{T_q}{v_p} \cdot \frac{v_q}{OQ} \qquad \text{(i)}$$

Locate the I-centre (instantaneous centre) of the link SP. It is at 31 as the directions of motions of points P and Q on it are known.

$$\frac{v_q}{v_p} = \frac{IQ}{IP} = \frac{OQ}{OS}$$

($\because$ Δs IQP and OQS are similar)

(i) becomes $F_p = \dfrac{T_q}{OS}$

When $\theta = 0°$, $OS = 80 + 120 = 200$ mm

$$F_p = \frac{100}{0.2} = 500 \text{ N}$$

When $\theta = 10°$,

$$OS = 80 \cos 10° + \sqrt{(120)^2 - (80 \sin 10°)^2}$$
$$= 198 \text{ mm}$$

$$F_p = \frac{100}{0.198} = \underline{505.05 \text{ N}}$$

When $\theta = 20°$,

$$OS = 80 \cos 20° + \sqrt{(120)^2 - (80 \sin 20°)^2}$$
$$= 192 \text{ mm}$$

$$F_p = \frac{100}{0.192} = \underline{520.8 \text{ N}}$$

As the angle θ increases, P moves in only approximate straight line and thus the calculations for F_p are not exact.

Watt Mechanism

It is a very simple mechanism. It has four links OQ, OA, QB and AB. OQ is the fixed link. Links OA and QB can oscillate about centres O and Q respectively. It is seen that if P is a point on the link AB such that $PA/PB = QB/OA$, then for small oscillations of OA and QB, P will trace an approximately straight line. This has been shown in Fig. 6.10 for three positions.

In earlier times, the mechanism was used by Watt to guide the piston, as it was difficult to machine plane surfaces.

6. Tchebicheff Mechanism

It consists of four links OA, QB, AB and OQ (fixed) as shown in Fig. 6.11. The links OA and QB are equal and crossed. P, the mid-point of AB, is the tracing point. The proportions of the links are taken in such a way that P, A and B lie on vertical lines when on extreme positions, i.e., when directly above O or Q.

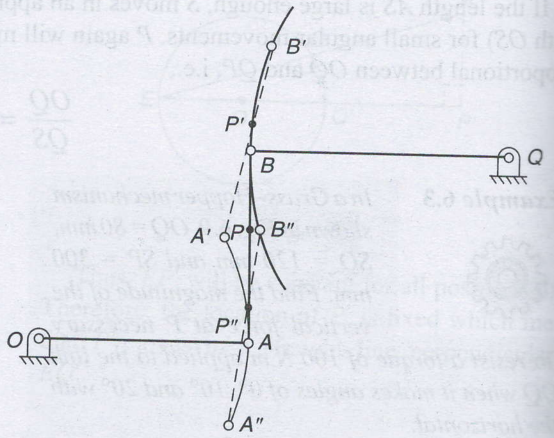


Fig. 6.10

Let $AB = 1$ unit

$OA = QB = x$ units

and $OQ = y$ units

When AB is on the extreme left position, A and B assume the positions A' and B', respectively.

In $\Delta OQB'$,

$$(QB')^2 - (OQ)^2 = (OB')^2$$

$$(QB)^2 - (OQ)^2 = (OB')^2 \qquad (OB' = OB)$$

$$x^2 - y^2 = (OA' - A'B')^2$$

$$= (x-1)^2$$

$$= x^2 - 2x + 1$$

or $\quad 2x = y^2 + 1$

$$x = \frac{y^2+1}{2} \qquad \text{(i)}$$

In ΔOAC,

$$(OA)^2 - (AC)^2 = (OC)^2$$

$$(OA)^2 - (OP')^2 = (AP')^2$$

$$(OA)^2 - (OA' - A'P')^2 = (PP' + AP)^2$$

$$x^2 - \left(x - \frac{1}{2}\right)^2 = \left(\frac{y}{2} + \frac{1}{2}\right)^2$$

or $\quad x^2 - \left(x^2 + \frac{1}{4} - x\right) = \frac{y^2}{4} + \frac{1}{4} + \frac{y}{2}$

$$x = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2} \qquad \text{(ii)}$$

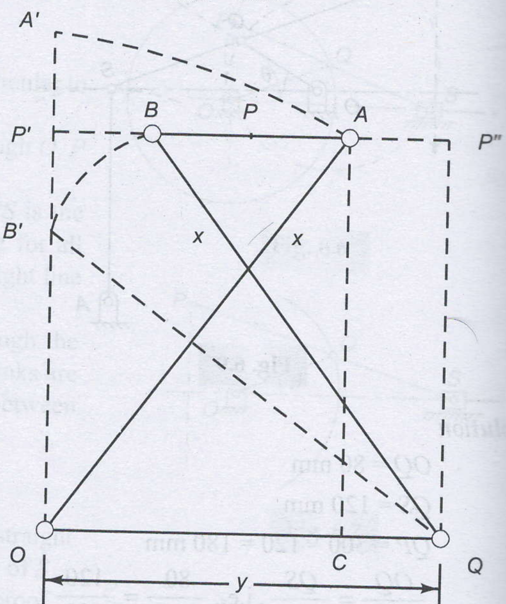


Fig. 6.11

From Eqs (i) and (ii),

$$\frac{y^2}{2} + \frac{1}{2} = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2}$$

$$\frac{y^2}{4} = \frac{y}{2}$$

or $\quad y = 2$

and $\quad x = \frac{y^2+1}{2} = 2.5$

Thus, $AB : OQ : OA = 1 : 2 : 2.5$

This ratio of the links ensures that P moves approximately in a horizontal straight line parallel to OQ.

7. Kempe's Mechanism

This mechanism consists of two identical mechanisms $ABCDEF$ and $A'B'C'D'EF'$. All pairs are turning pairs as shown in Fig. 6.12.

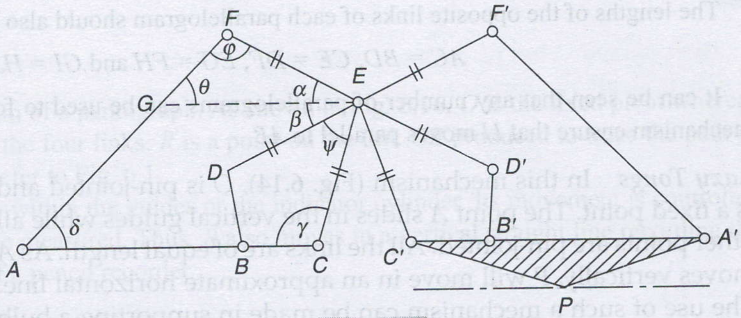


Fig. 6.12

The ratios of the links are

$AF = AC = 2(EC = ED = EF)$
$= 4(BD = BC)$

and

$A'F' = A'C' = 2(EC' = ED' =$
$EF') = 4(B'D' = B'C')$

Links DEF' and $D'EF$ are rigid links having turning pairs at E and at the ends.

It can be proved that if ABC is a fixed horizontal link, $A'B'C'$ also remains horizontal (in line with ABC) and thus any point on the link such as P will move in a horizontal straight line.

Quadrilaterals $ACEF$ and $EDBC$ are similar because,

$$\frac{AC}{ED} = \frac{CE}{DB} = \frac{EF}{BC} = \frac{FA}{CE} = 2$$

and $\angle\varphi = \angle\gamma$

(angles between corresponding sides when two pairs of adjacent sides are equal in the quadrilateral $ACEF$)

$\therefore$ $\angle\delta = \angle\psi$ (corresponding angles of two quadrilaterals)

Draw $EG \parallel CA$

In $\Delta\, EFG$,

$$\angle\alpha + \angle\varphi + \angle\theta = \pi$$

or

$$\begin{aligned}\angle\alpha &= \pi - \angle\varphi - \angle\theta \\ &= \pi - \angle\varphi - \angle\delta \qquad (\because EG \parallel CA) \\ &= \pi - \angle\gamma - \angle\psi \qquad \text{(i)}\end{aligned}$$

But as CE cuts EG and CA, two parallel lines,

$$\angle\gamma + \angle\psi + \angle\beta = \pi$$

or $\angle\gamma = \pi - \angle\psi - \angle\beta$

$\therefore$ from (i) $\angle\alpha = \pi - (\pi - \angle\psi - \angle\beta) - \angle\psi$
$= \angle\beta$

Thus, for all configurations, $\angle\alpha = \angle\beta$, i.e., inclination of ED and EF is same to EG or CA and the two identical parts of the mechanism always remain symmetrical.

Hence, if ABC is a fixed horizontal link, $A'B'C'$ also remains horizontal in line with ABC and any point P on it traces a horizontal path.

8. Parallel Linkages

If the opposite links of a four-link mechanism are made equal, the linkage will always form a parallelogram. The following types of parallel linkages are used universally.

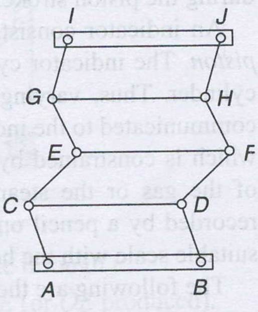


Fig. 6.13

Parallel Ruler As shown in Fig. 6.13, in a parallel ruler, all the horizontal links have the same length, i.e.,

$$AB = CD = EF = GH = IJ$$

The lengths of the opposite links of each parallelogram should also be equal, i.e.,

$$AC = BD, CE = DF, EG = FH \text{ and } GI = HJ$$

It can be seen that any number of parallelograms can be used to form this ruler. The dimensions of the mechanism ensure that IJ moves parallel to AB.

Lazy Tongs In this mechanism (Fig. 6.14), O is pin-jointed and is a fixed point. The point A slides in the vertical guides while all other points are pin-jointed. All the links are of equal length. As A moves vertically, P will move in an approximate horizontal line. The use of such a mechanism can be made in supporting a bulb (of a table lamp) or telephone, etc.

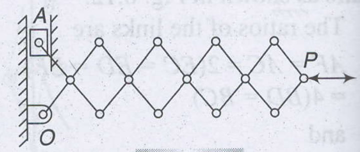


Fig. 6.14

Universal Drafting Machine In such a mechanism (Fig. 6.15), two parallelograms of the links are formed.

$$AB = CD \quad \text{and} \quad AC = BD$$

The link AB is fixed.

As $ABDC$ is a parallelogram, CD always remains parallel to AB. C and D are pin-jointed to a disc D_1. Thus, the disc D_1 can have translatory motion in a plane but not angular motions.

EF is another link on the disc D_1 pin-jointed at the ends E and F. As the orientation of the disc D_1 is fixed, the direction of EF is also fixed.

Also,

$$EG = FH \quad \text{and} \quad EF = GH$$

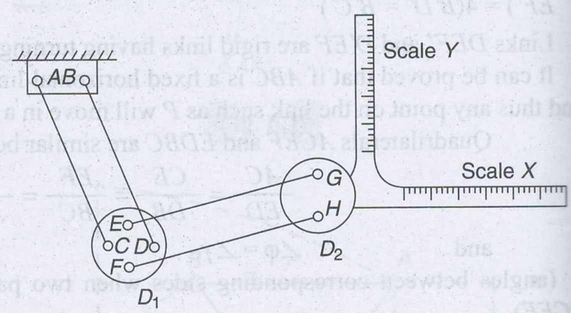


Fig. 6.15

Thus, the direction of GH is always parallel to EF or there is no angular movement of the disc D_2. Therefore, scales X and Y will always be along the horizontal and the vertical directions.

A universal drafting machine is extensively used as a substitute for T-square and set-square.

6.3 ENGINE INDICATORS

An *indicator* of a reciprocating engine is an instrument that keeps the graphical record of pressure inside the cylinder during the piston stroke.

An indicator consists of an *indicator cylinder* with a *piston*. The indicator cylinder is connected to the engine cylinder. Thus, varying pressure of the gas or steam is communicated to the indicator piston, the displacement of which is constrained by a spring to get a direct measure of the gas or the steam pressure. The displacement is recorded by a pencil on paper, wrapped on a drum, to a suitable scale with the help of a straight-line mechanism.

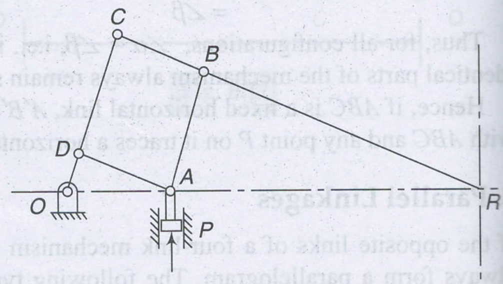


Fig. 6.16

The following are the usual types of indicators:

1. Simplex Indicator

This indicator employs the mechanism of a pantograph. As shown in Fig. 6.16, O is the fixed pivot whereas $ABCD$ is a parallelogram formed by the four links. R is a point on the link CB produced to trace the path of E or P (movement of piston). Also, refer to Fig. 6.1.

P moves in a vertical straight line within the guides or the indicator cylinder. Its movement is controlled by the steam or the gas pressure to be measured. Thus, R also moves in a vertical straight line recording the variation of pressure with the help of a pencil recorder.

Example 6.4 *Design a pantograph for an indicator to be used to obtain the indicator diagram of an engine. The distance between the fixed point and the tracing point is 180 mm. The indicator diagram should be three times the gas pressure inside the cylinder of the engine.*

Solution

Refer Fig. 6.16,

$$OR = 180 \text{ mm and } \frac{OR}{OA} = 3 \text{ (given)}$$

or $\frac{180}{OA} = 3$

or $OA = 60$ mm

The relationship of the different arms of a simplex indicator is as follows:

$$\frac{OR}{OA} = \frac{OC}{OD} = \frac{CR}{CB} = 3$$

Choose convenient dimensions of OD and DA. Let these be 30 mm and 50 mm respectively.

Thus, as $ABCD$ is to be a parallelogram and the above relation is to be fulfilled, the other dimensions will be

$$OC = 30 \times 3 = 90 \text{ mm}$$
$$CR = 50 \times 3 = 150 \text{ mm}$$

Construct the diagram as follows:

1. Locate D by making arcs of radii 30 mm and 50 mm with centres O and A respectively.
2. Produce OD to C such that $OC = 90$ mm.
3. Join CR.
4. Draw AB parallel to OC.

Thus, the required pantograph is obtained.

2. Crosby Indicator

This indicator employs a modified form of the pantograph. The mechanism has been shown in Fig. 6.17.

To have a vertical straight line motion of R, it must remain in line with O and P, and also the links OC and PB must remain approximately parallel.

As P lies on the link 3 and R on 5, locate the I-centres 31 and 51. If the directions of velocities of any two points on a link are known, the I-centre can be located easily which is the intersection of the perpendiculars to the directions of velocities at the two points.

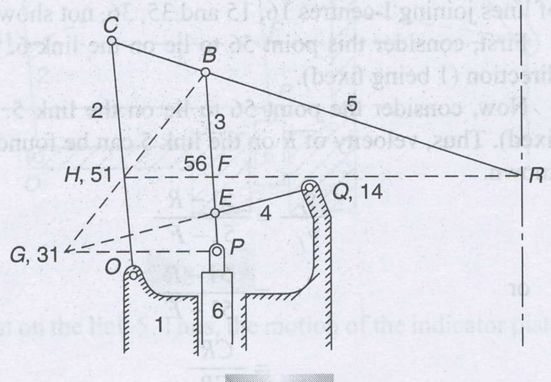


Fig. 6.17

First, locate 31 as the directions of velocities of P and E on the link 3 are known.

- The direction of velocity of P is vertical. Therefore, 31 lies on a horizontal line through P.
- The direction of velocity of E is perpendicular to QE. Therefore, 31 lies on QE (or QE produced).

The intersection of QE produced with the horizontal line through P locates the point 31.

Thus, the link 3 has its centre of rotation at 31 (link 1 is fixed) and the velocity of any point on the lin is proportional to its distance from 31, the direction being perpendicular to a line joining the point with th I-centre.

To locate 51, the directions of velocities of B and C are known.

- The direction of velocity of B is $\perp$ to $31 - B$. Therefore, 51 lies on $31 - B$.
- The direction of velocity of C is $\perp$ to OC. Therefore, 51 lies on OC.

Thus, 51 can be located.

Now, the link 5 has its centre of rotation at 51. The direction of velocity of the point R on this link will be perpendicular to 51-R. To have a vertical motion of R, it must lie on a horizontal line through 51.

The ratio of the velocities of R and P is given by,

$$\frac{v_r}{v_p} = \frac{v_r}{v_b} \cdot \frac{v_b}{v_p} \qquad (B \text{ is common to 3 and 5})$$

$$= \frac{51-R}{51-B} \cdot \frac{31-B}{31-P}$$

$$= \frac{51-R}{51-B} \cdot \frac{51-B}{51-F} \qquad (\because \Delta\text{s } BPG \text{ and } BFH \text{ are similar})$$

$$= \frac{51-R}{51-F}$$

$$= \frac{CR}{CB} \qquad (\because \Delta\text{s } CRH \text{ and } BRF \text{ are similar})$$

$$= \text{constant}$$

This shows that the velocity or the displacement of R will be proportional to that of P.

Alternatively, locate the I-centre 56 by using Kennedy's theorem. It will be at the point F (the intersection of lines joining I-centres 16, 15 and 35, 36, not shown in the figure).

First, consider this point 56 to lie on the link 6. Its absolute velocity is the velocity of 6 in the vertical direction (1 being fixed).

Now, consider the point 56 to lie on the link 5. The motion of 5 is that of rotation about 51 (1 being fixed). Thus, velocity of R on the link 5 can be found as the velocity of 56, another point on the same link is known.

$$\frac{v_r}{v_f} = \frac{51-R}{51-F}$$

or

$$= \frac{51-R}{51-F} \qquad (v_f = v_p)$$

$$= \frac{CR}{CB}$$

3. Thomson Indicator

A Thomson indicator employs a Grass–Hopper mechanism $OCEQ$. R is the tracing point which lies on CE produced as shown in Fig. 6.18.

The best position of the tracing point R is obtained as discussed below:

Locate the I-centres 31 and 51 as in case of a Crosby indicator. The directions of velocities of two points C and E on the link 5 are known; therefore, first locate the I-centre 51.

- The direction of velocity of C is $\perp$ to OC. Therefore 51 lies on OC.
- The direction of velocity of E is $\perp$ to QE, Therefore, 51 lies on QE (or QE produced).

Thus, 51 can be located.

Now, the directions of velocities of two points B and P on the link 3 are known.

The direction of velocity of B is $\perp$ to 51-B, (B is on the link 5 also)

Therefore, 31 lies on the line 51-B.

The direction of velocity of P is vertical,

Therefore, 31 lies on a horizontal line through P.

Thus, 31 can be located.

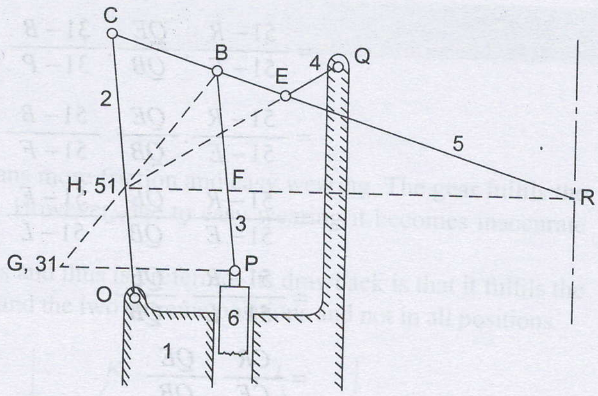


Fig. 6.18

As R is to move in a vertical direction, it must lie on a horizontal line through the I-centre of the link 5 on which the pointer lies.

Similar to the case of a Crosby indicator, the velocity ratio is given by,

$$\frac{v_r}{v_p} = \frac{CR}{CB} = \text{constant}$$

Therefore, the velocity or the displacement of R is proportional to that of P.

It is to be remembered that since OC and PB do not remain parallel for all positions, R moves in an approximate vertical line. However, the variations are negligible.

Alternatively, the I-centre 56 can be located by using Kennedy's theorem. It will be at the point F.

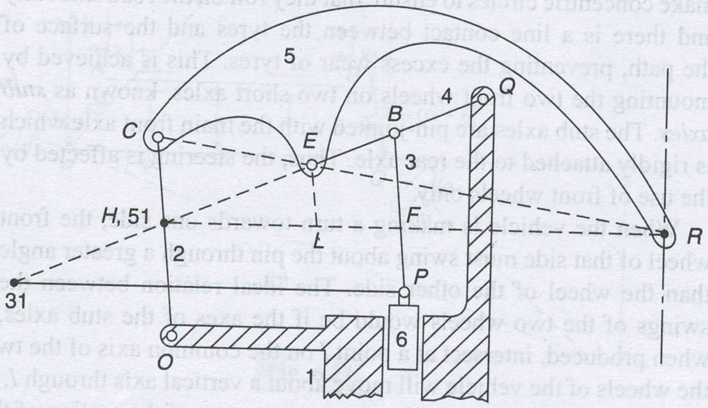


Fig. 6.19

4. Dobbie McInnes Indicator

This indicator is similar to a Thomson indicator, the difference being that the link 3 is pivoted to a point in the link 4 instead of a point on the link 5. Thus, the motion of the indicator piston is imparted to the link 4.

The indicator is shown in Fig. 6.19.

Locate the I-centre 31 as before.

Locate R by finding the intersection of CE and a horizontal line through 51.

Locate I-centre 31as usual.

Now

$$\frac{v_r}{v_p} = \frac{v_r}{v_e} \times \frac{v_e}{v_b} \times \frac{v_b}{v_p}$$

$$= \frac{51-R}{51-E} \cdot \frac{QE}{QB} \cdot \frac{31-B}{31-P}$$

$$= \frac{51-R}{51-E} \cdot \frac{QE}{QB} \cdot \frac{51-B}{51-F} \qquad (\because \Delta s\ BPG \text{ and } BFH \text{ are similar})$$

$$= \frac{51-R}{51-E} \cdot \frac{QE}{QB} \cdot \frac{51-E}{51-L} \qquad (\because \Delta s\ BFH \text{ and } ELH \text{ are similar})$$

$$= \frac{51-R}{51-L} \cdot \frac{QE}{QB}$$

$$= \frac{CR}{CE} \cdot \frac{QE}{QB}$$

$$= \text{constant}$$

This expression also gives approximately the ratio of the displacement of R to that of P.

6.4 AUTOMOBILE STEERING GEARS

When an automobile takes turns on a road, all the wheels should make concentric circles to ensure that they roll on the road smoothly and there is a line contact between the tyres and the surface of the path, preventing the excess wear of tyres. This is achieved by mounting the two front wheels on two short axles, known as *stub axles*. The stub axles are pin-jointed with the main front axle which is rigidly attached to the rear axle. Thus, the steering is affected by the use of front wheels only.

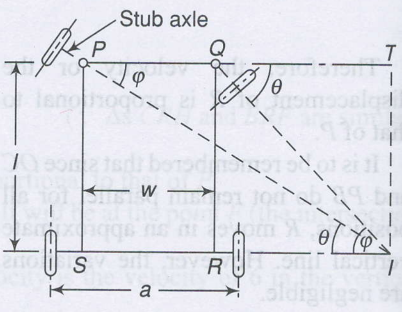


Fig. 6.20

When the vehicle is making a turn towards one side, the front wheel of that side must swing about the pin through a greater angle than the wheel of the other side. The ideal relation between the swings of the two wheels would be if the axes of the stub axles, when produced, intersect at a point I on the common axis of the two rear wheels Fig. (6.20). In that case, all the wheels of the vehicle will move about a vertical axis through I, minimizing the tendency of the wheels to skid. The point I is also the instantaneous centre of the motion of the four wheels.

Let θ and φ = angles turned by the stub axles

l = wheel base

w = distance between the pivots of front axles

Then,

$$\cot\varphi = \frac{PT}{Tl} \text{ and } \cot\theta = \frac{QT}{Tl}$$

$$\cot\varphi - \cot\theta = \frac{PT - QT}{Tl} = \frac{PQ}{Tl} = \frac{w}{l} \qquad (6.1)$$

This is known as the *fundamental equation of correct gearing*. Mechanisms that fulfil this fundamental equation are known as *steering gears*.

6.5 TYPES OF STEERING GEARS

There are two main types of steering gears:

1. Davis steering gear
2. Ackermann steering gear

A *Davis steering gear* has sliding pairs which means more friction and easy wearing. The gear fulfils the fundamental equation of gearing in all the positions. However, due to easy wearing it becomes inaccurate after some time.

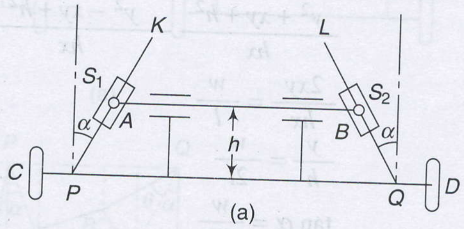
An *Ackermann steering gear* has only turning pairs and thus is preferred. Its drawback is that it fulfils the fundamental equation of correct gearing at the middle and the two extreme positions and not in all positions.

Davis Steering Gear

A Davis steering gear shown in Fig. 6.21(a) consists of two arms PK and QL fixed to the stub axles PC and QD to form two similar bell-crank levers CPK and DQL pivoted at P and Q respectively. A cross link or track arm AB, constrained to slide parallel to PQ, is pin-jointed at its ends to two sliders. The sliders S_1 and S_2 are free to slide on the links PK and QL respectively.

During the straight motion of the vehicle, the gear is in the mid-position with equal inclination of the arms PK and QL with PQ.

As the vehicle turns right, the cross-arm AB also moves right through a distance x from the mid-position as shown in Fig.6.21(b). The bell-crank levers assume the positions $C'PK'$ and $D'QL'$.



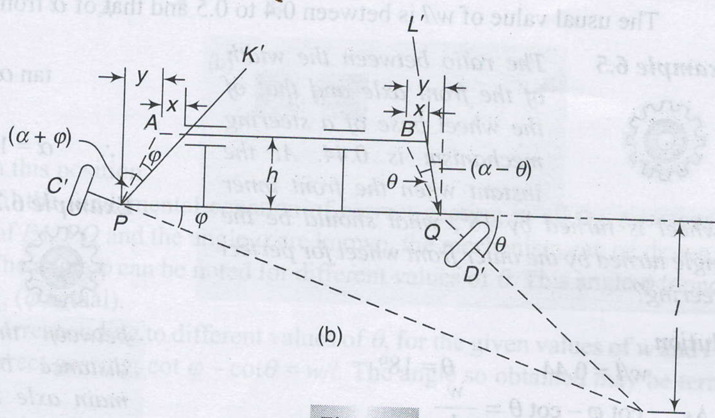


Fig. 6.21

Let h = vertical distance between AB and PQ

$$\tan(\alpha-\theta)=\frac{y-x}{h}$$

$$\frac{\tan\alpha-\tan\theta}{1+\tan\alpha\tan\theta}=\frac{y-x}{h}$$

$$\frac{\frac{y}{h}-\tan\theta}{1+\frac{y}{h}\tan\theta}=\frac{y-x}{h}\qquad\left(\tan\alpha=\frac{y}{h}\right)$$

$$\frac{y-h\tan\theta}{xh+y\tan\theta}=\frac{y-x}{h}$$

$$(y-h\tan\theta)h=(h+y\tan\theta)(y-x)$$

$$yh - h^2 \tan\theta = hy - hx + y^2 \tan\theta - xy\tan\theta$$
$$hx = (y^2 - xy + h^2)\tan\theta$$
$$\tan\theta = \frac{hx}{y^2 - xy + h^2}$$

Also, $\tan(\alpha + \varphi) = \dfrac{y + x}{h}$

and it can be proved that $\tan\varphi = \dfrac{hx}{y^2 + xy + h^2}$

For correct steering action, $\cot\varphi - \cot\theta = \dfrac{w}{l}$

or
$$\frac{y^2 + xy + h^2}{hx} - \frac{y^2 - xy + h^2}{hx} = \frac{w}{l}$$

or $\dfrac{2xy}{hx} = \dfrac{w}{l}$

or $\dfrac{y}{h} = \dfrac{w}{2l}$

or $\tan\alpha = \dfrac{w}{2l}$ (6.2)

The usual value of w/l is between 0.4 to 0.5 and that of α from 11 or 14 degrees.

Example 6.5 *The ratio between the width of the front axle and that of the wheel base of a steering mechanism is 0.44. At the instant when the front inner wheel is turned by 18°, what should be the angle turned by the outer front wheel for perfect steering?*

Solution

$w/l = 0.44 \qquad \theta = 18°$

As $\cot\varphi - \cot\theta = \dfrac{w}{l}$

$\therefore \quad \cot\varphi - \cot 18° = 0.44$

$\cot\phi = 0.44 + 3.078 = 3.518$

or $\quad \phi = \underline{15.9°}$

Example 6.6 *The distance between the steering pivots of a Davis steering gear is 1.3 m. The wheel base is 2.75 m. What will be the inclination of the track arms to the longitudinal axis of the vehicle if it is moving in a straight path?*

Solution

$w = 1.3$ m $\qquad l = 2.75$ m

$$\tan\alpha = \frac{w}{2l} = \frac{1.3}{2 \times 2.75} = 0.236$$

$\therefore \quad \alpha = 13.3°$ or $13°\,18'$

Example 6.7 *The track arm of a Davis steering gear is at a distance of 192 mm from the front main axle whereas the difference between their lengths is 96 mm. If the distance between steering pivots of the main axle is 1.4 m, determine the length of the chassis between the front and the rear wheels. Also, find the inclination of the track arms to the longitudinal axis of the vehicle.*

Solution

$w = 1.4$ m $\qquad h = 192$ mm $\qquad y = 96/2 = 48$ mm

$$\tan\alpha = \frac{y}{h} = \frac{48}{192} = 0.25$$

$\therefore \quad \alpha = \underline{14°}$

Also $\quad \tan\alpha = \dfrac{w}{2l}$

$\therefore \quad \tan 14° = \dfrac{1.4}{2l}$

or $\quad l = \underline{2.8 \text{ m}}$

Ackermann Steering Gear

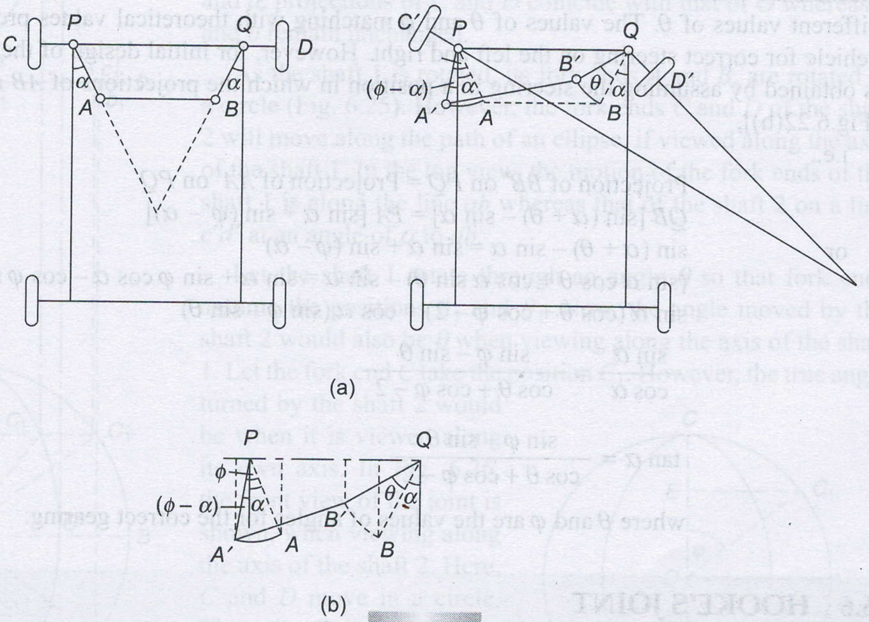


Fig. 6.22

This steering gear consists of a four-link mechanism *PABQ* having four turning pairs.

As shown in Fig. 6.22(a), two equal arms *PA* and *QB* are fixed to the stub axles *PC* and *QD* to form two similar bell-crank levers *CPA* and *DQB* pivoted at *P* and *Q* respectively. A cross link *AB* is pin-jointed at the ends to the two bell-crank levers.

During the straight motion of the vehicle, the gear is in the mid-position with equal inclination of the arms *PA* and *QB* with *PQ*. The cross link *AB* is parallel to *PQ* in this position.

An Ackermann gear does not fulfil the fundamental equation of correct gearing in all the positions but only in three positions. If the values of *PA, PQ* and the angle α are known, the mechanism can be drawn to a suitable scale in different positions. The angle ϕ can be noted for different values of θ. This angle ϕ found by drawing the gear may be termed as φ_a (φ actual).

Correct or theoretical values of φ corresponding to different values of θ, for the given values of *w* and *l* can be calculated from the relation for correct gearing, $\cot\varphi - \cot\theta = w/l$. The angle so obtained may be termed as φ_t (φ theoretical).

Comparing φ_a and φ_t, following observations are made:

1. For small values of θ, φ_a is marginally higher than φ_t
2. For larger values of θ, φ_a is lower than φ_t and the difference is substantial.

Thus, for larger values of θ or when the vehicle is taking a sharp turn, the wear of the tyres can be more due to slipping. However, to take sharp turns, the vehicle has to be slowed down, which reduces the wear of the tyres. Thus, the large difference between φ_a and φ_t does not affect much the life of tyres.

In an Ackermann gear, the instantaneous centre *I* does not lie on the rear axis but on a line parallel to the rear axis at an approximate distance of 0.3*l* above it.

Three positions of correct gearing are

1. when the vehicle moves straight,
2. when the vehicle moves at a correct angle to the right, and
3. when the vehicle moves at a correct angle to the left.

In all other positions, pure rolling is not possible due to slipping of the wheel.

Graphically, the two positions of the correct gearing are found by finding ($\cot\varphi - \cot\theta$) at different positions. The values that give the correct values of w/l ($w/l \approx 0.45$) correspond to correct gearing.

Angular Velocity Ratio

Let ω_1 = angular velocity of driving shaft $\left(= \frac{d\theta}{dt}\right)$

ω_2 = angular velocity of driving shaft $\left(= \frac{d\varphi}{dt}\right)$

Differentiating Eq. 6.4 with respect to time t,

$$\sec^2\theta \frac{d\theta}{dt} = \cos\alpha \sec^2\varphi \frac{d\varphi}{dt}$$

or

$$\frac{d\varphi/dt}{d\theta/dt} = \frac{\sec^2\theta}{\cos\alpha \sec^2\varphi}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos^2\theta \cos\alpha\,(1+\tan^2\varphi)}$$

$$= \frac{1}{\cos^2\theta\cos\alpha\left(1+\frac{\tan^2\theta}{\cos^2\alpha}\right)} \quad \ldots \left(\tan\varphi = \frac{\tan\theta}{\cos\alpha}\right)$$

$$= \frac{1}{\cos^2\theta\cos\alpha\left(1+\frac{\sin^2\theta}{\cos^2\theta\cos^2\alpha}\right)}$$

$$= \frac{\cos^2\theta\cos^2\alpha}{\cos^2\theta\cos\alpha\,(\cos^2\theta\cos^2\alpha+\sin^2\theta)}$$

$$= \frac{\cos\alpha}{\cos^2\theta\,(1-\sin^2\alpha)+\sin^2\theta}$$

$$= \frac{\cos\alpha}{\cos^2\theta-\cos^2\theta\sin^2\alpha+\sin^2\theta}$$

$$= \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta} \tag{6.5}$$

(i) $\frac{\omega_2}{\omega_1}$ is unity when $\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta} = 1$

or $\cos\alpha = 1-\sin^2\cos^2\theta$

or $\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha}$

$$= \frac{1-\cos\alpha}{1-\cos^2\alpha}$$

$$= \frac{1-\cos\alpha}{(1+\cos\alpha)(1-\cos\alpha)}$$

$$= \frac{1}{1+\cos\alpha}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{1+\cos\alpha}$$

$$= \frac{\cos^2\theta\left(\dfrac{\sin^2\theta}{\cos^2\theta}+1\right)}{1+\cos\alpha}$$

or
$$\frac{\sin^2\theta}{\cos^2\theta}+1 = 1+\cos\alpha$$

or
$$\tan^2\theta = \cos\alpha$$

$$\tan\theta = \pm\sqrt{\cos\alpha} \tag{6.6}$$

Thus, $\omega_2 = \omega_1$ or the velocities of the driven and the driving shafts are equal when the condition is fulfilled. This is possible once in all the four quadrants for particular values of θ if α is constant.

(ii) $\dfrac{\omega_2}{\omega_1}$ is minimum when the denominator of Eq. (6.5) is maximum,

i.e., $(1-\sin^2\alpha\cos^2\theta)$ is maximum.

This is so when $\cos^2\theta$ is minimum,

or $\theta = 90°$ or $270°$

Then,
$$\frac{\omega_2}{\omega_1} = \cos\alpha \tag{6.7}$$

(iii) $\dfrac{\omega_2}{\omega_1}$ is maximum when the denominator of Eq. (6.5) is minimum,

i.e. $(1-\sin^2\alpha\cos^2\theta)$ is minimum,

or $\theta = 0°$ or $180°$

and
$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\sin^2\alpha} = \frac{\cos\alpha}{\cos^2\alpha} = \frac{1}{\cos\alpha} \tag{6.8}$$

The variation in the speed of the driven shaft corresponding to the rotation of the driving shaft is shown in Fig. 6.27. Points 'e' correspond to the angular displacements of the driving shaft when the angular velocity of the driven shaft is equal to that of the driving shaft. Points 'min' and 'max' correspond to the angular displacements of the driving shaft when the angular speeds of the driven shaft are the minimum and the maximum respectively.

Graphically, the variation of angular velocity of the driven shaft can be represented by an ellipse whereas that of the driving shaft, by a circle (Fig 6.27). Such a diagram is known as a *polar velocity diagram.*

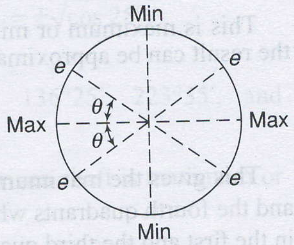


Fig. 6.27

Maximum variation of velocity of the driven shaft of its mean velocity

$$= \frac{\omega_{2\,max} - \omega_{2\,min}}{\omega_{mean}}$$

But ω_{mean} of the driven shaft is equal to the angular velocity ω_1 of the driving shaft as both the shafts complete one revolution in the same period of time.

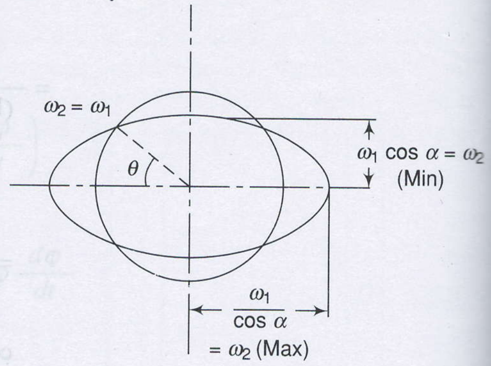


Fig. 6.28

$$\text{Maximum variation} = \frac{\omega_1 / \cos\alpha - \omega_1 \cos\alpha}{\omega_1} \tag{6.9}$$

$$= \frac{1-\cos^2\alpha}{\cos\alpha}$$

$$= \frac{\sin^2\alpha}{\cos\alpha}$$

$$= \tan\alpha \sin\alpha \tag{6.10}$$

If α is small, i.e., the angle between the axes of the two shafts is small,

$\sin\alpha \approx \tan\alpha \approx \alpha.$

Maximum variation $\approx \alpha^2$

(The mean speed ω is not equal to $\dfrac{\omega_{2\,max} + \omega_{2\,min}}{2}$ as the variation of speed is not linear throughout the rotation of the driven shaft.)

Angular Acceleration of Driven Shaft

Differentiating Eq. (6.5) with respect to time (ω_1 = constant)

$$\frac{d\omega_2}{dt} = \omega_1 \frac{d}{dt}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}\right)$$

or

$$\text{acceleration} = \omega_1 . \frac{d\theta}{dt}\frac{d}{d\theta}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}\right)$$

$$= \omega_1^2 \cos\alpha \frac{d}{d\theta}(1-\sin^2\alpha\cos^2\theta)^{-1}$$

$$= \omega_1^2 \cos\alpha\,(-1)\,(1-\sin^2\alpha\cos^2\theta)^{-2}$$

$$(-\sin^2\alpha)\,(2\cos\theta)\,(-\sin\theta)$$

$$= \frac{-\omega_1^2\cos\alpha\sin^2\alpha\sin 2\theta}{(1-\sin^2\alpha\cos^2\theta)^2} \tag{6.11}$$

This is maximum or minimum when $\dfrac{d\,(acc)}{d\theta} = 0$. The resulting expression being very cumbersome, the result can be approximated to

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha} \tag{6.12}$$

This gives the maximum acceleration of the driven shaft corresponding to two values of θ in the second and the fourth quadrants whereas the minimum acceleration (maximum retardation) corresponds to θ values in the first and the third quadrants.

Care is to be taken to keep the angle between the two shafts to the minimum possible and not to attach excessive masses to the driven shaft. Otherwise, very high alternating stresses due to the angular acceleration and retardation will be set up in the parts of the joint, which are undesirable.

Example 6.8 *Determine the maximum permissible angle between the shaft axes of a universal joint if the driving shaft rotates at 800 rpm and the total fluctuation of speed does not exceed 60 rpm. Also, find the maximum and the minimum speeds of the driven shaft.*

Solution:

$N_1 = 800$ rpm $\qquad \omega_{2\,max} - \omega_{2\,min} = 60$ rpm

We have

Maximum variation,

$$\frac{\omega_{2\,max} - \omega_{2\,min}}{\omega_{mean}} = \frac{1 - \cos^2 \alpha}{\cos \alpha}$$

or $$\frac{60}{800} = \frac{1 - \cos^2 \alpha}{\cos \alpha}$$

or $$1 - \cos^2 \alpha - 0.075 \cos \alpha = 0$$

$$\cos^2 \alpha + 0.075 \cos \alpha = 1$$

$$(\cos \alpha + 0.0375)^2 = 1 + (0.0375)^2$$

$$= 1.0014 = (1.000\ 703)^2$$

or

$$\cos \alpha = 1.000\ 703 - 0.0375 = 0.963\ 203$$

$$\alpha = \underline{15.6°}$$

Maximum speed of driven shaft

$$= \frac{N_1}{\cos \alpha} = \frac{800}{0.963\ 203}$$

$$= \underline{830.6 \text{ rpm}}$$

Minimum speed of driven shaft $= N_1 \cos \alpha = 800 \times 0.963\ 203 = \underline{770.6 \text{ rpm}}$

Example 6.9 *The driving shaft of a Hooke's joint rotates at a uniform speed of 400 rpm. If the maximum variation in speed of the driven shaft is ± 5% of the mean speed, determine the greatest permissible angle between the axes of the shafts. What are the maximum and the minimum speeds of the driven shaft?*

Solution:

$N_1 = 400$ rpm

Maximum variation in speed = 0.1
...(0.05 + 0.05 = 0.1)

or $$\frac{1 - \cos^2 \alpha}{\cos \alpha} = 0.1$$

$$1 - \cos^2 \alpha - 0.1 \cos \alpha = 0$$

$$\cos^2 \alpha + 0.1 \cos \alpha = 1$$

$$(\cos \alpha + 0.05)^2 = 1 + (0.05)^2 = 1.0025$$

$$= (1.001\ 25)^2$$

or

$$\cos \alpha = 1.001\ 25 - 0.05 = 0.951\ 25$$

$$\alpha = 17.96° \text{ or } 17°\ 58'$$

Minimum speed of driven shaft

$$= \frac{N_1}{\cos \alpha} = \frac{400}{0.95\ 125}$$

$$= \underline{420.5 \text{ rpm}}$$

Minimum speed of driven shaft

$$= N_1 \cos \alpha = 400 \times 0.951\ 25 = \underline{380.5 \text{ rpm}}$$

Example 6.10 *A Hooke's joint connects two shafts whose axes intersect at 25°. What will be the angle turned by the driving shaft when the*

(i) velocity ratio is maximum, minimum and unity?

(ii) acceleration of the driven shaft is maximum, minimum (negative) and zero?

Solution:

(i) (a) ω_2/ω_1 is maximum at $\theta = 0°$ and 180°

(b) ω_2/ω_1 is minimum at $\theta = 90°$ and 270°

(c) ω_2/ω_1 is unity when

$$\tan \theta = \pm\sqrt{\cos \alpha} = \pm\sqrt{\cos 25°}$$

$$= \pm\ 0.952$$

or θ = 43°35′, 136°25′, 223°35′, and 316°25′

(ii) Acceleration of driven shaft is maximum or minimum when

$$\cos 2\theta \approx \frac{2 \sin^2 \alpha}{2 - \sin^2 \alpha} \approx \frac{2 \sin^2 25°}{2 - \sin^2 25°} \approx 0.196$$

or $2\theta \approx 78°42'$, $(360° - 78°42')$, $(360° + 78°42')$, $(720° - 78°42')$

or $\theta \approx 39°21'$, $140°39'$, $219°21'$ and $320°39'$

Now,

$$\text{acceleration} = \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin\theta}{(1-\sin^2\alpha\cos^2\theta)^2}$$

Thus, acceleration is positive when $\sin 2\theta$ is negative and is negative when $\sin 2\theta$ is positive.

Corresponding to four values of 2θ found above, $\sin 2\theta$ will be +ve, –ve, +ve and –ve respectively.

Maximum acceleration will be at $140°39'$ and $320°39'$ and minimum acceleration (–ve) will be at $39°21'$ and $219°21'$.

Acceleration is zero when ω_2/ω_1 is maximum or minimum, i.e., at 0°, 90°, 180° and 270°.

Or acceleration is zero when 2θ is zero or when 2θ is 0°, 180°, 360°, 540° or when θ is 0°, 90°, 180° and 270°.

Example 6.11 *The angle between the axes of two shafts joined by Hooke's joint is 25°. The driving shaft rotates at a uniform speed of 180 rpm. The driven shaft carries a steady load of 7.5 kW. Calculate the mass of the flywheel of the driven shaft if its radius of gyration is 150 mm and the output torque of the driven shaft does not vary by more than 15% of the input shaft.*

Solution:

$\alpha = 25°$ $\qquad N_1 = 180$ rpm

$P = 7.5$ kW $\qquad \omega_1 = \dfrac{2\pi \times 180}{60} = 6\pi$

$k = 0.15$ m $\qquad \Delta T = 15\%$

Maximum torque on the driven shaft will be when the acceleration is maximum, i.e., when

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha} \approx \frac{2\sin^2 25°}{2-\sin^2 25°} \approx 0.196$$

or

$$2\theta = 78°42' \text{ or } 281°18'$$

$\therefore$ Maximum acceleration

$$= \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1-\sin^2\alpha\cos^2\theta)^2}$$

$$= \frac{-(6\pi)^2 \cos 25° \sin^2 25° \sin 281°18'}{(1-\sin^2 25°\cos^2 140°39')^2}$$

$= 70.677$ rad/s^2

$P = T\omega_1$

$7500 = T \times 6\pi$

Input torque, $T = 397.9$ N.m

Permissible variation = torque due to acceleration of driven shaft

$397.9 \times 0.15 = I\alpha = mk^2\,\alpha$

or $\quad 397.9 \times 0.15 = m \times (0.15)^2 \times 70.677$

$m = \underline{37.53 \text{ kg}}$

Example 6.12 *A Hooke's joint connects two shafts whose axes intersect at 18°. The driving shaft rotates at a uniform speed of 210 rpm. The driven shaft with attached masses has a mass of 60 kg and radius of gyration of 120 mm. Determine the*

(i) torque required at the driving shaft if a steady torque of 180 N.m resists rotation of the driven shaft and the angle of rotation is 45°

(ii) angle between the shafts at which the total fluctuation of speed of the driven shaft is limited to 18 rpm

Solution:

$m = 60$ kg $\qquad k = 120$ mm $\qquad N = 210$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

Maximum acceleration

$$= \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1-\sin^2\alpha\cos^2\theta)^2}$$

$$= \frac{-(22)^2 \cos 18° \sin^2 18° \sin 90°}{(1-\sin^2 18°\cos^2 45°)^2}$$

$$= -\frac{43.956}{0.907}$$

$= -48.47$ rad/s^2

The negative sign indicates that it is retardation at the instant.

Torque required for retardation of the driven shaft $= I\,\alpha = mk^2.\alpha$

$= 60 \times 0.12^2 \times (-48.47)$

$= -41.88$ N.m

Total torque required on the driven shaft, T_2

$$= \text{Steady torque} + \text{Accelerating torque}$$

$$= 180 + (-41.88)$$

$$= 138.12 \text{ N.m}$$

Now as $\quad P = T_1\omega_1 = T_2\omega_2$

$$\therefore\ T_1 = T_2\,\frac{\omega_2}{\omega_1} = T_2\,\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$$

$$= 138.12 \times \frac{\cos 18^\circ}{1-\sin^2 18^\circ \cos^2 45^\circ}$$

$$= \underline{137.99 \text{ N.m}}$$

Maximum variation $= \dfrac{1-\cos^2\alpha}{\cos\alpha}$

or $\quad \dfrac{\omega_{2\,max}-\omega_{2\,min}}{\omega_{mean}} = \dfrac{1-\cos^2\alpha}{\cos\alpha}$

or $\quad \dfrac{18}{180} = \dfrac{1-\cos^2\alpha}{\cos\alpha}$

or $\quad 1-\cos^2\alpha - 0.1\cos\alpha = 0$

$$\cos^2\alpha + 0.1\cos\alpha = 1$$

On solving,

$$\alpha = \underline{17.96^\circ}$$

6.7 DOUBLE HOOKE'S JOINT

In a single Hooke's joint, the speed of the driven shaft is not uniform although the driving shaft rotates at a uniform speed. To get uniform velocity ratio, a double Hooke's joint has to be used. In a double Hooke's joint, two universal joints and an intermediate shaft are used. If the angular misalignment between each shaft and the intermediate shaft is equal, the driving and the driven shafts remain in exact angular alignment, though the intermediate shaft rotates with varying speed.

A single Hooke's joint was analysed assuming the axes of the two shafts and the fork of the driving shaft to be horizontal. The results showed that the speed of the driven shaft is the same after an angular displacement of 180°. Therefore, it is immaterial whether the driven shaft makes the angle α with the axis of the driving shaft to its left or right.

Thus, to have a constant velocity ratio

- the driving and the driven shafts should make equal angles with the intermediate shaft, and
- the forks of the intermediate shaft should lie in the same plane.

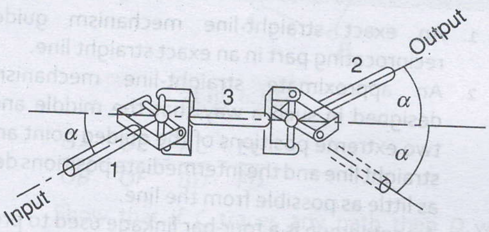


Fig. 6.29

Let γ be the angle turned by the intermediate shaft 3 while the angle turned by the driving shaft 1 and the driven shaft 2 be θ and φ respectively as before (Fig. 6.29).

Then,

$$\tan\theta = \cos\alpha\tan\gamma \qquad \text{(fork of shaft 1 horizontal)}$$

and

$$\tan\varphi = \cos\alpha\tan\gamma \qquad \text{(fork of shaft 2 horizontal)}$$

$$\therefore\quad \theta = \varphi$$

This type of joint can be used for two intersecting shafts as well as for two parallel shafts.

However, if somehow the forks of the intermediate shafts lie in planes perpendicular to each other, the variation of speed of the driven shaft will be there.

$$\left(\frac{\omega_3}{\omega_1}\right)_{min} = \cos\alpha \qquad \text{(fork of the shaft 1 horizontal)}$$

$$\left(\frac{\omega_2}{\omega_3}\right)_{min} = \cos\alpha \qquad \text{(fork of the shaft 1 horizontal)}$$

$$\therefore \quad \left(\frac{\omega_2}{\omega_1}\right)_{min} = \cos^2\alpha \qquad (6.13)$$

Similarly,

$$\left(\frac{\omega_2}{\omega_1}\right)_{max} = \frac{1}{\cos^2\alpha} \qquad (6.14)$$

Therefore, the maximum variation (fluctuation) of speed of the driven shaft is from $\cos^2\alpha$ to $1/\cos^2\alpha$.

Example 6.13 *The driving shaft of a double Hooke's joint rotates at 400 rpm. The angle of the driving and of the driven shaft with the intermediate shaft is 20°. If somehow the forks of the intermediate shaft lie in planes perpendicular to each other, determine the maximum and the minimum velocities of the driven shaft.*

Solution:

$$\omega_{2min} = \omega_1\cos^2\alpha$$

or

$$N_{2min} = N_1\cos^2\alpha = 400 \times \cos^2 20°$$

$$= \underline{353.2\text{ rpm}}$$

$$N_{2max} = \frac{N_1}{\cos^2\alpha} = \frac{400}{\cos^2 20°}$$

$$= \underline{453\text{ rpm}}$$

Summary

1. An exact straight-line mechanism guides a reciprocating part in an exact straight line.
2. An approximate straight-line mechanism is designed in such a way that the middle and the two extreme positions of the guided point are in a straight line and the intermediate positions deviate as little as possible from the line.
3. A *pantograph* is a four-bar linkage used to produce paths exactly similar to the ones traced out by a point on the linkage. The paths so produced are, usually, on an enlarged or a reduced scale.
4. An *indicator* of a reciprocating engine is an instrument that keeps the graphical record of pressure inside the cylinder during the piston stroke.
5. The fundamental equation of correct gearing for automobiles is, $\cot\varphi - \cot\theta = \frac{w}{l}$. Mechanisms that fulfill this fundamental equation are known as *steering gears*.
6. Two main types of steering gears are Davis steering gear, and Ackermann steering gear.
7. A *Davis steering gear* has sliding pairs which means more friction and easy wearing. The gear fulfils the fundamental equation of gearing in all the positions.
8. An *Ackermann steering gear* has only turning pairs and thus is preferred. Its drawback is that it fulfils the fundamental equation of correct gearing at the middle and the two extreme positions and not in all positions.
9. A *Hooke's joint* commonly known as a *universal joint*, is used to connect two non-parallel and intersecting shafts.
10. Speed of the driven shaft is minimum when $\theta = 90°$ or $270°$, and the minimum speed is given by $\omega_2 = \omega_1\cos\alpha$.
11. Speed of the driven shaft is maximum when $\theta = 0°$ or $180°$, and the maximum speed is given by $\omega_1/\cos\alpha$.

Exercises

1. What is a pantograph? Show that it can produce paths exactly similar to the ones traced out by a point on a link on an enlarged or a reduced scale.
2. Enumerate straight-line mechanisms. Why are they classified into exact and approximate straight-line mechanisms?
3. Sketch a Paucellier mechanism. Show that it can be used to trace a straight line.
4. Prove that a point on one of links of a Hart mechanism traces a straight line on the movement of its links.
5. What is a Scott–Russel mechanism? What is its limitation? How is it modified?
6. In what way is a Grass–Hopper mechanism a derivation of the modified Scott–Russel mechanism?
7. How can you show that a Watt mechanism traces an approximate straight line?
8. How can we ensure that a Tchebicheff mechanism traces an approximate straight line?
9. Prove that a Kempe's mechanism traces an exact straight line using two identical mechanisms.
10. Discuss some of the applications of parallel linkages.
11. What is an engine indicator? Describe any one of them.
12. With the help of neat sketch discuss the working of a Crosby indicator.
13. Describe the function of a Thomson or a Dobbie McInnes Indicator.
14. What is an automobile steering gear? What are its types? Which steering gear is preferred and why?
15. What is fundamental equation of steering gears? Which steering gear fulfils this condition?
16. An Ackermann steering gear does not satisfy the fundamental equation of a steering gear at all positions. Yet it is widely used. Why?
17. What is a Hooke's joint? Where is it used?
18. Derive an expression for the ratio of angular velocities of the shafts of a Hooke's joint.
19. Sketch a polar velocity diagram of a Hooke's joint and mark its salient features.
20. Design and dimension a pantograph to be used to double the size of a pattern.
(In Fig. 6.1, make $\frac{OC}{OD} = \frac{OR}{OP} = 2$. *Drawing tool at R; P traces the pattern*)
21. Design and dimension a pantograph which will decrease pattern dimensions by 30%.
(In Fig. 6.1, make $\frac{OC}{OD} = \frac{OR}{OP} = \frac{100}{100-30}$; *Drawing tool at R; P traces the pattern*)
22. Design and dimension a pantograph that can be used to decrease pattern dimensions by 15%. The fixed pivot should lie between the tracing point and the marking point (tool holder).
(In Fig. 6.1, make $\frac{CD}{OD} = \frac{RP}{OP} = \frac{100}{100-15}$; *P, the fixed pivot; Drawing tool at O; R traces the pattern*)
23. In Fig. 6.30, the dimensions of the various links are such that

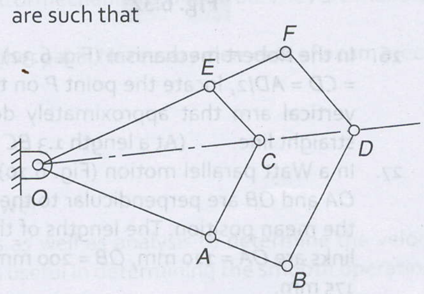


Fig. 6.30

$$\frac{OA}{OB} = \frac{OE}{OF} = \frac{AC}{BD} = \frac{EC}{FD}$$

Show that if C traces any path then D will describe a similar path and vice-versa.

24. Figure 6.31 shows a straight-line Watt mechanism. Plot the path of point P and mark and measure the straight line segment of the path of P.

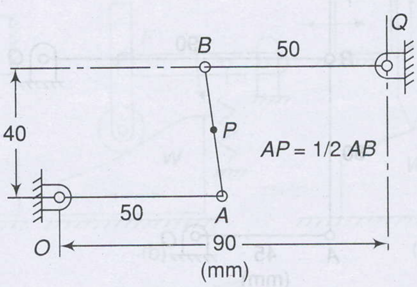


Fig. 6.31

25. Figure 6.32 shows a Robert straight-line mechanism in which *ABCD* is a four-bar linkage. The cranks *AB* and *DC* are equal and the connecting rod *BC* is one-half as long as the line of centres *AD*. *P* is a point rigidly attached to the connecting rod and lying on the midpoint of *AD* when *BC* is parallel to *AD*. Show that the point *P* moves in an approximately straight line for small displacement of the cranks.
(Note: *For better results take AB or DC* > 0.6 *AD*)

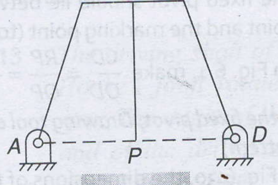


Fig. 6.32

26. In the Robert mechanism (Fig. 6.32) if $AB = BC = CD = AD/2$, locate the point *P* on the central vertical arm that approximately describes a straight line. (At a length 1.3 *BC* below *BC*)

27. In a Watt parallel motion (Fig. 6.10), the links *OA* and *QB* are perpendicular to the link *AB* in the mean position. The lengths of the moving links are $OA = 120$ mm, $QB = 200$ mm and $AB = 175$ mm.
Locate the position of a point *P* on *AB* to trace approximately a straight line motion. Also, trace the locus of *P* for all possible movements. ($AP = 109.3$ mm)

28. In a Watt mechanism of the type shown in Fig. 6.33, the links *OA* and *QB* are perpendicular to the link *AB* in the mean position. If $OA = 45$ mm, $QB = 90$ mm and $AB = 60$ mm, find the point *P* on the link *AB* produced for approximate straight-line motion of point *P*.

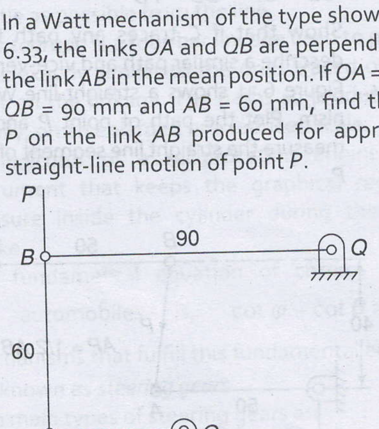


Fig. 6.33

($AP = 120$ mm)

29. In a Davis steering gear, the length of the car between axles is 2.4 m, and the steering pivots are 1.35 m apart. Determine the inclination of the track arms to the longitudinal axis of the car when the car moves in a straight path. (15°42

30. In a Hooke's joint, the angle between the two shafts is 15°. Find the angles turned by the driving shaft when the velocity of the driven shaft is maximum, minimum and equal to that of the driving shaft. Also, determine when the driven shaft will have the maximum acceleration and retardation.
(Max. vel. at 0° and 180°; min. at 90° and 270°; equal to 44°30′, 135°30′, 224°30′ and 315°30′; Max. acc. at 137° and 317°; and Max. ret. at 43° and 223°)

31. The driving shaft of a Hooke's joint has a uniform angular speed of 280 rpm. Determine the maximum permissible angle between the axes of the shafts to permit a maximum variation in speed of the driven shaft by 8% of the mean speed. (22.6

32. The two shafts of a Hooke's coupling have their axes inclined at 20°. The shaft *A* revolves at a uniform speed of 1000 rpm. The shaft *B* carries a flywheel of mass 30 kg. If the radius of gyration of the flywheel is 100 mm, find the maximum torque in shaft *B*. (411 N.m

33. In a double universal coupling joining two shafts, the intermediate shaft is inclined at 10° to each. The input and the output forks on the intermediate shaft have been assembled inadvertently at 90° to one another. Determine the maximum and the least velocities of the output shaft if the speed of the input shaft is 500 rpm. Also, find the coefficient of fluctuation in speed. (515.5 rpm; 484.9 rpm; 0.0