4 COMPUTER-AIDED ANALYSI OF MECHANISM

Introduction

The analyses of the velocity and the acceleration, given in chapters 2 and 3, depend upon the graphical appro are suitable for finding out the velocity and the acceleration of the links of a mechanism in one or two positio crank. However, if it is required to find these values at various configurations of the mechanism or to find the m values of maximum velocity or acceleration, it is not convenient to draw velocity and acceleration diagrar and again. In that case, analytical expressions for the displacement, velocity and acceleration in terms of the parameters are derived. A desk-calculator or digital computer facilitates the calculation work.

4.1 FOUR-LINK MECHANISM

Displacement Analysis

A four-link mechanism shown in Fig. 4.1 is in equilibrium. a, b, c and d represent the magnitudes of the links AB, BC, CD and DA respectively. θ, β and φ are the angles of AB, BC and DC respectively with the x-axis (taken along AD). AD is the fixed link. AB is taken as the input link whereas DC as the output link.

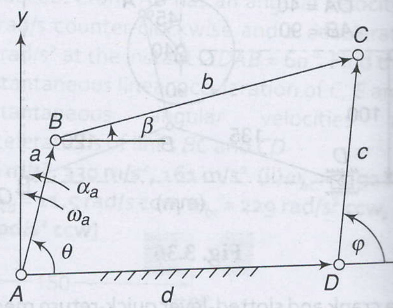

Fig. 4.1

As in any configuration of the mechanism, the figure must enclose, the links of the mechanism can be considered as vectors. Thus, vector displacement relationships can be derived as follows.

Displacement along x-axis

$$a \cos \theta + b \cos \beta = d + c \cos \phi$$

(The equation is valid for $\angle\phi$ more than 90° also.)

or

$$b \cos \beta = c \cos \varphi - a \cos \theta + d$$

or

$$(b \cos \beta)^2 = (c \cos \varphi - a \cos \theta + d)^2$$

$$= c^2 \cos^2 \varphi + a^2 \cos^2 \theta + d^2 - 2ac \cos \theta \cos \varphi - 2ad \cos \theta + 2cd \cos \varphi$$

Displacement along y-axis (4.3)

$$a \sin \theta + b \sin \beta = c \sin \phi$$

or
$$b \sin \beta = c \sin \varphi - a \sin \theta$$

or
$$(b \sin \beta)^2 = (c \sin \varphi - a \sin \theta)^2 \tag{4.4}$$
$$= c^2 \sin^2 \varphi + a^2 \sin^2 \theta - 2ac \sin \theta \sin \varphi$$

Adding equations (4.2) and (4.4), (4.5)

$$b^2 = c^2 + a^2 + d^2 - 2ac \cos \theta \cos \varphi - 2ad \cos \theta + 2cd \cos \varphi - 2ac \sin \theta \sin \varphi$$

Put
$$a^2 - b^2 + c^2 + d^2 = 2k$$

Then,
$$2cd \cos \varphi - 2ac \cos \theta \cos \varphi - 2ac \sin \theta \sin \varphi - 2ad \cos \theta + 2k = 0$$

or
$$cd \cos \varphi - ac \cos \theta \cos \varphi - ac \sin \theta \sin \varphi - ac \cos \theta + k = 0 \tag{4.6}$$

From trigonometric identities,

$$\sin \varphi = \frac{2 \tan\left(\dfrac{\varphi}{2}\right)}{1 + \tan^2\left(\dfrac{\varphi}{2}\right)}$$

$$\cos \varphi = \frac{1 - \tan^2\left(\dfrac{\varphi}{2}\right)}{1 + \tan^2\left(\dfrac{\varphi}{2}\right)}$$

$$cd\left[\frac{1 - \tan^2(\varphi/2)}{1 + \tan^2(\varphi/2)}\right] - ac \cos \theta \left[\frac{1 - \tan^2(\varphi/2)}{1 + \tan^2(\varphi/2)}\right] - ac \sin \theta \left[\frac{2 \tan(\varphi/2)}{1 + \tan^2(\varphi/2)}\right] - ad \cos \theta + k = 0$$

Multiplying throughout by $\left[1 + \tan^2\left(\dfrac{\varphi}{2}\right)\right]$

$$cd - cd \tan^2\left(\frac{\varphi}{2}\right) - ac \cos \theta + ac \cos \theta \tan^2\left(\frac{\varphi}{2}\right) - 2ac \sin \theta \tan\left(\frac{\varphi}{2}\right)$$
$$- ad \cos \theta - ad \cos \theta \tan^2\left(\frac{\varphi}{2}\right) + k + k \tan^2\left(\frac{\varphi}{2}\right) = 0$$

$$[k - a(d - c) \cos \theta - cd] \tan^2\left(\frac{\varphi}{2}\right) + [-2ac \sin \theta] \tan\left(\frac{\varphi}{2}\right) + [k - a(d + c) \cos \theta + cd] = 0$$

or

$$A\tan^2\left(\frac{\varphi}{2}\right) + B\tan\left(\frac{\varphi}{2}\right) + C = 0$$

where

$$A = k - a(d - c)\cos\theta - cd$$
$$B = -2ac\sin\theta$$
$$C = k - a(d + c)\cos\theta + cd$$

Equation (4.6) is a quadratic in $\tan\left(\frac{\varphi}{2}\right)$. Its two roots are

$$\tan\left(\frac{\varphi}{2}\right) = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

or

$$\varphi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \quad (4.7)$$

Thus, the position of the output link, given by angle φ, can be calculated if the magnitude of the links and the position of the input link are known, i.e., a, b, c, d and θ are known.

A relation between the coupler link position β and the input link position θ can also be found as below: Equations (4.1) and (4.3) can be written as,

$$c\cos\varphi = a\cos\theta + b\cos\beta - d \quad (4.8)$$

$$c\sin\varphi = a\sin\theta + b\sin\beta \quad (4.9)$$

Squaring and adding the two equations,

$$c^2 = a^2 + b^2 + d^2 + 2ab\cos\theta\cos\beta - 2bd\cos\beta - 2ad\cos\theta + 2ab\sin\theta\sin\beta$$

Put $\quad a^2 + b^2 - c^2 + d^2 = 2k'$

$$-2bd\cos\beta + 2ab\cos\theta\cos\beta + 2ab\sin\theta\sin\beta - 2ad\cos\theta + 2k' = 0$$

$$-bd\cos\beta + ab\cos\theta\cos\beta + ab\sin\theta\sin\beta - ad\cos\theta + k' = 0 \quad (4.10)$$

Equation (4.10) is identical to Eq. 4.6 and can be obtained from the same by substituting β for φ, $-b$ for c and k' for k.

Thus, the solution of Eq. (4.10) will be,

$$\beta = 2\tan^{-1}\left[\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right] \quad (4.11)$$

where $\quad D = k' - a(d + b)\cos\theta + bd$

$\quad E = 2ab\sin\theta$

$\quad F = k' - a(d - b)\cos\theta - bd$

β can also be found directly from relation (4.3) after calculating φ.

Velocity Analysis

Let ω_a, ω_b and ω_c be the angular velocities of the links *AB, BC* and *CD* respectively.

Rewriting Eq. (4.1),

$$a\cos\theta + b\cos\beta - c\cos\phi - d = 0 \tag{4.12}$$

Differentiating it with respect to time,

$$\frac{d}{dt}(a\cos\theta + b\cos\beta - c\cos\varphi - d) = 0$$

$$\frac{d}{d\theta}\frac{d\theta}{dt}(a\cos\theta) + \frac{d}{d\beta}\frac{d\beta}{dt}(b\cos\beta) - \frac{d}{d\varphi}\frac{d\varphi}{dt}(c\cos\varphi) - \frac{d}{dt}(d) = 0$$

$$\frac{d\theta}{dt}\frac{d}{d\theta}(a\cos\theta) + \frac{d\beta}{dt}\frac{d}{d\beta}(b\cos\beta) - \frac{d\varphi}{dt}\frac{d}{d\varphi}(c\cos\varphi) - 0 = 0 \qquad (d \text{ is constant})$$

$$-a\,\omega_a\sin\theta - b\omega_b\sin\beta + c\,\omega_c\sin\phi = 0 \tag{4.13}$$

Similarly, rewriting Eq. (4.3),

$$a\sin\theta + b\sin\beta - c\sin\phi = 0 \tag{4.14}$$

Differentiating it with respect to time,

$$a\,\omega_a\cos\theta + b\omega_b\cos\beta - c\,\omega_c\cos\varphi = 0 \tag{4.15}$$

Multiply Eq. (4.13) by $\cos\beta$ and Eq. (4.15) by $\sin\beta$ and add,

$$a\,\omega_a(\sin\beta\cos\theta - \cos\beta\sin\theta) - c\omega_c(\sin\beta\cos\varphi - \cos\beta\sin\varphi) = 0$$

or

$$a\,\theta_a\sin(\beta - \theta) - c\omega_c\sin(\beta - \phi) = 0$$

or

$$\omega_c = \frac{a\omega_\alpha\sin(\beta - \theta)}{c\sin(\beta - \varphi)} \tag{4.16}$$

Multiply Eq. (4.13) by $\cos\varphi$ and Eq. (4.15) by $\sin\varphi$ and add,

$$a\,\omega_a(\sin\varphi\cos\theta - \cos\varphi\sin\theta) + b\omega_b(\sin\varphi\cos\beta - \cos\varphi\sin\beta) = 0$$

or

$$a\,\omega_a\sin(\varphi - \theta) + b\omega_b\sin(\varphi - \beta) = 0$$

or

$$\omega_b = -\frac{a\omega_\alpha\sin(\varphi - \theta)}{b\sin(\varphi - \beta)} \tag{4.17}$$

Since a, b, c, θ, β, φ and ω_a are already known, ω_c and ω_b can be calculated from Eqs (4.16) and (4.17) respectively.

Acceleration Analysis

Let α_a, α_b, and α_c be the angular accelerations of the links a, b and c respectively.

Differentiating equations (4.13) and (4.15) with respect to time in the above manner or rewriting in t following form,

$$-a\,\omega_a \sin\omega_a t - b\omega_b \sin\omega_b t + c\,\omega_c \sin\omega_c t = 0 \qquad (4.1$$

$$a\,\omega_a \cos\omega_a t + b\omega_b \cos\omega_b t - c\,\omega_c \cos\omega_c t = 0 \qquad (4.1$$

Differentiating these equations with respect to time,

$$(-a\alpha_a \sin\theta - a\omega_a^2 \cos\theta) - (b\alpha_b \sin\beta - b\omega_b^2 \cos\beta) + (c\alpha_c \sin\varphi + c\omega_c^2 \cos\varphi) = 0 \tag{4.20}$$

$$(a\alpha_a \cos\theta - a\omega_a^2 \sin\theta) + (b\alpha_b \cos\beta - b\omega_b^2 \sin\beta) - (c\alpha_c \cos\varphi + c\omega_c^2 \sin\varphi) = 0 \tag{4.21}$$

where $\alpha_\alpha = \dfrac{d\omega_a}{dt}$, $\alpha_b = \dfrac{d\omega_b}{dt}$ and $\alpha_c = \dfrac{d\omega_c}{dt}$

Multiply Eq. (4.20) by $\cos\varphi$ and Eq. (4.21) by $\sin\varphi$ and add,

$$a\alpha_a (\sin\varphi\cos\theta - \cos\varphi\sin\theta) - a\omega_a^2 (\cos\theta\cos\varphi + \sin\theta\sin\varphi)$$
$$-b\alpha_b (\sin\beta\cos\varphi - \cos\beta\sin\varphi) - b\omega_b^2 (\cos\beta\cos\varphi + \sin\beta\sin\varphi) + c\omega_c^2 = 0$$

or

$$a\alpha_a \sin(\varphi-\theta) - a\omega_a^2 \cos(\varphi-\theta) - b\alpha_b \sin(\beta-\varphi) - b\omega_b^2 \cos(\varphi-\beta) + c\omega_c^2 = 0$$

or

$$\alpha_b = \frac{a\alpha_a \sin(\varphi-\theta) - a\omega_a^2 \cos(\varphi-\theta) - b\omega_b^2 \cos(\varphi-\beta) + c\omega_c^2}{b\sin(\beta-\varphi)} \tag{4.22}$$

Multiply Eq. (4.20) by $\cos\beta$ and Eq. (4.21) by $\sin\beta$ and add,

$$a\alpha_a (\sin\beta\cos\theta - \cos\beta\sin\theta) - a\omega_a^2 (\cos\beta\cos\theta + \sin\beta\sin\theta) - b\omega_b^2$$
$$+c\alpha_c (\sin\varphi\cos\beta - \cos\varphi\sin\beta) + c\omega_c^2 (\cos\beta\cos\varphi + \sin\beta\sin\varphi) = 0$$

or

$$a\alpha_a \sin(\beta-\theta) - a\omega_a^2 \cos(\beta-\theta) - b\omega_b^2 - c\alpha_c \sin(\beta-\varphi) + c\omega_c^2 \cos(\beta-\varphi) = 0$$

or

$$\alpha_c = \frac{a\alpha_a \sin(\beta-\theta) - a\omega_a^2 \cos(\beta-\theta) - b\omega_b^2 + c\omega_c^2 \cos(\beta-\varphi)}{c\sin(\beta-\varphi)} \tag{4.23}$$

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
void main()

  int i,j,iht,th,theta,limit,ins;
  float a,b,c,d,vela,acca,thet,aa,bb,cc,bet1,bet2,betd1,
  betd2,num1,num2,phi1,ph1,unroot,undroot,pi,k,phh,phi2,
  ph2,vel2,dthet;
  float num[2],phi[2],ph[2],bet[2],betd[2],b1[2],b2[2],
  b3[2],b4[2],c1[2],c2[2],c3[2],c4[2],accc[2],accb[2],
  velb[2],velc[2];
  clrscr();

  printf("enter values a,b,c,d,vela,acca,theta,limit\n");
  scanf("%f%f%f%f%f%f%d%d",&a,&b,&c,&d,&vela,&acca,
  &theta,&limit);
  printf( "   thet   vela   acca       phi       beta ");
  printf( "   velc    velb    accc    accb \n");
  ins=1;
  if(vela==0 && acca>0)ins=0;

  pi=4*atan(1);
  iht=360/theta;
  if(vela>0 && acca==0){ins=0;iht=360/theta; }
  if(ins==1)iht=theta;
  dthet=pi*2/iht;
  if(vela==0 && acca>0)iht=iht+limit/theta;
   for(j=0;j<iht+1;j++)
   {
    if(j>(iht-360/theta-1) && ins==0)acca=0;
    thet=j*dthet;
    if(ins==1){j=iht; thet=theta*pi/180;}
    th=theta*j;
    if(ins==1)th=theta;
    k=(a*a-b*b+c*c+d*d)/2;
    aa=k-a*(d-c)*cos(thet)-c*d;
    bb=-2*a*c*sin(thet);
    cc=k-a*(d+c)*cos(thet)+c*d;
    unroot=bb*bb-4*aa*cc;
     if(unroot>0)
```

```
         {
          undroot=sqrt(unroot);
          num[0]=-bb+undroot;
          num[1]=-bb-undroot;
            for(i=0;i<2;i++)
            {
              phi[i]=atan(num[i]*.5/aa)*2;
              ph[i]=phi[i]*180/pi;
              bet[i]=asin((c*sin(phi[i])-a*sin(thet))/b);
              betd[i]=bet[i]*180/pi;
              velc[i]=(a*vela*sin(bet[i]-thet))/(c*sin(bet[i]
              -phi[i]));
              velb[i]=(a*vela*sin(phi[i]-thet))/(b*sin(bet[i]
              -phi[i]));
              c1[i]=a*acca*sin(bet[i]-thet);
              c2[i]=a*pow(vela,2)*cos(bet[i]-thet)+
              b*pow(velb[i],2);
              c3[i]=c*pow(velc[i],2)*cos(phi[i]-bet[i]);
              c4[i]=c*sin(bet[i]-phi[i]);
              accc[i]=(c1[i]-c2[i]+c3[i])/c4[i];
              b1[i]=a*acca*sin(phi[i]-thet);
              b2[i]=a*pow(vela,2)*cos(phi[i]-thet);
              b3[i]=b*pow(velb[i],2)*cos(phi[i]-bet[i])
              -c*pow(velc[i],2);
              b4[i]=b*sin(bet[i]-phi[i]);
              accb[i]=(b1[i]-b2[i]-b3[i])/b4[i];
              printf( "%6.2d %6.2f%8.2f %8.2f %8.2f %6.2f
              %6.2f %6.2f %6.2f\n",th,vela,acca,ph[i],betd[i],
              velc[i],velb[i],accc[i],accb[i]);
             }
         }
        vela=sqrt(vela*vela+2*acca*dthet);
     }
       getch();
   }
```

Fig. 4.2

Figure 4.2 shows a program in *C* for solving such a problem. The program can be used to find the angular velocities and accelerations of the output and coupler links for the following cases:

1. Link *AB* is a crank and rotates at uniform angular velocity. In this case, the acceleration of the input link will be zero. If the link *AB* is not a crank but a rocker, the program will make the calculations only for feasible cases.
2. Link *AB* is a crank and starts from the stationary position. In this case, the initial velocity is zero and a value of the acceleration has to be provided along with the limit of the angle up to which the acceleration continues. At that angle when the maximum velocity is attained, the acceleration automatically reduces to zero and the onward the crank starts rotating at constant angular velocity. Further, calculations are made for one complete revolution.
3. For instant values of input velocity and acceleration, only one calculation is made for that specified position.

Various input variables are

`a, b, c, d`	Magnitudes of links *AB, BC, CD* and *DA* respectively (mm)
`vela`	Angular velocity of the input link *AB* (m/s)
`acca`	Angular acceleration of the input link (m/s^2)(acceleration is taken positive, deceleration negative)
`theta`	The interval of the input angle, i.e., the results are to be taken with a difference of 10°, 20° or 30°, etc., starting from zero
`limit`	Angle up to which acceleration continues (for the case 2; in the other cases any value may be given)

The output variables are

`thet`	Angular displacement of the input link *AB* (degrees)
`phi`	Angular displacement of the output link *DC* (degrees)
`beta`	Angular displacement of the coupler link *BC* (degrees)
`velc`	Angular velocity of the output link *DC* (rad/s)
`velb`	Angular velocity of the coupler link *BC* (rad/s)
`accc`	Angular acceleration of the output link (rad/s^2)
`accb`	Angular acceleration of the coupler link (rad/s^2)

The results are obtained in sets of two possible solutions for each position of the input link. In case the input *AB* is not a crank, the results are obtained for the possible positions only. The counter-clockwise direction is considered as positive and the clockwise as negative.

4.2 USE OF COMPLEX ALGEBRA

For a four-link mechanism, we can write

$$\mathbf{a} + \mathbf{b} - \mathbf{c} - \mathbf{d} = \mathbf{0} \tag{4.24}$$

Transforming it into complex polar form,

$$a\, e^{i\theta} + b\, e^{i\beta} - c\, e^{i\phi} - d = 0 \tag{4.25}$$

Now, we know, $e^{i\theta} = \cos\theta + i\sin\theta$

Thus, transforming this equation into complex rectangular form and separating the real and imaginary terms,

$$a\cos\theta + b\cos\beta = d + c\cos\phi \tag{4.26}$$

and

$$a\sin\theta + b\sin\beta = c\sin\phi \tag{4.27}$$

which are the same equations as 4.1 and 4.3 and thus can be solved to find β and θ.

Differentiating Eq. (4.25) with respect to t,

$$ia\dot{\theta}\, e^{i\theta} + ib\dot{\beta}\, e^{i\beta} - ic\dot{\varphi}\, e^{i\varphi} = 0 \tag{4.28}$$

or $$ia\ \omega_a\, e^{i\,\theta} + ib\ \omega_b\, e^{i\,\beta} - ic\ \omega_c\, e^{i\varphi} = 0 \tag{4.29}$$

Again, transforming this equation into complex rectangular form and separating the real and imaginary terms,

$$a\ \omega_a \cos\theta + b\omega_b \cos\beta - c\ \omega_c \cos\varphi = 0 \tag{4.30}$$

$$-a\ \omega_a \sin\theta - b\omega_b \sin\beta + c\ \omega_c \sin\varphi = 0 \tag{4.31}$$

which are the same equations as 4.13 and 4.15 and thus can be solved to find ω_b and ω_c.

Differentiating Eq. (4.28) with respect to t,

$$ia(\ddot{\theta}e^{i\theta} + i\dot{\theta}^2 e^{i\theta}) + ib(\ddot{\beta}\ e^{i\beta} + i\dot{\beta}^2 e^{i\beta}) - ic(\ddot{\varphi}e^{i\varphi} + i\dot{\varphi}^2 e^{i\varphi}) = 0 \tag{4.32}$$

or $$ia(\alpha_a e^{i\theta} + i\omega_a{}^2 e^{i\theta}) + ib(\alpha_b \ddot{\beta}\ e^{i\beta} + i\omega_b{}^2 e^{i\beta}) - ic(\alpha_c e^{i\varphi} + i\omega_c{}^2 e^{i\varphi}) = 0 \tag{4.33}$$

Transforming this equation into complex rectangular form and separating the real and imaginary terms,

$$-a\alpha_a \sin\theta - a\omega_a^2 \cos\theta - b\alpha_b \sin\beta - b\omega_b^2 \cos\beta + c\alpha_c \sin\varphi + c\omega_c^2 \cos\varphi = 0 \tag{4.34}$$

$$a\alpha_a \cos\theta - a\omega_a^2 \sin\theta + b\alpha_b \cos\beta - b\omega_b^2 \sin\beta - c\alpha_c \cos\varphi + c\omega_c^2 \sin\varphi = 0 \tag{4.35}$$

which are the same equations as 4.20 and 4.21 and can be solved as before.

4.3 THE VECTOR METHOD

We have

$$\mathbf{a} + \mathbf{b} - \mathbf{c} - \mathbf{d} = \mathbf{0}$$

Assuming that the angles β and ϕ have been determined by any of the above methods, differentiate the above equation with respect to time,

$$\omega_\mathbf{a} \times \mathbf{a} + \omega_\mathbf{b} \times \mathbf{b} - \omega_\mathbf{c} \times \mathbf{c} = \mathbf{0} \qquad (a, b, c \text{ and } d \text{ are constants}) \tag{4.36}$$

Let $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ be the unit vectors along **a**, **b** and **c** vectors. In plane–motion mechanisms, all the angular velocities are in the **k** direction. Therefore,

$$a\omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}) + b\ \omega_b\,(\mathbf{k} \times \hat{\mathbf{b}}) - c\ \omega_c\,(\mathbf{k} \times \hat{\mathbf{c}}) = \mathbf{0} \tag{4.37}$$

Take the dot product with $\hat{\mathbf{b}}$,

$$a\ \omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}).\ \hat{\mathbf{b}} + b\omega_b\,(\mathbf{k} \times \hat{\mathbf{b}})\ \hat{\mathbf{b}} - c\ \omega_c\,(\mathbf{k} \times \hat{\mathbf{c}})\ \hat{\mathbf{b}} = 0$$

$$a\ \omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}).\ \hat{\mathbf{b}} + \mathbf{0} - c\omega_c\,(\mathbf{k} \times \hat{\mathbf{c}})\ \hat{\mathbf{b}} = 0$$

or $$\omega_c = -\frac{a\omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}).\ \hat{\mathbf{b}}}{c(\mathbf{k} \times \hat{\mathbf{b}}).\ \hat{\mathbf{b}}} \tag{4.38}$$

Taking the dot product with $\hat{\mathbf{c}}$,

$$a\ \omega_a\,(\mathbf{k} \times \hat{\mathbf{a}}).\ \hat{\mathbf{c}} + b\ \omega_b\,(\mathbf{k} \times \mathbf{b}).\ \hat{\mathbf{c}} - c\ \omega_c\,(\mathbf{k} \times \hat{\mathbf{c}})\ \hat{\mathbf{c}} = 0$$

$$a\,\omega_a\,(\mathbf{k}\times\hat{\mathbf{a}})\,.\,\hat{\mathbf{c}} + b\,\omega_b\,(\mathbf{k}\times\hat{\mathbf{b}})\,.\,\hat{\mathbf{c}} - 0 = 0$$

or

$$\omega_b = -\frac{a\,\omega_a\,(\mathbf{k}\times\hat{\mathbf{a}})\,.\,\hat{\mathbf{c}}}{b(\,\mathbf{k}\times\hat{\mathbf{b}})\,.\,\hat{\mathbf{c}}} \tag{4.39}$$

It can be shown that Eqs 4.38 and 4.39 are the same as Eqs 4.16 and 4.17 as follows:

$$(\mathbf{k}\times\hat{\mathbf{a}})\,.\,\hat{\mathbf{b}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \end{vmatrix}\,.\,\hat{\mathbf{b}} = (-\sin\theta.\mathbf{i} + \cos\theta.\mathbf{j})\,.\,\hat{\mathbf{b}}$$

$$= (-\sin\theta.\mathbf{i} + \cos\theta.\mathbf{j})\,.\,(\cos\beta\,\mathbf{i} + \sin\beta\,\mathbf{j})$$

$$= -\sin\theta\cos\beta + \cos\theta\sin\beta$$

$$= \sin(\beta-\theta)$$

Similarly,

$$(\mathbf{k}\times\hat{\mathbf{c}})\,.\,\hat{\mathbf{b}} = \sin(\beta-\phi)$$

Therefore,

$$\omega_c = -\frac{a\omega_a\,(\mathbf{k}\times\hat{\mathbf{a}}).\hat{\mathbf{b}}}{c(\mathbf{k}\times\hat{\mathbf{c}}).\hat{\mathbf{b}}} = \frac{a\omega_\alpha\,\sin(\beta-\theta)}{c\sin(\beta-\varphi)} \tag{4.40}$$

In the same way,

$$w_b = -\frac{a\,\omega_a\,(\mathbf{k}\times\hat{\mathbf{a}})\,.\,\hat{\mathbf{c}}}{b(\,\mathbf{k}\times\hat{\mathbf{c}})\,.\,\hat{\mathbf{c}}} = -\frac{a\omega_\alpha\,\sin(\varphi-\theta)}{b\sin(\varphi-\beta)} \tag{4.41}$$

which are the same equations as equations 4.16 and 4.17.

Differentiating Eq. 4.36 with respect to time to get the accelerations,

$$\dot{\omega}_a\times\mathbf{a} + \omega_a\times(\omega_a\times\mathbf{a}) + \dot{\omega}_b\times\mathbf{b} + \omega_b\times(\omega_b\times\mathbf{b}) - \dot{\omega}_c\times\mathbf{c} - \omega_c\times(\omega_c\times\mathbf{c}) = 0$$

or

$$\alpha_a\times\mathbf{a} + \omega_a\times(\omega_a\times\mathbf{a}) + \alpha_b\times\mathbf{b} + \omega_b\times(\omega_b\times\mathbf{b}) - \alpha_c\times\mathbf{c} - \omega_c\times(\omega_c\times\mathbf{c}) = 0$$

or

$$a\,\alpha_a(\mathbf{k}\times\hat{\mathbf{a}}) - a\,\omega_a^2\,\hat{\mathbf{a}} + b\,\alpha_b(\mathbf{k}\times\hat{\mathbf{b}}) - b\,\omega_b^2\,\hat{\mathbf{b}} - c\alpha_c(\mathbf{k}\times\hat{\mathbf{c}}) + c\,\omega_c^2\,\hat{\mathbf{c}} = 0 \tag{4.42}$$

Take the dot product of this equation with $\hat{\mathbf{b}}$,

$$a\,\alpha_a(\mathbf{k}\times\hat{\mathbf{a}})\,.\,\hat{\mathbf{b}} - a\,\omega_a^2\,\hat{\mathbf{a}}\,\hat{\mathbf{b}} + 0 - b\,\omega_b^2\,\hat{\mathbf{b}}\,.\,\hat{\mathbf{b}} - c\alpha_c(\mathbf{k}\times\hat{\mathbf{c}})\,.\,\hat{\mathbf{b}} + c\,\omega_c^2\,\hat{\mathbf{c}}\,.\,\hat{\mathbf{b}} = 0$$

$$\alpha_c = \frac{a\,\alpha_a(\mathbf{k}\times\hat{\mathbf{a}})\,.\,\hat{\mathbf{b}} - a\,\omega_a^2\,\hat{\mathbf{a}}\,.\,\hat{\mathbf{b}} - b\omega_b^2 + c\omega_c^2\,\hat{\mathbf{c}}\,.\,\hat{\mathbf{b}}}{c(\mathbf{k}\times\hat{\mathbf{c}})\,.\,\hat{\mathbf{b}}} \tag{4.43}$$

Since,

$$(\mathbf{k} \times \hat{\mathbf{a}}) \,.\, \hat{\mathbf{b}} = \sin(\beta - \theta),$$
$$\hat{\mathbf{a}} \,.\, \hat{\mathbf{b}} = \cos(\beta - \theta),$$
$$\hat{\mathbf{b}} \,.\, \hat{\mathbf{b}} = 1$$
$$\hat{\mathbf{c}} \,.\, \hat{\mathbf{b}} = \cos(\beta - \varphi)$$
$$(\mathbf{k} \times \hat{\mathbf{c}}) \,.\, \hat{\mathbf{b}} = \text{win}\,(\beta - \varphi)$$

The above equation reduces to

$$\alpha_c = \frac{a\alpha_a \sin(\beta - \theta) - a\omega_a^2 \cos(\beta - \theta) - b\omega_b^2 + c\omega_c^2 \cos(\beta - \varphi)}{c \sin(\beta - \varphi)}$$

which is the same as Eq. 4.23.

Taking the dot product of Eq. 4.42 with $\hat{\mathbf{c}}$,

$$a\,\alpha_a(\mathbf{k} \times \hat{\mathbf{a}}) \,.\, \hat{\mathbf{b}} - a\,\omega_a^2\, \hat{\mathbf{a}} \,.\, \hat{\mathbf{b}} + b\,\alpha_b(\mathbf{k} \times \hat{\mathbf{b}}) \,.\, \hat{\mathbf{c}} - b\,\omega_b^2\, \hat{\mathbf{b}} \,.\, \hat{\mathbf{c}} + c\,\omega_c^2\, \hat{\mathbf{c}} \,.\, \hat{\mathbf{c}} = 0$$

or

$$b = \frac{a\,\alpha_a(\mathbf{k} \times \hat{\mathbf{a}}) \,.\, \hat{\mathbf{c}} - a\,\omega_a^2\, \hat{\mathbf{a}} \,.\, \hat{\mathbf{c}} - b\omega_b^2\, \hat{\mathbf{b}} \,.\, \hat{\mathbf{c}} + c\omega_c^2}{c(\mathbf{k} \times \hat{\mathbf{c}}) \,.\, \hat{\mathbf{b}}}$$

which can be shown to be the same as Eq. 4.22, i.e.,

$$\alpha_b = \frac{a\alpha_a \sin(\varphi - \theta) - a\omega_a^2 \cos(\varphi - \theta) - b\omega_b^2 \cos(\varphi - \beta) + c\omega_c^2}{b \sin(\beta - \varphi)}$$

Example 4.1 *In a four-link mechanism, the dimensions of the links are as under:*
AB = 50 mm, BC = 66 mm, CD = 56 mm and AD = 100 mm
AD is the fixed link. At an instant when DAC is 60°, the angular velocity of the input link AB is 10.5 rad/s in the counter-clockwise direction with an angular retardation of 26 rad/s². Determine analytically the angular displacements, angular velocities and angular accelerations of the output link DC and the coupler BC.

Solution We have,

$$2k = a^2 - b^2 + c^2 + d^2$$
$$k = (50^2 - 66^2 + 56^2 + 100^2)/2$$
$$= 5640$$
$$A = k - a\,(d - c)\cos\theta - cd$$
$$= 5640 - 50\,(100 - 56)\cos 60^\circ - 56 \times 100 = -1060$$
$$B = -2ac \sin\theta = -2 \times 50 \times 56 \sin 60^\circ = -4850$$
$$C = k - a\,(d + c)\cos\theta + cd$$
$$= 5640 - 50\,(100 + 56)\cos 60^\circ + 56 \times 100 =$$
$$\varphi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$$
$$= 2\tan^{-1}\left[\frac{4850 \pm \sqrt{(-4850)^2 - 4 \times (-1060)(7}}{2 \times (-1060)}\right.$$
$$= 2\tan^{-1}(1.199 \text{ or } -5.759)$$
$$= 100.35^\circ \text{ or } -160.3^\circ$$

Taking the first value, we have,

$$b \sin\beta = c \sin\varphi - a \sin\theta$$
$$66 \times \sin\beta = 56 \times \sin 100.35^\circ - 50 \times \sin 60^\circ$$
$$\sin\beta = 0.1786$$
$$\beta = 10.29^\circ$$
$$\omega_c = \frac{a\omega_\alpha \sin(\beta - \theta)}{c\sin(\beta - \varphi)}$$

$$= \frac{50 \times 10.5 \sin(10.29 - 60^\circ)}{56 \sin(10.29 - 100.35)} = 7.15 \text{ rad/s}$$

$$\omega_b = -\frac{a\omega_a \sin(\varphi - \theta)}{b\sin(\varphi - \beta)} =$$

$$\frac{50 \times 10.5 \sin(100.35^o - 60^o)}{66 \times \sin(100.35^o - 10.29^o)} = -5.15 \text{ rad/s}$$

$$\alpha_c = \frac{a\alpha_a \sin(\beta - \theta) - a\omega_a^2 \cos(\beta - \theta) - b\omega_b^2 + c\omega_c^2 \cos(\beta - \varphi)}{c\sin(\beta - \varphi)}$$

$$= \frac{50 \times (-26) \sin(10.29^\circ - 60^\circ) - 50 \times 10.5^2 \cos(10.29^\circ - 60^\circ) - 66 \times (5.15)^2 + 56^2 \cos(10.29^\circ - 100.35^\circ)}{56 \sin(10.29^\circ - 100.35^\circ)}$$

$= 77.26 \text{ rad/s}^2$

$$\alpha_b = \frac{a\alpha_a \sin(\varphi - \theta) - a\omega_a^2 \cos(\varphi - \theta) - b\omega_b^2 \cos(\varphi - \beta) + c\omega_c^2}{b \sin(\beta - \varphi)}$$

$$= \frac{50 \times (-26)\sin(100.35^\circ - 60^\circ) - 50 \times 10.5^2 \cos(100.35^\circ - 60^\circ) - 66 \times (5.15)^2 \cos(100.35^\circ - 10.29^\circ) + 56 \times 7.15^2)}{56\sin(10.29^\circ - 100.35^\circ)}$$

$= 32.98 \text{ rad/s}^2$

Using the other value of ϕ, ($\phi = -160.3^\circ$), another set of values of velocities and accelerations can be obtained.

The results obtained using the program of Fig. 4.2 are given in Fig. 4.3.

```
Enter values of a, b, c, d, vela, acca, theta, limit
50  66  56  100  10.5 -26  60  0
thet   vela      acca     phi      beta       velc     velb     accc     accb
60     10.5     -26.00  -160.35  -70.29     -7.15     5.15    50.04    94.32
60     10.50    -26.00   100.35   10.29      7.15    -5.15    77.26    32.98
```

Fig. 4.3

Compare these values of ω_b, ω_c, α_b and α_c at 60° with the values obtained graphically in Examples 2.1 and 3.1.

Example 4.2 *In a four-link mechanism, the dimensions of the links are as under:*
AB = 20 mm, BC = 66 mm, CD = 56 mm and AD = 80 mm

AD is the fixed link. The crank AB rotates at uniform angular velocity of 10.5 rad/s in the counter-clockwise direction. Determine using the program of Fig. 4.2, the angular displacements, angular velocities and angular accelerations of the output link DC and the coupler BC for a complete revolution of the crank at an interval of 40°.

Solution The results obtained using the program of Fig. 4.2 are given in Fig. 4.4.

```
Enter values of a, b, c, d, vela, acca, theta, limit
20  66  56  80  10.5  40  0
thet   vela     acca     phi      beta      velc     velb     aqccc    accb
0      10.5     0.0   -110.74  -52.51    -3.50    -3.50   -37.58    18.56
0      10.5     0.0    110.74   52.51    -3.50    -3.50    37.58   -18.56
40     10.5     0.0   -126.30  -61.47    -4.06    -0.83    15.50    50.99
40     10.5     0.0    103.82   38.99     0.07    -3.15    56.46    20.96
80     10.5     0.0   -139.02  -58.74    -2.51     2.03    26.17    31.04
80     10.5     0.0    110.16   29.87     2.92    -1.62    27.30    22.42
```

(contd.)

120	10.5	0.0	-145.28	-48.22	-0.77	3.20	26.64	4.54
120	10.5	0.0	123.49	26.44	3.77	-0.20	-0.66	21.44
160	10.5	0.0	-144.69	-36.44	1.12	2.75	29.94	-16.40
160	10.5	0.0	136.77	28.52	2.96	1.32	-22.41	23.99
200	10.5	0.0	-136.77	-28.52	2.96	1.32	22.41	-23.99
200	10.5	0.0	144.69	36.44	1.12	2.75	-29.94	16.40
240	10.5	0.0	-123.49	-26.44	3.77	-0.20	0.66	-21.44
240	10.5	0.0	145.28	48.22	-0.77	3.20	-26.64	-4.54
280	10.5	0.0	-110.16	-29.87	2.92	-1.62	-27.30	22.42
280	10.5	0.0	139.02	58.74	-2.51	2.03	-26.17	-31.04
320	10.5	0.0	-103.82	-38.99	0.07	-3.15	-56.46	-20.96
320	10.5	0.0	126.30	61.47	-4.06	-0.83	-15.50	-50.99

Fig. 4.4

4.4 SLIDER-CRANK MECHANISM

Figure 4.5 shows a slider-crank mechanism in which the strokeline of the slider does not pass through the axi of rotation of the crank. Angle β in clockwise direction from the x-axis is taken as negative.

Let e = eccentricity (distance CD).

Displacement along x-axis,

$$a \cos\theta + b \cos(-\beta) = d \tag{4.46}$$

or

$$b \cos\beta = d - a \cos\theta \tag{4.46a}$$

Displacement along y-axis,

$$a \sin\theta + b \sin(-\beta) + e \tag{4.47}$$

or

$$b \sin\beta = e - a \sin\theta \tag{4.47a}$$

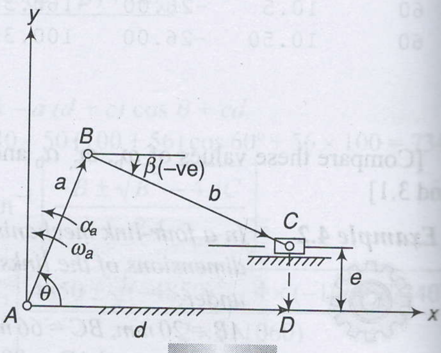

Fig. 4.5

Squaring Eqs (4.46a) and (4.47a) and adding,

$$b^2 = a^2 \cos^2\theta + d^2 - 2ad \cos\theta + a^2 \sin^2\theta + e^2 - 2ae \sin\theta$$
$$= a^2 + e^2 + d^2 - 2ae \sin\theta - 2ad \cos\theta$$

or

$$d^2 - (2a \cos\theta)d + a^2 - b^2 + e^2 - 2ae \sin\theta = 0$$

or

$$d^2 + C_1 d + C_2 = 0 \tag{4.48}$$

where $C_1 = -2a \cos\theta$

$C_2 = a^2 - b^2 + e^2 - 2ae \sin\theta$

Equation (4.48) is a quadric in d. Its two roots are,

$$d = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_2}}{2} \tag{4.49}$$

Thus, if the parameters a, b, e and θ of the mechanism are known, the output displacement can be computed.

Also, from Eq. (4.47a),

$$\beta = \sin^{-1}\frac{e - a \sin\theta}{b} \tag{4.50}$$

Velocity Analysis

Differentiating Eqs. (4.46) and (4.47) with respect to time,

$$-a\omega_a \sin\theta - b\,\omega_b \sin\beta - \dot{d} = 0 \tag{4.51}$$

$$a\omega_a \cos\theta + b\,\omega_b \cos\beta = 0 \tag{4.52}$$

Multiply Eq. (4.51) by $\cos\beta$ and Eq. (4.52) by $\sin\beta$ and add,

$$a\omega_a (\sin\beta \cos\theta - \cos\beta \sin\theta) - \dot{d}\cos\beta = 0$$

$$\dot{d} = \frac{a\omega_a \sin(\beta - \theta)}{\cos\beta} = \tag{4.53}$$

From Eq. (4.52),

$$\omega_b = -\frac{a\omega_a \cos\theta}{b\cos\beta} \tag{4.54}$$

ω_b provides the angular velocity of the coupler-link whereas $\dot{d}$ gives the linear velocity of the slider.

Acceleration Analysis

Differentiating Eqs (4.51) and (4.52) with respect to time,

$$-\left[a\alpha_a \sin\theta + a\omega_a^2 \cos\theta\right] - \left[b\alpha_b \sin\beta + b\omega_b^2 \cos\beta\right] - \ddot{d} = 0 \tag{4.55}$$

$$\left[a\alpha_a \cos\theta + a\omega_a^2 \sin\theta\right] - \left[b\alpha_b \cos\beta + b\omega_b^2 \sin\beta\right] = 0 \tag{4.56}$$

Multiply Eq. (4.55) by $\cos\beta$ and Eq. (4.56) by $\sin\beta$ and add,

$$a\alpha_a \sin(\beta - \theta) - a\omega_a^2 \cos(\beta - \theta) - b\omega_b^2 - \ddot{d}\cos\beta = 0$$

or

$$\ddot{d} = \frac{a\alpha_a \sin(\beta - \theta) - a\omega_a^2 \cos(\beta - \theta) - b\omega_b^2}{\cos\beta} \tag{4.57}$$

From Eq. (4.56)

$$\alpha_b = \frac{a\alpha_a \cos\theta - a\omega_a^2 \sin\theta - b\omega_b^2 \sin\beta}{b\cos\beta} \tag{4.58}$$

α_b provides the angular acceleration of the coupler-link whereas $\ddot{d}$ gives the linear acceleration of the slider.

Figure 4.6 shows a program to solve this type of problem. It can be used for the same type of three cases as for the four-link mechanism.

```
#include<stdio.h>
#include<conio.h>
#inculde<math.h>
void main()
{
    int j,iht,th,theta,limit,ins;
    float a,b,e,c1,c2,c3,c4,vela,acca,thet,pi,dthet,bet,
    velb,vels,accs,accb;
    clrscr();

    printf("enter values a,b,e,vela,acca,theta,limit\n");
    scanf("%f%f%f%f%f%d%d%",&a,&b,&e,&vela,&acca,&theta
    &li   mit);
    printf( " thet vela acca beta ");
    printf( " velc velb accc accb \n");
    ins=1;
    if(vela==0 && acca>0)ins=0;
    pi=4*atan(1);
    iht=360/theta;
    if(vela>0 && acca==00) {ins=0;iht=360/theta; }
    if(ins==1)iht=theta;
    dthet=pi*2/iht;
    if(vela==0 && acca>0)iht=iht+limit/theta;
        for(j=0;j<iht+1;j++)
        {
          if(j>(iht-360/theta-1) && ins==0)acca=0;
          thet=j*dthet;
          if(ins==1) {j=iht; thet=theta*pi/180;}
          th=theta*j;
          if(ins==1)th=theta;
          bet=asin((e-a*sin(thet))/b);
          vels=-a*vela*sin(thet-bet)/(cos(bet)*1000);
          velb=-a*vela*cos(thet)/b*cos(bet);
          c1=a*acca*sin(bet-thet)-b*pow(velb,2);
          c2=a*pow(vela,2)*cos(bet-thet);
          accs=(c1-c2)/(cos(bet)*1000);
          c3=a*acca*cos(thet)-a*pow(vela,2)*sin(thet);
          c4=b*pow(velb,2)*sin(bet);
          accb=-(c3-c4)/(b*cos(bet));
          printf( "%6.2d %6.2f %6.2f %6.2f %6.2f%8.2f
          %8.2f %8.2f\n",th,vela,acca,bet*180/pi,vels,
          velb,accs,accb);
          vela=sqrt(vela*vela+2*acca*dthet);
        }
   getch();
}
```

Fig. 4.6

The input variables are

[illegible]	The magnitudes *a, b* and *e* respectively (mm)
[illegible]	Angular velocity of the input link *AB* (m/s)
[illegible]	Angular acceleration of the input link (m/s^2)
[illegible]	The interval of the input angle (degrees)
[illegible]	Angle up to which acceleration continues, in case the crank starts from stationary position (in other cases any value may be given)

The output variables are

[illegible]	Angular displacement of the input link *AB* (degrees)
[illegible]	Angular displacement of link *AB* (rad/s)
[illegible]	Linear velocity of the slider (m/s)
[illegible]	Angular velocity of link *BC* (rad/s)
[illegible]	Linear acceleration of the slider (m/s^2)
[illegible]	Angular acceleration of link *BC* (rad/s^2)

Example 4.3 *In a slider-crank mechanism, the lengths of the crank and the connecting rod are 480 mm and 1.6 m respectively. It has an eccentricity of 100 mm. Assuming a velocity of 20 rad/s of the crank OA, calculate the following at an interval of 30°:*

(i) Velocity and the acceleration of the slider

(ii) Angular velocity and angular acceleration of the connecting rod

Solution The input and the output have been shown in Fig. 4.7. The results have been obtained at an interval of 30° of the input link (crank).

```
Enter values of a, b, e, vela, acca, theta, limit
480 1600 100 20 0 30 0
```

theta	vela	acca	beta	vels	velb	accs	accb
[illegible]	20.0	0.0	3.58	0.60	-5.99	-249.49	2.25
[illegible]	20.0	0.0	-5.02	-5.53	-5.18	-200.88	57.88
[illegible]	20.0	0.0	-11.38	-9.28	-2.94	-76.65	104.27
[illegible]	20.0	0.0	-13.74	-9.60	0.00	46.94	123.53
[illegible]	20.0	0.0	-11.38	-7.35	2.94	115.35	104.27
[illegible]	20.0	0.0	-5.02	-4.07	5.18	131.67	57.88
[illegible]	20.0	0.0	3.58	-0.60	5.99	134.51	2.25
[illegible]	20.0	0.0	12.27	2.99	5.08	144.94	-55.80
[illegible]	20.0	0.0	18.80	6.68	2.84	138.98	-107.04
[illegible]	20.0	0.0	21.25	9.60	-0.00	74.68	-128.76
[illegible]	20.0	0.0	18.80	9.95	-2.84	-53.02	-107.04
[illegible]	20.0	0.0	12.27	6.61	-5.08	-187.61	-55.80

Fig. 4.7

4.5 COUPLER CURVES

A coupler curve is the locus of a point on the coupler link. A four-link mechanism *ABCD* with a coupler point *E* (offset) is shown in Fig. 4.8. Let the *x*-axis be along the fixed link *AD*.

Let $BE = e$ and $\angle CBE = \alpha$

Angles β and γ are defined as shown in the diagram.

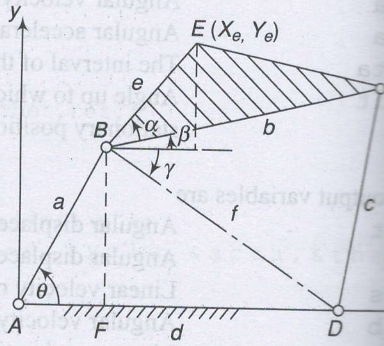

Fig. 4.8

Let X_e and Y_e be the coordinates of the point *E*.

Then,

$$X_e = a\cos\theta + e\cos(\alpha + \beta) \quad (4.59)$$

$$Y_e = a\sin\theta + e\sin(\alpha + \beta) \quad (4.60)$$

In these equations a, e, θ and α are known. To know the coordinates X_e and Y_e, it is necessary to express β in terms of known parameters, i.e., a, b, c, d, e, θ

In ΔBDC, applying cosine law,

$$\cos(\beta + \gamma) = \frac{b^2 + f^2 - c^2}{2bf}$$

or $$\beta + \gamma = \cos^{-1}\left[\frac{b^2 + f^2 - c^2}{2bf}\right]$$

$$\beta = \cos^{-1}\left[\frac{b^2 + f^2 - c^2}{2bf}\right] - \gamma$$

where $$\tan\gamma = \frac{BF}{FD} = \frac{BF}{AD - AF} = \frac{a\sin\theta}{d - a\cos\theta}$$

or $$\gamma = \tan^{-1}\left[\frac{a\sin\theta}{d - a\cos\theta}\right]$$

f^2 can be found by applying the cosine law to ΔABD, i.e.,

$$f^2 = a^2 + d^2 - 2ad\cos\theta$$

Having found the value of the angle β, the coordinates of the point *E* can be known for different values of θ from Eqs (4.59) and (4.60).

A coupler curve can also be obtained in case of a slider-crank mechanism (Fig. 4.9). The angle *CBE* is α and the eccentricity is *e*.

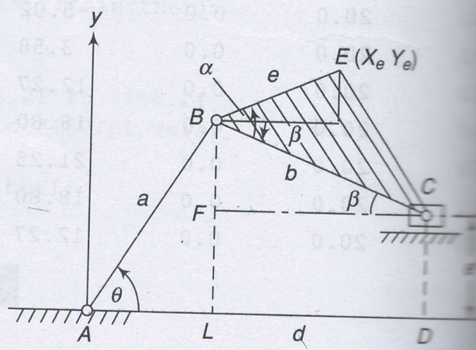

Fig. 4.9

Draw $BL \perp AD$ and $CF \perp BL$

$$X_e = a\cos\theta + e\cos(\alpha - \beta) \quad (4.63)$$

$$Y_e = a\sin\theta + e\sin(\alpha - \beta) \quad (4.64)$$

Angle β can be expressed in terms of known parameters as below:

$$\sin\beta = \frac{BF}{BC} = \frac{BL - FL}{BC} = \frac{a\sin\theta - e'}{b}$$

$$\beta = \sin^{-1}\left[\frac{a\sin\theta - e'}{b}\right] \qquad (4.65)$$

Figure 4.10 shows a program to find the coordinates of the coupler point for both the above cases.

The input variables are

a, b, e — The magnitudes *a*, *b* and *e* respectively (mm)
cas — 1, in case of a four-link mechanism
2, in case of a slider-crank mechanism
c — The magnitude *c* (case 1) or eccentricity *e′* (case 2)
d — The magnitude *d* (case 1) or 0 (case 2)
alph — The angle α (degrees)

The output variables are:

thet — Angular displacement of the link *AB* (degrees)
xe — X-coordinates of the point *E*
ye — Y-coordinates of the point *E*

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
void main()
{
 int cas;
 float a,b,c,d,e,f,alph,gamm,bet,squ,pi,thet,theta,
 xe,ye;
 clrscr();

 printf("enter values of a,b,c,d,e,alph,cas,theta\n");
 scanf("%f%f%f%f%f%f%d%f",&a,&b,&c,&d,&e,&alph,
 &cas,&theta);
 printf("      theta         xe            ye\n");
 thet=0;
 pi=4*atan(1);
    while(thet<359*pi/180)
    {
          gamm=atan(a*sin(thet)/(d-a*cos(thet)));
          squ=a*a+d*d-2*a*d*cos(thet);
          f=pow(squ,.5);
          if (cas==1)bet=acos((b*b+f*f-c*c)/2*b*f))-gamm;
          if (cas==2) bet=asin((c-a*sin(thet))/b);
          xe=a*cos(thet)+e*cos(alph*pi/180+bet);
          ye=a*sin(thet)+e*sin(alph*pi/180+bet);
          printf( "%10.2f %10.2f %10.2f \n",
          thet*180/pi,xe,ye);thet=thet+theta*pi/180;
    }
 getch();
}
```

Fig. 4.10

Example 4.4 *Draw a coupler curve of the coupler point E of a four-link mechanism having the following data:*

AB = 50 mm, BC = 66 mm, CD = 90 mm,

AD = 100 mm, BE = 30 mm ∠CBE = 40°

(refer to Fig. 4.8)

Solution The input and the output have been show in Fig. 4.11 using the program of Fig. 4.10 for th given date. The required coupler curve has bee shown in Fig. 4.12.

```
Enter values of a, b, c, d, e, alph,
cas theta
50  66  90  100  30  40  1  30
  theta          xe          ye
   0.0          26.73       18.93
  30.0          35.27       53.90
  60.0          29.79       72.92
  90.0          11.70       77.62
 120.0          -9.51       68.99
 150.0         -26.10       49.58
 180.0         -34.41       25.63
 210.0         -35.43        3.95
 240.0         -28.72      -13.53
 270.0         -15.08      -24.06
 300.0           1.75      -24.35
 330.0          16.54      -11.44
```

Fig. 4.11

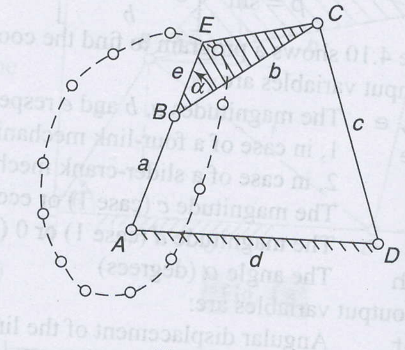

Fig. 4.12

Summary

1. To draw velocity and acceleration diagrams again and again for different positions of the crank is not convenient. Analytical methods prove to be very helpful.
2. In analytical methods, the links of the mechanism are considered as vectors.
3. In a four-link mechanism,
   (i) The angle of the output link is given by

$$\varphi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$$

where
$$2k = a^2 - b^2 + c^2 + d^2$$
$$A = k - a(d - c)\cos\theta - cd$$
$$B = -2ac\sin\theta$$
$$C = k - a(d + c)\cos\theta + cd$$

   (ii) The angle of the coupler link is given by

$$\beta = 2\tan^{-1}\left[\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right]$$

where
$$2k' = a^2 + b^2 - c^2 + d^2$$
$$D = k' - a(d + b)\cos\theta + bd$$
$$E = 2ab\sin\theta$$
$$F = k' - a(d - b)\cos\theta - bd$$

   (iii) The velocities of the output and coupler links are given by

$$\omega_c = \frac{a\omega_\alpha \sin(\beta - \theta)}{c\sin(\beta - \varphi)} \text{ and } \omega_b = -\frac{a\omega_\alpha \sin(\varphi - \theta)}{b\sin(\varphi - \beta)}$$

   (iv) The accelerations of the output and coupler links are given by

$$a_c = \frac{a\alpha_a \sin(\beta - \theta) - a\omega_a^2 \cos(\beta - \theta) - b\omega_b^2 + c\omega_c^2 \cos(\beta - \varphi)}{c\sin(\beta - \varphi)}$$

and

$$a_b = \frac{a\alpha_a \sin(\varphi - \theta) - a\omega_a^2 \cos(\varphi - \theta) - b\omega_b^2 \cos(\varphi - \beta) + c\omega_c^2}{b\sin(\beta - \varphi)}$$

4. In a slider-crank mechanism,
   (i) The displacement of the slider is given by

$$d = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_2}}{2}$$

where $C_1 = -2a\cos\theta$

$C_2 = a^2 - b^2 + e^2 - 2ae\sin\theta$

(ii) The angle of the coupler, $\beta = \sin^{-1}\dfrac{e - a\sin\theta}{b}$

(iii) The velocities of the slider and the angular velocity of the coupler are given by

$$\dot{d} = \frac{a\omega_a \sin(\beta-\theta)}{\cos\beta} \quad \text{and} \quad \omega_b = \frac{a\omega_a\cos\theta}{b\cos\beta}$$

(iv) The accelerations of the slider and the angular velocity of the coupler are given by

$$\ddot{d} = \frac{a\alpha_a\sin(\beta-\theta) - a\omega_a^2\cos(\beta-\theta) - b\omega_b^2}{\cos\beta} \text{ and}$$

$$\alpha_b = \frac{a\alpha_a\cos\theta - a\omega_a^2\sin\theta - b\omega_b^2\sin\beta}{b\cos\beta}$$

Exercises

1. Find expressions to determine the angles of the output link and coupler of a four-link mechanism. Deduce relations for the angular velocity and accelerations of the same links.
2. Deduce expressions to find the linear velocity and acceleration and angular velocity and angular acceleration of the coupler of a slider-crank mechanism.
3. What are coupler curves? Deduce expressions to show the same in case of a four-link mechanism and slider-crank mechanism.
4. Derive expressions for the displacement, velocity and acceleration analyses of an inverted slider-crank mechanism.
5. In a four-link mechanism (Fig. 4.1), the dimensions of the links are AB = 30 mm, BC = 80 mm, CD = 40 mm and AD = 75 mm. If OA rotates at a constant angular velocity of 30 rad/s in the clockwise direction, calculate the angular velocities and the angular accelerations of links BC and CD for values of θ at an interval of 30°.
6. In a slider-crank mechanism (Fig. 4.5), the crank AB = 50 mm, BC = 160 mm and eccentricity e = 15 mm. For the angle θ = 45°, angular velocity of AB = 8 rad/s with an angular acceleration of 12 rad/s² (both clockwise), find the linear velocity and the acceleration of the slider and the angular velocity and the angular acceleration of the connecting rod analytically.

   (0.32 m/s, 1.98 m/s², 1.78 rad/s, 16.53 rad/s²)
7. Derive expressions to find the angular displacement, angular velocity and the angular acceleration of the link EF of a six-link mechanism shown in Fig. 4.13. AB is the input link having an angular velocity of ω rad/s in the counter-clockwise direction.

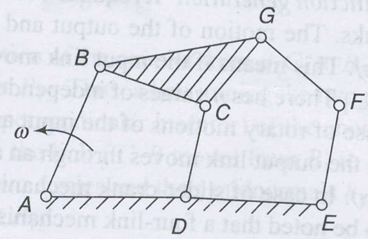

Fig. 4.13

8. Derive expressions for the coupler curves of an inverted slider-crank mechanism.
9. For the data of Example 4.3, take some more coupler points by taking different values of BE and $\angle\alpha$ and draw coupler curves for the same. Make a cardboard model of the mechanism and obtain the coupler curve by rotating the crank through 360°.