1. Design of Shaft

Table 1-1: Deterministic ASTM Minimum Tensile and Yield strengths for some hot-rolled (HR) and cold – drawn (CD) steels [The strengths listed area estimates ASTM minimum values in the size range 18 t0 32 mm. These strengths are suitable for use with the design factor defined in sec. 1-10, provided the materials conform to ASTM A6 or A586 requirements or are required in the purchase specifications. Remember that a numbering systems is not specifications.] Source: 1986 SAE Handbook, p 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in,%	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (57)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	50	137
		CD	520 (76)	440 (64)	12	40	149
G10350	1035	HR	500 (72)	270 (39.5)	18	42	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201
G10800	1080	HR	770 (112)	420 (61.5)	10	25	229
G10950	1095	HR	830 (120)	460 (66)	10	23	248

Table 1-2 : First Iteration Estimates for stress concentration factors K_t and K_{ts}. Warning: These factors are only estimates for use when actual dimensions are not yet determined. Do not use these once actual dimensions are available.

	Bending	Torsionsal	Axial
Shoulder fillet – sharp (r/d = 0.02)	2.7	2.2	3.0
Shoulder fillet – well rounded (r/d = 0.1)	1.7	1.5	1.9
End milled kyeseat (r/d 0.02)	2.14	3.0	-
Sled runner keyseat	1.7	-	-
Retaining ring groove	5.0	3.0	5.0

Table 1-3: Parameters for Marin Surface Modification Factor.

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	-0.085
Machined or cold drawn	2.70	4.51	-0.265
Hot-rolled	14.4	57.7	-0.718
As-forged	39.9	272	-0.995

From C.J. Noll and C.Lipson,"Allowable Working Strsses", Society for Experimental Stress Analysis, vol.3, no.2,1946 p. 29. Reproduced by O.J. Horger(ed.) Metals Engineering Design ASME Handbook, McGraw-Hill, New York, Copyright© 1953 by The McGraw-Hill Companies, Inc.Reprinted by permission.

Table 1-4: Reliability factors K_e corresponding to 8% standard deviation of the endurance limit.

Reliability%	Transformation Variate Z_a	Reliability Factor K_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Table 1-5: Typical maximum ranges for slopes and transverse deflections.

Slopes	
Tapered roller	0.0005 – 0.0012 rad
Cylindrical roller	0.008 – 0.0012 rad
Deep-groove ball	0.001 – 0.003 rad
Spherical ball	0.026 – 0.052 rad
Self-align bal	0.026 – 0.052 rad
Uncrowned spur gear	<0.0005 rad
Transverse Deflections	
Spur gears with P< 10 teeth/cm	0.25 mm
Spur gears with 11 <P<19	0.125 mm
Spur gears with 20 <P<50	0.075 mm

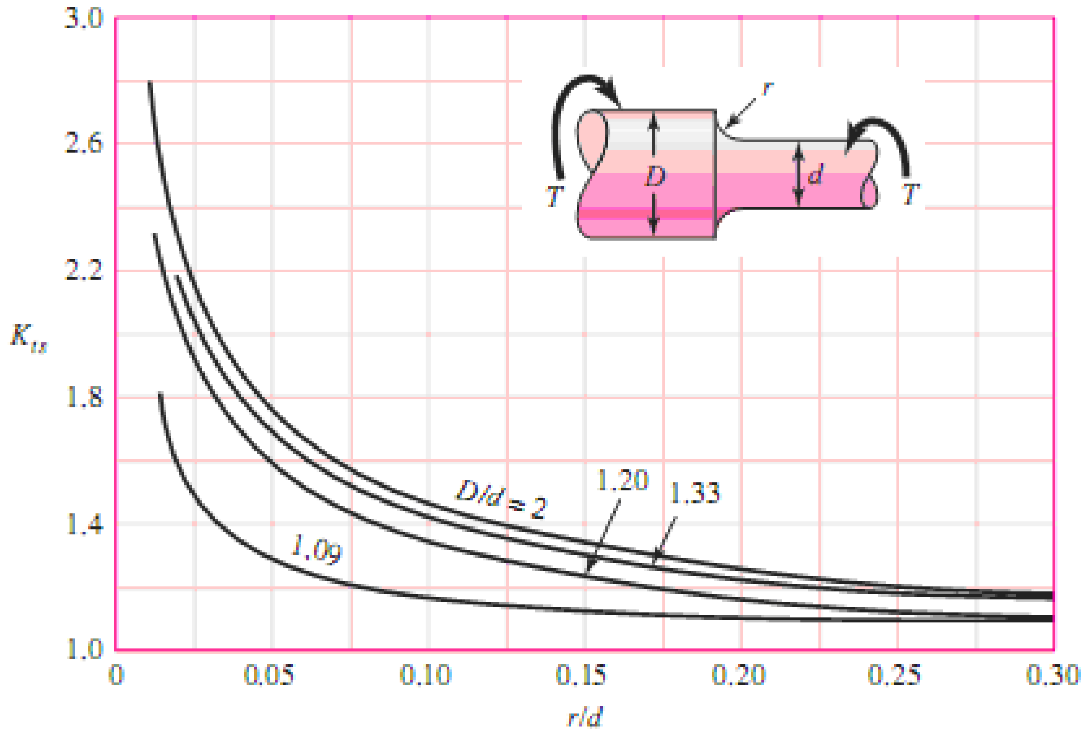


Figure 1-1: Round shaft with shoulder fillet in torsion: τ_0 = TC/J, where c = d/2 and J = Лd⁴/32.

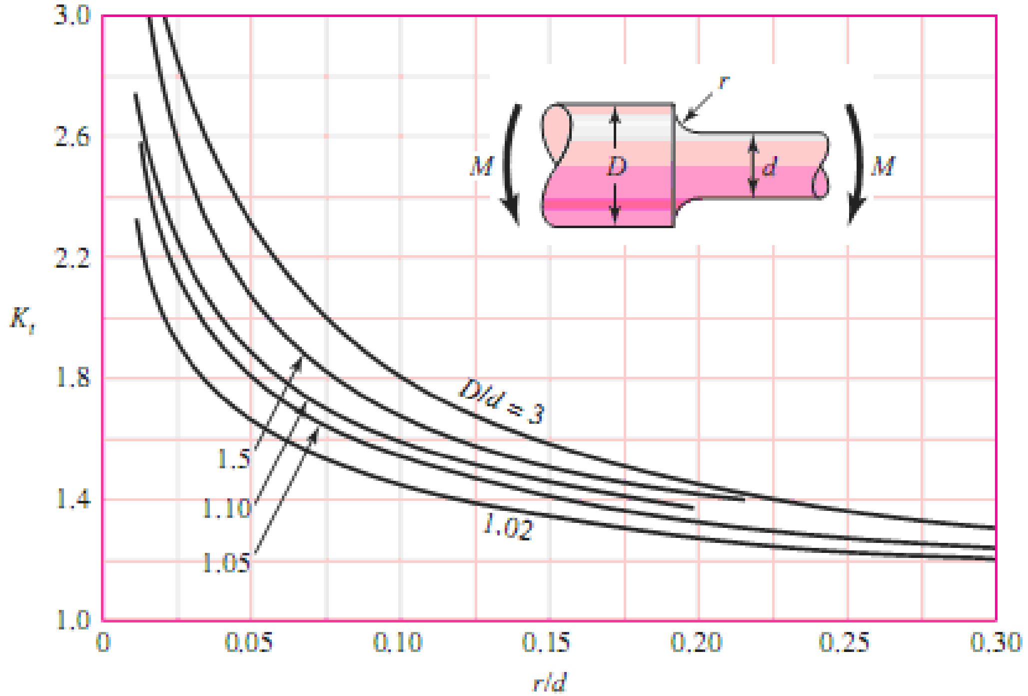


Figure 1-2: Round shaft with shoulder fillet in bending. σ_0 = MC/I, where c = d/2 and J = Лd⁴/64.

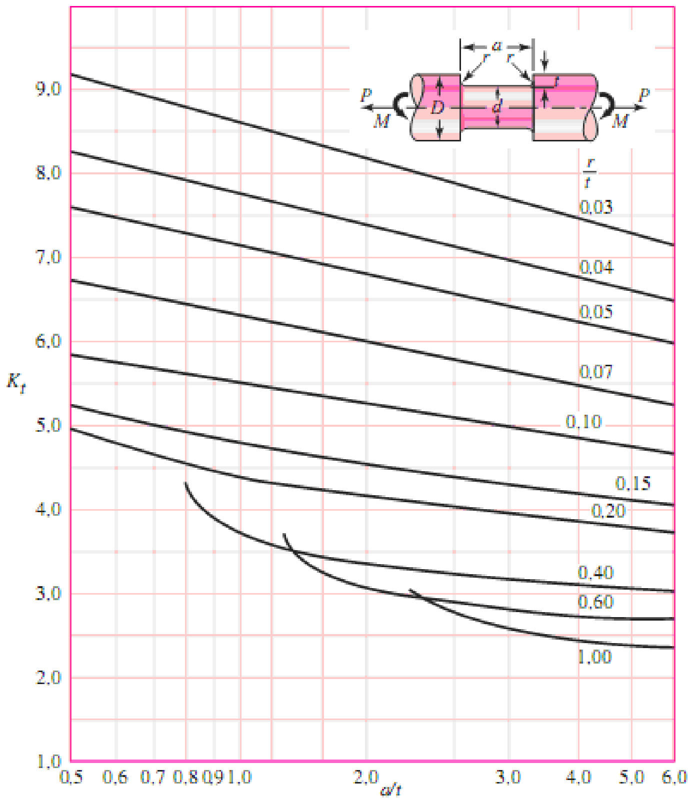


Figure 1-3: Round shaft with flat-bottom grove in bending and / or tension. $\sigma_0 = 4P/\text{Л}d^2 + 32M/\text{Л}d^3$.

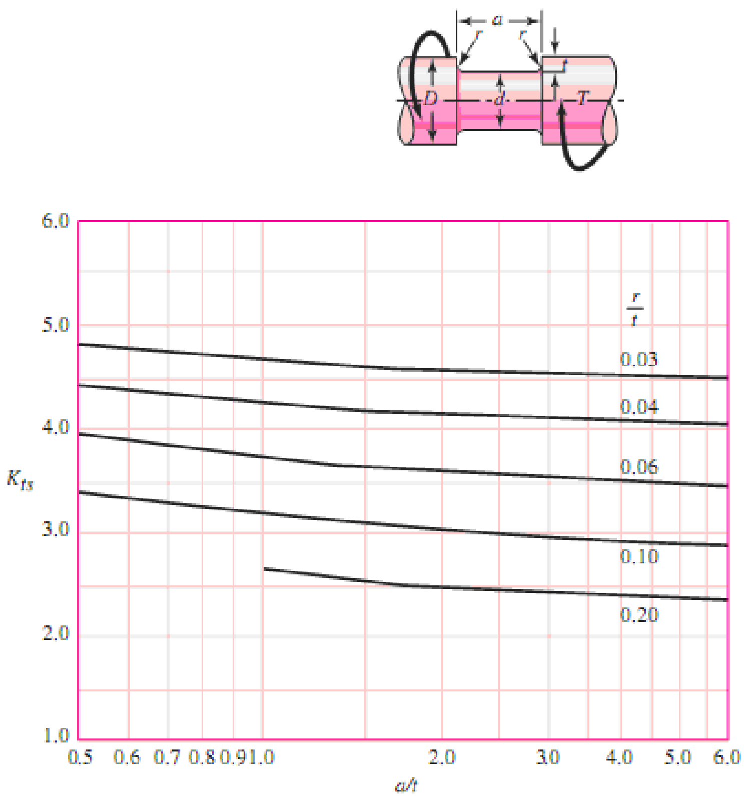


Figure 1-4: Round shaft with flat-bottom grove in torsion. $\tau_0 = 16T/\text{Л}d^3$.

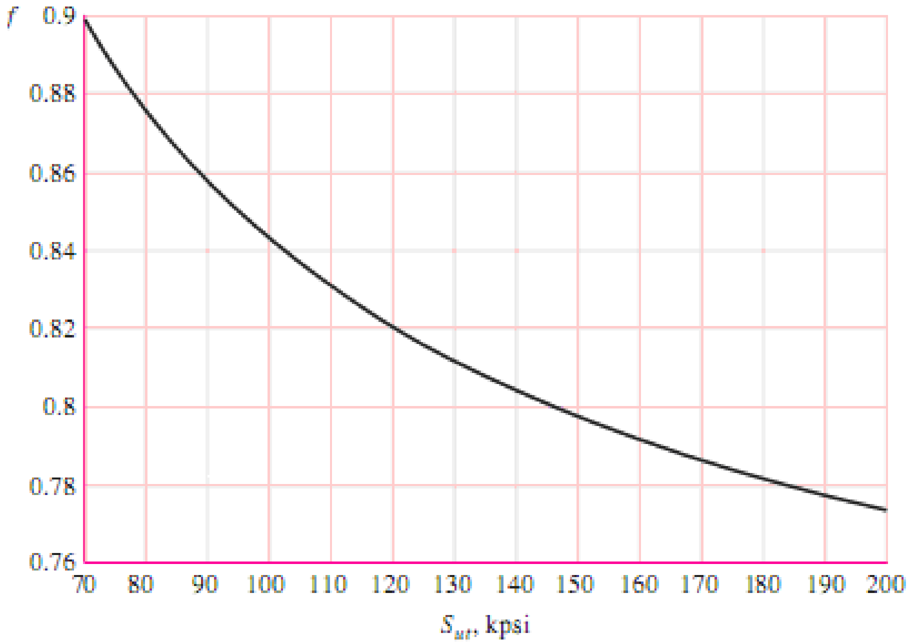


Figure 1-5: Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S_e' = 0.5S_{ut}$.

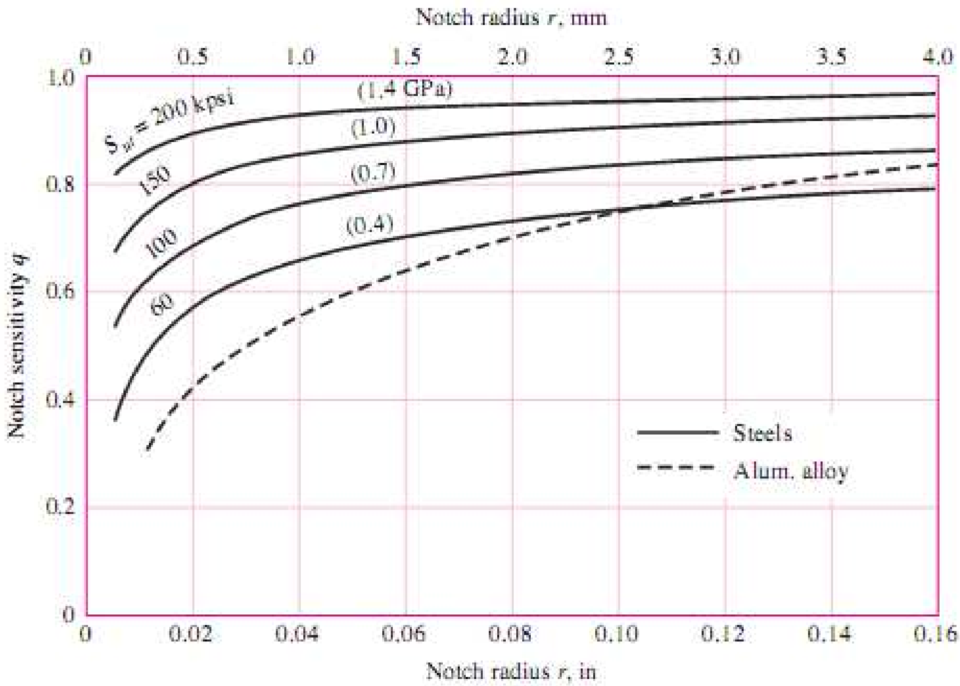


Figure 1-6: Notch sensitivity for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads for large notch radii, use the values of q corresponding to the t = 4 mm ordinate.

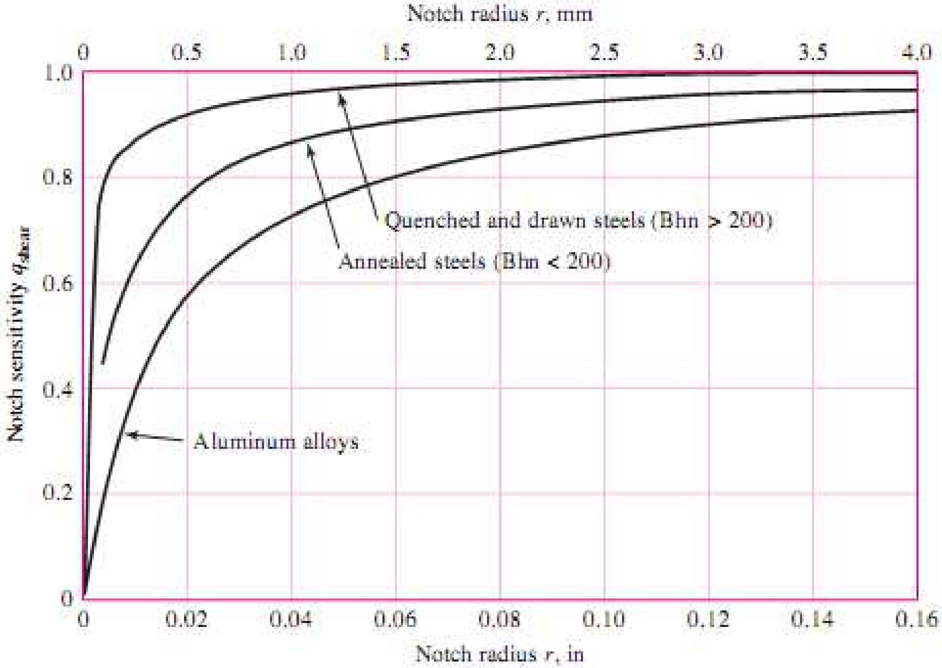


Figure 1-7: Notch sensitivity curves for materials in reversed torsion. For large notch radii, use the values of q_{shear} corresponding to t = 4 mm.

1. Road maps and important design equations for the stress life method.

1.1 Endurance limit

The important procedures and equations for deterministic stress-life are presented here.

Completely Reversed Simple loading

Determine $S_e^{'}$ either from test data or $S_e^{'} = \begin{cases} 0.5S_{ut} & S_{ut} \le 1400\text{MPa} \\ 700\,\text{MPa} & S_{ut} > 1400\text{MPa} \end{cases}$

1.2 Fatigue life constants a and b.

If $S_{ut} \ge 490$ MPa, determine f from Figure 1-5. If $S_{ut} < 490$ MPa, let f = 0.9.

$$a = \frac{\left(f\ S_{ut}\right)^2}{S_e}$$

$$b = -\frac{\left[\log \frac{\left(f\ S_{ut}\right)}{S_e}\right]}{3}$$

1.3 Fatigue Strength S_f at N cycles, or N cycles to failure at reversing stress σ_{rev}

$$S_f = aN^b$$

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b}$$

1.4 For finite-life fatigue strength,

Mod-Goodman $$S_{rev} = \frac{\sigma_a}{1-\left(\frac{\sigma_m}{S_{ut}}\right)}$$

Gerber $$S_{rev} = \frac{\sigma_a}{1-\left(\frac{\sigma_m}{S_{ut}}\right)^2}$$

If determining the finite line N with a factor of safety n, substitute S_{rev}/n for σ_{rev}, that is,

$$N = \left(\frac{\sigma_{rev}/n}{a}\right)^{1/b}$$

1.5 Endurance limit modifying factor, Marin Equation

$$S_e = k_a k_b k_c k_d k_e k_f S_e^{'}$$

where k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor

k_f = miscellaneous effects odification factor

$S_e^{'}$ = rotary beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use.

Modify $S_e^{'}$ to determine S_e.

$k_a = aS_{ut}^b$, for a and b refer Table 1-5.

Rotating shaft: For bending or torsion,

$$k_b = \begin{cases} 1.24d^{-0.107} & 2.79 \le d \le 51\,\text{mm} \\ 1.51d^{-0.157} & 51 < d \le 254\,\text{mm} \end{cases}$$

For axial,

$$k_b = 1$$

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

$$k_d = 0.975 + 0.432\left(10^{-3}\right)T_F - 0.115\left(10^{-5}\right)T_F^2 + 0.104\left(10^{-8}\right)T_F^3 - 0.595\left(10^{-12}\right)T_F^4$$

1.6 Determine fatigue stress concentration factor K_f or K_{fs}

First, find K_t or K_{ts} from Table 1-2, Figure 1- 1 to Figure 1-3. For not sensitivity q, use Figure 1-6 or Figure 1-7.

$$K_f = 1 + q\left(K_t - 1\right) \quad \text{or} \quad K_{fs} = 1 + q\left(K_{ts} - 1\right)$$

Alternatively, **for reversed bending or axial loads**,

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a / r}}$$

where $\sqrt{a}$ is the Neuber constant and is a material constant.

For S_{ut} in kpsi, and $\sqrt{a}$ uint is $\sqrt{\text{inch}}$

$$\sqrt{a} = 0.245799 - 0.307794\left(10^{-2}\right)S_{ut} + 0.150874\left(10^{-4}\right)S_{ut}^2 - 0.266978\left(10^{-7}\right)S_{ut}^3$$

For torsion for low alloy steels,

$$\sqrt{a} = 0.19 - 2.51\left(10^{-3}\right)S_{ut} + 1.35\left(10^{-5}\right)S_{ut}^2 - 2.67\left(10^{-8}\right)S_{ut}^3$$

1.7 Fluctuating simple load

For S_e, K_f or K_{fs}, refer section 1-3 and section 1-4.

1. Calculate midrange stress σ_m and alternating stress σ_a. Apply K_f to both stresses.

$$\sigma_m = \left(\sigma_{max} + \sigma_{min}\right)/2 \qquad \sigma_a = \left(\sigma_{max} - \sigma_{min}\right)/2$$

2. Apply to a fatigue failure criterion,

$\sigma_m \ge 0$

Soderburg $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

Mod-Goodman $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

Gerber $$n\frac{\sigma_a}{S_e} + \left(n\frac{\sigma_m}{S_{ut}}\right)^2 = 1$$

ASME-elliptic $$\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = \frac{1}{n^2}$$

$\sigma_m < 0$, $$\sigma_a = \frac{S_e}{n}$$

Torsion. Use the same equations as apply for $\sigma_m \geq 0$, except replace σ_m and σ_a with τ_m and τ_m, use $k_c = 0.59$ for S_e, replace S_{ut} with $S_{su} = 0.67\ S_{ut}$, and replace S_y with $S_{sy} = 0.577S_y$.

Check for localized yielding.

$$\sigma_a + \sigma_m = \frac{S_y}{n}$$

or, for torsion,
$$\tau_a + \tau_m = 0.577S_y / n$$

1.8 Combination of loading modes

i. Calculate von Mises stresses for alternating and midrange stress states, $\sigma_a^{'}$ and $\sigma_m^{'}$. When determining S_e, do not use k_c nor divided by K_f or K_{fs}. Apply K_f and/or K_{fs} directly to each specific alternating axial stress. If axial stress is present divide the alternating axial stress by $k_c = 0.85$. For the special case of combined bending, torsional shear, and axial stresses

$$\sigma_a^{'} = \left\{\left[(K_f)_{bending}(\sigma_a)_{bending} + (K_f)_{axial}\frac{(\sigma_a)_{axial}}{0.85}\right]^2 + 3\left[(K_{fs})_{torsion}(\tau_a)_{torsion}\right]^2\right\}^{1/2}$$

$$\sigma_m^{'} = \left\{\left[(K_f)_{bending}(\sigma_m)_{bending} + (K_f)_{axial}(\sigma_m)_{axial}\right]^2 + 3\left[(K_{fs})_{torsion}(\tau_m)_{torsion}\right]^2\right\}^{1/2}$$

ii. Apply stresses to gatigue criterion.

iii. Conservative check for localized yielding using von Mises stresses.

$$\sigma_a^{'} + \sigma_m^{'} = \frac{S_y}{n}$$

ii. Shaft stresses

Bending, torsion, and axial stresses may be present in both midrange and alternating components. For analysis, it is simple enough to combine the different types of stresses into alternating and midrange von Mises stresses. It is sometimes convenient to customize the equations specially for shaft applications. Axial loads are usually comparatively very small at critical locations where bending and torsion dominate, so they will be left out of the following equations. The fluctuating stresses due to bending and torsion are given by

$$\sigma_a = K_f\frac{M_a c}{I} \qquad \sigma_m = K_f\frac{M_m c}{I}$$

$$\tau_a = K_{fs}\frac{T_a c}{J} \qquad \tau_m = K_{fs}\frac{T_m c}{J}$$

where M_m and M_a are the midrange and alternating bending moments, T_m and T_a are the midrange and alternating torques, and K_f and K_{fs} are the fatigue stress concentration factors for bending and torsion, respectively.

Assuming a solid shaft with round cross section, appropriate geometry terms can be introduced for c, I, and J in

$$\sigma_a = K_f\frac{32M_a}{\pi d^3} \qquad \sigma_m = K_f\frac{32M_m}{\pi d^3}$$

$$\tau_a = K_{fs}\frac{16T_a}{\pi d^3} \qquad \tau_m = K_{fs}\frac{16T_m}{\pi d^3}$$

Combining these stresses in accordance with the distortion energy failure theory, the von Mises stresses for rotating round, solid shafts, neglecting axial loads, are given by

$$\sigma_a' = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma_m' = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

Note that the stress concentration factors are sometimes considered optional for the midrange components with ductile materials, because of the capacity of the ductile material to yield locally at the discontinuity.

These equivalent alternating and midrange stresses can be evaluated using an appropriate failure curve on the modified Goodman diagram. For example, the fatigue failure criteria for the modified Goodman line as expresses is

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$

Goodman

Substitute of σ_a' and σ_m' results in

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{1/2}\right\}$$

For design purpose, it is also desirable to solve the equation for the diameter.
This results in

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{1/2}\right\}\right)^{1/3}$$

Similar expressions can be obtained for any of the common failure criteria by substituting the von Mises stresses into any of the failure criteria. The resulting equations for several of the commonly used failure curves are summarized below. The names given to each set of equations identifies the significant failure theory, followed by a fatigue failure locus name. For example, DE-Gerber indicates the stresses are combined using the distortion energy (DE) theory, and the Gerber criteria is used for the fatigue failure.

DE- Gerber

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3}$$

where

$$A = \sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2}$$

$$B = \sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2}$$

DE-ASME Elliptic

$$\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}$$

$$d=\left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2+3\left(\frac{K_{fs} T_a}{S_e}\right)^2+4\left(\frac{K_f M_m}{S_y}\right)^2+3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3}$$

DE-Soderberg

$$\frac{1}{n}=\frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2+3\left(K_{fs} T_a\right)^2\right]^{1/2}+\frac{1}{S_{yt}}\left[4\left(K_f M_m\right)^2+3\left(K_{fs} T_m\right)^2\right]^{1/2}\right\}$$

$$d=\left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2+3\left(K_{fs} T_a\right)^2\right]^{1/2}+\frac{1}{S_{yt}}\left[4\left(K_f M_m\right)^2+3\left(K_{fs} T_m\right)^2\right]^{1/2}\right\}\right)^{1/3}$$

For a rotating shaft with constant bending and torsion, the bending stress is completely reversed and the torsion is steady. At such situations, setting M_m and T_a equal to zero drops out some of the terms.

Note that in an analysis situation in which the diameter is known and the factor of safety is desired, as an alternative to using the specialized equations above, it is always still valid to calculate the alternating and mid-range stresses, and substitute them into one of the equations for the failure criteria, then solve directly for n using above equations.

It is always necessary to consider the possibility of static failure in the first load cycle. The Soderberg criteria inherently guards against yielding, as can be seen by noting that its failure curve is conservatively within the yield (Langer) line. The ASME Elliptic also takes yielding into account, but is not entirely conservative throughout its entire range. This is evident by noting that it crosses the yield line. Ther Gerber and modified Goodman criteria do not guard against yielding, requiring a separate check for yielding. A von Mises maximum stress is calculated for this purpose.

$$\sigma'_{max}=\left[\left(\sigma_m+\sigma_a\right)^2+3\left(\tau_m+\tau_a\right)^2\right]^{1/2}$$

$$\sigma'_{max}=\left[\left(\frac{32K_f\left(\sigma_m+\sigma_a\right)}{\pi d^3}\right)^2+3\left(\frac{16K_{fs}\left(T_m+T_a\right)}{\pi d^3}\right)^2\right]^{1/2}$$

To check for yielding, this von Mises maximum stress is compared to the yield strength, as usual,

$$n_y=\frac{S_y}{\sigma'_{max}}$$

For a quick, conservative, an estimate for σ'_{max} can be obtained by simply adding σ'_a and σ'_a, ($\sigma'_a+\sigma'_a$) will always be greater than or equal to σ'_{max}, and will therefore be conservative.

1.9 Deflection considerations

$$d_{new}=d_{old}\left|\frac{n_d y_{old}}{y_{all}}\right|^{1/4},\quad d_{new}=d_{old}\left|\frac{n_d\left(dy/dx\right)_{old}}{(slope)_{all}}\right|^{1/4}$$ where d is the diameter, yall is allowable deflection, nd is the design factor, (slope)all is the allowable slope.

2. Design of bearing

2.1 Rolling – contacts bearings

(a) For 90 % reliability

$$C_{10}\left(L_R n_R 60\right)^{1/a} = F_D\left(L_D n_D 60\right)^{1/a}$$

where,

C_{10} = catalog rating, kN — F_D = Desired radial load, kN

L_R = rating life in hours — L_D = desired life, hours

n_R = rating speed, rev,min — n_D = desired speed, rev/min

(b) For reliability greater or equals to 90%, assume a = 3 for ball & angular contact bearing and a = 10/3 for roller bearing (cylindrical and tapered roller).

$$C_{10} = F_D\left[\frac{x_D}{x_0 + (\theta - x_0)(1-R_D)^{1/b}}\right]^{1/a} \qquad R \geq 0.9$$

$$\text{and, } x_D = \frac{L}{L_{10}} = \frac{60 L_D n_D}{60 L_R n_R}$$

Loads are often non-steady, so that the desired load is multiplied by an application factor a_f . The steady load $a_f F_D$ does the same damage as the variable load F_D does to the rolling surfaces.

Manufacturer	Rating Life, Revolution	Weibull parameter Rating lives X_o	θ	b
1	$90(10^6)$	0	4.48	1.15
2	$1(10^6)$	0.02	4.459	1.483

Table 2-1: Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearing

Bore, Mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm d_s	d_H	Loading Rating, kN Deep Groove C_{10}	C_0	Angular Contact C_{10}	C_0
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

Table 2-2: Dimensions and Load Ratings for Cylindrical Roller Bearings.

			02- Series Loading Rating, kN				02- Series Loading Rating, kN	
Bore, Mm	OD, mm	Width, mm	C_{10}	C_0	OD, mm	Width, mm	C_{10}	C_0
25	52	15	16.8	8.8	62	17	28.6	15
30	62	16	22.4	12.0	72	19	36.9	20
35	72	17	31.9	17.6	80	21	44.6	27.1
40	80	18	41.8	24.0	90	23	56.1	32.5
45	85	19	44.0	25.5	100	25	72.1	45.4
50	90	20	45.7	27.5	110	27	88.0	52.0
55	100	21	56.1	34.0	120	29	102	67.2
60	110	22	64.4	43.1	130	31	123	76.5
55	120	23	76.5	51.2	140	33	138	85.0
70	125	24	79.2	51.2	150	35	151	102
75	130	25	93.1	63.2	160	37	183	125
80	140	26	106	69.4	170	39	190	125
85	150	28	119	78.3	180	41	212	149
90	160	30	142	100	190	43	242	160
95	170	32	165	112	200	45	264	189
100	180	34	183	125	215	47	303	220
110	200	38	229	167	240	50	391	304
120	215	40	260	183	260	55	457	340
130	230	40	270	193	280	58	539	408
140	250	42	319	240	300	62	682	454
150	270	45	446	260	320	65	781	502

2.2 Combined Radial and Thrust Loading

Equivalent radial load that does the same damage as the combined radial and thrust load

$$F_e = X_i V F_r + Y_i F_a$$

where F_r – radial load, F_a – axial load, inner ring rotating V = 1, outer ring rotating V =1.2

Table 2-3: Equivalent radial load factors for ball bearings.

		$F_a/(VF_r) \le e$		$F_a/(VF_r) > e$	
F_a/C_0	E	X_1	Y_1	X_2	Y_2
0.014	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

2.3 Tapered roller bearings

i. Induced thrust load from a radial load, $F_i = \dfrac{0.47F_r}{K}$, K factor is geometry – specific (1.5 for a radial bearing and 0.75 for step angle bearing)

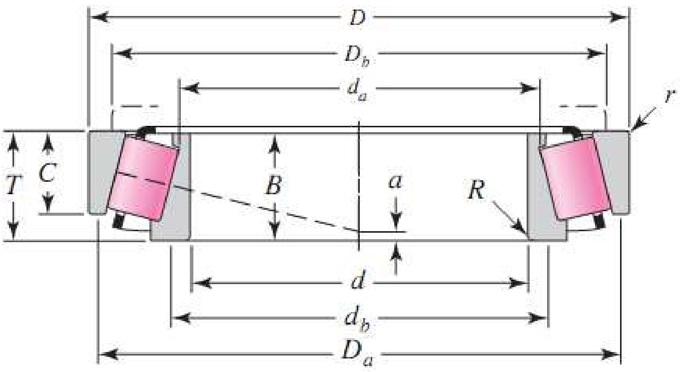
ii. Equivalent load, $F_e = 0.4F_r + KF_a$

iii. If $F_{iA} \leq (F_{iB} + F_{ae})$ $\begin{cases} F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) \\ F_{eB} = F_{rB} \end{cases}$

If $F_{iA} > (F_{iB} + F_{ae})$ $\begin{cases} F_{eB} = 0.4F_{rB} + K_B(F_{iA} - F_{ae}) \\ F_{eA} = F_{rA} \end{cases}$

Table 2-4: Single row straight bore roller bearing.

SINGLE-ROW STRAIGHT BORE



bore d	outside diameter D	width T	rating at 500 rpm for 3000 hours L_{10}: one-row radial N lbf	rating at 500 rpm for 3000 hours L_{10}: thrust N lbf	factor K	eff. load center a②	part numbers: cone	part numbers: cup	cone: max shaft fillet radius R①	cone: width B	cone: backing shoulder diameters d_b	cone: backing shoulder diameters d_a	cup: max housing fillet radius r①	cup: width C	cup: backing shoulder diameters D_b	cup: backing shoulder diameters D_a
25.000 0.9843	52.000 2.0472	16.250 0.6398	8190 1840	5260 1180	1.56	−3.6 −0.14	◆30205	◆30205	1.0 0.04	15.000 0.5906	30.5 1.20	29.0 1.14	1.0 0.04	13.000 0.5118	46.0 1.81	48.5 1.91
25.000 0.9843	52.000 2.0472	19.250 0.7579	9520 2140	9510 2140	1.00	−3.0 −0.12	◆32205-B	◆32205-B	1.0 0.04	18.000 0.7087	34.0 1.34	31.0 1.22	1.0 0.04	15.000 0.5906	43.5 1.71	49.5 1.95
25.000 0.9843	52.000 2.0472	22.000 0.8661	13200 2980	7960 1790	1.66	−7.6 −0.30	◆33205	◆33205	1.0 0.04	22.000 0.8661	34.0 1.34	30.5 1.20	1.0 0.04	18.000 0.7087	44.5 1.75	49.0 1.93
25.000 0.9843	62.000 2.4409	18.250 0.7185	13000 2930	6680 1500	1.95	−5.1 −0.20	◆30305	◆30305	1.5 0.06	17.000 0.6693	32.5 1.28	30.0 1.18	1.5 0.06	15.000 0.5906	55.0 2.17	57.0 2.24
25.000 0.9843	62.000 2.4409	25.250 0.9941	17400 3910	8930 2010	1.95	−9.7 −0.38	◆32305	◆32305	1.5 0.06	24.000 0.9449	35.0 1.38	31.5 1.24	1.5 0.06	20.000 0.7874	54.0 2.13	57.0 2.24
25.159 0.9905	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07096	07196	1.5 0.06	14.260 0.5614	31.5 1.24	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85
25.400 1.0000	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07100	07196	1.0 0.04	14.260 0.5614	30.5 1.20	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85
25.400 1.0000	50.005 1.9687	13.495 0.5313	6990 1570	4810 1080	1.45	−2.8 −0.11	07100-S	07196	1.5 0.06	14.260 0.5614	31.5 1.24	29.5 1.16	1.0 0.04	9.525 0.3750	44.5 1.75	47.0 1.85

(continued on next page)

			rating at 500 rpm for 3000 hours L_{10}				part numbers		cone				cup			
bore	outside diameter	width	one-row radial	thrust	factor	eff. load center	cone	cup	max shaft fillet radius	width	backing shoulder diameters		max housing fillet radius	width	backing shoulder diameters	
d	D	T	N lbf	N lbf	K	a②			R①	B	d_b	d_a	r①	C	D_b	D_a
25.400 1.0000	**65.088** 2.5625	**22.225** 0.8750	**13100** 2950	**16400** 3690	**0.80**	**−2.3** −0.09	**23100**	**23256**	**1.5** 0.06	**21.463** 0.8450	**39.0** 1.54	**34.5** 1.36	**1.5** 0.06	**15.875** 0.6250	**53.0** 2.09	**63.0** 2.48
25.400 1.0000	**66.421** 2.6150	**23.812** 0.9375	**18400** 4140	**8000** 1800	**2.30**	**−9.4** −0.37	**2687**	**2631**	**1.3** 0.05	**25.433** 1.0013	**33.5** 1.32	**31.5** 1.24	**1.3** 0.05	**19.050** 0.7500	**58.0** 2.28	**60.0** 2.36
25.400 1.0000	**68.262** 2.6875	**22.225** 0.8750	**15300** 3440	**10900** 2450	**1.40**	**−5.1** −0.20	**02473**	**02420**	**0.8** 0.03	**22.225** 0.8750	**34.5** 1.36	**33.5** 1.32	**1.5** 0.06	**17.462** 0.6875	**59.0** 2.32	**63.0** 2.48
25.400 1.0000	**72.233** 2.8438	**25.400** 1.0000	**18400** 4140	**17200** 3870	**1.07**	**−4.6** −0.18	**HM88630**	**HM88610**	**0.8** 0.03	**25.400** 1.0000	**39.5** 1.56	**39.5** 1.56	**2.3** 0.09	**19.842** 0.7812	**60.0** 2.36	**69.0** 2.72
25.400 1.0000	**72.626** 2.8593	**30.162** 1.1875	**22700** 5110	**13000** 2910	**1.76**	**−10.2** −0.40	**3189**	**3120**	**0.8** 0.03	**29.997** 1.1810	**35.5** 1.40	**35.0** 1.38	**3.3** 0.13	**23.812** 0.9375	**61.0** 2.40	**67.0** 2.64
26.157 1.0298	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15103**	**15245**	**0.8** 0.03	**20.638** 0.8125	**33.0** 1.30	**32.5** 1.28	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
26.162 1.0300	**63.100** 2.4843	**23.812** 0.9375	**18400** 4140	**8000** 1800	**2.30**	**−9.4** −0.37	**2682**	**2630**	**1.5** 0.06	**25.433** 1.0013	**34.5** 1.36	**32.0** 1.26	**0.8** 0.03	**19.050** 0.7500	**57.0** 2.24	**59.0** 2.32
25.400 1.0000	**50.292** 1.9800	**14.224** 0.5600	**7210** 1620	**4620** 1040	**1.56**	**−3.3** −0.13	**L44642**	**L44610**	**3.5** 0.14	**14.732** 0.5800	**36.0** 1.42	**29.5** 1.16	**1.3** 0.05	**10.668** 0.4200	**44.5** 1.75	**47.0** 1.85
25.400 1.0000	**50.292** 1.9800	**14.224** 0.5600	**7210** 1620	**4620** 1040	**1.56**	**−3.3** −0.13	**L44643**	**L44610**	**1.3** 0.05	**14.732** 0.5800	**31.5** 1.24	**29.5** 1.16	**1.3** 0.05	**10.668** 0.4200	**44.5** 1.75	**47.0** 1.85
25.400 1.0000	**51.994** 2.0470	**15.011** 0.5910	**6990** 1570	**4810** 1080	**1.45**	**−2.8** −0.11	**07100**	**07204**	**1.0** 0.04	**14.260** 0.5614	**30.5** 1.20	**29.5** 1.16	**1.3** 0.05	**12.700** 0.5000	**45.0** 1.77	**48.0** 1.89
25.400 1.0000	**56.896** 2.2400	**19.368** 0.7625	**10900** 2450	**5740** 1290	**1.90**	**−6.9** −0.27	**1780**	**1729**	**0.8** 0.03	**19.837** 0.7810	**30.5** 1.20	**30.0** 1.18	**1.3** 0.05	**15.875** 0.6250	**49.0** 1.93	**51.0** 2.01
25.400 1.0000	**57.150** 2.2500	**19.431** 0.7650	**11700** 2620	**10900** 2450	**1.07**	**−3.0** −0.12	**M84548**	**M84510**	**1.5** 0.06	**19.431** 0.7650	**36.0** 1.42	**33.0** 1.30	**1.5** 0.06	**14.732** 0.5800	**48.5** 1.91	**54.0** 2.13
25.400 1.0000	**58.738** 2.3125	**19.050** 0.7500	**11600** 2610	**6560** 1470	**1.77**	**−5.8** −0.23	**1986**	**1932**	**1.3** 0.05	**19.355** 0.7620	**32.5** 1.28	**30.5** 1.20	**1.3** 0.05	**15.080** 0.5937	**52.0** 2.05	**54.0** 2.13
25.400 1.0000	**59.530** 2.3437	**23.368** 0.9200	**13900** 3140	**13000** 2930	**1.07**	**−5.1** −0.20	**M84249**	**M84210**	**0.8** 0.03	**23.114** 0.9100	**36.0** 1.42	**32.5** 1.27	**1.5** 0.06	**18.288** 0.7200	**49.5** 1.95	**56.0** 2.20
25.400 1.0000	**60.325** 2.3750	**19.842** 0.7812	**11000** 2480	**6550** 1470	**1.69**	**−5.1** −0.20	**15578**	**15523**	**1.3** 0.05	**17.462** 0.6875	**32.5** 1.28	**30.5** 1.20	**1.5** 0.06	**15.875** 0.6250	**51.0** 2.01	**54.0** 2.13
25.400 1.0000	**61.912** 2.4375	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15243**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**2.0** 0.08	**14.288** 0.5625	**54.0** 2.13	**58.0** 2.28
25.400 1.0000	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15100**	**15245**	**3.5** 0.14	**20.638** 0.8125	**38.0** 1.50	**31.5** 1.24	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
25.400 1.0000	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15245**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
25.400 1.0000	**62.000** 2.4409	**19.050** 0.7500	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15102**	**15245**	**1.5** 0.06	**20.638** 0.8125	**34.0** 1.34	**31.5** 1.24	**1.3** 0.05	**14.288** 0.5625	**55.0** 2.17	**58.0** 2.28
25.400 1.0000	**62.000** 2.4409	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15244**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.3** 0.05	**15.875** 0.6250	**55.0** 2.17	**58.0** 2.28
25.400 1.0000	**63.500** 2.5000	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15250**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.3** 0.05	**15.875** 0.6250	**56.0** 2.20	**59.0** 2.32
25.400 1.0000	**63.500** 2.5000	**20.638** 0.8125	**12100** 2730	**7280** 1640	**1.67**	**−5.8** −0.23	**15101**	**15250X**	**0.8** 0.03	**20.638** 0.8125	**32.5** 1.28	**31.5** 1.24	**1.5** 0.06	**15.875** 0.6250	**55.0** 2.17	**59.0** 2.32
25.400 1.0000	**64.292** 2.5312	**21.433** 0.8438	**14500** 3250	**13500** 3040	**1.07**	**−3.3** −0.13	**M86643**	**M86610**	**1.5** 0.06	**21.433** 0.8438	**38.0** 1.50	**36.5** 1.44	**1.5** 0.06	**16.670** 0.6563	**54.0** 2.13	**61.0** 2.40

(continued on next page)

bore d	outside diameter D	width T	rating at 500 rpm for 3000 hours L_{10}: one-row radial N lbf	rating at 500 rpm for 3000 hours L_{10}: thrust N lbf	factor K	eff. load center a②	part numbers: cone	part numbers: cup	cone: max shaft fillet radius R①	cone: width B	cone: backing shoulder diameters d_b	cone: backing shoulder diameters d_a	cup: max housing fillet radius r①	cup: width C	cup: backing shoulder diameters D_b	cup: backing shoulder diameters D_a
26.975 1.0620	58.738 2.3125	19.050 0.7500	11600 2610	6560 1470	1.77	−5.8 −0.23	1987	1932	0.8 0.03	19.355 0.7620	32.5 1.28	31.5 1.24	1.3 0.05	15.080 0.5937	52.0 2.05	54.0 2.13
† 26.988 † 1.0625	50.292 1.9800	14.224 0.5600	7210 1620	4620 1040	1.56	−3.3 −0.13	L44649	L44610	3.5 0.14	14.732 0.5800	37.5 1.48	31.0 1.22	1.3 0.05	10.668 0.4200	44.5 1.75	47.0 1.85
† 26.988 † 1.0625	60.325 2.3750	19.842 0.7812	11000 2480	6550 1470	1.69	−5.1 −0.20	15580	15523	3.5 0.14	17.462 0.6875	38.5 1.52	32.0 1.26	1.5 0.06	15.875 0.6250	51.0 2.01	54.0 2.13
† 26.988 † 1.0625	62.000 2.4409	19.050 0.7500	12100 2730	7280 1640	1.67	−5.8 −0.23	15106	15245	0.8 0.03	20.638 0.8125	33.5 1.32	33.0 1.30	1.3 0.05	14.288 0.5625	55.0 2.17	58.0 2.28
† 26.988 † 1.0625	66.421 2.6150	23.812 0.9375	18400 4140	8000 1800	2.30	−9.4 −0.37	2688	2631	1.5 0.06	25.433 1.0013	35.0 1.38	33.0 1.30	1.3 0.05	19.050 0.7500	58.0 2.28	60.0 2.36
28.575 1.1250	56.896 2.2400	19.845 0.7813	11600 2610	6560 1470	1.77	−5.8 −0.23	1985	1930	0.8 0.03	19.355 0.7620	34.0 1.34	33.5 1.32	0.8 0.03	15.875 0.6250	51.0 2.01	54.0 2.11
28.575 1.1250	57.150 2.2500	17.462 0.6875	11000 2480	6550 1470	1.69	−5.1 −0.20	15590	15520	3.5 0.14	17.462 0.6875	39.5 1.56	33.5 1.32	1.5 0.06	13.495 0.5313	51.0 2.01	53.0 2.09
28.575 1.1250	58.738 2.3125	19.050 0.7500	11600 2610	6560 1470	1.77	−5.8 −0.23	1985	1932	0.8 0.03	19.355 0.7620	34.0 1.34	33.5 1.32	1.3 0.05	15.080 0.5937	52.0 2.05	54.0 2.13
28.575 1.1250	58.738 2.3125	19.050 0.7500	11600 2610	6560 1470	1.77	−5.8 −0.23	1988	1932	3.5 0.14	19.355 0.7620	39.5 1.56	33.5 1.32	1.3 0.05	15.080 0.5937	52.0 2.05	54.0 2.13
28.575 1.1250	60.325 2.3750	19.842 0.7812	11000 2480	6550 1470	1.69	−5.1 −0.20	15590	15523	3.5 0.14	17.462 0.6875	39.5 1.56	33.5 1.32	1.5 0.06	15.875 0.6250	51.0 2.01	54.0 2.13
28.575 1.1250	60.325 2.3750	19.845 0.7813	11600 2610	6560 1470	1.77	−5.8 −0.23	1985	1931	0.5 0.03	19.355 0.7620	34.0 1.34	33.5 1.32	1.3 0.05	15.875 0.6250	52.0 2.05	55.0 2.17

3. Design of lubrication and journal bearings

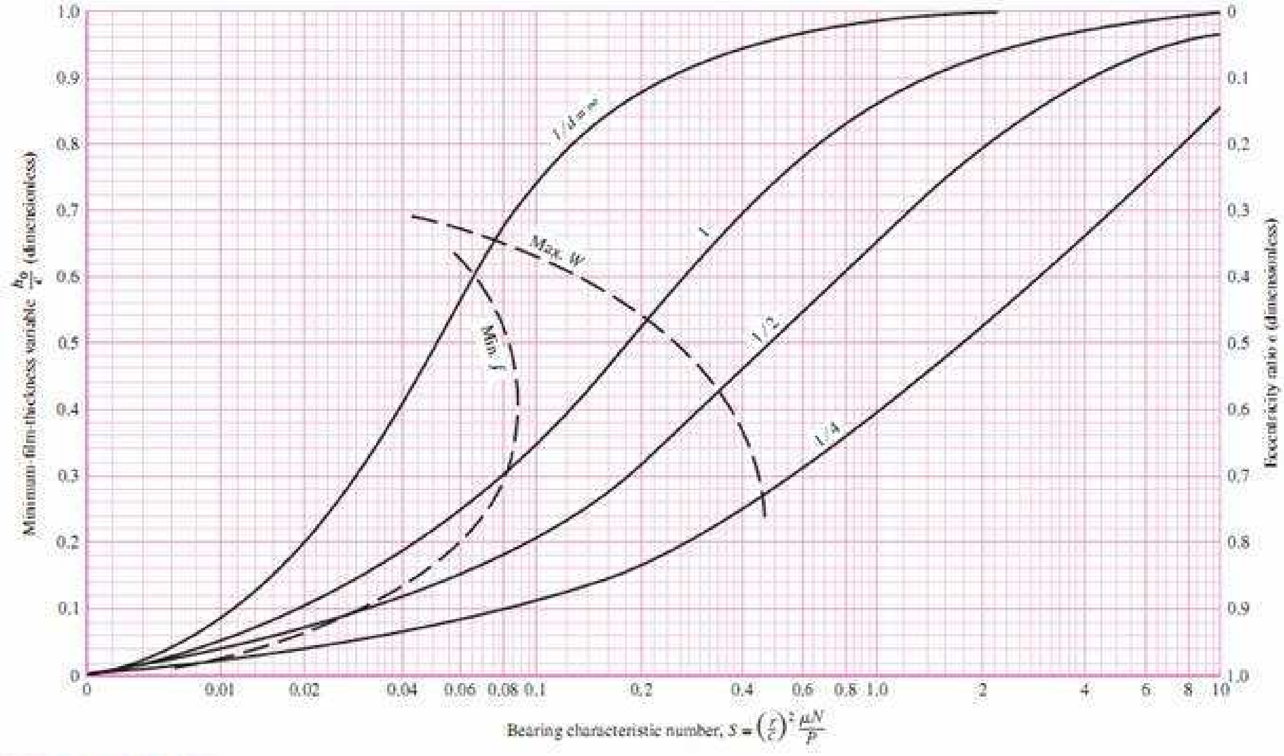


Figure 3-1: Chart for minimum film-thickness variable and eccentricity ratio. The left boundary of the zone defines the optimal h0 for minimum friction; the right boundary is optimum h_0 for load. (Raimondi and Boyd.)

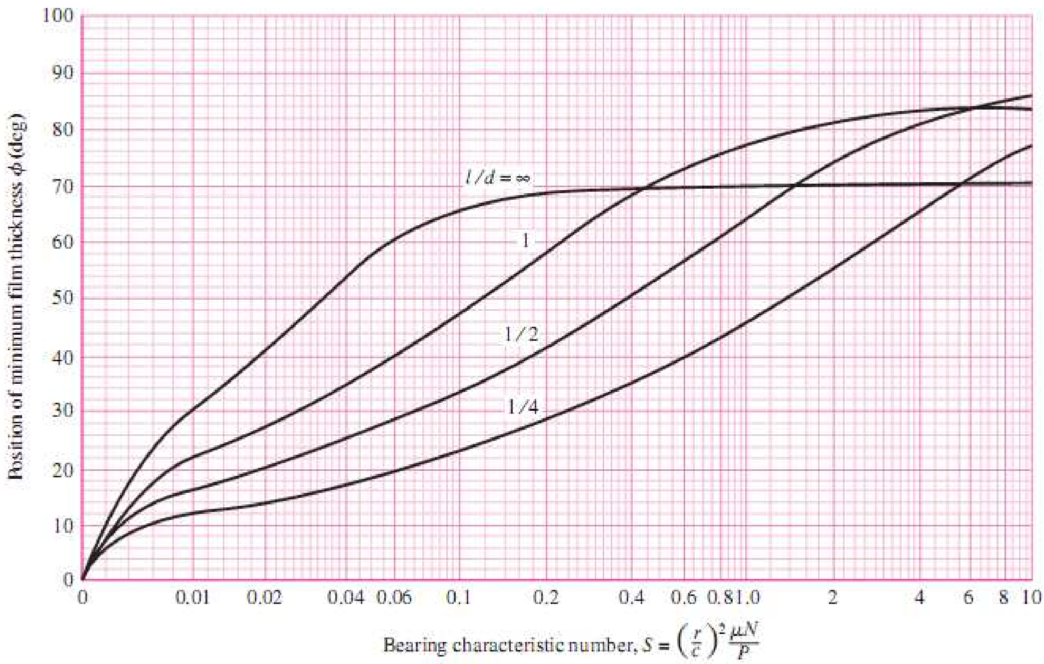


Figure 3-2: Chart for determining the position of the minimum film thickness h_0. (Raimondi and Boyd)

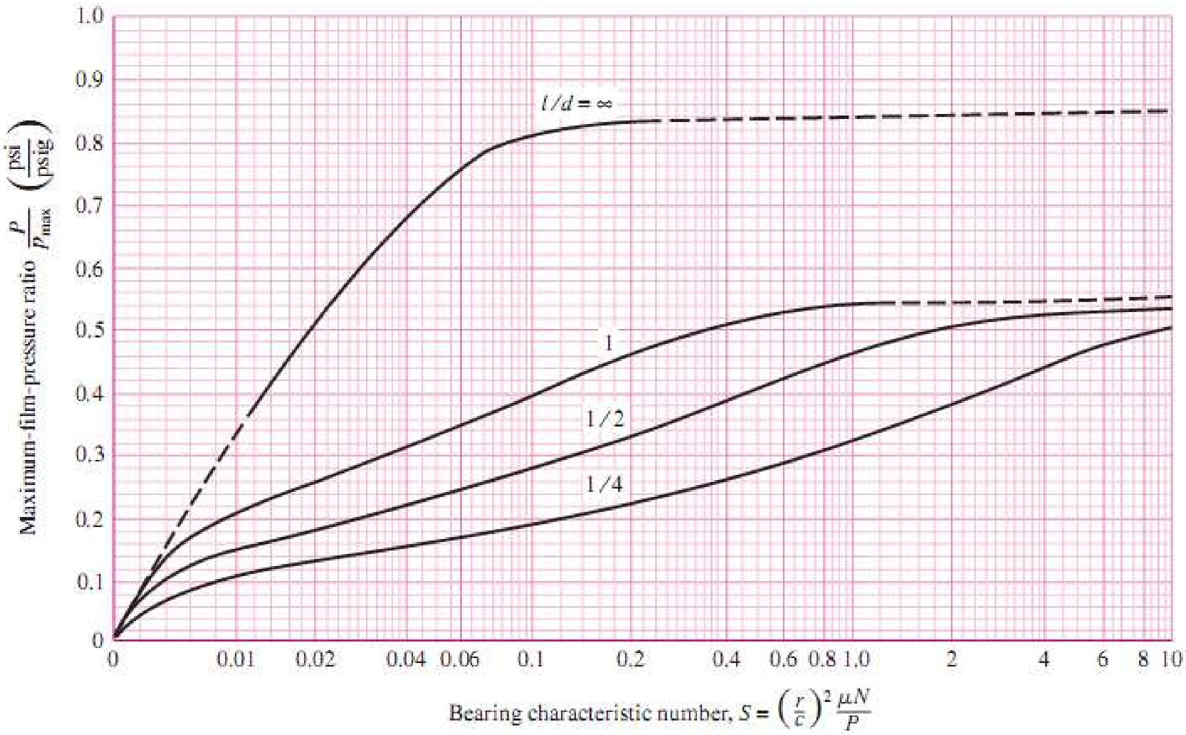


Figure 3-3: Chart for determining the maximum film pressure. Note: not for the pressure fed bearings. (Raimondi and Boyd).

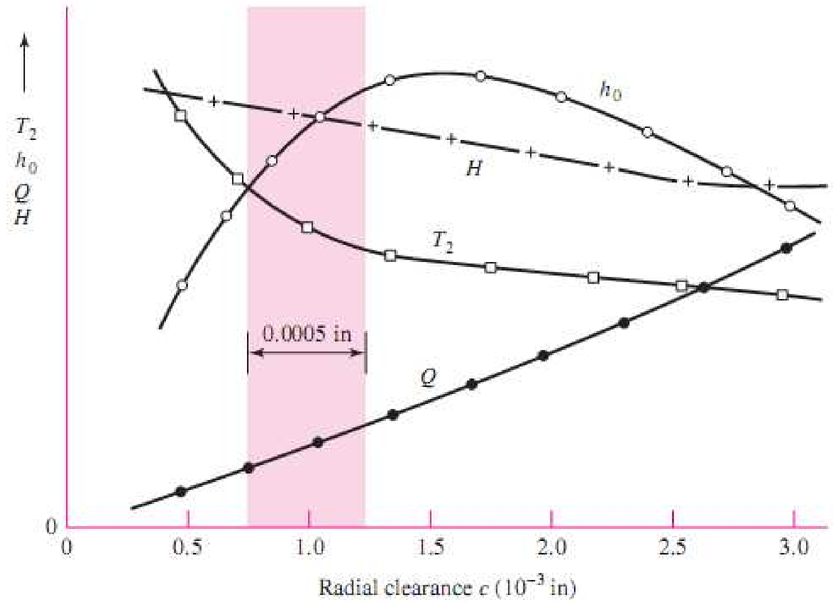


Figure 3-4: A plot of some performance characteristics of the bearings of Exs.12-1 to 12-4 for radial clearances of 0.0005 to 0.0003 in. The bearing outlet temperature is designated T2. New bearings should be designed for the shaded zone, because wear will move the operating point to the right.

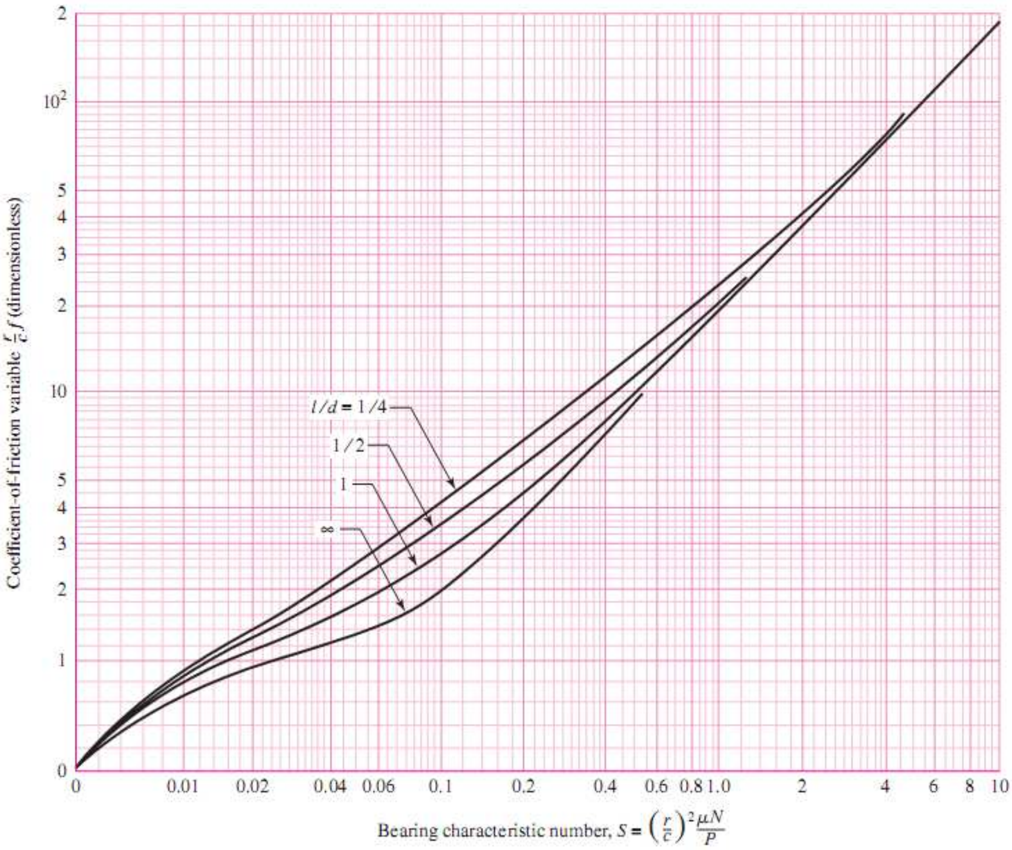


Figure 3-5: Chart for coefficient of friction variable, note that Petroff's equation is the asymptote (Raymondi and Boyd).

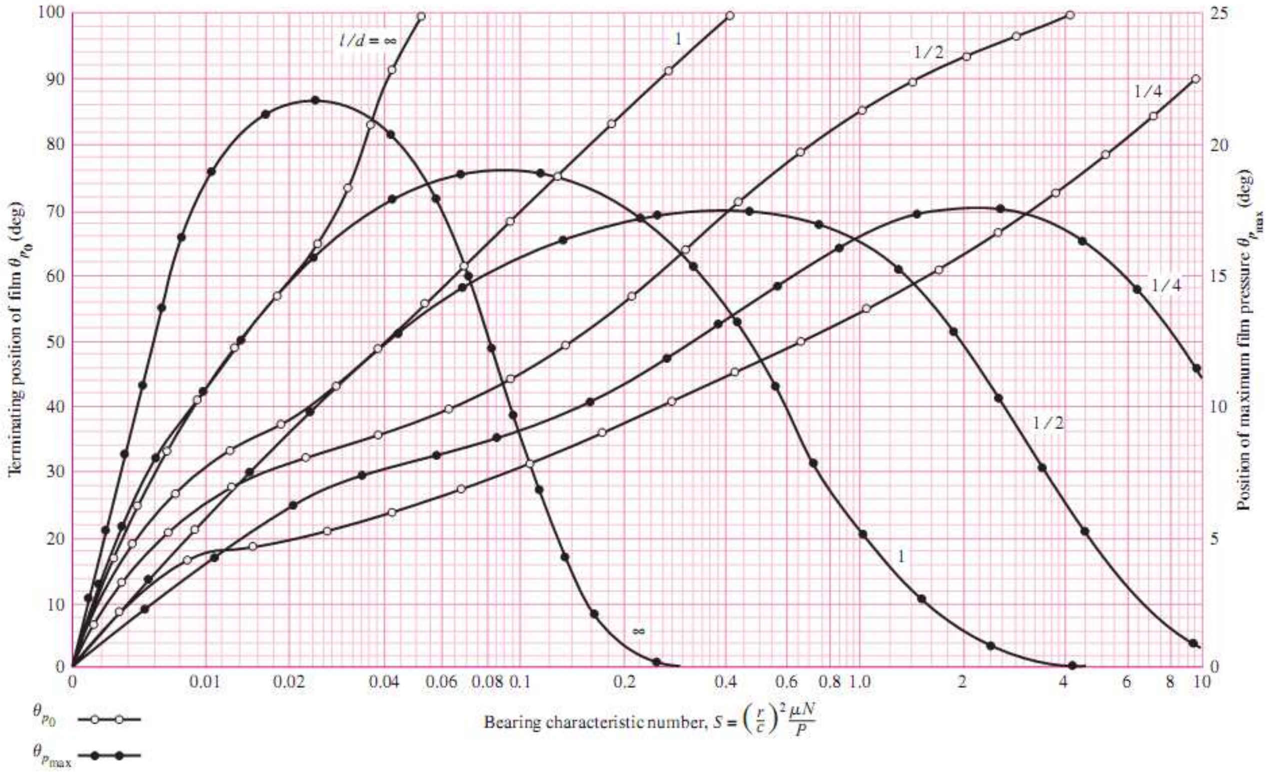


Figure 3-6: Chart for finding the terminating position of the lubricant film and the position of maximum film pressure. (Raimondi and Body.)

Basic design equations

3.1 The nominal bearing pressure (in projected area of the journal)

$$P = \frac{W}{2r\ell}$$

W – bearing force (N); r – radius of the shaft; ℓ - length of the bearing

3.2 Sommerfeld number

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu N}{P}\right)$$

c- clearance; N – shaft rotation (rev/sec), μ – viscosity

3.3 The friction torque on the journal

$$T = fWr$$

f- coefficient of friction

3.4 The power loss, P = 2 Л N T

4. Design of flat belts

Table 4-1: Properties of some Flat and Rounded-Belt Materials (Diameter=d, thickness=t, width=w)

Material	Specification	Size, mm	Minimum Pulley Diameter, mm	Allowable Tension per unit Width of 3m/s, (10^3)N/mm	Specific Weight, KN/m^3	Coefficient of Friction
Leather	1 ply	t=4.5	75	5	9.5-12.2	0.4
		t=5	90	6	9.5-12.2	0.4
	2 ply	t=7	115	7	9.5-12.2	0.4
		t=8	150	9	9.5-12.2	0.4
		t=9	230	10	9.5-12.2	0.4
Polyamide[b]	F-0[c]	t=0.8	15	1.8	9.5	0.5
	F-1[c]	t=1.3	25	6	9.5	0.5
	F-2[c]	t=1.8	60	10	13.8	0.5
	A-2[c]	t=2.8	60	10	10.0	0.8
	A-3[c]	t=3.3	110	18	11.4	0.8
	A-4[c]	t=5.0	240	30	10.6	0.8
	A-5[c]	t=6.4	340	48	10.6	0.8
Urethane[d]	w=12.7	t=1.6		1.0[e]	10.3-12.2	0.7
	w=19	t=2.0	See Table 4-2	1.7[e]	10.3-12.2	0.7
	w=32	t=2.3		3.3[e]	10.3-12.2	0.7
	Round	d=6		1.4[e]	10.3-12.2	0.7
		d=10		3.3[e]	10.3-12.2	0.7
		d=12	See Table 4-2	5.8[e]	10.3-12.2	0.7
		d=20		13[e]	10.3-12.2	0.7

[a]Add 2 in to pulley size for belts 8 in wide or more
[b]Source: Habasit Engineering Manual, Habasit Belting, Inc., Chamblee (Atlanta), Ga
[c]Friction cover of acrylonitrile –butadiene rubber on both sides
[d]Source: Eagle Belting Co. Des Plaines, III
[e]At 6% elongation: 12% is maximum allowable value

Table 4-2: Minimum Pulley Sizes for Flat and Round Urethane Belts (Listed are Pulley Diameters in mm)

		Ratio of Pulley Speed to Belt Length, rev/(m.s)		
Belt Style	**Belt Size,mm**	**Up to 14**	**14 to 27**	**28-55**
Flat	12.7*1.6	9.7	11.2	12.7
	19*2.0	12.7	16	19
	32*2.3	12.7	16	19
Round	6	38.1	44.5	50.8
	10	57.1	66.5	76.2
	12	76.2	88.9	101.6
	20	127	152	177.8

Table 4-3: Pulley Correction Factor C_p for Flat Belts*

Material	**40-100**	**115-200**	**220-310**	**355-405**	**460-800**	**Over 800**
Leather	0.5	0.6	0.7	0.8	0.9	1.0
PolyamideF-0	0.95	1.0	1.0	1.0	1.0	1.0
PolyamideF-1	0.70	0.92	0.95	1.0	1.0	1.0
PolyamideF-2	0.73	0.86	0.96	1.0	1.0	1.0
PolyamideA-2	0.73	0.86	0.96	1.0	1.0	1.0
PolyamideA-3	-	0.70	0.87	0.94	0.96	1.0
PolyamideA-4	-	-	0.71	0.80	0.85	0.92
PolyamideA-5	-	-	-	0.72	0.77	0.91

*Average Value of C_p for the given ranges was approximated from curves in Habasit Engineering manual, Habasit Belting. Inc. Chamblee(Atlanta), Ga.

Table 4-4: Crown Height and ISO Pulley Diameter for Flat Belts*

ISO Crown Pulley Diameter, mm	ISO Height, mm	Pulley Diameter, mm	Crown Height ,in W≤250mm	W>250
40, 50, 62	0.3	315, 355	0.75	0.75
70, 80	0.3	315, 355	1.0	1.0
90, 100, 115	0.3	570, 635, 710	1.3	1.3
125, 142	0.4	800, 900	1.3	1.5
160, 180	0.5	1015	1.3	1.5
200, 230	0.6	1140, 1270, 1420	1.5	2.0
250,285	0.75	1600, 1800, 2030	1.8	2.5

*crown should be rounded, not angled; maximum roughness is R_a=AA 1500 µmm.

4.1 Design steps:

The steps in analyzing a flat belt drive can include

4.1.1 Find $\exp(f\phi)$ from belt-drive geometry and friction.

4.1.2 From belt geometry and speed find F_c.

4.1.3 From $T = H_{nom}K_s n_d / (2\pi n)$ find the necessary torque..

4.1.4 From torque T find the necessary $(F_1)_a - F_2 = 2T/D$.

4.1.5 From data book, determine $(F_1)_a$.

4.1.6 Find F_2 from $(F_1)_a - [(F_1)_a - F_2]$.

4.1.7 Find the necessary initial tension from, $F_i = \dfrac{F_1 + F_2}{2} - F_c$.

4.1.8 Check the friction development, $f' < f$, with $f' = \frac{1}{\phi}\ln\frac{(F_1)_a - F_c}{F_2 - F_c}$

4.1.9 Find the factor of safety from $n_{fs} = H_a / (H_{nom} K_s)$

4.1.10 Open-belt drive: contact angles:

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

where D = diameter of large pulley

d = diameter of small pulley

C = center distance

θ = angle of contact

4.1.11 Open-belt drive: Belt length

$$L = \left[4C^2 - (D-d)^2\right]^{1/2} + \frac{1}{2}(D\theta_D + d\theta_d)$$

4.1.12 Cross-belt drive: contact angles , the angle of wrap is the same for both pulleys

$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

4.1.13 Cross-belt drive: Belt length

$$L = \left[4C^2 - (D-d)^2\right]^{1/2} + \frac{1}{2}(D+d)\theta$$

4.2 Important formulas::

4.2.1 belt speed, V = Лdn (m/s)

4.2.2 weight w of a meter of belt

4.2.3 w = γbt

4.2.4 γ – weight density (N/m^3); b – belt width; t - belt thickness

4.2.5 centrifugal force , $F_c = (wV^2)/g$

4.2.6 Torque, T = (H_{nom} K_s n_d)/(2Лn)

4.2.7 H_{nom} – power, K_s – shock factor or service factor; n_d – design or safety factor; n –

4.2.8 rpm of driving pulley

4.2.9 $(F_1)_a - F_2 = 2T/d$

4.2.10 T – torque; d - diameter of driving pulley

4.2.11 $F_1 = (F_1)_a = b\ F_a\ C_p\ C_v$

4.2.12 F_a – manufacturer's allowed tension (from table 17-2) ; C_p – pulley correction factor (Table 17-4); C_v – velocity correction factor.

For polyamide, $C_v = 1$.

4.2.13 $F_2 = (F_1)_a - [(F_1)_a - F_2]$

4.2.14 $F_i = \frac{F_1 + F_2}{2} - F_c$

4.2.15 Allowable power or design power

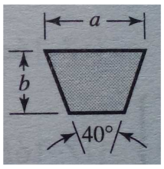
$$H_a = H_{nom} \times K_s \times n_d$$

4.2.16 check for friction development, $f' < f$, $f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c}$

4.2.17 factor safety ; $n_{fs} = \frac{H_a}{H_{nom} K_s}$

5. Design of V belts

Table 5-1: Standard V-Belt Sections



Belt Section	Width a, mm	Thickness b, mm	Minimum sheave Diameter, mm	KW Range, One or More Belts
A	12	8.5	75	0.2-7.5
B	16	11	135	0.7-18.5
C	22	13	230	11-75
D	30	19	325	37-186
E	38	25	540	75 and up

Table 5-2: Inside Circumferences of Standard V Belts.

Section	Circumference, mm
A	650, 775, 825, 875, 950, 1050, 1150, 1200, 1275, 1325, 1375, 1425, 1500, 1550, 1600, 1700, 1775, 1875, 1950, 2000, 2125, 2250, 2400, 2625, 2800, 3000, 3200
B	875, 950, 1050, 1150, 1200, 1275, 1325, 1375, 1425, 1500, 1550, 1600, 1650, 1700, 1775, 1875, 1950, 2000, 2125, 2250, 2400, 2625, 2800, 3000, 3200, 3275, 3400, 3450, 3950, 4325, 4500, 4875, 5250, 6000, 6750, 7500
C	1275, 1500, 1700, 1875, 2025, 2125, 2250, 2400, 2625, 2800, 3000, 3200, 3400, 3600, 3950, 4050, 4350, 4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10500
D	3000, 3200, 3600, 3950, 4050, 4350, 4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10 500, 12000, 13500, 15000, 16500
E	4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10500, 12000, 13500, 15000, 16500

Table 5-3: Length Conversion Dimensions (Add the Listed Quantities to the Inside Circumference to Obtain the Pitch Length in mm)

Belt Section	A	B	C	D	E
Quantity to be added	32	45	72	82	112

Table 5-4: Power (kW) Ratings of Standard V Belts.

		Belt Speed (m/s)				
Belt Section	**Sheave Pitch Diameter, mm**	**5**	**10**	**15**	**20**	**25**
A	65	0.35	0.46	0.40	0.11	
	75	0.49	0.75	0.84	0.69	0.28
	85	0.60	0.98	1.17	1.64	0.84
	95	0.69	1.16	1.43	1.49	1.28
	105	0.77	1.30	1.64	1.78	1.63
	115	0.83	1.41	1.82	2.01	1.93
	125 and up	0.83	1.51	1.97	2.21	2.16
B	105	0.80	1.51	1.97	2.21	2.16
	115	0.95	1.18	1.25	0.94	0.16
	125	1.07	1.48	1.71	1.55	0.92
	135	1.19	1.74	2.09	2.06	1.57
	145	1.28	1.95	2.42	2.49	2.10
	155	1.36	2.14	2.69	2.87	2.57
	165	1.43	2.31	2.94	3.19	2.98
	175 and up	1.50	2.45	3.16	3.48	3.34
C	150	1.37	1.98	2.03	1.40	
	175	1.85	2.94	3.46	3.31	2.33
	200	2.21	3.66	4.54	4.74	4.12
	225	2.49	4.21	5.38	5.36	5.51
	250	2.72	4.66	6.05	7.16	6.63
	275	2.89	5.03	6.59	7.46	7.53
	300 and up	3.05	5.33	7.06	8.13	8.28
D	250	3.09	4.57	4.89	3.80	1.01
	275	3.73	5.84	6.80	6.34	4.19
	300	4.26	6.91	8.36	8.50	6.85
	325	4.71	7.83	9.70	10.30	9.10
	350	5.09	8.58	10.89	11.79	11.04
	375	5.42	9.25	11.86	13.13	12.68
	400	5.71	9.85	12.76	14.32	14.17
	425 and up	5.98	10.37	13.50	15.37	15.44
E	400	6.48	10.44	13.06	13.50	11.41
	450	7.40	12.46	15.82	17.16	16.04
	500	8.13	13.95	18.05	20.07	19.69
	550	8.73	15.14	19.84	22.53	22.75
	600	9.25	16.11	21.34	24.54	25.22
	650	9.70	17.01	22.60	26.19	27.38
	700 and up	10.00	17.68	23.72	27.68	29.17

Table 5-5: Angle of Contact Correction Factor K_1 for VV* and V-Flat Drives.

		K_1	
(D-d)/C	**θ, deg**	**VV**	**V Flat**
0.00	180	1.00	0.75
0.10	174.3	0.99	0.76
0.20	166.5	0.97	0.78
0.30	162.7	0.96	0.79
0.40	156.9	0.94	0.80
0.50	151.0	0.93	0.81
0.60	145.1	0.91	0.83
0.70	139.0	0.89	0.84
0.80	132.8	0.87	0.85
0.90	126.5	0.85	0.85
1.00	120.0	0.82	0.82
1.10	113.3	0.80	0.80
1.20	106.3	0.77	0.77
1.30	98.9	0.73	0.73
1.40	91.1	0.70	0.70
1.50	82.8	0.65	0.65

*A curve fit for the VV column in terms of θ is

$$K_1 = 0.143543 + 0.0074680\theta - 0.0000150520\theta^2$$

In the range $90° \le \theta \le 180°$

Table 5-6: Belt-Length Correction Factor K_2^*.

	Nominal Belt Length, (m)				
Length Factor	**A Belts**	**B Belts**	**C belts**	**D Belts**	**E Belts**
0.85	Up to 0.88	Up to 1.15	Up to 1.88	Up to 3.2	
0.9	0.95-1.15	1.2-1.5	2.03-2.4	3.6-4.05	Up to 4.88
0.95	1.2-1.38	1.55-1.88	2.63-3.0	4.33-5.25	5.25-6.0
1	1.5-1.88	1.95-2.43	3.2-3.95	6	6.75-7.5
1.05	1.95-2.25	2.63-3.0	4.05-4.88	6.75-8.25	8.25-9.75
1.1	2.4-2.8	3.2-3.6	5.25-6.0	9.0-10.5	10.5-12.0
1.15	3.0 and up	3.95-4.5	6.75-7.5	12	13.5-15.0
1.2		4.88 and up	8.25 and up	13.5 and up	16.5

Table 5-7: Suggested Service Factors Ks for V-Belt Drives.

	Source Of Power	
Driven Machinery	Normal Torque Characteristic	High Or Nonuniform Torque
Uniform	1.0-1.2	1.1-1.3
Light shock	1.1-1.3	1.2-1.4
Medium shock	1.2-1.4	1.4-1.6
Heavy shock	1.3-1.5	1.5-1.8

Table 5-8: Some V-belt parameters*

Belt Section	K_b	K_c
A	220	0.561
B	576	0.965
C	1600	1.716
D	5680	3.498
E	10850	5.041
3V	230	0.425
5V	1098	1.217
8V	4830	3.288

Table 5-9: Durability Parameters for Some V-Belts Sections

Source: M.E. Spotts, Design of Machine Elements, 6th ed. Prentice Hall, Englewood Cliffs, N.J., 1985

Belt Section	10^8 to 10^9 Force Peaks		10^8 to 10^9 Force Peaks		Minimum Sheave Diameter, mm
	K	b	K	b	
A	2999	11.089			75
B	5309	10.926			125
C	9069	11.173			215
D	18 726	11.105			325
E	26 791	11.100			540
3V	3240	12.464	4726	10.153	66
5V	7360	12.593	10 653	10.293	177
8V	16 189	12.629	23 376	10.319	312

Design Equations:

5.1 Pitch length , $L_p = 2C + \pi(D+d)/2 + (D-d)^2/(4C)$

5.2 Center to center distance, $C = 0.25\left\{\left[L_p - \frac{\pi}{2}(D+d)\right] + \sqrt{\left[L_p - \frac{\pi}{2}(D+d)\right]^2 - 2(D-d)^2}\right\}$

where D = pitch diameter of the large sheave and d = pitch diameter of the small sheave.

5.3. Allowable power , $H_a = K_1 K_2 H_{tab}$

where K_1 = angle of wrap correction factor

K_2 = belt length correction factor

5.4 $\frac{F_1 - F_c}{F_2 - F_c} = \exp(0.5123\phi)$

5.5 Design power, $H_d = H_{nom} K_s n_d$

where H_{nom} = nominal power,

K_s = service factor

N_d = design factor

N_b = number of belts (usually the next higher integer to Hd/Ha)

That is, $N_b \geq \frac{H_d}{H_a} \qquad N_b = 1, 2, 3...$

Designers work on a per-belt basis:

5.6 Centrifugal tension Fc is given by

$$F_c = K_c \left(\frac{V}{2.4} \right)^2 \qquad \text{V in m/s}$$

where K_c is selected from table.

5.7 The power that is transmitted per belt is based on $\Delta F - F_1 - F_2$, where

$$\Delta F = \frac{H_d / N_b}{\pi n d}$$

5.8 The largest tension, $F_1 = F_c + \frac{\Delta F \exp(f\phi)}{\exp(f\phi) - 1}$

5.9 The least tension, $F_2 = F_1 - \Delta F$

5.10 Initial tension, $F_i = \frac{F_1 + F_2}{2} - F_c$

5.11 The factor of safety, $n_{fs} = \frac{H_a + N_b}{H_{nom} K_s}$

5.12 Life

Durability (life) correlations are complicated by the fact that the bending induces flexural stresses in the belt; the corresponding belt tension that induces the same maximum tensile stress is F_{b1} at the driving heave and F_{b2} at the driven pulley. These equivalent tensions are added to F_1 as

$$\text{In SI units,} \quad T_1 = F_1 + (F_b)_1 = F_1 + \frac{4.45}{39.37} \frac{K_b}{d}$$

$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{4.45}{39.37}\frac{K_b}{D}$$

where K_b is given in Table. The equation for the tension versus pass trade-off used by the Gates Rubber Company is of the form

$$T^b N_P = K^b$$

where Np = the number of passes

b = approximately 11, see Table 5-9.

The Miner rule is used to sum damage incurred by the two tension peaks as

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1}$$

The lifetime in hours , $t = \frac{N_p L_P}{3600}$

The constants K and b have their ranges of validity. If $N_p > 10^9$, report that $N_p = 10^9$.

And $t > \frac{N_p L_P}{3600V}$ without placing confidence in numerical values beyond the validity interval.

Summary of Design Steps

The analysis of a V-belt drive can consist of the following steps:

- Find V, L_p, C, ϕ and $\exp(0.5123\phi)$
- Find H_d, H_a, and N_b from H_d / H_a and round up
- Find $F_c, \Delta F, F_1, F_2$, and F_i, and n_{fs}
- Find belt life in number of passes, or hours, if possible

6. Design of Gears

6.1 Gear train

Pinion and driven gears are represented as

Pinion – 2

Driven gear – 3

The speed of the driven gear is

$$n_3 = \left|\frac{N_2}{N_3} n_2\right| = \left|\frac{d_2}{d_3} n_2\right|$$

where,

n = revolutions or rev/min

N = number of teeth

D = pitch diameter

Train value,

$$e = \frac{\text{product of driving tooth number}}{\text{product of driven tooth number}}$$

Finally,

$$n_L = e n_F$$

where,

n_L – the speed of the last gear in the train

n_F – the speed of the first.

6.2 Force Analysis – Spur Gearing

6.2.1 Notations: Gears

1- the frame of the machine

2 – input gear or pinion

3, 4, etc – succeeding gears

6.2.2 Notations: Shaft

Using lowercase letters of the alphabet a, b, c, etc.

6.2.3 Notations: Force exerted

F_{23} – the force exerted by gear 2 against gear 3.

F_{2a} - the force of gear 2 against a shaft ‘a’.

F_{a2} - the force of a shaft ‘a’ against gear 2.

6.2.4 Notations: Coordinates

x, y ,z coordinates

6.2.5 Notations: radial and tangential direction (Superscripts)

r - radial direction

t - tangential direction

F_{43}^{t} - the tangential component of the force of gear 4 acting against gear 3.

6.2.6 Notations: Torque

T_{a2} - The torque of a shaft 'a' against gear 2.

- Pinion mounted on a shaft 'a' rotating clockwise at n_2 rev/min.
- Driving a gear on shaft b at n3 rev/min.
- The reaction between the mating teeth occur along the pressure line.

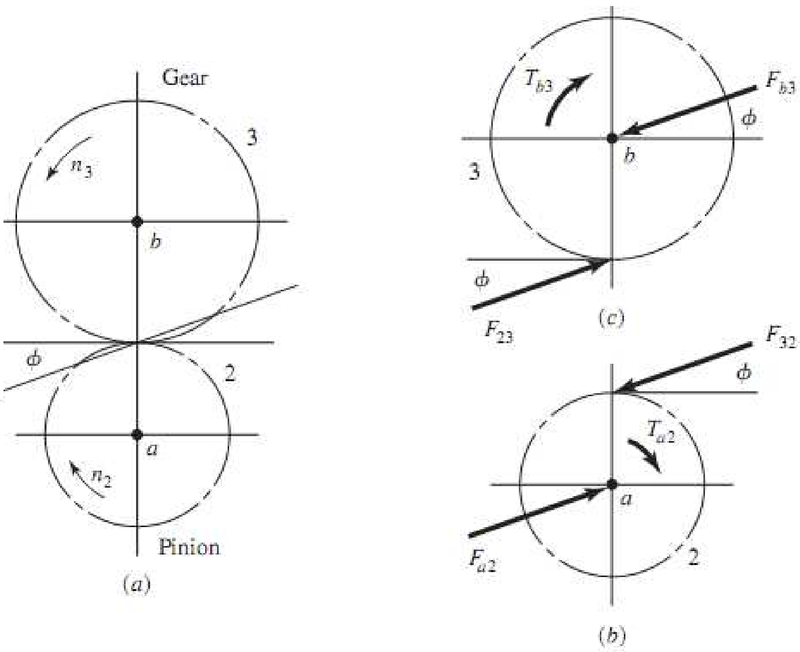


Figure 6-1: freebody diagrams of the forces and moments acting upon two gears of a simple gear train

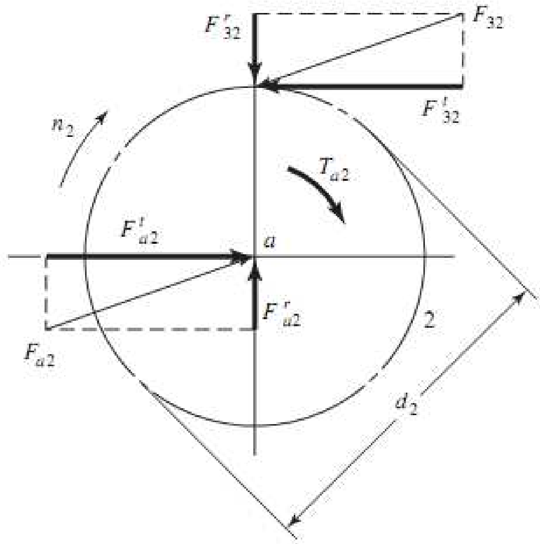


Figure 6-2 : Resolution of gear forces

I. Transmitted load,

$$W_t = F_{32}^t$$

II. Torque

$$T = \frac{d}{2} W_t$$

where, $T = T_{a2}$ and $d = d_2$ to obtain a general relation.

III. Power transmitted, H, through a rotating gear can be obtained

$$H = T\omega = \left(W_t \frac{d}{2} \right) \omega$$

where, ω is the angular veloticity.

IV. Pitch line velocity at radius of pitch circle

$$v = \omega \frac{d}{2}$$

and

$$v = \frac{\pi d n}{60} (m/s)$$

where, d = gear diameter, m
n = gear speed, rev/min

V. Transmitted load

$$W_t = 60000 \frac{H}{\pi d n}$$

where,
W_t = transmitted load, kN, H = power, kW, d = gear diameter, mm, n = speed, rev/min

6.3 Bending of spur gear

I. Velocity,

$$\begin{aligned} v &= \pi d n (m/s) \\ &= \pi d_p n_p, \text{pinion} (m/s) \\ &= \pi d_G n_G, \text{gear} (m/s) \end{aligned}$$

II. Transmitted load, $W^t = \frac{H}{V} = \frac{0.7457 \times 1000 \times H(hp)}{V}$

III. K_o, Overload factors

$K_o = 1$, for uniform loading.

Overload factor K_o is allowance for all externally applied loads in excess of the nominal tangential load Wt in a particular applications. Eg: variations in torque from the mean value due to firing of cylinder in an IC engine or reaction to torque variations in a piston pump drive.

Table 6-1: Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate Shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

IV. K_V, Dynamic factor

Dynamic factors are used to account for inaccuracies in the manufacturer and meshing of gear teeth in action. Transmission errors is defined as the departure from uniform angular velocity of the gear pair. Some of the effects that produce transmission error are

- tooth profile: error in tooth spacing, profile lead, and run out.
- Vibration of the tooth during meshing due to the tooth stiffness.
- Magnitude of the pitch line velocity
- Dynamic unbalance of the rotating members
- Wear and permanent deformation of contacting portions of the teeth
- Gearshaft misalignment and the linear and angular deflection of the shaft
- Tooth friction.

$$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B \qquad v \text{ in } (m/s)$$

where,

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_v)^{2/3}$$

Q_v ranges from 6 to 12, with lower numbers representing greater accuracy.

V. K_s, Size factor

The size factor reflects non uniformity of material properties due to size. It depends upon

1. tooth size 2. diameter of part 3. ratio of tooth size to diameter of part.

$$K_s = 0.904\left(bm\sqrt{Y}\right)^{0.0535}$$

If K_s in above equation is less than 1, use K_s = 1.

For Y, Lewis form factor, use the following table.

Table 6-2: Values of the Lewis form factor Y (these values are for a normal pressure angle 200, full depth teeth , and a diameter pitch of unity in the plane of rotation.

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rock	0.485

VI. K_H – load distribution factor

The load distribution factor modified the stress equations to reflect non uniform distribution of load across the line of contact.

$$K_H = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

where

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$

$$C_{pf} = \begin{cases} \dfrac{b}{10d} - 0.025 & b \le 25\text{mm} \\ \dfrac{b}{10d} - 0.0375 + 4.92\left(10^{-4}\right)b & 25 < b \le 425\text{mm} \\ \dfrac{b}{10d} - 0.1109 + 8.15\left(10^{-4}\right)b - 3.53\left(10^{-7}\right)b^2 & 425 < b \le 1000\text{mm} \end{cases}$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle – mounted pinion with } S_1/S < 0.175 \\ & \text{or (bearings immediately adjacent)} \\ 1.1 & \text{for straddle – mounted pinion with } S_1/S \ge 0.175 \end{cases}$$

$$C_{ma} = \left\{ A + Bb + Cb^2 \right.$$

Table 6-3: Empirical constants A, B, C for C_{ma}, face width b in inch

Condition	A	B	C
Open Gearing	0.247	0.0167	$-0.7659(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision , enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extra precision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20-22, for SI formulation

VII. K_B, Rim thickness

Assuming constant thickness gear, the rim thickness factor is

$K_B = 1$

VIII. Y_J = J, Bending strength geometry factor

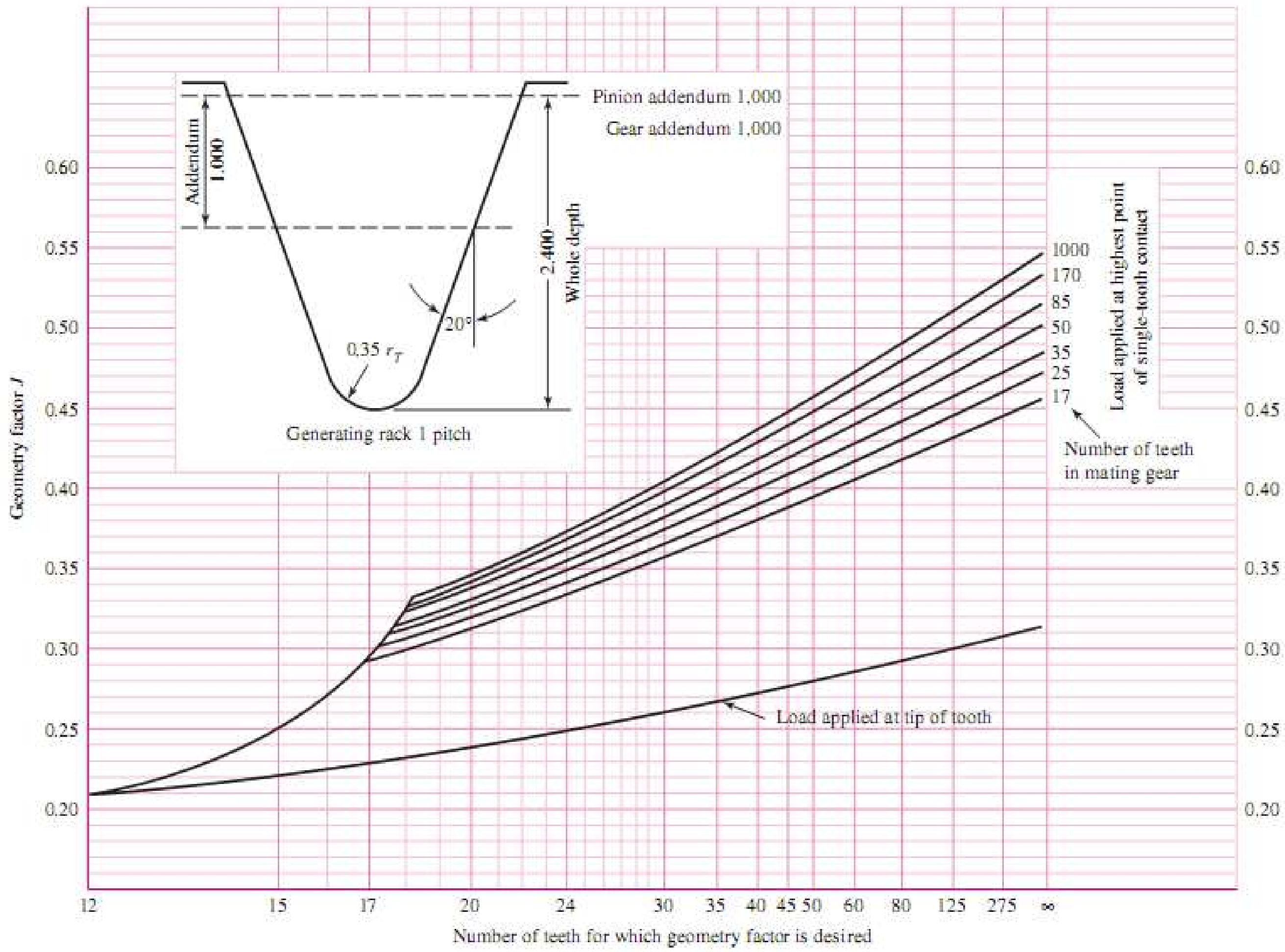


Figure 6-3: Spur gear geometry factors J. Source: The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

IX. Bending stress

$$\sigma = W^t K_o K_V K_s \frac{1}{bm_t} \frac{K_H K_B}{Y_J}$$

6.4 Helical Gearing: Force Analysis

The three dimensional view of the forces acting against a helical gear tooth is shown in figure. Usually tangential component, also called transmitted load, is given and other forces are desired.

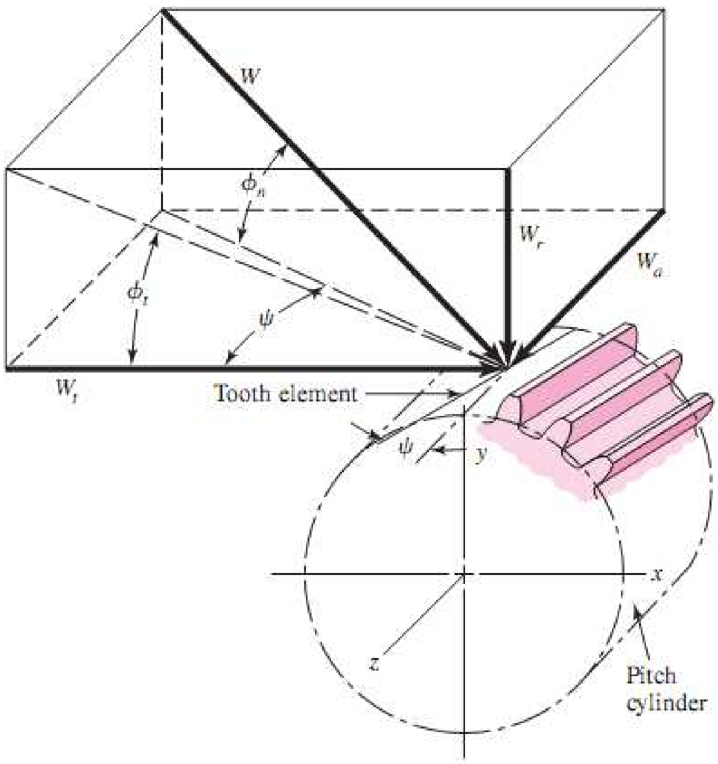


Figure 6-4: 3D forces on a helical gear.

I. Tangential component, $W_t = \dfrac{H}{V}$

where, H = given power, and

II. Pitch line velocity, $V = \pi dn$ in m/s

III. Radial component, $W_r = W_t \tan\phi_t$

IV. Axial component, $W_a = W_t \tan\psi$

V. Total force, $W = \dfrac{W_t}{\cos\phi_n \cos\psi}$

VI. Relation between angles, $\cos\psi = \dfrac{\tan\phi_n}{\tan\phi_t}$

where ψ is helix angle , ϕ_n is normal pressure angle and ϕ_t is the transverse or pressure angle in the direction of rotation.

7. Transverse module, $m_t = \dfrac{m_n}{\cos\psi}$, where m_n is normal module.

6.5 Bevel Gear: Force Analysis

Bevel gears are used to transmit motion between interesting shafts. Although bevel gears are usually made for a shaft angle of 90°, they may be produced for almost any angle.

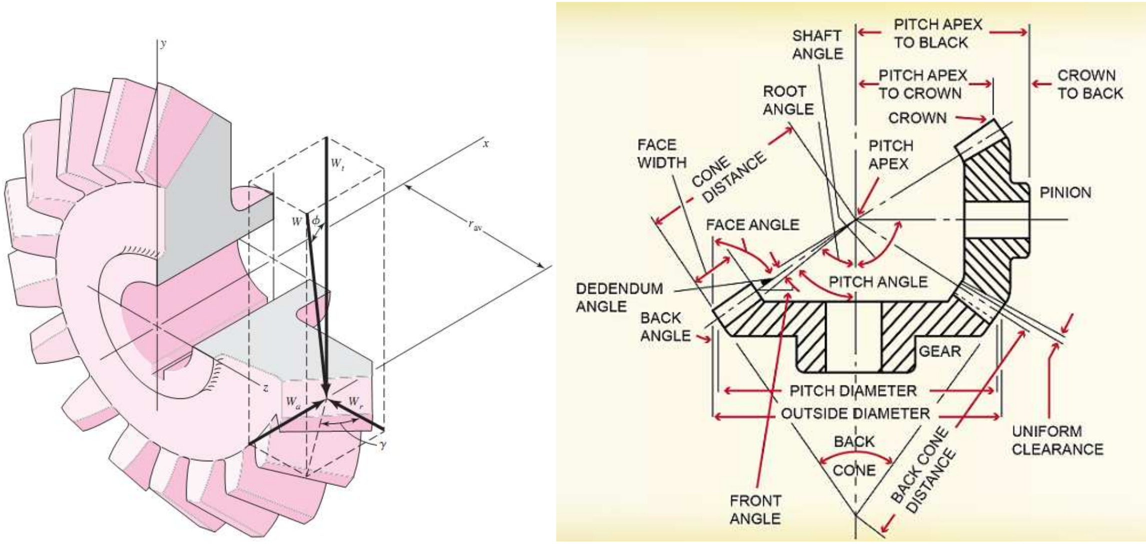


Figure 6-5: Bevel gear tooth forces and terminology.

Pitch angles are related to the tooth numbers as follows:

I. Pinion pitch angel, $\tan\gamma = \dfrac{N_P}{N_G} = \dfrac{\text{Ext diameter of pinion}}{\text{Ext diameter of gear}}$

II. Gear pitch angel, $\tan\Gamma = \dfrac{N_G}{N_P} = \dfrac{\text{Ext diameter of gear}}{\text{Ext diameter of pinion}}$

where N_P is tooth numbers of pinion and N_G is the tooth numbers of gear

III. The transmitted load, $W_t = \dfrac{T}{r_{av}} = \dfrac{H}{V}$

where, T is torque, r_{av} is average pitch radius, H is power, V is pitch line velocity.

IV. Pitch line velocity, $V = \pi dn$ in m/s.

For pinion:

a. Radial force, $W_r = W_t \tan\phi \cos\gamma$

b. Axial force, $W_a = W_t \tan\phi \sin\gamma$

For gear:

a. Radial force, $W_r = W_t \tan\phi \cos\Gamma$

b. Axial force, $W_a = W_t \tan\phi \sin\Gamma$

6.6 Worm Gearing: Force Analysis

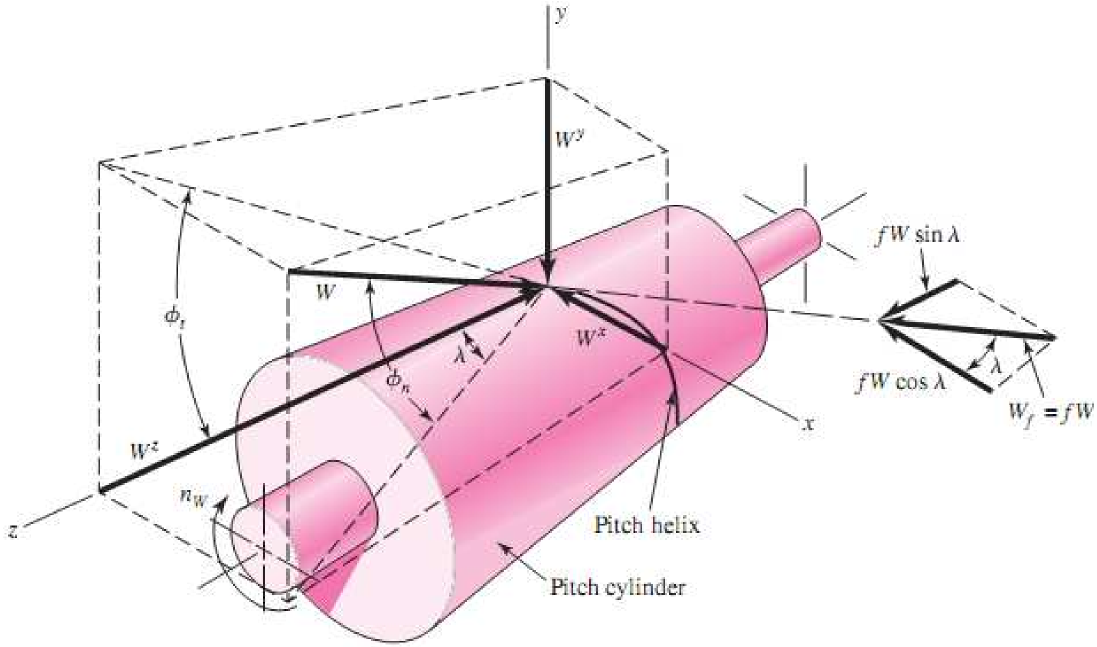


Figure 6-6: Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.

a) Neglecting friction

The force exerted by the gear is the force W. The force W has three orthogonal components

i) $W^x = W\cos\phi_n \sin\lambda$

ii) $W^y = W\sin\phi_n$

iii) $W^z = W\cos\phi_n \cos\lambda$

b) Using subscripts
W – Worm and
G - Gear

Forces acting against the worm and gear are

i) $W_{wt} = -W_{Ga} = W^x$

ii) $W_{wr} = -W_{Gr} = W^y$

iii) $W_{wa} = -W_{Gt} = W^z$

c) Including friction at the contact point

i) $W^x = W(\cos\phi_n \sin\lambda + f\cos\lambda)$

ii) $W^y = W\sin\phi_n$

iii) $W^z = W(\cos\phi_n \cos\lambda - f\sin\lambda)$

d) Transverse circular pitch, $p_t = \pi m_t = \dfrac{\pi}{P}$

e) Axial pitch, $p_x = p_t$

f) Lead angle, $\lambda = \tan^{-1}\dfrac{L}{\pi d_w}$

g) Sliding Velocity, $V_s = \dfrac{V_w}{\cos\lambda}$

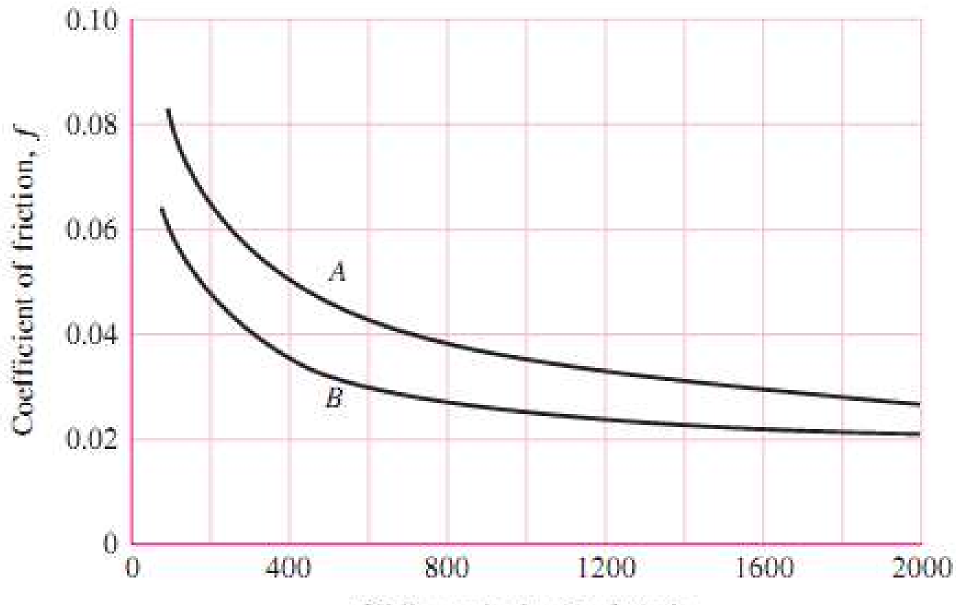


Figure 6-7: Representative values of the coefficient for worm gearing. These values are based on good lubrication. Use curve B for high quality materials, such as a case hardened steel worm mating with a phosphor- bronze gear. Use curve A when more friction is expected, as with cast-iron worm mating with a cast-iron worm gear.

6.7 Safety factor

6.7.1 Bending fatigue failure, S_F

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} = \text{fully corrected bending strength}$$

where

a) S_t - allowable bending stress

i) Use following table and figure for determining S_t

$$S_t = 0.533 H_B + 88.3 \text{ MPa}, \text{ grade } 1$$

$S_t = 0.703 H_B + 113 \text{ MPa}, \text{ grade } 2$

Table 6-4: Repeatedly applied bending strength S_t at 107 cycles and 0.99 reliability for steel. gears

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number Grade 1	Grade 2	Grade 3
Steel [3]	Through hardened[4]	See Fig. 6-8	See Fig. 6-8	See Fig. 6-8	_
	Flame[4] or induction Hardened[4] with type A pattern[5]	See Table 8*	45 000	22 000	_
	Flame[4] or induction Hardened[4] with type B pattern[5]	See Table 8*	22 000	65 000 or 70 000	_
	Carburized and hardened[5]	See Table 9*	55 000	225 000	75 000
	Nitrided[4.7] (Through hardened steels)	83.5 HR 15N	See Fig. 14-3	See Fig. 14-3	_
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminium)	Nitrided[4.7]	83.5 HR 15N	See Fig. 14-4	See Fig. 14-4	See Fig. 14-4

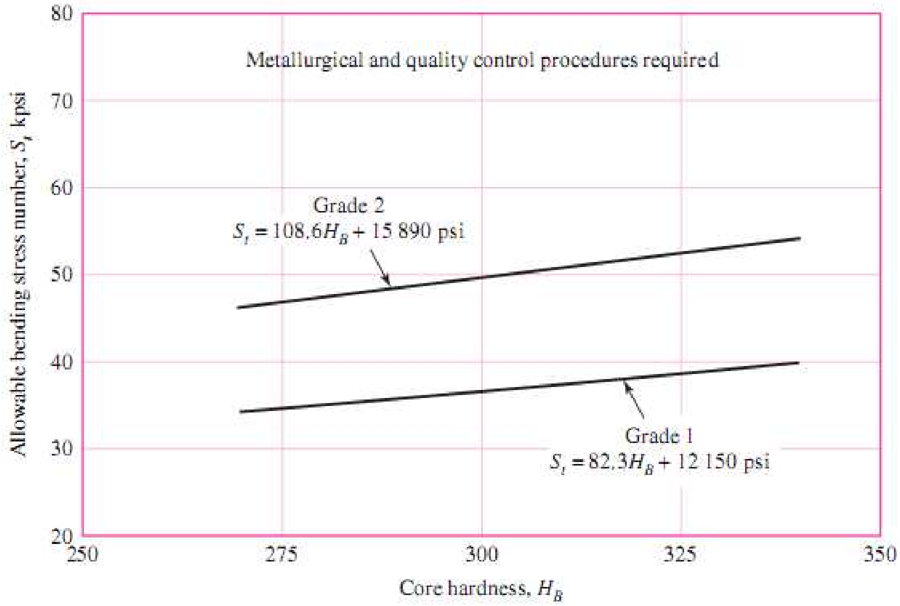


Figure 6-8: Allowable bending stress number for through hardened steels.

b) Y_N, Stress – cycle factor

The load cycle factors will be given in the question statement, with
N(pinion) = N cycles
N(Gear) = N/m_G , and $m_G = N_G/N_P$
As an example: For grade 1 steel,

$$(Y_N)_P = 1.3558 \times (N)^{-0.0178}$$

$$(Y_N)_G = 1.3558 \times (N/m_G)^{-0.0178}$$

Or use following figure for appropriate equations based on the number of cycles.

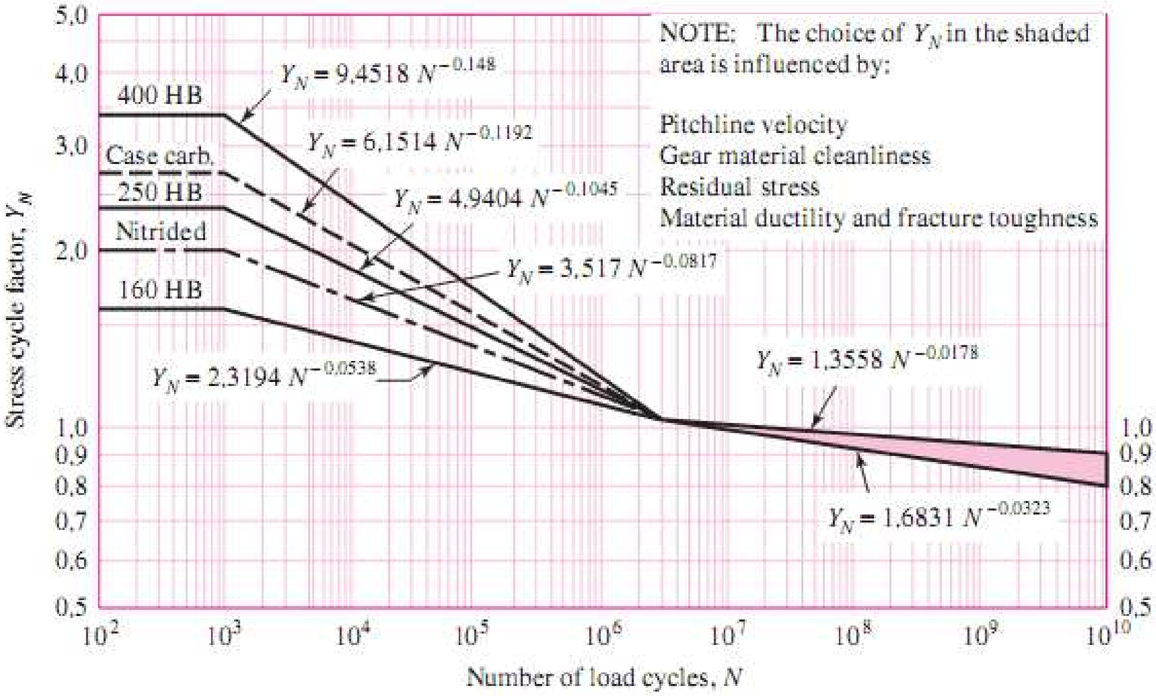


Figure 6-9: Repeatedly applied bending strength stress-cycle factor Y_N.

c) Temperature factor, K_T
$K_T = 1$, if T < 120

d) Reliability factor, K_R

Reliability	$K_R(Y_Z)$
0.9999	1.50
0.999	1.25
0.99	1.00
0.9	0.85
0.50	0.70

e) σ, bending stress from section 6.3.

6.7.2 Pitting failure or Wear, S_H

$$(S_H) = \left(\frac{S_C Z_N / (K_T K_R)}{\sigma_C} \right)$$

a) Repeatedly applied contact strength, S_C
Use following table and graph to estimate S_C

Table 6-5: Repeatedly applied contact strength S_C at 10^7 cycles and 0.99 reliability for steel gears.

Material Designation	Heat treatment	Minimum surface Hardness[1]	Allowable contact stress Number[2], S_c, psi		
			Grade1	Grade2	Grade 3
Steel[3]	Through Hardened[4]	See figure 14-5	See figure 14-5	See figure 14-5	-
	Flame[5] or Induction Hardened[5]	50 HRC 54 HRC	170 000 175 000	190 000 195 000	-
	Carburized and hardened[5]	See Table 9[*]	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N 84.5 HR15N	150 000 155 000	163 000 168 000	175 000 180 000
2.5% chrome(no aluminum)	Nitrided[5]	87.5HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0HR15N	172 000	188 000	205 000
2.5% chrome(no aluminum)	Nitrided[5]	90.0HR15N	176 000	196 000	216 000

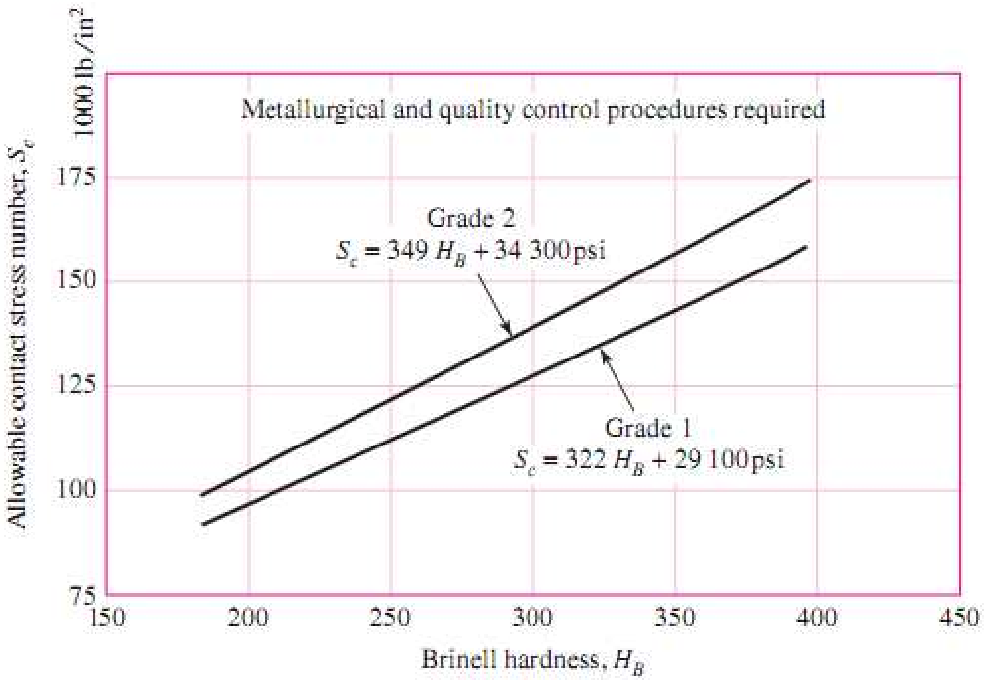


Figure 6-10: Contact fatigue strength S_C at 10^7 cycles and 0.99 reliability for through-hardened steel gear. The SI equations are

$$S_C = 2.22H_B + 200 \text{ MPa}, \text{ grade 1}$$
$$S_C = 2.41H_B + 237 \text{ MPa}, \text{ grade 2}$$

b) Pitting resistance stress cycle factor, Z_N
Generally given in question statement or use following figure to estimate Z_N

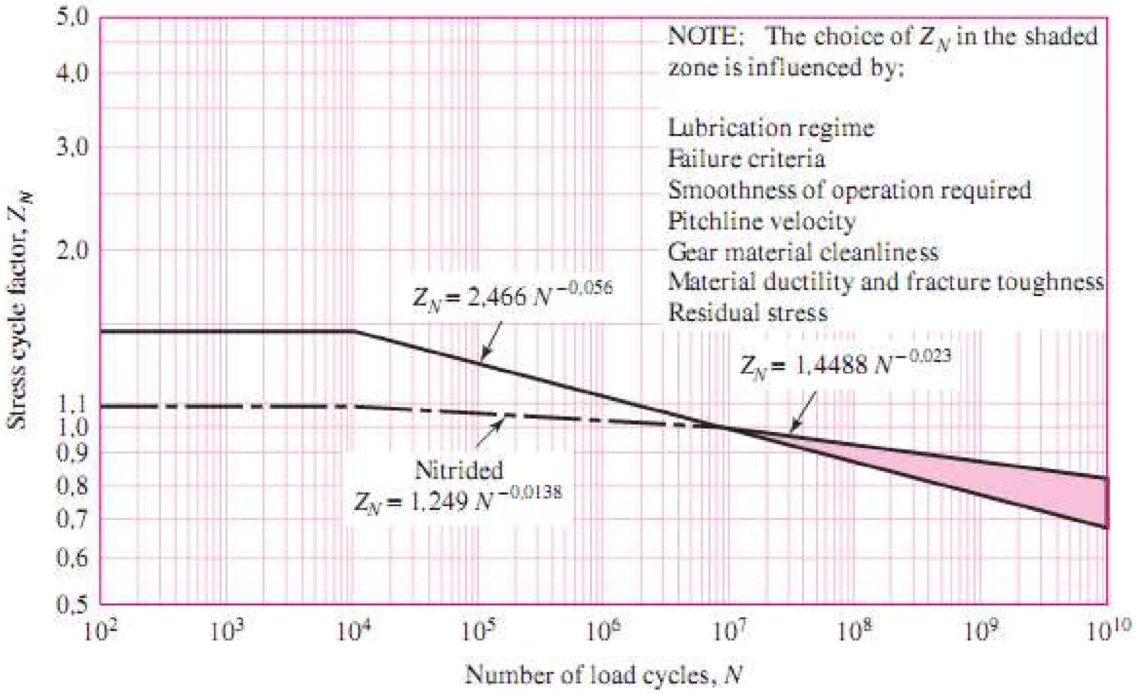


Figure 6-11: Pitting resistance stress-cycle factor, Z_N

c) Temperature factor, K_T

$K_T = 1$, if $T < 120$

d) Reliability factor, K_R

Reliability	$K_R(Y_Z)$
0.9999	1.50
0.999	1.25
0.99	1.00
0.9	0.85
0.50	0.70

e) Pitting resistance (contact stress) σ_C

$$\sigma_C = Z_E \sqrt{W^t K_o K_v K_s \frac{K_H}{d_{w1} b} \frac{Z_R}{Z_I}}$$

where,

Z_E - elastic coefficient $\sqrt{N/mm^2}$

Z_R - surface condition factor

d_{w1} - pitch diameter of the pinion , in (mm)

Z_I - geometry factor for pitting resisitance

And ,

W^t, K_o, K_v, K_s, K_H, b the same terms as defined for the bending stress section 6.3.

i) Elastic coefficient, Z_E

Elastic coefficient $Z_E \sqrt{N/mm^2}$

Pinion Material	**Table 6-6**: Gear Material and Modulus of Elasticity E_G, lbf/in^2 (MPa)*						
	Pinion Modulus of Elasticity E_D psi (MPa)*	Steel $30*10^6$ ($2*10^5$)	Malleable Iron $25*10^6$ ($1.7*10^5$)	Nodular Iron $24*10^6$ ($1.7*10^5$)	Cast Iron $22*10^6$ ($1.5*10^5$)	Aluminum Bronze $17.5*10^6$ ($1.2*10^5$)	Tin Bronze $16*10^6$ ($1.1*10^5$)
Steel	$30*10^6$ ($2*10^5$)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable Iron	$25*10^6$ ($1.7*10^5$)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular Iron	$24*10^6$ ($1.7*10^5$)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast Iron	$22*10^6$ ($1.5*10^5$)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum Bronze	$17.5*10^6$ ($1.2*10^5$)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1700 (145)	1650 (141)
Tin Bronze	$16*10^6$ ($1.1*10^5$)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

ii) Surface condition factor, Z_R

Standard surface conditions for gear teeth have not yet been established. Thus, use $Z_R = 1$.

iii) Geometry factor for pitting resisitance, Z_I

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G + 1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N} \dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases}$$

For spur gear, $m_N = 1$

iv) Now you can find σ_C.

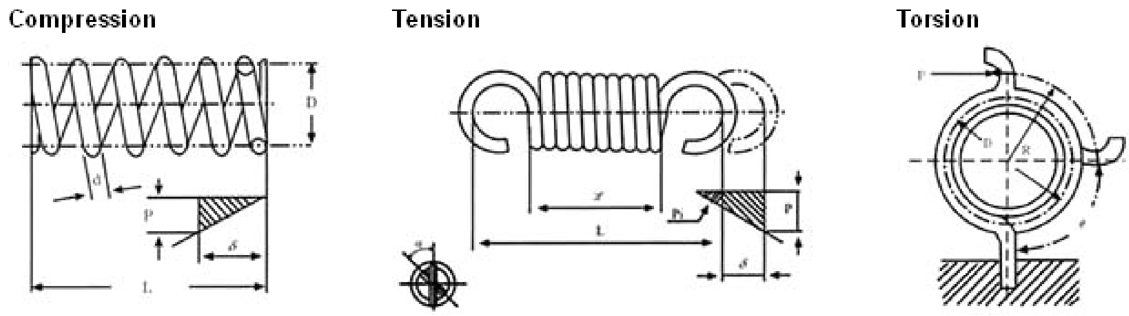
v) Finally , find pittinf failure factor S_H

6.7.3 Compare factor of safety

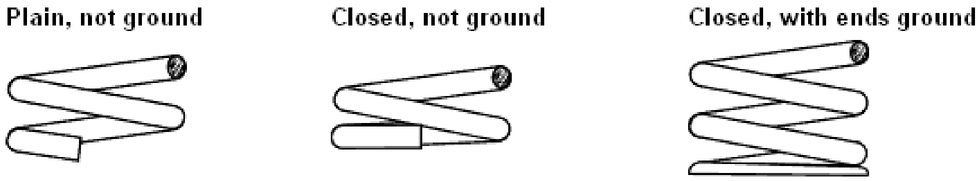
Compare (S_F) with $(S_H)^2$ both for pinion and gear.

7. Mechanical Springs

The most commonly used springs are of the three types below:



There are many different types of compression springs. The following figure illustrate the difference between them.



Notations

D	Mean coil diameter
D	Wire diameter
F	Direct shear force
T	Torque
K_B	Stress correction factor
C	Spring index
N or N_a	Number of active coils
A	End condition constant
λ_{eff}	Slenderness ratio

Formulas:

7.1 Shear stresses in Helical springs

$$\tau = K_B \frac{8FD}{\pi d^3}$$

7.2 Shear stress correction factor

$$K_B = \frac{4C+2}{4C-3}$$

7.3 Spring index

$$C = \frac{D}{d}$$

7.4 Total deflection

$$y = \frac{F}{k} = \frac{8FD^3N}{d^4G}$$

7.5 Spring rate or scale of the spring

$$k = \frac{F}{y} = \frac{d^4 G}{8D^3 N}$$

7.6 Stability

$$\text{a)}\quad L_o < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{\frac{1}{2}}$$

b) For steel,

$$L_o < 2.63\frac{D}{\alpha}$$

c) For squared and ground ends,

$$\alpha = 0.5 \text{ and } L_o < 5.26D$$

d) Slenderness ratio

$$\lambda_{eff} = \frac{\alpha L_o}{D}$$

7.7 Critical frequency of Helical Springs

a) Spring placed between two flat plate and parallel plates

$$\text{fundamental frequency, } f = \frac{1}{2}\sqrt{\frac{kg}{W}}$$

b) Spring placement: one end against a flat plate and the other end free

$$\text{fundamental frequency, } f = \frac{1}{4}\sqrt{\frac{kg}{W}}$$

where,
k = spring rate
g = acceleration due to gravity
W = weight of spring

c) $$W = AL\gamma = \frac{\pi^2 d^2 D N_a \gamma}{4}$$

$\gamma =$ Specific weight

7.8 Fatigue Loading of Helical compression Springs

a) Endurance strength components for infinite life for: Zimmerli data

Unpeened:

S_{sa} = 241 MPa $\quad$ S_{sm} = 379 MPa

Peened:

S_{sa} = 398 MPa $\quad$ S_{sm} = 534 MPa

b) Torsional modulus of rupture or shearing ultimate strength

$$S_{su} = 0.67 S_{ut}$$

where S_{ut} is ultimate tensile strength

c) Shear forces

$$\text{Alternating shear force, } F_a = \frac{F_{max} - F_{min}}{2}$$

$$\text{Midrange shear force, } F_m = \frac{F_{max} + F_{min}}{2}$$

d) Shear stresses

$$\text{Alternating shear stress, } \tau_a = K_B \frac{8F_a D}{\pi d^3}$$

$$\text{Midrange shear stress, } \tau_m = K_B \frac{8F_m D}{\pi d^3}$$

e) The Gerber ordinate intercept for shear

$$S_{se} = \frac{S_{sa}}{1 - \left(\frac{S_{sm}}{S_{su}}\right)^2}$$

The alternating component of strength S_{sa}

$$S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{rS_{su}}\right)^2} \right]$$

The fatigue factor of safety

$$n_f = \frac{S_{sa}}{\tau_a}$$

f) The Sines failure criterion

$$S_{sm} = 0$$

$$S_{sa} = \text{minimum for the Zimmerli data.}$$

The fatigue factor of safety

$$n_f = \frac{S_{sa}}{\tau_a}$$

g) The Goodman ordinate intercept

$$S_{se} = \frac{S_{sa}}{1 - \left(\frac{S_{sm}}{S_{su}}\right)}$$

The alternating component of strength S_{sa}

$$S_{sa} = \frac{rS_{se}S_{su}}{rS_{su} + S_{se}}$$

The fatigue factor of safety

$$n_f = \frac{S_{sa}}{\tau_a}$$

Table 7-1: Decimal Equivalents of Wire and Sheet Metal Gauges* (All sizes are given in Millimeters).

Name of Gauge:	American or Brown & Sharpe	Birmingham or Stubs Iron Wire	United States Standard†	Manu-facturers Standard	Steel Wire or Washburn & Moen	Music Wire	Stubs Steel Wire	Twist Drill
Principal Use:	Nonferous Sheet, Wire and Rod	Tubing, Ferrous Strip, Flat Wire, and Spring Steel	Ferrous Sheet and Plate, 75.4 kN/m^3	Ferrous Sheet	Ferrous Wire Except Music wire	Music Wire	Steel Drill Rod	Twist Drills and Drill Steel
18	1.024	1.245	1.27	1.265	1.206	1.041	4.267	4.305
19	0.912	1.067	1.111	1.062	1.041	1.092	4.166	4.216
20	0.812	0.889	0.952	0.912	0.884	1.143	4.089	4.089
21	0.723	0.813	0.873	0.836	0.805	1.194	3.988	4.039
22	0.644	0.711	0.794	0.759	0.726	1.245	3.937	3.988
23	0.573	0.635	0.714	0.683	0.655	1.295	3.886	3.912
24	0.511	0.559	0.635	0.607	0.584	1.397	3.835	3.861
25	0.455	0.508	0.556	0.531	0.518	1.499	3.759	3.797
26	0.405	0.457	0.476	0.455	0.46	1.6	3.708	3.734
27	0.361	0.406	0.437	0.417	0.439	1.702	3.632	3.658
28	0.321	0.356	0.397	0.378	0.411	1.803	3.531	3.556
29	0.286	0.33	0.357	0.343	0.381	1.905	3.404	3.454
30	0.255	0.305	0.318	0.305	0.356	2.032	3.226	3.264
31	0.227	0.254	0.278	0.267	0.335	2.159	3.048	3.048
32	0.202	0.229	0.258	0.246	0.325	2.286	2.921	2.946
33	0.18	0.203	0.238	0.229	0.3	2.413	2.845	2.87
34	0.16	0.178	0.128	0.208	0.264		2.794	2.819
35	0.143	0.127	0.198	0.19	0.241		2.743	2.794
36	0.127	0.102	0.179	0.17	0.229		2.692	2.705
37	0.113		0.169	0.163	0.216		2.616	2.642
38	0.101		0.159	0.152	0.203		2.565	2.578
39	0.09				0.19		2.515	2.527
40	0.08				0.178		2.464	2.489

*Specify sheet, wire and plate by stating the gauge number, the gauge name, and the decimal equivalent in parenthesis.
†Reflects present average and weights of sheet steel.

Table 7-2: Formula for the dimensional characteristics of compression – springs. (Na = Number of active coils. Source: Design handbook, 1987, p.32. courtesy of associated spring.)

	Types of Spring Ends			
Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total Coils, N_t	N_a	N_a+1	N_a+2	N_a+2
Free Length, L_o	$p\ N_a + d$	$p(N_a+1)$	$p\ N_a+3d$	pN_a+2d
Solid Length, L_s	$d(N_t+1)$	dN_t	$d(N_t+1)$	dN_t
Pitch, p	$(L_0-d)/N_a$	$L_0/(N_a+1)$	$(L_0-3d)/N_a$	$(L_0-2d)/N_a$

Table 7-3: End condition constants α for Helical compression springs*.

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); Other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

Table 7-4: High carbon and alloy spring steels. Source. Harold C.R. Carlson. "Selection and application of spring materials," Mechanical engineering, vol.78.1956,pp.331-334.

Name of Material	**Similar Specifications**	**Description**
Music wire, 0.80–0.95C	UNS G10850 AISI 1085 ASTM A228-51	This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3 mm (0.005 to 0.125 in). Do not use above 120°C (250°F) or at subzero temperatures
Oil-tempered wire, 0.60–0.70C	UNS G10650 AISI 1065 ASTM 229-41	This general-purpose spring steel is used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger than available in music wire. Not for shock or impact loading. Available in diameters 3 to 12 mm (0.125 to 0.5000 in), but larger and smaller sizes may be obtained. Not for use above 180°C (350°F) or at subzero temperatures.
Hard-drawn wire, 0.60–0.70C	UNS G10660 AISI 1066 ASTM A227-47	This is the cheapest general-purpose spring steel and should be used only where life, accuracy, and deflection are not too important. Available in diameters 0.8 to 12 mm (0.031 to 0.500 in). Not for use above 120°C (250°F) or at subzero temperatures.
Chrome-vanadium	UNS G61500 AISI 6150 ASTM 231-41	This is the most popular alloy spring steel for conditions involving higher stresses than can be used with the high-carbon steels and for use where fatigue resistance and long endurance are needed. Also good for shock and impact loads. Widely used for aircraft-engine valve springs and for temperatures to 220°C (425°F). Available in annealed or pretempered sizes 0.8 to 12 mm (0.031 to 0.500 in) in diameter.
Chrome-silicon	UNS G92540 AISI 9254	This alloy is an excellent material for highly stressed springs that require long life and are subjected to shock loading. Rockwell hardnesses of C50 to C53 are quite common, and the material may be used up to 250°C (475°F). Available from 0.8 to 12 mm (0.031 to 0.500 in) in diameter.

Table 7-5: Constants A and m of Sut = A/dm for estimating minimum tensile strength of common spring wires. Source: Design handbook, 1987, p.19, Courtesy of associated spring.

Materials	ASTM NO.	Exponent m	Diameter in	A kpsi. in^m	Diameter mm	A MPa. mm^{mm}	Relative Cost of Wire
Music Wire *	A228	0.145	0.004-0.256	201	0.10-6.5	2211	2.6
OQ & T wire[1]	A229	0.187	0.020-0.500	147	0.5-12.7	1855	1.3
Hard Drawn wire[2]	A227	0.190	0.032-0.437	140	0.7-12.7	1783	1.0
Chrome-Vanadium wire[3]	A232	0.168	0.063-0.375	169	0.8-11.7	2005	3.1
Chrome silicon wire [4]	A401	0.108	0.013-0.10	202	1.6-9.5	1974	4.0
302 Stainless Wire[5]	A313	0.146	0.20-0.40	169	0.3-2.5	1867	7.6-11
		0.263	0.10-0.20	128	2.5-5	2065	
		0.478	0.2-0.40	90	5-10	2911	
Phosphor-Bronze wire [6]	B159	0	0.004-0.022	145	0.1-0.6	1000	8.0
		0.028	0.022-0.075	121	0.6-2	913	
		0.064	0.075-0.30	110	2-2.75	932	

*Surface is smooth, free of defects, and has a bright, lustrous finish

1- Has a slight heat-treating scale which must be removed before plating

2-Surface smooth and bright with no visible marks

3- Aircraft Quality tempered wire, can also be obtained annealed

4- Tempered to Rockwell C49, but may be obtained untendered

5- Type 302 Stainless Steel

6- Temper CA510.

Table 7-6:Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications.

Source:Robert E. Joerres, "Springs", Chap. 6 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), Standard Handbook of Machine Design, 3rd ed, McGraw-Hill, New York, 2004.

	Maximum Percent of Tensile Strength	
Material	Before Set Removed (includes K_w or K_B)	After Set Removed (includes K_s)
Music wire and cold drawn carbon steel	45	60-70
Hardened and tempered carbon and low-alloy steel	50	65-75
Austenitic staninless steel	35	55-65
Nonferrous alloys	35	55-65

Table 7-7: Mechanical properties of some spring wires.

Material	Elastic Percent tension	Limit, of S_{ut} Torsion	Diameter d, mm	E Mpsi	E GPa	G Mpsi	G GPa
Music wire A228	65-75	45-60	<.8	29.5	203.4	12.0	82.7
			.8-1.6	29.0	200	11.85	81.7
			1.61-3	28.5	196.5	11.75	81.0
			>3	28.0	193	11.6	80.0
HD spring A227	60-70	45-55	<.8	28.8	198.6	11.7	80.7
			.8-1.6	28.7	197.9	11.6	80.0
			1.61-3	28.6	197.2	11.5	79.3
			>3	28.5	196.5	11.4	78.6
Oil tempered A239	85-90	45-50		28.5	196.5	11.2	77.2
Valve spring A230	85-90	50-60		29.5	203.4	11.2	77.2
Chrome-vanadium A231	88-93	65-75		29.5	203.4	11.2	77.2
A232	88-93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85-93	65-75		29.5	203.4	11.2	77.2
Stainless steel							
A313*	65-75	45-55		28	193	10	69.0
17-7PH	75-80	55-60		29.5	208.4	11	75.8
414	65-70	42-55		29	200	11.2	77.2
420	65-75	45-55		29	200	11.2	77.2
431	72-76	50-55		30	206	11.5	79.3
Phosphor-bronze B159	75-80	45-50		15	103.4	6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
	75	50-55		19	131	7.3	50.3
Inconel alloy X-750	65-70	40-45		31	213.7	11.2	77.2

Table 7-8: Maximum Recommended Bending Stresses (KB Corrected) for Helical Torsion Springs in Cyclic Applications as Percent of S_{ut}

	ASTM A228 and Type 302 Stainless Steel		**ASTM A230 and A232**	
Fatigue Life, Cycles	**Not Shot-Peened**	**Short- Peened**	**Not Short-Peened**	**Short peened**
10^5	53	62	55	64
10^6	50	60	53	62

7.9 Helical coil torsion spring

a) Stress correction factor, $K_i = \dfrac{4C^2 - C - 1}{4C(C-1)}$

b) Bending stress, round wire, $\sigma = K_i \dfrac{32Fr}{\pi d^3}$

c) Static strength from distortion energy theory

$$S_y = \begin{cases} 0.78S_{ut} \\ 0.87S_{ut} \\ 0.61S_{ut} \end{cases}$$

d) Angular deflection of the body of the coil in number of turns,

$$\theta_c^{'} = \frac{10.8MDN_b}{d^4E}$$

e) Number of active turns N_a, $N_a = N_b + \dfrac{l_1 + l_2}{3\pi D}$

N_b = body turns ,l_1 and l_2 are lengths from applied force to origin.

f) Spring rate , $K^{'} = \dfrac{d^4E}{10.8DN_a}$

g) Helix diameter of the coil

$$D^{'} = \frac{N_bD}{N_b + \theta_c^{'}}$$

h) Diametral clearance,

$$\Delta = D^{'} - d - D_p$$

D_p = pin diameter

i) Fatigue

i) Alternating moment, $M_a = \left(M_{max} - M_{min}\right)/2$

ii) Midrange moment, $M_m = \left(M_{max} + M_{min}\right)/2$

iii) Load line ratio, $r = \dfrac{M_a}{M_m}$

iv) Alternating stress, $\sigma_a = K_i \dfrac{32M_a}{\pi d^3}$

v) Midrange stress, $\sigma_m = K_i \dfrac{32M_m}{\pi d^3}$

vi) Gerber fatigue failure, $S_e = \dfrac{S_r/2}{1 - \left(\dfrac{S_r/2}{S_{ut}}\right)^2}$, $S_a = \dfrac{r^2S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\dfrac{2S_e}{rS_{ut}}\right)^2}\right]$

Factor of safety, $n_f = \dfrac{S_a}{\sigma_a}$

8. Clutches and Brakes

8.1 Internal Expanding Rim Clutches and Brakes

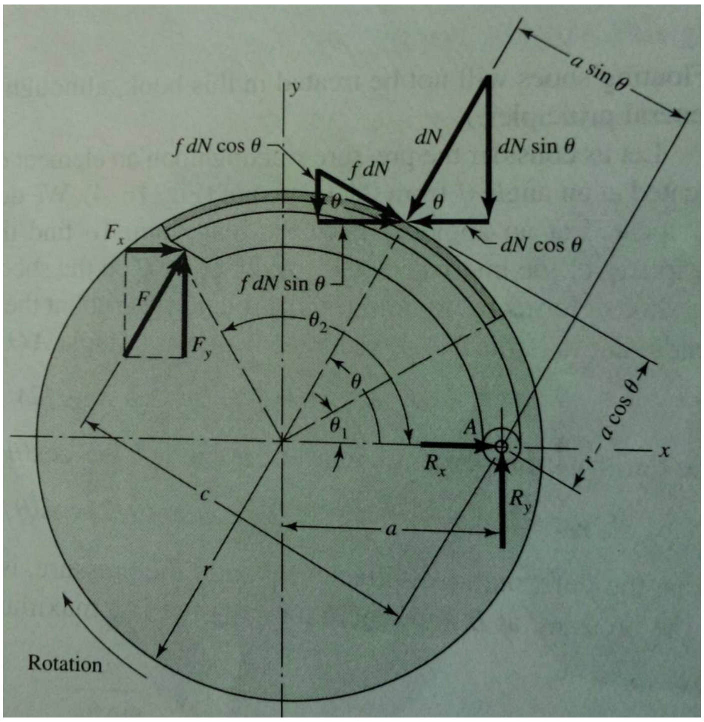


Figure 8-1: Forces on the shoe

A) Clockwise roataiton

1) Pressure distribution $p=\dfrac{p_a}{\sin\theta_a}\sin\theta$

p_a = maximum pressure

Short shoe: p_a occurring at the end of the shoe, θ_2

Long shoe: p_a occurring $\theta = 90^o$.

2) Moment of frictional forces, $M_f=\dfrac{fp_a br}{\sin\theta_a}\displaystyle\int_{\theta_1}^{\theta_2}\sin\theta(r-a\cos\theta)d\theta$

3) Moment of normal forces, $M_N=\dfrac{p_a bra}{\sin\theta_a}\displaystyle\int_{\theta_1}^{\theta_2}\sin^2\theta d\theta$

4) Actuating force, $F=\dfrac{M_N-M_f}{c}$

5) Torque, $T=\dfrac{fp_a br^2\left(\cos\theta_1-\cos\theta_2\right)}{\sin\theta_a}$

6) Hinge pin reactions,

$$R_x=\frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2}\sin\theta\cos\theta d\theta-f\int_{\theta_1}^{\theta_2}\sin^2\theta\right)-F_x$$

$$R_y = \frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin^2\theta + f\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta d\theta\right) - F_y$$

B) Counterclockwise roataiton

1) Actuating force, $F = \dfrac{M_N + M_f}{c}$

2) Hinge pin reactions,

$$R_x = \frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta d\theta + f\int_{\theta_1}^{\theta_2} \sin^2\theta\right) - F_x$$

$$R_y = \frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin^2\theta - f\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta d\theta\right) - F_y$$

8.2 External Contracting Rim Clutches and Brakes

A) Unsymmetric external contracting shoe - Clockwise rotation

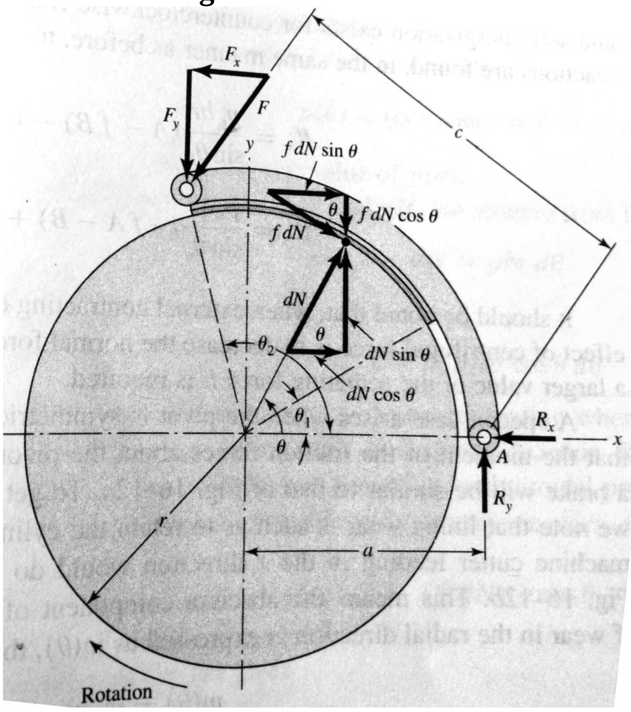


Figure 8-2: Notation of external contracting shoes.

1) Moment of frictional forces, $M_f = \dfrac{fp_a br}{\sin\theta_a}\displaystyle\int_{\theta_1}^{\theta_2} \sin\theta(r - a\cos\theta)d\theta$

2) Moment of normal forces, $M_N = \dfrac{p_a bra}{\sin\theta_a}\displaystyle\int_{\theta_1}^{\theta_2} \sin^2\theta d\theta$

3) Actuating force, $F = \dfrac{M_N + M_f}{c}$

4) Hinge pin reactions,

$$R_x = \frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta d\theta + f\int_{\theta_1}^{\theta_2} \sin^2\theta\right) - F_x$$

$$R_y = \frac{p_a br}{\sin\theta_a}\left(f\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta d\theta - \int_{\theta_1}^{\theta_2} \sin^2\theta \right) + F_y$$

B) Unsymmetric external contracting shoe - Counterclockwise rotation

1) Moment of frictional forces, $M_f = \frac{fp_a br}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin\theta(r - a\cos\theta)d\theta$

2) Moment of normal forces, $M_N = \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin^2\theta d\theta$

3) Actuating force, $F = \frac{M_N - M_f}{c}$

4) Hinge pin reactions,

$$R_x = \frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta d\theta - f\int_{\theta_1}^{\theta_2} \sin^2\theta \right) - F_x$$

$$R_y = \frac{p_a br}{\sin\theta_a}\left(-f\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta d\theta - \int_{\theta_1}^{\theta_2} \sin^2\theta \right) + F_y$$

C) Symmetric external contracting shoe - Clockwise rotation

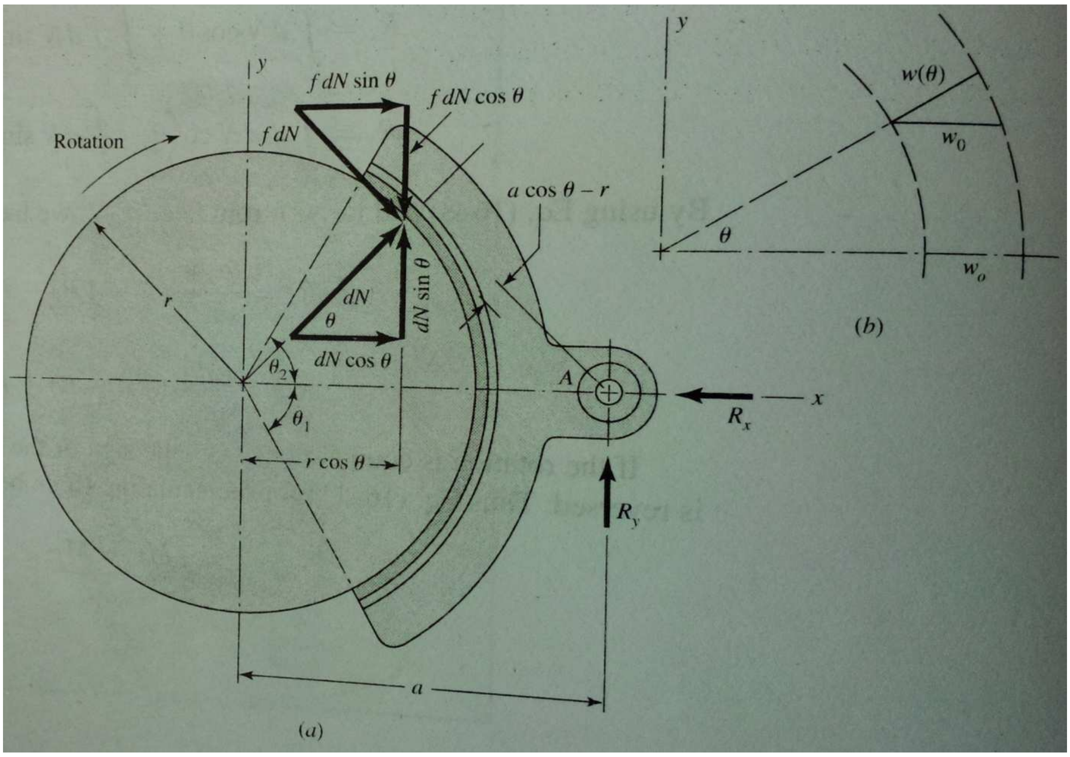


Figure 8-3: (a) Brake with symmetrical pivoted shoe (b) wear of brake lining

1) Pressure distribution $p = p_a \cos\theta$

2) distance a, $a = \frac{4r\sin\theta_2}{2\theta_2 + \sin 2\theta_2}$

3) Hinge pin reactions,

$$R_x = \frac{p_a br}{2}\left(2\theta_2 + \sin 2\theta_2\right)$$

$$R_y = \frac{p_a brf}{2}\left(2\theta_2 + \sin 2\theta_2\right)$$

8.3 Band-type clutches and brakes

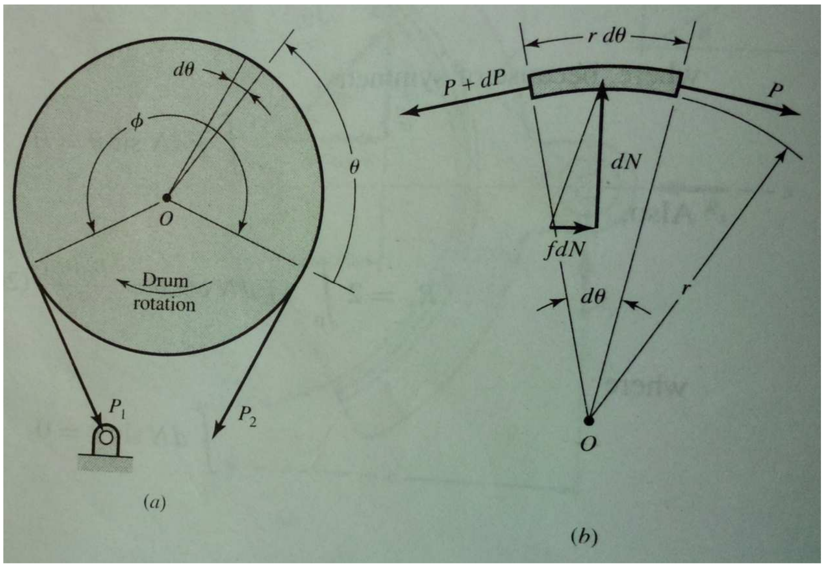


Figure 8-4: Forces on a brake band.

1) Actuating force P_2 and and pin reaction P_1

$$\frac{P_1}{P_2} = f\phi$$

2) Torque , $T = \left(P_1 - P_2\right)\frac{D}{2}$

3) Pressure, $p = \frac{2P}{bD}$

4) Maximum Pressure, $p_a = \frac{2P_1}{bD}$

8.4 Frictional contact axial clutch

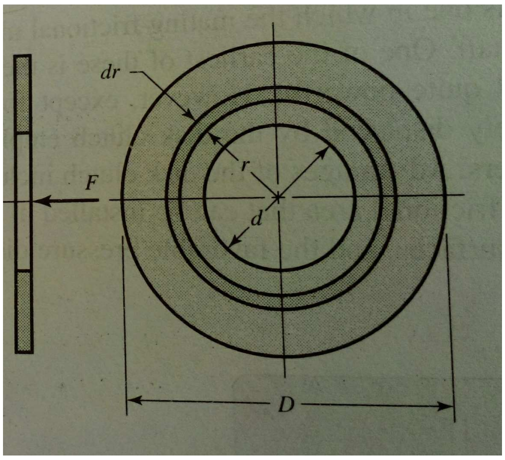


Figure 8-5: Disk friction member.

8.4.1 Uniform wear – single plate

a) Actuating force, $F = \frac{\pi p_a d}{2}(D - d)$

b) Torque, $T = \frac{Ff}{4}(D + d)$

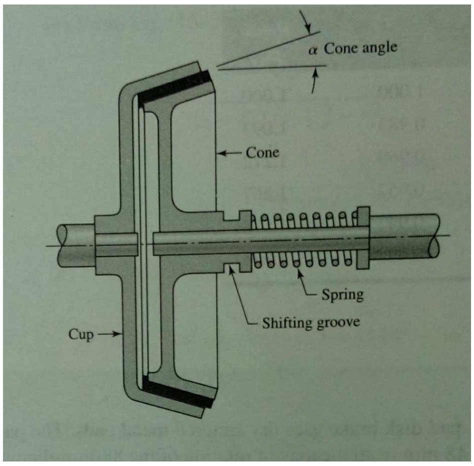
8.4.2 Uniform pressure – single plate

a) Actuating force, $F = \frac{\pi p_a}{4}(D^2 - d^2)$

b) Torque, $T = \frac{Ff}{3}\frac{(D^3 - d^3)}{(D^2 - d^2)}$

Note: For multiple pair of mating surfaces the force and torque values must be multiplied by the number of pairs of surfaces in contact.

8.5 Cone clutches and brakes



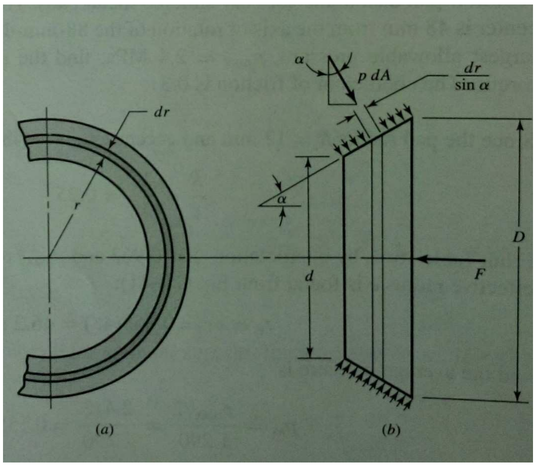


Figure 8-6: i) Cross section of a cone clutch ii) Contact area of a cone clutch.

8.5.1 Uniform wear

a) Pressure relation, $p = p_a \frac{d}{2r}$

b) Actuating force, $F = \frac{\pi p_a d}{2}(D - d)$

b) Torque, $T = \frac{Ff}{4 \sin \alpha}(D + d)$

8.5.2 Uniform pressure – single plate

a) Using $p = p_a$

a) Actuating force, $F = \frac{\pi p_a}{4}(D^2 - d^2)$

b) Torque, $T = \frac{Ff}{3 \sin \alpha}\frac{(D^3 - d^3)}{(D^2 - d^2)}$

8.6 Temperature rise

1. Temperature rise of clutch or brake, $\Delta T = \frac{E}{C_p m}$

where, ΔT = temperature rise, ºC

C_p = specific heat capacity, use 500 J/kg. ºC for steel or cast iron

m = mass of clutch or brake parts, kg

2. Energy dissipated, $E = \dfrac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$

Energy absorbed by the brake stopping equivalent rotary inertia I, $E = \dfrac{I(\omega_o^2 - \omega_f^2)}{2}$

3. Newton's cooling, $\dfrac{T - T\infty}{T_1 - T\infty} = \exp\left(-\dfrac{h_{CR} A}{m C_p} t\right)$

where, T= temperature at time t, ºC

T_1= initial temperature, ºC

$T\infty$ = environmental temperature, ºC

h_{CR} = overall coefficient of heat transfer, W/(m^2. ºC)

A = lateral surface area, m^2

m = mass of the object, kg

C_p = specific heat capacity of the object, J/kg. ºC

m = mass of clutch or brake parts, kg

4. Heat loss, $H_{loss} = h_{CR} A (T - T_\infty)$

5. $h_{CR} = (h_r + f_v h_c)$

h_r = radiation component of h_{CR}, W/(m^2. ºC)

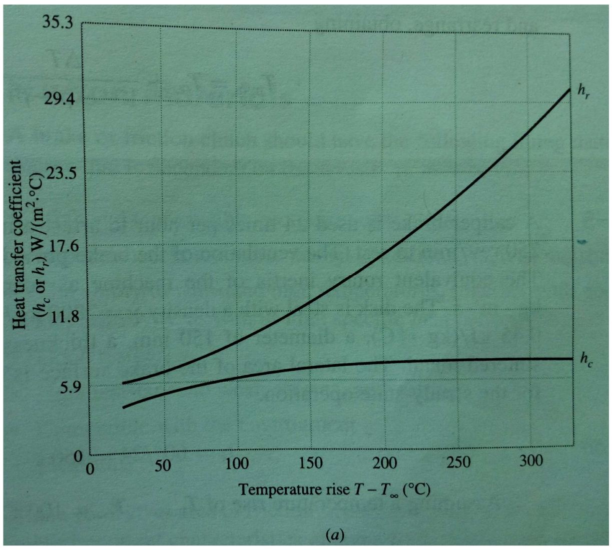
h_c = convection component of h_{CR}, W/(m^2. ºC)

f_v = ventilation factor

6. $T_{max} = T_\infty + \dfrac{\Delta T}{1 - \exp(-\beta t_1)}$

$\beta = \dfrac{h_{CR} A}{m C_p}$

7. $T_{max} - T_{min} = \Delta T$



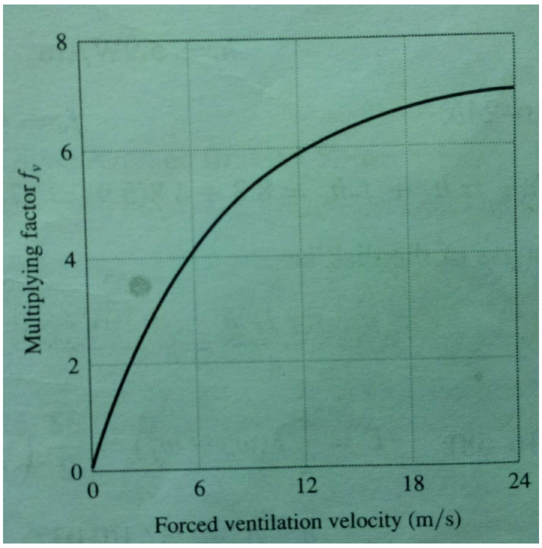


Figure 8-7: (a) Heat-transfer coefficient in still air.(b) Ventilation factors.

9. Power Screw

Terminology and definitions

The pitch is the distance between adjacent thread forms and measured parallel to the thread axis.

The major diameter d is the largest diameter of a screw thread.

The minor (or root diameter) dr is the smallest diameter of a screw thread.

The pitch diameter dp is a theoretical diameter between the major and minor diameters.

The lead l is the distance the nut moves parallel to the screw axis when the nut is given one turn.

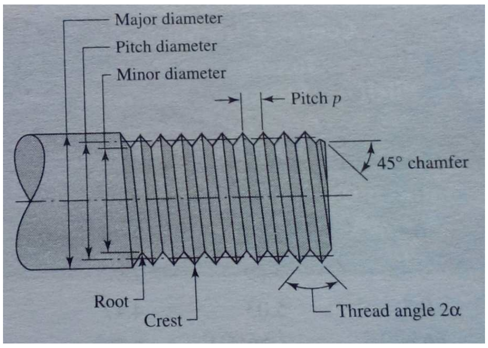


Figure 9-1: Terminology of Screw threads. Sharp vee threads shown for clarity; the crests and roots are actually flattened or rounded during the forming operation.

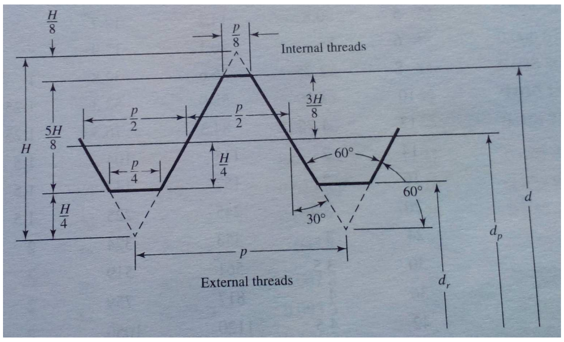


Figure 9-2: Basic profile for metric M threads.
d = major diameter
d_r = minor diameter
d_p = pitch diameter
p = pitch

$$H = \frac{\sqrt{3}}{2} p$$

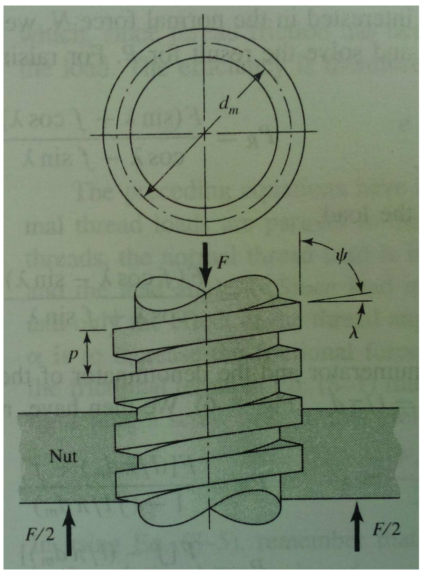


Figure 9-3: Portion of a power screw.

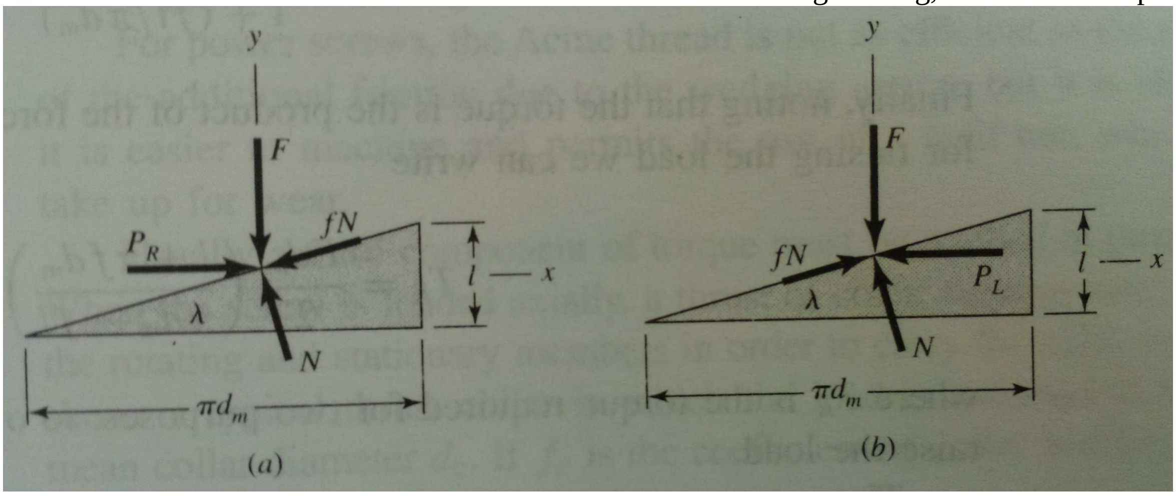


Figure 9-4: Force diagrams: (a) lifting the load; (b) lowering the load.

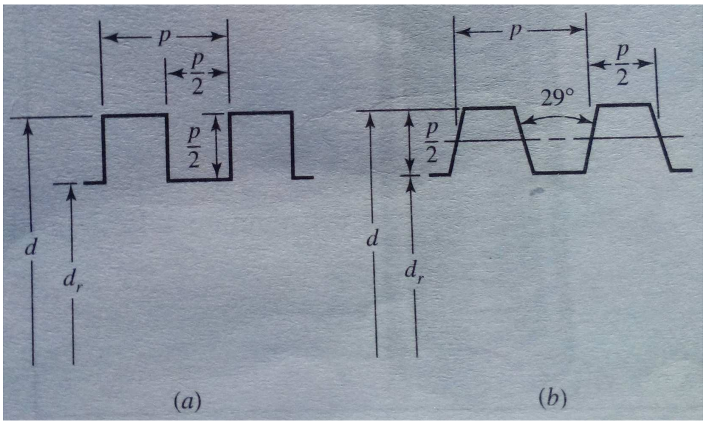


Figure 9-5: (a) Square thread; (b) Acme thread.

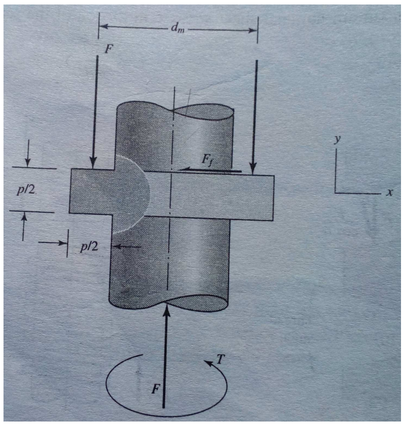


Figure 9-6: Geometry of square thread useful in finding bending and transverse shear stresses at the thread root.

9.1 Formulas for Square thread:

1. Force required for rising the load, $P_R = \dfrac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda}$

$$\text{or } P_R = \frac{F\left[(\ell/\pi d_m) + f\right]}{1 - (f\ell/\pi d_m)}$$

2. Torque required for rising the load, $T_R = \dfrac{Fd_m(\sin\lambda + f\cos\lambda)}{2}$

3. Force required for lowering the load, $P_L = \dfrac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda}$

$$\text{or } P_L = \frac{F\left[f - (\ell/\pi d_m)\right]}{1 + (f\ell/\pi d_m)}$$

4. Torque required for lowering the load, $T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - \ell}{\pi d_m + f\ell}\right)$

5. Self locking, $\pi f d_m > \ell$

or $f > \tan\lambda$

where $\tan\lambda = \frac{\ell}{\pi d_m}$

6. Torque required to raise the load assuming f = 0, $T_0 = \frac{F\ell}{2\pi}$

7. Efficiency of power screw, $e = \frac{T_0}{T_R} = \frac{F\ell}{2\pi T_R}$

9.2 Acme thread

1. For raising the load, or for tightening a screw or bolt, $T_R = \frac{Fd_m}{2}\left(\frac{\ell + \pi f d_m \sec\alpha}{\pi d_m - f\ell \sec\alpha}\right)$

9.3 Thrust collar

1. Torque, $T_c = \frac{F f_c d_c}{2}$

where f_c is coefficient of collar friction.

9.4 Nominal body stresses

1. The maximum nominal shear stress, $\tau = \frac{16T}{\pi d_r^3}$

2. The axial stress (in the absence of column action), $\sigma = \frac{4F}{\pi d_r^2}$

3. The bearing stress, $\sigma_B = \frac{2F}{\pi d_m n_t p}$

where n_t is the number of engaged threads

4. The bending stress at the root of the thread, $\sigma_b = \frac{6F}{\pi d_r n_t p}$

5. The transverse shear stress at the center of the root of the thread due to load F,

$$\tau = \frac{3F}{\pi d_r n_t p}\text{; but at top of the root, } \tau = 0$$

6. The von Mises stress σ' at the root plane, the coordinate system of Figure 9-6,

$$\sigma_x = \frac{6F}{\pi d_r n_t p} \qquad \tau_{xy} = 0$$

$$\sigma_y = -\frac{4F}{\pi d_r^2} \qquad \tau_{yz} = \frac{16T}{\pi d_r^3}$$

$$\sigma_z = 0 \qquad \tau_{zx} = 0$$

Then for three dimensional stress, von Mises stress

$$\sigma' = \frac{1}{\sqrt{2}}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]^{1/2}$$

And for plane stress, $\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

Formulas related to Elective course: System Design and Simulation

1. Equivalent stiffness, $k_{eq} = \frac{\left(A_{i+1} + A_i\right)E}{2l}$

2. Cross sectional area variation of tapered bar in the y-direction, $A(y) = 0.25 - 0.0125y$

3. One dimensional elements,

Linear element

Global Shape functions S_i and S_j according to the equation,

$$S_i = \frac{X_j - X}{l} \text{ and } S_j = \frac{X - X_i}{l}$$

$$\psi^{(e)} = S_i\psi_i + S_j\psi_j$$

Quadratic Elements

Global shape functions S_i , S_j and S_k according to the equation,

$$S_i = \frac{2}{l^2}\left(X - X_j\right)\left(X - X_k\right) \text{ , } S_j = \frac{2}{l^2}\left(X - X_i\right)\left(X - X_k\right)\text{, and } S_k = -\frac{4}{l^2}\left(X - X_i\right)\left(X - X_j\right)$$

$$\psi^{(e)} = S_i\psi_i + S_j\psi_j + S_k\psi_k$$

4. Global coordinates, local and natural coordinates

$$X = X_i + x$$

X = global coordinate and

x = local coordinate

5. Shape functions in terms of local coordinate x

$$S_i = 1 - \frac{X}{\ell} \text{ and } S_j = \frac{X}{\ell} \qquad 0 \le x \le \ell$$

6. Shape functions in terms of natural coordinate ξ

$$S_i = \frac{1}{2}(1 - \xi) \text{ and } S_j = \frac{1}{2}(1 + \xi) \quad \text{at} \quad \begin{cases} \xi = -1, & \psi = \psi_i \\ \xi = 1, & \psi = \psi_j \end{cases}$$

7. Heat transfer

(a) The conductance matrix and thermal load matrix for all elements, excluding the last element $[K]^{(e)} = \left\{\frac{kA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hpl}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right\}$ & $\{F\}^{(e)} = \frac{hplT_f}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ and

(b) The conductance matrix and thermal load matrix for the last element or tip element if the heat loss through the tip is considered

$$[K]^{(e)} = \left\{\frac{kA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hpl}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & hA \end{bmatrix}\right\} \text{ \& } \{F\}^{(e)} = \frac{hplT_f}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ hAT_f \end{Bmatrix}$$

(c) $[K]^{(G)}\{T\} = \{F\}^{(G)}$.

8. Fluid mechanicas

1. For piping flow with variation of viscosity

(a) Element flow resistance matrix $[K]^{(e)} = \frac{\mu}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(b) Forcing matrix $\{F\}^{(e)} = \frac{\text{Pressure drop} \times \ell}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

(c) $[K]^{(G)}\{u\}^{(G)} = \{F\}^{(G)}$

2. For piping flow with constant viscosity

(a) Element flow resistance matrix $[R]^{(e)} = \begin{bmatrix} \frac{\pi D^4}{128L\mu} & -\frac{\pi D^4}{128L\mu} \\ -\frac{\pi D^4}{128L\mu} & \frac{\pi D^4}{128L\mu} \end{bmatrix}$

(b) $[R]^{(G)}\{P\}^{(G)} = \{P\}^{(L)}$

Where $\{P\}^{(G)}$ - nodal pressure and $\{P\}^{(L)}$ - load pressure.

9. Structural

(a) Element stiffness matrix $[K]^{(e)} = \frac{AE}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(b) $[K]^{(G)}\{u\}^{(G)} = \{F\}^{(G)}$

10. fracture

i. stress intensity factor of mode I

$K_I = \beta\sigma\sqrt{\pi a}$

β - stress intensity modification factor, σ - stress , a – crack length

ii. yields

$$S_y = \frac{F}{A}$$

A – sectional area, F – force

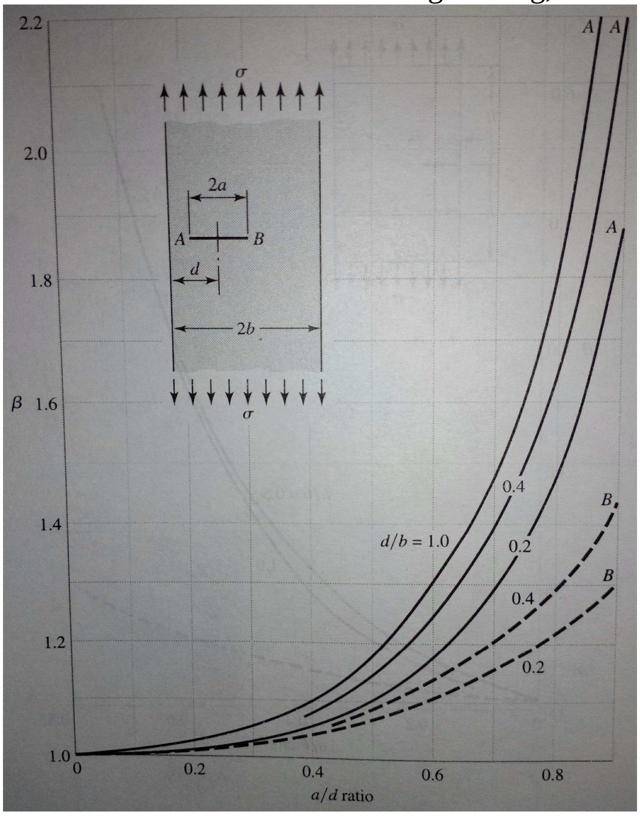


Figure A-1: Off-center crack in a plate longitudinal tension; solid curves are for the crack tip at A; dashed curves are for the tip at B.

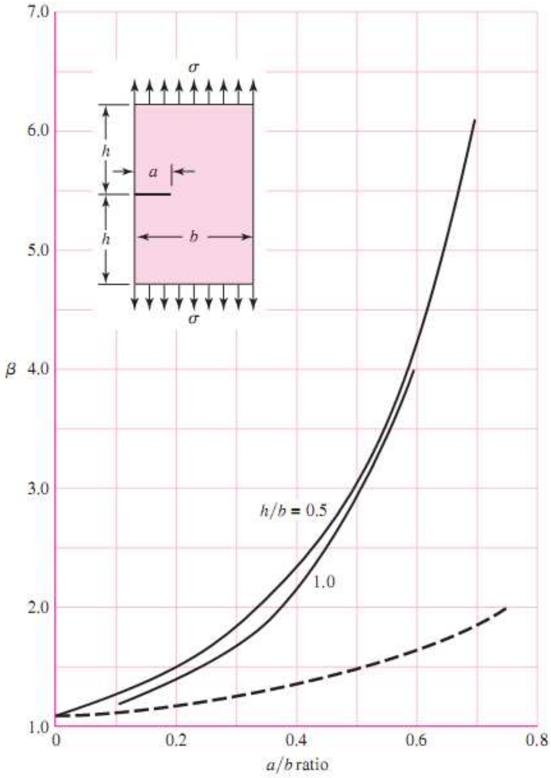


Figure A-2: Plate loaded in longitudinal tension with a crack at the edge; for the solid curve there are no constraints to bending; the dashed curve was obtained with bending constraints added.

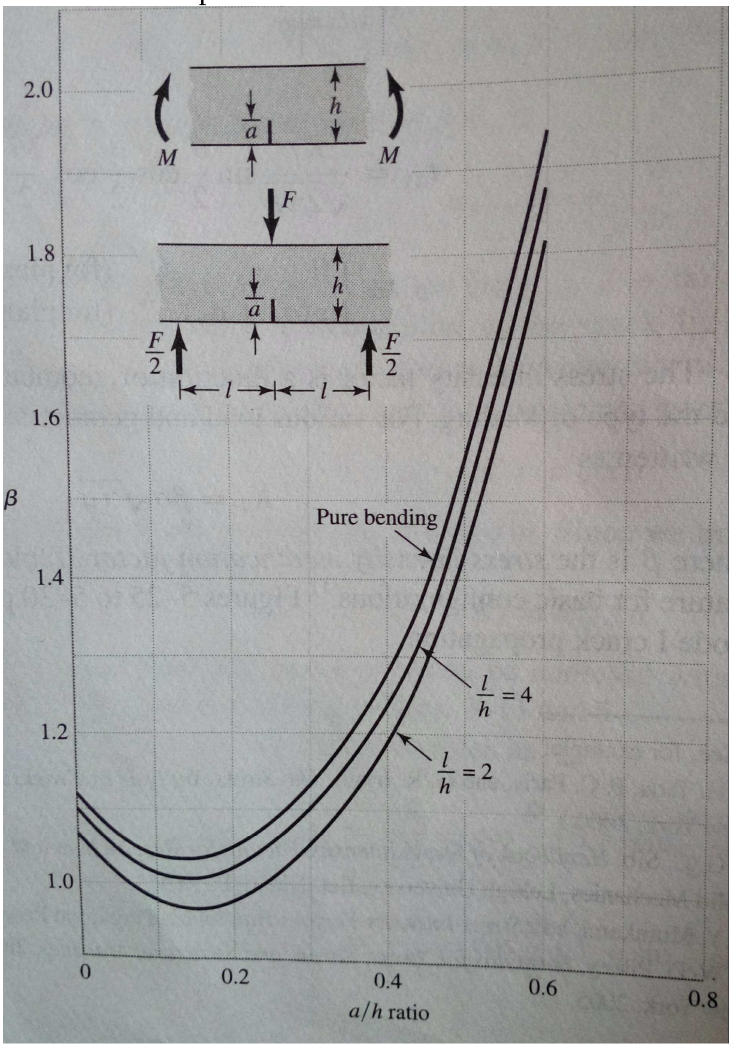


Figure A-3: Beams of rectangular cross section having an edge crack.

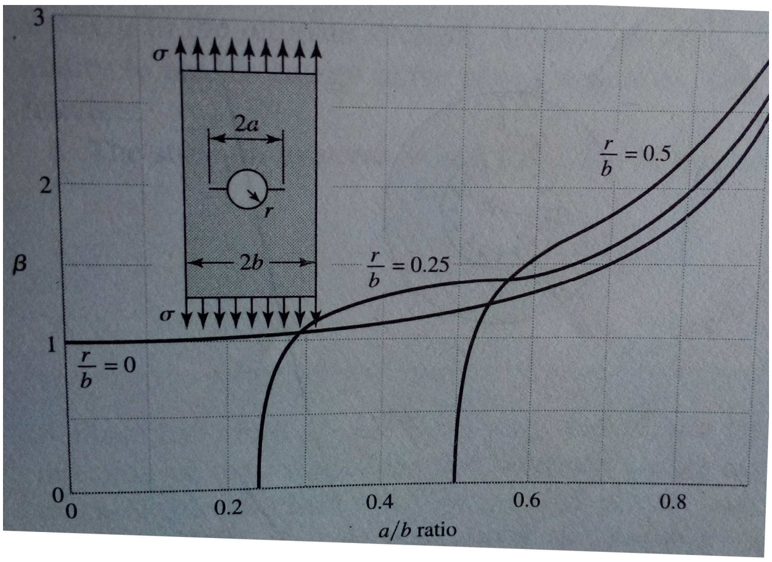


Figure A-4: Plate in tension containing a circular hole with two cracks.

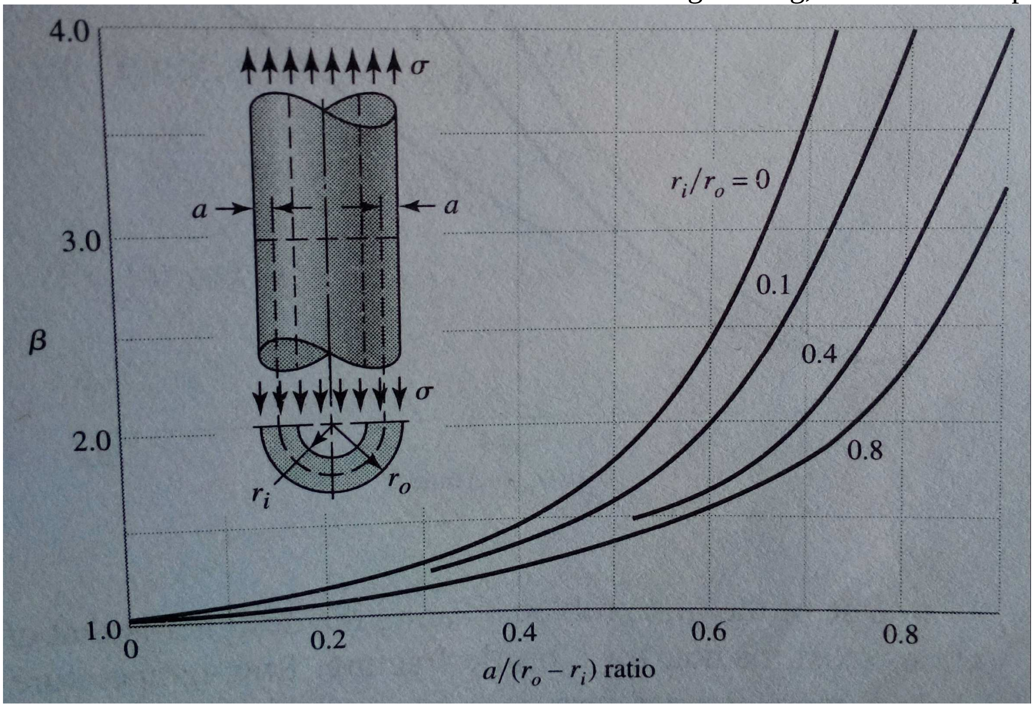


Figure A-5: A cylindrical loading in axial tension having a radial crack of hepth a extending completely around the circumference of the cylinder.

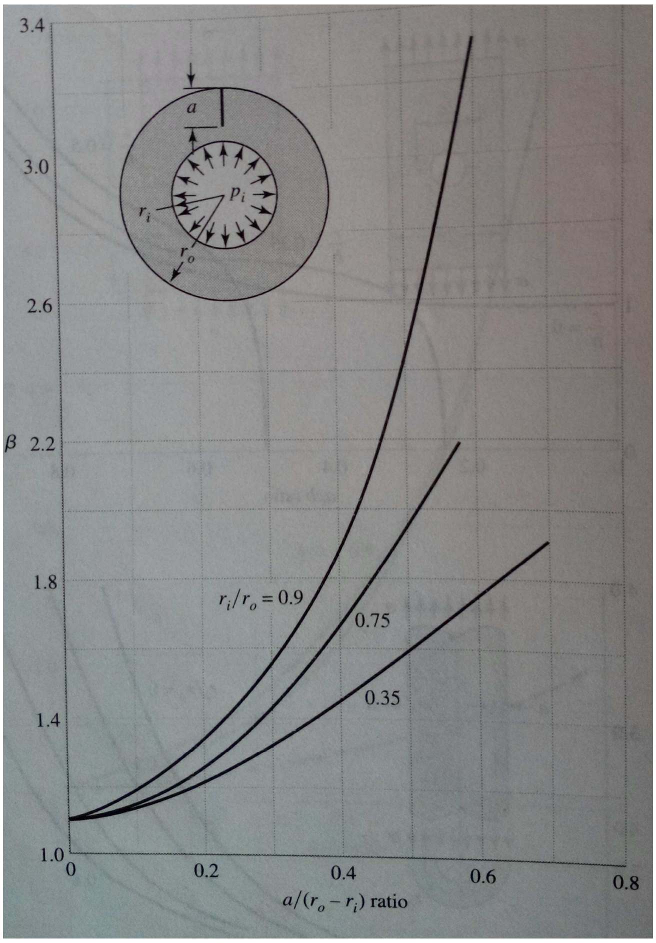


Figure A-6: Cylinder subjected to internal pressure p, having a radial crack in the longitudinal direction of depth a. Use

11. Fatigue

i. Axial loading

Nominal alternating component, $\sigma_{ao} = \frac{F_a}{A}$

Alternating component, $\sigma_a = K_f \sigma_{ao}$

Nominal midrange component, $\sigma_{mo} = \frac{F_m}{A}$

Midrange component, $\sigma_m = K_f \sigma_{mo}$

ii. Gerber factor of safety,

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

iii. Langer static yield

$$n_f = \frac{S_y}{\sigma_a + \sigma_m}$$

Unit Conversion

Multiply input X	By factor A	To get output Y
Inch, in	2.54	centimeter, cm
Inch, in	25.4	millimeter, mm
Pound, lbf	4.45	newton, N
Pound/in^2, psi (lbf/in^2)	6.89	kilopascal, kPa

1. module (m) = 1/ diametral pitch (P)

. References

[1] Budynas and Nisbett, Shigley's Mechanical Engineering Design, Eight edition, Mc Graw Hill, ISBN: 0-390-76487-6.

[2] Budynas and Nisbett, Shigley's Mechanical Engineering Design, Ninth edition in SI Units, Mc Graw Hill, ISBN: 13:978-0-07-107783-5 (ISBN 10: 0-07-107783-9).

[3] Saeed Moaveni, Finite Element Analysis Theory and Application with ANSYS, 3rd Edition, ISBN: 0131890808 (ISBN13: 9780131890800).