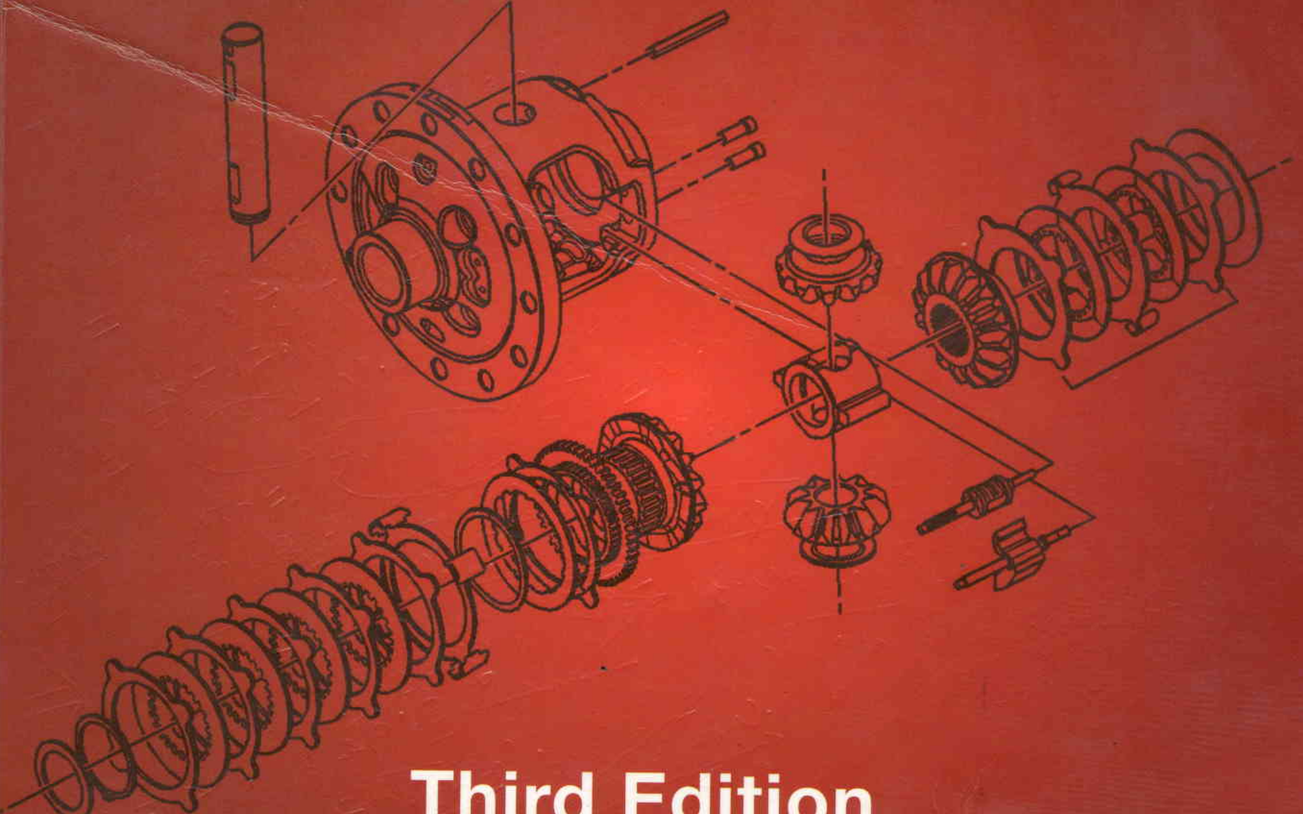

Third Edition

THEORY OF MACHINES

S S RATTAN

THEORY OF MACHINES

Third Edition

ABOUT THE AUTHOR

S S Rattan is currently serving as a Professor in the Department of Mechanical Engineering at the National Institute of Technology, Kurukshetra. He did his post-graduation in Mechanical Engineering from Punjab Engineering College, Chandigarh, in 1981. Since then he has been engaged in teaching and research. He obtained his PhD degree from Kurukshetra University in 1995. He has authored several research papers which have been published in national and international journals as well as in the proceedings of conferences in India and abroad. Besides these, Dr Rattan is the author of two other books titled *Strength of Materials* and *Fluid Mechanics and Fluid Machines*. He is also a member of the Indian Society of Theoretical and Applied Mechanics.

THEORY OF MACHINES

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In
the memory of
My Father

CONTENTS

PREFACE

Mechanisms and machines have considerable fascination for most students of mechanical engineering since the theoretical principles involved have immediate applications to practical problems. The main objective of writing this book has been to give a clear understanding of the concepts underlying engineering design. A sincere effort has been made to maintain the physical perceptions in the various derivations and to give the shortest comprehending solution to a variety of problems. The parameters kept in mind while writing the book are the coverage of contents, prerequisite knowledge of students, lucidity of writing, clarity of diagrams and the variety of solved and unsolved numerical problems.

The book is meant to be useful to the degree-level students of mechanical engineering as well as those preparing for AMIE and various other competitive examinations. However, diploma-level students will also find the book to be highly useful. The book will also benefit postgraduate students to some extent as it also contains advanced topics like curvature theory, analysis of rigid and elastic cam systems, complex number and vector methods, force balancing of linkages and field balancing. The salient features of the book are

- Concise and compact covering all major topics
- Presentation of concepts in a logical, innovative and lucid manner
- Evolving the basic theory from simple and readily understood principles
- A balanced presentation of the graphical and analytical approaches
- Computer programs in user-friendly C-language
- Large number of solved examples
- Summary, review questions as well as a number of unsolved problems at the end of each chapter
- An appendix containing objective-type questions
- Another appendix containing important relations and results

It is expected that the students using this book might have completed a course in applied mechanics. The book is divided broadly into two sections, kinematics and dynamics of machines. Kinematics involves study from the geometric point of view to know the displacement, velocity and acceleration of various components of mechanisms, whereas dynamics is the study of the effects of the applied and inertia forces. Chapters 1 to 11 are devoted to the study of the kinematics and the rest to that of dynamics. **Chapter 1** introduces the concepts of mechanisms and machines. **Chapters 2 and 3** describe graphical methods of velocity and acceleration analysis whereas the analytical approach is discussed in **Chapter 4**. Synthesis or designing of mechanisms is important to have the desirable motion of various components of machinery—the detail procedures for the same, both graphical and analytical, are given in **Chapter 5**. Various types of mechanisms with higher number of links are discussed in **Chapter 6**. Friction in various components of machines is very important as it affects their efficiency and is described in **Chapter 8**. Cams, belts, gears, gear trains are meant to transmit power from one shaft to another and are discussed in **chapters 7, 9, 10 and 11** respectively.

Forces are mainly of static and dynamic nature. **Chapters 12 and 13** are devoted to their effects on the components of the mechanisms. **Chapter 13** also includes the topic of flywheels which are essential components for rotary machines to regulate speeds. Speed regulation is also affected by governors which are described in **Chapter 16**. Unbalanced forces and vibrations in various components of rotating machines are mostly undesirable since the efficiency is reduced. A detailed study of these is undertaken in **chapters 14 and 18**. Brakes are essential for any moving components of machinery and are discussed in **Chapter 15**.

Moving bodies like aeroplanes, ships, two- and four-wheelers, etc., experience gyroscopic effect while taking turns. It is described in **Chapter 17**. Automatic control of machinery is very much desirable these days and an introduction of the same is given in **Chapter 19**.

The first edition of the book aimed at providing the fundamentals of the subject in a simple manner for easy comprehension by students. Simple mathematical methods were preferred instead of more elegant but less obvious methods so that those with limited mathematical skills could easily understand the expositions. However, to make the book more purposeful and acceptable to a wider section of users, the second edition also consisted of methods involving vector and complex numbers usually preferred by those who excel in mathematical skills. Such methods frequently lead to computer-aided solutions of the problems. The computer programs were rewritten in the more user-friendly C language. A Summary of each chapter was added at the end and theoretical questions were added to the exercises. One appendix containing objective-type questions was also included. All the previous figures were redrawn.

The present edition is aimed at making the book more exhaustive. Many more worked examples as well as unsolved problems have been added. Many new sections have been added in most of the chapters apart from rewriting some previous sections. Another appendix containing important relations and results has also been added. Effort has been made to remove all sorts of errors and misprints as far as possible. In spite of addition of a large amount of material, care has been taken to let the book remain concise and compact. Hints to most of the numerical problems at the end of each chapter have been provided at the publisher's website of the book for the benefit of average and weak students. Full solutions of the same are available to the faculty members at the same site. The facility can be availed by logging on to http://www.mhhe.com/rattan/tom3e.

I am grateful to all those teachers and students who pointed out errors and mistakes of the previous editions and also gave many valuable suggestions. I acknowledge the efforts of the editorial staff of Tata McGraw Hill Education Private Limited for bringing out the new edition in an excellent format.

Finally, I make an affectionate acknowledgement to my wife, Neena, and my children, Ravneet and Jasmeet, for their patience, support and putting up with it all so cheerfully. But for their sacrifice, I would not have been able to complete this work in the most satisfying way.

For further improvement of the book, readers are requested to post their comments and suggestions at ss_rattan@hotmail.com.

S S Rattan

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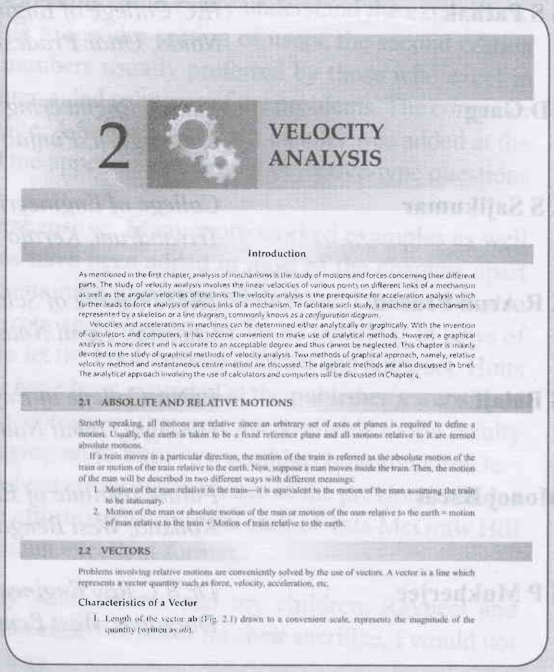
VISUAL WALKTHROUGH

Introduction at the beginning of each chapter sums up the aim and contents of the chapter.



2 VELOCITY ANALYSIS

Introduction

As mentioned in the first chapter, analysis of mechanisms is the study of motions and forces concerning their different parts. The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as the angular velocities of the links. The velocity analysis is the prerequisite for acceleration analysis which further leads to force analysis of various links of a mechanism. To facilitate such study, a machine or a mechanism is represented by a skeleton or a line diagram, commonly knows as a *configuration diagram*.

Velocities and accelerations in machines can be determined either analytically or graphically. With the invention of calculators and computers, it has become convenient to make use of analytical methods. However, a graphical analysis is more direct and is accurate to an acceptable degree and thus cannot be neglected. This chapter is mainly devoted to the study of graphical methods of velocity analysis. Two methods of graphical approach, namely, relative velocity method and instantaneous centre method are discussed. The algebraic methods are also discussed in brief. The analytical approach involving the use of calculators and computers will be discussed in Chapter 4.

2.1 ABSOLUTE AND RELATIVE MOTIONS

Strictly speaking, all motions are relative since an arbitrary set of axes or planes is required to define a motion. Usually, the earth is taken to be a fixed reference plane and all motions relative to it are termed absolute motions.

If a train moves in a particular direction, the motion of the train is referred as the absolute motion of the train or motion of the train relative to the earth. Now, suppose a man moves inside the train. Then, the motion of the man will be described in two different ways with different meanings:

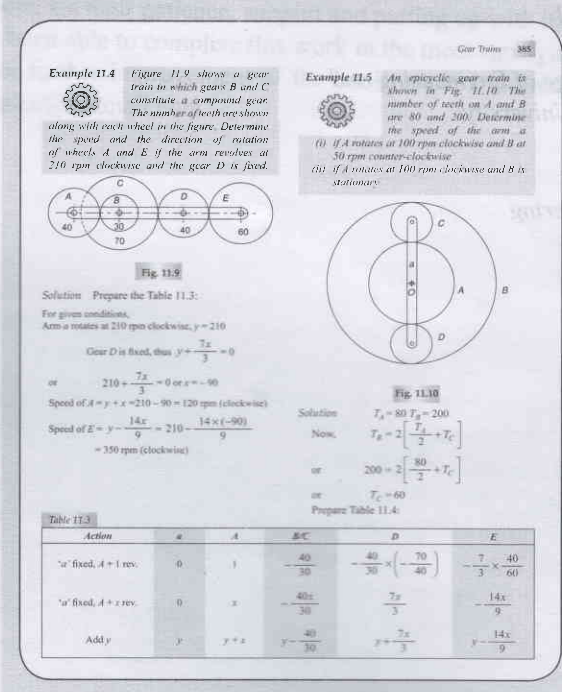
1. Motion of the man relative to the train—it is equivalent to the motion of the man assuming the train to be stationary.
2. Motion of the man or absolute motion of the man or motion of the man relative to the earth = motion of man relative to the train + Motion of train relative to the earth.

2.2 VECTORS

Problems involving relative motions are conveniently solved by the use of vectors. A vector is a line which represents a vector quantity such as force, velocity, acceleration, etc.

Characteristics of a Vector

1. Length of the vector ab (Fig. 2.1) drawn to a convenient scale, represents the magnitude of the quantity (written as *ab*).



Example 11.4 *Figure 11.9 shows a gear train in which gears B and C constitute a compound gear. The number of teeth are shown along with each wheel in the figure. Determine the speed and the direction of rotation of wheels A and E if the arm revolves at 210 rpm clockwise and the gear D is fixed.*

Fig. 11.9

Solution Prepare the Table 11.3:

For given conditions,
Arm *a* rotates at 210 rpm clockwise, $y = 210$

Gear *D* is fixed, thus $y + \frac{7x}{3} = 0$

or $210 + \frac{7x}{3} = 0$ or $x = -90$

Speed of $A = y + x = 210 - 90 = 120$ rpm (clockwise)

Speed of $E = y - \frac{14x}{9} = 210 - \frac{14 \times (-90)}{9}$

$= 350$ rpm (clockwise)

Table 11.3

Action	*a*	*A*	*B/C*	*D*	*E*
'*a*' fixed, *A* + 1 rev.	0	1	$-\frac{40}{30}$	$-\frac{40}{30} \times \left(-\frac{70}{40}\right)$	$-\frac{7}{3} \times \frac{40}{60}$
'*a*' fixed, *A* + *x* rev.	0	*x*	$-\frac{40x}{30}$	$\frac{7x}{3}$	$-\frac{14x}{9}$
Add *y*	*y*	$y + x$	$y - \frac{40}{30}$	$y + \frac{7x}{3}$	$y - \frac{14x}{9}$

Example 11.5 *An epicyclic gear train is shown in Fig. 11.10. The number of teeth on A and B are 80 and 200. Determine the speed of the arm a*
(i) if A rotates at 100 rpm clockwise and B at 50 rpm counter-clockwise
(ii) if A rotates at 100 rpm clockwise and B is stationary.

Fig. 11.10

Solution $T_A = 80$ $T_B = 200$

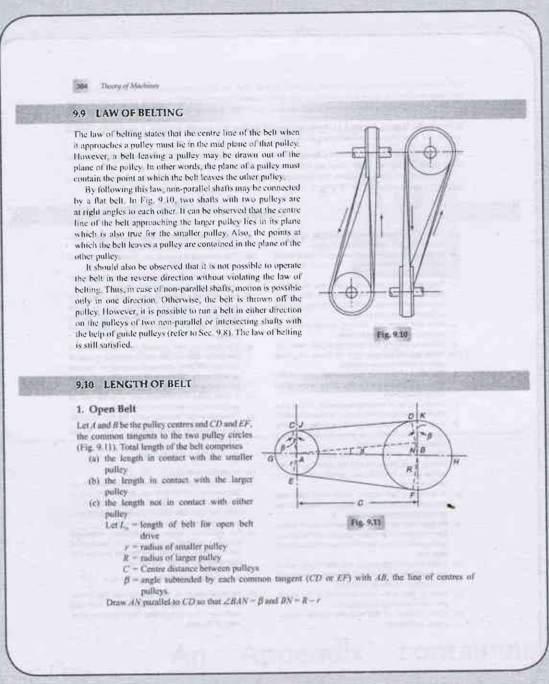
Now, $T_B = 2\left[\frac{T_A}{2} + T_C\right]$

or $200 = 2\left[\frac{80}{2} + T_C\right]$

or $T_C = 60$

Prepare Table 11.4:

A variety of solved examples are given to reinforce the concepts.



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9.9 LAW OF BELTING

The law of belting states that the centre line of the belt when it approaches a pulley must lie in the mid plane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley. In other words, the plane of a pulley must contain the point at which the belt leaves the other pulley.

By following this law, non-parallel shafts may be connected by a flat belt. In Fig. 9.10, two shafts with two pulleys are at right angles to each other. It can be observed that the centre line of the belt approaching the larger pulley lies in its plane which is also true for the smaller pulley. Also, the points at which the belt leaves a pulley are contained in the plane of the other pulley.

It should also be observed that it is not possible to operate the belt in the reverse direction without violating the law of belting. Thus, in case of non-parallel shafts, motion is possible only in one direction. Otherwise, the belt is thrown off the pulley. However, it is possible to run a belt in either direction on the pulleys of two non-parallel or intersecting shafts with the help of guide pulleys (refer to Sec. 9.8). The law of belting is still satisfied.

Fig. 9.10

9.10 LENGTH OF BELT

1. Open Belt

Let A and B be the pulley centres and CD and EF, the common tangents to the two pulley circles (Fig. 9.11). Total length of the belt comprises

(a) the length in contact with the smaller pulley

(b) the length in contact with the larger pulley

(c) the length not in contact with either pulley

Let L_o = length of belt for open belt drive

r = radius of smaller pulley

R = radius of larger pulley

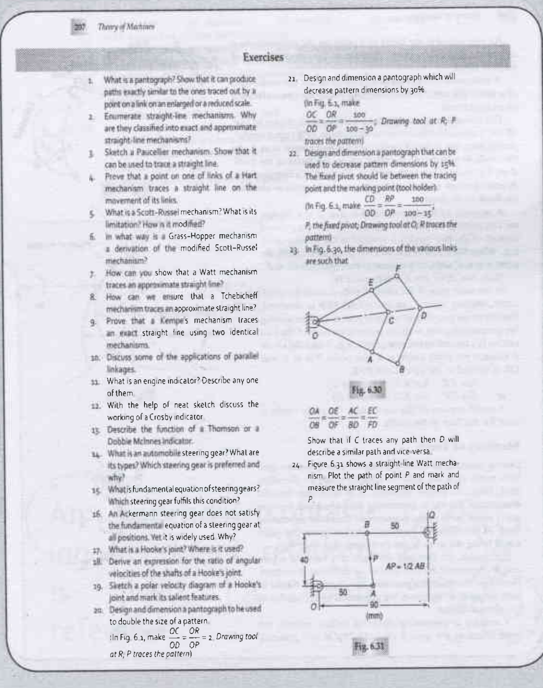
C = Centre distance between pulleys

β = angle subtended by each common tangent (CD or EF) with AB, the line of centres of pulleys.

Draw AN parallel to CD so that $\angle BAN = \beta$ and $BN = R - r$

Fig. 9.11

Each chapter has a concise and comprehensive treatment of topics with emphasis on fundamental concepts.



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Exercises

1. What is a pantograph? Show that it can produce paths exactly similar to the ones traced out by a point on a link on an enlarged or a reduced scale.
2. Enumerate straight-line mechanisms. Why are they classified into exact and approximate straight-line mechanisms?
3. Sketch a Paucellier mechanism. Show that it can be used to trace a straight line.
4. Prove that a point on one of links of a Hart mechanism traces a straight line on the movement of its links.
5. What is a Scott-Russel mechanism? What is its limitation? How is it modified?
6. In what way is a Grass-Hopper mechanism a derivation of the modified Scott-Russel mechanism?
7. How can you show that a Watt mechanism traces an approximate straight line?
8. How can we ensure that a Tchebicheff mechanism traces an approximate straight line?
9. Prove that a Kempe's mechanism traces an exact straight line using two identical mechanisms.
10. Discuss some of the applications of parallel linkages.
11. What is an engine indicator? Describe any one of them.
12. With the help of neat sketch discuss the working of a Crosby indicator.
13. Describe the function of a Thomson or a Dobbie McInnes indicator.
14. What is an automobile steering gear? What are its types? Which steering gear is preferred and why?
15. What is fundamental equation of steering gears? Which steering gear fulfils this condition?
16. An Ackermann steering gear does not satisfy the fundamental equation of a steering gear at all positions. Yet it is widely used. Why?
17. What is a Hooke's joint? Where is it used?
18. Derive an expression for the ratio of angular velocities of the shafts of a Hooke's joint.
19. Sketch a polar velocity diagram of a Hooke's joint and mark its salient features.
20. Design and dimension a pantograph to be used to double the size of a pattern.
(In Fig. 6.1, make $\frac{OC}{OD} = \frac{OR}{OP} = 2$. *Drawing tool at R; P traces the pattern*)
21. Design and dimension a pantograph which will decrease pattern dimensions by 30%.
(In Fig. 6.1, make $\frac{OC}{OD} = \frac{OR}{OP} = \frac{100}{100-30}$; *Drawing tool at R; P traces the pattern*)
22. Design and dimension a pantograph that can be used to decrease pattern dimensions by 15%. The fixed pivot should lie between the tracing point and the marking point (tool holder).
(In Fig. 6.1, make $\frac{CD}{OD} = \frac{RP}{OP} = \frac{100}{100-15}$, *P, the fixed pivot; Drawing tool at O; R traces the pattern*)
23. In Fig. 6.30, the dimensions of the various links are such that

Fig. 6.30

$$\frac{OA}{OB} = \frac{OE}{OF} = \frac{AC}{BD} = \frac{EC}{FD}$$

Show that if C traces any path then D will describe a similar path and vice versa.

24. Figure 6.31 shows a straight-line Watt mechanism. Plot the path of point P and mark and measure the straight line segment of the path of P.

Fig. 6.31

A number of theoretical questions and unsolved exercises are given for practice to widen the horizon of comprehension of the topic.

A Summary at the end of each chapter recapitulates the inferences for quick revision.

to turn on the bushing. Figure 9.17 shows this type of chain in place on the sprocket. A good roller chain is quieter and wears less as compared to a block chain.

(iii) *Silent Chain (Inverted Tooth Chain)* Though roller chains can run quietly at fairly high speeds, the silent chains or inverted tooth chains are used where maximum quietness is desired.

Silent chains do not have rollers. The links are so shaped as to engage directly with the sprocket teeth. The included angle is either 60° or 75° [Fig. 9.21(c)].

Summary

1. Power is transmitted from one shaft to another by means of belts, ropes, chains and gears.
2. Belts, ropes and chains are used where the distance between the shafts is large. For small distances, gears are preferred.
3. Belts and ropes transmit power due to friction between them and the pulleys. If the power transmitted exceeds the force of friction, the belt or rope slips over the pulley.
4. Belts and ropes are strained during motion as tensions are developed in them.
5. Owing to slipping and straining action, belts and ropes are not positive type of drives, i.e., their velocity ratios are not constant.
6. The effect of slip is to decrease the speed of the belt on the driving shaft and to decrease the speed of the driven shaft.
7. A belt may be of rectangular section, known as a *flat* belt or of trapezoidal section, known as a V-belt.
8. In case of a flat belt, the rim of the pulley is slightly *crowned* which helps to keep the belt running centrally on the pulley rim.
9. The groove on the rim of the pulley of a V-belt drive is made deeper to take the advantage of the wedge action. The belt does not touch the bottom of the groove.
10. A multiple V-belt system, using more than one belt in the two pulleys, can be used to increase the power transmitting capacity.
11. An open-belt drive is used when the driven pulley is to be rotated in the same direction as the driving pulley and a crossed-belt drive when in the opposite direction.
12. While transmitting power, one side of the belt is more tightened (known as tight side) as compared to the other (known as slack side).
13. Velocity ratio is the ratio of speed of the driven pulley to that of the driving pulley.
14. Usual materials of flat belts are leather, canvas, cotton and rubber.
15. V-belts are made of rubber impregnated fabric with angle of V between 30 to 40 degrees.
16. The materials for ropes are cotton, hemp, manila or wire.
17. The main types of pulleys are *idler, intermediate* (or *countershaft*), *loose and fast and guide*.
18. *Law of belting* states that the centre line of the belt when it approaches a pulley must lie in the mid-plane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley.
19. The length of belt depends only on the sum of the pulley radii and the centre distance in case of crossed-belt drive whereas it depends on the sum as well as the difference of the pulley radii apart from the centre distance in case of open-belt drive.
20. A cone pulley has different sets of pulley radii to give varying speeds of the driven shaft.
21. The ratio of belt tensions when the belt is on the point of slipping on the pulleys, $\frac{T_1}{T_2} = e^{\mu\theta}$ for flat belt drive,
    $\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha}$ for V-belt drive
22. Power transmitted is, $P = (T_1 - T_2) \times v$
23. The centrifugal force produces equal tensions on the two sides of the belt, i.e., on the tight side as well as on the slack side. It is independent of the tight and slack side tensions and depends only on the velocity of the belt over the pulley.
24. For maximum power transmission, centrifugal tension in the belt must be equal to one-third of the maximum allowable belt tension and the belt should be on the point of slipping.
25. Initial tension in the belt is given by, $T_0 = \frac{T_1 + T_2}{2}$
26. As more length of belt approaches the driving pulley than the length that leaves, the belt slips back over the driving pulley. This slip is known as *creep* of the belt.

An *epicycloid* is the locus of a point on the circumference of a circle that rolls without slipping on the circumference of another circle.

A *hypocycloid* is the locus of a point on the circumference of a circle that rolls without slipping inside the circumference of another circle.

The formation of a cycloidal tooth has been shown in Fig. 10.18. A circle H rolls inside another circle APB (pitch circle). At the start, the point of contact of the two circles is at A. As the circle H rolls inside the pitch circle, the locus of the point A on the circle H traces the path ALP which is a hypocycloid. A small portion of this curve near the pitch circle is used for the flank of the tooth.

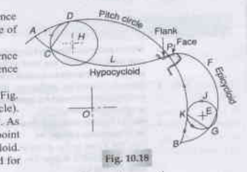


Fig. 10.18

A property of the hypocycloid is that at any instant, the line joining the generating point (A) with the point of contact of the two circles is normal to the hypocycloid, e.g., when the circle H touches the pitch circle at D, the point A is at C and CD is normal to the hypocycloid ALP.

Also, Arc AD = Arc CD (on circle H)

In the same way, if the circle E rolls outside the pitch circle, starting from P, an epicycloid PFB is obtained. Similar to the property of a hypocycloid, the line joining the generating point with the point of contact of the two circles is a normal to the epicycloid, e.g., when the circle E touches the pitch circle at K, the point P is at G and GK is normal to the epicycloid PFB.

Arc PK = Arc KJG (on circle E)

or Arc BK = Arc KG (on circle E)

A small portion of the curve near the pitch circle is used for the face of the tooth.

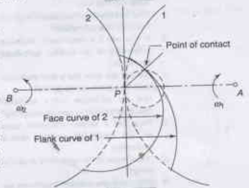


Fig. 10.19

Meshing of Teeth

During meshing of teeth, the face of a tooth on one gear is to mesh with the flank of another tooth on the other gear. Thus, for proper meshing, it is necessary that the diameter of the circle generating face of a tooth (on one gear) is the same as the diameter of the circle generating flank of the meshing tooth (on another gear); the pitch circle being the same in the two cases (Fig. 10.19).

Of course, the face and the flank of a tooth of a gear can be generated by two circles of different diameters. However, for interchangeability, the faces and flanks of both the teeth in the mesh are generated by the circles of the same diameter.

Consider a generating circle G rolling outside the pitch circle of the gear 2 (Fig. 10.20). It will generate

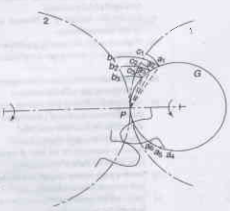
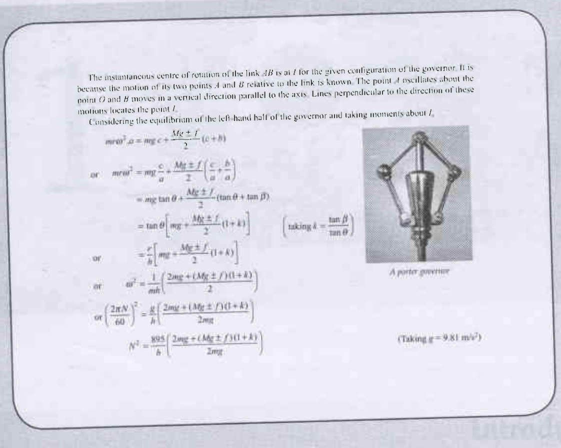


Fig. 10.20

Simple diagrams are given for easy visualization of the explanations.



The instantaneous centre of rotation of the link AB is at I for the given configuration of the governor. It is because the motion of its two points A and B relative to the link is known. The point A oscillates about the point O and B moves in a vertical direction parallel to the axis. Lines perpendicular to the direction of these motions locates the point I.

Considering the equilibrium of the left-hand half of the governor and taking moments about I,

$$mr\omega^2 . a = mg\, c + \frac{Mg \pm f}{2}(c+b)$$

or $$mr\omega^2 = mg\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a}+\frac{b}{a}\right)$$

$$= mg \tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)$$

$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}(1+k)\right] \qquad \left(\text{taking } k = \frac{\tan\beta}{\tan\theta}\right)$$

or $$= \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1+k)\right]$$

or $$\omega^2 = \frac{1}{mh}\left(\frac{2mg + (Mg \pm f)(1+k)}{2}\right)$$

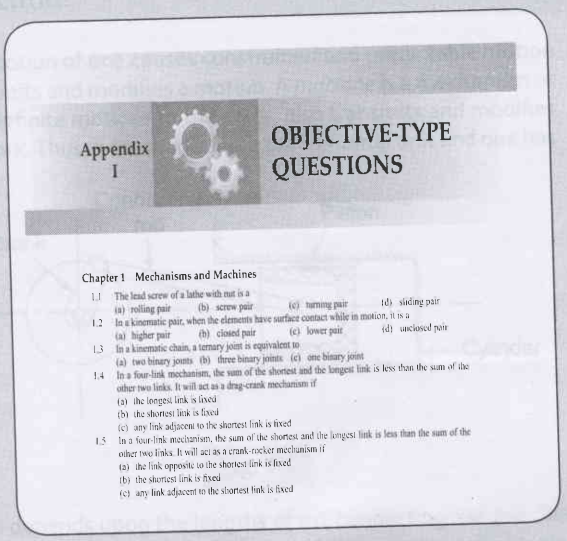
or $$\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}\left(\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right)$$

$$N^2 = \frac{895}{h}\left(\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right) \qquad (\text{Taking } g = 9.81 \text{ m/s}^2)$$

A porter governor

A number of photographs are given to emphasize the factual shape of various components.

An Appendix containing multiple choice questions is given at the end to help students prepare for competitive examinations.



Appendix I — OBJECTIVE-TYPE QUESTIONS

Chapter 1 Mechanisms and Machines

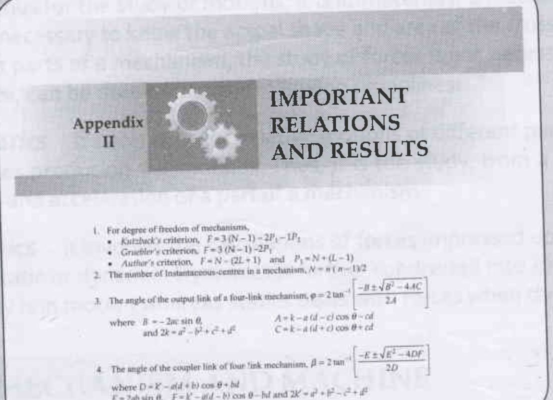
1.1 The lead screw of a lathe with nut is a
(a) rolling pair (b) screw pair (c) turning pair (d) sliding pair

1.2 In a kinematic pair, when the elements have surface contact while in motion, it is a
(a) higher pair (b) closed pair (c) lower pair (d) unclosed pair

1.3 In a kinematic chain, a ternary joint is equivalent to
(a) two binary joints (b) three binary joints (c) one binary joint

1.4 In a four-link mechanism, the sum of the shortest and the longest link is less than the sum of the other two links. It will act as a drag-crank mechanism if
(a) the longest link is fixed
(b) the shortest link is fixed
(c) any link adjacent to the shortest link is fixed

1.5 In a four-link mechanism, the sum of the shortest and the longest link is less than the sum of the other two links. It will act as a crank-rocker mechanism if
(a) the link opposite to the shortest link is fixed
(b) the shortest link is fixed
(c) any link adjacent to the shortest link is fixed



Appendix II — IMPORTANT RELATIONS AND RESULTS

1. For degree of freedom of mechanisms,
   - *Kutzback's criterion*, $F = 3(N-1) - 2P_1 - 1P_2$
   - *Gruebler's criterion*, $F = 3(N-1) - 2P_1$
   - *Author's criterion*, $F = N - (2L+1)$ and $P_1 = N + (L-1)$
2. The number of Instantaneous-centres in a mechanism, $N = n(n-1)/2$
3. The angle of the output link of a four-link mechanism, $\varphi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$

   where $B = -2ac \sin\theta$, and $2k = a^2 - b^2 + c^2 + d^2$; $A = k - a(d-c)\cos\theta - cd$, $C = k - a(d+c)\cos\theta + cd$
4. The angle of the coupler link of four link mechanism, $\beta = 2\tan^{-1}\left[\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right]$

   where $D = K' - a(d+b)\cos\theta + bd$, $E = 2ab\sin\theta$, $F = k' - a(d-b)\cos\theta - bd$ and $2k' = a^2 + b^2 - c^2 + d^2$

An Appendix containing important relations is given for ready reference.