Theory of Machines and Mechanism

Kinematics Fundamentals

Lecture 1

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Kinematics Fundamentals

- Degrees of Freedom
- Types of Motion
- Links, Joints, and Kinematic chains
- Determining Degree of Freedom
  - Degree of Freedom in Planar Mechanisms
- Mechanisms and Structures
- Number Synthesis

Types of Motion

- A rigid body free to move within a reference frame will, in the general case, have **complex motion**, which is a simultaneous combination of **rotation** and **translation**.
- In three-dimensional space, there may be rotation about any axis and also simultaneous translation which can be resolved into components along three axes. In a plane, or two-dimensional space, complex motion becomes a combination of simultaneous rotation about one axis (perpendicular to the plane) and also translation resolved into components along two axes in the plane.
- we will limit our present discussions to the case of **planar (2-D) kinematic systems**.

Types of Motion

- **Pure rotation**

  the body possesses one point (center of rotation) which has no motion with respect to the "stationary" frame of reference. All other points on the body describe arcs about that center. A reference line drawn on the body through the center changes only its angular orientation.

- **Pure translation**

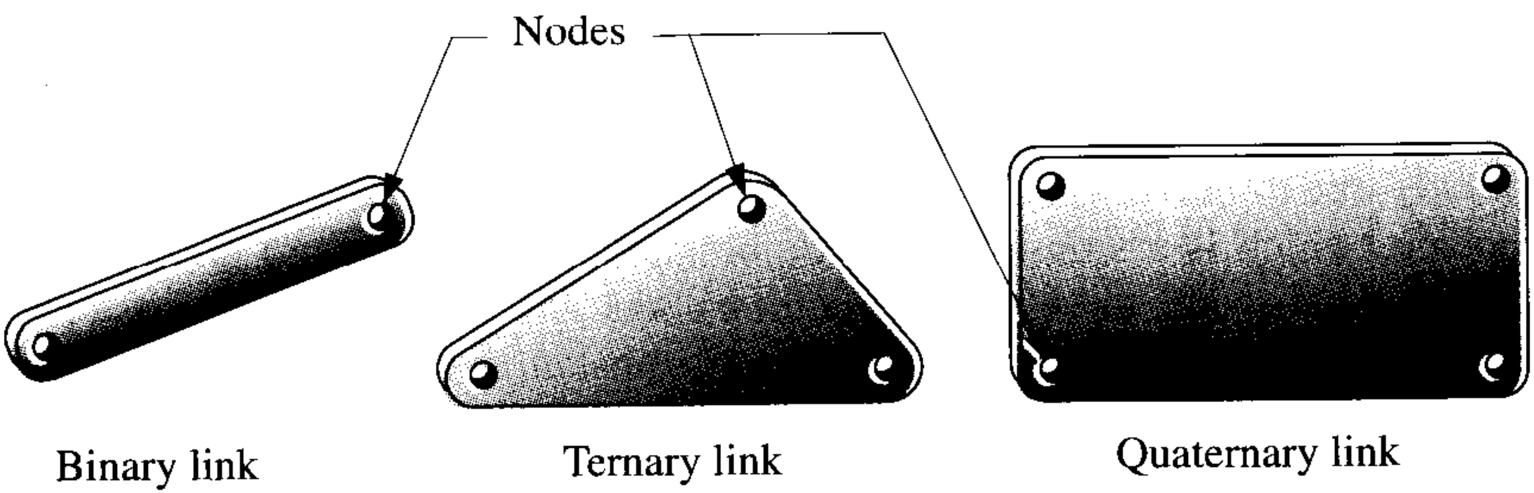
  all points on the body describe parallel (curvilinear or rectilinear) paths. A reference line drawn on the body changes its linear position but does not change its angular orientation.

- **Complex motion**

  a simultaneous combination of rotation and translation. Any reference line drawn on the body will change both its linear position and its angular orientation. Points on the body will travel nonparallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.

LINKS, JOINTS, AND KINEMATIC CHAINS

- A **link** is an (assumed) rigid body which possesses at least two **nodes** which are *points for attachment to other links*.



LINKS, JOINTS, AND KINEMATIC CHAINS

- Some of the common types of links are:

- **Binary link** - *one with two nodes.*
- **Ternary link** - *one with three nodes.*
- **Quaternary link** - *one with four nodes.*
- ***Pentagonals*** *– one with five nodes.*
- ***Hexagonals*** *– one with six nodes*

LINKS, JOINTS, AND KINEMATIC CHAINS

- A **joint** is *a connection between two or more links (at their nodes), which allows some motion, or potential motion, between the connected links. Joints (also called* ***kinematic*** **pairs**) can be classified in several ways:

1. By the type of contact between the elements, line, point, or surface.

2. By the number of degrees of freedom allowed at the joint.

3. By the type of physical closure of the joint: either **force** or **form** closed.

4. By the number of links joined (order of the joint).

LINKS, JOINTS, AND KINEMATIC CHAINS

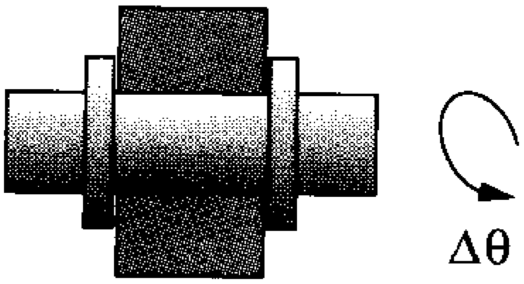
- **1. Classification by the Type of Contact**

  We can classify Joints by the type of contact as **Lower Pairs** or **Higher Pairs**. If joints have surface contact, they are called **Lower pair** (as with a pin surrounded by a hole). If joints have point or line contact, they are called **Higher pair**.

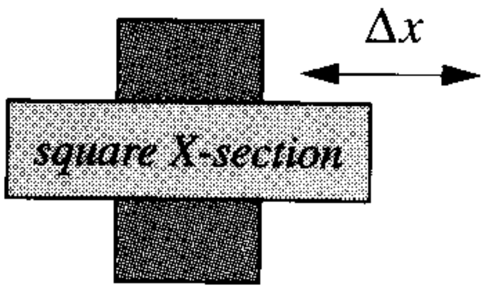
- The main practical advantage of lower pairs over higher pairs is their better ability to trap lubricant between their enveloping surfaces.

LINKS, JOINTS, AND KINEMATIC CHAINS

- The six possible **lower pairs** are: Revolute (R), Prismatic (P), Screw/Helical (H), Cylindric (C), Spherical (S), and Flat (F).

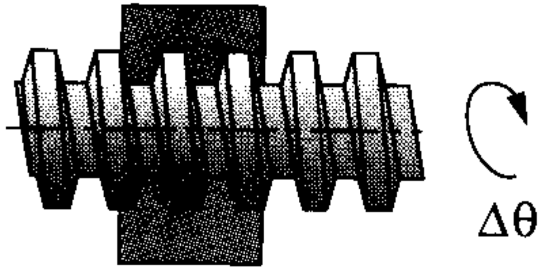


Revolute (R) joint—1 *DOF*

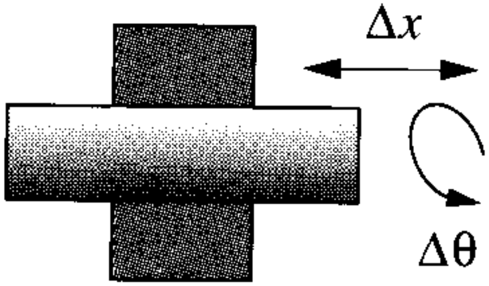


Prismatic (P) joint—1 *DOF*

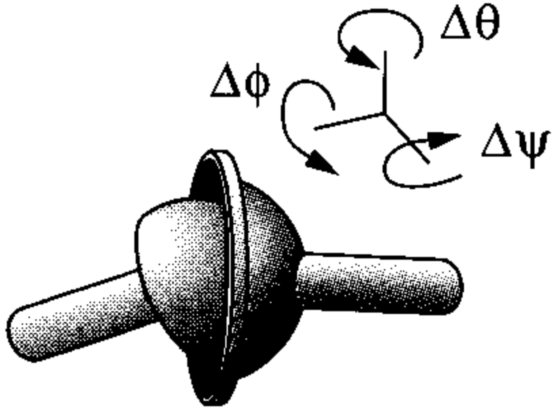
LINKS, JOINTS, AND KINEMATIC CHAINS



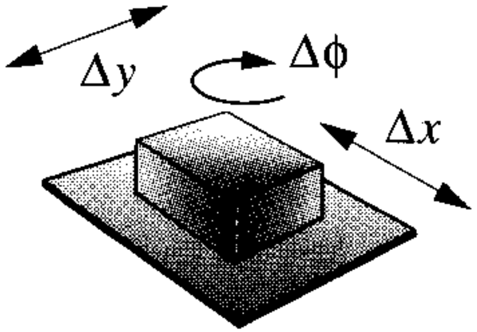
Helical (H) joint—1 *DOF*



Cylindric (C) joint—2 *DOF*

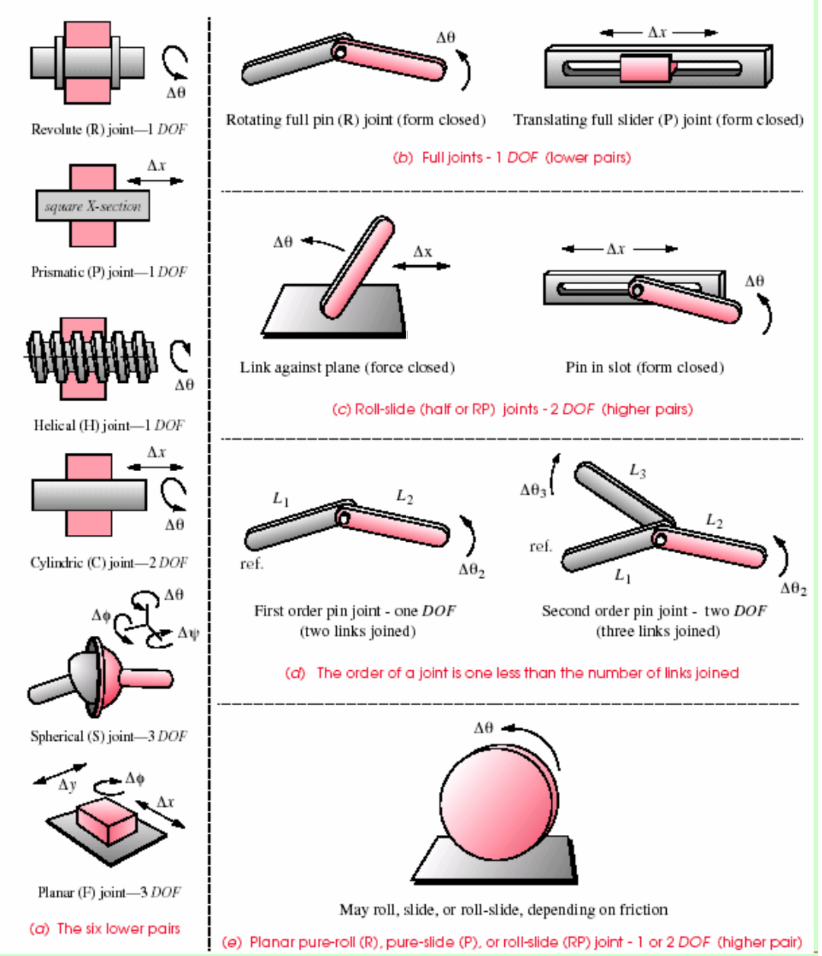


Spherical (S) joint—3 *DOF*



Planar (F) joint—3 *DOF*

Joints allow DOF between links

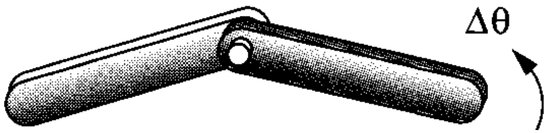


LINKS, JOINTS, AND KINEMATIC CHAINS

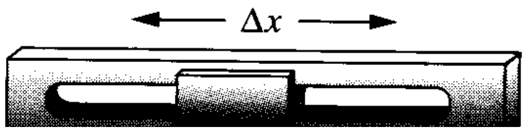
- **2. Classification by the Number of Degrees of Freedom allowed at the joint**

  We can classify Joints by the number of degrees of freedom allowed at the joint as **One-Freedom** Joints or **Full Joints, Two Freedom** Joints or **Half Joints** and **Three Freedom Joints.**
- Examples of one freedom joints are : a rotating pin joint (R) and a translating slider Joint (P).
- Examples of two freedom joints are: link against plane and a pin in slot.
- Examples of three freedom joints are: a spherical, or ball-and-socket joints.

LINKS, JOINTS, AND KINEMATIC CHAINS

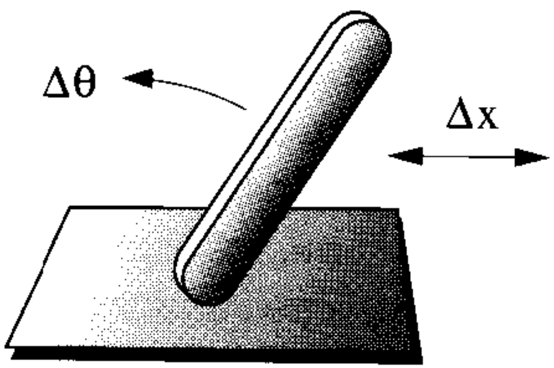


Rotating full pin (R) joint

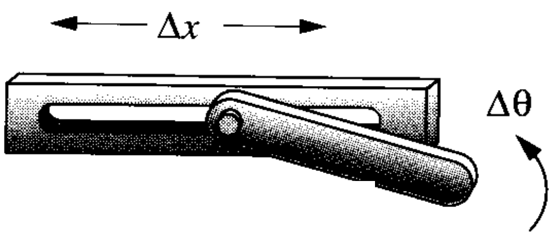


Translating full slider (P) joint

Full joints - 1 *DOF*



Link against plane



Pin in slot

Roll-slide (half or RP) joints - 2 *DOF*

LINKS, JOINTS, AND KINEMATIC CHAINS

- **3. Classification by the Type of Physical Closure of the Joint**

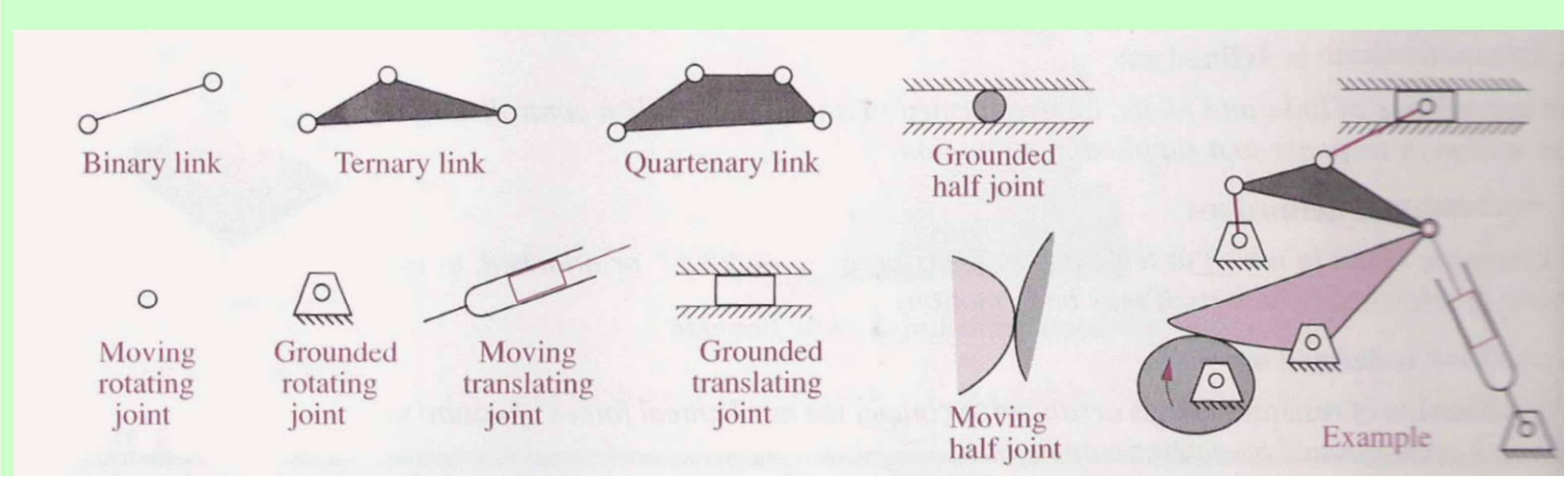
  We can classify Joints by the type of physical closure of the joint as **Force** closed or **Form** closed.

- A **form-closed** joint is kept together or *closed by its geometry. A pin in a hole or a slider in* a two-sided slot are form closed. In contrast, a **force-closed** joint, such as a pin in a half-bearing or a slider on a surface, *requires some external force to keep it together or closed. This force could be supplied by gravity, a spring, or any external means.*

LINKS, JOINTS, AND KINEMATIC CHAINS

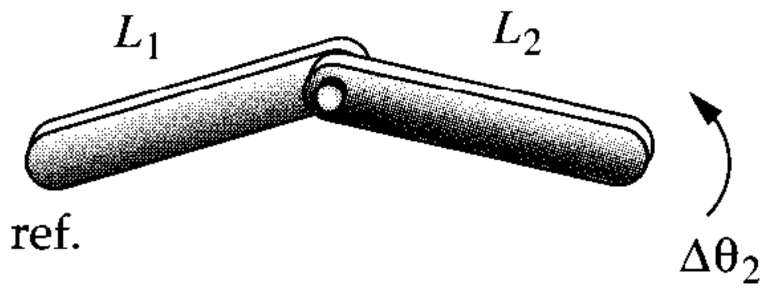
- **4. Classification by the Order of Joints**

  We can classify Joints by the order of joints as **1st order**, **2nd order** and so on.
- ***Order*** *is defined as the number of links joined minus one.*
- It takes two links to make a single joint; thus the simplest joint combination of two links has order one.
- Joint order has significance in the proper determination of overall degree of freedom for the assembly.

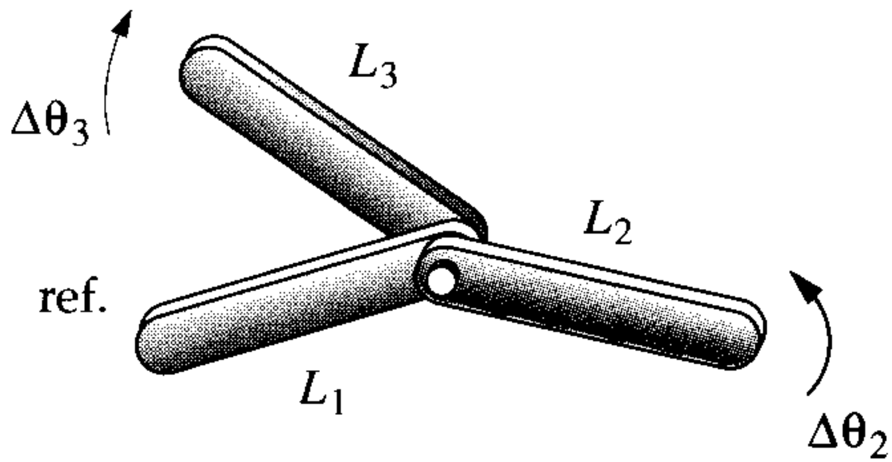


Binary link
Ternary link
Quartenary link
Grounded half joint
Moving rotating joint
Grounded rotating joint
Moving translating joint
Grounded translating joint
Moving half joint
Example

LINKS, JOINTS, AND KINEMATIC CHAINS



First order pin joint - one *DOF*



Second order pin joint - two *DOF*
(three links joined)

LINKS, JOINTS, AND KINEMATIC CHAINS

- **Kinematic Chain**

- A **kinematic chain** is defined as:

  An assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied input motion.

- A **mechanism** is defined as:

  A kinematic chain in which at least one link has been "grounded," or attached, to the frame of reference (which itself may be in motion).

- A **machine** is defined as:

  A combination of resistant bodies arranged to compel the mechanical forces of nature to do work accompanied by determinate motions.

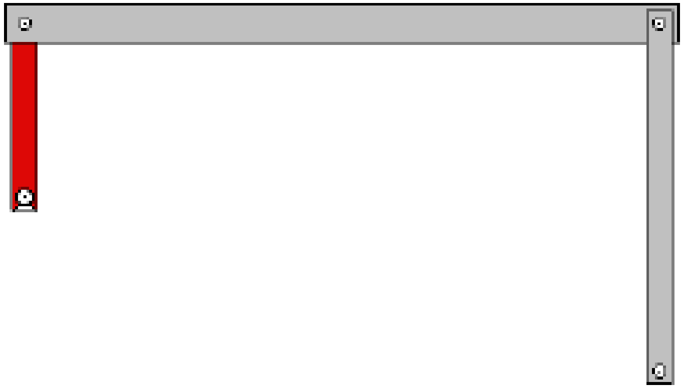
LINKS, JOINTS, AND KINEMATIC CHAINS

- **Ground**

  Any link or links that are fixed with respect to the reference frame

- **Crank**

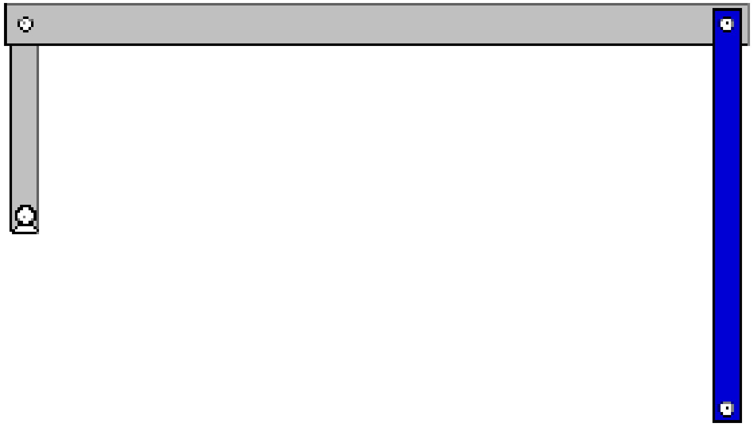
  A link which makes a complete revolution and is pivoted to ground



LINKS, JOINTS, AND KINEMATIC CHAINS

- **Rocker**

  A link which has oscillatory (back and forth) rotation and pivoted to ground



LINKS, JOINTS, AND KINEMATIC CHAINS

- **Coupler (connecting Rod)**

  A link which has complex motion and is pivoted to ground

Determining Degree of Freedom

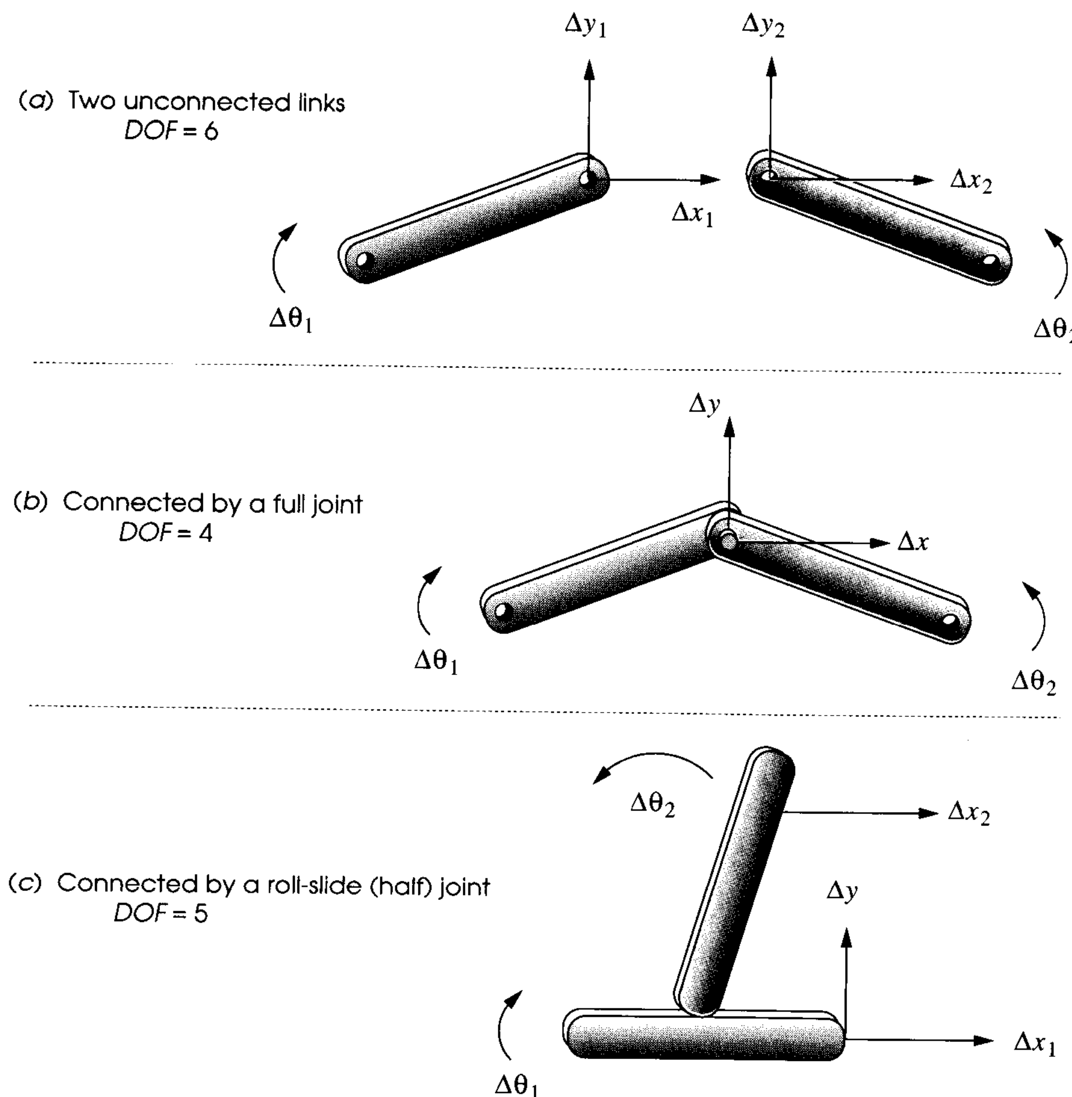
- **Degree of freedom** (also called the **mobility** *M) of a system can be defined as:*

  the number of inputs which need to be provided in order to create a predictable output or the number of independent coordinates required to define its position.

- **Degree of Freedom in Planar Mechanisms**

  To determine the overall *DOF of any mechanism, we must account for the number of* links and joints, and for the interactions among them.

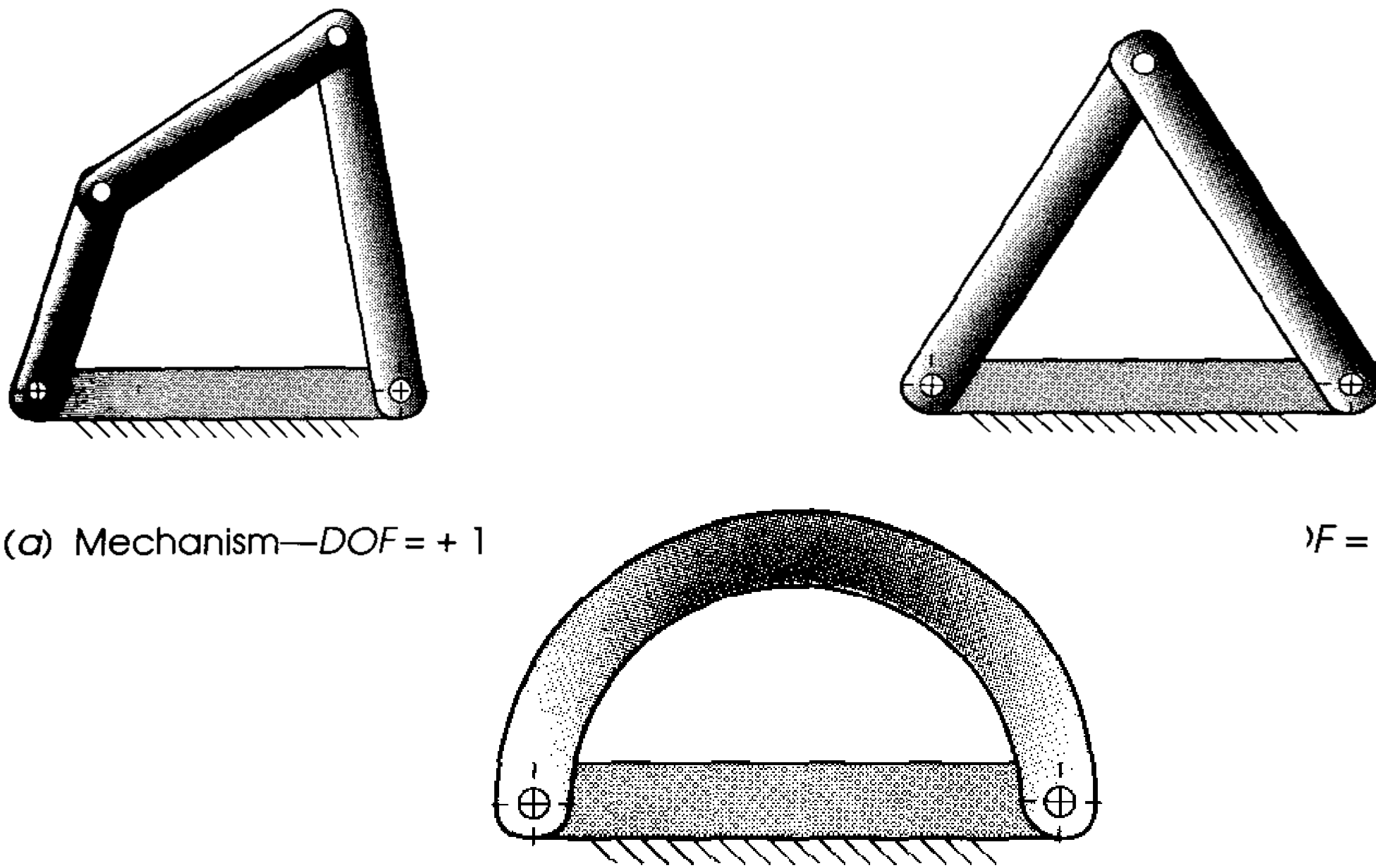
Determining Degree of Freedom



Mechanisms and Structures

- The degree of freedom of an assembly of links completely predicts its character.
- Thereare only three possibilities. *If the DOF is positive, it will be a **mechanism**, and the links* will have relative motion. *If the DOF is exactly zero, then it will be a **structure**, and no* motion is possible. *If the DOF is negative, then it is a **preloaded structure**, which means* that no motion is possible and some stresses may also be present at the time of assembly.

Mechanisms and Structures



(a) Mechanism—*DOF* = + 1

F = 0

(c) Preloaded structure—*DOF* = −1

GRUBLER'S CRITERION

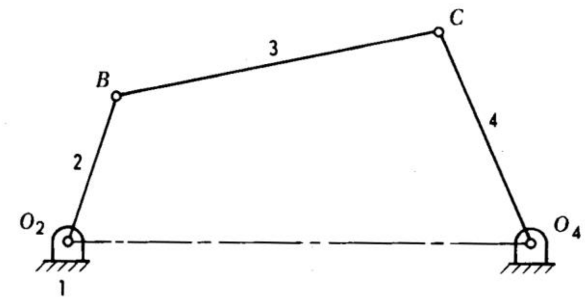
Number of degrees of freedom of a mechanism is given by

$$F = 3(n-1)-2l-h.$$

Where,

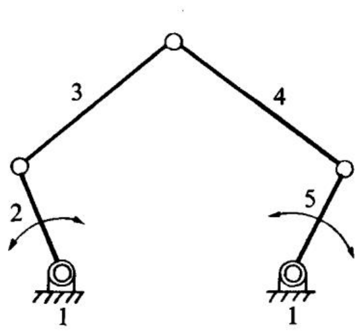
- F = Degrees of freedom
- n = Number of links in the mechanism.
- l = Number of lower pairs, which is obtained by counting the number of joints. If more than two links are joined together at any point, then, one additional lower pair is to be considered for every additional link.
- h = Number of higher pairs

Examples - DOF



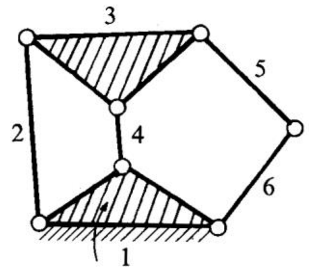
- F = 3(n-1)-2l-h
- Here, n = 4, l = 4 & h = 0.
- F = 3(4-1)-2(4) = 1
- I.e., one input to any one link will result in definite motion of all the links.

Examples - DOF



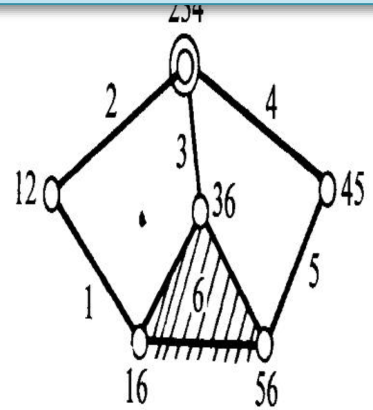
- $F = 3(n-1)-2l-h$
- Here, $n = 5$, $l = 5$ and $h = 0$.
- $F = 3(5-1)-2(5) = 2$
- I.e., two inputs to any two links are required to yield definite motions in all the links.

Examples - DOF



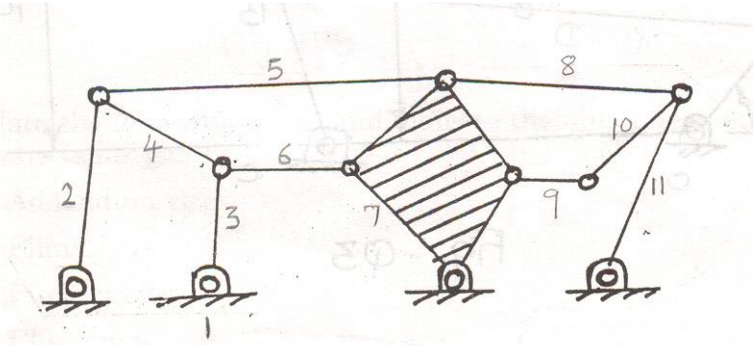
- $F = 3(n-1)-2l-h$
- Here, $n = 6$, $l = 7$ and $h = 0$.
- $F = 3(6-1)-2(7) = 1$
- I.e., one input to any one link will result in definite motion of all the links.

Examples - DOF



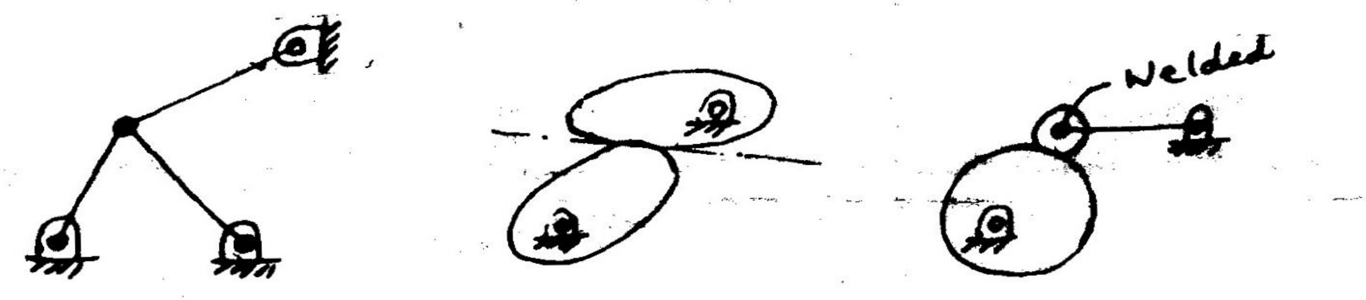
- F = 3(n-1)-2l-h
- Here, n = 6, l = 7 (at the intersection of 2, 3 and 4, two lower pairs are to be considered) and h = 0.
- F = 3(6-1)-2(7) = 1

Examples - DOF



- $F = 3(n-1)-2l-h$
- Here, $n = 11$, $l = 15$ (two lower pairs at the intersection of 3, 4, 6; 2, 4, 5; 5, 7, 8; 8, 10, 11) and $h = 0$.
- $F = 3(11-1)-2(15) = 0$

Examples - DOF



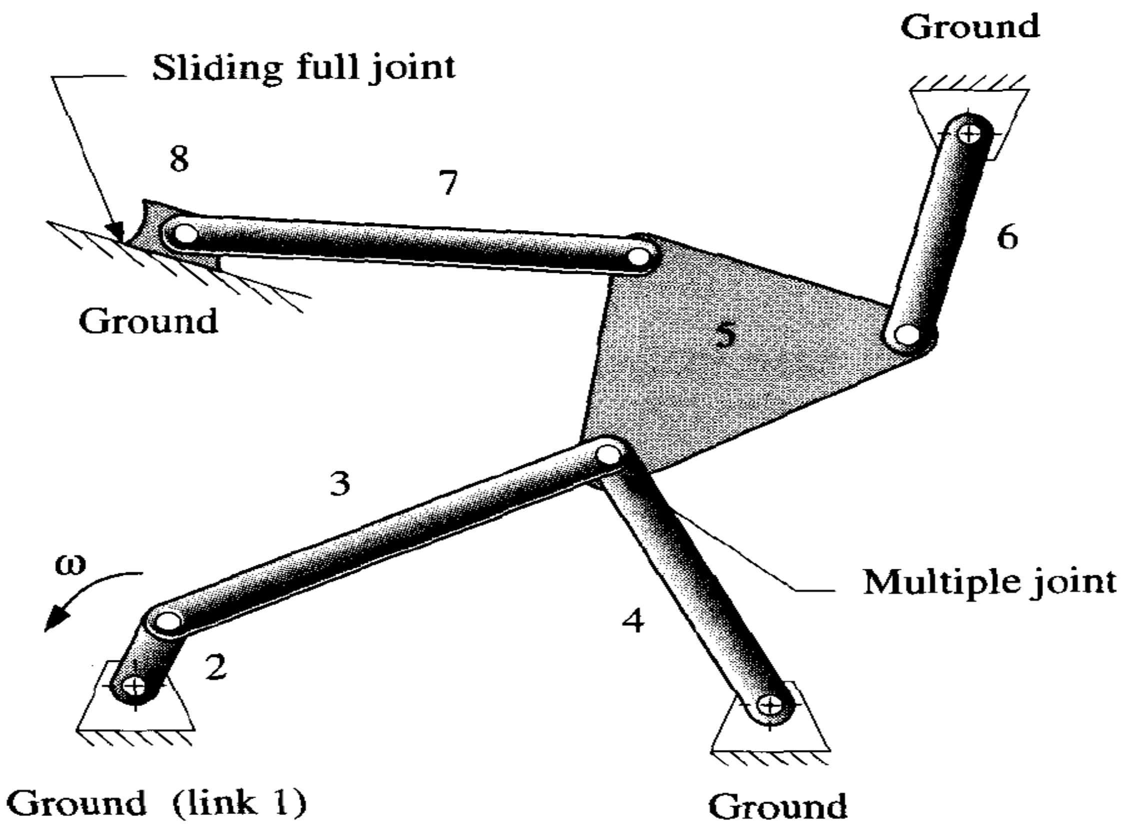
(a)
F = 3(n-1)-2l-h
Here, n = 4, l = 5 and h = 0.
F = 3(4-1)-2(5) = -1
I.e., it is a structure

(b)
F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

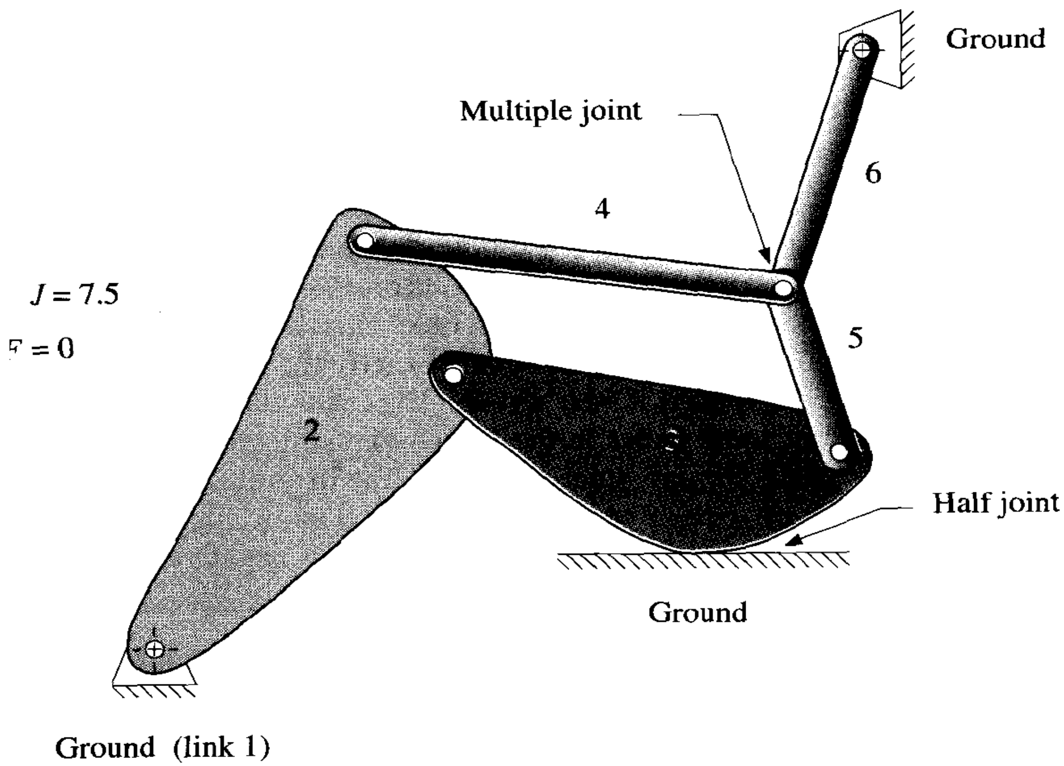
(c)
F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

Exercise

- Compute the *DOF of the following examples with* ***Kutzbach's equation.***



Exercise



Ground
Multiple joint
6
4
J = 7.5
F = 0
5
2
3
Half joint
Ground
Ground (link 1)