12 STATIC FORCE ANALYSIS

Introduction

In all types of machinery, forces are transmitted from one component to the other such as from a belt to a pulley, from a brake drum to a brake shoe, from a gear to shaft. In the design of machine mechanisms, it is necessary to know the magnitudes as well as the directions of forces transmitted from the input to the output. The analysis helps in selecting proper sizes of the machine components to withstand the stresses developed in them. If proper sizes are not selected, the components may fail during the machine operations. On the other hand, if the members are designed to have more strength than required, the machine may not be able to compete with others due to more cost, weight, size, etc.

If the components of a machine accelerate, inertia forces are produced due to their masses. However, if the magnitudes of these forces are small compared to the externally applied loads, they can be neglected while analysing the mechanism. Such an analysis is known as *static-force analysis*. For example, in lifting cranes, the bucket load and the static weight loads may be quite high relative to any dynamic loads due to accelerating masses, and thus static-force analysis is justified.

When the inertia effect due to the mass of the components is also considered, it is called *dynamic-force analysis* which will be dealt in the next chapter.

12.1 CONSTRAINT AND APPLIED FORCES

A pair of action and reaction forces which constrain two connected bodies to behave in a particular manner depending upon the nature of connection are known as *constraint forces* whereas forces acting from outside on a system of bodies are called *applied forces*.

Constraint forces As the constraint forces at a mechanical contact occur in pairs, they have no net force effect on the system of bodies. However, for an individual body isolated from the system, only one of each pair of constraint forces has to be considered.

Applied forces Usually, these forces are applied through direct physical or mechanical contact. However, forces like electric, magnetic and gravitational are applied without actual physical contact.

12.2 STATIC EQUILIBRIUM

A body is in static equilibrium if it remains in its state of rest or motion. If the body is at rest, it tends to remain at rest and if in motion, it tends to keep the motion. In static equilibrium

- the vector sum of all the forces acting on the body is zero, and
- the vector sum of all the moments about any arbitrary point is zero.

Mathematically,

$$\Sigma \mathbf{F} = 0 \tag{12.1}$$

$$\Sigma \mathbf{T} = 0 \tag{12.2}$$

In a planer system, forces can be described by two-dimensional vectors and, therefore,

$$\Sigma \mathbf{F}_x = 0 \tag{12.3}$$

$$\Sigma \mathbf{F}_y = 0 \tag{12.4}$$

$$\Sigma \mathbf{T}_z = 0 \tag{12.5}$$

12.3 EQUILIBRIUM OF TWO- AND THREE-FORCE MEMBERS

A member under the action of two forces will be in equilibrium if

- the forces are of the same magnitude,
- the forces act along the same line, and
- the forces are in opposite directions.

Figure 12.1 shows such a member.

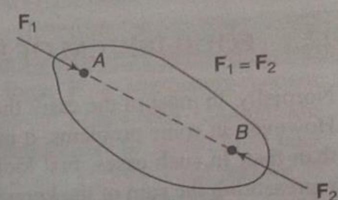

Fig. 12.1

A member under the action of three forces will be in equilibrium if

- the resultant of the forces is zero, and
- the lines of action of the forces intersect at a point (known as *point of concurrency*).

Figure 12.2 (a) shows a member acted upon by three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ and is in equilibrium as the lines of action of forces intersect at one point O and the resultant is zero. This is verified by adding the forces vectorially [Fig. 12.2 (b)]. As the head of the last vector $\mathbf{F}_3$ meets the tail of the first vector $\mathbf{F}_1$, the resultant is zero. It is not necessary to add the three vectors in order to obtain the resultant as is shown in Fig. 12.2 (c) in which $\mathbf{F}_2$ is added to $\mathbf{F}_3$ and then $\mathbf{F}_1$ is taken.

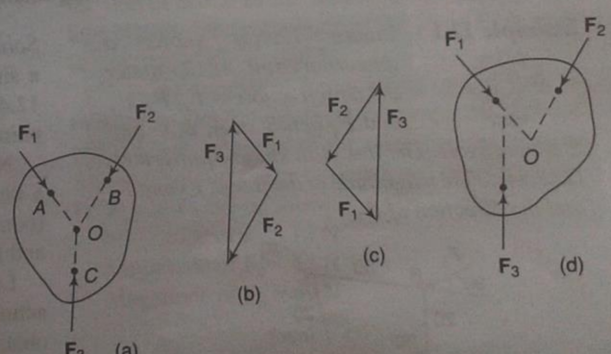

Fig. 12.2

Figure 12.2 (d) shows a case where the magnitudes and directions of the forces are the same as before, but the lines of action of the forces do not intersect at one point. Thus, the member is not in equilibrium.

Consider a member in equilibrium in which the force $\mathbf{F}_1$ is completely known, $\mathbf{F}_2$ is known in direction only and $\mathbf{F}_3$ is completely unknown. The point of applications of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are A, B and C respectively. To solve such a problem, first find the point of concurrency O from the two forces with known directions, i.e., from $\mathbf{F}_1$ and $\mathbf{F}_2$. Joining O with C gives the line of action of the third force $\mathbf{F}_3$. To know the magnitudes of the forces $\mathbf{F}_2$ and $\mathbf{F}_3$, take a vector of proper magnitude and direction to represent the force $\mathbf{F}_1$. From its two ends, draw lines parallel to the lines of action of the forces $\mathbf{F}_2$ and $\mathbf{F}_3$ forming a force triangle [Figs 12.2 (b) or (c). Mark arrowheads on $\mathbf{F}_2$ and $\mathbf{F}_3$ so that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are in the same order.

If the lines of action of two forces are parallel then the point of concurrency lies at infinity and, therefore, the third force is also parallel to the first two.

12.4 MEMBER WITH TWO FORCES AND A TORQUE

A member under the action of two forces and an applied torque will be in equilibrium if

- the forces are equal in magnitude, parallel in direction and opposite in sense, and
- the forces form a couple which is equal and opposite to the applied torque.

Figure 12.3 shows a member acted upon by two equal forces $\mathbf{F}_1$ and $\mathbf{F}_2$ and an applied torque $\mathbf{T}$. For equilibrium,

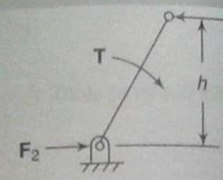

Fig. 12.3

$$T = F_1 \times h = F_2 \times h \tag{12.6}$$

where T, F_1 and F_2 are the magnitudes of $\mathbf{T}$, $\mathbf{F}_1$ and $\mathbf{F}_2$ respectively. $\mathbf{T}$ is clockwise whereas the couple formed by $\mathbf{F}_1$ and $\mathbf{F}_2$ is counter-clockwise.

12.5 EQUILIBRIUM OF FOUR-FORCE MEMBERS

Normally, in most of the cases the above conditions for equilibrium of a member are found to be sufficient. However, in some problems, it may be found that the number of forces on a member is four or even more than that. In such cases, first look for the forces completely known and combine them into a single force representing the sum of the known forces. This may reduce the number of forces acting on a body to two or three. However, in planer mechanisms, a four-force system is also solvable if one force is known completely along with lines of action of the others. The following examples illustrate the procedure.

Example 12.1 *Figure 12.4(a) shows a quaternary link ABCD under the action of forces F_1, F_2, F_3 and F_4 acting at A, B, C and D respectively. The link is in static equilibrium. Determine the magnitude of the forces F_2 and F_3 and the direction of F_3.*

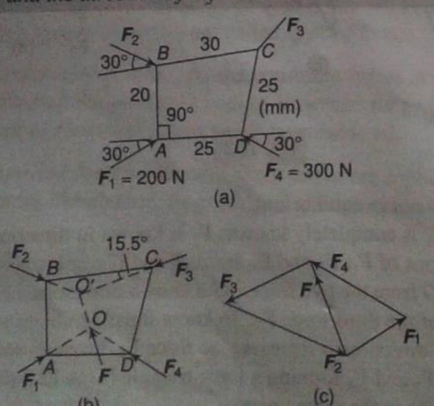

Fig. 12.4

Solution The forces $\mathbf{F}_1$ and $\mathbf{F}_4$ can be combined into a single force $\mathbf{F}$ by obtaining their resultant [Figs 12.4(b) and (c)]. The force $\mathbf{F}$ acts through O, the point where lines of action of $\mathbf{F}_1$ and $\mathbf{F}_4$ meet.

Now, the four-force member $ABCD$ is reduced to a three-force member under the action of forces $\mathbf{F}$ (completely known), $\mathbf{F}_2$ (only the direction known) and $\mathbf{F}_3$ (completely unknown).

Let $\mathbf{F}$ and $\mathbf{F}_2$ meet at O'. Then CO' is the line of action of force $\mathbf{F}_3$. By completing the force triangle, obtain the magnitude of $\mathbf{F}_2$ and $\mathbf{F}_3$.

Magnitude of $\mathbf{F}_2$ = <u>380 N</u>

Magnitude of $\mathbf{F}_3$ = <u>284 N</u>

Line of action of force $\mathbf{F}_3$ makes an angle of 15.5° with CB.

Example 12.2 *Figure 12.5(a) shows a cam with a reciprocating-roller follower system. Various forces acting on the follower are indicated in the figure. At the instant, an external force F_1 of 40 N, a spring force F_2 of*

15 N and cam force $\mathbf{F}_5$ *of unknown magnitude act on it along the lines of action as shown.* $\mathbf{F}_3$ *and* $\mathbf{F}_4$ *are the bearing reactions. Determine the magnitudes of the forces* $\mathbf{F}_3$*,* $\mathbf{F}_4$ *and* $\mathbf{F}_5$*. Assume no friction.*

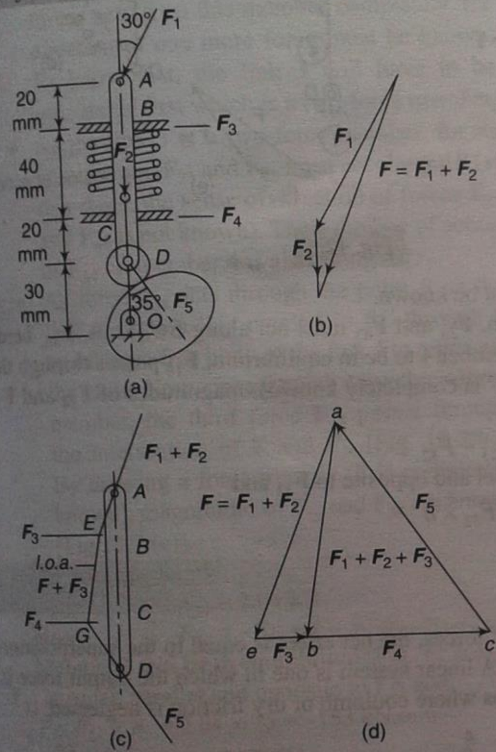

Fig. 12.5

Solution As in the previous example, forces $\mathbf{F}_1$ and $\mathbf{F}_2$ can be combined into a single force $\mathbf{F}$ by obtaining their resultant [Figs 12.5(b)]. Their resultant must pass through point A, the point of intersection of $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus, the number of forces acting on the body is reduced to four.

Now, assume that the magnitude of force $\mathbf{F}_3$ is known and the force $\mathbf{F}$ is to be combined with it. Then the resultant must pass through their point of intersection, i.e., the point E [Fig. 12.5(c)]. This way, the body becomes under the action of three forces which must be concurrent for the equilibrium of the body. Thus, the resultant of $\mathbf{F}$ and $\mathbf{F}_3$ must pass through the point G, the point of intersection of the forces $\mathbf{F}_4$ and $\mathbf{F}_5$. Therefore, the line of action of the resultant of $\mathbf{F}$ and $\mathbf{F}_3$ is EG.

Now since the force $\mathbf{F}$ is completely known and the lines of action of $\mathbf{F}_3$ and their resultant are known, the force diagram can be made. First take the force $\mathbf{F}$ and then to add $\mathbf{F}_3$ draw a line parallel to its line of action through the head of $\mathbf{F}$ [Fig. 12.5(d)]. Through the tail of vector $\mathbf{F}$ draw a line parallel to the line of action of the resultant. The triangle *aeb* thus provides the magnitude of the force $\mathbf{F}_3$ as well as resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.

Now the number of forces acting on the body is reduced to three. One force is completely known and the lines of action of the other two are known. A triangle of forces can be drawn and magnitudes of $\mathbf{F}_3$, $\mathbf{F}_4$ and $\mathbf{F}_5$ can be found.

Magnitude of $\mathbf{F}_3 = 12$ N
Magnitude of $\mathbf{F}_4 = 42$ N
Magnitude of $\mathbf{F}_5 = 60$ N

12.6 FORCE CONVENTION

The force exerted by the member i on the member j is represented by $\mathbf{F}_{ij}$.

12.7 FREE-BODY DIAGRAMS

A free-body diagram is a sketch or diagram of a part isolated from the mechanism in order to determine the nature of forces acting on it.

Figure 12.6(a) shows a four-link mechanism. The free-body diagrams of its members 2, 3 and 4 are shown in Figs 12.6 (b) (c) and (d) respectively. Various forces acting on each member are also shown. As the mechanism is in static equilibrium, each of its members must be in equilibrium individually.

Member 4 is acted upon by three forces **F**, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$.

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$.

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T**.

Initially, the direction and the sense of some of the forces may not be known.

Assume that the force **F** on the member 4 is known completely. To know the other two forces acting on this member completely, the direction of one more force must be known.

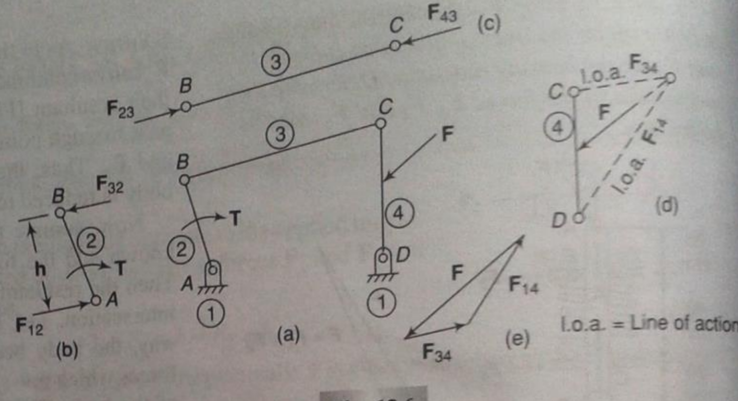

Fig. 12.6

Link 3 is a two-force member and for its equilibrium, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ must act along *BC*. Thus, $\mathbf{F}_{34}$, being equal and opposite to $\mathbf{F}_{43}$, also acts along *BC*. For the member 4 to be in equilibrium, $\mathbf{F}_{14}$ passes through the intersection of **F** and $\mathbf{F}_{34}$. By drawing a force triangle (**F** is completely known), magnitudes of $\mathbf{F}_{14}$ and $\mathbf{F}_{34}$ can be known [Fig.12.6 (e)].

Now $$F_{34} = F_{43} = F_{23} = F_{32}$$

Member 2 will be in equilibrium if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

$$T = F_{12} \times h = F_{32} \times h$$

12.8 SUPERPOSITION

In linear systems, if a number of loads act on a system of forces, the net effect is equal to the superposition of the effects of the individual loads taken one at a time. A linear system is one in which the output force is directly proportional to the input force, i.e., in mechanisms where coulomb or dry friction is neglected.

Example 12.3 *A slider-crank mechanism with the following dimensions is acted upon by a force F = 2 kN at B as shown in Fig. 12.7(a):*

OA = 100 mm, AB = 450 mm.

Determine the input torque T on the link OA for the static equilibrium of the mechanism for the given configuration.

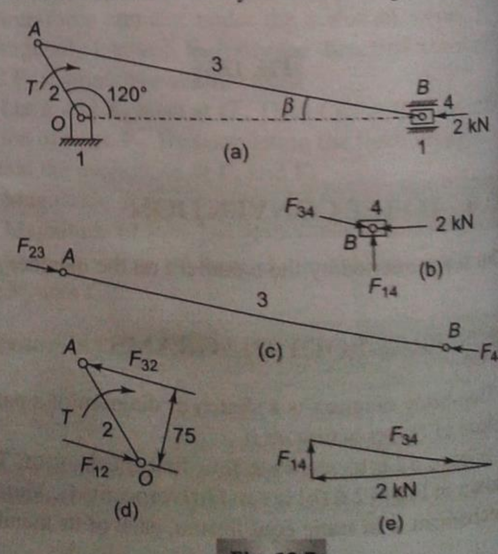

Fig. 12.7

Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

Member 4 is acted upon by three forces **F**, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ [Fig. 12.7(b)].

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ [Fig. 12.7(c)].

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T** [Fig. 12.7(d)].

Initially, the direction and the sense of some of the forces are not known.

Now, adopt the following procedure:

- Force **F** on member 4 is known completely (= 2 kN, horizontal). To know the other two forces acting on this member completely, the direction of one more force must be known. To know that, the link 3 will have to be considered first which is a two-force member.
- As the link 3 is a two-force member, for its equilibrium, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ must act along AB (at this stage, the sense of direction of forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ is not known). Thus, the line of action of $\mathbf{F}_{34}$ on member 4 is also along AB.
- As force $\mathbf{F}_{34}$ acts through the point B on the link 4, draw a line parallel to BC through B by taking a free body of the link 4 to represent the same. Now, since the link 4 is a three-force member, the third force $\mathbf{F}_{14}$ passes through the intersection of **F** and $\mathbf{F}_{34}$ [Fig. 12.7(b)]. By drawing a force triangle (**F** is completely known), magnitudes of $\mathbf{F}_{14}$ and $\mathbf{F}_{34}$ are known [Fig.12.7 (e)].

From force triangle,

$$F_{34} = 2.04 \text{ kN}$$

Now, $\qquad F_{34} = -F_{43} = F_{23} = -F_{32}$

Member 2 will be in equilibrium [Fig. 12.7(e)] if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

$$T = F_{32} \times h = 2.04 \times 75 = -153 \text{ kN.mm}$$

($h = 75$ mm on measurement)

The input torque has to be equal and opposite to this couple i.e.,

$T =$ <u>153 kN.mm</u> or <u>153 N.m</u> (clockwise)

Analytical solution

$$\cos\beta = \frac{1}{n}\sqrt{n^2 - \sin^2\theta} = \frac{1}{4.5}\sqrt{4.5^2 - \sin^2 120^\circ}$$

$= 0.981$

or $\beta = 11.1^\circ$ (Refer Section 13.5)

$F_{34}\cos 11.1^\circ = 2$ or $F_{34} = 2.04$ kN

$\angle OAB = 180^\circ - 120^\circ - 11.1^\circ = 48.9^\circ$

$\therefore T = F_{32} \times h = 2.04 \times 100 \sin 48.9^\circ$

$= 153.7$ kN.mm

- The direction and senses of forces in the analytical solution can be known by drawing rough figures instead of drawing these to the scale.

Example 12.4 *A four-link mechanism with the following dimensions is acted upon by a force 80 ∠150° N on the link DC [Fig. 12.8(a)]:*

AD = 500 mm, AB = 400 mm, BC = 1000 mm, DC = 750 mm, DE = 350 mm

Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration.

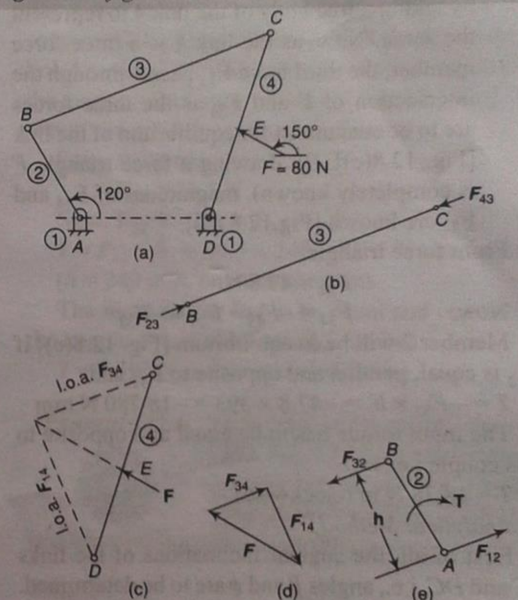

Fig. 12.8

Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

Member 4 is acted upon by three forces **F**, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$.

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$.

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T**.

Initially, the direction and the sense of some of the forces are not known.

Now, adopt the following procedure:

- Force **F** on the member 4 is known completely. To know the other two forces acting on this member completely, the direction of one more force must be known. To know that, the link 3 will have to be considered first which is a two-force member.

- As the link 3 is a two-force member (Fig. 12.8b), for its equilibrium, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ must act along BC (at this stage, the sense of direction of forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ is not known). Thus, the line of action of $\mathbf{F}_{34}$ is also along BC.
- As the force $\mathbf{F}_{34}$ acts through the point C on the link 4, draw a line parallel to BC through C by taking a free body of the link 4 to represent the same. Now, as the link 4 is a three-force member, the third force $\mathbf{F}_{14}$ passes through the intersection of $\mathbf{F}$ and $\mathbf{F}_{34}$ as the three forces are to be concurrent for equilibrium of the link [Fig. 12.8(c)]. By drawing a force triangle F is completely known), magnitudes of $\mathbf{F}_{14}$ and $\mathbf{F}_{34}$ are known [Fig.12.8 (d)].

From force triangle,

$$F_{34} = 47.8 \text{ N}$$

Now, $\quad F_{34} = -F_{43} = F_{23} = -F_{32}$

Member 2 will be in equilibrium [Fig. 12.8(e)] if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

$$T = -F_{32} \times h = -47.8 \times 393 = -18\,780 \text{ N.mm}$$

The input torque has to be equal and opposite to this couple i.e.,

$T =$ 18.78 N.m (clockwise)

Analytical Method

First of all, the angular inclinations of the links BC and DC, i.e., angles β and ϕ are to be determined. This may be done by drawing the configuration or by analytical means (Section 4.1).

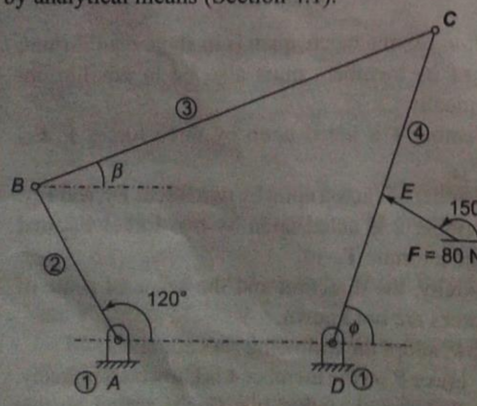

Fig. 12.9

We have (Fig. 12.9),

$$2k = a^2 - b^2 + c^2 + d^2$$

$$k = (0.4^2 - 1^2 + 0.75^2 + 0.5^2)/2 = -0.01375$$

$$A = k - a(d-c)\cos\theta - cd = -0.01375 - 0.4\,(0.5 - 0.75)\cos 120° - 0.75 \times 0.5 = -0.439$$

$$B = -2ac\sin\theta = -2 \times 0.4 \times 0.75 \sin 120° = -0.52$$

$$C = k - a(d+c)\cos\theta + cd = -0.01375 - 0.4\,(0.5 + 0.75)\cos 120° + 0.75 \times 0.5 = 0.611$$

$$\varphi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \qquad \text{(Eq. 4.7)}$$

$$= 2\tan^{-1}\left[\frac{0.52 \pm \sqrt{(-0.52)^2 - 4 \times (-0.439)(0.611)}}{2 \times (-0.439)}\right]$$

$$= 2\tan^{-1}(0.727 \text{ or } -0.439)$$

$$= 72° \text{ or } -47.4°$$

Taking the first value (value in the first quadrant),

We have,

$$a\sin\theta + b\sin\beta = c\sin\varphi \qquad \text{(Eq. 4.3)}$$

$$0.4 \times \sin 120° + 1 \times \sin\beta = 0.75 \times \sin 72°$$

or $\sin\beta = 0.712$ or $\beta = 21.5°$

Position vectors

$\mathbf{AB} = 0.4\,\angle 120°$, $\mathbf{BC} = 1.0\,\angle 21.5°$, $\mathbf{DC} = 0.75\,\angle 72°$, $\mathbf{DE} = 0.35\,\angle 72°$

The direction of $\mathbf{F}_{34}$ is along BC since it is a two-force member,

$$\mathbf{F}_{34} = F_{34}\,\angle 21.5°$$

As the link DC is in static equilibrium, no resultant forces or moments are acting on it.

Taking moments of the forces about point D,

$$M_d = \mathbf{F}_4 \times \mathbf{DE} + \mathbf{F}_{34} \times \mathbf{DC} = \mathbf{0} \qquad \text{(i)}$$

Moments are the cross-multiplication of the vector, so it should be done in rectangular coordinates.

$$\mathbf{F}_4 = 80\,\angle 150° = -69.28\,\mathbf{i} + 40\,\mathbf{j}$$

$$\mathbf{DE} = 0.35\,\angle 72° = 0.108\,\mathbf{i} + 0.333\,\mathbf{j}$$

$$\mathbf{F}_{34} = F_{34}\,\angle 21.5° = F_{34}(0.93\,\mathbf{i} + 0.367\,\mathbf{j})$$

$$\mathbf{DC} = 0.75\,\angle 72° = 0.232\,\mathbf{i} + 0.713\,\mathbf{j}$$

Inserting the values of vectors in (i),

$$(-69.28\,\mathbf{i} + 40\,\mathbf{j}) \times (0.108\,\mathbf{i} + 0.333\,\mathbf{j}) + F_{34}(0.93\,\mathbf{i} + 0.367\,\mathbf{j}) \times (0.235\,\mathbf{i} + 0.712\,\mathbf{j}) = 0$$

$$\text{or}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -69.28 & 40 & 0 \\ 0.108 & 0.333 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.93F_{34} & 0.367F_{34} & 0 \\ 0.232 & 0.713 & 0 \end{vmatrix} = 0$$

or $(-69.28 \times 0.333 - 40 \times 0.108) + (0.93\ F_{34} \times 0.713 - 0.367F_{34} \times 0.232)$

or $-27.4 + 0.58\ F_{34} = 0$ or $F_{34} = 47.3$ N

Thus, $\mathbf{F}_{34} = 47.3\angle 21.5°$

Now, $\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34} = 47.3\angle 21.5°$

$\mathbf{F}_{12} = -\mathbf{F}_{32} = 47.3\ \angle 21.5°$

$T_{2c} = \mathbf{F}_{12} \times \mathbf{AB} = 47.3\angle 21.5° \times 0.4\ \angle 120°$
$= 18.9$ N.m

Example 12.5 *A four-link mechanism with the following dimensions is acted upon by a force of 50 N on the link DC at the point E (Fig. 12.10a):*

AD = 300 mm, AB = 400 mm, BC = 600 mm, DC = 640 mm, DE = 840 mm

Determine the input torque T on the link AB for the static equilibrium of the mechanism for the given configuration.

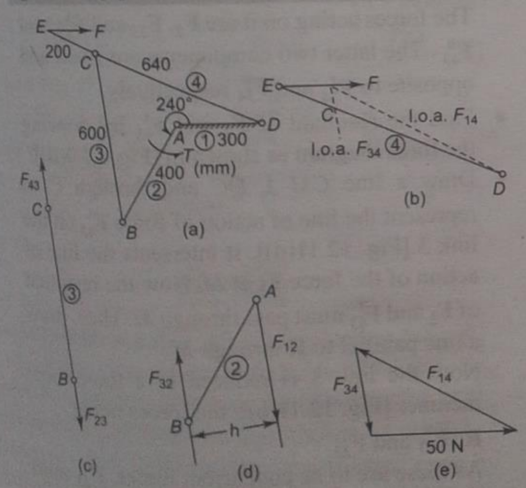

Fig. 12.10

Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

Member 4 is acted upon by three forces **F**, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ [Fig. 12.10(b)]

Member 3 is acted upon by two forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ [Fig. 12.10(c)]

Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T** [Fig. 12.10(d)]

Initially, the direction and the sense of some of the forces are not known.

The procedure to solve the problem graphically is exactly similar to the previous example. In brief, the link 3 is a two-force member, so it provides the line of action of force $\mathbf{F}_{34}$ on the link 4. Since the link 4 is a three-force member and forces are to be concurrent, the lines of action of all the forces on the link 4 can be drawn. Then the force diagram provides the magnitude of various forces [Fig. 12.10(e)]. The rest of the procedure is self-explanatory.

From force triangle,

$F_{34} = 30.5$ N

Now, $\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34}$ or $F_{32} = 30.5$ N

$T = F_{32} \times h = 30.5 \times 249 = 7595$ N.mm

($h = 249$ mm, on measurement)

The input torque has to be equal and opposite to the couple obtained by parallel forces i.e.,

$T = \underline{7.595 \text{ N.m}}$ (counter clockwise)

Example 12.6 *For the mechanism shown in Fig. 12.11a, determine the torque on the link AB for the static equilibrium of the mechanism.*

Solution

(i) Composite Graphical Solution As the mechanism is in static equilibrium, each of its members must also be in equilibrium individually.

- Member 4 is acted upon by three forces $\mathbf{F}_1$, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ [Fig. 12.11(b)].
- Member 3 is acted upon by three forces $\mathbf{F}_2$, $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$.
- Member 2 is acted upon by two forces $\mathbf{F}_{32}$ and $\mathbf{F}_{12}$ and a torque **T**.

To solve the problem graphically, proceed as follows:

- Force $\mathbf{F}_1$ on the member 4 is known completely. To know the other two forces acting on this member completely, the direction of one more force must be known. However, as the link 3 now is a three-force member, it is not possible to know the direction of the force $\mathbf{F}_{34}$ from that also.

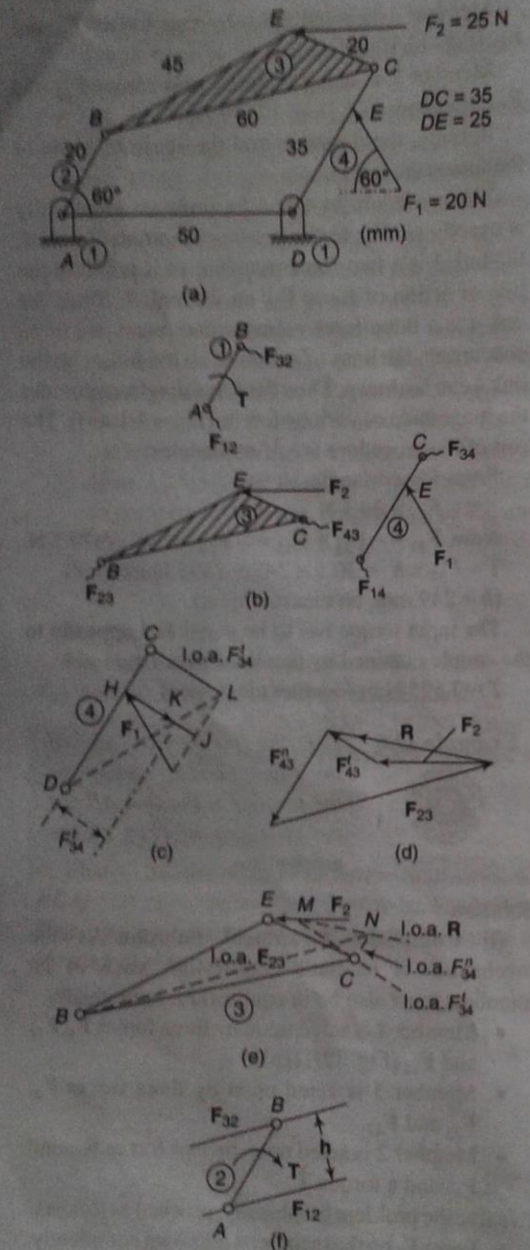

Fig. 12.11

- Consider two components, normal F^n_{34} and tangential F^t_{34} of the force F_{34}. Assume F^n_{34} to be along DC and F^t_{34} perpendicular to DC through C. Also, take the components of force F_1, i.e., F^n_1 and F^t_1 along the same directions.
- Now as the link 4 is in equilibrium, no moments are acting on it. Taking moments of all the forces acting on it about pivot point D,

$$M = F^t_{34} \times DC + F^t_1 \times DE = 0$$

(No moments are to be there due to forces F^n_{34}, F^n_1 and F_{14} as these forces pass through the point D)

or $F^t_{34} = -\, F^t_1 \times \dfrac{DE}{DC}$

Graphically, the above value of F^t_{34} can be obtained by taking F_1 on the link 4 to some convenient scale and then taking two components of it, the normal component along DC and the tangential component perpendicular to DC being shown by JH in Fig. 12.11(c). Also, draw $CL \perp DC$. Draw JL parallel to HC. Join DL which intersects JH at K. Now, KH is the component F^t_{34} the direction being towards K.
- Now consider the equilibrium of the link 3. The forces acting on it are F_2, F_{23} and F^t_{43} and F^n_{43}. The latter two components are equal and opposite to F^t_{34} and F^n_{34} respectively.
- Find the resultant of F_2 and F^t_{43} by drawing the force diagram as shown in [Fig. 12.9(d)].
- Draw a line $CM \perp DC$ and through C to represent the line of action of force F^t_{43} on the link 3 [Fig. 12.11(d)]. It intersects the line of action of the force F_2 at M. Now the resultant of F_2 and F^t_{43} must pass through M. Thus, draw a line parallel to **R** through M.

Now the link 3 is reduced to a three-force member [Fig. 12.11(e)], the forces being:

R, F^n_{43} and F_{23}.

As these are to be concurrent forces, F_{23} must pass through the intersection of lines of forces F^n_{43} and **R**. Draw a line parallel to DC and through C to represent the line of action of force F^n_{43}. This intersects the line of action of **R** at N. Join BN. Now BN represents the line of action of force F_{23}.

- Complete the force diagram and find the magnitude of $\mathbf{F}_{23}$ and $\mathbf{F}^{n}_{43}$.
- Draw line parallel to line *BN* through *B* on link 2 [Fig. 12.11(f)] to represent the line of action of force $\mathbf{F}_{32}$ and a parallel line through *A* to represent the line of action of force $\mathbf{F}_{12}$.

From force diagram,

$$F_{23} = 49.4 \text{ N}$$

Now, $F_{32} = -F_{23} = -49.4$

Member 2 will be in equilibrium if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

$T = -F_{32} \times h = -49.8 \times 14.3 = -706.4$ N.mm

The input torque has to be equal and opposite to this couple, i.e.,

$T = 706.4$ N.mm (clockwise)

(ii) Graphical Solution by Superposition method

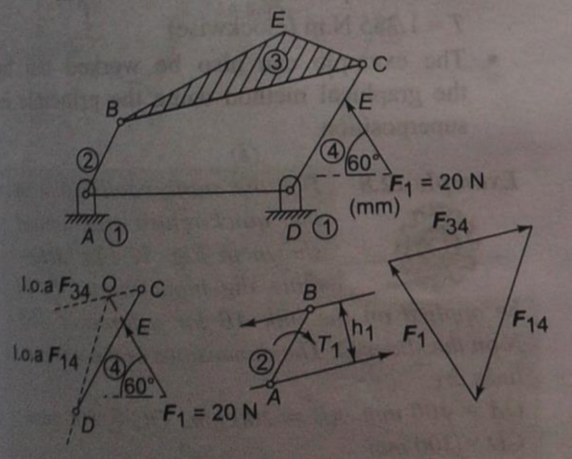

Fig. 12.12

Subproblem a (Fig. 12.12) Neglecting force $\mathbf{F}_2$

Link 4 is a three-force member in which only one force $\mathbf{F}_1$ is known. However, the line of action of $\mathbf{F}_{34}$ can be obtained from the equilibrium of the link 3 which is a two-force member and is acted upon by forces $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$. Thus, lines of action of forces $\mathbf{F}_{43}$ or $\mathbf{F}_{34}$ are along *BC*. If $\mathbf{F}_1$ and $\mathbf{F}_{34}$ intersect at *O* then line of action of $\mathbf{F}_{14}$ will be along *OD* since the three forces are to be concurrent. Draw the force triangle ($\mathbf{F}_1$ is completely known) and obtain the magnitudes of forces $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$.

$$F_{34} = 17.6 \text{ N}$$

Also, $F_{34} = -F_{43} = F_{23} = -F_{32} = -17.6$ N

So, the direction of F_{32} is opposite to that of F_{23}.

Link 2 is subjected to two forces and a torque $\mathbf{T}_1$.

For equilibrium, $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$.

$T_1 = F_{32} \times h_1 = 17.6 \times 14.9 = 262$ N.mm clockwise

Subproblem b (Fig. 12.13) Neglecting force $\mathbf{F}_1$.

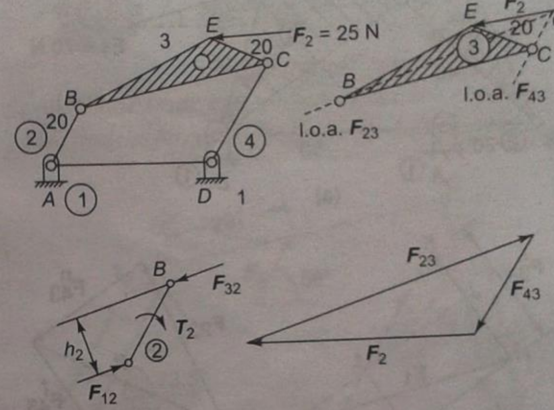

Fig. 12.13

Link 4 is a two-force member. The two forces $\mathbf{F}_{14}$ and $\mathbf{F}_{34}$ are to be equal and opposite and their line of action is to be the same which shows that the line of action is along *DC*. Thus, the line of action of $\mathbf{F}_{43}$ is also along *DC*.

Link 3 is a three-force member in which $\mathbf{F}_2$ is completely known, only the direction of $\mathbf{F}_{43}$ is known (parallel to *DC*) and $\mathbf{F}_{23}$ is completely unknown. If the line of action of $\mathbf{F}_2$ and $\mathbf{F}_{43}$ meet at *O*, the line of action of $\mathbf{F}_{23}$ will be along *OB* as the three forces are to be concurrent. Draw the force triangle ($\mathbf{F}_2$ is completely known) by taking $\mathbf{F}_2$ to a suitable scale and two lines parallel to lines of action of $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$. Mark arrowheads on $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ to know the directions.

$$F_{23} = 33.2 \text{ N}$$

and $F_{23} = -F_{32} = -33.2$ N

So, direction of F_{32} is opposite to that of F_{23}.

Link 2 is subjected to two forces and a torque $\mathbf{T}_2$.

For equilibrium, $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$.

$T_2 = F_{32} \times h_2 = 33.2 \times 13.2 = 438$ N.mm lockwise

Total torque $= 262 + 438 = 700$ N.mm

Example 12.7 *For the static equilibrium of the mechanism of [Fig. 12.14(a)], find the torque to be applied on link AB.*

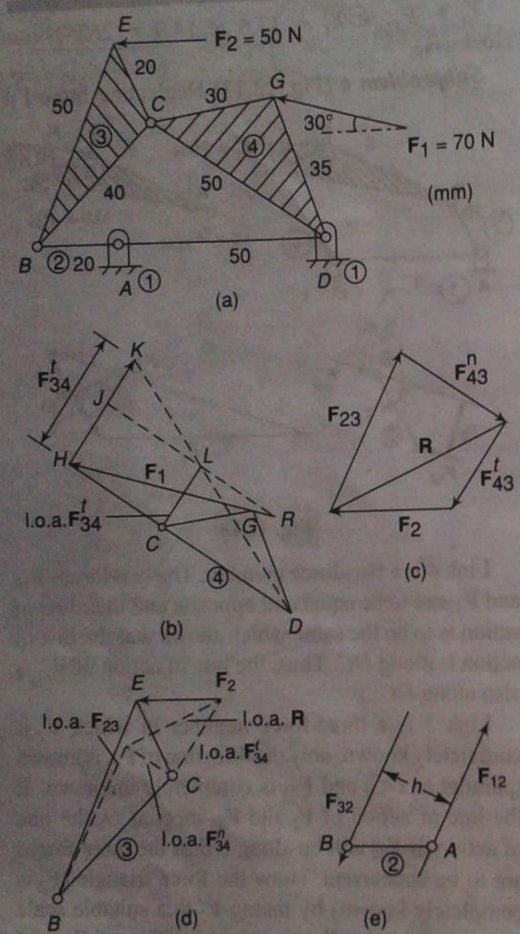

Fig. 12.14

Solution The point of action of force $\mathbf{F}_1$ on the link 4 is an offset point *G*. If *DC* is extended and let the line of action of force $\mathbf{F}_1$ meet at *H* then the force $\mathbf{F}_1$ may be considered to be acting on a virtual point *H* on the link *DC* as the magnitude of force as well as the magnitude couple effect is not going to vary.

Now, the problem can be solved by adopting the procedure given in the previous example. In brief:

- Take vector *RH* to represent force $\mathbf{F}_1$ to some scale.
- Find force $\mathbf{F}^t_{34}$. Its magnitude is given by *HK* and it acts through *C*.
- Find the resultant of $\mathbf{F}_2$ and $\mathbf{F}^t_{43}$ and its point of application in the free body diagram.
- Through point *C*, draw line for the vector $\mathbf{F}^n_{43}$ and then find the line of application of $\mathbf{F}_{23}$. From force diagram,

$$F_{23} = 68.9 \text{ N}$$

Now, $\quad F_{32} = -F_{23} = -49.4$

Member 2 will be in equilibrium if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and

$$T = -F_{32} \times h = -68.9 \times 18.65 = -1285 \text{ N.mm}$$

The input torque has to be equal and opposite to this couple, i.e.,

$$T = 1.285 \text{ N.m (clockwise)}$$

- The example can also be worked out by the graphical method using the principle of superposition.

Example 12.8 *For the static equilibrium of the quick-return mechanism shown in Fig. 12.15a, determine the input torque T_2 to be applied on the link AB for a force of 300 N on the slider D. The dimensions of the various links are*

OA = 400 mm, AB = 200 mm, OC = 800 mm, CD = 300 mm

Solution The slider at *D* or the link 6 is a three-force member. Lines of action of the forces are [Fig.12.15(b)]

- **F**, 300 N as given
- $\mathbf{F}_{56}$ along *CD*, as link 5 is a two force member
- $\mathbf{F}_{16}$, normal reaction, perpendicular to slider motion

Draw the force diagram and determine the direction sense of forces $\mathbf{F}_{56}$ and $\mathbf{F}_{16}$. From the force $\mathbf{F}_{56}$, the directions of forces $\mathbf{F}_{65}$, $\mathbf{F}_{35}$ and $\mathbf{F}_{53}$ are known. Now, the link 3 is a three-force member. Lines of action of the forces are

- $\mathbf{F}_{53}$, known completely through *C*

- F_{43}, perpendicular to slider motion through *B*
- F_{13}, unknown through *A*.

As the lines of action of forces acting through *B* and *C* are known, the line of action of $\mathbf{F}_{13}$ through *A* must also pass through the point of intersection of the other two forces. Find the sense of the direction of force $\mathbf{F}_{43}$ by drawing the force triangle.

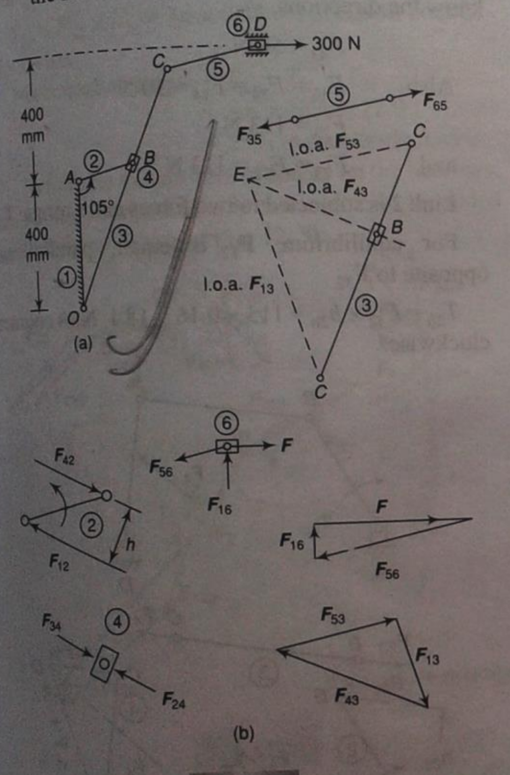

Fig. 12.15

Considering the equilibrium of the slider 4, the direction of $\mathbf{F}_{24}$ is known which is equal and opposite to $\mathbf{F}_{34}$. Considering the equilibrium of the link 2, the lines of action of $\mathbf{F}_{42}$ and $\mathbf{F}_{12}$ are drawn and the perpendicular distance between them is measured.

Then, torque on the link 2,

$T_2 = F_{42} \times h = 403 \times 120 = 48\ 360$ N counter-clockwise

Example 12.9 *A four-link mechanism is subjected to the following external forces (Fig.12.16 & Table 12.1): Determine the shaft torque $\mathbf{T}_2$ on the input link AB for static equilibrium of the mechanism. Also find the forces on the bearings A, B, C and D.*

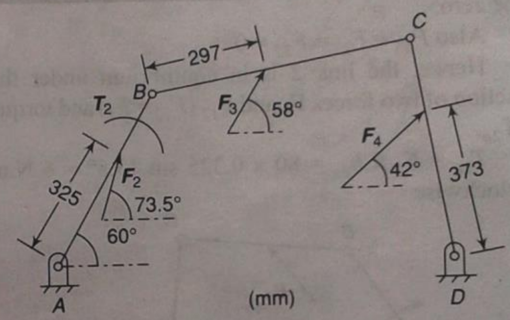

Fig. 12.16

Solution

The solution of the stated problem is worked out by

(i) graphical solution by using theorem of superposition, i.e., dividing the problem into subproblems by considering only one force on a member and ignoring the other forces on other members
(ii) a composite graphical solution
(iii) analytical solution

(i) Graphical Method by Superposition

Subproblem a (Fig. 12.17) Neglecting forces $\mathbf{F}_3$ and $\mathbf{F}_4$.

Table 12.1

Link	Length	Force	Magnitude	Point of application force (r)
AB (2)	500 mm	F_2	80 ∠ 73.5°N	325 mm from *A*
AB (3)	660 mm	F_3	144 ∠ 58°N	297 mm from B
AB (4)	560 mm	F_4	60 ∠ 42°N	373 mm from *D*
AB (1)	1000 mm	-	(Fixed link)	

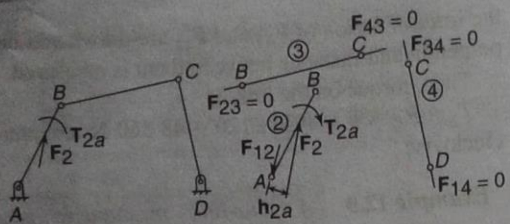

Fig. 12.17

Links 3 and 4 are both two-force members. Therefore, $\mathbf{F}_{43}$ can be along BC and $\mathbf{F}_{43}$, along DC. As $\mathbf{F}_{43}$ is to be equal and opposite of $\mathbf{F}_{34}$, both must be zero.

Also $F_{43} = F_{23} = F_{32} = 0$

Hence, the link 2 is in equilibrium under the action of two forces $\mathbf{F}_2$ and $\mathbf{F}_{12}$ $(F_{12} = F_2)$ and torque $\mathbf{T}_{2a}$.

$T_{2a} = F_2 \times h_{2a} = 80 \times 0.325 \sin 13.5° = 6$ N.m clockwise

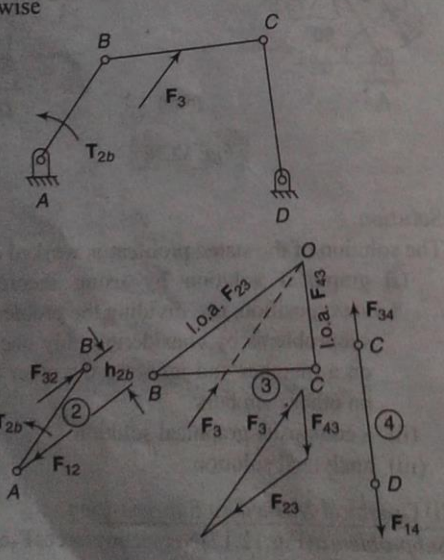

Fig. 12.18

Subproblem b (Fig. 12.18) Neglecting forces $\mathbf{F}_2$ and $\mathbf{F}_4$.

Link 4 is a two-force member.

$\therefore$ $\mathbf{F}_{34} = \mathbf{F}_{14}$ magnitudes unknown, directions parallel to DC

Link 3 is a three-force member in which $\mathbf{F}_3$ is completely known, only the direction of $\mathbf{F}_{43}$ is know (parallel to DC) and $\mathbf{F}_{23}$ is completely unknown. If the line of action of $\mathbf{F}_3$ and $\mathbf{F}_{43}$ meet at O, the line of action of $\mathbf{F}_{23}$ will be along OB. Draw the force triangle ($\mathbf{F}_3$ is completely known) by taking $\mathbf{F}_3$ to a suitable scale and two lines parallel to lines of action of $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$. Mark arrowheads on $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ to know the directions.

$$F_{43} = 50 \text{ N}$$

Also, $F_{43} = F_{34} = F_{14} = 50$ N

$$F_{23} = 113 \text{ N}$$

and $F_{23} = F_{32} = 113$ N

Link 2 is subjected to two forces and a torque $\mathbf{T}_{2b}$.

For equilibrium, $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$.

$T_{2b} = F_{32} \times h_{2b} = 113 \times 0.16 = 18.1$ N.m counter-clockwise.

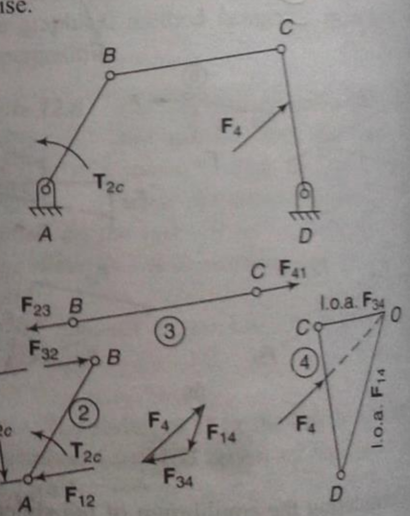

Fig. 12.19

Subproblem c (Fig. 12.19) Neglecting forces $\mathbf{F}_2$ and $\mathbf{F}_3$.

Link 4 is a three-force member in which only one force $\mathbf{F}_4$ is known. However, the line of action of $\mathbf{F}_{34}$ can be obtained from the equilibrium of the link 3 which is a two-force member. $\mathbf{F}_{34}$ will be equal and opposite to $\mathbf{F}_{43}$ which is along BC. If $\mathbf{F}_4$ and $\mathbf{F}_{14}$ intersect at O then the line of action of $\mathbf{F}_{14}$ will be along OD. Draw the force triangle ($\mathbf{F}_4$ is completely known) and obtain the magnitudes of forces $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$.

$$F_{14} = 34.8 \text{ N}$$

Also, $F_{34} = F_{43} = F_{23} = F_{32} = 34$ N

Link 2 is subjected to two forces and a torque $\mathbf{T}_{2c}$. For equilibrium,

$\mathbf{F}_{12} = \mathbf{F}_{32}$.

$T_{2c} = F_{32} \times h_{2c} = 34 \times 0.38 = 12.9$ N.m counter-clockwise.

$$\text{Net crankshaft torque} = T_{2a} + T_{2b} + T_{2c}$$
$$= -6 + 18.1 + 12.9$$
$$= 25 \text{ N.m counter-clockwise}$$

To find the magnitudes of forces on the bearings, the results obtained in a, b and c have to be added vectorially as shown in Fig. 12.20.

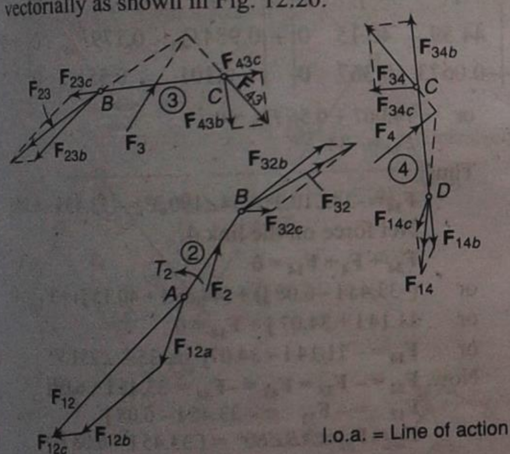

Fig. 12.20

$F_{14} = 80$ N

$F_{34} = F_{43} = 60$ N

$F_{23} = F_{32} = 137$ N

$F_{12} = 204$ N

(ii) Composite graphical solution

The problem can be solved by following the same procure as in examples 12.6 and 12.7. The solution is worked out in Fig. 12.21 which is self-explanatory. After obtaining the force $\mathbf{F}_{32}$, the resultant R' of this force with the force $\mathbf{F}_2$ can be obtained by drawing a force diagram. This resultant passes through the intersection of the lines of action of $\mathbf{F}_2$ and $\mathbf{F}_{23}$.

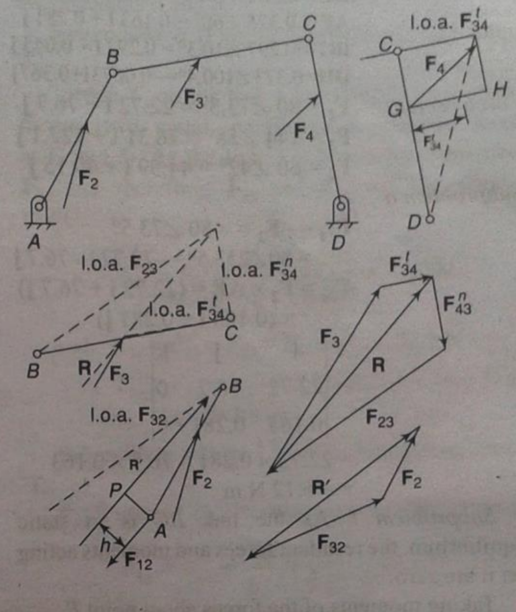

Fig. 12.21

$$T = \mathbf{R}' \times h = \mathbf{F}_{12} \times h$$
$$= 208.8 \times 117 = 24\,430 \text{ N.mm or } \underline{24.43 \text{ N.m}}$$

(iii) Analytical Method

First of all, determine the angular inclinations of the links BC and DC, i.e., angles β and ϕ. This may be done by drawing the configuration or by analytical means (section 4.2). Angles β and ϕ are found to be 10.3° and 100.4° (Fig. 12.22) respectively using analytical means.

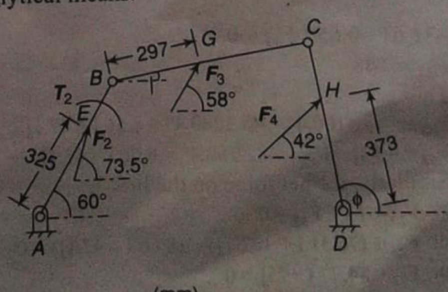

Fig. 12.22

Position vectors $\mathbf{AB} = 0.5\ \angle 60^\circ = 0.25\mathbf{i} + 0.433\mathbf{j}$

$\mathbf{BC} = 0.66\ \angle 10.3^\circ = 0.649\ \mathbf{i} + 0.118\ \mathbf{j}$

$\mathbf{DC} = 0.56\ \angle 100.4^\circ = -0.101\ \mathbf{i} + 0.551\ \mathbf{j}$

$\mathbf{AE} = 0.325\ \angle 60^\circ = 0.163\ \mathbf{i} + 0.281\ \mathbf{j}$

$\mathbf{BG} = 0.297\ \angle 10.3^\circ = 0.292\ \mathbf{i} + 0.053\ \mathbf{j}$

$\mathbf{DH} = 0.373 \angle 100.4^\circ = -0.0673\mathbf{i} + 0.367\mathbf{j}$

Force vectors $\mathbf{F}_2 = 80\ \angle 73.5^\circ = 22.72\ \mathbf{i} + 76.7\ \mathbf{j}$

$\mathbf{F}_3 = 144\ \angle 58^\circ = 76.31\ \mathbf{i} + 122.1\ \mathbf{j}$

$\mathbf{F}_4 = 60\ \angle 42^\circ = 44.59\ \mathbf{i} + 40.15\ \mathbf{j}$

Subproblem a

$$\mathbf{F}_{14} = -\mathbf{F}_2 = -80\ \angle 73.5^\circ = 80\angle 253.5^\circ = -22.72\mathbf{i} - 76.7\ \mathbf{j}$$

$$T_{2a} = \mathbf{F}_2 \times \mathbf{AE} = (22.72\ \mathbf{i} + 76.7\ \mathbf{j}) \times (0.163\ \mathbf{i} + 0.281\ \mathbf{j})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 22.72 & 76.7 & 0 \\ 0.163 & 0.281 & 0 \end{vmatrix}$$

$$= 22.72 \times 0.281 - 76.7 \times 0.163$$

$$= -6.12 \text{ N.m}$$

Subproblem b As the link *BC* is in static equilibrium, the resultant forces and moments acting on it are zero.

Taking moments of the forces about point *B*,

$$M_b = \mathbf{F}_3 \times \mathbf{BG} + \mathbf{F}_{43} \times \mathbf{BC} = \mathbf{0} \qquad \text{(i)}$$

As the direction of $\mathbf{F}_{43}$ is along *DC* if force $\mathbf{F}_4$ is ignored,

$\therefore\ \mathbf{F}_{43} = F_{43}\ \angle 100.4^\circ = -0.181\ F_{43}\ \mathbf{i} + 0.983\ F_{43}\mathbf{j}$

Inserting the values of vectors in (i),

$(76.31\ \mathbf{i} + 122.1\mathbf{j}) \times (0.292\ \mathbf{i} + 0.053\mathbf{j})$
$+(-0.181F_{43}\mathbf{i} + 0.983F_{43}\mathbf{j}) \times (0.649\mathbf{i} + 0.118\mathbf{j}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 76.3 & 122.1 & 0 \\ 0.292 & 0.053 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.181F_{43} & 0.983F_{43} & 0 \\ 0.649 & 0.118 & 0 \end{vmatrix} = 0$$

$-31.61 - 0.659\ F_{43} = 0$

$F_{43} = -48$

Thus

$\mathbf{F}_{43} = -48\angle 100.4^\circ = 48\angle 280.4^\circ = 8.66\ \mathbf{i} - 47.1\mathbf{j}$

$\mathbf{F}_{14} = -\mathbf{F}_{34} = \mathbf{F}_{43} = 48\angle 280.4^\circ = 8.66\ \mathbf{i} - 47.1\mathbf{j}$

Similarly, the net force on the link 3,

$\mathbf{F}_{23} + \mathbf{F}_3 + \mathbf{F}_{43} = 0$

or $\mathbf{F}_{23} + (76.31\ \mathbf{i} + 122.1\mathbf{j}) + (8.66\ \mathbf{i} - 47.1\mathbf{j}) = 0$

or $\mathbf{F}_{23} + 84.97\ \mathbf{i} + 75\mathbf{j} = 0$

or $\mathbf{F}_{23} = -84.97\ \mathbf{i} - 75\mathbf{j}$ or $113.3\ \angle 221.4^\circ$

or $\mathbf{F}_{32} = 113.3\ \angle 41.4^\circ = 84.97\ \mathbf{i} + 75\mathbf{j}$

$\mathbf{F}_{12} = -\mathbf{F}_{32} = -84.97\ \mathbf{i} - 75\ \mathbf{j}$

$T_{2b} = \mathbf{F}_{32} \times AB\angle 60^\circ = (84.97\ \mathbf{i} + 75\ \mathbf{j}) \times (0.25\ \mathbf{i} + 0.433\ \mathbf{j})$

$= 18$ N.m

Subproblem c As the link *DC* is in static equilibrium, no forces and no moments are acting on it. Taking moments of the forces about point *D*,

$$M_d = \mathbf{F}_4 \times \mathbf{DH} + \mathbf{F}_{34} \times \mathbf{DC} = \mathbf{0} \qquad \text{(ii)}$$

As the direction of $\mathbf{F}_{34}$ is along *BC* if the force $\mathbf{F}_3$ is ignored,

$\therefore\ \mathbf{F}_{34} = F_{34}\ \angle 10.3^\circ = 0.984\ F_{34}\ \mathbf{i} + 0.179\ F_{34}\mathbf{j}$

Inserting the values of vectors in (ii),

$(44.59\ \mathbf{i} + 40.15\ \mathbf{j}) \times (-0.0673\ \mathbf{i} + 0.367\ \mathbf{j})$
$+(0.984F_{34}\mathbf{i} + 0.179F_{34}\mathbf{j}) \times (-0.101\ \mathbf{i} + 0.551\ \mathbf{j}) = 0$

or

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 44.59 & 40.15 & 0 \\ -0.0673 & 0.367 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.984F_{43} & 0.179F_{43} & 0 \\ -0.101 & 0.551 & 0 \end{vmatrix} = 0$$

or $19.067 + 0.56\ F_{34} = 0$

$F_{34} = -34$

Thus

$\mathbf{F}_{34} = -34\angle 10.3^\circ = 34\angle 190.3^\circ = -33.45\ \mathbf{i} - 6.08\mathbf{j}$

Net force on the link 4,

$\mathbf{F}_{34} + \mathbf{F}_4 + \mathbf{F}_{14} = 0$

or $(-33.45\ \mathbf{i} - 6.08\ \mathbf{j}) + (44.59\ \mathbf{i} + 40.15\ \mathbf{j}) + \mathbf{F}_{14} = 0$

or $11.14\ \mathbf{i} + 34.07\ \mathbf{j} + \mathbf{F}_{14} = 0$

or $\mathbf{F}_{14} = -11.14\ \mathbf{i} - 34.07\mathbf{j}$ or $35.8\ \angle 251.9^\circ$

Now, $\mathbf{F}_{32} = -\mathbf{F}_{23} = \mathbf{F}_{43} = -\mathbf{F}_{34} = 33.45\ \mathbf{i} + 6.08\mathbf{j}$

$\mathbf{F}_{12} = -\mathbf{F}_{32} = -33.45\ \mathbf{i} - 6.08\ \mathbf{j}$

$T_{2c} = \mathbf{F}_{32} \times AB\angle 60^\circ = (33.45\ \mathbf{i} + 6.08\ \mathbf{j}) \times (0.25\ \mathbf{i} + 0.433\ \mathbf{j})$

$= 12.96$ N.m

Net crankshaft torque $= T_{2a} + T_{2b} + T_{2c}$

$= -6.12 + 18 + 12.96$

$=$ <u>24.84 N.m</u> counter-clockwise

Forces on the bearings

On *D*, $\mathbf{F}_{14} = (8.66\mathbf{i} - 47.1\ \mathbf{j}) + (-11.14\mathbf{i} - 34.07\mathbf{j})$

$= -2.48\ \mathbf{i} - 81.17\ \mathbf{j}$

$= 81.2\ \angle 268.2^\circ$N

or it can be stated as $\mathbf{F}_{41} = 81.2\ \angle 88.2^\circ$N

On *C*, $\mathbf{F}_{43} = (8.66\ \mathbf{i} - 47.1\ \mathbf{j}) + (33.45\ \mathbf{i} + 6.08\ \mathbf{j})$

$= 41.45\ \mathbf{i} - 41.02\ \mathbf{j}$

$= 58.8\ \angle 315.8^\circ$N

On B, $\mathbf{F}_{23} = (-84.97\,\mathbf{i} - 75\,\mathbf{j}) + (-33.45\,\mathbf{i} - 6.08\,\mathbf{j})$
$= 118.42\,\mathbf{i} - 81.08\,\mathbf{j}$
$= 143.5\,\angle 214.4°\text{N}$

On A, $\mathbf{F}_{12} = (-22.72\,\mathbf{i} - 76.7\,\mathbf{j}) + (-84.97\,\mathbf{i} - 75\,\mathbf{j}) + (-33.45\,\mathbf{i} - 6.08\,\mathbf{j})$
$= -141.14\,\mathbf{i} - 157.78\,\mathbf{j}$
$= 211.7\angle 228.2°\text{N}$

Example 12.10 *In a four-link mechanism shown in Fig. 12.23(a), torque $\mathbf{T}_3$ and $\mathbf{T}_4$ have magnitudes of 30N.mand20N.mrespectively. The link lengths are AD = 800 mm, AB = 300 mm, BC = 700 mm and CD = 400 mm. For the static equilibrium of the mechanism, determine the required input torque $\mathbf{T}_2$.*

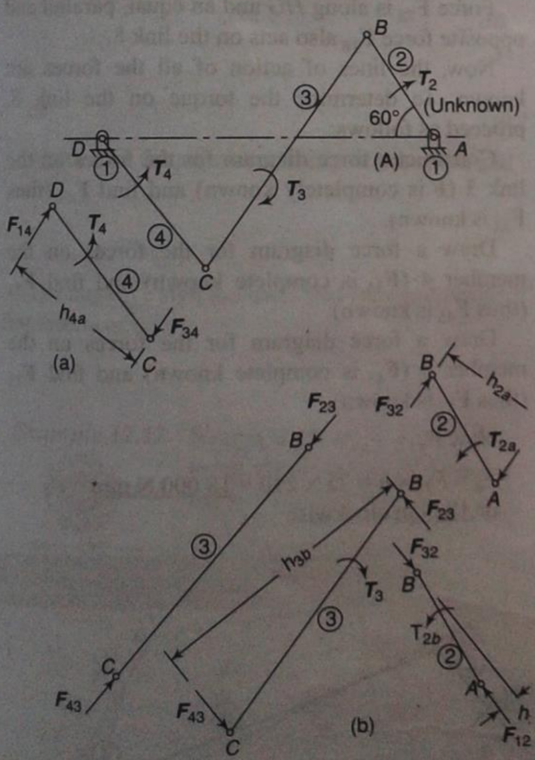

Fig. 12.23

Solution The solution of the stated problem can be obtained by superposition of the solutions of subproblems *a* and *b*.

Subproblem a [Fig. 12.23(a)] Neglecting torque $\mathbf{T}_3$

Torque $\mathbf{T}_4$ on the link 4 is balanced by a couple having two equal, parallel and opposite forces at C and D. As the link 3 is a two-force member, $\mathbf{F}_{43}$ and therefore, $\mathbf{F}_{34}$ and $\mathbf{F}_{14}$ will be parallel to BC.

$$F_{34} = F_{14} = \frac{T_4}{h_{4a}} = \frac{20}{0.383} = 52.2\text{ N}$$

and $F_{34} = F_{43} = F_{23} = F_{32} = F_{12} = 52.2\text{ N}$

$T_{2a} = F_{32} \times h_{2a} = 52.2 \times 0.274 = 14.3\text{ N.m}$ counter-clockwise.

Subproblem b [Fig. 12.23(b)] Neglecting torque $\mathbf{T}_4$.

$\mathbf{F}_{43}$ is along CD. The diagram is self-explanatory.

$$F_{43} = F_{23} = \frac{T_3}{h_{3b}} = \frac{30}{0.67} = 44.8\text{ N}$$

$F_{23} = F_{32} = F_{12} = 44.8\text{ N}$

$T_{2b} = F_{32} \times h_{2b} = 44.8 \times 0.042 = 1.88\text{ N.m}$

counter-clockwise.

$T_2 = T_{2a} + T_{2b} = 14.3 + 1.88 = \underline{16.18\text{ N}}$

counter-clockwise

Example 12.11 *Figure 12.24 shows a schematic diagram of an eight-link mechanism. The link lengths are*

AB = 450 mm	*OF = FC = 250 mm*
AC = 300 mm	*CG = 150 mm*
BD = 400 mm	*HG = 600 mm*
BE = 200 mm	*QH = 300 mm*

Determine the required shaft torque on the link 8 for static equilibrium against an applied load of 400 N on the link 3.

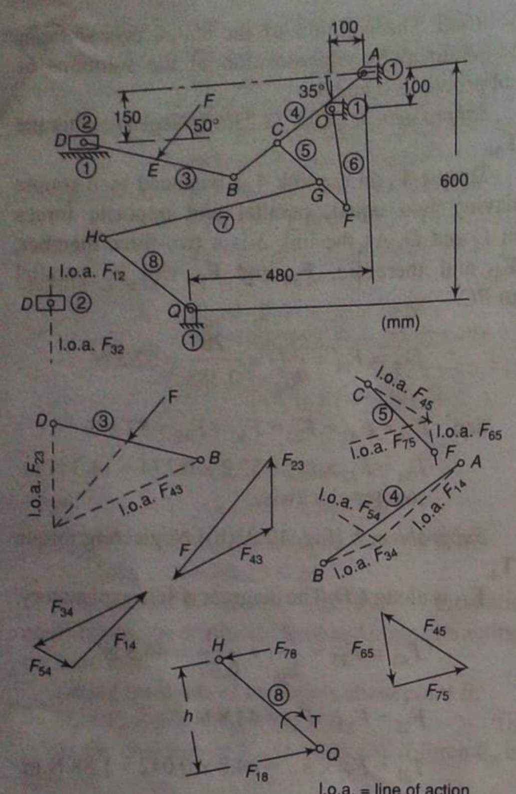

Fig. 12.24

Solution Links 2, 6 and 7 are two-force members. Since their lines of action can easily be visualised, it is not necessary to draw their free-body diagrams. Links 3, 4 and 5 are three-force members and 8 is a member with two forces and a torque.

Slider 2 is a two-force member. If friction is neglected, the forces on it $\mathbf{F}_{12}$ and $\mathbf{F}_{32}$ must act perpendicular to the guide path.

Considering the link 3, concurrency point can be found from the lines of action of $\mathbf{F}_{23}$ and $\mathbf{F}$, and thus the line of action of $\mathbf{F}_{43}$ is established.

The equilibrium of the link 4 cannot be considered at this stage as the line of action of only one force $\mathbf{F}_{34}$ is known (from $\mathbf{F}_{43}$).

Taking the link 5 which is a three-force member, the line of action of force at F is along OF and of force at G along HG. Establishing the point of concurrency from these two forces, the line of action of force at C, i.e., of the force $\mathbf{F}_{45}$ is known.

Now, take the link 4 and determine the line of action of the force at A since the lines of action of forces at B and C are known.

Force $\mathbf{F}_{78}$ is along HG and an equal, parallel and opposite force $\mathbf{F}_{18}$ also acts on the link 8.

Now, the lines of action of all the forces are known. To determine the torque on the link 8, proceed as follows:

Construct a force diagram for the forces on the link 3 ($\mathbf{F}$ is completely known) and find $\mathbf{F}_{43}$ (thus $\mathbf{F}_{34}$ is known).

Draw a force diagram for the forces on the member 4 ($\mathbf{F}_{34}$ is complete known) and find $\mathbf{F}_{54}$ (thus $\mathbf{F}_{45}$ is known).

Draw a force diagram for the forces on the member 5 ($\mathbf{F}_{45}$ is complete known) and find $\mathbf{F}_{75}$ (thus $\mathbf{F}_{57}$ is known).

Now $F_{57} = F_{87} = F_{78} = F_{18}$

$F_8 = F_{78} \times h = 75 \times 240 = \underline{18\ 000 \text{ N.mm}}$

or $\underline{18 \text{ N.m}}$ clockwise

12.9 PRINCIPLE OF VIRTUAL WORK

The principle of virtual (imaginary) work can be stated as 'the work done during a virtual displacement from the equilibrium is equal to zero'. Virtual displacement may be defined as an imaginary infinitesimal displacement of the system. By applying this principle, an entire mechanism is examined as a whole and there is no need of dividing it into free bodies.

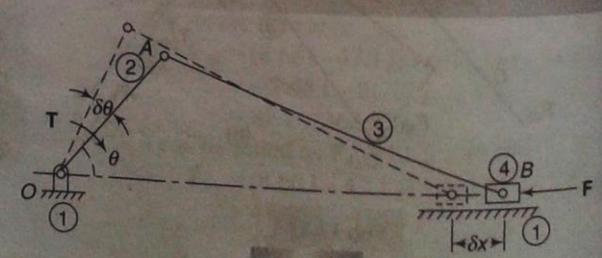

Fig. 12.25

Consider a slider-crank mechanism shown in Fig. 12.25. It is acted upon by the external piston force **F**, the external crankshaft torque **T** and the force at the bearings. As the crank rotates through a small angular displacement $\delta\theta$, the corresponding displacement of the piston is δx. the various forces acting on the system are

- Bearing reaction at O (performs no work)
- Force of cylinder on piston, perpendicular to piston displacement (produces no work)
- Bearing forces at A and B, being equal and opposite (AB is a two-force member), no work is done
- Work done by torque $T = T\delta\theta$
- Work done by force $F = F\,\delta x$

Work done is positive if a force acts in the direction of the displacement and negative if it acts in the opposite direction.

According to the principle of virtual work,

$$W = T\,\delta\theta + F\,\delta x = 0 \tag{12.7}$$

As virtual displacement must take place during the same interval δt,

$$\therefore \quad T\frac{d\theta}{dt} + F\frac{dx}{dt} = 0$$

or

$$T\omega + Fv = 0 \tag{12.8}$$

where ω is the angular velocity of the crank and v, the linear velocity of the piston.

$$T = -\frac{F}{\omega}v$$

The negative sign indicates that for equilibrium, **T** must be applied in the opposite direction to the angular displacement.

Example 12.12 *Solve Example 12.9 by using the principle of virtual work.*

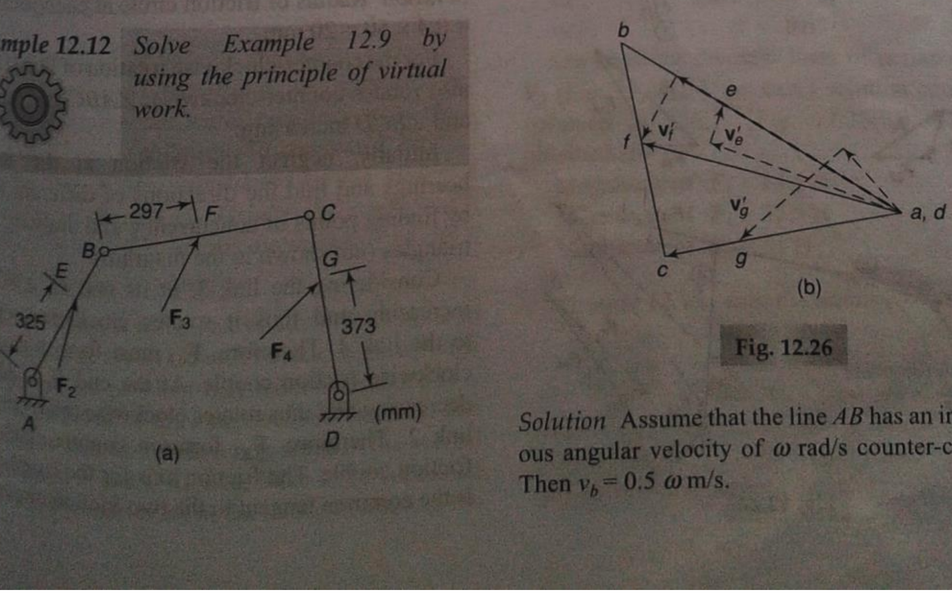

Fig. 12.26

Solution Assume that the line AB has an instantaneous angular velocity of ω rad/s counter-clockwise. Then $v_b = 0.5\,\omega$ m/s.

From the configuration diagram [Fig. 12.26(a)], draw the velocity diagram [Fig. 12.26(b)]. Locate the points E, F and G on the velocity diagram and locate the velocity vectors for the same. Take their components parallel and perpendicular to the direction of forces.

$v'_e = 0.0745$ rad/s (parallel to $\mathbf{F}_2$)
$v'_f = 0.124$ rad/s (parallel to $\mathbf{F}_3$)
$v'_g = 0.205$ rad/s (parallel to $\mathbf{F}_4$)

Assuming T to be counter-clockwise and applying the principle of virtual work,

$$T \times \omega + F_2 \times 0.0745\omega - F_3 \times 0.124\omega - F_4 \times 0.205\omega = 0$$

or $$T + 80 \times 0.0745 - 144 \times 0.124 - 60 \times 0.205 = 0$$

or $$T = -6 + 17.3 + 12.3$$
$$= 23.5 \text{ N.m counter-clockwise}$$

12.10 FRICTION IN MECHANISMS

When two members of a mechanism move relative to each other, friction occurs at the joints. The presence of friction increases the energy requirements of a machine.

Friction at the bearing is taken into account by drawing friction circles and at the sliding pairs by considering the angle of friction (Refer sections 8.13 and 8.14).

Example 12.13 *In a four-link mechanism ABCD, AB = 350 mm, AD = 700 mm, BC = 500 mm, DE = 150 mm, CD = 400 mm, ∠DAB = 60° (AD is the fixed link). A force of 35 N (F_4) acts at E on link DC as shown in Fig. 12.27a. Determine the force on the link AB required at the midpoint in the direction shown in the diagram for the static equilibrium of the mechanism. The coefficient of friction is 0.4 for each revolving pair. Assume impending motion of AB to be counter-clockwise. The radius of each journal is 50 mm.*

Also, find the torque on AB for its impending clockwise motion. (A very high value of coefficient of friction has been assumed to obtain a clear diagram).

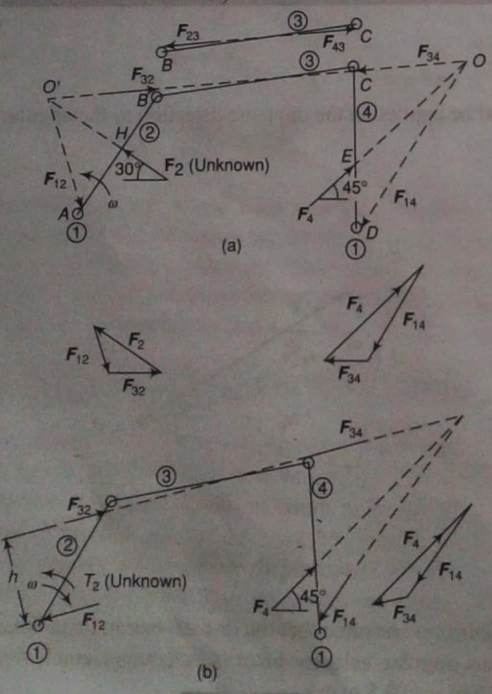

Fig. 12.27

Solution Radius of friction circle at each joint = μr = 0.4 × 50 = 20 mm.

For the counter-clockwise rotation of link AB, DC also rotates counter-clockwise; ∠ABC is deceasing and ∠BCD increasing.

Initially, neglect the friction at the journal bearings and find the directions of different forces by finding points of concurrency and drawing force triangles (not shown in the diagram).

Considering the link 3, at its end C, ∠BCD is increasing and thus it rotates clockwise relative to the link 4. Therefore, $\mathbf{F}_{43}$ must form a counter-clockwise friction couple. At the end B, ∠ABC is decreasing and thus rotates clockwise relative to the link 2. Therefore, $\mathbf{F}_{23}$ forms a counter-clockwise friction couple. The friction axis for the coupler BC is the common tangent to the two friction circles.

Now, consider the link 4. The line of action of the force $\mathbf{F}_{34}$ will be opposite to that of $\mathbf{F}_{43}$. Intersection of this line with the line of action of $\mathbf{F}_4$ gives the point of concurrency O for the forces acting on the link 4. As the link 4 rotates counter-clockwise, the tangent to the friction circle at D drawn from point O is such that a clockwise friction couple is obtained.

By drawing a force triangle for the forces acting on link 4 ($\mathbf{F}_4$ is completely known), $\mathbf{F}_{34}$ is obtained.

$$F_{34} = F_{43} = F_{23} = F_{32}$$

The point of concurrency for the forces acting on the link 2 is at O' which is the intersection of $\mathbf{F}_{32}$ and $\mathbf{F}_2$. As the link 1 rotates counter-clockwise, draw a tangent to the friction circle at A from O' such that a clockwise friction couple is obtained.

Draw a force diagram for the forces acting on the link 2 ($\mathbf{F}_{32}$ is completely known) and obtain the value of $\mathbf{F}_2$.

$$F_2 = 20.3 \text{ N}$$

When the motion of AB is clockwise, DC also moves clockwise. For the equilibrium of the link 4, the friction couples at D and C are to be counter-clockwise. For the equilibrium of the link 2, friction couples at A and B are also to be counter-clockwise. Obtain $\mathbf{F}_{32}$ in the manner discussed above and shown in Fig. 12.27(b) $\mathbf{F}_{12}$ will be equal, parallel and opposite to $\mathbf{F}_{32}$.

$$T_2 = F_{32} \times h = 8.6 \times 208 = 1789 \text{ N.mm}$$

or $\underline{1.789 \text{ N. m}}$

Example 12.14 *Find the minimum value of force $\mathbf{F}_5$ to be applied for the static equilibrium of the follower of Example 12.2 if the friction is also considered of the sliding bearings at B and C. Assume the coefficient of friction as 0.15. Ignore the thickness of the follower.*

Solution When a force analysis with friction is to be made, it is always convenient to seek a rough solution of the problem first without friction. This may be obtained by drawing freehand sketches. The purpose is to know the direction-sense of the normal reactions at B and C as these have to be combined with the friction forces at the sliders. Adopting the procedure of Example 12.2, the forces $\mathbf{F}_3$ and $\mathbf{F}_4$ at the bearings are found to be towards right.

As the force $\mathbf{F}_5$ required for the static equilibrium is to be the least, i.e., any force smaller than that will make the follower move down due to the applied force. Thus, the impending motion of the follower is downwards. (If it is desired to have the maximum force for the static equilibrium, any force greater than that will make the follower move up and the impending motion of the follower will be upwards).

Now, as the impending motion of the follower is downwards, the friction forces at the bearings are upwards. Combining these forces with the reaction forces which are towards right, the lines of action of both the forces $\mathbf{F}_3$ and $\mathbf{F}_4$ are tilted through an angle ϕ given by

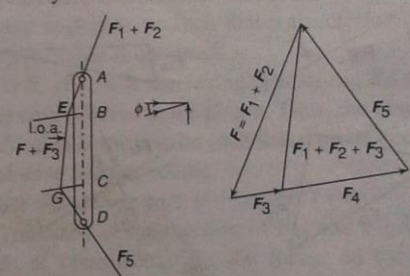

Fig. 12.28

$$\mu = 0.15$$

or $\tan\phi = 0.15$

or $\phi = 8.5°$

On knowing the new lines of action of $\mathbf{F}_3$ and $\mathbf{F}_4$ [Fig. 12.28(a)], the exact solution can be easily obtained as before [Fig. 12.28(b)]. The values obtained are

Magnitude of $\mathbf{F}_3 = 14.5$ N

Magnitude of $\mathbf{F}_4 = 35.5$ N

Magnitude of $\mathbf{F}_5 = 51$ N

Example 12.15 *For the static equilibrium of the quick-return mechanism shown in Fig. 12.29a, find the maximum input torque T_2 required for a force of 300 N on the slider D. Angle θ is 105°. Coefficient of friction $\mu = 0.15$ for each sliding pair.*

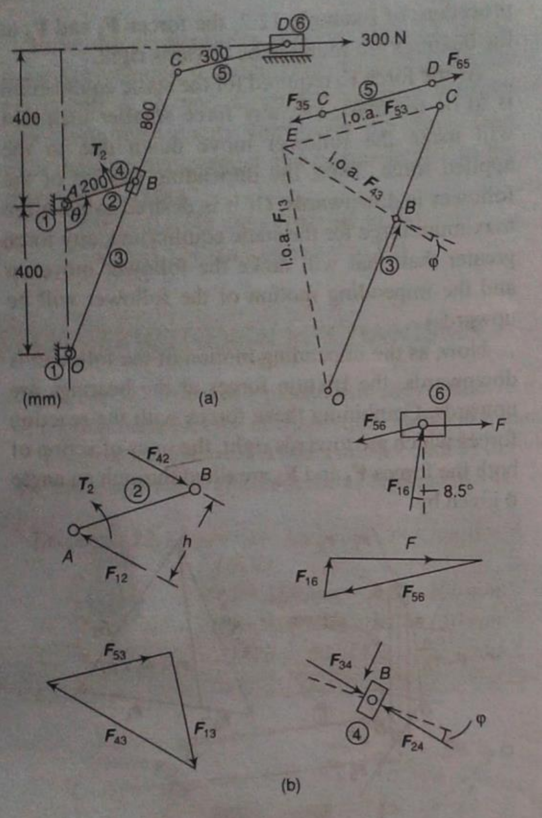

Fig. 12.29

Solution As mentioned in the previous example, to analyse a problem with friction, it is always convenient to seek a rough solution of the problem first without friction which may be obtained by drawing freehand sketches. This is needed to know the direction-sense of the normal reactions at the two sliders which are to be combined with the friction forces.

As the torque required for the static equilibrium is to be the maximum, i.e., any torque more than that will make the slider at *D* move left. Thus, the impending motion of the slider *D* is to the left.

Now,

$$\mu = 0.15 \quad \text{or} \quad \tan\phi = 0.15$$

or $\quad \phi = 8.5°$

Solving the problem first without friction,

Slider at *D* or the link 6 is a three-force member. Lines of action of the forces are

- **F**, as given
- $\mathbf{F}_{56}$ along *CD*, as link 5 is a two-force member
- $\mathbf{F}_{16}$, normal reaction, perpendicular to slider motion

Draw the force diagram and determine the direction sense of forces $\mathbf{F}_{56}$ and $\mathbf{F}_{16}$ from it (the diagrams may not be to scale). From the force $\mathbf{F}_{56}$, the directions of forces $\mathbf{F}_{65}$, $\mathbf{F}_{35}$ and $\mathbf{F}_{53}$ are known. Now link 3 is a three-force member. Lines of action of the forces are

- $\mathbf{F}_{53}$, known completely through *C*
- $\mathbf{F}_{43}$, perpendicular to slider motion through *B*
- $\mathbf{F}_{13}$, unknown through *A*.

As the lines of action of forces acting through *B* and *C* are known, the line of action of $\mathbf{F}_{13}$ through *A* must also pass through the point of intersection of the other two forces. Find the sense of the direction of force $\mathbf{F}_{43}$ by drawing the force triangle.

After obtaining the sense of direction of the normal forces $\mathbf{F}_{16}$ (upwards) and $\mathbf{F}_{43}$ (towards left), solve the problem by considering the force of friction also. Now the diagrams must be to the scale.

The force of friction at the slider *D* is towards right as the impending motion of the slider is towards the left. Combining this force with the normal force $\mathbf{F}_{16}$, it is tilted towards left as shown in the figure. Now draw the force triangle by modifying the line of action of force $\mathbf{F}_{16}$. Repeat the above procedure and obtain magnitude as well the direction of the force $\mathbf{F}_{53}$.

The motion of the slider 4 on the link 3 is upwards for impending motion of the slider *D* towards left. It implies that the motion of the link 3 relative to the link 4 is downwards. Thus, force of friction on the link 3 is upwards (on slider it is downwards). Combining this with the normal force $\mathbf{F}_{43}$ which is towards left, the force $\mathbf{F}_{43}$ is tilted through an angle ϕ as shown in the figure. Now again draw the force triangle with the modified direction of the force $\mathbf{F}_{43}$ for the forces on the link 3 and obtain the magnitude of this force also.

Now,

$$F_{34} = F_{43}$$

As the slider B is a two-force member with forces $\mathbf{F}_{24}$ and $\mathbf{F}_{34}$. Therefore,

$$F_{34} = F_{24} = F_{42} = F_{12}$$

Thus, as the link 2 is acted upon by two forces and a torque,

$$T = F_{42} \times h = 437 \times 147 = 64240 \text{ N.m}$$
$$= \underline{64.24 \text{ N.m}} \text{ counter-clockwise}$$

Example 12.16 *Solve Example 8.28 using graphical method. Take coefficient of friction for the journals as 0.4 instead of 0.05. (A fictitious high value of coefficient of friction is taken so that friction circles of reasonable diameter may be drawn on a smaller scale).*

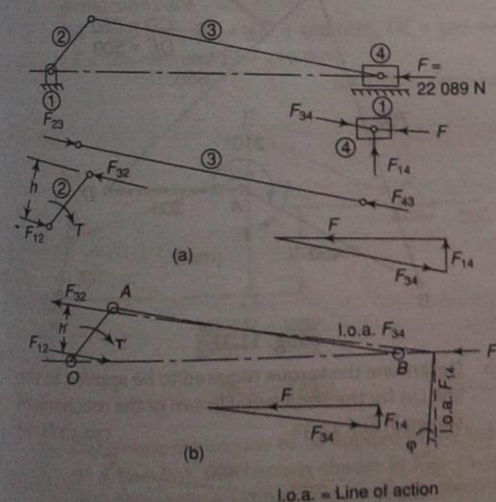

Fig. 12.30

Solution Figure 12.30(a) shows the solution of the problem neglecting the friction. From the force triangle for the forces on the slider,

$$F_{34} = 22\,500 \text{ N}$$

Now,

$$F_{34} = -F_{43} = F_{23} = F_{32}$$
$$T = F_{23} \times h = 22\,500 \times 0.261$$
$$= 5872.5 \text{ N.m clockwise}$$

When friction is considered [Fig.12.30(b)],

$$\text{Radius of friction circle at } O = 0.4 \times \frac{140}{2} = 28 \text{ mm}$$

$$\text{Radius of friction circle at } A = 0.4 \times \frac{120}{2} = 24 \text{ mm}$$

$$\text{Radius of friction circle at } B = 0.4 \times \frac{80}{2} = 16 \text{ mm}$$

As the crank moves counter-clockwise, $\angle OAB$ decreases. AB rotates clockwise relative to OA. Thus, tangent at A is to be such that a counter-clockwise friction couple is obtained.

At B, $\angle OBA$ is increasing. Therefore, BA rotates clockwise relative to the piston. Thus, the tangent to the friction circle is to be such that it gives a counter-clockwise friction couple.

For the sliding pair, $\varphi = \tan^{-} 00.7 = 4°$

The point of intersection of $\mathbf{F}_{34}$ and $\mathbf{F}$ gives the point of concurrency for the forces on the slider. Force $\mathbf{F}_{14}$, i.e., the reaction of the guide, is inclined to the perpendicular to the slider path, and passes through the point of concurrency.

By drawing a force triangle for the forces acting on the slider, $\mathbf{F}_{34}$ is obtained.

The force at A is equal, parallel and opposite to $\mathbf{F}_{32}$ and tangent to the friction circle such that a clockwise friction couple is obtained.

$$T' = \mathbf{F}_{32} \times h' = 22\,200 \times 0.202 = 4484 \text{ N.m clockwise}$$

Summary

1. A pair of action and reaction forces which constrain two connected bodies to behave in a particular manner are known as *constraint forces* whereas forces acting from outside on a system of bodies are called *applied forces*.
2. A member under the action of two forces will be in equilibrium if the forces are of the same magnitude, act along the same line and are in opposite directions.
3. A member under the action of three forces will be in equilibrium if the resultant of the forces is zero and the lines of action of the forces intersect at a point, known as the *point of concurrency*.

4. A member under the action of two forces and an applied torque is in equilibrium if the forces are equal in magnitude, parallel in direction and opposite in sense and the forces form a couple which is equal and opposite to the applied torque.
5. The force exerted by the member *i* on the member *j* is represented by $\mathbf{F}_{ij}$.
6. A *free-body diagram* is a sketch or diagram of a part isolated from the mechanism in order to determine the nature of forces acting on it.
7. In linear systems, if a number of loads act on a system of forces, the net effect is equal to the superposition of the effects of the individual loads taken one at a time. A linear system is one in which the output force is directly proportional to the input force, i.e., in mechanisms in which coulomb or dry friction is neglected.
8. The principle of virtual (imaginary) work can be stated as 'the work done during a virtual displacement from the equilibrium is equal to zero'. Virtual displacement may be defined as an imaginary infinitesimal displacement of the system. By applying this principle, an entire mechanism is examined as a whole and there is no need of dividing it into free bodies.
9. Friction at the bearing is taken into account by drawing friction circles and at the sliding pairs by considering the angle of friction.

Exercises

1. What do you mean by applied and constraint forces? Explain.
2. What are conditions for a body to be in equilibrium under the action of two forces, three forces and two forces and a torque?
3. What are free-body diagrams of a mechanism? How are they helpful in finding the various forces acting on the various members of the mechanism?
4. Define and explain the superposition theorem as applicable to a system of forces acting on a mechanism.
5. What is the principle of virtual work? Explain.
6. How is the friction at the bearings and at sliding pairs of a mechanism is taken into account?
7. The dimensions of a four-link mechanism are: AB = 400 mm, BC = 600 mm, CD = 500 mm, AD = 900 mm, and $\angle DAB = 60°$. AD is the fixed link. E is a point on the link BC such that BE = 400 mm and CE = 300 mm (BEC clockwise)
   A force of 150 $\angle 45°$ N acts on DC at a distance of 250 mm from D. Another force of magnitude 100 $\angle 180°$ N acts at point E. Find the required input torque on the link AB for static equilibrium of the mechanism. (4.6 N.m clockwise)
8. Determine the required input torque on the crank of a slider-crank mechanism for the static equilibrium when the applied piston load is 1500 N. The lengths of the crank and the connecting rod are 40 mm and 100 mm respectively and the crank has turned through 45° from the inner-dead centre. (55 N.m)
9. Find the torque required to be applied to link AB of the linkage shown in Fig. 12.31 to maintain the static equilibrium. (8.85 N.m)

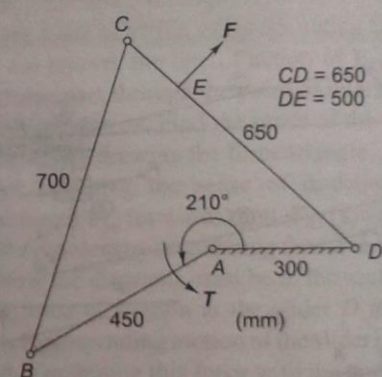

Fig. 11.31

10. Determine the torque required to be applied to the link OA for the static equilibrium of the mechanism shown in Fig. 12.32. (30.42 N.m)

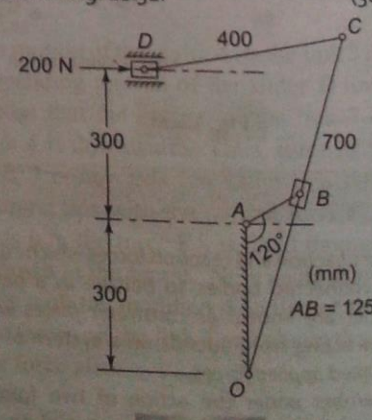

Fig. 12.32

11. For the mechanism shown in Fig. 12.33, find the required input torque for the static equilibrium. The lengths OA and AB are 250 mm and 650 mm respectively. $F = 500$ N. (68 N.m clockwise)

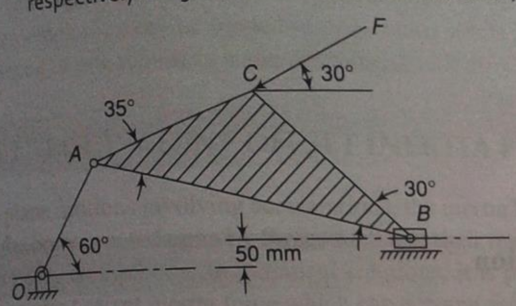

Fig. 12.33

12. For the static equilibrium of the mechanism of Fig. 12.34, find the required input torque. The dimensions are
$AB = 150$ mm, $BC = AD = 500$ mm, $DC = 300$ mm, $CE = 100$ mm and $EF = 450$ mm.
(45.5 N.m clockwise)

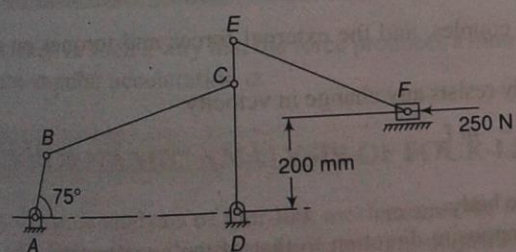

Fig. 12.34

13. Determine the torque to be applied to the link AB of a four link mechanism shown in Fig. 12.35 to maintain static equilibrium at the given position. (44 N.m)

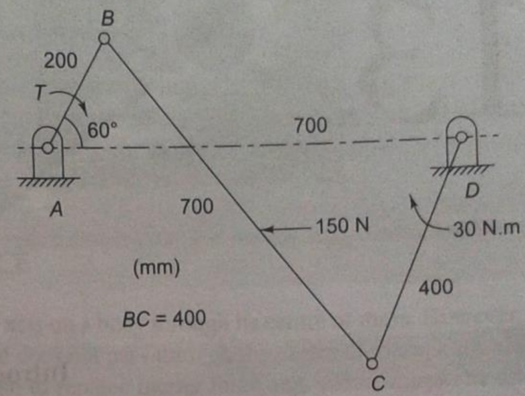

Fig. 12.35

14. A two-cylinder engine shown in Fig. 12.36 is in static equilibrium. The dimensions are $OA = OB =$ 50 mm, $AC = BD = 250$ mm, $\angle AOB = 90°$. Determine the torque on the crank OAB.
(106 N.m clockwise)

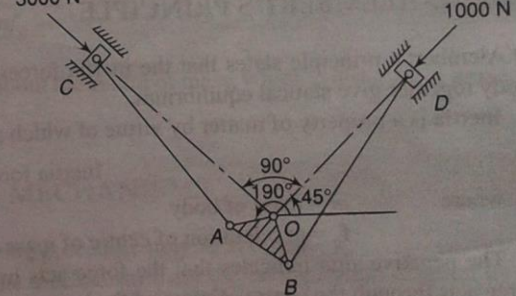

Fig. 12.36