

THEORY OF MACHINES AND MECHANISM

Lecture 2

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Grashoff's Law

The sum of the shortest and longest link of a planar four-bar linkage cannot be greater than the sum of remaining two links if there is to be continuous relative motion between the links. Below are the possible types of pinned, four-bar linkages

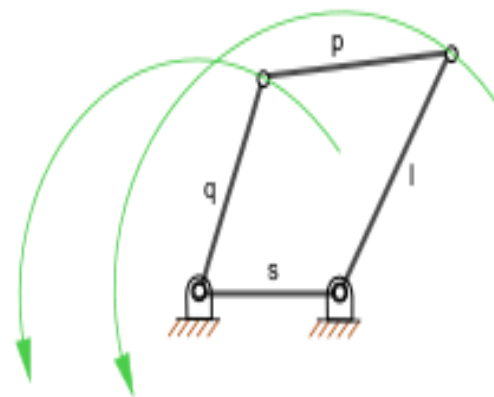
$$S + L \leq P + Q$$

- L = length of longest link
- P = length of one remaining link
- Q = length of other remaining link

if $L + S > P + Q$ then the linkage is a **Non-Grashof** Type

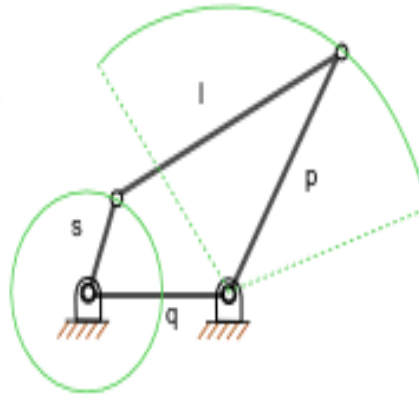
if $L + S < P + Q$ then the linkage is a **Grashof** type

Diagrams for Grashoff's Law

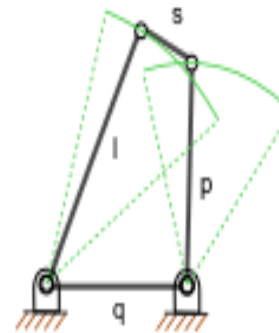


(full revolution,
both links)

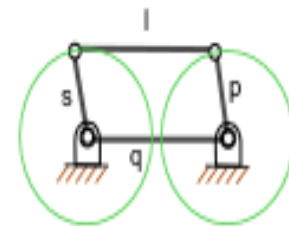
Drag-link
 $s+l \neq p+q$
(continuous motion)



Crank-rocker
 $s+l \neq p+q$
(continuous motion)



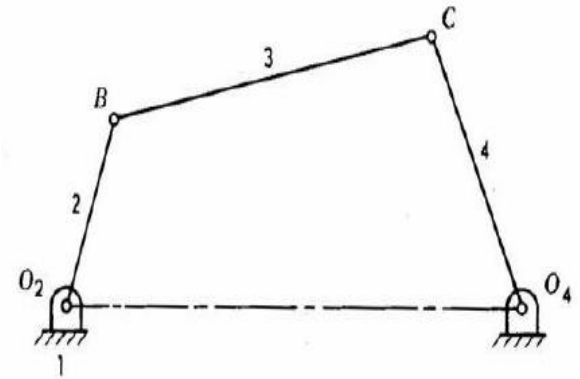
Double-rocker
 $s+l > p+q$
(no continuous motion)



Parallelogram linkage
 $s+l \neq p+q$
(continuous motion)

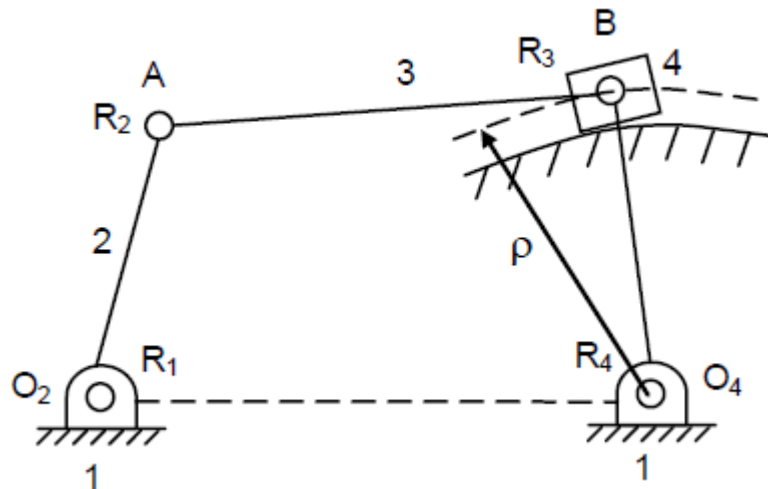
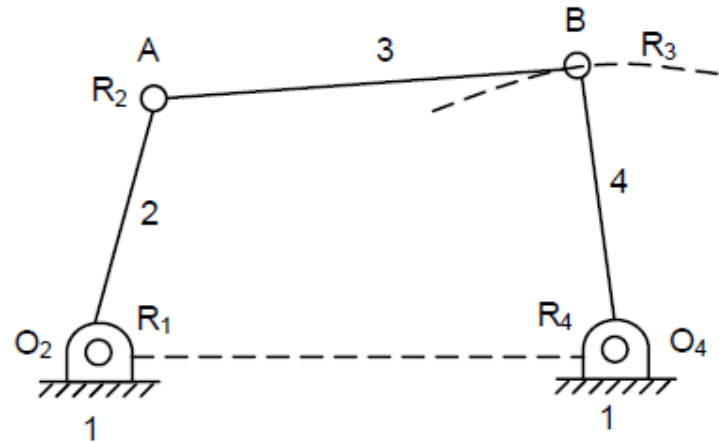
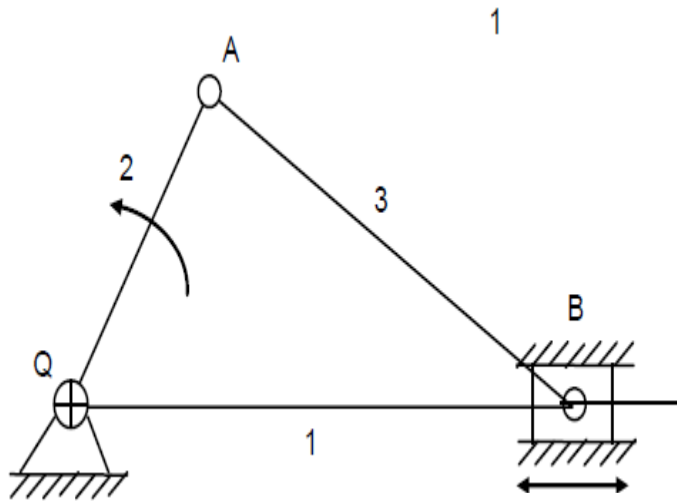
Inversions of a Four-Bar Mechanism

- Kinematic chain of four members connected by revolute joints.
- four possible configurations with a different link fixed as frame each time.



Typical four bar mechanism

Different four bar mechanism



Inversion of kinematic chains

- Mainly three type of inversion of four bar mechanism
- 1. 4R-kinematic chain which has all the four kinematic pairs as revolute pairs.
- 2. (b) 3R-1 P kinematic chain which has three revolute pairs and one prismatic pair. This is also called as single slider crank chain.
- 3. (c) 2R-2P kinematic chain which has two revolute pairs and two prismatic pairs. This is also called as double slider crank chain.

4R-kinematic chain

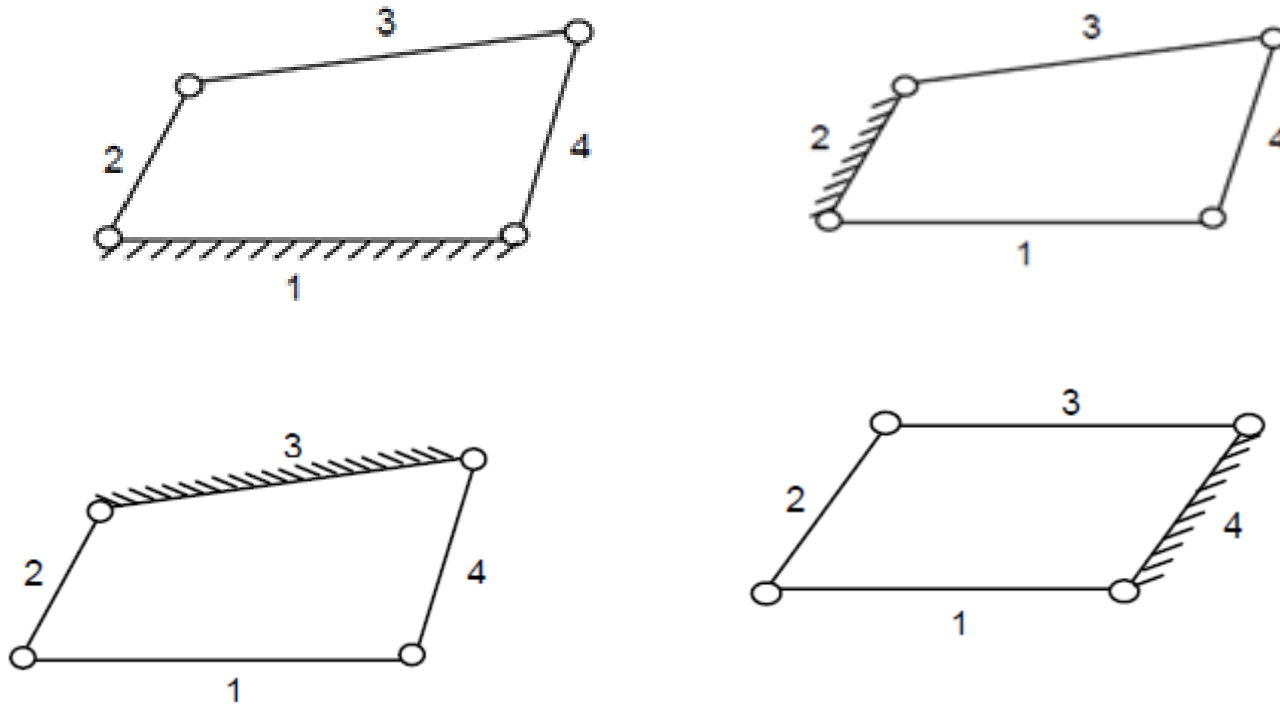
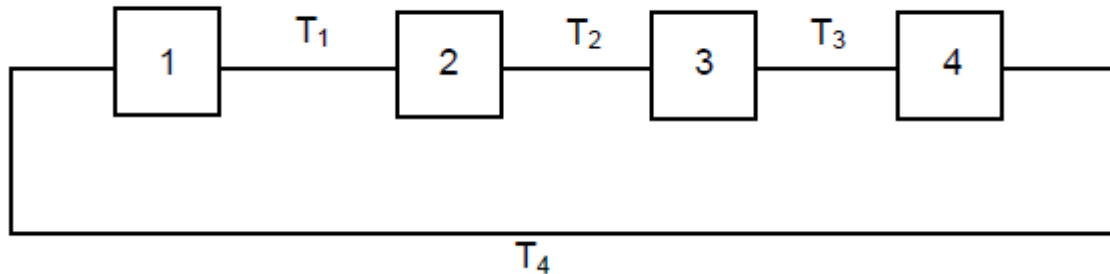


Figure Inversion of Kinematic Chain

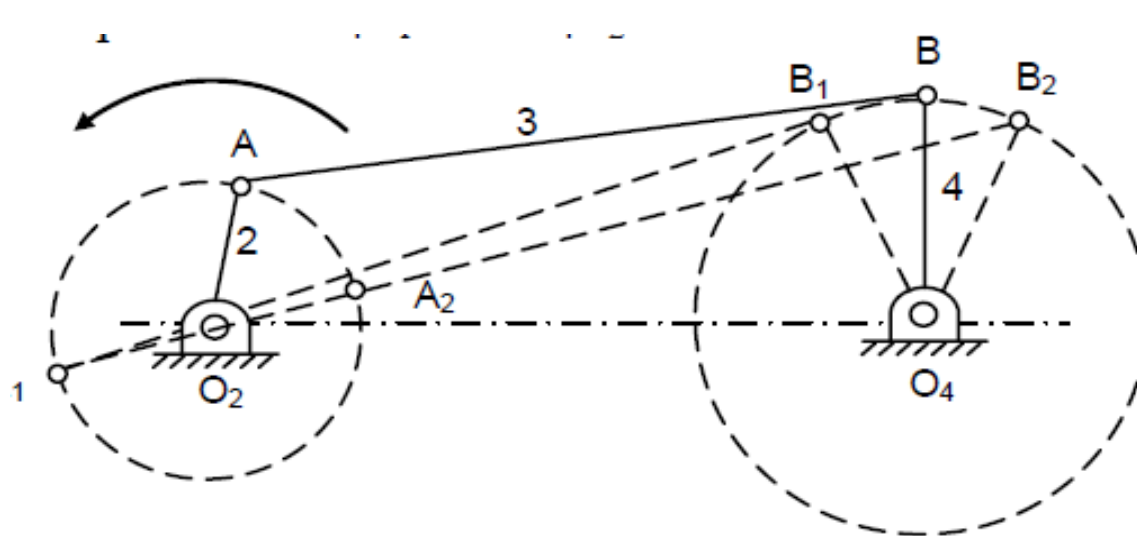
1. 4R-Kinematic Chain Inversions

- Kinematically speaking, all four inversions of 4R-kinematic chain are identical. However, by suitably altering the proportions of lengths of links 1, 2, 3 and 4 respectively several mechanisms are obtained.



a. Crank-lever Mechanism or Crank-rocker Mechanism

- Every complete rotation of link 2 (called a crank), the link 4 (called a lever or rocker), makes oscillation between extreme positions O_4B_1 and O_4B_2 .
- Crank angles for the two strokes (forward and backward) of oscillating link O_4B are *not* same.
- The proportions of the link may be as follows

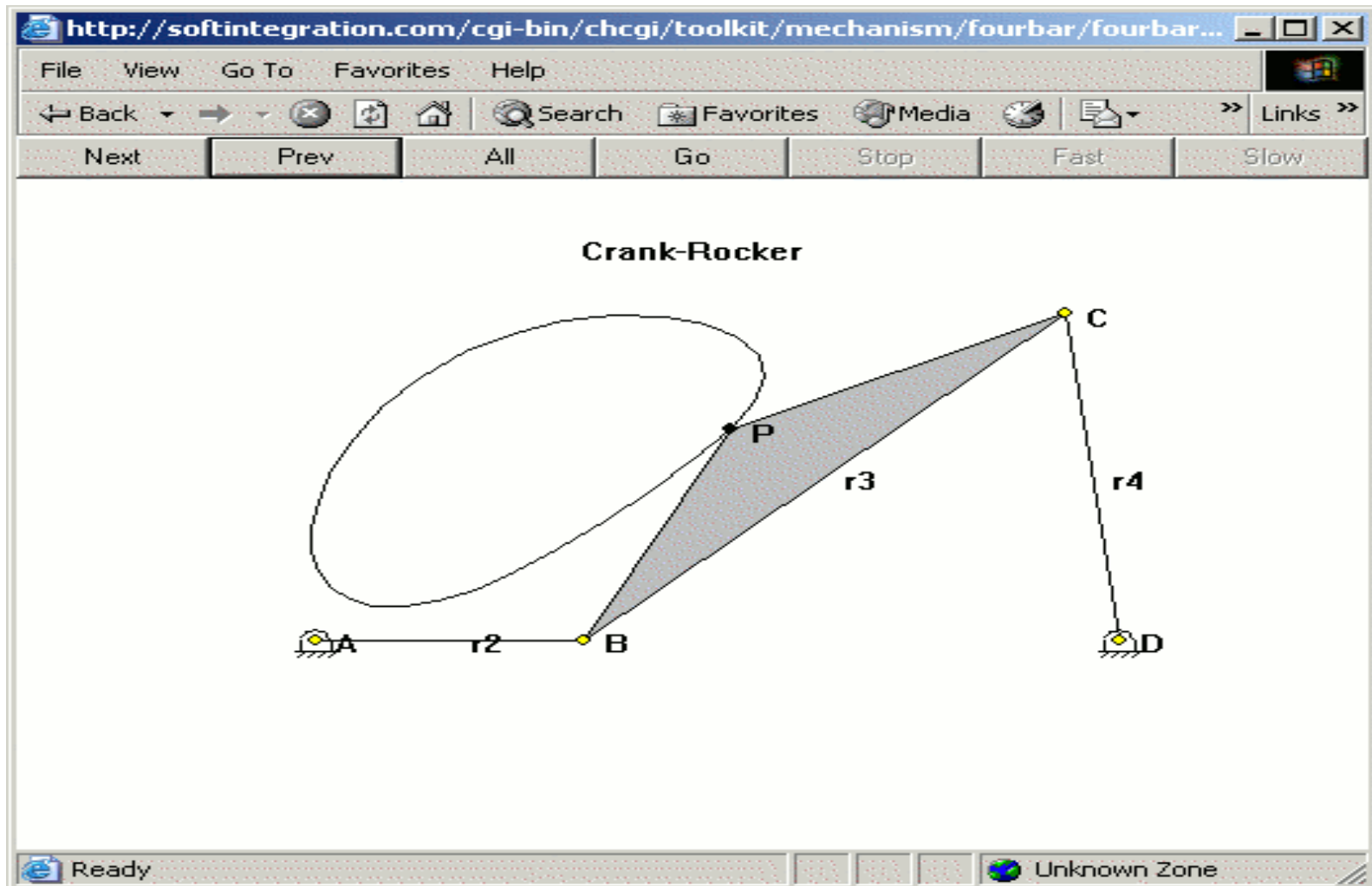


$$(l_1 + l_2) < (l_3 + l_4)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

Fig. Crank-rocker Mechanism

Example



b. Double-leaver Mechanism or Rocker-Rocker Mechanism

- In this mechanism, both the links 2 and 4 can only oscillate.
- This mechanism must satisfy the following relations.

$$(l_3 + l_4) < (l_1 + l_2)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

If links 2 and 4 are of equal lengths and $l_1 > l_3$, this mechanism forms automobile steering gear

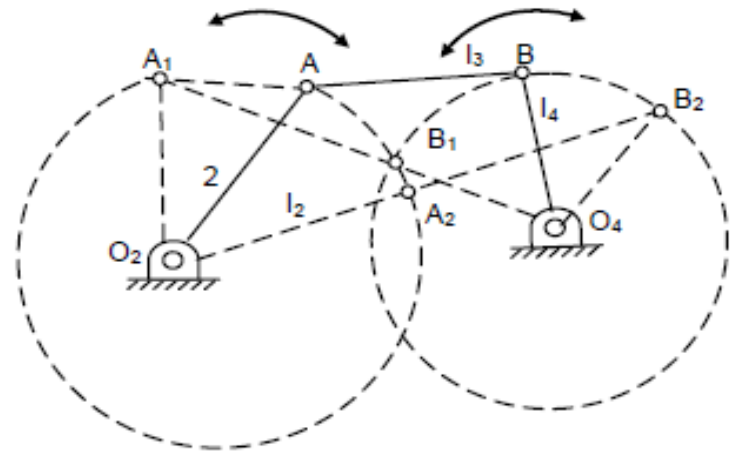
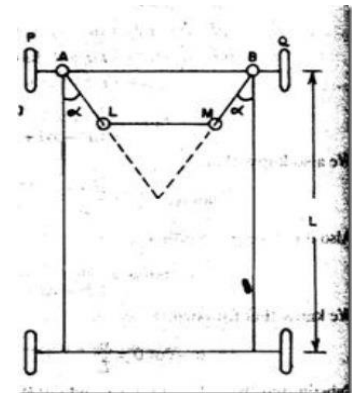
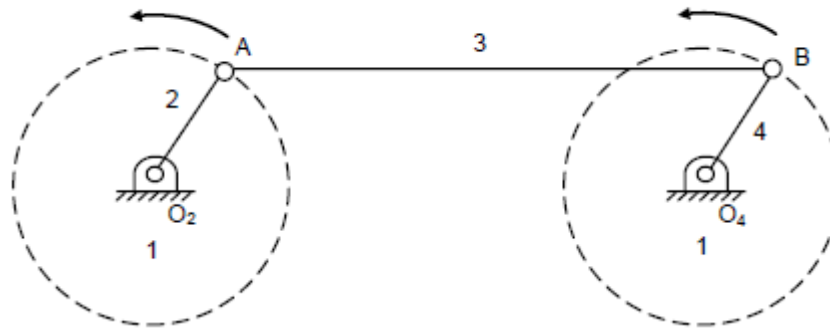


Fig. Double-lever Mechanism



c. Double Crank Mechanism

- The links 2 and 4 of the double crank mechanism make complete revolutions. There are two forms of this mechanism.



i. Parallel Crank Mechanism

In this mechanism, lengths of links 2 and 4 are equal. Lengths of links 1 and 3 are also equal. It is shown in Figure

ii. Drag Link Mechanism

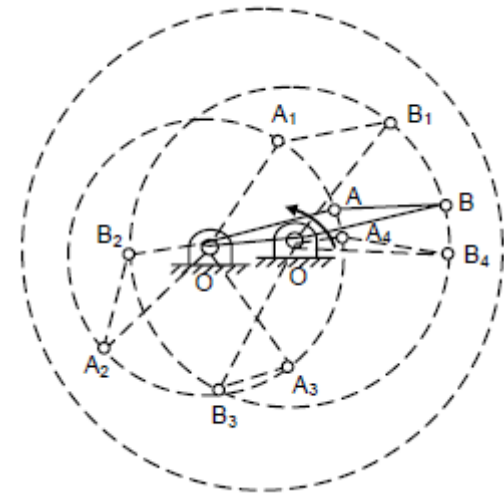
Example

- Coupling of the locomotive wheels where wheels act as cranks of equal length and length of the coupling rod is equal to centre distance between the two coupled wheels



ii. Drag Link Mechanism

- In this mechanism also links 2 and 4 make full rotation. As the link 2 and 4 rotate sometimes link 4 rotate faster and sometimes it becomes slow in rotation.
- Used in the automobile bar steering sysem



The proportions of this mechanism

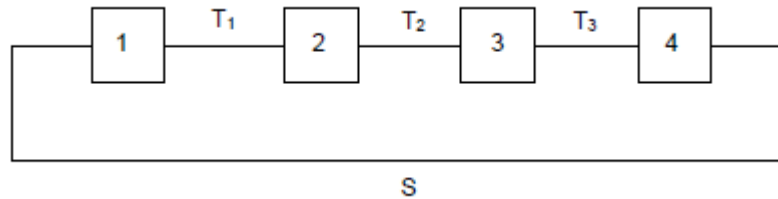
$$l_3 > l_1 \ ; \ l_4 > l_2$$

$$l_3 > (l_1 + l_4 - l_2)$$

$$l_3 < (l_2 + l_4 - l_1)$$

2. Inversions of 3R-1P Kinematic Chain or Inversions of Slider crank

- In this four bar kinematic chain, four links shown by blocks are connected through three revolute pairs T_1 , T_2 and T_3 and one prismatic pair.



a. First Inversion

- In this mechanism, link 1 is fixed, link 2 works as crank, link 4 works as a slider and link 3 connects link 2 with 4. It is called connecting rod. Between links 1 and 4 sliding pair has been provided.
- This mechanism is also known as slider crank chain or reciprocating engine mechanism because it is used in internal combustion engines. It is also used in reciprocating pumps as it converts rotatory motion into reciprocating motion and vice-versa.

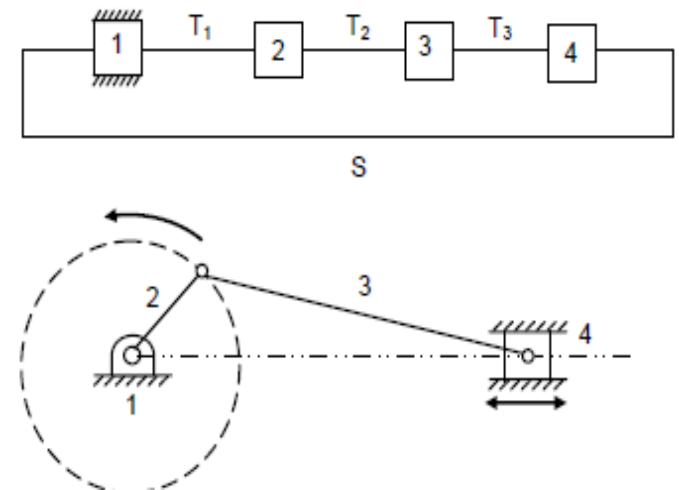


Figure : first inversion

b. Second Inversion

- In this case link 2 is fixed and link 3 works as crank. Link 1 is a slotted link which facilitates movement of link 4 which is a slider. This arrangement gives quick return motion mechanism.
- this mechanism and it is called Whitworth Quick Return Motion Mechanism.

$$\begin{aligned}
 \text{Quick Return Ratio} &= \frac{\text{Time taken in forward stroke}}{\text{Time taken in return stroke}} \\
 &= \frac{(2\pi - \theta)}{\theta} \\
 &= \frac{\omega}{\omega} \\
 &= \frac{2\pi - \theta}{\theta}
 \end{aligned}$$

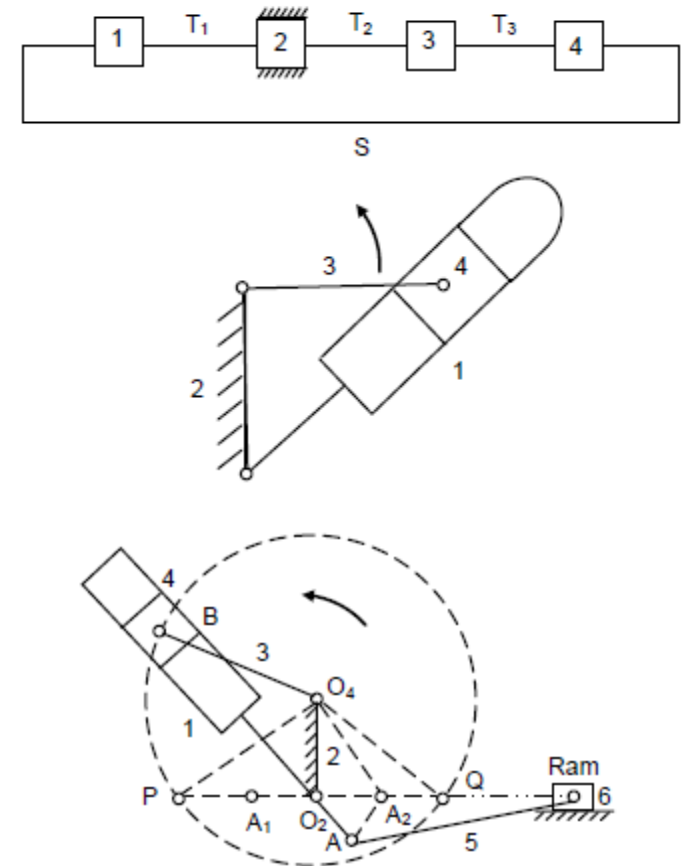


Figure : Second Inversion

c. Third Inversion

- This inversion is obtained by fixing link 3. Some applications of this inversion are oscillating cylinder engine and crank and slotted lever quick return motion mechanism of a shaper machine.

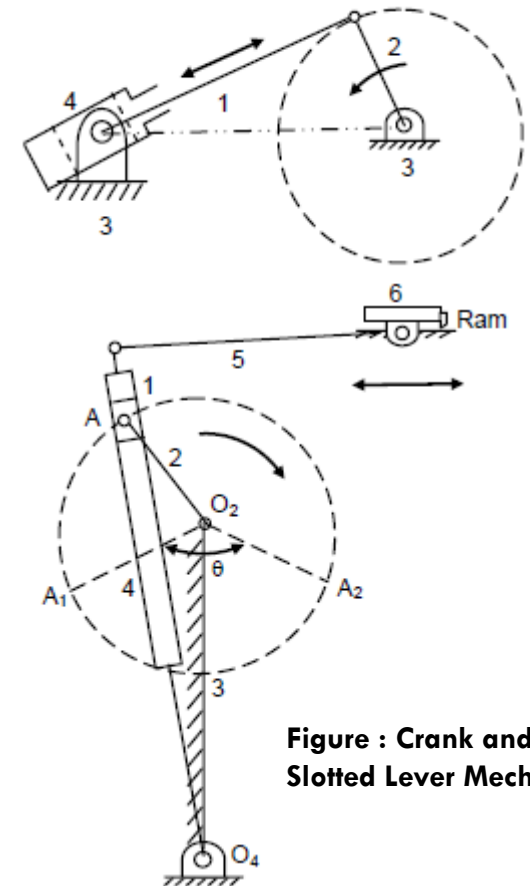
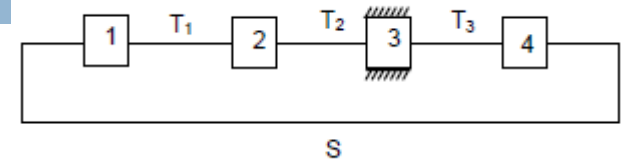


Figure : Crank and Slotted Lever Mechanism

d. Fourth Inversion – Pendulum Pump

- It is obtained by fixing link 4 which is slider.
- Application of this inversion is limited.
- examples Pendulum pump and hand pump .
- pendulum pump, link 3 oscillates like a pendulum and link 1 has translatory motion which can be used for a pump.

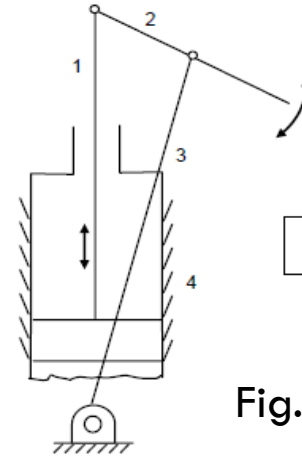


Fig. Hand pump

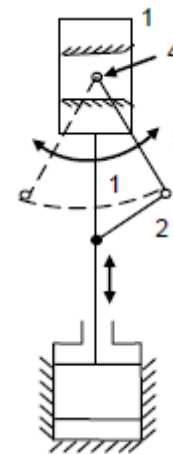
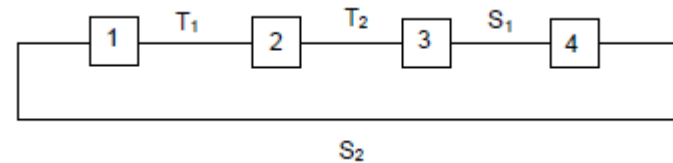


Fig . Pendulum pump

I2R-2P inversion or Double Slider Crank Chain

- This four bar kinematic chain has two revolute or turning pairs – T_1 and T_2 and two prismatic or sliding pairs – S_1 and S_2 . This chain provides three different mechanisms



A. First Inversion

- The first inversion is obtained by fixing link 1. By doing so a mechanism called Scotch Yoke is obtained. The link 1 is a slider similar to link 3. Link 2 works as a crank. Link 4 is a slotted link. When link 2 rotates, link 4 has simple harmonic motion for angle ' θ ' of link 2, the displacement of link 4 is given by
- $X = OA \cos \theta$

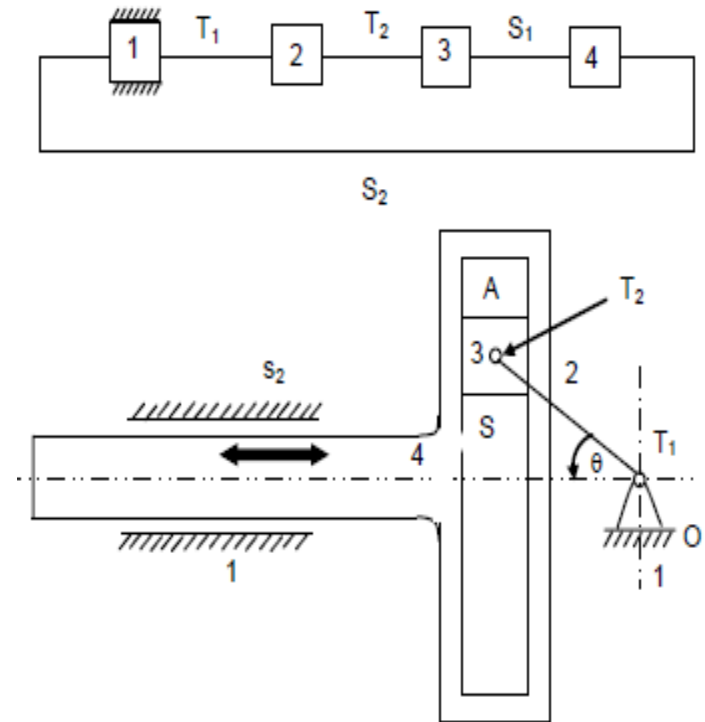


Figure : Scotch Yoke Mechanism

B. Second Inversion

- In this case, link 2 is fixed and a mechanism called Oldham's coupling is obtained.
- This coupling is used to connect two shafts which have eccentricity ' θ '. The axes of the two shafts are parallel but displaced by distance θ . The link 4 slides in the two slots provided in links 3 and 1. The centre of this link will move on a circle with diameter equal to eccentricity.

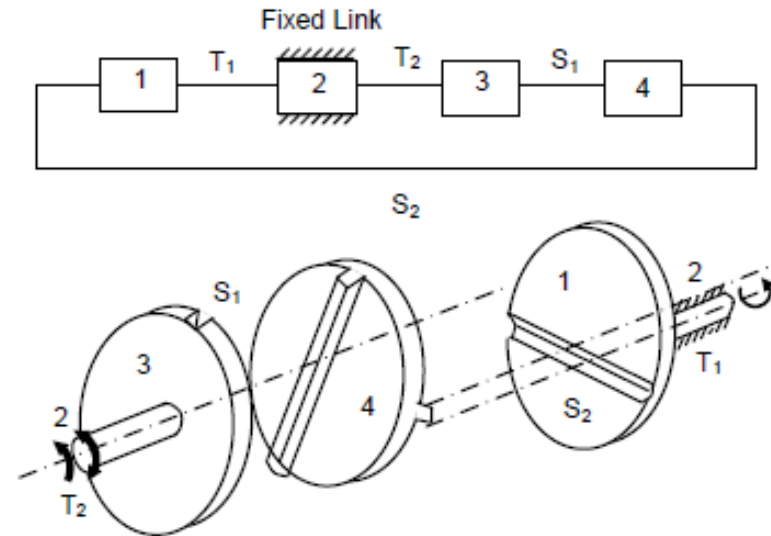


Figure: Oldham's Coupling

- This inversion is obtained by fixing link 4. The mechanism so obtained is called elliptical trammel



Straight Line Motion Mechanisms

- A mechanism built in such a manner that a particular point in it is constrained to trace a straight line path within the possible limits of motion, is known as a *straight line motion mechanism*.

The Scott Russel Mechanism is straight line mechanism

- This mechanism is shown in Figure. It consists of a crank OC, connecting rod CP, and a slider block P which is constrained to move in a horizontal straight line passing through O. The connecting rod PC is extended to Q such that
- $PC = CQ = CO$
- It will be proved that for all horizontal movements of the slider P, the locus of point Q will be a straight line perpendicular to the line OP.

Scott Russel Mechanism

- Draw a circle of diameter PQ as shown. It is well known that diameter of a circle always subtends a right angle or any point on the circle. Thus, at point O , the angle QOP is a right angle. For any position of P , the line connecting O with P will always be horizontal. Therefore, line joining the corresponding position of Q with O will always be a straight line perpendicular to OP . Thus, the locus of point Q will be a straight line perpendicular to OP . Thus, a horizontal straight line motion of slider block P will enable point Q to generate a vertical straight line, both passing through O .

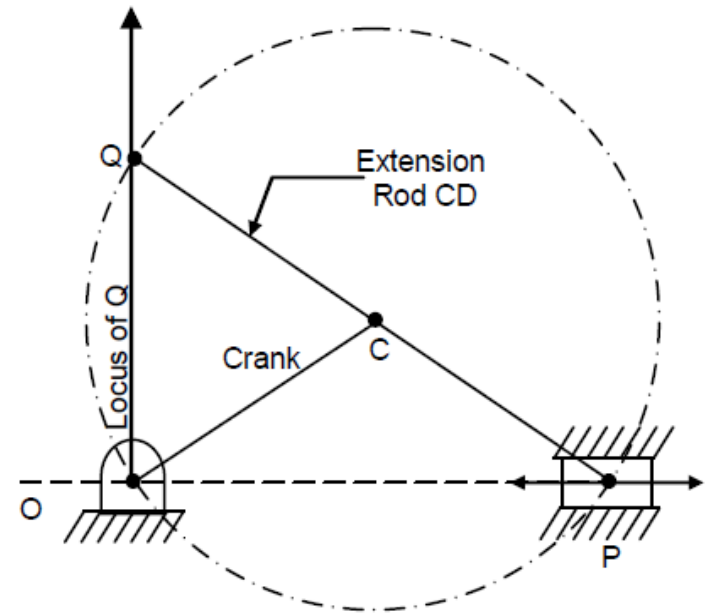
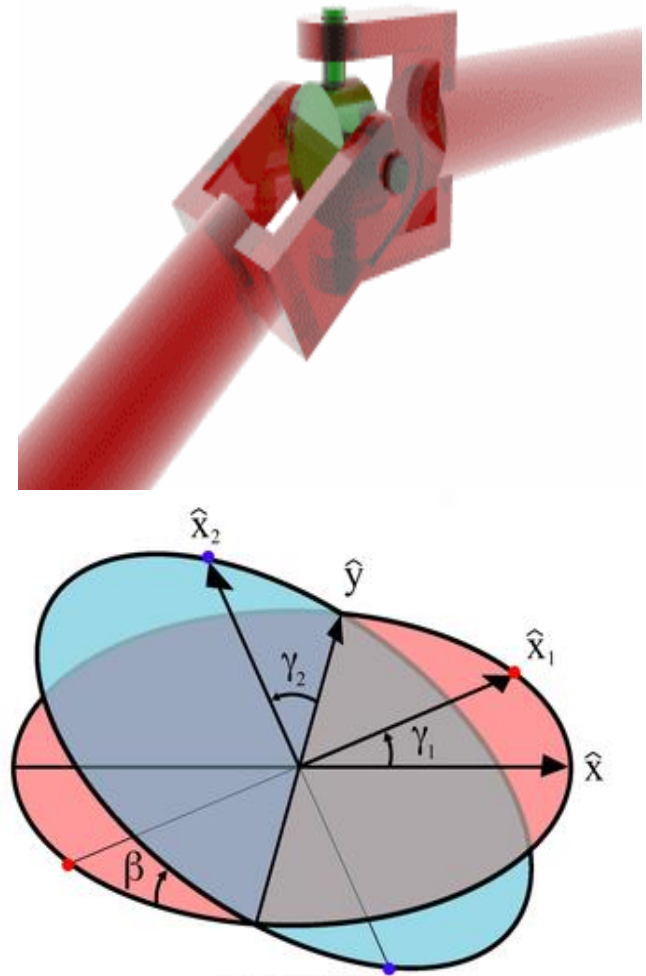


Figure :Scott Russel Mechanism

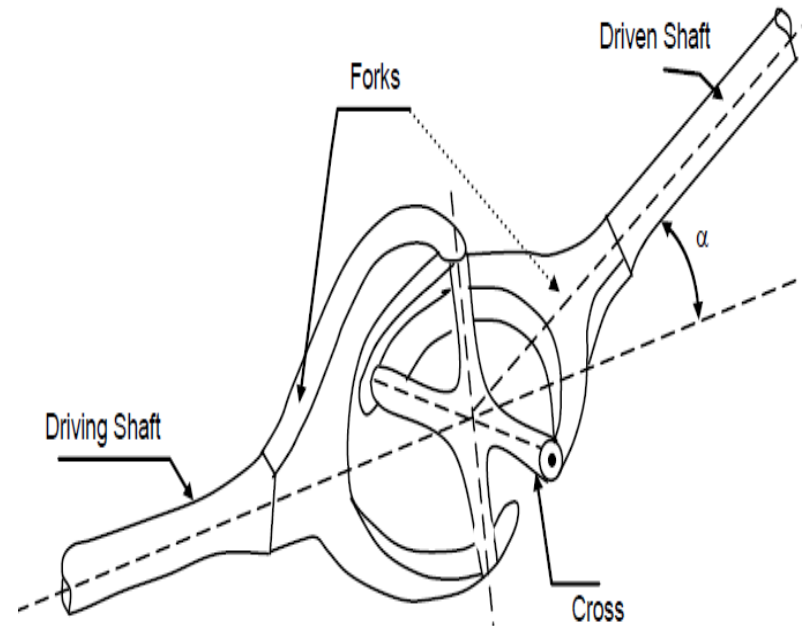
Hook's Joint or Universal Coupling

- It is also known as *universal joint*.
- It is used for connecting two shafts whose axes are non-parallel but intersecting as shown in Figure. Both the shafts, driving and the driven, are forked at their ends. Each fork provides for two bearings for the respective arms of the cross. The cross has two mutually perpendicular arms. In fact, the cross acts as an intermediate link between the two shafts. In the figure, the driven shaft has been shown as inclined at an angle α with the driving shaft



Parts of universal joints

- transmission of motion from the gear box to the back axle of automobile
- drive to the spindles in a multi-spindle drilling machines
- motion is required to be transmitted in non-parallel shafts with their axes intersecting



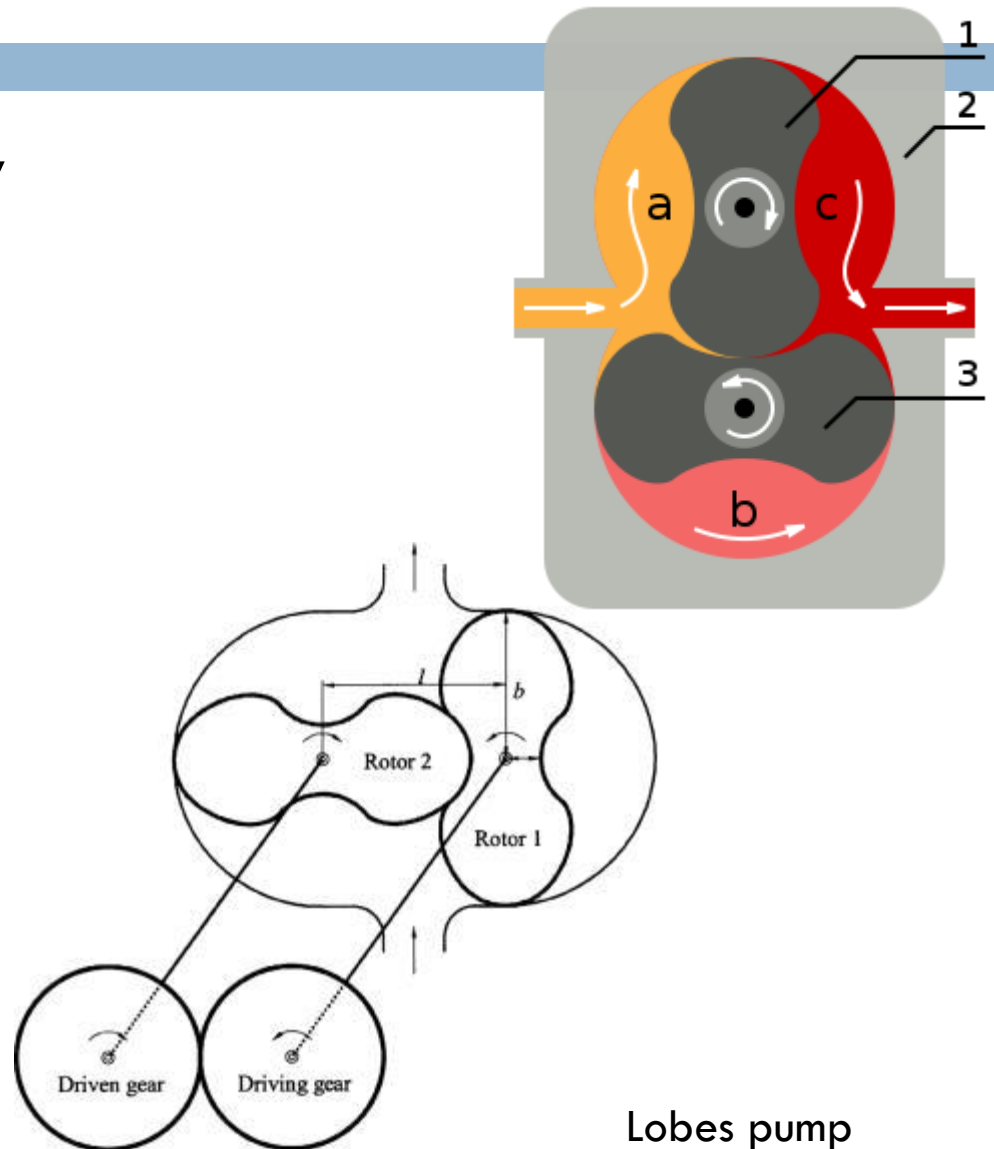
Chamber wheels

In this mechanism mainly the two types

i. Lobe pump

Two lobes operates inside the casing.

pressure, temperature, and density of air supplied to an internal combustion engine.



Lobes pump

Operation of Rankle engine

- In the Wankel engine, the four strokes of a typical Otto cycle occur
- Wankel engines are considerably lighter, simpler, and contain far fewer moving parts than piston engines
- In the racing world, Mazda has had substantial success with two-rotor, three-rotor, and four-rotor cars. Private racers have also had considerable success with stock and modified Mazda Wankel-engine car

