2 VELOCITY ANALYSIS

Introduction

As mentioned in the first chapter, analysis of mechanisms is the study of motions and forces concerning their different parts. The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as the angular velocities of the links. The velocity analysis is the prerequisite for acceleration analysis which further leads to force analysis of various links of a mechanism. To facilitate such study, a machine or a mechanism is represented by a skeleton or a line diagram, commonly knows as a *configuration diagram*.

Velocities and accelerations in machines can be determined either analytically or graphically. With the invention of calculators and computers, it has become convenient to make use of analytical methods. However, a graphical analysis is more direct and is accurate to an acceptable degree and thus cannot be neglected. This chapter is mainly devoted to the study of graphical methods of velocity analysis. Two methods of graphical approach, namely, relative velocity method and instantaneous centre method are discussed. The algebraic methods are also discussed in brief. The analytical approach involving the use of calculators and computers will be discussed in Chapter 4.

2.1 ABSOLUTE AND RELATIVE MOTIONS

Strictly speaking, all motions are relative since an arbitrary set of axes or planes is required to define a motion. Usually, the earth is taken to be a fixed reference plane and all motions relative to it are termed absolute motions.

If a train moves in a particular direction, the motion of the train is referred as the absolute motion of the train or motion of the train relative to the earth. Now, suppose a man moves inside the train. Then, the motion of the man will be described in two different ways with different meanings:

1. Motion of the man relative to the train—it is equivalent to the motion of the man assuming the train to be stationary.
2. Motion of the man or absolute motion of the man or motion of the man relative to the earth = motion of man relative to the train + Motion of train relative to the earth.

2.2 VECTORS

Problems involving relative motions are conveniently solved by the use of vectors. A vector is a line which represents a vector quantity such as force, velocity, acceleration, etc.

Characteristics of a Vector

1. Length of the vector **ab** (Fig. 2.1) drawn to a convenient scale, represents the magnitude of the quantity (written as *ab*).

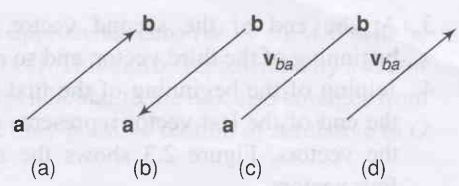


Fig. 2.1

Direction of the line is parallel to the direction in which the quantity acts.

The initial end **a** of the line is the tail and the final end **b**, the head. An arrowhead on the line indicates the direction-sense of the quantity which is always from the tail to the head, i.e., **a** to **b**.

If the sense is as shown in Fig. 2.1(a), the vector is read as **ab** and if the sense is opposite [Fig. 2.1 (b)], the vector is read as **ba**. This implies that **ab** = – **ba**

2. Vector **ab** may also represent a vector quantity of a body *B* relative to a body *A* such as velocity of *B* relative to *A*.

   If the body *A* is fixed, **ab** represents the absolute velocity of *B*. If both the bodies *A* and *B* are in motion, the velocity of *B* relative to *A* means the velocity of *B* assuming the body *A* to be fixed for the moment.

   The vector **ab** can also be shown as $\mathbf{v}_{ba}$ [Fig. 2.1(c)], meaning the velocity of *B* relative to *A* provided *a* and *b* are indicated at the ends or an arrowhead is put on the vector [Fig. 2.1(d)].

3. Vector **ab** may also represent a vector quantity of a point *B* relative to a point *A* in the same body.

   If a vector $\mathbf{v}_{ba}$ or **ab** represents the velocity of *B* relative to *A*, the same vector in the opposite sense represents the velocity of *A* relative to *B* and will be read as $\mathbf{v}_{ab}$ or **ba**.

2.3 ADDITION AND SUBTRACTION OF VECTORS

Let

$\mathbf{v}_{ao}$ = velocity of *A* relative to *O*

$\mathbf{v}_{ba}$ = velocity of *B* relative to *A*

$\mathbf{v}_{bo}$ = velocity of *B* relative to *O*

The law of vector addition states that the velocity of *B* relative to *O* is equal to the vectorial sum of the velocity of *B* relative to *A* and the velocity of *A* relative to *O*.

Velocity of *B* relative to *O* = velocity of *B* relative to *A* + velocity of *A* relative to *O* (2.1)

i.e. $\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$

$= \mathbf{v}_{ao} + \mathbf{v}_{ba}$

or $\mathbf{ob} = \mathbf{oa} + \mathbf{ab}$

Take the vector **oa** and place the vector **ab** at the end of the vector **oa.** Then **ob** is given by the closing side of the two vectors (Fig. 2.2).

Note that the arrows of the two vectors to be added are in the same order and that of the resultant is in the opposite order.

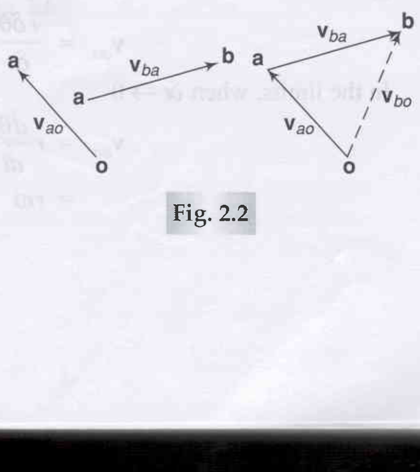


Fig. 2.2

Any number of vectors can be added as follows:

1. Take the first vector.
2. At the end of the first vector, place the beginning of the second vector.

3. At the end of the second vector, place the beginning of the third vector, and so on.
4. Joining of the beginning of the first vector and the end of the last vector represents the sum of the vectors. Figure 2.3 shows the addition of four vectors.

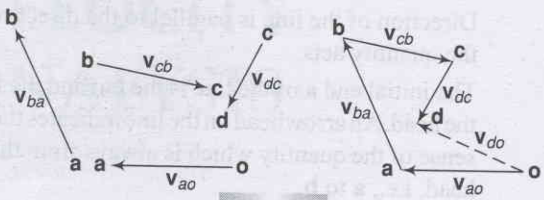


Fig. 2.3

$$\begin{aligned} \mathbf{v}_{do} &= \mathbf{v}_{dc} + \mathbf{v}_{cb} + \mathbf{v}_{ba} + \mathbf{v}_{ao} \\ &= \mathbf{v}_{ao} + \mathbf{v}_{ba} + \mathbf{v}_{cb} + \mathbf{v}_{dc} \\ \mathbf{od} &= \mathbf{oa} + \mathbf{ab} + \mathbf{bc} + \mathbf{cd} \end{aligned} \tag{2.2}$$

Equation 2.1 may be written as,

Vel. of B rel. to A = Vel. of B rel. to O – Vel. of A rel. to O

$$\mathbf{v}_{ba} = \mathbf{v}_{bo} - \mathbf{v}_{ao} \tag{2.3}$$

or
$$\mathbf{ab} = \mathbf{ob} - \mathbf{oa}$$

This shows that in Fig. 2.2, **ab** also represents the subtraction of **oa** from **ob** [Fig. 2.4(a)]

Also
$$\mathbf{v}_{ab} = -\mathbf{v}_{ba} = \mathbf{v}_{ao} - \mathbf{v}_{bo}$$
or
$$\mathbf{ba} = \mathbf{oa} - \mathbf{ob}$$

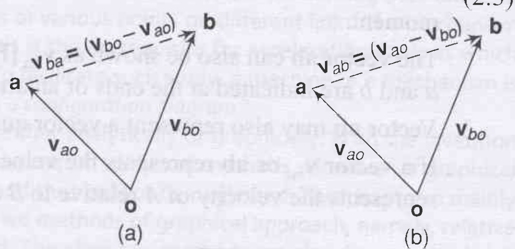


Fig. 2.4

This has been shown in Fig. 2.4 (b).

Thus, the difference of two vectors is given by the closing side of a triangle, the other two sides of which are formed by placing the two vectors tail to tail, the sense being towards the vector quantity from which the other is subtracted.

2.4 MOTION OF A LINK

Let a rigid link OA, of length r, rotate about a fixed point O with a uniform angular velocity ω rad/s in the counter-clockwise direction [Fig.2.5 (a)]. OA turns through a small angle $\delta\theta$ in a small interval of time δt. Then A will travel along the arc AA' as shown in [Fig.2.5(b)].

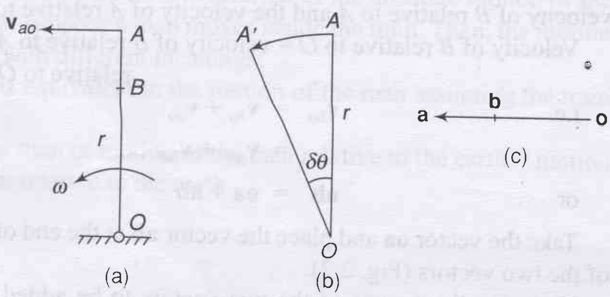


Fig. 2.5

Velocity of A relative to $O = \dfrac{\text{Arc}AA'}{\delta t}$

or

$$v_{ao} = \frac{r\delta\theta}{\delta t}$$

In the limits, when $\delta t \to 0$

$$\begin{aligned} v_{ao} &= r\frac{d\theta}{dt} \\ &= r\omega \end{aligned} \tag{2.4}$$

The direction of v_{ao} is along the displacement of A. Also, as δt approaches zero ($\delta t \to 0$), AA' will be perpendicular to OA. Thus, velocity of A is ωr and is perpendicular to OA. This can be represented by a vector **oa** (Fig. 2.5 c). The fact that the direction of the velocity vector is perpendicular to the link also emerges from the fact that A can neither approach nor recede from O and thus, the only possible motion of A relative to O is in a direction perpendicular to OA.

Consider a point B on the link OA.

Velocity of $B = \omega \,.\, OB$ perpendicular to OB.

If **ob** represents the velocity of B, it can be observed that

$$\frac{\mathbf{ob}}{\mathbf{oa}} = \frac{\omega OB}{\omega OA} = \frac{OB}{OA} \tag{2.5}$$

i.e., **b** divides the velocity vector in the same ratio as B divides the link.

Remember, the velocity vector v_{ao} [Fig. 2.5(c)] represents the velocity of A at a particular instant. At other instants, when the link OA assumes another position, the velocity vectors will have their directions changed accordingly.

Also, the magnitude of the instantaneous linear velocity of a point on a rotating body is proportional to its distance from the axis of rotation.

2.5 FOUR-LINK MECHANISM

Figure 2.6(a) shows a four-link mechanism (quadric-cycle mechanism) $ABCD$ in which AD is the fixed link and BC is the coupler. AB is the driver rotating at an angular speed of ω rad/s in the clockwise direction if it is a crank or moving at this angular velocity at this instant if it is a rocker. It is required to find the absolute velocity of C (or velocity of C relative to A).

Writing the velocity vector equation,

Vel. of C rel. to A = Vel. of C rel. to B + vel. of B rel. to A

$$\mathbf{v}_{ca} = \mathbf{v}_{cb} + \mathbf{v}_{ba} \tag{2.6}$$

The velocity of any point relative to any other point on a fixed link is always zero. Thus, all the points on a fixed link are represented by one point in the velocity diagram. In Fig. 2.6(a), the points A and D, both lie on the fixed link AD. Therefore, the velocity of C relative to A is the same as velocity of C relative to D.

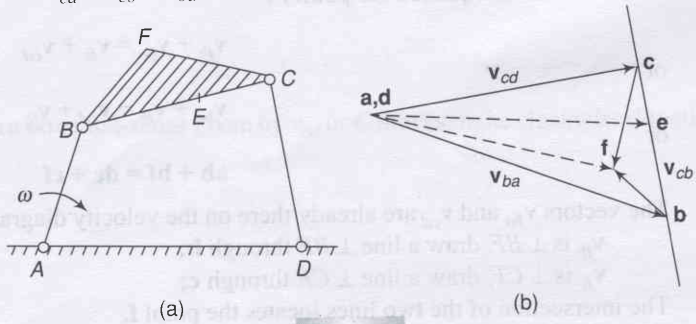


Fig. 2.6

Equation (2.6) may be written as,

$$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

or

$$\mathbf{dc} = \mathbf{ab} + \mathbf{bc}$$

where $\mathbf{v}_{ba}$ or $\mathbf{ab} = \omega AB$; $\perp$ to AB

$\mathbf{v}_{cb}$ or **bc** is unknown in magnitude ; $\perp$ to BC

$\mathbf{v}_{cd}$ or **dc** is unknown in magnitude ; ⊥ to DC

The velocity diagram is constructed as follows:

1. Take the first vector **ab** as it is completely known.
2. To add vector **bc** to **ab**, draw a line ⊥ BC through **b**, of any length. Since the direction-sense of **bc** is unknown, it can lie on either side of **b. A** convenient length of the line can be taken on both sides of **b.**
3. Through **d,** draw a line ⊥ DC to locate the vector **dc.** The intersection of this line with the line of vector **bc** locates the point **c**.
4. Mark arrowheads on the vectors **bc** and **dc** to give the proper sense. Then **dc** is the magnitude and also represents the direction of the velocity of C relative to A (or D). It is also the absolute velocity of the point C (A and D being fixed points).
5. Remember that the arrowheads on vector **bc** can be put in any direction because both ends of the link BC are movable. If the arrowhead is put from **c** to **b**, then the vector is read as **cb**. The above equation is modified as

$$\mathbf{dc} = \mathbf{ab} - \mathbf{cb} \qquad (\mathbf{bc} = -\,\mathbf{cb})$$

or

$$\mathbf{dc} + \mathbf{cb} = \mathbf{ab}$$

Intermediate Point

The velocity of an intermediate point on any of the links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides the link. For point E on the link BC,

$$\frac{\mathbf{be}}{\mathbf{bc}} = \frac{BE}{BC}$$

ae represents the absolute velocity of E.

Offset Point

Write the vector equation for point F,

$$\mathbf{v}_{fb} + \mathbf{v}_{ba} = \mathbf{v}_{fc} + \mathbf{v}_{cd}$$

or

$$\mathbf{v}_{ba} + \mathbf{v}_{fb} = \mathbf{v}_{cd} + \mathbf{v}_{fc}$$

or

$$\mathbf{ab} + \mathbf{bf} = \mathbf{dc} + \mathbf{cf}$$

The vectors $\mathbf{v}_{ba}$ and $\mathbf{v}_{cd}$ are already there on the velocity diagram.

$\mathbf{v}_{fb}$ is ⊥ BF, draw a line ⊥ BF through **b;**

$\mathbf{v}_{fc}$ is ⊥ CF, draw a line ⊥ CF through **c;**

The intersection of the two lines locates the point **f.**

af or **df** indicates the velocity of F relative to A (or D) or the absolute velocity of F.

2.6 VELOCITY IMAGES

Note that in Fig. 2.6, the triangle **bfc** is similar to the triangle BFC in which all the three sides **bc**, **cf** and **fb** are perpendicular to BC, CF and FB respectively. The triangles such as **bcf** are known as velocity images and are found to be very helpful devices in the velocity analysis of complicated shapes of the linkages. Thus, any offset point on a link in the configuration diagram can easily be located in the velocity diagram by drawing the velocity image. While drawing the velocity images, the following points should be kept in mind:

1. The velocity image of a link is a scaled reproduction of the shape of the link in the velocity diagram from the configuration diagram, rotated bodily through 90° in the direction of the angular velocity.
2. The order of the letters in the velocity image is the same as in the configuration diagram.
3. In general, the ratio of the sizes of different images to the sizes of their respective links is different in the same mechanism.

2.7 ANGULAR VELOCITY OF LINKS

1. Angular Velocity of BC

(a) Velocity of C relative to B, v_{cb} = **bc** (Fig. 2.6)

Point C relative to B moves in the direction-sense given by $\mathbf{v}_{cb}$ (upwards). Thus, C moves in the counter-clockwise direction about B.

$$\mathbf{v}_{cb} = \boldsymbol{\omega}_{cb} \times BC = \omega_{cb} \times CB$$

$$\omega_{cb} = \frac{v_{cb}}{CB}$$

(b) Velocity of B relative to C, $\mathbf{v}_{bc}$ = **cb**

B relative to C moves in a direction-sense given by v_{bc} (downwards, opposite to **bc**), i.e., B moves in the counter-clockwise direction about C with magnitude ω_{bc} given by

$$\frac{v_{bc}}{BC}$$

It can be seen that the magnitude of $\omega_{cb} = \omega_{bc}$ as $v_{cb} = v_{bc}$ and the direction of rotation is the same. Therefore, angular velocity of a link about one extremity is the same as the angular velocity about the other. In general, the angular velocity of link BC is ω_{bc} (= ω_{cb}) in the counter-clockwise direction.

2. Angular Velocity of CD

Velocity of C relative to D,

v_{cd} = **dc**

It is seen that C relative to D moves in a direction-sense given by v_{cd} or C moves in the clockwise direction about D.

$$\omega_{cd} = \frac{v_{cd}}{CD}$$

2.8 VELOCITY OF RUBBING

Figure 2.7 shows two ends of the two links of a turning pair. A pin is fixed to one of the links whereas a hole is provided in the other to fit the pin. When joined, the surface of the hole of one link will rub on the surface of the pin of the other link. The velocity of rubbing of the two surfaces will depend upon the angular velocity of a link relative to the other.

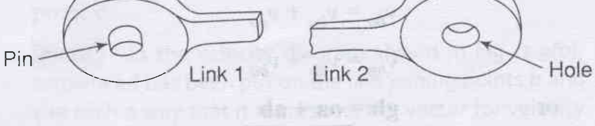


Fig. 2.7

Pin at A (Fig. 2.6a)

The pin at A joins links AD and AB. AD being fixed, the velocity of rubbing will depend upon the angular velocity of AB only.

Let r_a = radius of the pin at A

Then velocity of rubbing $= r_a . \omega$

Pin at D

Let r_d = radius of the pin at D

Velocity of rubbing $= r_d . \omega_{cd}$

Pin at B

$\omega_{ba} = \omega_{ab} = \omega$ clockwise

$\omega_{bc} = \omega_{cb} = \dfrac{v_{bc}}{BC}$ counter-clockwise

Since the directions of the two angular velocities of links AB and BC are in the opposite directions, the angular velocity of one link relative to the other is the sum of the two velocities.

Let r_b = radius of the pin at B

Velocity of rubbing $= r_b\,(\omega_{ab} + \omega_{bc})$

Pin at C

$\omega_{bc} = \omega_{cb}$ counter-clockwise

$\omega_{dc} = \omega_{cd}$ clockwise

Let r_c = radius of the pin at C

Velocity of rubbing $= r_c\,(\omega_{bc} + \omega_{dc})$

In case it is found that the angular velocities of the two links joined together are in the same direction, the velocity of rubbing will be the difference of the angular velocities multiplied by the radius of the pin.

2.9 SLIDER-CRANK MECHANISM

Figure 2.8(a) shows a slider-crank mechanism in which OA is the crank moving with uniform angular velocity ω rad/s in the clockwise direction. At point B, a slider moves on the fixed guide G. AB is the coupler joining A and B. It is required to find the velocity of the slider at B.

Writing the velocity vector equation,

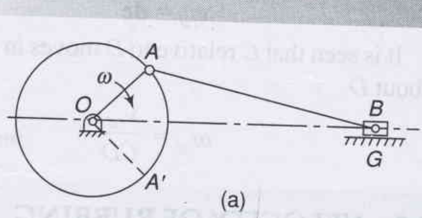
Vel. of B rel. to O = Vel. of B rel. to A + Vel. of A rel. to O

$$v_{bo} = v_{ba} + v_{ao}$$

$$v_{bg} = v_{ao} + v_{ba}$$

or $\qquad$ **gb = oa + ab**

v_{bo} is replaced by v_{bg} as O and G are two points on a fixed link



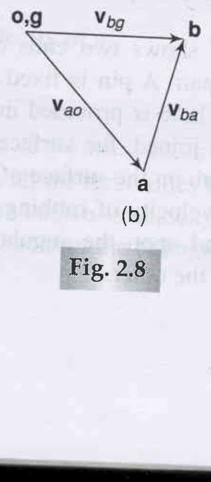


Fig. 2.8

with zero relative velocity between them.

Take the vector $\mathbf{v}_{ao}$ which is completely known.

$\mathbf{v}_{ao} = \omega . OA$; $\perp$ to OA

$\mathbf{v}_{ba}$ is $\perp$ AB, draw a line $\perp$ AB through **a**;

Through **g** (or **a**), draw a line parallel to the motion of B (to locate the vector $\mathbf{v}_{bg}$).

The intersection of the two lines locates the point **b**.

gb (or **ob**) indicates the velocity of the slider B relative to the guide G. This is also the absolute velocity of the slider (G is fixed). The slider moves towards the right as indicated by **gb.** When the crank assumes the position OA' while rotating, it will be found that the vector **gb** lies on the left of **g** indicating that B moves towards left.

For the given configuration, the coupler AB has angular velocity in the counter-clockwise direction, the magnitude being $\dfrac{v_{ba}}{BA(\text{or } AB)}$

Example 2.1 *In a four-link mechanism, the dimensions of the links are as under:*

AB = 50 mm, BC = 66 mm, CD = 56 mm and AD = 100 mm

At the instant when ∠DAB = 60°, the link AB has an angular velocity of 10.5 rad/s in the counter-clockwise direction. Determine the

(i) velocity of the point C

(ii) velocity of the point E on the link BC when BE = 40 mm

(iii) angular velocities of the links BC and CD

(iv) velocity of an offset point F on the link BC if BF = 45 mm, CF = 30 mm and BCF is read clockwise

(v) velocity of an offset point G on the link CD if CG = 24 mm, DG = 44 mm and DCG is read clockwise

(vi) velocity of rubbing at pins A, B, C and D when the radii of the pins are 30, 40, 25 and 35 mm respectively.

Solution The configuration diagram has been shown in Fig. 2.9(a) to a convenient scale.

Writing the vector equation,

Vel. of C rel. to A = Vel. of C rel. to B + Vel. of B rel. to A

$$\mathbf{v}_{ca} = \mathbf{v}_{cb} + \mathbf{v}_{ba}$$

or $$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

or $$\mathbf{dc} = \mathbf{ab} + \mathbf{bc}$$

We have,

$$v_{ba} = \omega_{ba} \times BA = 10.5 \times 0.05 = 0.525 \text{ m/s}$$

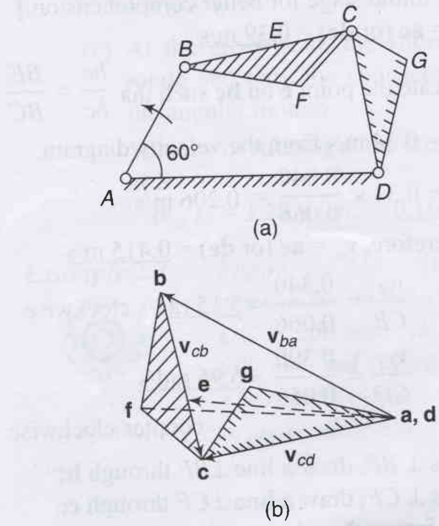


Fig. 2.9

Take the vector $\mathbf{v}_{ba}$ to a convenient scale in the proper direction and sense [Fig. 2.9(b)].

$\mathbf{v}_{cb}$ is $\perp$ BC, draw a line $\perp$ BC through **b;**

$\mathbf{v}_{cd}$ is $\perp$ DC, draw a line $\perp$ DC through **d;**

The intersection of the two lines locates the point **c.**

Note In the velocity diagram shown in Fig. 2.9(b), arrowhead has been put on the line joining points b and c in such a way that it represents the vector for velocity of C relative to B. This satisfies the above equation. As the same equation

$$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

can also be put as

$$\mathbf{v}_{cd} + \mathbf{v}_{bc} = \mathbf{v}_{ba}$$

$$\mathbf{dc} + \mathbf{cb} = \mathbf{ab}$$

This shows that on the same line joining **b** and **c**, the arrowhead should be marked in the other direction so that it represents the vector of velocity of B relative to C to satisfy the latter equation.

Thus, it implies that in case both the ends of a link are in motion, the arrowhead may be put in either direction or no arrowhead is put at all. This is because every time it is not necessary to write the velocity equation. The velocity equation is helpful only at the initial stage for better comprehension.]

(i) v_c = **ac** (or **dc**) = <u>0.39 m/s</u>

(ii) Locate the point **e** on **bc** such tha $\dfrac{be}{bc} = \dfrac{BE}{BC}$

bc = 0.34 m/s from the velocity diagram.

$$\mathbf{be} = 0.34 \times \frac{0.040}{0.066} = 0.206 \text{ m/s}$$

Therefore, v_e = **ae** (or **de**) = <u>0.415 m/s</u>

(iii) $\omega_{cb} = \dfrac{v_{cb}}{CB} = \dfrac{0.340}{0.066} = \underline{5.15 \text{ rad/s}}$ clockwise

$\omega_{cd} = \dfrac{v_{cd}}{CD} = \dfrac{0.390}{0.056} = \underline{6.96 \text{ rad/s}}$ counter-clockwise

(iv) $\mathbf{v}_{fb}$ is $\perp BF$, draw a line $\perp BF$ through **b**;
$\mathbf{v}_{fc}$ is $\perp CF$, draw a line $\perp CF$ through **c**;
The intersection locates the point **f**.
v_f (i.e., v_{fa} or v_{fd}) = **af** = <u>0.495 m/s</u>

The point **f** can also be located by drawing the velocity image **bcf** of the triangle BCF as discussed earlier.

(v) $\mathbf{v}_{gd}$ is $\perp DG$, draw **dg** $\perp DG$ through **d**;
$\mathbf{v}_{gc}$ is $\perp CG$, draw **cg** $\perp CG$ through **c**.
The intersection locates the point **g**.
v_g = **dg** = <u>0.305 m/s</u>

However, the velocity of G could be found directly since G is a point on the link DC which rotates about a fixed point D and the velocity of C is already known.

$$\frac{v_g}{v_c} = \frac{DG}{DC}$$

or

$$v_g = 0.390 \times \frac{0.044}{0.056} = \underline{0.306 \text{ m/s}}$$

The point **g** can also be located by drawing the velocity image **dcg** of the triangle dCG.

(vi) (a) ω_{ba} (or ω_{ab}) is counter-clockwise and ω_{cb} (or ω_{bc}) is clockwise,

Velocity of rubbing at pin $B = (\omega_{ab} + \omega_{cb})r_b$
$= (10.5 + 5.15) \times 0.040$
$= \underline{0.626 \text{ m/s}}$

(b) ω_{dc} is counter-clockwise and ω_{bc} is clockwise,

Velocity of rubbing at the pin C
$= (\omega_{dc} + \omega_{bc})r_c$
$= (6.96 + 5.15) \times 0.025$
$= \underline{0.303 \text{ m/s}}$

(c) Velocity of rubbing at the pin A
$= \omega_{ba}\, r_a = 10.5 \times 0.03 = \underline{0.315 \text{ m/s}}$

(d) Velocity of rubbing at the pin D
$= \omega_{cd}\, r_d = 6.96 \times 0.035 = \underline{0.244 \text{ m/s}}$

Example 2.2 *In a slider-crank mechanism, the crank is 480 mm long and rotates at 20 rad/s in the counter-clockwise direction The length of the connecting rod is 1.6 m. When the crank turns 60° from the inner-dead centre, determine the*

(i) velocity of the slider

(ii) velocity of a point E located at a distance 450 mm on the connecting rod extended

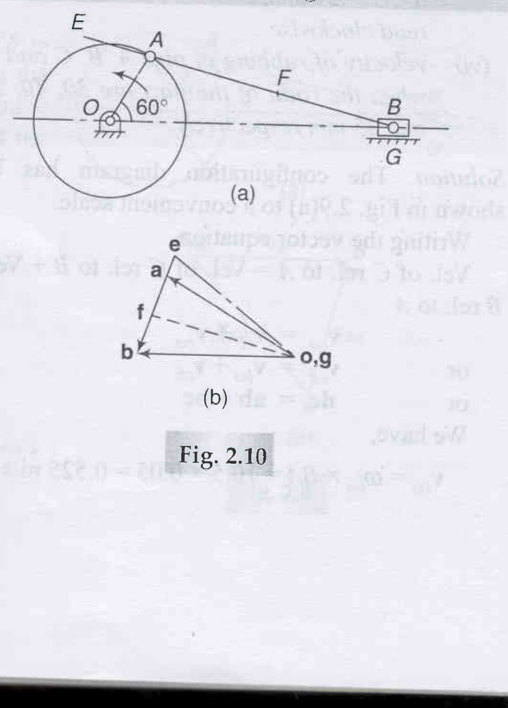


Fig. 2.10

(iii) *position and velocity of a point F on the connecting rod having the least absolute velocity*
(iv) *angular velocity of the connecting rod*
(v) *velocities of rubbing at the pins of the crankshaft, crank and the cross-head having diameters 80, 60 and 100 mm respectively.*

Solution Figure 2.10(a) shows the configuration diagram to a convenient scale.

$v_{ao} = \omega_{ao} \times OA = 20 \times 0.48 = 9.6$ m/s

The vector equation is $\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$

or $\quad \mathbf{v}_{bg} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$

or $\quad \mathbf{v}_{bg} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$

or $\quad \mathbf{gb} = \mathbf{oa} + \mathbf{ab}$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale in the proper direction and sense [Fig. 2.10 (b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a**;

The slider B has a linear motion relative to the guide G. Draw a line parallel to the direction of motion of the slider through **g** (or **o**). Thus, the point **b** is located.

(i) Velocity of the slider, $v_b = \mathbf{ob} = 9.7$ m/s

(ii) Locate the point **e** on **ba** extended such that

$$\frac{\mathbf{ae}}{\mathbf{ba}} = \frac{AE}{BA}$$

ba = 5.25 m/s on measuring from the diagram.

$$\therefore \mathbf{ae} = 5.25 \times \frac{0.45}{1.60} = 1.48 \text{ m/s}$$

$v_e = \mathbf{oe} = \underline{10.2 \text{ m/s}}$

(iii) To locate a point F on the connecting rod which has the least velocity relative to the crankshaft or has the least absolute velocity, draw **of** $\perp$ **ab** through **o**.

Locate the point F on AB such the $\dfrac{AF}{AB} = \dfrac{\mathbf{af}}{\mathbf{ab}}$

or

$$AF = 1.60 \times \frac{2.76}{5.25} = 0.84 \text{ m}$$

$v_f = \mathbf{of} = \underline{9.4 \text{ m/s}}$

(iv) $\omega_{ba} = \dfrac{v_{ba}}{AB} = \dfrac{5.25}{1.60} = 3.28$ rad/s clockwise

(v) (a) Velocity of rubbing at the pin of the crankshaft (at O)

$= \omega_{ao} r_o = 20 \times 0.04 = 0.8$ m/s $\quad \left(r_o \dfrac{80}{2} = 40 \text{ mm} \right)$

(b) ω_{oa} is counter-clockwise and ω_{ba} is clockwise.

Velocity of rubbing at the crank pin $A = (\omega_{oa} + \omega_{ba})\, r_a$

$= (20 + 3.28) \times 0.03$

$= 0.698$ m/s

(c) At the cross-head, the slider does not rotate and only the connecting rod has the angular motion.

Velocity of rubbing at the cross-head pin at B

$= \omega_{ab}\, r_b = 3.28 \times 0.05 = \underline{0.164 \text{ m/s}}$

Example 2.3 *Figure 2.11a shows a mechanism in which OA = QC = 100 mm, AB = QB = 300 mm and CD = 250 mm. The crank OA rotates at 150 rpm in the clockwise direction. Determine the*

(i) *velocity of the slider at D*
(ii) *angular velocities of links QB and AB*
(iii) *rubbing velocity at the pin B which is 40 mm in diameter*

Solution $\quad \omega_{ao} = \dfrac{2\pi \times 150}{60} = 15.7$ rad/s

$v_{ao} = \omega_{ao} \times OA = 15.7 \times 0.1 = 1.57$ m/s

The vector equation for the mechanism $OABQ$,

$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$

or $\quad \mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$ or $\mathbf{qb} = \mathbf{oa} + \mathbf{ab}$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale in the proper direction and sense [Fig. 2.11 (b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a**;

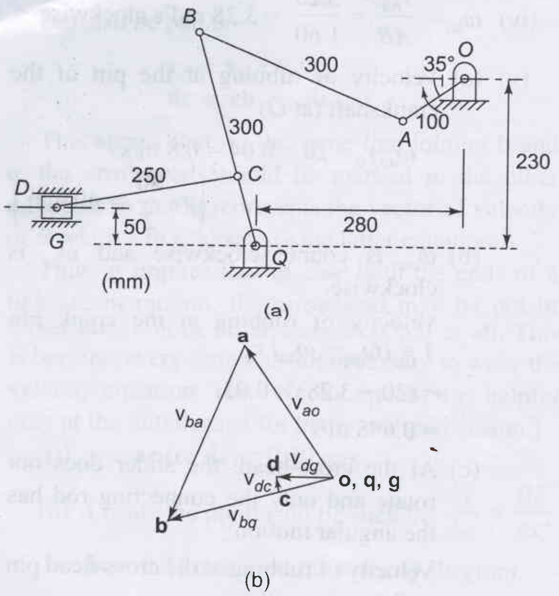


Fig. 2.11

$\mathbf{v}_{bq}$ is $\perp QB$, draw a line $\perp QB$ through **q;**

The intersection of the two lines locates the point **b.**

Locate the point **c** on **qb** such that

$$\frac{\mathbf{qc}}{\mathbf{qb}} = \frac{100}{300} = 0.3$$

The vector equation for the mechanism QCD,

$$\mathbf{v}_{dq} = \mathbf{v}_{dc} + \mathbf{v}_{cq} \quad \text{or} \quad \mathbf{v}_{dg} = \mathbf{v}_{cq} + \mathbf{v}_{dc}$$

or $\quad \mathbf{gd} = \mathbf{qc} + \mathbf{cd}$

$\mathbf{v}_{dc}$ is $\perp DC$, draw a line $\perp DC$ through **c;**

For $\mathbf{v}_{dg}$, draw a line through **g**, parallel to the line of stroke of the slider in the guide G.

The intersection of the two lines locates the point **d.**

(i) The velocity of slider at D, v_d = **gd** = 0.56 m/s

(vi) $\omega_{bq} = \dfrac{v_{bq}}{QB} = \dfrac{1.69}{0.3} = 5.63$ rad/s counter-clockwise

(vii) $\omega_{ba} = \dfrac{v_{ba}}{AB} = \dfrac{1.89}{0.3} = 6.3$ rad/s counter-clockwise

As both the links connected at B have counter-clockwise angular velocities,

velocity of rubbing at the crank pin

$$B = (\omega_{ba} - \omega_{bq})\, r_b$$
$$= (6.3 - 5.63) \times 0.04 = 0.0268 \text{ m/s}$$

Example 2.4 *An engine crankshaft drives a reciprocating pump through a mechanism as shown in Fig. 2.12(a). The crank rotates in the clockwise direction at 160 rpm. The diameter of the pump piston at F is 200 mm. Dimensions of the various links are*

OA = 170 mm (crank) CD = 170 mm
AB = 660 mm DE = 830 mm
BC = 510 mm

For the position of the crank shown in the diagram, determine the

(i) velocity of the crosshead E

(ii) velocity of rubbing at the pins A, B, C and D, the diameters being 40, 30, 30 and 50 mm respectively

(iii) torque required at the shaft O to overcome a pressure of 300 kN/m² at the pump piston at F

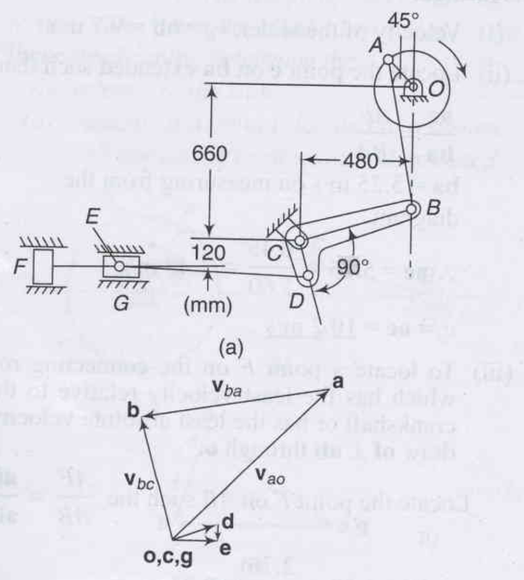


Fig. 2.12

Solution:

$$\omega_{ao} = \frac{2\pi N}{60} = \frac{2\pi \times 160}{60} = 16.76 \text{ rad/s}$$

$$v_{ao} = \omega_{ao} \times OA = 16.76 \times 0.17 = 2.85 \text{ m/s}$$

Writing the vector equation for the mechanism *OABC*,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{bc} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or

$$\mathbf{cb} = \mathbf{oa} + \mathbf{ab}$$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.12(b)]

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a;**

$\mathbf{v}_{bc}$ is $\perp BC$, draw a line $\perp BC$ through **c.**

The intersection of the two lines locates the point *b*. Velocity of *B* relative to *C* is upwards for the given configuration. Therefore, the link *BCD* moves counter-clockwise about the pivot *C*.

$$\frac{v_{dc}}{v_{bc}} = \frac{DC}{BC}$$

or $v_{dc} = 1.71 \times \dfrac{0.17}{0.51} = 0.57$ m/s $\quad (\perp DC)$

Writing the vector equation for the mechanism *CDE*,

$$\mathbf{v}_{ec} = \mathbf{v}_{ed} + \mathbf{v}_{dc}$$

or

$$\mathbf{v}_{eg} = \mathbf{v}_{dc} + \mathbf{v}_{ed}$$

or

$$\mathbf{ge} = \mathbf{cd} + \mathbf{de}$$

Take $\mathbf{v}_{dc}$ in the proper direction and sense from **c** assuming *D* in the configuration diagram as an offset point on link *CB;*

$\mathbf{v}_{ed}$ is $\perp DE$, draw a line $\perp DE$ through **d**.

For $\mathbf{v}_{eg}$, draw a line through **g**, parallel to the direction of motion of the slider *E* in the guide *G*.

This way the point **e** is located.

(i) The velocity of the crosshead,

$$v_e = \mathbf{oe} = \underline{0.54 \text{ m/s}}$$

(ii) (a) ω_{oa} and ω_{ba} both are clockwise.

$$\omega_{ba} = \frac{\mathbf{ab}}{AB} = \frac{2.49}{0.66} = 3.77 \text{ rad/s}$$

Velocity of rubbing at the pin $A = (\omega_{oa} - \omega_{ba})\, r_a$

$$= (16.76 - 3.77) \times \frac{0.04}{2}$$

$$= \underline{0.26 \text{ m/s}}$$

(b) ω_{ab} is clockwise and ω_{cd} is counter-clockwise.

$$\omega_{cb} = \frac{v_{cb}}{CB} = \frac{1.71}{0.51} = 3.35 \text{ rad/s}$$

Velocity of rubbing at $B = (\omega_{ab} + \omega_{cb})\, r_b$

$= (3.77 + 3.35) \times 0.015 \;\ldots.\; (\omega_{ab} = \omega_{ba})$

$= \underline{0.107 \text{ m/s}}$

(c) Velocity of rubbing at $C = \omega_{bc} . r_c$

$$= 3.35 \times \frac{0.03}{2} = \underline{0.05 \text{ m/s}}$$

(d) ω_{cd} and $\omega_{ed,}$ both are counter-clockwise

$\omega_{cd} = \omega_{bc} = 3.35$ rad/s … (*BCD* is one link)

$$= \omega_{ed} = \frac{v_{ed}}{ED} = \frac{0.15}{0.83} = 0.18 \text{ rad/s}$$

Velocity of rubbing at $D = (\omega_{cd} - \omega_{ed})\, r_d$

$$= (3.35 - 0.18) \times \frac{0.05}{2}$$

$$= \underline{0.079 \text{ m/s}}$$

(iii) Work input = work output

$T.\omega = F.v$

where T = torque on the crankshaft

ω = angular velocity of the crank

F = force on the piston

v = velocity of the piston $= v_f = v_e$

Thus, neglecting losses,

$$T = \frac{F.v}{\omega} = \frac{\pi}{4}(0.02)^2 \times 300 \times 10^3 \times \frac{0.54}{16.76}$$

$$= \underline{303.66 \text{ N.m}}$$

Example 2.5 *Figure 2.13(a) shows a mechanism in which OA = 300 mm, AB = 600 mm, AC = BD = 1.2 m. OD is horizontal for the given configuration. If OA rotates at 200 rpm in the clockwise direction, find*

(iv) linear velocities of C and D

(v) angular velocities of links AC and BD

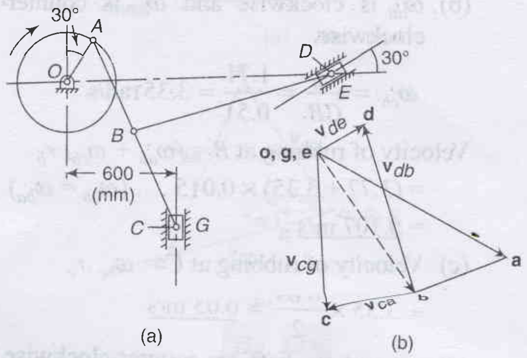


Fig. 2.13

Solution: $\omega_a = \dfrac{2\pi \times 200}{60} = 20.94$ rad/s

$v_a = \omega_a OA = 20.94 \times 0.3 = 6.28$ m/s

Writing the vector equation for the mechanism *OAC*,

$$\mathbf{v}_{co} = \mathbf{v}_{ca} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{cg} = \mathbf{v}_{ao} + \mathbf{v}_{ca}$$

or

$$\mathbf{gc} = \mathbf{oa} + \mathbf{ac}$$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.13(b)].

$\mathbf{v}_{ca}$ is $\perp AC$, draw a line $\perp AC$ through **a**;

$\mathbf{v}_{cg}$ is vertical, draw a vertical line through **g** (or **o**).

The intersection of the two lines locates the point **c**. Locate the point **b** on **ac** as usual. Join **ob** which gives $\mathbf{v}_{bo}$. Writing the vector equation for the mechanism *OABD*,

$$\mathbf{v}_{do} = \mathbf{v}_{db} + \mathbf{v}_{bo}$$

or

$$\mathbf{v}_{de} = \mathbf{v}_{bo} + \mathbf{v}_{db}$$

or

$$\mathbf{ed} = \mathbf{ob} + \mathbf{bd}$$

$\mathbf{v}_{db}$ is $\perp BD$, draw a line $\perp BD$ through **b**;

For $\mathbf{v}_{de}$, draw a line through **e**, parallel to the line of stroke of the piston in the guide *E*.

The intersection locates the point **d**.

$v_c = \mathbf{oc} = \underline{5.2 \text{ m/s}}$

$v_d = \mathbf{od} = \underline{1.55 \text{ m/s}}$

$$\omega_{ac} = \omega_{ca} = \frac{v_{ca}}{AC} = \frac{5.7}{1.20} = \underline{4.75 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{bd} = \omega_{db} = \frac{v_{db}}{BD} = \frac{5.17}{1.20} = \underline{4.31 \text{ rad/s}} \text{ clockwise}$$

Example 2.6 *For the position of the mechanism shown in Fig. 2.14(a), find the velocity of the slider B for the given configuration if the velocity of the slider A is 3 m/s.*

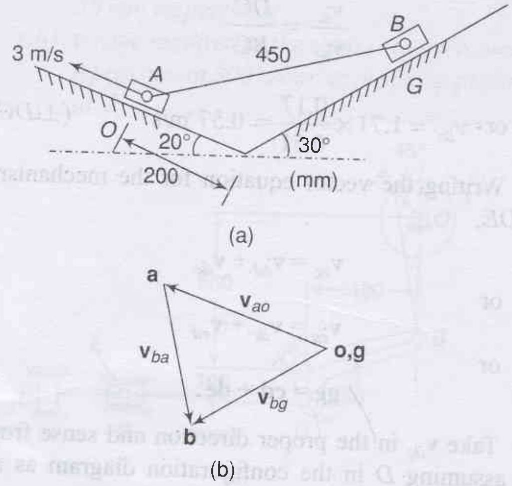


Fig. 2.14

Solution The velocity vector equation is

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{bg} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or

$$\mathbf{gb} = \mathbf{oa} + \mathbf{ab}$$

Take the vector $\mathbf{v}_{ao}$ (= 3 m/s) to a convenient scale [Fig. 2.14(b)]

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line AB through **a**;

For $\mathbf{v}_{bg}$, draw a line through **g** parallel to the line of stroke of the slider B on the guide G.

The intersection of the two lines locates the point **b**.

Velocity of B = **gb** = 2.67 m/s.

Example 2.7 *In a mechanism shown in Fig. 2.15(a), the angular velocity of the crank OA is 15 rad/s and the slider at E is constrained to move at 2.5 m/s. The motion of both the sliders is vertical and the link BC is horizontal in the position shown. Determine the*

(i) rubbing velocity at B if the pin diameter is 15 mm

(ii) velocity of slider D.

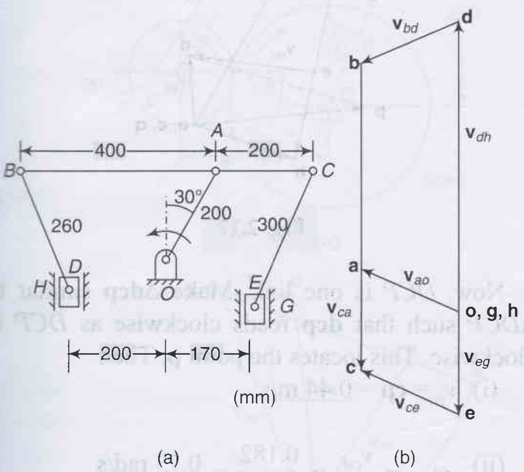


Fig. 2.15

Solution $v_a = \omega_a OA = 15 \times 0.2 = 3$ m/s

Draw the velocity diagram as follows:

- Take vector **oa** to a suitable scale (2.15b).
- Consider two points G and H on the guides of sliders E and F respectively. In the velocity diagram, the points **g** and **h** coincide with **o**. Through **g**, take a vector **ge** parallel to direction of motion of the slider E and equal to 2.5 m/s using some scale.
- Through **e** draw a line $\perp EC$ and through **a**, a line $\perp AC$, the intersection of these two lines locates the point **c**.
- Locate the point **b** on the vector **ca** so that **ca**/**cb** = CA/CB.
- Through **b,** draw a line $\perp BD$ and through **h**, a line parallel to direction of motion of the slider D, the intersection of these two lines locates the point **d**.

(i) Angular velocity of link BD = **bd**/BD = 2.95/0.26 =11.3 rad/s (counter-clockwise)
Angular velocity of link BC = **bc**/BC = 8.4/0.6 =14 rad/s (clockwise)
Thus velocity of rubbing at

$$B = (\omega_{bd} + \omega_{bc})r_b$$
$$= (11.3 + 14) \times 0.015$$
$$= \underline{0.38 \text{ m/s}}$$

(ii) The velocity of the slider D = **hd** = $\underline{8.3 \text{ m/s}}$

Example 2.8 *The lengths of various links of a mechanism shown in Fig. 2.16(a) are as follows:*

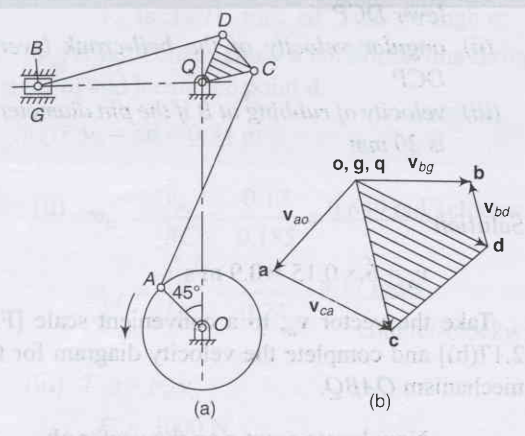


Fig. 2.16

OA = 150 mm	*CD = 125 mm*
AC = 600 mm	*BD = 500 mm*
CQ = QD = 145 mm	*OQ = 625 mm*

The crank OA rotates at 60 rpm in the counter-clockwise direction. Determine the velocity of the slider B and the angular velocity of the link BD when the crank has turned an angle of 45° with the vertical.

Solution

$$v_a = \frac{2\pi N}{60} \times OA = \frac{2\pi \times 60}{60} \times 0.15 = 0.94 \text{ m/s}$$

Take the vector $\mathbf{v}_a$, to a convenient scale [Fig. 2.16(b)] and complete the velocity diagram for the mechanism *OACQ*.

Now *CQD* is one link. Make Δ **cqd** similar to Δ *CQD* such that **cqd** reads clockwise as *CQD* is clockwise. This locates the point **d**. Complete the velocity diagram for the mechanism *QDB*.

$v_b = \mathbf{ob} = \underline{0.9 \text{ m/s}}$

$$\omega_{bd} = \frac{v_{bd}}{BD} = \frac{0.49}{0.50} = \underline{0.98 \text{ rad/s}} \text{ clockwise}$$

Example 2.9 *The configuration diagram of a wrapping machine is given in Fig. 2.17(a). The crank OA rotates at 6 rad/s clockwise. Determine the*

(i) velocity of the point P on the bell-crank lever DCP

(ii) angular velocity of the bell-crank lever DCP

(iii) velocity of rubbing at B if the pin diameter is 20 mm

Solution

$$v_a = 6 \times 0.15 = 0.9 \text{ m/s}$$

Take the vector $\mathbf{v}_a$, to a convenient scale [Fig. 2.17(b)] and complete the velocity diagram for the mechanism *OABQ*.

Now locate point **e** on the vector **ab**.

$\mathbf{v}_{de}$ is ⊥ *DE*, draw **de** ⊥ *DE* through **e**;

$\mathbf{v}_{dc}$ is ⊥ *CD*, draw **cd** ⊥ *CD* through **c**.

The intersection locates the point **d**.

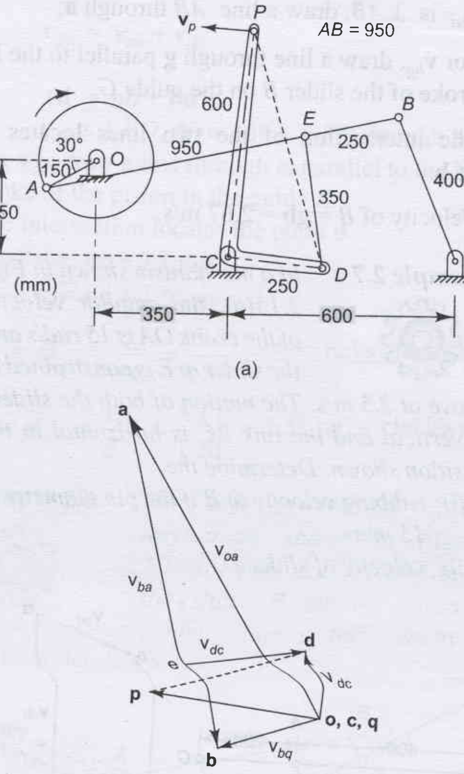


Fig. 2.17

Now, *DCP* is one link. Make Δ**dcp** similar to Δ*DCP* such that **dcp** reads clockwise as *DCP* is clockwise. This locates the point **p**. Then

(i) $v_p = \mathbf{cp} = \underline{0.44 \text{ m/s}}$

(ii) $$\omega_{cd} = \frac{v_{cd}}{CD} = \frac{0.182}{0.25} = \underline{0.73 \text{ rad/s}}$$

counter clockwise

(iii) $$\omega_{ab} = \frac{v_{ab}}{AB} = \frac{0.91}{0.95} = \underline{0.96 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{qb} = \frac{v_{qb}}{QB} = \frac{0.28}{0.4} = \underline{0.7 \text{ rad/s}}$$

counter-clockwise

Thus, velocity of rubbing at $B = (\omega_{ab} + \omega_{bq})r_b$

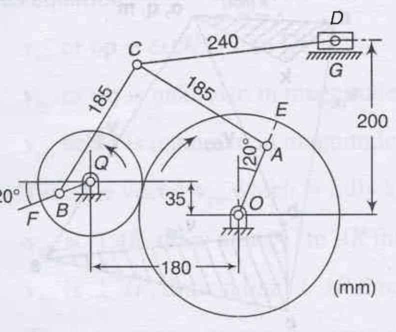
$= (0.96 + 0.7) \times 0.02 = \underline{0.0332 \text{ m/s}}$

Example 2.10 *Figure 2.18(a) shows an Andrew variable-stroke-engine mechanism. The lengths of the cranks OA and QB are 90 mm and 45 mm respectively. The diameters of wheels with centres O and Q are 250 mm and 120 mm respectively. Other lengths are shown in the diagram in mm. There is a rolling contact between the two wheels. If OA rotates at 100 rpm, determine the*

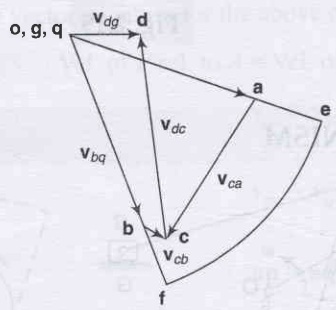
(i) velocity of the slider D

(ii) angular velocities of links BC and CD

(iii) torque at QB when force required at D is 3 kN



(a)



(b)

Fig. 2.18

Solution

$$v_a = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} \times 0.09 = 0.943 \text{ m/s}$$

$$v_e = v_a \frac{OE}{OA} = 0.943 \times \frac{0.125}{0.09} = 1.309 \text{ m/s}$$

$$v_f = v_e = 1.309 \text{ m/s}$$

$$v_b = v_f \frac{QB}{QF} = 1.309 \times \frac{0.045}{0.06} = 0.982 \text{ m/s}$$

v_b can also be obtained graphically as follows:

Take vector $\mathbf{v}_a$ to a convenient scale [Fig. 2.18(b)]. Produce **oa** to **e** such that **oe/oa** = *OE/OA*. Rotate **oe** to **of** so that **of** is perpendicular to *QF*. Mark the point **b** on **qf** such that **qb/qf** = *QB/Qf*.

Now, $\mathbf{v}_{co} = \mathbf{v}_{cq}$

$$\mathbf{v}_{ca} + \mathbf{v}_{ao} = \mathbf{v}_{cb} + \mathbf{v}_{bq}$$

or

$$\mathbf{v}_{ao} + \mathbf{v}_{ca} = \mathbf{v}_{bq} + \mathbf{v}_{cb}$$

or

oa + ac = qb + bc

$\mathbf{v}_{ao}$ and $\mathbf{v}_{bq}$ are already there in the velocity diagram.

$\mathbf{v}_{ca}$ is $\perp AC$, draw a line $\perp AC$ through **a**;

$\mathbf{v}_{cb}$ is $\perp BC$, draw a line $\perp BC$ through **b**;

Thus, the point **c** is located.

Further, $\mathbf{v}_{do} = \mathbf{v}_{dc} + \mathbf{v}_{co}$

or

$$\mathbf{v}_{dg} = \mathbf{v}_{co} + \mathbf{v}_{dc}$$

or

gd = oc + cd

$\mathbf{v}_{co}$ already exists in the diagram.

$\mathbf{v}_{dc}$ is $\perp CD$, draw **cd** $\perp CD$ through **c**;

$\mathbf{v}_{dg}$ is horizontal. Draw a horizontal line through **g** (or **o**) and locate the point **d**.

(i) v_d = **od** = <u>0.34 m/s</u>

(ii) $\omega_{bc} = \frac{v_{bc}}{BC} = \frac{0.12}{0.185} = \underline{0.649 \text{ rad/s}}$ clockwise

$\omega_{cd} = \frac{v_{dc}}{DC} = \frac{1.0}{0.24} = \underline{4.17 \text{ rad/s}}$ counter-clockwise

(iii) $T.\omega = F_d.v_d$

$F_d = 3000$ N

v_d = **od** (=**gd**) = 0.34 m/s

$$T = \frac{3000 \times 0.34}{(2\pi \times 100)/60} = \underline{97.4 \text{ N.m}}$$

Example 2.11 *The mechanism of a stone-crusher is shown in Fig. 2.19a along with various dimensions of links in mm. If crank OA rotates at a uniform velocity of 120 rpm, determine the velocity of the point K (jaw) when the crank OA is inclined at an angle of 30° to the horizontal. What will be the torque required at the crank OA to overcome a horizontal force of 40 kN at K?*

Solution $$\omega_{ao} = \frac{2\pi \times 120}{60} = 12.6 \text{ rad/s}$$

$$v_{ao} = \omega_{ao} \times OA = 12.6 \times 0.1 = 1.26 \text{ m/s}$$

Write the vector equation for the mechanism $OABQ$ and complete the velocity diagram as usual [(Fig. 2.19(b)]. Make Δ**bac** similar to ΔBAC (both are read clockwise).

Write the vector equation for the mechanism $OACDM$ and complete the velocity diagram. Make Δ**dmk** similar to ΔDMK (both are read clockwise).

v_k = **ok** = 0.45 m/s

v_k (horizontal) = 0.39 m/s

$T.\omega_{ao} = F_k v_k$(horizontal)

$$T = \frac{40\,000 \times 0.39}{12.6} = \underline{1242 \text{ N.m}}$$

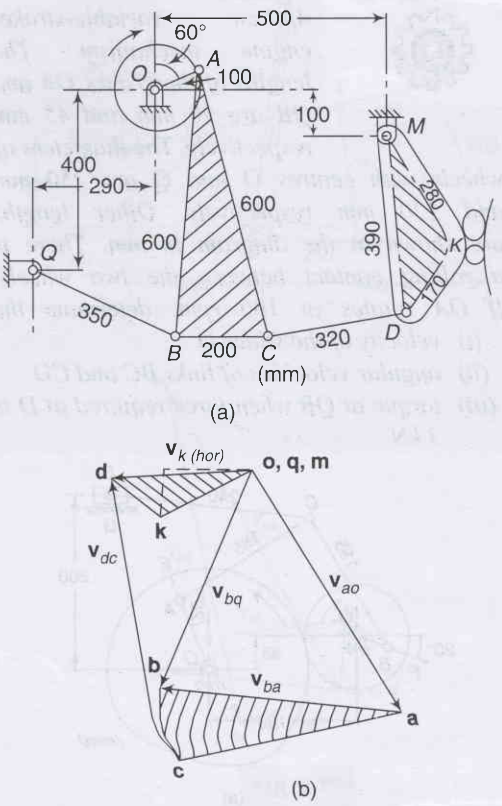


Fig. 2.19

2.10 CRANK- AND SLOTTED-LEVER MECHANISM

While analysing the motions of various links of a mechanism, sometimes we are faced with the problem of describing the motion of a moveable point on a link which has some angular velocity. For example, the motion of a slider on a rotating link. In such a case, the angular velocity of the rotating link along with the linear velocity of the slider may be known and it may be required to find the absolute velocity of the slider.

A crank and slotted-lever mechanism, which is a form of quick-return mechanism used for slotting and shaping machines, depicts the same form of motion [Fig. 2.20(a)]. OP is the crank rotating at an angular velocity of ω rad/s in the

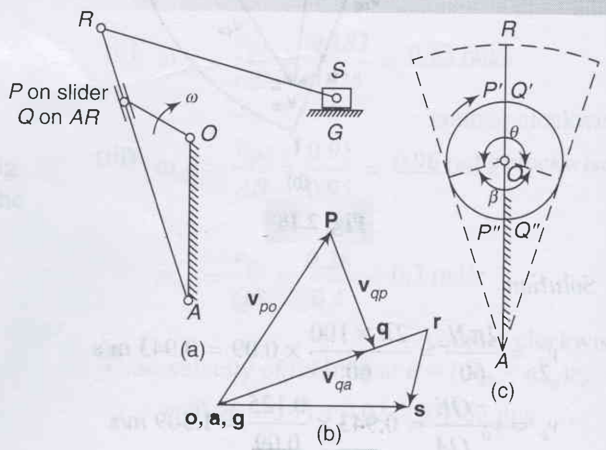


Fig. 2.20

clockwise direction about the centre O. At the end of the crank, a slider P is pivoted which moves on an oscillating link AR.

In such problems, it is convenient if a point Q on the link AR immediately below P is assumed to exist (P and Q are known as coincident points). As the crank rotates, there is relative movement of the points P and Q along AR.

Writing the vector equation for the mechanism OPA,

Vel. of Q rel. to O = Vel. of Q rel. to P + Vel. of P rel. to O

$$\mathbf{v}_{qo} = \mathbf{v}_{qp} + \mathbf{v}_{po}$$

or

$$\mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp}$$

or

$$\mathbf{aq} = \mathbf{op} + \mathbf{pq}$$

In this equation,

$\mathbf{v}_{po}$ or $\mathbf{op} = \omega.OP$; $\perp$ to OP

$\mathbf{v}_{qp}$ or $\mathbf{pq}$ is unknown in magnitude ; $||$ to AR

$\mathbf{v}_{qa}$ or $\mathbf{aq}$ is unknown in magnitude ; $\perp$ to AR

Take the vector $\mathbf{v}_{po}$ which is fully known [Fig. 2.20 (b)].

$\mathbf{v}_{qp}$ is $||$ AR, draw a line $||$ to AR through **p**;

$\mathbf{v}_{qa}$ is $\perp AR$, draw a line $\perp AR$ through **a** (or **o**).

The intersection locates the point **q**.

The vector equation for the above could also have been written as

Vel. of P rel. to A = Vel. of P rel. to Q + Vel. of Q rel. to A

$$\mathbf{v}_{pa} = \mathbf{v}_{pq} + \mathbf{v}_{qa}$$

or

$$\mathbf{v}_{po} = \mathbf{v}_{qa} + \mathbf{v}_{pq}$$

or

$$\mathbf{op} = \mathbf{aq} + \mathbf{qp}$$

Take the vector $\mathbf{v}_{po}$ which is completely known.

$\mathbf{v}_{qa}$ is $\perp AR$, draw a line $\perp AR$ through **a**;

$\mathbf{v}_{pq}$ is $||$ AR, draw a line $||$ AR through **p**.

The intersection locates the point **q**. Observe that the velocity diagrams obtained in the two cases are the same except that the direction of $\mathbf{v}_{pq}$ is the reverse of that of $\mathbf{v}_{qp}$.

As the vectors **oq** and **qp** are perpendicular to each other, the vector $\mathbf{v}_{po}$ may be assumed to have two components, one perpendicular to AR and the other parallel to AR.

The component of velocity along AR, i.e., **qp** indicates the relative velocity between Q and P or the velocity of sliding of the block on link AR.

Now, the velocity of R is perpendicular to AR. As the velocity of Q perpendicular to AR is known, the point **r** will lie on vector **aq** produced such that **ar**/**aq** = AR/AQ

To find the velocity of ram S, write the velocity vector equation,

$$\mathbf{v}_{so} = \mathbf{v}_{sr} + \mathbf{v}_{ro}$$

or

$$\mathbf{v}_{sg} = \mathbf{v}_{ro} + \mathbf{v}_{sr}$$

or

$$\mathbf{gs} = \mathbf{or} + \mathbf{rs}$$

$\mathbf{v}_{ro}$ is already there in the diagram. Draw a line through **r** perpendicular to RS for the vector $\mathbf{v}_{sr}$ and a line through **g**, parallel to the line of motion of the slider S on the guide G, for the vector $\mathbf{v}_{sg}$. In this way the point **s** is located.

The velocity of the ram S = **os** (or **gs**) towards right for the given position of the crank.

Also, $\omega_{rs} = \dfrac{v_{rs}}{RS}$ clockwise

Usually, the coupler RS is long and its obliquity is neglected.

Then **or** ≈ **os**

Referring Fig. 2.20 (c),

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$

When the crank assumes the position OP' during the cutting stroke, the component of velocity along AR (i.e., **pq**) is zero and **oq** is maximum (= **op**)

Let r = length of crank (= OP)

l = length of slotted lever (= AR)

c = distance between fixed centres (= AO)

ω = angular velocity of the crank

Then, during the cutting stroke,

$$v_{s\,\max} = \omega \times OP' \times \frac{AR}{AQ} = \omega r \times \frac{l}{c+r}$$

This is by neglecting the obliquity of the link RS, i.e. assuming the velocity of S equal to that of R.

Similarly, during the return stroke,

$$v_{s\,\max} = \omega \times OP'' \times \frac{AR}{AQ''} = \omega r \times \frac{l}{c-r}$$

$$\frac{v_{s\,\max}\,(\text{cutting})}{v_{s\,\max}\,(\text{return})} = \frac{\omega r \dfrac{1}{c+r}}{\omega r \dfrac{1}{c-r}} = \frac{c-r}{c+r}$$

Example 2.12 *Figure 2.21(a) shows the link mechanism of a quick return mechanism of the slotted lever type, the various dimensions of which are*

OA = 400 mm, OP = 200 mm, AR = 700 mm, RS = 300 mm

For the configuration shown, determine the velocity of the cutting tool at S and the angular velocity of the link RS. The crank OP rotates at 210 rpm.

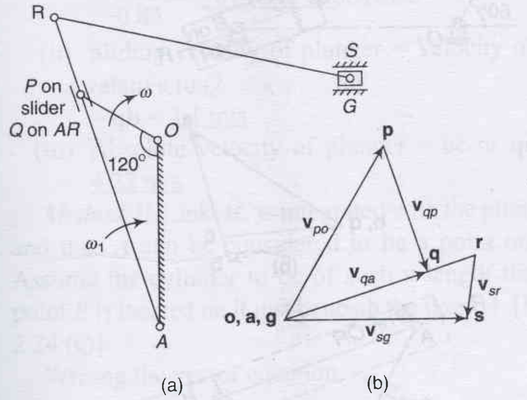


Fig. 2.21

Solution $\omega_{po} = \dfrac{2\pi \times 210}{60} = 22$ rad/s

Draw the configuration to a suitable scale. The vector equation for the mechanism *OPA*,

$$\mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp} \quad \text{or} \quad \mathbf{aq} = \mathbf{op} + \mathbf{pq}$$

In this equation,

$$\mathbf{v}_{po} \text{ or } \mathbf{op} = \omega . OP = 22 \times 0.2 = 4.4 \text{ m/s}$$

Take the vector $\mathbf{v}_{po}$ which is fully known [Fig. 2.21(b)].

$\mathbf{v}_{qp}$ is | | *AR*, draw a line | | to *AR* through **p;**

$\mathbf{v}_{qa}$ is $\perp AR$, draw a line $\perp AR$ through **a** (or **o**).

The intersection locates the point **q.** Locate the point **r** on the vector **aq** produced such that **ar/aq** = *AR/AQ*.

Draw a line through **r** perpendicular to *RS* for the vector $\mathbf{v}_{sr}$ and a line through **g**, parallel to the line of motion of the slider *S* on the guide *G*, for the vector $\mathbf{v}_{sg}$. In this way the point **s** is located.

The velocity of the ram *S* = **os** (or **gs**) = 4.5 m/s

It is towards right for the given position of the crank.

Angular velocity of link *RS*,

$$\omega_{rs} = \frac{v_{rs}}{RS} = \frac{1.4}{0.3} = 4.67 \text{ rad/s clockwise}$$

Example 2.13 *For the inverted slider-crank mechanism shown in Fig. 2.22(a), find the angular velocity of the link QR and the sliding velocity of the block on the link QR. The crank OA is 300 mm long and rotates at 20 rad/s in the clockwise direction. OQ is 650 mm and $\angle QOA = 40°$*

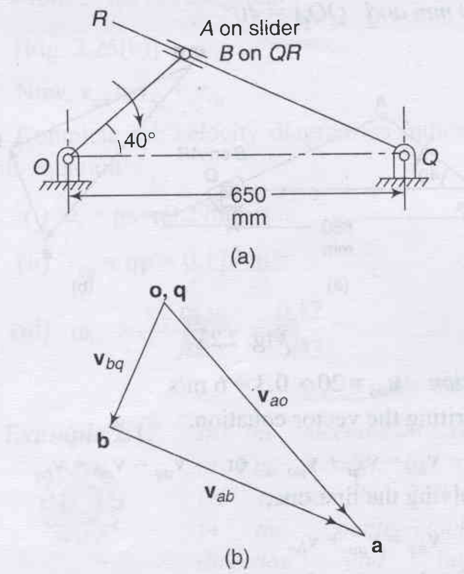


Fig. 2.22

Solution The velocity vector equation can be written as usual.

$$\mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq} \quad \text{or} \quad \mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$
$$\mathbf{v}_{ao} = \mathbf{v}_{bq} + \mathbf{v}_{ab} \qquad \mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$
$$\mathbf{oa} = \mathbf{qb} + \mathbf{ba} \qquad \mathbf{qb} = \mathbf{oa} + \mathbf{ab}$$

$\mathbf{v}_{ao}$ is fully known and after taking this vector, draw lines for $\mathbf{v}_{bq}$ and $\mathbf{v}_{ab}$ (or $\mathbf{v}_{ba}$) and locate the point **b**. obviously, the direction-sense of $\mathbf{v}_{ab}$ is opposite to that of $\mathbf{v}_{ba}$. Figure 2.22 (b) shows the solution of the first equation.

$$\omega_{qr} = \omega_{qb} = \frac{v_{qb} \text{ or } v_{bq}}{BQ}$$

$$= \frac{2.55}{0.46} \qquad (BQ = 0.46 \text{ m on measuring})$$

$$= \underline{5.54 \text{ rad/s}} \text{ counter} - \text{clockwise}$$

Sliding velocity of block = v_{ba} or **ab** = 5.45 m/s

Example 2.14 *For the position of the mechanism shown in Fig. 2.23(a), calculate the angular velocity of the link AR. OA is 300 mm long and rotates at 20 rad/s in the clockwise direction. OQ = 650 mm and QOA = 40°*

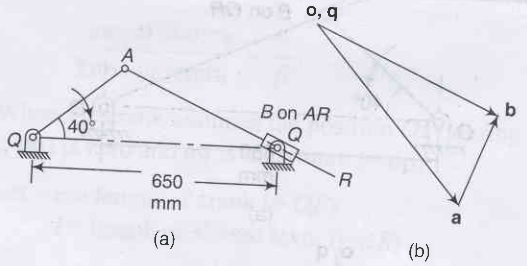


Fig. 2.23

Solution $v_{ao} = 20 \times 0.3 = 6$ m/s

Writing the vector equation,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao} \quad \text{or} \quad \mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

Solving the first one,

$$\mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or **qb = oa + ab**

Take $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.23(b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a**;

$\mathbf{v}_{bq}$ is along AB, draw a line || to AB through **q**.

The intersection locates the point **b**.

$$\omega_{ar} = \omega_{ab} = \frac{v_{ab} \text{ or } v_{ba}}{AB} = \frac{2.55}{0.46}$$

$= \underline{5.54 \text{ rad/s}}$ counter–clockwise

Example 2.15 *In the pump mechanism shown in Fig. 2.24(a), OA = 320 mm, AC = 680 mm and OQ = 650 mm. For the given configuration, determine the*

(i) angular velocity of the cylinder
(ii) sliding velocity of the plunger
(iii) absolute velocity of the plunger

The crank OA rotates at 20 rad/s clockwise.

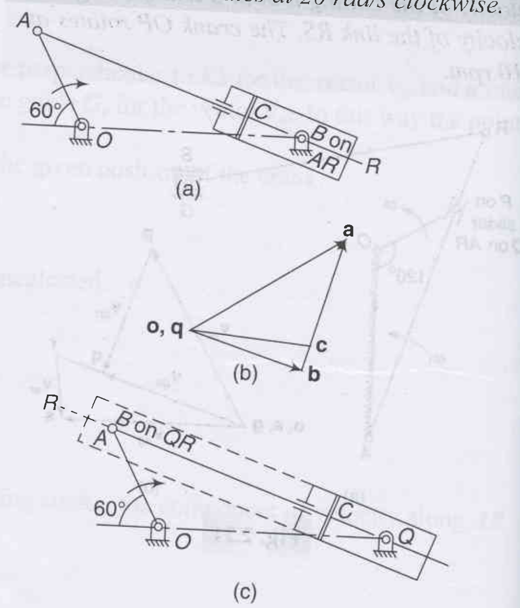


Fig. 2.24

Solution $v_{ao} = 0.32 \times 20 = 6.4$ m/s

Method I Produce AC to R. Line AC passes through the pivot Q. Let B be a point on AR beneath Q.

Writing the vector equation,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao} \quad \text{or} \quad \mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

Solving any of these equations leads to same velocity diagram except for the direction of $\mathbf{v}_{ba}$ and $\mathbf{v}_{ab}$.

Taking the latter equation,

$$\mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

or

$$\mathbf{v}_{ao} = \mathbf{v}_{bq} + \mathbf{v}_{ab}$$

or $\qquad$ **oa = qb + ba**

Complete the velocity triangle as usual [Fig. 2.24(b)]

Locate point **c** on **ab** such that $\frac{\mathbf{ac}}{\mathbf{ab}} = \frac{AC}{AB}$

(i) Angular velocity of cylinder = Angular velocity of AR or AB

$$= \frac{v_{ab}}{AB}$$

$$= \frac{4.77}{0.85} = \underline{5.61 \text{ rad/s}} \text{ clockwise}$$

(ii) Sliding velocity of plunger = Velocity of B relative to Q

= **qb** = <u>4.1 m/s</u>

(iii) Absolute velocity of plunger = **oc** or **qc** = <u>4.22 m/s</u>

Method II Link AC is integrated with the plunger and thus A can be considered to be a point on it. Assume the cylinder to be of such a length that a point B is located on it just beneath the point A. [Fig. 2.24 (c)].

Writing the vector equation,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao} \quad \text{or} \quad \mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

$$\mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba} \qquad \mathbf{v}_{ao} = \mathbf{v}_{bq} + \mathbf{v}_{ab}$$

qb = oa + ab $\qquad$ **oa = qb + ba**

Thus, the same equations have been obtained as in Method-I and thus can be solved easily.

Example 2.16 *A Whitworth quick-return mechanism has been shown in Fig. 2.25(a). The dimensions of the links are OP (crank) = 240 mm, OA =150 mm, AR = 165 mm and RS = 430 mm. The crank rotates at an angular velocity of 2.5 rad/s. At the moment when the crank makes an angle of 45° with the vertical, calculate the*

(i) velocity of the ram S

(ii) velocity of the slider P on the slotted lever

(iii) angular velocity of the link RS

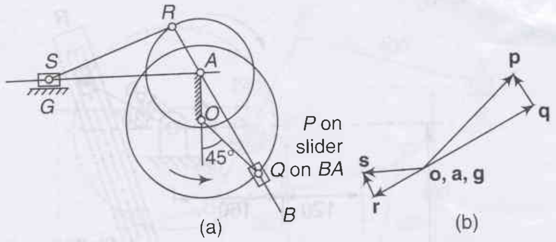


Fig. 2.25

Solution $\quad v_p = 2.5 \times 0.24 = 0.6$ m/s

Locate a point Q on AB beneath point P on the slider.

Solve any of the following velocity vector equations,

$$\mathbf{v}_{pa} = \mathbf{v}_{pq} + \mathbf{v}_{qa} \quad \text{or} \quad \mathbf{v}_{qo} = \mathbf{v}_{qp} + \mathbf{v}_{po}$$

Produce **qa** to **r** such that $\frac{\mathbf{ar}}{\mathbf{qa}} = \frac{AR}{QA}$

[Fig. 2.25(b)]

Now, $\mathbf{v}_{sa} = \mathbf{v}_{sr} + \mathbf{v}_{ra}$

Complete the velocity diagram as indicated by this equation

(i) v_s = **gs** = <u>0.276 m/s</u>

(ii) v_{pq} = **qp** = <u>0.177 m/s</u>

(iii) $\omega_{rs} = \frac{v_{rs} \text{ or } v_{sr}}{RS} = \frac{0.12}{0.43} =$

<u>0.279 rad/s</u> clockwise

Example 2.17 *In the mechanism shown in Fig. 2.26(a), the crank OP rotates at 210 rpm in the counter-clockwise direction and imparts vertical reciprocating motion to rack through a toothed quadrant. Slotted bar and the quadrant oscillate about the fixed pivot A. Determine for the given position the*

(i) linear speed of the rack

(ii) ratio of the times of raising and lowering of the rack

(iii) stroke of the rack

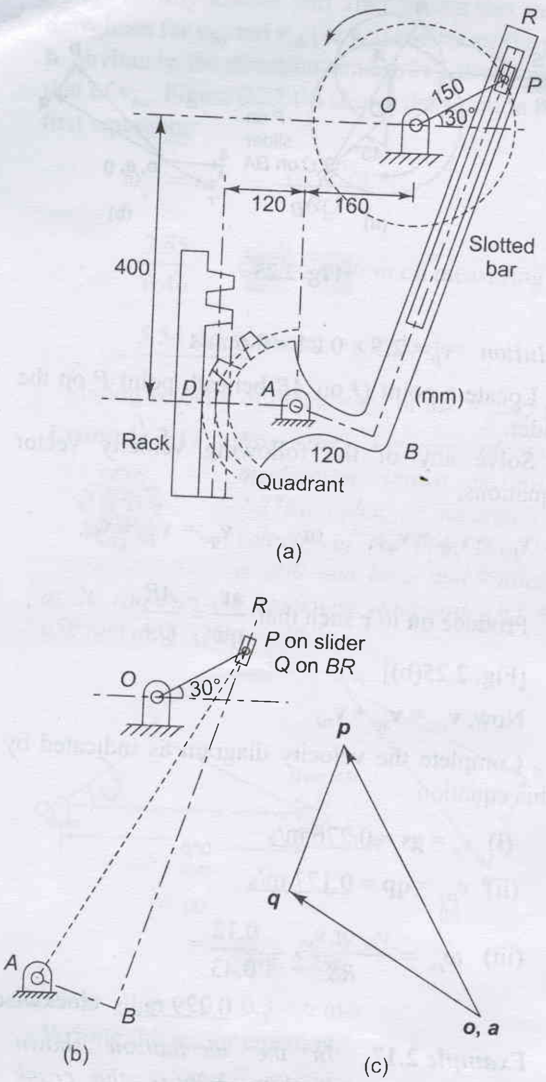


Fig. 2.26

Solution $\omega_{po} = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 210}{60} = 22$ rad/s

$v_{po} = 22 \times 0.15 = 3.3$ m/s

Draw the configuration diagram to a suitable scale [Fig. 2.26(b)].

Locate a point Q on BR beneath point P on the slider.

Then the vector equation is

$$\mathbf{v}_{qo} = \mathbf{v}_{qp} + \mathbf{v}_{po} \quad \text{or} \quad \mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp}$$

Take the vector $\mathbf{v}_{po}$ to a convenient scale in the proper direction and sense [Fig. 2.26(c)].

$\mathbf{v}_{qp}$ is along BR, draw a line parallel to BR through **p**;

Now, Q is a point on the link ABR which is pivoted at point A. The direction of velocity of any point on the link is perpendicular to the line joining that point with the pivoted point A.

v_{qa} is $\perp QA$, draw a line $\perp QA$ through **a**;

The intersection of the two lines locates the point **q.**

Now angular velocity of the quadrant and the lever ABQ,

$$\omega_{aq} = \frac{v_{aq}}{AQ} = \frac{2.5}{0.577} = 4.33 \text{ rad/s}$$

counter-clockwise

(i) The linear velocity of the rack will be equal to the tangential velocity of the quadrant at the teeth, i.e.,

$$v_r = \omega \times AD = \omega \times 120 = 4.33 \times 120 = \underline{519.6 \text{ mm/s}}$$

(ii) The reciprocating rack changes the direction when the crank OP assumes a position such that the tangent at P to the circle at O is also a tangent to the circle at A with radius AB as shown in Fig. 2.27. The rack is lowered during the rotation of the crank from P to P' and is raised when P' moves to P counter-clockwise.

Thus,

$$\frac{\text{Time of lowering}}{\text{Time of raising}} = \frac{\theta}{\beta} = \frac{215^\circ}{135^\circ} = 1.59$$

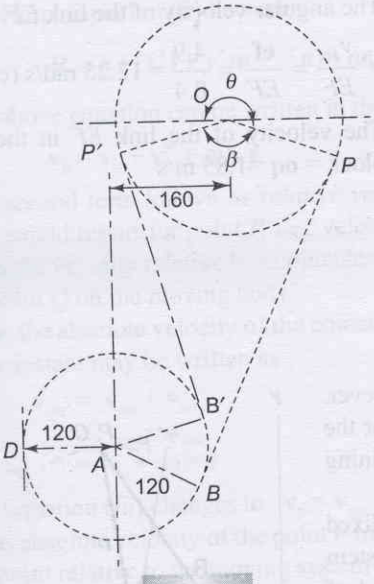


Fig. 2.27

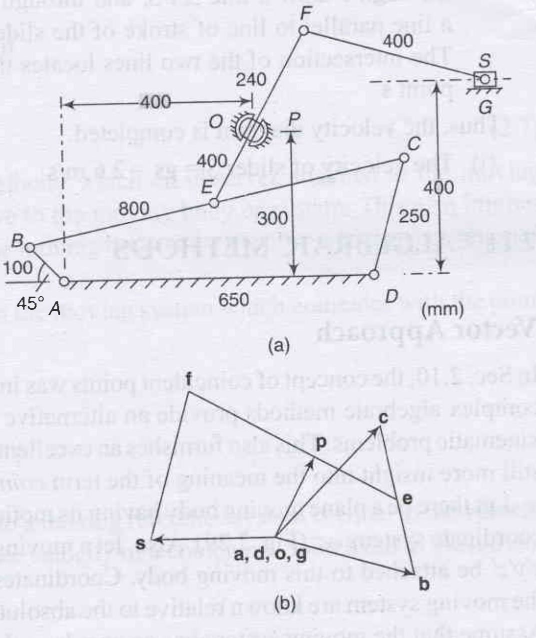


Fig. 2.28

(iii) Stroke of the rack = angular displacement of the quadrant × its radius

$= \text{angle } BAB' \times AB$

$= 44 \times \dfrac{\pi}{180} \times 120 = 92.2$ mm

($\angle BAB' = 44°$ by measurement)

Example 2.18 *In the swiveling-joint mechanism shown in Fig. 2.28(a), AB is the driving crank rotating at 300 rpm clockwise.*

The lengths of the various links are

AD = 650 mm, AB = 100 mm, BC = 800 mm, DC = 250 mm, BE = CE, EF = 400 mm, OF = 240 mm, FS = 400 mm

For the given configuration of the mechanism, determine the

(i) velocity of the slider block S

(ii) angular velocity of the link EF

(iii) velocity of the link EF in the swivel block

Solution $\omega_{ba} = \dfrac{2\pi \times 300}{60} = 31.4$ rad/s

$v_b = 31.4 \times 0.1 = 3.14$ m/s

The velocity diagram is completed as follows:

- Draw the velocity diagram of the four-link mechanism *ABCD* as usual starting with the vector **ab** as shown in Fig. 2.28(b).
- Locate the point **e** in the velocity diagram at the midpoint of **bc** as the point *E* is the midpoint of *BC*. Let *Q* be a point on the link *EF* at the joint *O*. Draw a line ⊥ *EQ* through **e**, a point on which will represent the velocity of *Q* relative to *E*.
- The sliding velocity of link *EF* in the joint at the instant is along the link. Draw a line parallel to *EF* through **o**, the intersection of which with the previous line locates the point **q**.
- Extend the vector **eq** to **f** such that **ef**/**eq** = *EF*/*EQ*.

- Through **f** draw a line $\perp FS$, and through **g** a line parallel to line of stroke of the slider. The intersection of the two lines locates the point **s**.

Thus, the velocity diagram is completed.

(i) The velocity of slider S = **gs** = 2.6 m/s

(ii) The angular velocity of the link EF

$$= \frac{v_{fe}}{EF} = \frac{\mathbf{ef}}{EF} = \frac{4.9}{0.4} = 12.25 \text{ rad/s (ccw)}$$

(iii) The velocity of the link EF in the swi block = **oq** =1.85 m/s

2.11 ALGEBRAIC METHODS

Vector Approach

In Sec. 2.10, the concept of coincident points was introduced. However, complex algebraic methods provide an alternative formulation for the kinematic problems. This also furnishes an excellent means of obtaining still more insight into the meaning of the term *coincident points.*

Let there be a plane moving body having its motion relative to a fixed coordinate system xyz (Fig. 2.29). Also, let a moving coordinate system $x'y'z'$ be attached to this moving body. Coordinates of the origin A of the moving system are known relative to the absolute reference system. Assume that the moving system has an angular velocity ω also.

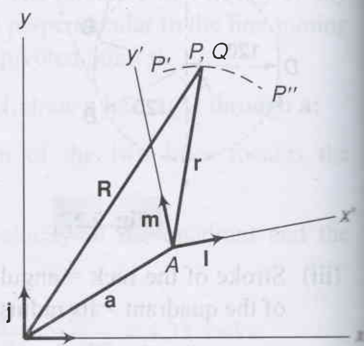


Fig. 2.29

Let

i,j,k	unit vectors for the absolute system
l,m,n	unit vectors for the moving system
ω	angular velocity of rotation of the moving system
R	vector relative to fixed system
r	vector relative to moving system

Let a point P move along path $P'PP''$ relative to the moving coordinate system $x'y'z'$. At any instant, the position of P relative to the fixed system is given by the equation

$$\mathbf{R} = \mathbf{a} + \mathbf{r}$$

where $\mathbf{r} = x'\mathbf{l} + y'\mathbf{m} + z'\mathbf{n}$

Thus, (i) may be written as, $\mathbf{R} = \mathbf{a} + x'\mathbf{l} + y'\mathbf{m} + z'\mathbf{n}$

Taking the derivatives with respect to time to find the velocity,

$$\dot{\mathbf{R}} = \dot{\mathbf{a}} + (\dot{x}'\,\mathbf{l} + \dot{y}'\,\mathbf{m} + \dot{z}'\,\mathbf{n}) + (x'\,\dot{\mathbf{l}} + y'\,\dot{\mathbf{m}} + z'\,\dot{\mathbf{n}})$$

The first term in this equation indicates the velocity of the origin of the moving system. The second term refers to the velocity of P relative to the moving system. The third term is due to the fact that the reference system has also rotary motion with angular velocity ω.

Also, $\dot{\mathbf{l}} = \omega \times \mathbf{l}$, $\quad \dot{\mathbf{m}} = \omega \times \mathbf{m}$, $\quad \dot{\mathbf{n}} = \omega \times \mathbf{n}$

Therefore, Equation (ii) becomes

$$\dot{\mathbf{R}} = \dot{\mathbf{a}} + (\dot{x}'\mathbf{l} + \dot{y}'\mathbf{m} + \dot{z}'\mathbf{n}) + \omega(x'\mathbf{l} + y'\mathbf{m} + z'\mathbf{n}$$

The above equation can be written in the form,

$$\mathbf{v}_p = \mathbf{v}_a + \mathbf{v}^R + \omega \times \mathbf{r} \tag{2.7}$$

The second term known as *relative velocity* is the velocity which an observer attached to the moving system would report for point P, i.e., velocity of P relative to the moving body or system. This also implies that it is the velocity relative to a coincident point Q on the moving body since the observer may be stationed at the point Q on the moving body.

Now, the absolute velocity of the coincident point Q on the moving system which coincides with the point P at the instant may be written as

$$\begin{aligned} \mathbf{v}_{qo} &= \mathbf{v}_{qa} + \mathbf{v}_{ao} \\ &= \mathbf{v}_{ao} + \mathbf{v}_{qa} \\ &= \mathbf{v}_a + \omega \times \mathbf{r} \end{aligned}$$

and equation (iii) changes to $\mathbf{v}_p = \mathbf{v}_{qo} + \mathbf{v}^R$

Thus absolute velocity of the point P moving relative to a moving reference system is equal to the velocity of the point relative to the moving system plus the absolute velocity of a coincident point fixed to the moving reference system.

The above equation may be written as

$$\begin{aligned} \mathbf{v}_{po} &= \mathbf{v}_{qo} + \mathbf{v}_{pq} \\ \mathbf{v}_{po} &= \mathbf{v}_{pq} + \mathbf{v}_{qo} \end{aligned}$$

Vel. of P rel. to O = Vel. of P rel. to Q + Vel. of Q rel. to O

Use of Complex Numbers

In a complex number system, a vector connecting two points O and P (Fig. 2.30) may be expressed as

$\mathbf{r} = a + ib$ in the rectangular form

where a and b are known as *real* and *imaginary* parts of $\mathbf{r}$. The real part is always taken along the X-axis from the origin, to the right if positive and to the left if negative. The symbol i prefixed to b indicates that it is to be taken at an angle of 90° in the counter-clockwise direction from the positive x-direction.

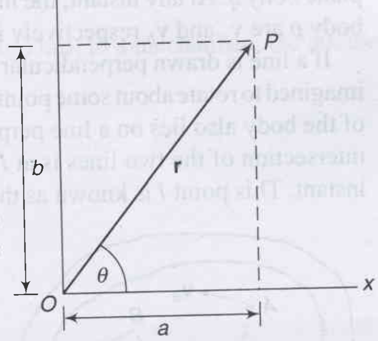


Fig. 2.30

As $i = \sqrt{-1}$ indicates 90° counter-clockwise direction,

Therefore,

$i^2 = (\sqrt{-1})^2 = -1$ is 180° counter-clockwise

$i^3 = (\sqrt{-1})^3 = (\sqrt{-1})^2\sqrt{-1} = (-1)i = -i$ is 270° counter-clockwise or 90° clockwise

In the polar form **r** can be expressed as

$$\mathbf{r} = r\cos\theta + ir\sin\theta = r(\cos\theta + i\sin\theta)$$

In this equation r is said to be magnitude of **r**, denoted by $|\mathbf{r}|$ and θ is called the argument of **r**, de by arg(**r**).

Since r is the magnitude of vector **r**, the term in the parenthesis in the above equation plays the ro unit vector which points in the direction of OP.

From trigonometry, it can be written that

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Therefore,

$$\mathbf{r} = re^{i\theta} \text{ which is the complex polar form.}$$

Complex numbers are assumed to follow all the formal rules of real algebra.

Velocity

Differentiating Eq. (i) with respect to time,

$$\mathbf{v} = \dot{r}e^{i\theta} + ir\dot{\theta}e^{i\theta}$$
$$= (\dot{r} + ir\dot{\theta})e^{i\theta}$$

2.12 INSTANTANEOUS CENTRE (I-CENTRE)

Let there be a plane body p having a non-linear motion relative to another plane body q. At any instant, the linear velocities of two points A and B on the body p are $\mathbf{v}_a$ and $\mathbf{v}_b$ respectively in the directions as shown in Fig. 2.31.

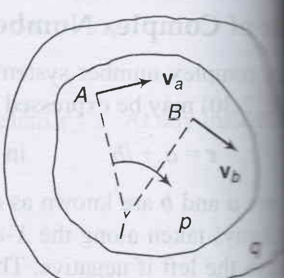
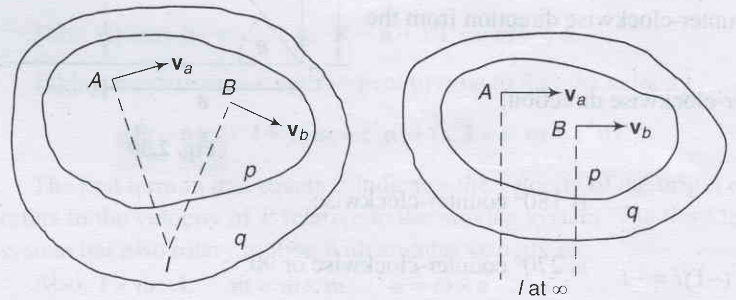


Fig. 2.31

If a line is drawn perpendicular to the direction of $\mathbf{v}_a$ at A, the body can be imagined to rotate about some point on this line. Similarly, the centre of rotation of the body also lies on a line perpendicular to the direction of $\mathbf{v}_b$ at B. If the intersection of the two lines is at I, the body p will be rotating about I at the instant. This point I is known as the *instantaneous centre of velocity* or more



(a) (b)

Fig. 2.32

commonly *instantaneous cen* *rotation* for the body p. This pro is true only for an instant and a point will become the instantan centre at the next instant. Thus, i misnomer to call this point the ce of rotation, as generally this poi not located at the centre of curva of the apparent path taken by a p of one body with respect to the other body. However, even with this limitation, the instantaneous centre is a useful tool for understanding the kinematics of planar motion. In our further discussions, this centre will be called the *I-centre*.

In case the perpendiculars to $\mathbf{v}_a$ and $\mathbf{v}_b$ at A and B respectively meet outside the body p, the I-centre will lie outside the body p [Fig. 2.32(a)]. If the directions of $\mathbf{v}_a$ and $\mathbf{v}_b$ are parallel and the perpendiculars at A and B meet at infinity, the I-centre of the body lies at infinity. This is the case when the body has a linear motion [Fig. 2.32(b)].

As the body p rotates about the point I at the instant and the velocity of any point on the body is proportional to the distance of the point from I, the velocity of the point I itself would be zero (the distance being zero). This implies that the two bodies p and q are relatively at rest or there is no relative motion between the two at the I-centre.

Now imagine that the body q is also in motion relative to a third body r (Fig. 2.33). Then the motion of the point I relative to the third body would be the same whether this point is considered on the body p or q.

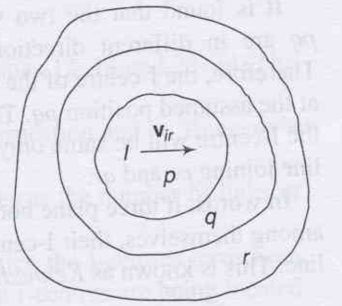


Fig. 2.33

Notation

An I-centre is a centre of rotation of a moving body relative to another body. If a body p is in motion relative to a fixed body q, the centre of rotation (I-centre) may be named as pq. However, in case of relative motions, the body q can also be imagined to rotate relative to body p (i.e., as if the body p is fixed for the moment) about the same centre. Thus, centre of rotation or I-centre can be named qp also.

This shows that the I-centre of the two bodies p and q in relative motion can be named either pq or qp meaning the same thing. In general, the I-centre will be named in the ascending order of the alphabets or digits, i.e., 13, 35, pq, eg, etc.

Number of I-Centres

For two bodies having relative motion between them, there is an I-centre. Thus, in a mechanism, the number of I-centres will be equal to possible pairs of bodies or links.

Let N = Number of I-centres

n = number of bodies or links

Then, $$N = \frac{n(n-1)}{2}$$

2.13 KENNEDY'S THEOREM

Consider three plane bodies p, q and r; r being a fixed body. p and q rotate about centre pr and qr respectively relative to the body r. Thus, pr is the I-centre of bodies p and r whereas qr is the I-centre of bodies q and r. Assume the I-centre of the bodies p and q at the point pq as shown in Fig. 2.34.

Now, p and q both are moving relative to a third fixed body r. Therefore, the motion of their mutual I-centre pq is to be the same whether this point is considered in the body p or q. (Refer to Sec. 2.12).

If the point pq is considered on the body p, its velocity v_p is perpendicular to the line joining pq and pr.

If the point pq is considered on the body q, its velocity v_q is perpendicular to the line joining pq and qr.

It is found that the two velocities of the I-centre pq are in different directions which is impossible. Therefore, the I-centre of the bodies p and q cannot be at the assumed position pq. The velocities v_p and v_q of the I-centre will be same only if this centre lies on the line joining pr and qr.

In words, if three plane bodies have relative motion among themselves, their I-centre must lie on a straight line. This is known as *Kennedy's theorem.*

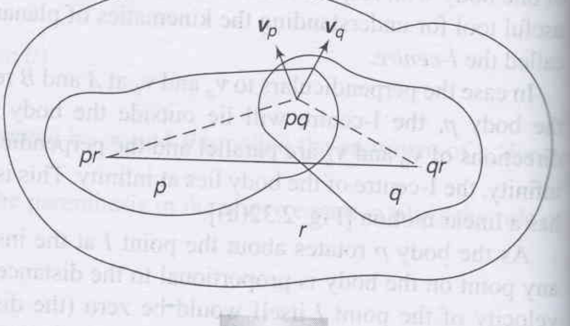


Fig. 2.34

2.14 LOCATING I-CENTRES

The procedure to locate I-centres of a mechanism is being illustrated with the help of the following example of a four-link mechanism.

Figure 2.35 shows a four-link mechanism $ABCD$, the links of which have been named as 1, 2, 3 and 4. The number of I-centres is given by

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

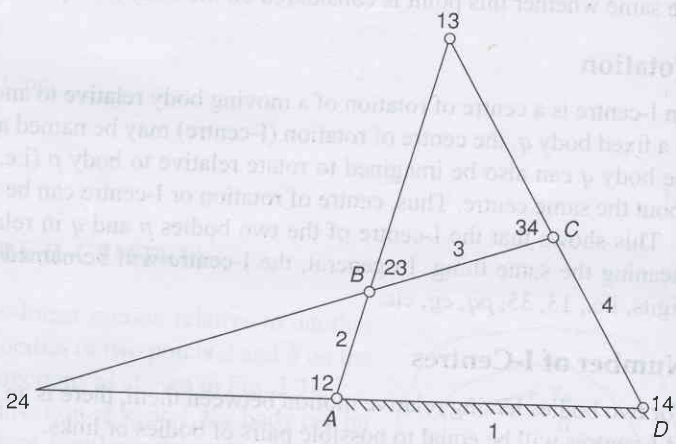


Fig. 2.35

Now, as the centre of rotation of 2 relative to 1 is at A, the I-centre 12 for the links 1 and 2 lies at A. Also, the location of A is not going to change with the rotation of the link 2. Therefore, this I-centre is referred as the *fixed I-centre*. Similarly, 14 is another fixed I-centre for the links 1 and 4 located at D.

Link 3 rotates about B relative to the link 2 and thus the I-centre 23 for links 2 and 3 lies at B. With the movement of the links, the position of the pin-joint B will change and so will the position of the I-centre. However, at all times, the I-centre will be located at the pin joint. Thus, 23 is known as a *permanent* but not a fixed I-centre. Similarly, 34 is another permanent but not a fixed I-centre for the links 3 and 4.

The above I-centres have been located by inspection only. The other two I-centres 13 and 24 which are neither fixed nor permanent can be located easily by applying Kennedy's theorem as explained below.

I-Centre 13

First, consider three links 1, 2 and 3. One more link 2 has been added to links 1 and 3 with the condition that the I-centres 12 and 23 are already known and the third I-centre 13 is to be located.

Now, as the three links 1, 2 and 3 have relative motions among themselves, their I-centres lie on a straight line. Thus, I-centre 13 lies on the line joining 12 and 13 (or line AB).

Similarly, consider the links 1, 4 and 3. Their I-centres are 14, 34 and 13. Out of these, 14 and 34 are already known. Therefore, I-centre 13 lies on the line joining 14 and 34 (or *DC*). The intersection of the line joining 12 and 23 (or produced) with the line joining 14 and 34 (or produced) locates the I-centre 13.

I-Centre 24

Considering two sets of links 2, 1, 4 and 2, 3, 4; the I-centre would lie on the lines 12–14 and 23–34. The interaction locates the I-centre 24.

There is a convenient way of keeping the track of the I-centres located by inspection and by Kennedy's theorem.

Mark points as the corners of a regular polygon having same number of sides as the number of links in the mechanism.

Name them according to the links of the mechanism. Join the points of which the I-centres have been located by inspection, by firm lines. Then go on joining the points, of which the I-centres are being located by Kennedy's theorem, by dotted lines.

For example, for a four-link mechanism, mark the points 1, 2, 3 and 4 as shown in Fig. 2.36(a). Join 12, 23, 34 and 14 (or 41) by firm lines after locating these I-centres by inspection [Fig. 2.36(b)]. In Fig. 2.29(c) these centres have been encircled for the record.

To find the I-centre 13, join 1 to 3 by a dotted line [Fig. 2.36(b)]. The construction shows that the I-centre lies on the line joining I-centres 12 and 23, and the line joining 14 and 34 (or 43). Locate the I-centre actually on the intersection of the two lines in the configuration diagram of the mechanism. In Fig. 2.36(c), 13 is underlined to note that the I-centre has been located by Kennedy's theorem. Similarly, find the I-centre 24 by joining 2 and 4 and locate the point on the intersection of the lines 12–14 and 23–34.

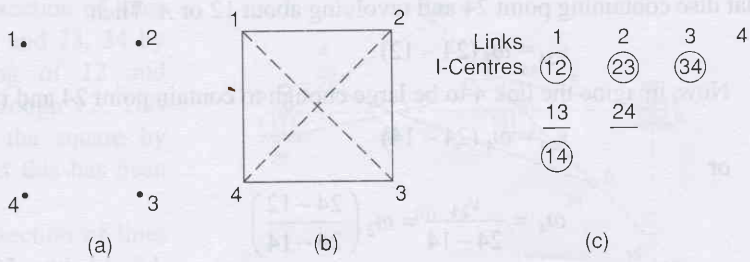


Fig. 2.36

It was mentioned in Section 2.15 that the I-centre is generally not located at the centre of curvature of the apparent path taken by a point of one body with respect to the other links. In the above example of a four-link mechanism, the I-centre of the pivot point *B* on the coupler relative to the fixed link is at 13, whereas its apparent path is a circular curve about the fixed pivot *A* which means *A* is its centre of curvature and the length *AB* is the radius of curvature. Also, the I-centre of the pivot point *C* on the coupler relative to the fixed link is again at 13, whereas its apparent path is rotation about the fixed pivot *D*.

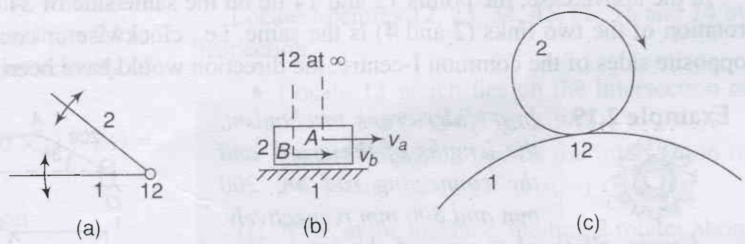


Fig. 2.37

Rules to Locate I-Centres by Inspection

1. In a pivoted joint, the centre of the pivot is the I-centre for the two links of the pivot [Fig. 2.37 (a)].
2. In a sliding motion, the I-centre lies at infinity in a direction perpendicular to the path of motion of the slider. This is because the sliding motion is equivalent to a rotary motion of the links with the radius of curvature as infinity [Fig. 2.37(b)].

3. In a pure rolling contact of the two links, the I-centre lies at the point of contact at the given instant [Fig. 2.37(c)]. It is because the two points of contact on the two bodies have the same linear velocity and thus there is no relative motion of the two at the point of contact which is the I-centre (Refer Sec. 2.12).

2.15 ANGULAR-VELOCITY-RATIO THEOREM

When the angular velocity of a link is known and it is required to find the angular velocity of another link, locate their common I-centre. The velocity of this I-centre relative to a fixed third link is the same whether the I-centre is considered on the first or the second link (Sec. 2.13). First consider the I-centre to be on the first link and obtain the velocity of the I-centre. Then consider the I-centre to be on the second link and find its angular velocity.

For example, if it is required to find the angular velocity of the link 4 when the angular velocity of the link 2 of a four-link mechanism is known, locate the I-centre 24 (Fig. 2.35). Imagine link 2 to be in the form of a flat disc containing point 24 and revolving about 12 or A. Then

$$v_{24} = \omega_2\,(24 - 12)$$

Now, imagine the link 4 to be large enough to contain point 24 and revolving about 14 or D. Then

$$v_{24} = \omega_4\,(24 - 14)$$

or

$$\omega_4 = \frac{v_{24}}{24-14} = \omega_2\left(\frac{24-12}{24-14}\right)$$

or

$$\frac{\omega_4}{\omega_2} = \frac{24-12}{24-14}$$

The above equation is known as the *angular-velocity-ratio theorem*. In words, the angular velocity ratio of two links relative to a third link is inversely proportional to the distances of their common I-centre from their respective centres of rotation.

In the above case, the points 12 and 14 lie on the same side of 24 on the line 24–14 and the direction of rotation of the two links (2 and 4) is the same, i.e., clockwise or counter-clockwise. Had they been on the opposite sides of the common I-centre, the direction would have been opposite.

Example 2.19 *In a slider-crank mechanism, the lengths of the crank and the connecting rod are 200 mm and 800 mm respectively. Locate all the I-centres of the mechanism for the position of the crank when it has turned 30° from the inner dead centre. Also, find the velocity of the slider and the angular velocity of the connecting rod if the crank rotates at 40 rad/s.*

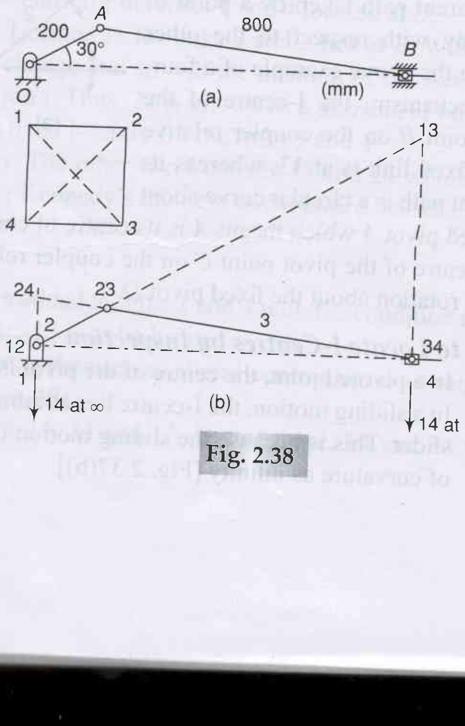


Fig. 2.38

Solution The slider-crank mechanism is shown in Fig. 2.38(a). Name the four links as 1, 2, 3 and 4. Locate the various I-centres as follows:

(i) Locate I-centres 12, 23 and 34 by inspection. They are at the pivots joining the respective links. As the line of stroke of the slider is horizontal, the I-centre 14 lies vertically upwards or downwards at infinity as shown in Fig. 2.38(b).

(ii) Take four points in the form of a square and mark them as 1, 2, 3 and 4. Join 12, 23, 34 and 14 by firm lines as these have been located by inspection.

(iii) I-centre 24 lies at the intersection of lines joining the I-centres 12, 14 and 23, 34 by Kennedy's theorem. Joining of 12 and 14 means a vertical line through 12. This I-centre can be shown in the square by a dotted line to indicate that this has been located by inspection.

(iv) I-centre 13 lies at the intersection of lines joining the I-centres 12, 23 and 14, 34. Joining of 34 and 14 means a vertical line through 34. Show this I-centre in the square by a dotted line.

Thus, all the I-centres are located.

As velocity of the link 2 is known and the velocity of the link 4 is to be found, consider the I-centre 24. The point 24 has the same velocity whether it is assumed to lie in link 2 or 4. First, assume 24 to lie on the link 2 which rotates at angular velocity of 40 rad/s.

Linear velocity of I-centre 24 = 40 × (12–24) = 40 × 0.123

= 4.92 m/s in the horizontal direction

Now, when this point is assumed in the link 4, it will have the same velocity which means the linear velocity of the slider is the same as of the point 24.

Thus, linear velocity of the slider = <u>4.92 m/s</u>

Example 2.20 *Figure 2.39(a) shows a six-link mechanism. The dimensions of the links are OA = 100 mm, AB = 580 mm, BC = 300 mm, QC = 100 mm and CD = 350 mm. The crank OA rotates clockwise at 150 rpm. For the position when the crank OA makes an angle of 30° with the horizontal, determine the*

(i) linear velocities of the pivot points B, C and D

(ii) angular velocities of the links AB, BC and CD

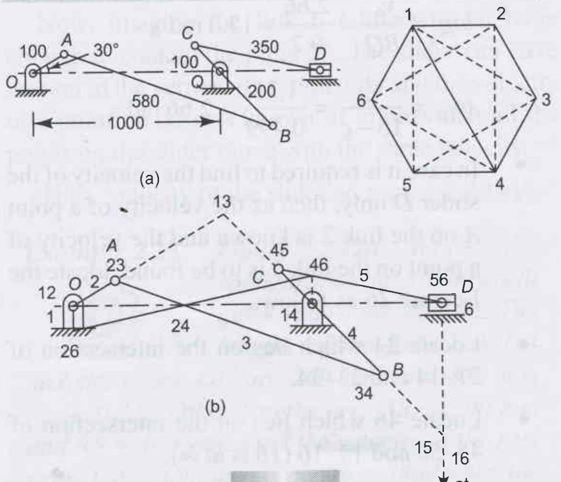


Fig. 2.39

Solution $\omega_2 = \dfrac{2\pi \times 150}{60} = 15.7$ rad/s

$$v_a = \omega_2 .OA = 15.7 \times 0.1 = 1.57 \text{ m/s}$$

Locate I-centres 12, 23, 34, 45, 56, 16 and 14 by inspection.

- Locate 13 which lies on the intersection of 12–23 and 14–34 [Fig. 2.39(b)]
- Locate 15 which lies on the intersection of 14–45 and 56–16 (16 is at ∞)

(i) Now, at the instance, the link 3 rotates about the I-centre 13.

Thus, $\dfrac{v_b}{v_a} = \dfrac{13-34}{13-23}$ or $v_b = \dfrac{453}{265} \times 1.57 = 2.66$ m/s

and $\dfrac{v_c}{v_b} = \dfrac{QC}{QB}$ or $v_c = \dfrac{100}{200} \times 2.66 = 1.33$ m/s

At the instance, the link 5 rotates about the I-centre 15.

Thus, $\dfrac{v_d}{v_c} = \dfrac{15-56}{15-45}$ or

$$v_d = \frac{300}{506} \times 1.33 = 0.788 \quad \text{m/s}$$

(ii) $\omega_{ab} = \dfrac{v_a}{13-A} = \dfrac{1.57}{0.267} = 5.88$ rad/s

$$\omega_{bc} = \frac{v_b}{BQ} = \frac{2.66}{0.2} = 13.3 \text{ rad/s}$$

$$\omega_{cd} = \frac{v_c}{15-c} = \frac{1.33}{0.499} = 2.66 \text{ rad/s}$$

* In case it is required to find the velocity of the slider D only, then as the velocity of a point A on the link 2 is known and the velocity of a point on the link 6 is to be found, locate the I-centre 26 as follows:

- Locate 24 which lies on the intersection of 21–14 and 23–34.
- Locate 46 which lies on the intersection of 45–56 and 14–16 (16 is at ∞).
- Locate 26 which is the intersection of 24–46 and 21–16.

First, imagine the link 2 to be in the form of a flat disc containing the point 26 and revolving about O with an angular velocity of 15.7 rad/s.

Then, $v_{26} = \omega_2 \times (12\text{–}26) = 15.7 \times 50 = 785$ mm/s or 0.785 m/s

The velocity of the point 26 is in the horizontally left direction if OA rotates clockwise.

Now, imagine the link 6 (slider) to be large enough to contain the point 26. The slider can have motion in the horizontal direction only and the velocity of a point 26 on it is known; it implies that all the points on the slider move with the same velocity.

Thus, velocity of the slider, $v_d = v_{26} = \underline{785 \text{ mm/s}}$

Example 2.21 *Figure 2.40(a) shows a six-link mechanism. The dimensions of the links are OA = 100 mm, AB = 450 mm, BD = 200 mm, QB = 400 mm, DE = 200 mm, CE = 200 mm. Find the angular velocity of the link CE by the instantaneous centre method if the link OA rotates at 20 rad/s.*

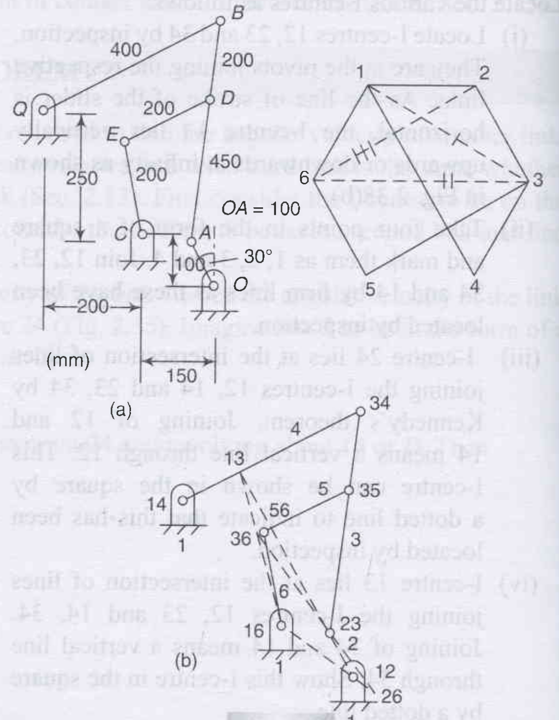


Fig. 2.40

Solution Name the six links by numbers as 1, 2, 3, 4, 5 and 6 as shown in Fig. 2.40(b).

The velocity of the link OA (2) is known and the velocity of the link CE (6) is to be found. Therefore, the I-centre 26 is required to be located.

- First mark the I-centres which can be located by inspection. They are 12, 23, 34, 56 16 and 14.
- Locate the I-centre 13 which is at the intersection of lines joining I-centres 12, 23 and 16, 36. Similarly, Locate the I-centre 36 which is at the intersection of lines joining I-centres 13,16 and 35, 56.
- Locate the I-centre 26 at the intersection of lines joining I-centres 12, 16 and 23, 36.
- Now, as the velocity of the I-centre 26 is the same whether it is considered to lie on link 2 or 6,

$$v_{26} = \omega_2 .(12\text{–}26) = \omega_6 .(16\text{–}26)$$

$$\text{or } \omega_6 = \frac{\omega_2 (12-26)}{(16-26)} = \frac{20 \times 53}{235} = \underline{4.5 \text{ rad/s}}$$

Example 2.22 *Figure 2.41(a) shows a six-link mechanism. The dimensions of the links are OA = 220 mm, AB = 485 mm, BQ = 310 mm, BC = 590 mm and CD = 400 mm. For the position when the crank OA makes an angle of 60° with the vertical, find the velocity of the slider D. The crank OA rotates clockwise at 150 rpm.*

Solution $\omega_2 = \dfrac{2\pi \times 150}{60} = 5\pi$ rad/s

The velocity of a point A on the link 2 is known. It is required to find the velocity of a point on the link 6. Thus, locate the I-centre 26 as follows:

- Locate I-centres 12, 23, 34, 45, 56, 16 and 14 by inspection.
- Locate 24 which lies on the intersection of 21–14 and 23–34 [Fig. 2.41(b)].
- Locate 46 which lies on the intersection of 45–56 and 14–16 (16 is at ∞).
- Locate 26 which is the intersection of 24–46 and 21–16.

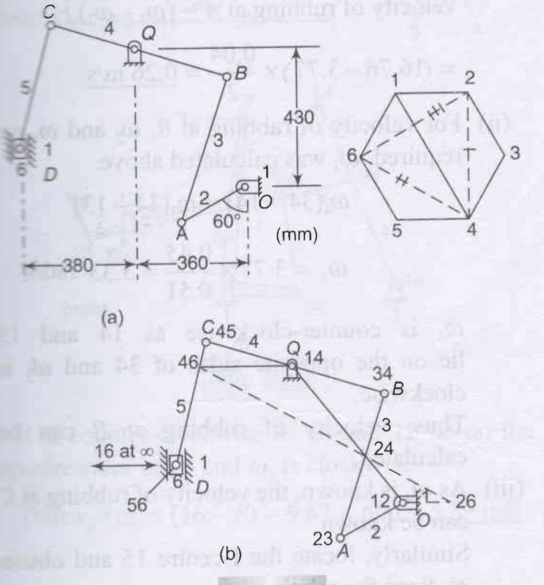


Fig. 2.41

First, imagine the link 2 to be in the form of a flat disc containing the point 26 and revolving about O with an angular velocity of 5π rad/s.

Then, $v_{26} = \omega_2 \times (12\text{–}26) = 5\pi \times 0.145 = 2.28$ m/s.

The velocity of the point 26 is in the vertically downward direction if OA rotates clockwise.

Now, imagine the link 6 (slider) to be large enough to contain the point 26. The slider can have motion in the vertical direction only and the velocity of a point 26 on it is known; it implies that all the points on the slider move with the same velocity.

Thus, velocity of the slider, $v_d = v_{26} = \underline{2.28 \text{ m/s}}$

Example 2.23 *Figure 2.42a shows the configuration of a Whitworth quick return mechanism. The lengths of the fixed link OA and the crank OP are 200 mm and 350 mm respectively. Other lengths are: AR = 200 mm and RS = 400 mm. Find the velocity of the ram using the instantaneous centre method when the crank makes an angle of 120° with the fixed link and rotates at 10 rad/s.*

Solution I-centre 26 is needed to be located as the velocity of the link 2 is known and that of 6 is to be found.

- Locate I-centres 12, 23, 34, 45, 56 , 16 and 14 by inspection [Fig. 2.42(b)]
- Locate I-centre 24 at the intersection of lines joining I-centres 23, 34 and 12, 14.
- Locate I-centre 46 at the intersection of lines joining I-centres 14, 16 and 45, 56. I-centre 16 is perpendicular to AS and lies at infinity. Joining of 12 and 16 means a line passing through OA.
- Now, while locating I-centre 26 at the intersection of lines joining I-centres 12, 16 and 24, 46 it is observed that they lie on the same vertical line OA. Thus, I-centre 26 cannot be located using this path.
- Locate I-centres 15 and then 25 using Kennedy's theorem.
- Now, locate I-centre 26 which lies at the

intersection of lines joining I-centres 12, 16 and 25, 56.

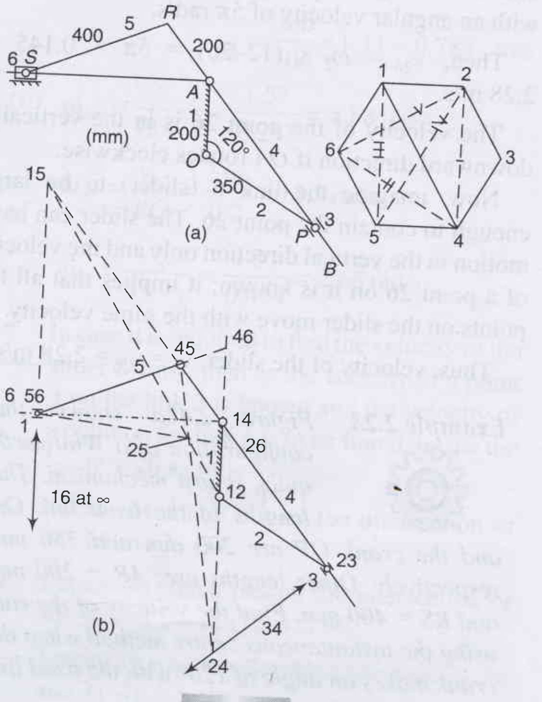


Fig. 2.42

Now, as the velocity of the I-centre 26 is the same whether it is considered to lie on the link 2 or 6,

$$v_{26} = \omega_2 .(12–26) = v_s$$

or $\quad v_s = \omega_2 .(12–26) = 10 \times 0.137 = 1.37$ m/s

Example 2.24 *Solve Example 2.4 by the instantaneous centre method.*

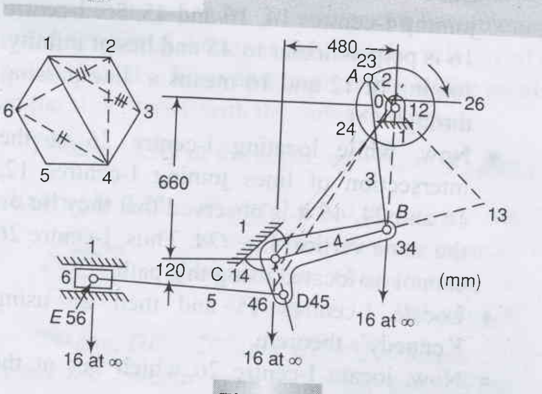


Fig. 2.43

Solution Draw the configuration to a suitab[le] scale as shown in Fig. 2.43.

(a) To find the velocity of E or the link 6, [it] is required to locate the I-centre 26 as the velocity of a point A on the link 2 is known. After locating I-centres by inspection, locate I-centres 24, 46 and 26 by Kennedy's theorem.

First consider 26 to be on the crank 2.

$v_{26} = \omega\,(12–26) = 16.76 \times 0.032 = 0.536$ m/s (horizontal)

When the point 26 is considered on the link 6, all points on it will have the same velocity as the point 26.

Velocity of the crosshead = $\underline{0.536 \text{ m/s}}$

(b) (i) To find the velocity of rubbing at A (or 23), ω_2 and ω_3 are required.

Locate I-centre 13. Then

$$\omega_3(23–13) = \omega_2\,(23–12)$$

$$\therefore \quad \omega_3 = 16.76 \times \frac{0.17}{0.756}$$

$$= \underline{3.77 \text{rad/s}}$$

ω_3 is clockwise as 13 and 12 lie on the same side of 23.

Velocity of rubbing at $A = (\omega_3 - \omega_2)\,r_a$

$$= (16.76 - 3.77) \times \frac{0.04}{2} = \underline{0.26 \text{ m/s}}$$

(ii) For velocity of rubbing at B, ω_2 and ω_4 are required. ω_3 was calculated above.

$$\omega_4(34 - 14) = \omega_3(34 - 13)$$

$$\omega_4 = 3.77 \times \frac{0.45}{0.51} = 3.33 \text{ rad/s}$$

ω_4 is counter-clockwise as 14 and 13 lie on the opposite sides of 34 and ω_3 is clockwise.

Thus, velocity of rubbing at B can be calculated.

(iii) As ω_4 is known, the velocity of rubbing at C can be known.

Similarly, locate the I-centre 15 and obtain ω_5 from the relation,

$$\omega_5 = \omega_4 \left(\frac{45-14}{45-15} \right)$$ and determine the velocity of rubbing at D.

Torque is determined in the same way as in Example 2.3.

Example 2.25 *The configuration diagram of a wrapping machine is given in Fig. 2.44(a). Determine the velocity of the point P on the bell-crank lever DCP if the crank OA rotates at 80 rad/s.*

Solution ω_2 is known, ω_6 is required.

Locate the I-centre 26 by first finding the 13 and 16 by Kennedy's theorem. [Fig. 2.44(b)].

Then $\omega_6(26-16) = \omega_2(26-12)$

$$\omega_6 = \omega_2 \times \frac{26-12}{26-16} = 80 \times \frac{47}{383} = 9.82 \text{ rad/s}$$

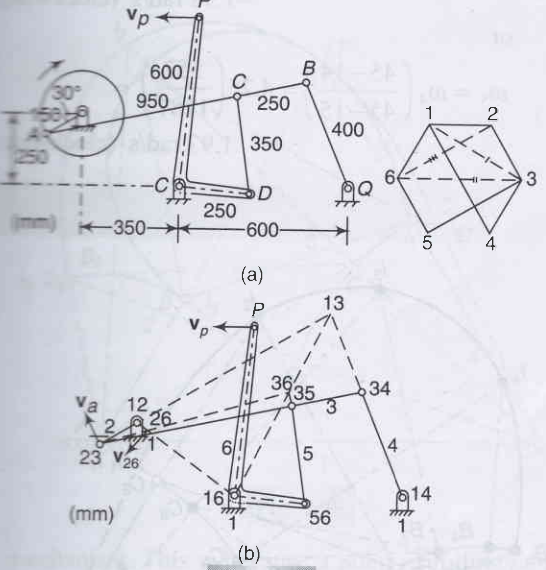


Fig. 2.44

It is counter-clockwise as 16 and 12 lie on the opposite sides of 26 and ω_2 is clockwise.

Thus $v_p = \omega_6 \times (16 - P) = 9.82 \times 600 = \underline{5.89 \text{ m/s}}$

Example 2.26 *Figure 2.45(a) shows the mechanism of a sewing machine needle box. For the given configuration, find the velocity of the needle fixed to the slider D when the crank OA rotates at 40 rad/s.*

Solution Locate the I-centre 26 (Fig. 2.45b).

Consider 26 to lie on the link 2.

$v_{26} = \omega_2 \times (12 - 26) = 40 \times 22.4 = 896$ mm/s vertically downwards.

Consider 26 to lie on the link 6.

Velocity of needle = Velocity of slider = v_{26} = $\underline{896 \text{ mm/s}}$

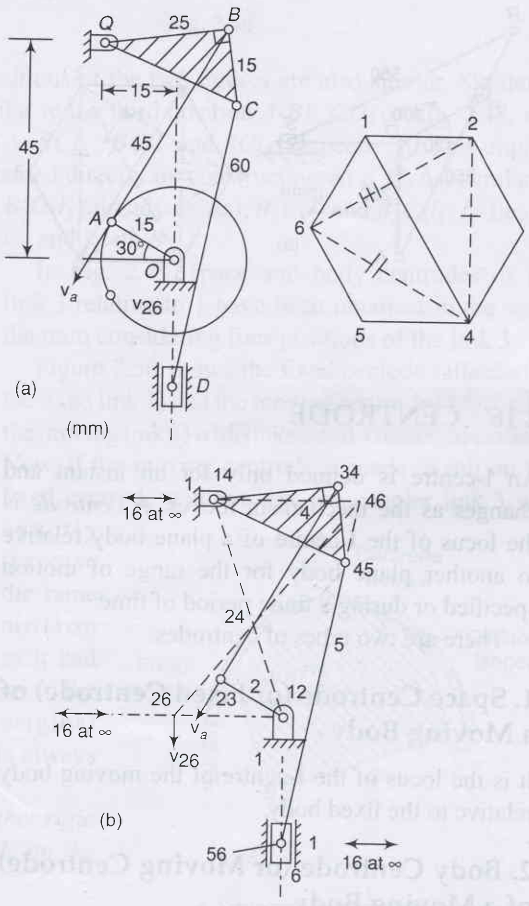


Fig. 2.45

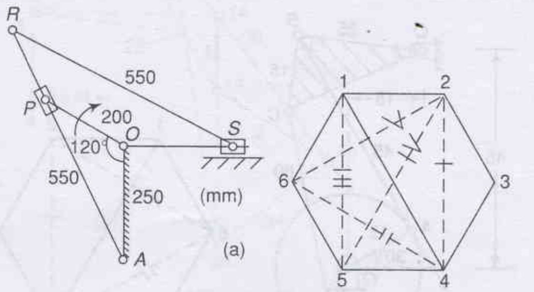
Example 2.27 *Figure 2.46(a) represents a shaper mechanism. For the given configuration, what will be the velocity of the cutting tool at S and the angular velocities of the links AR and RS. Crank OP rotates at 10 rad/s.*

Solution Locate the I-centre 26 [Fig. 2.46(b)]

(i) $v_6 = v_{26} = \omega_2 \times (12 - 26) = 10 \times 0.166 = 1.66$ m/s

(ii) $\omega_4 = \omega_2 \left(\dfrac{24-12}{24-14}\right) = 10 \times \left(\dfrac{183}{430}\right)$

$= 4.25$ rad/s (clockwise)



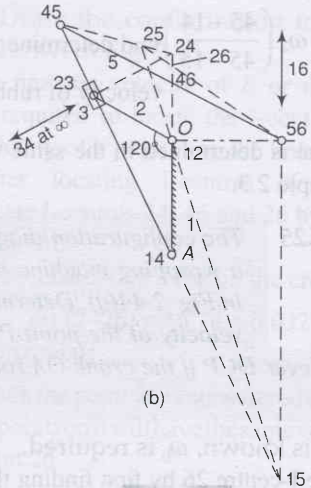


Fig. 2.46

Similarly,

$$\omega_5 = \omega_2 \left(\frac{25-12}{25-15}\right) = 10 \times \left(\frac{210}{1060}\right)$$

$= 1.98$ rad/s (clockwi

or

$$\omega_5 = \omega_4 \left(\frac{45-14}{45-15}\right) = 4.25 \left(\frac{552}{1187}\right) =$$

1.97 rad/s (clockwi

2.16 CENTRODE

An I-centre is defined only for an instant and changes as the mechanism moves. A *centrode* is the locus of the I-centre of a plane body relative to another plane body for the range of motion specified or during a finite period of time.

There are two types of centrodes:

1. Space Centrode (or Fixed Centrode) of a Moving Body

It is the locus of the I-centre of the moving body relative to the fixed body.

2. Body Centrode (or Moving Centrode) of a Moving Body

It is the locus of the I-centre of the fixed body relative to the movable body, i.e., the locus of the

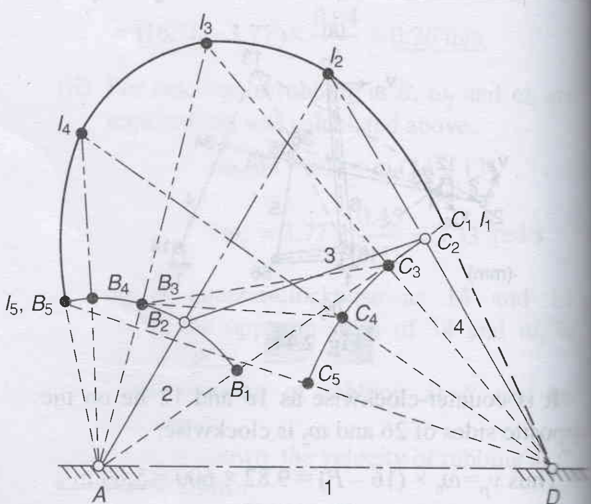


Fig. 2.47

I-centre assuming the movable body to be fixed and the fixed body to be movable.

In a four-link mechanism shown in Fig. 2.47, the link 1 is fixed. The locus of the I-centre of links 1 and 3 over a range of motion of the link 3 is the space centrode. Five positions of the I-centre, i.e., I_1, I_2, I_3, I_4 and I_5 have been obtained and joined with a smooth curve which is the space centrode. If the link 3 is assumed to be fixed and 1 movable, the locus of the I-centre of 1 and 3 is the body centrode. This has been shown in Fig. 2.43 for five positions of the link 1.

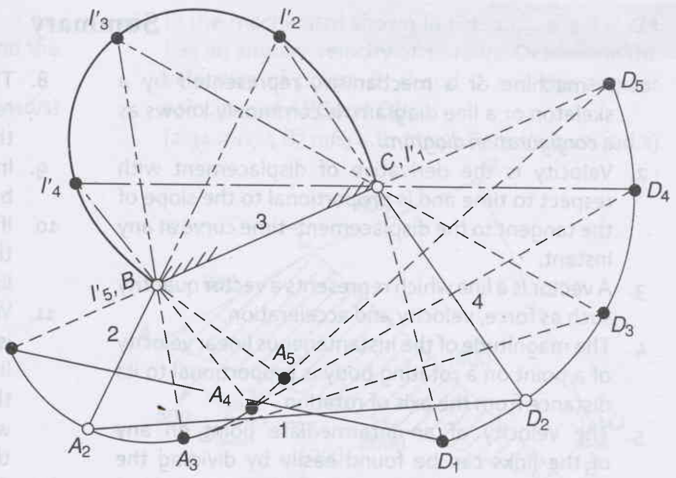


Fig. 2.48

Comparing Figs 2.47 and 2.48, observe that the first position AB_1C_1 (I_1) D of Fig. 2.47 is exactly similar to the first position A_1BC $(I_1')D_1$ of Fig. 2.48. The second positions of the two figures are also similar. Similarly, the third position $AB_3I_3C_3D$ of Fig. 2.47 is exactly similar to the third position $A_3BI_3'CD_3$ of Fig. 2.48, and so on. Thus, Δs $B_2C_2I_2$, $B_3C_3I_3$ and $B_4C_4I_4$ are similar to Δs BCI_2', BCI_3' and BCI_4' respectively. This implies that the positions of the I-centre of Fig. 2.43 can be obtained directly by constructing on B_2C_2 Δs, similar to $B_2C_2I_2$ (already exists), $B_3C_3I_3$ and $B_4C_4I_4$. I_1' lies on C_2 and I_5' on B_2.

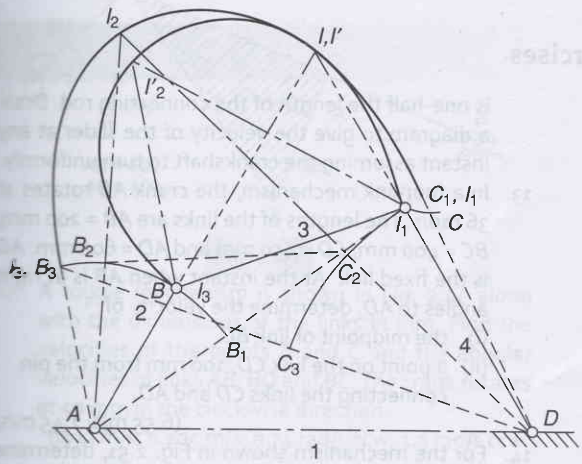


Fig. 2.49

In Fig. 2.49, space and body centrodes of the link 3 relative to 1 have been obtained in the same diagram considering four positions of the link 3.

Figure 2.50 shows the fixed centode (attached to the fixed link 1) and the moving centrode (attached to the moving link 3) with links 2 and 4 removed entirely. Now, if the moving centrode is made to roll on the fixed centrode without slip, the coupler link 3 will exactly traverse the same motion as it had in the original mechanism. This is because a point of rolling contact is always an I-centre in different positions of the link 3.

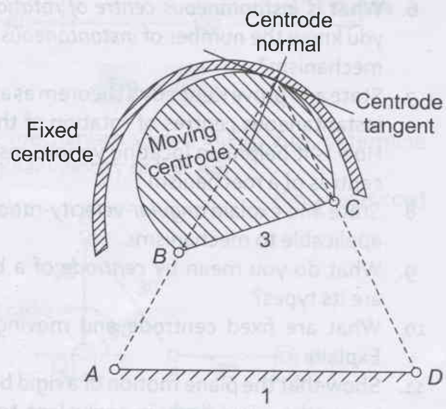


Fig. 2.50

Thus, *the plane motion of a rigid body relative to another rigid body is equivalent to the rolling motion of one centrode on the other.*

The instant point of rolling contact is the instantaneous centre. The common tangent and the common normal to the two centrodes are known as the *centrode tangent* and the *centrode normal* respectively.

Summary

1. A machine or a mechanism, represented by a skeleton or a line diagram, is commonly knows as a *configuration diagram*.
2. Velocity is the derivative of displacement with respect to time and is proportional to the slope of the tangent to the displacement–time curve at any instant.
3. A vector is a line which represents a vector quantity such as force, velocity and acceleration.
4. The magnitude of the instantaneous linear velocity of a point on a rotating body is proportional to its distance from the axis of rotation.
5. The velocity of an intermediate point on any of the links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides the link.
6. *Velocity images* are found to be very helpful devices in the velocity analysis of complicated linkages. The order of the letters in the velocity image is the same as in the configuration diagram.
7. The angular velocity of a link about one extremity is the same as the angular velocity about the other.
8. The *instantaneous centre of rotation* of a body relative to another body is the centre about which the body rotates at the instant.
9. In a mechanism, the number of I-centres is given by $N = n(n-1)/2$
10. If three plane bodies have relative motion among themselves, their I-centres must lie on a straight line. This is known as *Kennedy's theorem*.
11. When the angular velocity of a link is known and it is required to find the angular velocity of another link, locate their common I-centre. The velocity of this I-centre relative to a fixed third link is the same whether the I-centre is considered on the first or the second link.
12. A *centrode* is the locus of the I-centre of a plane body relative to another plane body for the range of motion specified or during a finite period of time.
13. The plane motion of a rigid body relative to another rigid body is equivalent to the rolling motion of one centrode on the other.

Exercises

1. What is a configuration diagram? What is its use?
2. Describe the procedure to construct the diagram of a four-link mechanism.
3. What is a velocity image? State why it is known as a helpful device in the velocity analysis of complicated linkages.
4. What is velocity of rubbing? How is it found?
5. What do you mean by the term 'coincident points'?
6. What is *instantaneous centre of rotation?* How do you know the number of *instantaneous centres* in a mechanism?
7. State and prove Kennedy's theorem as applicable to instantaneous centres of rotation of three bodies. How is it helpful in locating various instantaneous centres of a mechanism?
8. State and explain *angular-velocity-ratio theorem* as applicable to mechanisms.
9. What do you mean by *centrode* of a body? What are its types?
10. What are fixed centrode and moving centrode? Explain.
11. Show that the plane motion of a rigid body relative to another rigid body is equivalent to the rolling motion of one centrode on the other.
12. In a slider-crank mechanism, the stroke of the slider is one-half the length of the connecting rod. Draw a diagram to give the velocity of the slider at any instant assuming the crankshaft to turn uniformly.
13. In a four-link mechanism, the crank *AB* rotates at 36 rad/s. The lengths of the links are *AB* = 200 mm, *BC* = 400 mm, *CD* = 450 mm and *AD* = 600 mm. *AD* is the fixed link. At the instant when *AB* is at right angles to *AD*, determine the velocity of
    (i) the midpoint of link *BC*
    (ii) a point on the link *CD*, 100 mm from the pin connecting the links *CD* and *AD*.
    (6.55 m/s; 1.45 m/s)
14. For the mechanism shown in Fig. 2.51, determine the velocities of the points *C*, *E* and *F* and the angular velocities of the links *BC*, *CDE* and *EF*.

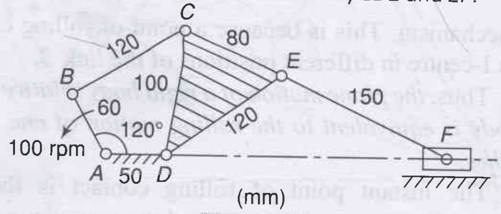


Fig. 2.51

(0.83 m/s; 0.99 m/s; 0.81 m/s; 5.4 rad/s ccw; 8.3 rad/s ccw; 6.33 rad/s ccw)

15. For the four-link mechanism shown in Fig. 2.52, find the linear velocities of sliders *C* and *D* and the angular velocities of links *AC* and *BD*.
(2.1 m/s; 0.38 m/s; 1.14 rad/s; 5.93 rad/s)

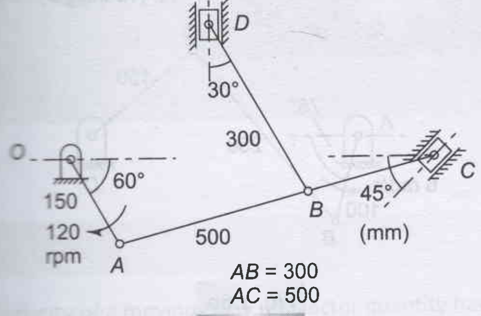


Fig. 2.52

16. An offset slider-crank mechanism is shown in Fig. 2.53. The crank is driven by the slider *B* at a speed of 15 m/s towards the left at given instant. Find the velocity of the offset point *D* on the coupler *AB* and the angular velocities of links *OA* and *AB*.
(18 m/s; 60 rad/s; 16.4 rad/s)

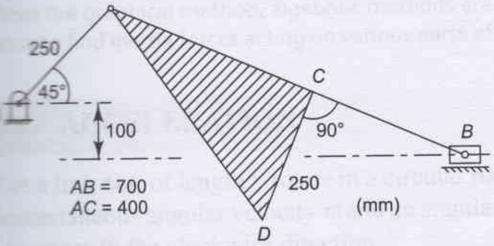


Fig. 2.53

17. A toggle mechanism is shown in Fig. 2.54 along with the dimensions of the links in mm. Find the velocities of the points *B* and *C* and the angular velocities of links *AB*, *BQ* and *BC*. The crank rotates at 50 rpm in the clockwise direction.
(0.13 m/s; 0.105 m/s; 0.74 rad/s ccw; 1.3 rad/s ccw; 1.33 rad/s cw)

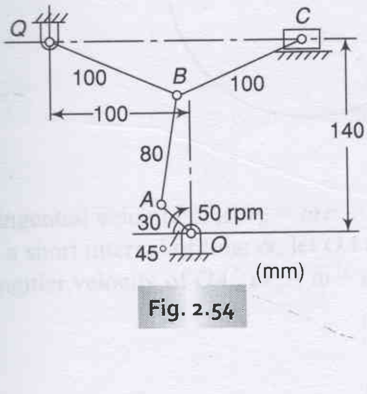


Fig. 2.54

18. In the mechanism shown in Fig. 2.55, the link *OA* has an angular velocity of 10 rad/s. Determine the velocities of points *B*, *C* and *D* and the angular velocities of *ABC* and *QD*.
(2.34 m/s; 4.87 m/s; 4.87 m/s; 1.87 rad/s; 4.06 rad/s)

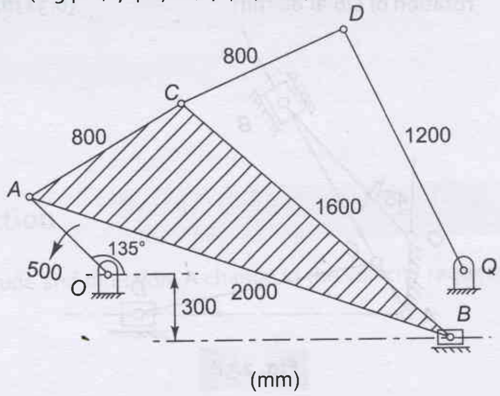


Fig. 2.55

19. Draw the velocity polygon for the mechanism shown in Fig. 2.56. Find the angular velocity of link *AB*. (1.75 rad/s cw)
(*Hint*: Assume the length of vector $\mathbf{v}_{ef}$ and complete the velocity polygon. Determine the velocity scale.)

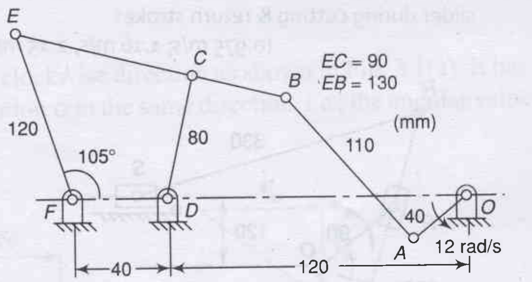


Fig. 2.56

20. For the mechanism shown in Fig. 2.57, determine the angular velocity of link *AB*.
(60.3 rad/s ccw)

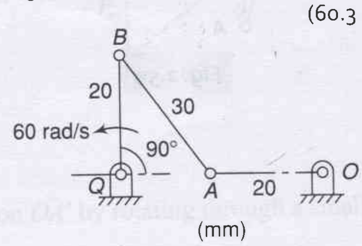


Fig. 2.57

21. In the mechanism shown in Fig. 2.58, O and A are fixed. CD = 200 mm, OA = 60 mm, AC = 50 mm and OB(crank) = 150 mm. OAD = 90°. Determine the velocity of the slider D for counter-clockwise rotation of OB at 80 rpm. (0.32 m/s)

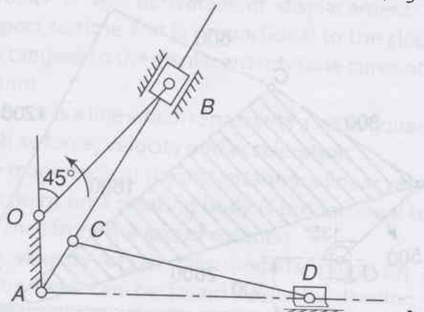


Fig. 2.58

22. The crank OP of a crank- and slotted-lever mechanism (Fig. 2.59) rotates at 100 rpm in the counter-clockwise direction. Various lengths of the links are OP = 90 mm, OA = 300 mm, AR = 480 mm and RS = 330 mm. The slider moves along an axis perpendicular to AO and is 120 mm from O. Determine the velocity of the slider when the AOP is 135°. Also, find the maximum velocity of the slider during cutting & return strokes.
(0.975 m/s; 1.16 m/s, 2.15 m/s)

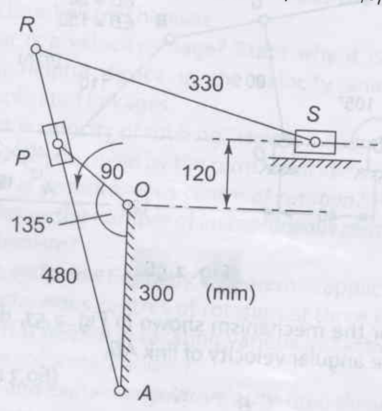


Fig. 2.59

23. For the four-link mechanism shown in Fig. 2.60, find the angular velocities of the links BC and CD using the instantaneous centre method.
(1.3 rad/s, 3.07 rad/s)

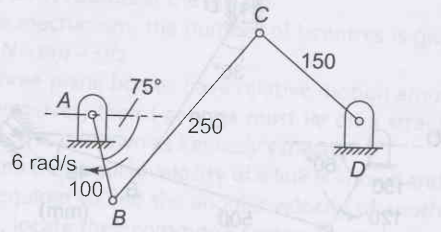


Fig. 2.60

24. Solve Problem 17 (Fig. 2.54) using the I-centre method.
25. Solve Problem 21 (Fig. 2.58) using the I-centre method.
26. Solve Example 2.5 (Fig. 2.13) using the I-centre method.
27. Solve Example 2.11 (Fig. 2.19) using the I-centre method.