SCALER BASIC MATHS CERTIFICATION

* **Arithematic progression** :
  1) $\boxed{a_n = a + (n-1)d}$
  2) $\boxed{sum_n = \frac{n}{2} + (a + a_n)}$

d = difference b/t two conse cutive terms

a_n : value of term at n^{th} pos.

a = first term

n = number of terms in progression

* **Geometric progression** :
  1) $\boxed{a_n = a r^{(n-1)}}$
  2) $sum_n = \boxed{\frac{a(r^n - 1)}{r-1}}$ if $\boxed{r>1}$

  OR

  $sum_n = \boxed{\frac{a(1-r^n)}{1-r}}$ if $\boxed{r<1}$

r = common ratio

a = first term

n = number of terms in progression

Q1. for an AP: $\left.\begin{array}{l} a_5 = 23 \\ a_{12} = 50 \end{array}\right\}$ Given

Find: a, d

$a_5 = a + (5-1)d$

$= a + 4d$

$a_{12} = a + 11d$

$a_{12} - a_5 = 50 - 23$

$= 27$

$27 = 11d - 4d$

$27 = 7d \Rightarrow \frac{27}{7} = d$ ✓

$23 = a + 4d$

$23 - 4d = a$.

$23 - 4\left(\frac{27}{7}\right) = a$.

$\frac{161 - 108}{7} = a \Rightarrow a = \frac{53}{7}$ ✓

Q2. wooden log → cuts into half each step. i.e. N, $\frac{N}{2}$, $\frac{N}{4}$, $\frac{N}{8}$ -

So,

progression = N, $\frac{N}{2^1}$, $\frac{N}{2^2}$, $\frac{N}{2^3}$ ...

geometric

How long will it take to become 1?

So, $a_n = 1$

$a_n = ar^{n-1}$ where we know $r = \frac{1}{2}$

and $a = N$

So, $a_n = N\left(\frac{1}{2}\right)^{n-1}$ or $\frac{N}{2^{n-1}}$

Now,

$1 = \frac{N}{2^{n-1}} \Rightarrow 2^{n-1} = N$

apply $\log_2$ on both sides,

$\log_2 2^{n-1} = \log N$

$n - 1 = \log N$

$n = \log N + 1$ ✓

* **Permutations** (or **arrangements**)

Arranging elements: (a, b, c)

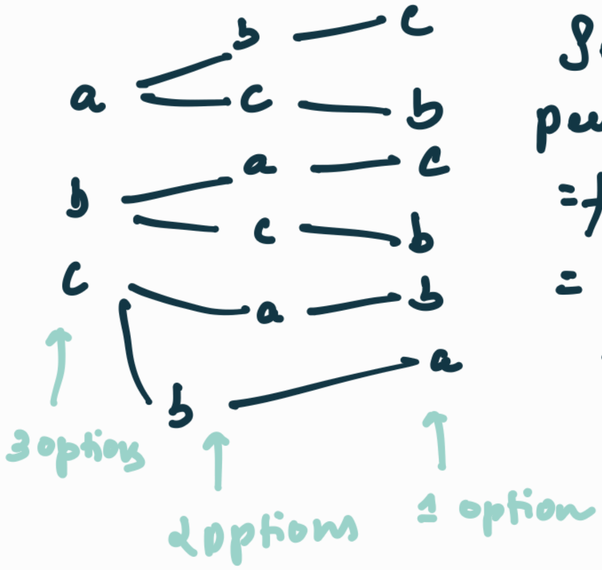
↓

possible ways = (a b c) = (c a b)

↳ So, 3 terms = (a c b) = (c b a)

6 unique ways = (b a c)

= (b c a)



So, permutations = factorials

= i.e. 3!

= 3×2×1

= 6.

another scenario,

↳ n items → use only r spots / items

i.e. there are 5 elements but you can form only a 3 elements arrangement

working : 1^{st} choice = 5 options

2^{nd} choice = 4 options

3^{rd} choice = 3 options

So, ways = 5×4×3

$= \frac{5!}{2!} = \boxed{{}^5P_3}$

$= \frac{5!}{(5-3)!}$

permutations for total n items, but r choices where n > r : $\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$

↓

all unique arrangents

if $\boxed{n=r}$ the $\boxed{{}^nP_r = n!} = {}^nP_n$

For repetitive arrangements, formula changes!!

if elements = abbcccdddde

the permutations $= \frac{11!}{2! \times 3! \times 4!}$

where 11 = no. of elements (here, letters)

2! = for b = 2 (no. of 'b')

3! = for c = 3 (no of 'c')

4! = for d = 4 (no. of 'd')

1! = 1×1 = 1 = for a & e

So, for repetition: $P = \frac{n!}{r_1! \times r_2! \times \ldots}$

where r_1 = one repeated element.

(summary formula in last page)

* Combinations : SELECTIONS

For n items and r selections :

$$\boxed{{}^nC_r = \frac{n!}{r! \times (n-r)!}}$$

Permutation vs Combination :

For 5 elements : a, b, c, d, e

Selection of 3 : $\frac{5!}{3! \times 2!} = 10$

But,

arrangement of 3 : $\frac{5!}{2!} = 60$

reduced by 3!

looking at the formulas : ${}^nP_r = \frac{n!}{(n-r)!}$

$${}^nC_r = \frac{n!}{r! \times (n-r)!}$$

$$\boxed{{}^nC_r = \frac{{}^nP_r}{r!}}$$

permutations : selection + arrangement } duplicates in results

↓

more options

where as combinations : selection

↓ all unique results

↓ then arrangements

So, the addition r! = for the duplication factor

* Binomial terms : $(a+b)^n$ → related to combinations

↳ ${}^nC_0 \, a^n b^0$, ${}^nC_1 \, a^{n-1} b^1$, ${}^nC_2 \, a^{n-2} b^2$ .... ${}^nC_n \, a^0 b^n$

Combinations with repetition :

eg. 10 candies for 3 children

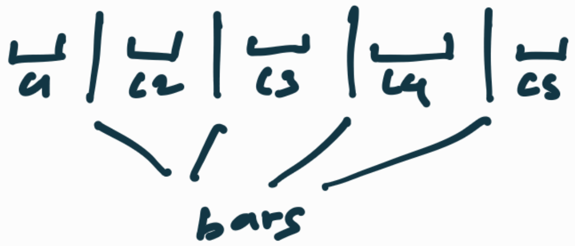
↓

Use method : Stars & Bars

n = stars = 10 here,

r = bars = 2 ?

if there were 5 children & 10 candies, then there will be 4 bars :



So, final formula is :

$$\boxed{{}^{\text{stars + bars}}C_{\text{bars}}}$$

↓

or if there are 'n' total elements and 'r' selections with repetitions, then combination results as

$$\boxed{{}^{n+r-1}C_{r-1}}$$

Questions practiced below :-

Q1. 8 acts → only 5 can be performed
↓
+ arrangement is needed for creating an engaging flow

So, permutation = needed (no repetition)

$= {}^{8}P_5 = \frac{8!}{3!} = 8 \times 7 \times 6 \times 5 \times 4$

Q3) How many distinct permutations of the word 'MISSISSIPPI' exist if the characters appear consecutively?

$M \to 1$ $S \to 4$
$I \to 4$ $P \to 2$ } frequency

Eg: Sets = ① M
= ② IIII (all together)
= ③ SSSS ④ PP
} 4 comb.

So, if M is the start,

word = M
then, word = M $\underbrace{\{IIII\}}_{\text{all together}}$

So, answer
$= 4! = 4 \times 3 \times 2$
$= 6 \times 4 = 24$

Q6) n = 52 → card deck
4 aces in the deck.
Player randomly draws = 5

Find : P(player draws 2 aces out of 5 cards in their hand)

Prob
$= \frac{\text{Favourable outcomes}}{\text{Total}} = \frac{\text{Favourable}}{{}^{52}C_5}$

Favourable : 2 aces out of 4 in the deck × (52−4) normal cards → out of which 3 are selected by user

Q2. 'MYSTERY' → How many distinct permutations of the string exists?

$M \to 1$
$Y \to 2$
$S \to 1$
$T \to 1$
$E \to 1$
$R \to 1$
} freq.
7 = total

${}^{7}P_2 = \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2}}{\cancel{2}}$
for repetition

$= 7 \times 6 \times 5 \times 4 \times 3$
$= 42 \times 60$
$= 2520$

Q4) 12 different types of flowers
painting (each) = 5 unique combinations of flowers
How many combinations?

A: ${}^{12}C_5 = \frac{12!}{5! \times 7!}$

$= \frac{12 \times 11 \times \cancel{10} \times \cancel{9} \times \cancel{8} \times \cancel{4 \times 6 \times 5 \times 4 \times 3 \times 2}}{\cancel{5 \times 4 \times 3 \times 2} \times \cancel{7 \times 6 \times 5 \times 4 \times 3 \times 2}}$

$= 12 \times 11 \times 6 = 132 \times 6$
$= 792$

Q5) Ice cream → 10 different toppings
per customer = 4 toppings + repeated!
How many unique combinations?

Stars & bars → 4 stars and 9 bars.

So, ${}^{9+4}C_4 = {}^{13}C_4 = \frac{13!}{4! \times (13-4)!}$

$= \frac{13 \times \cancel{12} \times 11 \times 10 \times \cancel{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}}{\cancel{4} \times \cancel{3} \times 2 \times \cancel{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}}$

$= 13 \times 11 \times 10 / 2 = 143 \times 5 = 715$

So, $= {}^{4}C_2 \times {}^{48}C_3$
= favourable outcomes

$P = \frac{{}^{4}C_2 \times {}^{48}C_3}{{}^{52}C_5}$

GCD and LCM

multiples of 5 → 5·1 = 5
→ 5·2 = 10

Factors of 5 → 1 and 5 as only the can divide 5.

Factors of 18 = 3, 6, 9, 2, 1, 18
They all can divide 18

LCM = lowest common multiple

eg. 5 and 20's LCM

5 = 5, 10, 15, 20, 25 } multiples.
20 = 20, 40, 60, 80 ...

↑ LCM

Commanly, if n_1 & n_2 are no.s from which we must find the LCM and GCD for, then :-

$$\boxed{LCM \times GCD = n_1 \times n_2}$$

• GCD
= greatest common divisor
↳ 2 methods to find

method: 1) Dividing larger no. by smaller

eg. for finding GCD for 20 and 12

i)

```
       1
  12 ) 20
       12
           1
       8 ) 12
            8
               2
           4 ) 8
               8
               0
```

GCD: 2

Summary Formula Sheet

1. AP: $a_n = a + (n-1)d$

2. $Sum_n = \frac{n}{2}(a + a_n)$
(for AP)

3. GP: $a_n = a(r^{n-1})$

4. $Sum_n = \frac{a(r^n - 1)}{r-1}$ if $r > 1$

OR

$sum_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$

5. Permutations : Arrangements
if unique
↳ a. if $n = r$, then
$^nP_n = n!$
↳ b. if $n > r$, then
$^nP_r = \frac{n!}{(n-r)!}$

6. permutations : repetitive :

$$P = \frac{n!}{r_1! \times r_2! \times \ldots}$$

where $r_1, r_2 \ldots$ = are repetitive terms frequency.

7. Relationship b/t ^nP_r & ^nC_r

$\Rightarrow {}^nC_r = \frac{^nP_r}{r!}$ } for no repetition only

8. Combinations with binomial factors:
$(a+b)^n = {}^nC_r[a^{n-r}b^r]$

$r = 1$ then, $^nC_1 a^{n-1} b^1$

9. Combinations & repetitions:
$^{n+r-1}C_{r-1}$