Title: Predict the win probability of basketball games by alternating renewal reward process model

Xuhui Liu xul019@ucsd.edu

Abstract: We used alternating renewal reward process to model the basketball games and Monte Carlo Simulation to predict the win probability. Our data are possession-based data from 2020-2021 season NBA regular season games. Different from previous work, our model considers the dependent relationship between possession time and scores per possession and makes special adjustment to the effect of home court advantages under the influence of pandemics by incorporating ELO rating system. Results show that compared with pure ELO forecast our model produces a better forecast indicated by 40% less MSE.

1. Introduction

Predicting win probability of sports games has long been an interesting topic, especially today, when the betting market is thriving. Betting companies offer variety of betting strategies including betting at any time of the game, on any statistics of the game, and on the outcome of the game. Considering the popularity of sports betting and my interest in basketball, we develop a new alternating renewal reward process model based on Conrad's (2013) **[1]** work to predict the win probability of teams in a basketball game.

As far as 1978, Albert Elo**[2]** proposed the famous ELO rating system to predict the win probability of chess games. He gave each chess player a rating and used this rating to calculate the win probability of a certain chess game. The ELO rating system is also widely used in other sports win probability forecast, including basketball. The 538 websites used the ELO rating system to produce a win probability forecast. In 2007, Shirley**[3]** proposed a model by using transitional probability matrix to predict the win probability. This model captures the dependent nature of possessions in a basketball game. Given the information in the last possession, we can have the probability of what will happen in the next possession. This model looks great, but it is almost impossible to customize the transitional probability matrix for each game (between two different teams). So, Shirley considered all the home teams as a whole and all the away team as another one. Then, the data would be enough to construct the transitional probability matrix and make the prediction. However, if the differences between teams are ignored, the prediction would not produce very accurate result. In 2012, Štrumbelj**[4]** improved on Shirley's model. Štrumbelj compared the result from transitional probability matrix model with others like ELO and bookmaker's odds and found that the transitional probability matrix model is an appropriate approach. In 2020, Kai Song

[5] proposed using gamma process combined with bookmaker's betting line to make predictions on the real time win probability. They derived a useful formula in predicting the probability. In the long run, they claimed that they had a positive return against the betting company. In 2013, Conrad**[1]** proposed using alternating renewal reward process to forecast the game outcome. This model captures the progression of alternating possessions in basketball games, but it does not consider the dependent relationship between possessions. More recently there are also some novel approaches in predicting the win probability. In 2018, Alison and Varol**[6]** proposed a "Data snapshot" approach in predicting the real time win probability. It utilizes 20 seasons NBA data. For each moment in a game, this model finds similar moment in the last 20 seasons and estimate the win probability. This similar moment includes information of scores difference and home and away team information. It also tests including information about team strength, but the model does not perform better. This approach is good. However, some games like 20 seasons away are too far from now, which may not be representative. The rules might change a lot in the last 20 years. In 2020, machine learning has also been used to predict the win probability. Deokar**[7]** used supervised learning to predict the real time win probability. It utilizes the running box score and score difference. This is a good idea, but the problem is also that the model doesn't emphasize the differences between team.

Our work is built on model proposed by Conrad in 2013. Conrad suggested using the alternating renewal reward process to forecast the game outcomes. Conrad's model is purely based on the simulation of random process. In this paper, we will improve the model by incorporating the impact of possession time on the scores per possession and by utilizing the ELO rating system to make adjustment to the influence of home court advantages under the situation of pandemic. We will use our final model to predict the win probability by Monte Carlo simulations and compare our result with other model predictions.

2. Data

There are mainly four kinds of data needed to build our model. The first is 2020-2021 NBA season all games' possession data. This is not only the most important data, but also the data that most difficult to collect. We will need the possession time to fit the time distribution of the random process, and we also need the points per possession and opponent point per possession to fit the reward distribution of the random process. The second kind of data is the ELO rating data. This is relatively easy to get. We simply use the data on the 538 websites. The third kind of data is the Moneyline data from betting companies. This is also not hard to get. We get the data from oddsportal.com. These two kinds of data above are used as the benchmark for our model. The fourth kind of data we need is the attendance data for all the 2020-2021 season NBA games. They are used to quantify the home court advantages. We got this data from basketball-reference.com.

We will focus on introducing the collection of the first part of the data, that is, how we get the possession data. We first need the play-by-play data of all the NBA games. The play-by-play data describes what has happened on the court in a very detailed manner and is available on the ESPN. The followings are some sample play-by-play data.

TIME	TEAM	PLAY	SCORE
12:00		Kevon Looney vs. Andre Drummond (LeBron James gains possession)	0 - 0
11:37		Anthony Davis misses 15-foot two point shot	0 - 0
11:34		Stephen Curry defensive rebound	0 - 0
11:23		**Andrew Wiggins makes driving layup (Draymond Green assists)**	**2 - 0**
11:10		**Andre Drummond makes dunk (Dennis Schroder assists)**	**2 - 2**
10:55		**Kent Bazemore makes 27-foot three point jumper (Stephen Curry assists)**	**5 - 2**
10:35		Dennis Schroder misses 12-foot pullup jump shot	5 - 2

We decide to collect all the play-by-play data from the ESPN. After collecting the play-by-play data, we cleaned and transformed the data to the format of Table 1.

Table 1

Sample possession-based data with the possession time and scores

Game_details	Game_time	Away_score	Home_score	pos2
Deandre Ayton vs. Brook Lopez (Devin Booker ga...	0.0	0	0	START
Giannis Antetokounmpo defensive rebound	21.0	0	0	PHX
Jae Crowder defensive rebound	40.0	0	0	MIL
Khris Middleton defensive rebound	47.0	0	0	PHX
Giannis Antetokounmpo makes 4-foot two point s...	54.0	0	2	MIL
Devin Booker makes 15-foot two point shot	72.0	2	2	PHX
shot clock turnover	96.0	2	2	MIL

The key part in the data transformation is to transform the play-by-play data to the possession data. Generally, we have 4 ways to end a possession:

1. The team makes a shot
2. The team makes a turnover
3. The team misses a shot, and the other team gets the defensive rebound
4. The quarter ends

By using the 4 principles above, we identify the end of possession from the *Game_details* column. Then, we get the following Point Per Possession (PPP) table and Opponent Point Per Possession (OPPP) table. The Table 2 are some parts of the PPP and OPPP table.

Table 2

Sample PPP table and OPPP table

	Team	Time	Score	Time_left
1	BKN*	10.0	0.0	11:50
4	GS	12.0	2.0	11:38
5	BKN*	16.0	2.0	11:22
6	GS	11.0	2.0	11:11
7	BKN*	22.0	3.0	10:49

	Team	Time	Score
1	GS-	10.0	0.0
4	BKN*-	12.0	2.0
5	GS-	16.0	2.0
6	BKN*-	11.0	2.0
7	GS-	22.0	3.0

We have collected a total of 21,3085 possessions data. They are from all games of 2020-2021 NBA season. The followings are some summary statistics of the possession data.

Table 3

PPP (points per possession) summary statistics

Team	Time	Score
LEB	10.802913	1.407767
DUR*	10.111765	1.235294
LAC*	15.068420	1.208600
UTAH*	14.604061	1.207035
BKN	14.352571	1.206519
POR	15.060063	1.199543
MIL*	13.458294	1.194406

Team	Time	Score
HOU*	14.360418	1.063319
ORL	15.151321	1.054277
OKC*	14.343534	1.054002
CLE	15.545552	1.047522
OKC	14.507291	1.039922

The table 3 is the average point per possession for the top five teams and last five teams (excluded the LEB and DUR*, they are all-star teams). The "*" means that it is the team at home. So, the LAC at home has the highest average point per possession, while the OKC has the lowest. That is to say LAC can score 1.21 points in a possession on average, and OKC at away can only score 1.04 points in a possession on average.

The table 4 summarize team's OPPP data. The UTAH at home has the highest defensive power. It only allows its opponent to score 1.06 in a possession on average. The SAC at home has the worst defensive power, which allows its opponent to score 1.21 in a possession on average.

Table 4

OPPP (opponent points per possession) summary statistics

Team	Time	Score
UTAH*-	14.591161	1.060057
PHI*-	14.329014	1.086656
NY-	14.654641	1.089031
LAL-	14.599482	1.089132
LAL*-	14.631800	1.090017

Team	Time	Score
CLE*-	14.044356	1.172335
POR-	14.238284	1.173603
OKC*-	14.177913	1.174370
BKN-	14.833249	1.177329
SAC*-	14.830681	1.206130

Table 5

Possession time summary statistics

Team	Time	Scor
DUR*	10.111765	1.23529
LEB	10.802913	1.40776
WSH*	13.331962	1.13538
MIL	13.381241	1.17418
MIL*	13.458294	1.19440
GS	13.491929	1.08614

Team	Time	Score
NY	15.260505	1.093732
DET*	15.417745	1.090308
CLE*	15.418511	1.088027
NY*	15.469128	1.136403
CLE	15.545552	1.047522

The WSH has the fastest pace, which finishes its possession in 13.33 seconds on average, while CLE has a slowest pace, which finishes its possession in more than 15.5 seconds. This result corresponds to our common sense that WSH is the team with most possessions per game.

3. Model

3.1 Introduction to alternating renewal reward process

Conrad proposed using alternating renewal reward process to model the basketball games in 2013**[1]**. We will just continue to use his notation and definition in this paper.

If we let S_i be I.I.D. random variable with distribution F, and let U_i be I.I.D. random variable with distribution G. An alternating renewal process spends an amount of time S_i in state 1, an amount of time U_i in state 2, then repeats the cycle again. The alternating renewal reward process is defined as followed, where S_i and U_i are time function and A_t and H_t are reward function.

$$J_n = \sum_{i=1}^{n} S_i + \sum_{i=1}^{n-1} U_i$$

$$K_n = \sum_{i=1}^{n} (S_i + U_i)$$

$$X_t = \sup\{n: J_n \leq t\}$$

$$Y_t = \sup\{n: K_n \leq t\}$$

$$A_t = \sum_{i=1}^{X_t} V_i$$

$$H_t = \sum_{i=1}^{Y_t} W_i$$

Similar to what Conrad did, we will fit S_i and U_i as the possession time of the two teams and fit A_t and H_t as the scoring distribution of the two teams.

3.2 An overview of all models used in this paper

In this paper, we have 4 models derived from the alternating renewal reward process. The first basic model is called simple no home defense model (SNHD). This is the model that does not take team's defensive improvement into account when playing at home. The second model is simple home defense model (SHD) which is the model that does take team's defensive improvement into account when playing at home. Then, the third model is called "complex". This is a model that considers the dependent relationship between the time distribution (team's possession time) and reward distribution (team's scoring distribution). Our final model is just called "Final", this is a model that uses ELO system to correct team's home court advantage. We do so because the sample data from the regular season games mostly contain game with few spectators while in the games that we intend to predict, the number of spectator surges. Before introducing these four models in more detail, we also need to introduce other two benchmark models: they are latent strength model (ELO rating system) and Moneyline odds from the betting company. In the following chapter, we will introduce these models in more detail.

3.3 ELO rating system

The ELO rating system is widely used in all kinds of sports including chess, football and basketball etc. We can estimate the team's win probability from two teams' ELO rating. This model was developed initially by Arpad Elo in 1978 **[2]**. Here, we use the pure ELO prediction from 538 website.

Team's ELO ratings are calculated as followed:

$R_n = R_0 + K\ (W - W_e)$

R_n is the new rating after the event.

R_0 is the pre-event rating.

K is the rating point value of a single game score. (538 chooses 20 here)

W is 1 or 0, where 1 means wining and 0 means losing.

W_e is the expected win probability based on R_0.

We calculated the win probability as followed:

$$Win\ Probability = \frac{1}{1 + 10^{-\frac{Team\ Rating\ Differential + Bonus\ Differential}{400}}}$$

The Bonus differential represents home court advantage. 538 uses 100 in their pure ELO forecast model.

3.4 Win probability prediction from Moneyline odds

Another benchmark model we use is the Moneyline given out by the www.oddsportal.com. Odds are generally considered as the benchmark model in win probability prediction. So, in this paper, we also use the odds as the main indicator of the prediction accuracy of our morel.

02 Jun 2021 - Play Offs			1	2	B's
02:00	**Phoenix Suns** - Los Angeles Lakers	**115:85**	-179	+156	14
01:00	**Denver Nuggets** - Portland Trail Blazers	**147:140 OT**	-123	+106	14
01 Jun 2021 - Play Offs			**1**	**2**	**B's**
23:30	**Brooklyn Nets** - Boston Celtics	**123:109**	-909	+644	14
01:30	Memphis Grizzlies - **Utah Jazz**	**113:120**	+189	-222	14
31 May 2021 - Play Offs			**1**	**2**	**B's**
23:00	**Washington Wizards** - Philadelphia 76ers	**122:114**	+305	-370	14
01:30	Dallas Mavericks - **Los Angeles Clippers**	**81:106**	+142	-164	14

In the Moneyline odds, a negative number -α means that you need to invest α in order to earn 100 back, while a positive number β means that if you invest 100, you will earn β back. For example, In 02 Jun 2021 PHX vs. LAL, the number -179 means that you need to invest 179 dollars to earn 100 dollars back if PHX wins. The number +156 means that if you invest 100 dollars on LAL and LAL wins eventually, you will earn 156 dollars.

We will use this Moneyline odds to reconstruct the win probability back. Suppose the Moneyline odds for home team is -α and for away team is β.

The first step is to convert the Moneyline odds to decimal odds. A decimal odd like 1.7 means that if you invest 1 dollar, you will get an extra 0.7 dollar if you win. Let's use D_{home} to represent the decimal odds for home team and D_{away} to represent the decimal odds for away team.

$$D_{home} = 1 - \frac{100}{-\alpha} \qquad D_{away} = 1 + \frac{\beta}{100}$$

The second step is to calculate the profit percentage of the betting company. When calculating the odds, betting company will multiply the raw odds by a percentage γ, mostly around 0.95, to ensure that they have positive profit in the long run. Let P_{home} be the probability that the home team win and P_{away} be the probability that the away team win.

We know that

$$D_{home} = \frac{1}{P_{home}} * \gamma \qquad \text{and} \qquad D_{away} = \frac{1}{P_{away}} * \gamma$$

So,

$$\gamma = \frac{D_{home} * D_{away}}{D_{home} + D_{away}}$$

Then, we calculate the win probability as followed:

$$P_{home} = \frac{\gamma}{D_{home}} \qquad \text{and} \qquad P_{away} = \frac{\gamma}{D_{away}}$$

3.5 The simple no home defense model (SNHD)

In our basic model (SNHD), the team's possession time is modeled by gamma distribution. We will fit data into this distribution in the next chapter. The reward distribution is modeled by team's point per possession (PPP) and opponent point per possession (OPPP). Let's use AP as a random variable to represent the away team's point per possession distribution, AO as a random variable to represent the away team's opponent point per possession, HP as a random variable to represent the home team's PPP distribution and HO as a random variable to represent the home team's OPPP distribution. We take the average value of the away team's offensive power and the defensive power of the opponent of the away team. (This method is proposed by Conrad 2013) **[1]** Then the reward distribution of away team is

$$AR = \frac{AP + HO}{2}$$

The AR is the random variable that models the reward distribution for the away team, which is number of points scored by away team in this possession.

Similarly, we use the same model for the home team. The reward distribution of the home team is:

$$HR = \frac{HP + AO}{2}$$

In this model, when we are fitting the distribution of AP and HP, we consider a team at home and at away as two different teams. For example, if LAL is playing at away, we fit AP with only past game data where LAL is playing at away and exclude those data where LAL is playing at home. However, when fitting the AO and HO, we do not consider a team at home and at away as two different teams. For example, whether LAL is playing at home or at away, we use the same OPPP for them.

3.6 The simple home defense model (SHD)

In the simple no home defense model, we use separate PPP distribution for team at home and away. This model is similar to the simple no home defense model, but we are also using different OPPP for a single team at home and at away. Basically, in this model, we believe that team performs differently in defensive side when playing at home and at away. In Jason's research in 2014 **[10]**, the home court advantage indeed has an influence on team's defense. Thus, in this model, with other simulating parameters being the same, we use different OPPP distribution for team at home and away.

3.7 The complex model

Based on the simple home defense model, we do some further improvement in this model. In this model, we consider the possible dependent relationship between the time distribution (team's possession time) and the reward distribution (team's scoring distribution).

Considering the nature of basketball games, using the same PPP distribution for all possession time is not accurate enough. For example, if the possession time is less than 1 seconds, then the time for a complete play may not be enough, which makes the probability of scoring 0 in this possession much higher. As a result, we decide to use different scoring distributions for different time of possession. We divide the possession time into three categories: less than 2 seconds, between 2 and 8 seconds, and greater than 8 seconds. When the possession time is less 2 seconds, then the possession might be completed in a hurry and the expected points will be lower in this possession. When the possession time is between 2 and 8 seconds, the possession might be a fast break. In a fast break the expect points per possession will be higher. When the possession time is greater than 8 seconds, it tends to be a long possession and the scoring distribution is different. So, in this model, we use a different point per possession distribution as compared to models above.

Here is a more detailed explanation of how we model the reward distribution in this model. Note that AT represents away team's possession time and HT represents home team's possession time, and they are modeled by the gamma distribution.

$$
\begin{aligned}
AR = &\left(\frac{(AP|AT < 2) + (HO|AT < 2)}{2}\right) \\
&\times p(AT < 2) \\
&+ \left(\frac{(AP|2 \leq AT < 8) + (HO|AT < 2 \leq AT < 8)}{2}\right) \\
&\times p(2 \leq AT < 8) \\
&+ \left(\frac{(AP|AT > 8) + (HO|AT > 8)}{2}\right) \times p(AT > 8)
\end{aligned}
$$

$$
\begin{aligned}
HR = &\left(\frac{(HP|HT < 2) + (AO|HT < 2)}{2}\right) \\
&\times p(HT < 2) \\
&+ \left(\frac{(HP|2 \leq HT < 8) + (AO|HT < 2 \leq HT < 8)}{2}\right) \\
&\times p(2 \leq HT < 8) \\
&+ \left(\frac{(HP|HT > 8) + (AO|HT > 8)}{2}\right) \times p(HT > 8)
\end{aligned}
$$

3.8 The final model

The final model is built on the complex model above. In this model, we consider the situation especially for the 2020-2021 season.

2020-2021 NBA is a season heavily impacted by COVID-19. In regular season, due to the pandemic, the attends in home court tend to be very low, but in the play-off, the attends increase sharply. Then, our model cannot make an accurate prediction in this case. The table 6 is the average number of attends for each team in the regular season.

Table 6

Average attends for each team in the regular season

Team	Attends
Utah Jazz	4203

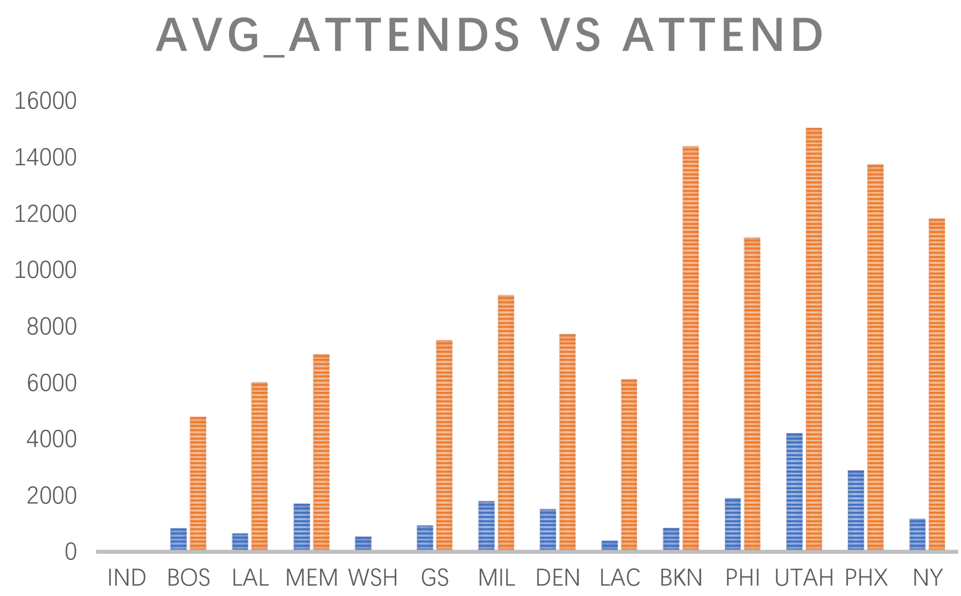
Orlando Magic	3513
Houston Rockets	3250
Phoenix Suns	2890
Dallas Mavericks	2635
...	...
Portland Trail Blazers	162
Oklahoma City Thunder	0
Sacramento Kings	0
Indiana Pacers	0
Miami Heat	0

However, in our test set games, the attends are generally very high. Thus, we need to modify the win probability based on the different influence of the home court advantage. The following is a table that compares the team's average attends in regular season and in the play-offs that we will predict.

Table 7

Teams' average attends in regular season vs the actual attends

AWAY	HOME	AVG_ATTEND	ATTEND
CHA	IND	0	0
WSH	BOS	835	4789
GS	LAL	647	6022
SA	MEM	1707	7019
IND	WSH	533	0
MEM	GS	929	7505
MIA	MIL	1799	9107
POR	DEN	1516	7732
DAL	LAC	386	6117
BOS	BKN	847	14391
WSH	PHI	1905	11160
MEM	UTAH	4208	15047
LAL	PHX	2890	13750
ATL	NY	1170	11824



It is not reasonable to simply add a fraction to team's win probability. Besides, it's hard to quantify the influence of home court advantage the PPP distribution and OPPP distribution. Therefore, we combine our model with ELO model to make the prediction. First, we use the win probability from the complex model to solve the implied ELO rating (latent strength). **[2]** Since we have ELO win probability formula as followed, we will solve the Team Rating Differential from our probability calculated by the Final model

$$Win\ Probability = \frac{1}{1 + 10^{-\frac{Team\ Rating\ Differential}{400}}}$$

Then, we add a modifier to the team's ELO rating based on the number of attends. If the number of attends is less than 3,000, we do not add the modifier. If the number of attends is greater than 3,000 but less than 10,000, we add a latent strength of 20. If the number of attends is greater than 10,000, we add a latent strength of 40. Then, we calculate the win probability based on the modified ELO value.

4. Fit the alternating renewal reward process

4.1 Fit the time distribution

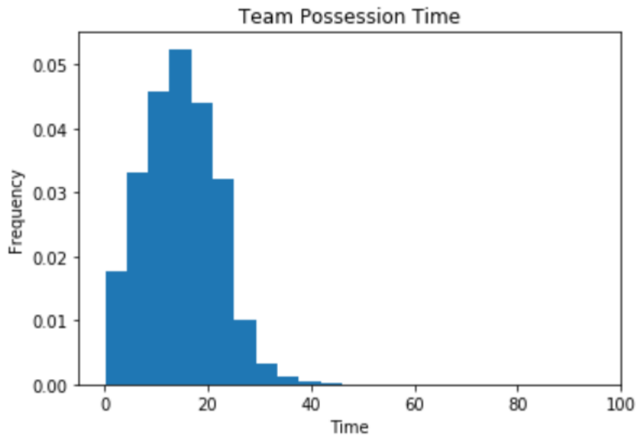
We will fit the time distribution for all NBA teams from our possession data.

Table 8

the average possession time for all teams

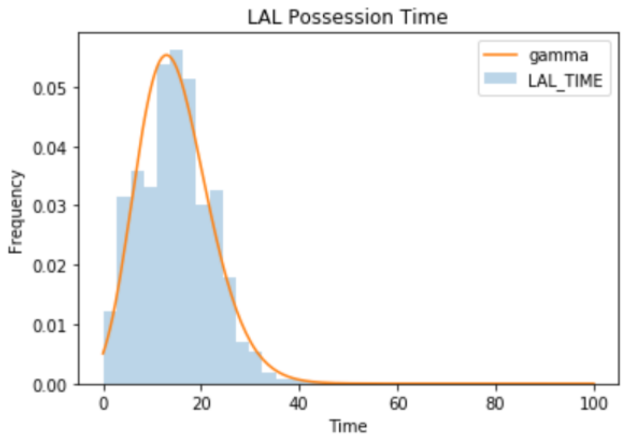
Team*	Time	Team*	Time
CLE	15.4813	UTAH	14.6462
NY	15.3649	PHI	14.6261

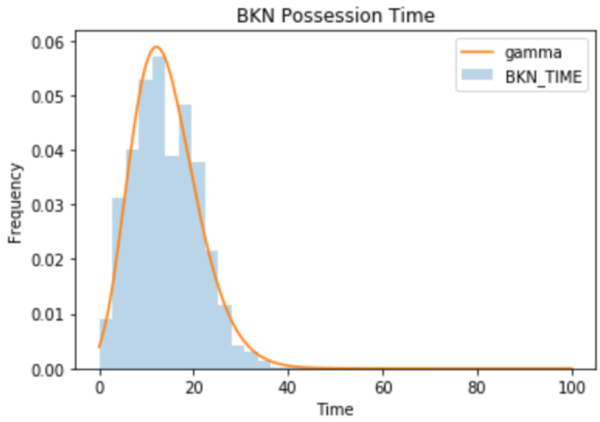
DET	15.2873	CHA	14.4653
ORL	15.1423	OKC	14.4254
DEN	15.1185	TOR	14.4147
DAL	15.0831	HOU	14.3202
POR	15.0428	SAC	14.2331
LAC	15.0278	NO	14.2203
ATL	14.9090	MIN	14.2134
MIA	14.8973	BKN	14.2117
BOS	14.7752	MEM	14.1182
SSA	14.7524	IND	13.7701
CHI	14.7457	GS	13.6376
PHX	14.7413	WSH	13.4691
LAL	14.7399	MIL	13.4197



We will model the possession time distribution by gamma distribution. We use the MLE (maximum likelihood estimator).

We take LAL and BKN for examples to check our model performance.





We can see from the graph that the gamma distribution is a pretty good fit for the time distribution.

4.2 Fit the reward distribution

We will also fit each team's reward distribution (scoring distribution) from our possession data by categorical distributions and estimate the distribution by frequency.

Let's take LAL at home for example. We count the number of possession that LAL scores 0 points, the number of possessions that LAL scores 1 points etc. Then, we normalize it and get the table 9.

Table 9

Probability distribution of points per possession for LAL

# of points	Probability
0	0.48932
1	0.042718
2	0.336477
3	0.130374
4	0.000832
5	0.000277

So, for LAL at home, the probability of scoring 0 in a certain possession is 0.489, the probability of scoring 2 in a certain possession is 0.336, and the probability of scoring 3 in a certain possession is 0.130.

We do similar thing on the defensive side. For example, if LAL is playing at home vs BKN, in a LAL's possession, we not only need LAL's scoring distribution, but also need the BKN's opponent point per possession distribution. We estimate it by using the OPPP data.

Table 10

Probability distribution of the opponent's points per possession for BKN

# of points	Probability
0	0.475589
1	0.032267
2	0.333614
3	0.156566
4	0.001684
5	0.000281

So, for BKN's opponent, the probability of scoring 0 is 0.476, the probability of scoring 2 is 0.334, and the probability of scoring 3 is 0.157.

By using these two distributions, we can fit the home team's reward distribution. We also use the similar approach to fit away team's reward distribution.

In the complex model, we fit the reward distribution in a similar way. We estimate the scoring distribution in each time interval with frequency.

5. Simulation and results

In this section, we use Monte Carlo simulation to simulate the alternating renewal reward process and estimate the win probability. We simulate these four models above. Since we use data from all 2020-2021 NBA regular season games, the play-off games will be a good test set for our model efficiency. So, we choose all the play-in tournaments and the first game of the play-off series to test our model. We did not choose other games in the play-off series since those games are heavily dependent on the former games which does not fit our model quite well. Since the win probability is not a deterministic number, we use latent strength model (ELO rating predictions) and Moneyline odds as the benchmarks. Our goal is to let our win probability be as closed as the Moneyline odds, since the Moneyline odds are generally viewed as the most accurate model. For each game, we run the simulation 2000 times (cannot simulate more due to the limitation of calculation power).

5.1 Simple no home defense model (SNHD)

Table 11

Simulation results of SNHD model

ELO	SNHD	ODDS	ELO_SQ_DIFF	SNHD_SQ_DIFF
0.7100	0.5220	0.5402	0.0288	0.0003
0.6400	0.6125	0.5450	0.0090	0.0046
0.6800	0.5855	0.6846	0.0000	0.0098
0.7500	0.5590	0.6270	0.0151	0.0046

0.6500	0.4505	0.5937	0.0032	0.0205
0.6200	0.5310	0.6450	0.0006	0.0130
0.7100	0.6710	0.6643	0.0021	0.0000
0.6600	0.5560	0.5120	0.0219	0.0019
0.7000	0.5905	0.6846	0.0002	0.0089
0.7500	0.5750	0.7650	0.0002	0.0361
0.7700	0.7150	0.7450	0.0006	0.0009
0.7600	0.6475	0.7510	0.0001	0.0107
0.7300	0.5965	0.5490	0.0328	0.0023
0.6800	0.5440	0.5198	0.0257	0.0006

MSE_ELO	MSE_SNHD
0.01003	0.00816

The column SNHD is our simulated probability. The column ELO and ODDS are our other two benchmark probabilities. The ELO_SQ_DIFF is the square difference between ELO and ODDS, and the SNHD_SQ_DIFF is the square difference between SNHD and ODDS. In general, we have a smaller MSE for the SNHD model compared with the ELO model.

5.2 Simple home defense model

Table 12

Simulation results of SHD model

AWAY	HOME	ELO	SNHD	SHD	ODDS
CHA	IND	0.71	0.522	0.538	0.5402
WSH	BOS	0.64	0.6125	0.631	0.545
GS	LAL	0.68	0.5855	0.582	0.6846
SA	MEM	0.75	0.559	0.5165	0.627
IND	WSH	0.65	0.4505	0.4655	0.5937
MEM	GS	0.62	0.531	0.525	0.645
MIA	MIL	0.71	0.671	0.642	0.6643
POR	DEN	0.66	0.556	0.5865	0.512
DAL	LAC	0.7	0.5905	0.569	0.6846
BOS	BKN	0.75	0.575	0.631	0.765
WSH	PHI	0.77	0.715	0.7075	0.745
MEM	UTAH	0.76	0.6475	0.706	0.751
LAL	PHX	0.73	0.5965	0.6295	0.549
ATL	NY	0.68	0.544	0.572	0.5198

MSE_ELO	MSE_SNHD	MSE_SHD
0.01003	0.00816	0.00793

The column SHD is our simulated probability. The column ELO and ODDS are our other two benchmark probabilities. The simple home defense model indeed decreases the MSE a little bit.

5.3 complex model

Table 13

Simulation results of complex model

AWAY	HOME	ELO	SNHD	SHD	COMPLEX	ODDS
CHA	IND	0.71	0.522	0.538	0.534	0.5402
WSH	BOS	0.64	0.6125	0.631	0.5825	0.545
GS	LAL	0.68	0.5855	0.582	0.585	0.6846
SA	MEM	0.75	0.559	0.5165	0.5145	0.627
IND	WSH	0.65	0.4505	0.4655	0.465	0.5937
MEM	GS	0.62	0.531	0.525	0.564	0.645
MIA	MIL	0.71	0.671	0.642	0.6525	0.6643
POR	DEN	0.66	0.556	0.5865	0.6165	0.512
DAL	LAC	0.7	0.5905	0.569	0.566	0.6846
BOS	BKN	0.75	0.575	0.631	0.646	0.765
WSH	PHI	0.77	0.715	0.7075	0.672	0.745
MEM	UTAH	0.76	0.6475	0.706	0.6865	0.751
LAL	PHX	0.73	0.5965	0.6295	0.5995	0.549
ATL	NY	0.68	0.544	0.572	0.5655	0.5198

MSE_ELO	MSE_SNHD	MSE_SHD	MSE_COMPLEX
0.01003	0.00816	0.00793	0.00718

The column complex is our simulated probability. The column ELO and ODDS are our other two benchmark probabilities. It turns out that the MSE decreases compared to the SHD model.

5.4 Final Model

Table 14

Simulation results of the final model

AWAY	HOME	ATTEND	COMPLEX	ELO_DIFF	MODIFIER	MODIFIED_ELO	FINAL
CHA	IND	0	0.5340	23.6621	0	23.6621	0.5340
WSH	BOS	4789	0.5825	57.8558	20	77.8558	0.6102
GS	LAL	6022	0.5850	59.6431	20	79.6431	0.6126
SA	MEM	7019	0.5145	10.0785	20	30.0785	0.5432
IND	WSH	0	0.4650	-24.3603	0	-24.3603	0.4650
MEM	GS	7505	0.5640	44.7170	20	64.7170	0.5921

MIA	MIL	9107	0.6525	109.4503	20	129.4503	0.6781
POR	DEN	7732	0.6165	82.4671	20	102.4671	0.6433
DAL	LAC	6117	0.5660	46.1307	20	66.1307	0.5940
BOS	BKN	14391	0.6460	104.4917	40	144.4917	0.6967
WSH	PHI	11160	0.6720	124.5982	40	164.5982	0.7206
MEM	UTA H	15047	0.6865	136.1612	40	176.1612	0.7338
LAL	PHX	13750	0.5995	70.0747	40	110.0747	0.6533
ATL	NY	11824	0.5655	45.7771	40	85.7771	0.6210

Using the model described in section 3.10. we calculated the win probability in table 14. From column COMPLEX, we solve the implied ELO ratings difference. Then, we adjust the ELO rating difference by a constant based on different attends. Finally, we solve the final win probability based on the modified ELO ratings. We will analyze and compare each model performance in more detail in the next chapter.

6. Model performance comparison

Table 14

simulated win probability from each model.

AWAY	HOME	ELO	SNHD	SHD	COMPLEX	FINAL	ODDS
CHA	IND	0.7100	0.5220	0.5380	0.5340	0.5340	0.5402
WSH	BOS	0.6400	0.6125	0.6310	0.5825	0.6102	0.5450
GS	LAL	0.6800	0.5855	0.5820	0.5850	0.6126	0.6846
SA	MEM	0.7500	0.5590	0.5165	0.5145	0.5432	0.6270
IND	WSH	0.6500	0.4505	0.4655	0.4650	0.4650	0.5937
MEM	GS	0.6200	0.5310	0.5250	0.5640	0.5921	0.6450
MIA	MIL	0.7100	0.6710	0.6420	0.6525	0.6781	0.6643
POR	DEN	0.6600	0.5560	0.5865	0.6165	0.6433	0.5120
DAL	LAC	0.7000	0.5905	0.5690	0.5660	0.5940	0.6846
BOS	BKN	0.7500	0.5750	0.6310	0.6460	0.6967	0.7650
WSH	PHI	0.7700	0.7150	0.7075	0.6720	0.7206	0.7450
MEM	UTAH	0.7600	0.6475	0.7060	0.6865	0.7338	0.7510
LAL	PHX	0.7300	0.5965	0.6295	0.5995	0.6533	0.5490
ATL	NY	0.6800	0.5440	0.5720	0.5655	0.6210	0.5198

The following is each model's MSE with respect to the ODDS.

MSE_ELO	MSE_SNHD	MSE_SHD	MSE_COMPLEX	MSE_FINAL
0.01003	0.00816	0.00793	0.00718	0.00630

Clearly, as the model improves, the MSE decreases. The sharpest decreases are from SHD to Complex model and Complex model to the Final model. The complex model captures the dependent relationship between the possession time and scoring distributions. It is not surprising that complex model can increase the model performance, since it is the nature of basketball games. The improvement made by the Final model is also reasonable, since we consider the influence of pandemic on the home court advantages. Although we see a clear decrease trend of MSE here, we cannot guarantee that our Final model is superior, since we are not able to calculate the confidence interval due to the limit in calculation power. Further, in order to evaluate our model more detailly, we need to check and explain the abnormality in the win probability prediction.

One key abnormality comes from the 5th game: IND vs WSH

AWAY	HOME	ELO	SNHD	SHD	COMPLEX	FINAL	ODDS
IND	WSH	0.6500	0.4505	0.4655	0.4650	0.4650	0.5937

In this game, all our model predictions seem to be failed. To explain this abnormality, we need to check the WSH's performance across the season. We observe that in the first half of the season, WSH has winning percentage of 38.9% while in the second half of the quarter, WSH has winning percentage of 55.6%. Even more surprisingly, WSH won 17 games in its first 48 games, while it also won 17 games in its last 24 games, which means that WSH has win winning percentage of 35.4% in the first 48 games but 70.8 in the last 24 games. Considered the exceptional performance of WSH in its last 24 games, it is hard not to expect WSH to have a higher win probability when playing with IND. However, our model cannot capture this abnormality since our model weighs each game the same without considering the time. Considering this limitation, we will work on this problem by including a factor decreasing with time in the future. Other prediction results are not that surprising. So, in general, our models reach a pretty high prediction quality. We achieve a MSE of 0.00630 by the Final model in contrast with 0.01003 by the ELO model.

7. Model Limitations

There are three major limitations of our model. The biggest issue in this model is that the model does not capture the dependence relationship between possessions. The alternating renewal reward process assumes that possessions are independent from each other, but it is not. Imagine that the home team just misses a shot, and the away team quickly gets the defensive rebound. It is very likely that the away team will start a fast break. In the fast break, the away team has a higher probability of scoring some points in the possession. Our model cannot capture this situation. In order to address this problem, we can turn to Shirley's work (2007) or Strumbelj's work (2012). They use transitional probability matrix to predict the win probability, but the lack of data is another limitation. The second issue is that our model cannot take injuries and substitution into account. Injuries and substitutions are major part of the basketball games. If one of the major players of a team cannot play in a game, the win probability prediction will vary a lot. The third limitation is that when the sample data amount is small, our model cannot make a reliable prediction. Suppose in the beginning of a season, a team only play one game. It is not reasonable to predict the second game of the team based only on the result from the first game. We may need to combine with other model, like ELO, to make a more reliable prediction when the data amount is small.

8. Conclusion

Based on Conrad's work, we develop a more complex model using the alternating renewal reward process. Our model captures the dependent relationship between the possession time and the scoring distribution and combines with ELO rating system to make special adjustment to the predictions under the situation of pandemic. The prediction results are better than the pure ELO forecast, but there are still some abnormalities that can be further improved.

[1] Peuter, C. D. *Modeling Basketball Games as Alternating Renewal-Reward Processes and Predicting Match Outcomes*. Diss. Master's thesis, Duke University, 2013.

[2] Elo, A. E. (1978). *The Rating of Chessplayers, Past and Present*. New York: Arco Pub.. ISBN: 0668047216 9780668047210

[3] Shirley, Kenny. "A Markov model for basketball." *New England Symposium for Statistics in Sports*. 2007.

[4] Štrumbelj, Erik, and Petar Vračar. "Simulating a basketball match with a homogeneous Markov model and forecasting the outcome." *International Journal of Forecasting* 28.2 (2012): 532-542.

[5] Song, Kai, Yiran Gao, and Jian Shi. "Making real-time predictions for NBA basketball games by combining the historical data and bookmaker's betting line." *Physica A: Statistical Mechanics and its Applications* 547 (2020): 124411.

[6] Kayhan, Varol Onur, and Alison Watkins. "A data snapshot approach for making real-time predictions in basketball." *Big data* 6.2 (2018): 96-112.

[7] Deokar, Viraj. "Win Probability Modelling in the National Basketball Association." (2020).

[8] Kotecki, Jason. "Estimating the effect of home court advantage on wins in the NBA." (2014).