6.6 Three-dimensional transformation

- Three-dimensional translation,
- Rotation,
- Scaling,
- Reflection,
- Shear transformation,
- 3D composite transformation,
- 3D viewing pipeline,
- Projection concepts (Orthographic, parallel, perspective projection)

Three dimensional graphics:

- Three-dimensional space is a geometric 3-parameters model of the physical universe (without considering time) in which all known matter exists.
- These three dimensions can be labeled by a combination of length, breadth, and depth. Any three directions can be chosen, provided that they do not all lie in the same plane.

3D Transformation

- Three dimensional geometric transformations are extended from two dimensional methods by including considerations for z coordinate.
- Like two dimensional transformations, these transformations are formed by composing the basic transformations of translation, scaling, and rotation.
- Each of these transformations can be represented as a matrix transformation with homogeneous coordinates.
- Therefore, any sequence of transformations can be represented as a single matrix, formed by combining the matrices for the individual transformations in the sequence.

1. Translation:

- Translation in 3D is similar to translation in the 2D except that there is one more direction parallel to the z-axis. If, tx, ty, and tz are used to represent the translation vectors.
- Then the translation of the position P(x, y, z) into the point P' (x', y', z') is done by:

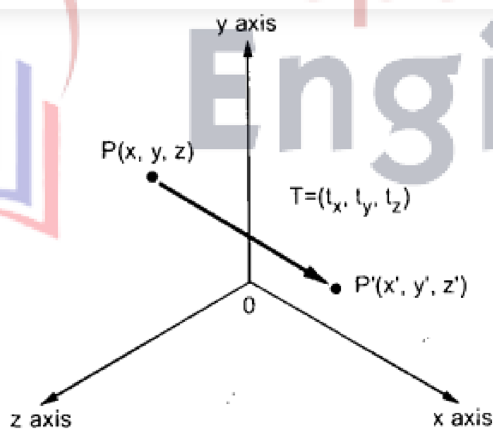
$$x' = x + t_x, \quad y' = y + t_y, \quad z' = z + t_z$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

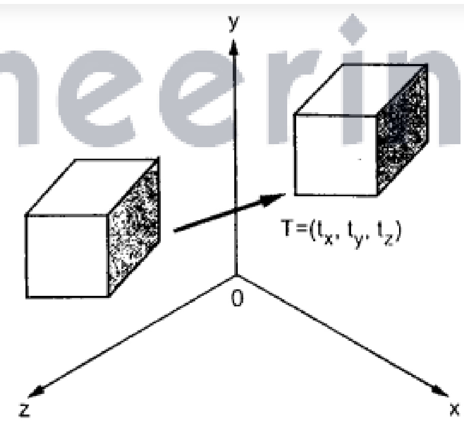
$$\therefore \quad P' = P \cdot T$$

$$\therefore \quad [x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$= [x + t_x \quad y + t_y \quad z + t_z \quad 1]$$



(a) Translating point



(b) Translating object

Scaling

- Three dimensional transformation matrix for scaling with homogenous coordinates is as given below.

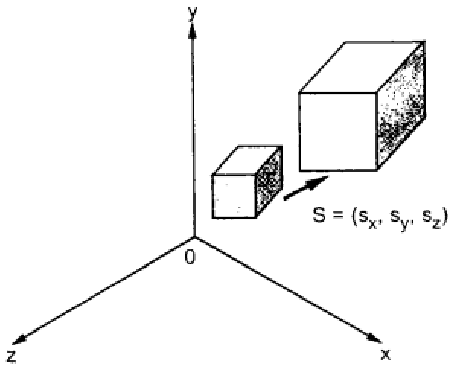


Fig: 3D Scaling

- It specifies three coordinates with their own scaling factor.

$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P' = P \cdot S$$

$$\therefore \; [x' \;\; y' \;\; z' \;\; 1] = [x \;\; y \;\; z \;\; 1] \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \cdot S_x & y \cdot S_y & z \cdot S_z & 1 \end{bmatrix}$$

- A scaling of an object with respect to a selected fixed position can be represented with the following transformation sequence.
  1. Translate the fixed point to the origin.
  2. Scale the object
  3. Translate the fixed point back to its original position.

Rotation

- For three dimensional rotation, we have to specify an axis of rotation about which the object is to be rotated along with the angle of rotation.
- The easiest rotation axes to handle are those that are parallel to the coordinate axes.
- It is possible to combine the coordinate axis rotations to specify any general rotation

Coordinate Axis Rotation

Three dimensional transformation matrix for each coordinate axes rotations with homogeneous coordinate are as given below

$$R_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a)

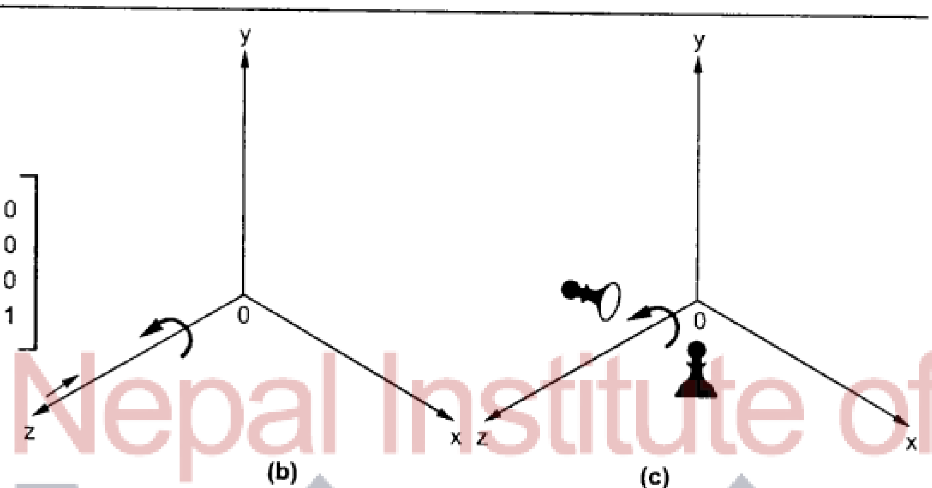


Fig: Rotation about z axis

- The positive value of angle θ indicates counterclockwise rotation. For clockwise rotation value of angle θ is negative.

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a)

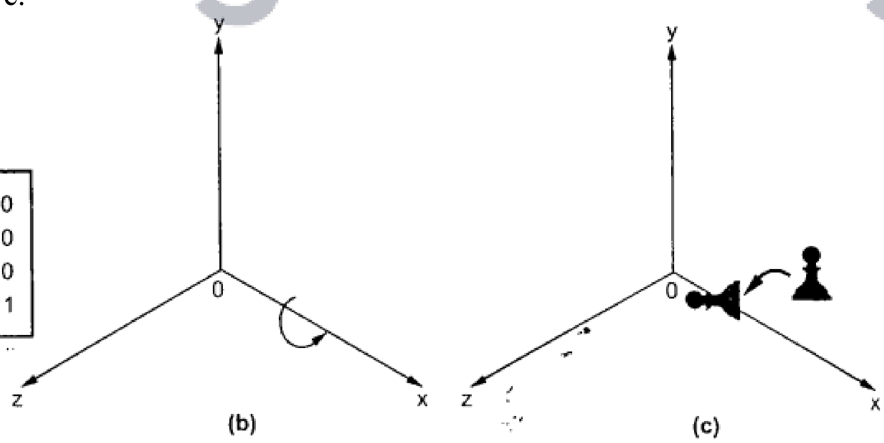


Fig: Rotation about x axis

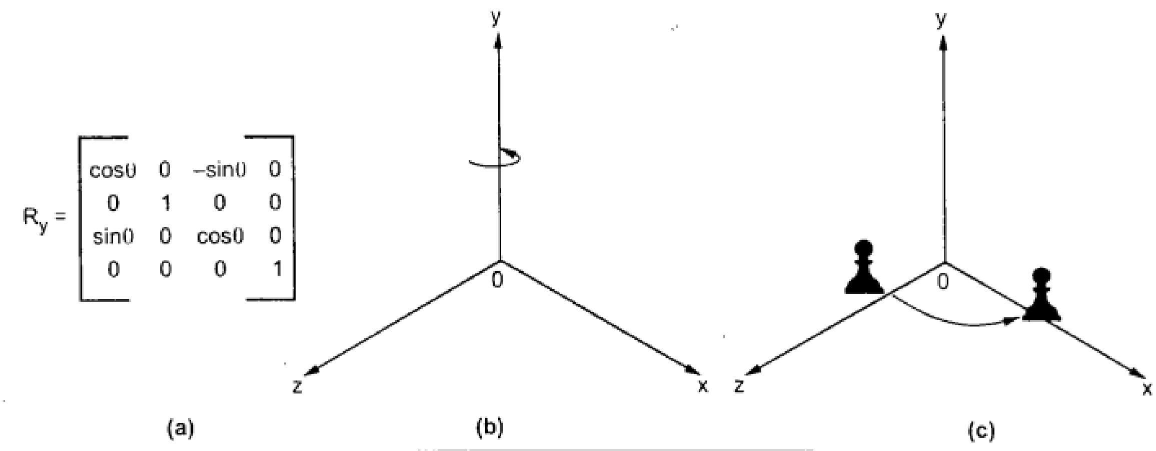


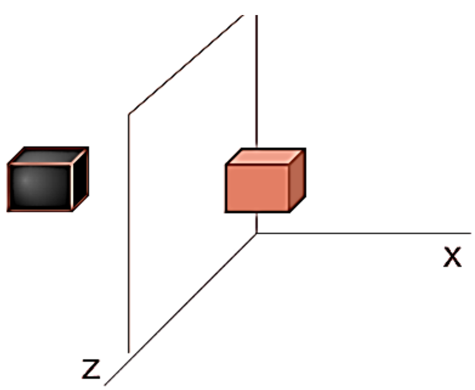
Fig: Rotation about y axis

Reflection:

- A three-dimensional reflection can be performed relative to selected reflection axis or with respect to a selected reflection plane.
- The three dimensional reflection matrices are set up similarly to those for two dimensions.

i) Reflection about yz plane

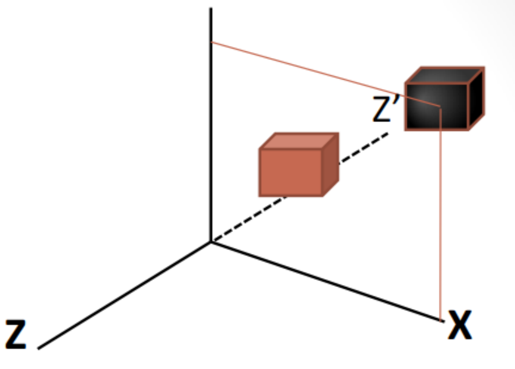
X'=-X
Y'=Y
Z'=Z



$$T_{yz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii) Reflection about XY plane

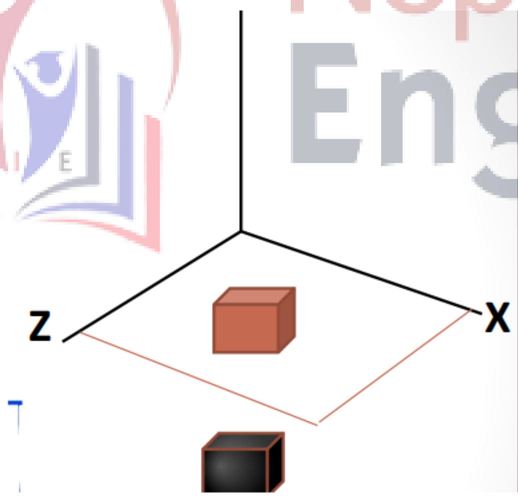
X'=X
Y'=Y
Z'=-Z



$$T_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii) **Reflection about XZ plane**

X' = X
Y' = -Y
Z' = Z



$$T_{xz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Shearing:

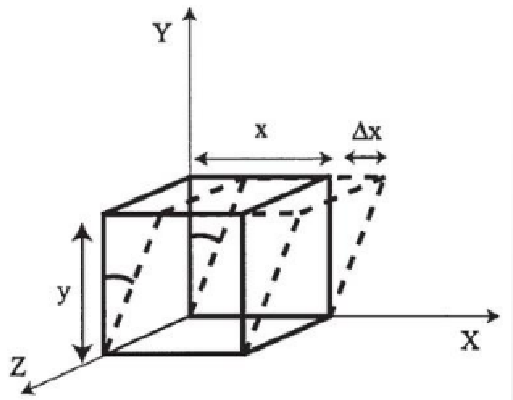
- Shearing transformations can be used to modify object shapes.

Z- axis Shear:

- This transformation alters x and y coordinate values by an amount that is proportional to z value while leaving z coordinate unchanged i.e**,**

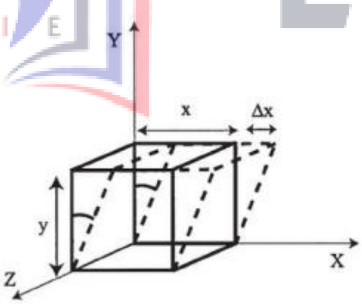
$$x' = x + S_{hx} . z$$
$$y' = y + S_{hy} . z$$
$$z' = z$$

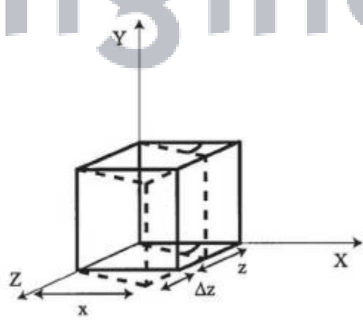
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & S_{hx} & 0 \\ 0 & 1 & S_{hy} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

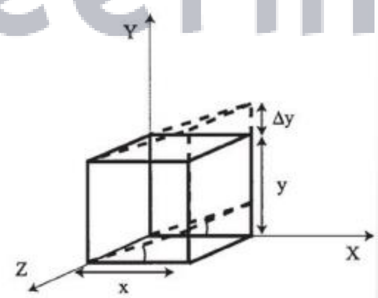


Similarly, we can find X-axis shear and Y-axis shear.

$$SH_z = \begin{bmatrix} 1 & 0 & S_{hx} & 0 \\ 0 & 1 & S_{hy} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad SH_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ S_{hx} & 1 & 0 & 0 \\ S_{hx} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad SH_y = \begin{bmatrix} 1 & S_{hx} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & S_{hx} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$







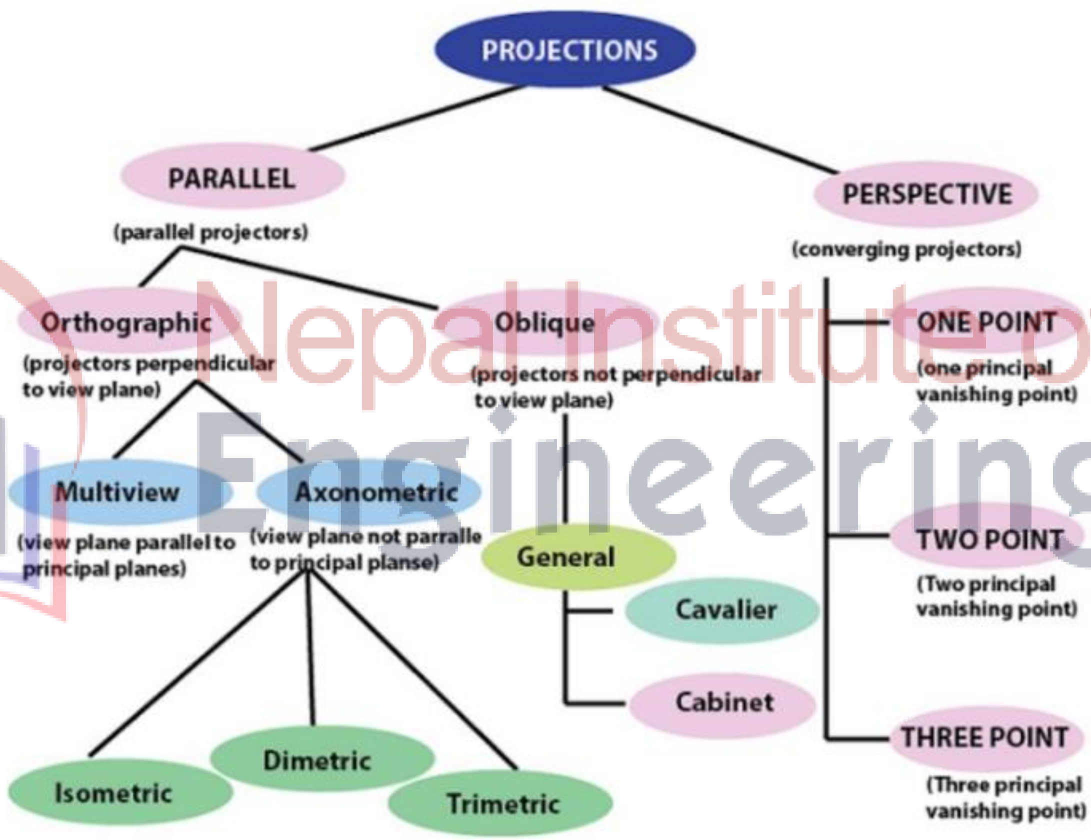
Projection:

• Transformation that changes a point in n-dimensional coordinate system into a point in a coordinate system that has dimension less than n.

• Converts 3-D viewing co-ordinates to 2-D projection coordinates

• View Plane or Projection Plane: Two dimensional plane in which 3D objects are projected is called the view plane or projection plane. Simply it is a display plane on an output device.

Types of Projection

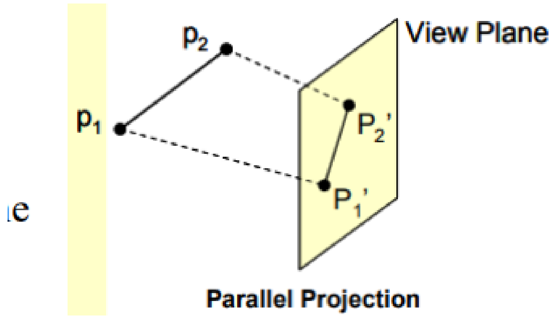


Parallel Projection:

• Coordinate positions are transformed to view plane along parallel lines (projection lines)

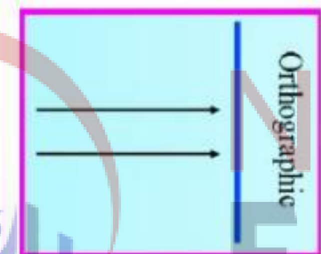
• Preserves relative proportions of objects

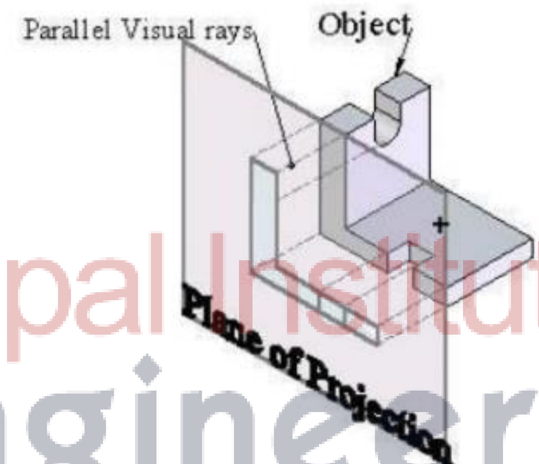
• Accurate views of various sides of an object are obtained.



Orthographic Parallel Projection:

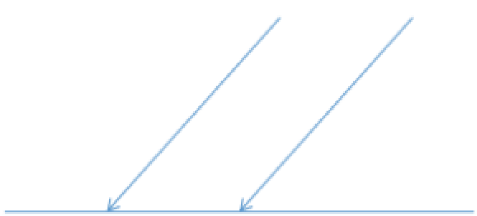
- When projection is perpendicular to view plane then it is called orthographic parallel projection.

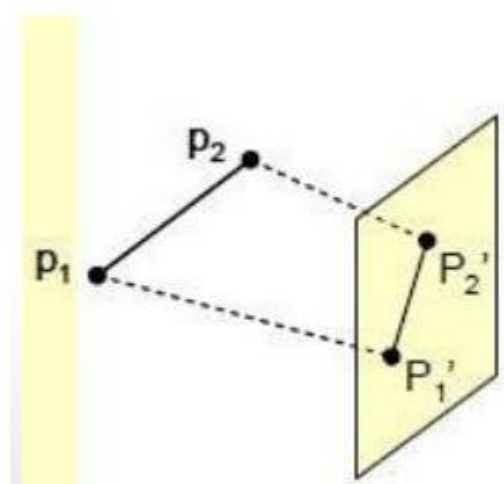




Oblique Parallel Projection:

- Projectors (projection vectors) are not perpendicular to the projection plane.
- It preserves 3D nature of an object.





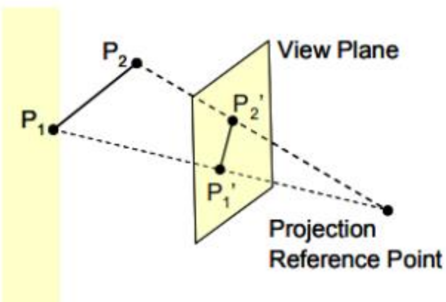
Perspective Projection:

• Coordinate positions are transformed to view plane along lines (projection lines) that converges to a point called projection reference point (center of projection)

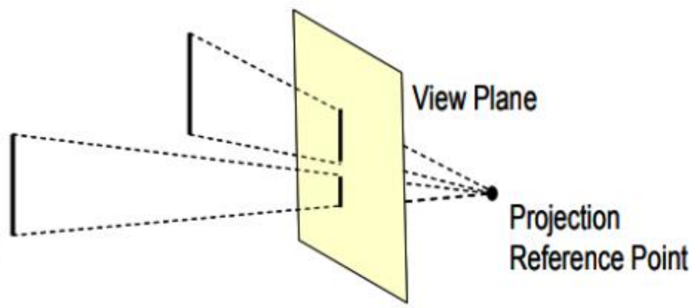
• Produce realistic view

• Does not preserve relative proportions

• Equal sized object appears in different size according as distance from view plane.



Perspective Projection



Perspective Projection of equal sized objects at different distances from the view plane

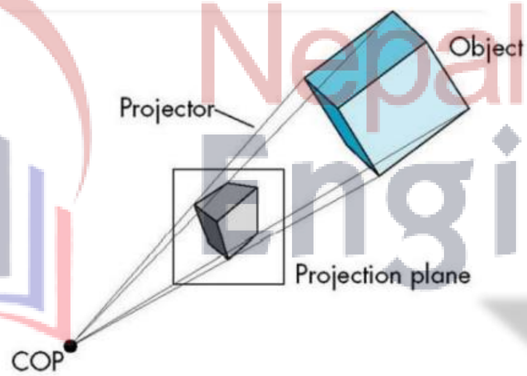


Figure. Perspective View

Vanishing Point:

- A set of parallel lines that are not parallel to the view plane pare projected as converging lines that appear to converge at a point called vanishing point.
- A set of parallel lines that are parallel to view plane are projected as parallel lines.
- More than one set of parallel lines form more than one vanishing point in the scene.