Nepal Engineering Council Registration Examination Preparation Class

Computer Engineering

Theorems

Significance of Network Theorems

- Network theorems are fundamental principles in electrical engineering and circuit analysis that help simplify and analyze complex electrical networks. These theorems provide mathematical tools and techniques to determine various circuit parameters, such as current, voltage, power, and resistance.

Superposition Theorem:

Statement :

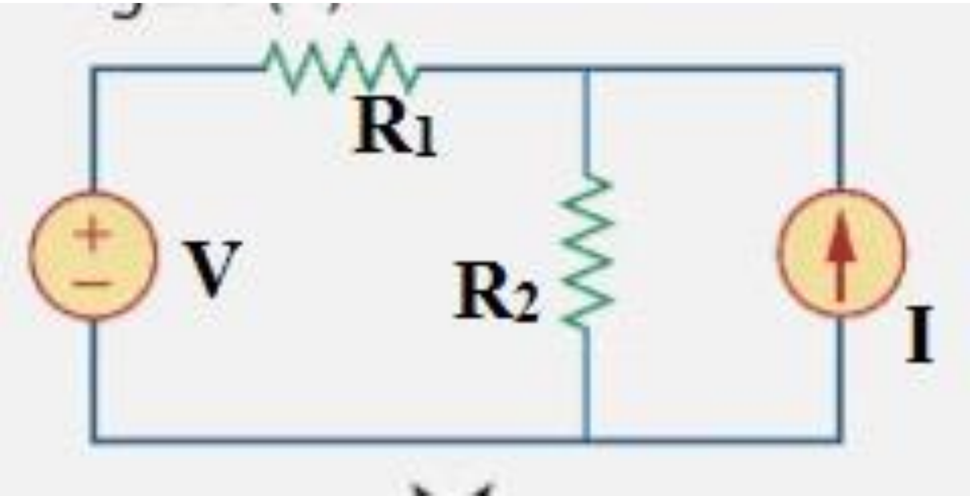
When a number of voltage and current sources are acting in a linear network simultaneously, the resultant current in any branch of the circuit is the algebraic sum of current flowing through it taking one source at a time while deactivating the other sources.

The superposition theorem allows complex circuits to be analyzed by breaking them down into simpler parts.

By analyzing each input signal separately,

engineers can focus on one input at a time

and simplify the circuit to a more manageabl.



Steps to Analyse Superposition Theorem

1. Consider the various independent sources in a given circuit.

2. Select and retain one of the independent sources and replace all other sources with their internal resistances or else replace the current sources with open circuits and voltage sources with short circuits.

3. To avoid confusion re-label the voltage and current notations suitably.

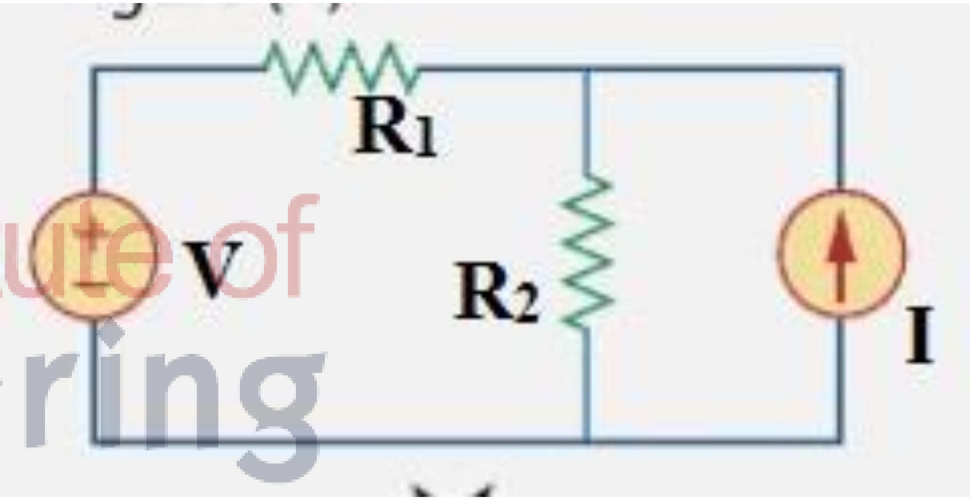
4. Find out the desired voltage/currents due to the one source acting alone using various circuit reduction techniques (equivalent resistance/ mesh analysis/ nodal analysis).

5. Repeat the steps 2 to 4 for each independent source in the given circuit.

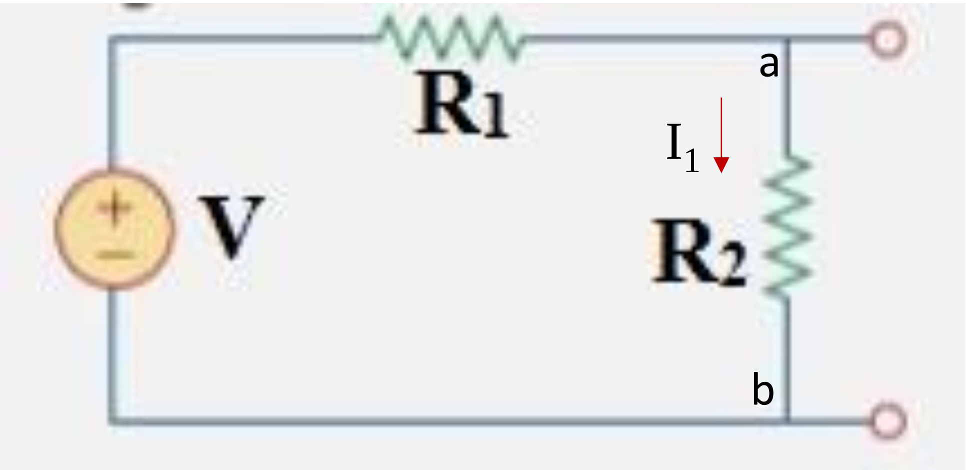
6. Algebraically add all the voltages/currents that are obtained from each individual source (Consider the voltage signs and current directions while adding).

Explanation :

Let u consider a circuit as shown in figure. Let current flowing through R_2 is to be determine using superposition theorem, then



Considering Voltage source alone, replacing Current source by open circuit, Current through R_2 due to Voltage source V acting alone is

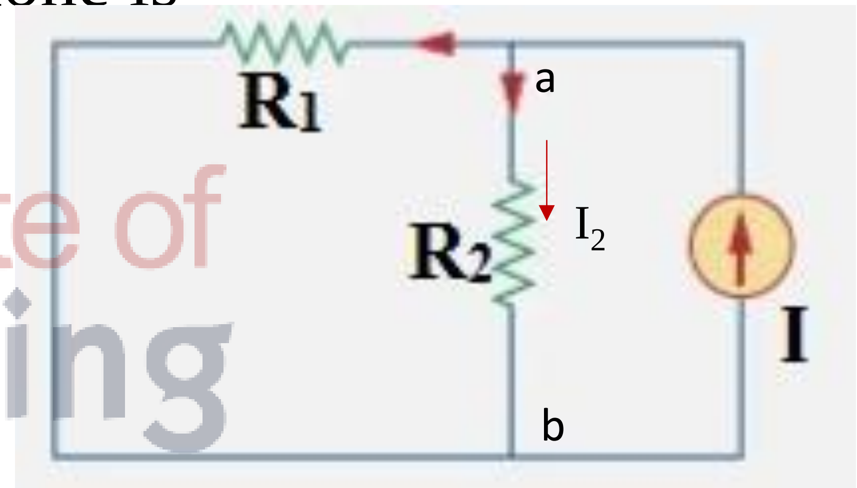


$I_1 = V/(R_1+R_2)$ (a to b)

Considering Current source alone, replacing Voltage source by Short circuit,

Current through R_2 due to current source I acting alone is

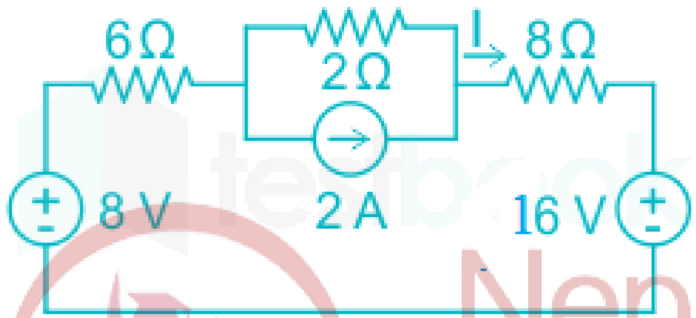
$I_2 = I*R_1/(R_1 + R_2)$ (a to b)



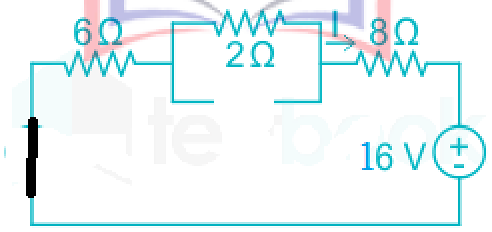
Net current through R_2 is,

$I = I_1 + I_2$

The I in the following circuit is

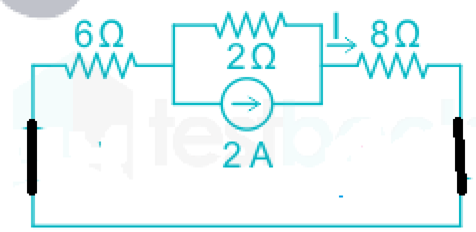


Considerinf 16 V alone



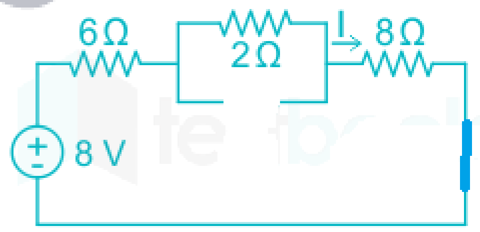
I1=-1A

Considering 2A alone



I2=0.25 A

Considering 8V alone

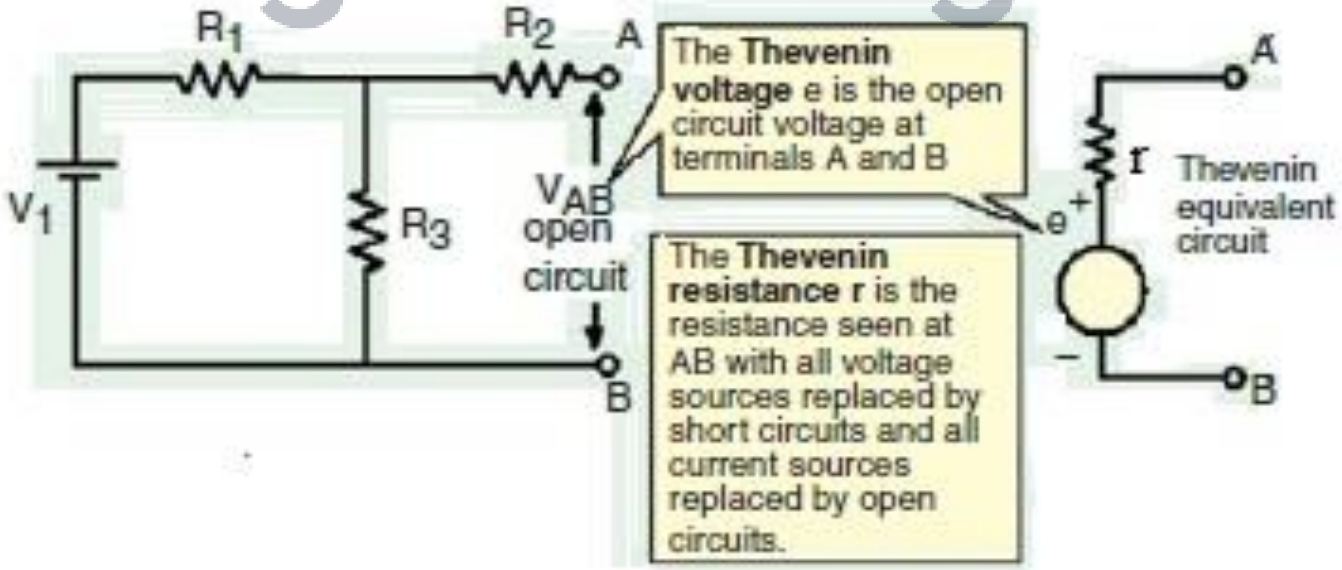


I3=0.5A

I=I1+I2+I3 = -1+0.25+0.5 = -0.25A

Thevenin's Theorem

- Any linear active bilateral network can be replaced by an equivalent circuit consisting of a voltage Source in series with a resistance. The voltage source is an open circuit voltage across the open circuited load terminals and the resistance being the internal resistance of the source is the equivalent resistance of the network looking from the open circuited load terminals.



Steps for Solving Thevenin’s Theorem

Step 1 – First of all remove the load resistance $\mathbf{r_L}$ of the given circuit.

Step 2 – Determine Open terminal voltage (i.e. Voltage across open circuited load terminals) , by applying any of the network simplification technique. This will be V_{th} .

Step 3 – Find the equivalent resistance at the load terminals by replacing all the sources by their internal resistance (If sources are ideal then short circuit the voltage source and open circuit the current source). This will be R_{th}.

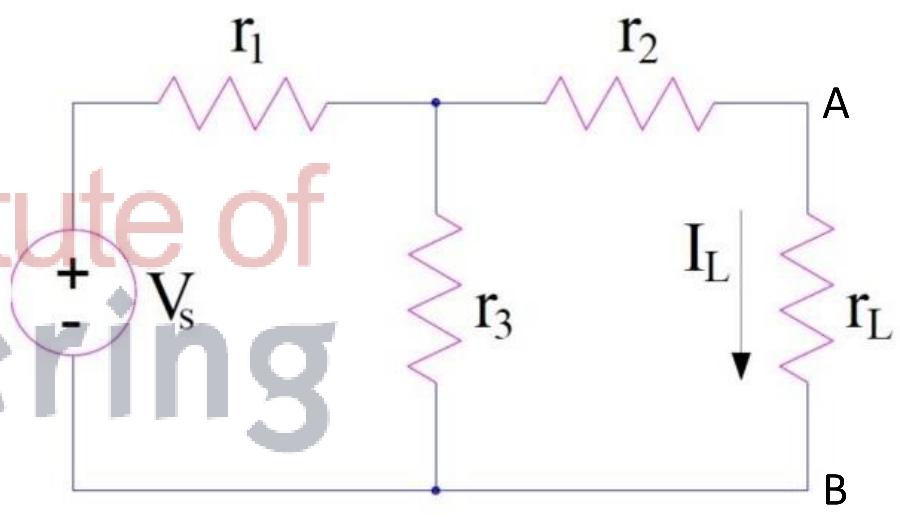
Step 4 - Draw the Thevenin’s equivalent circuit with source V_{th} , equivalent resistance R_{th} in series with V_{th} , across terminals of branch of interest.

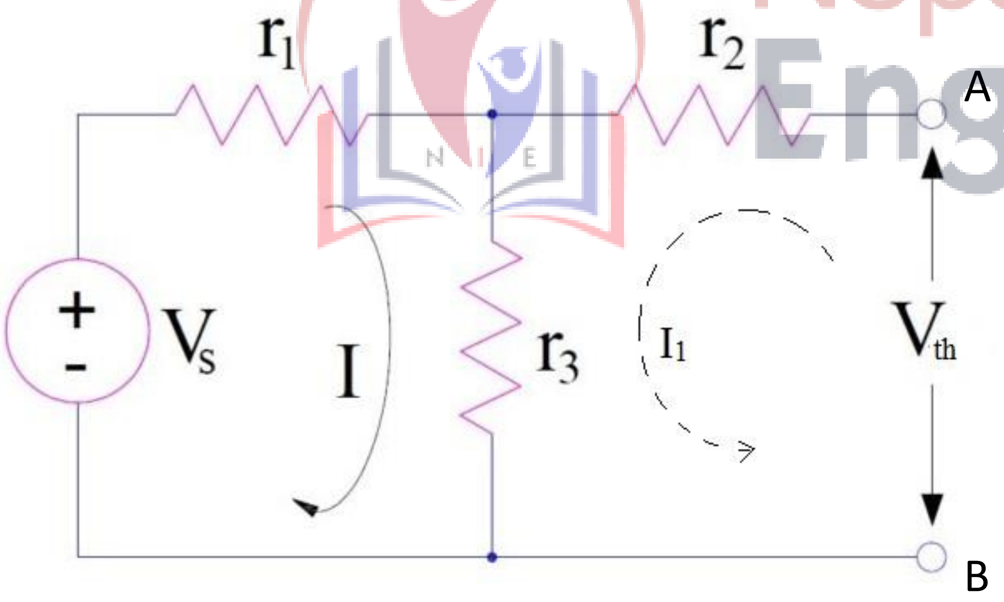
Step 5 – Draw the Thevenin’s equivalent circuit by connecting the load resistance and after that determine the desired response.

Explanation :

Let us consider a circuit as shown in figure. Let current flowing through r_L is to be determine using Thevenin's theorem, then

Remove r_L and find the open circuited voltage across open circuited load terminals.





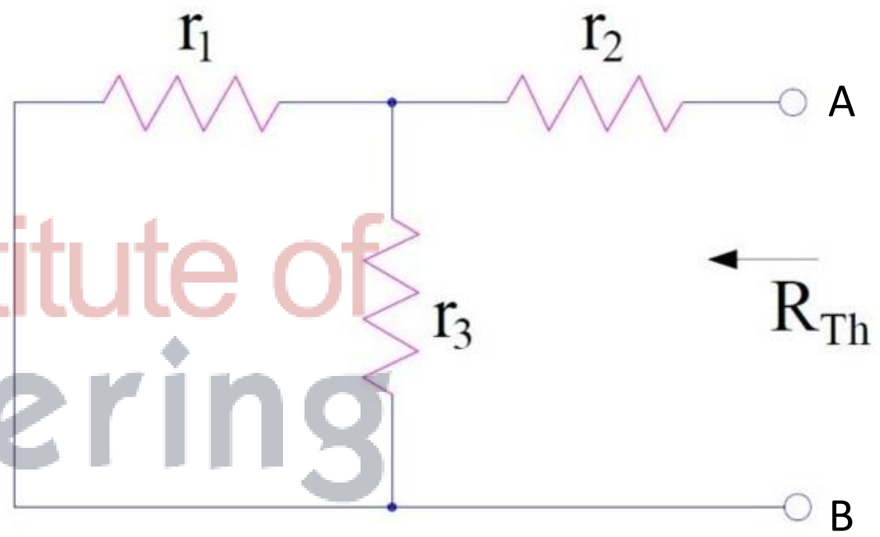
$$I = \frac{Vs}{r1+r3}$$

Applying KVL, $V_{th} - Ir_3 = 0$

$V_{th} = Ir_3$

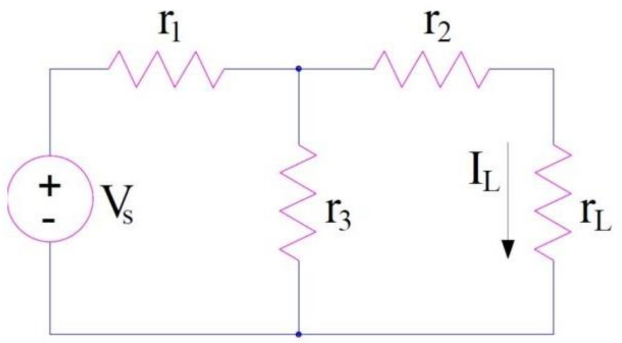
Now to find the equivalent resistance at the load terminals, replacing all the sources by their internal resistance i.e. short circuit the voltage source.

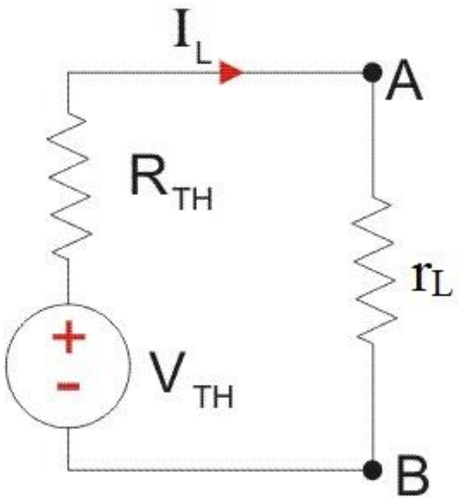
$$R_{Th} = r_2 + \frac{r1 * r3}{r1 + r3}$$



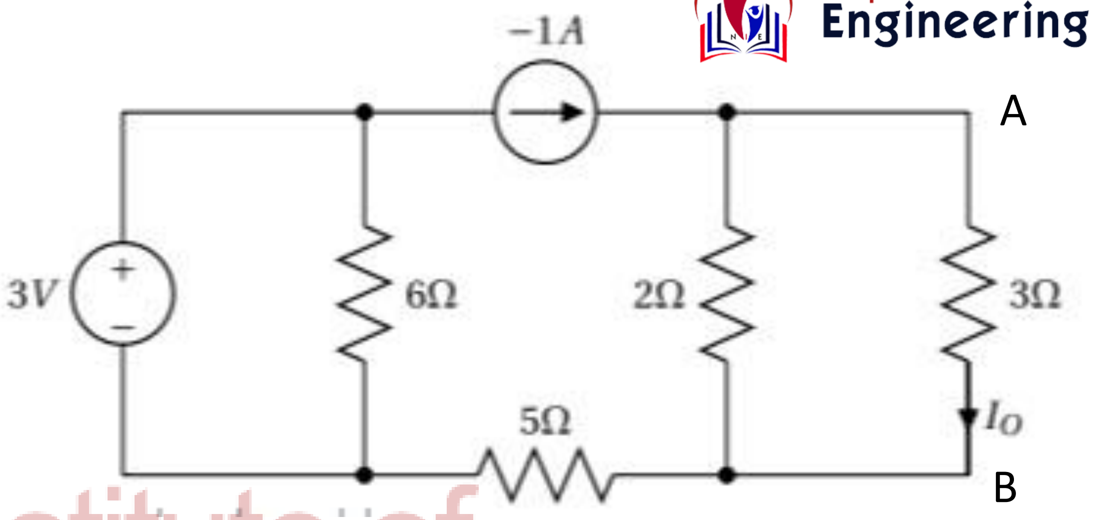
Draw the Thevenin's equivalent circuit with source V_{th} , equivalent resistance R_{th} in series with V_{th} and connect the load resistance.

$$I_L = \frac{VTh}{RTh + rL}$$





- Find I_o using Thevenin's Theorem.



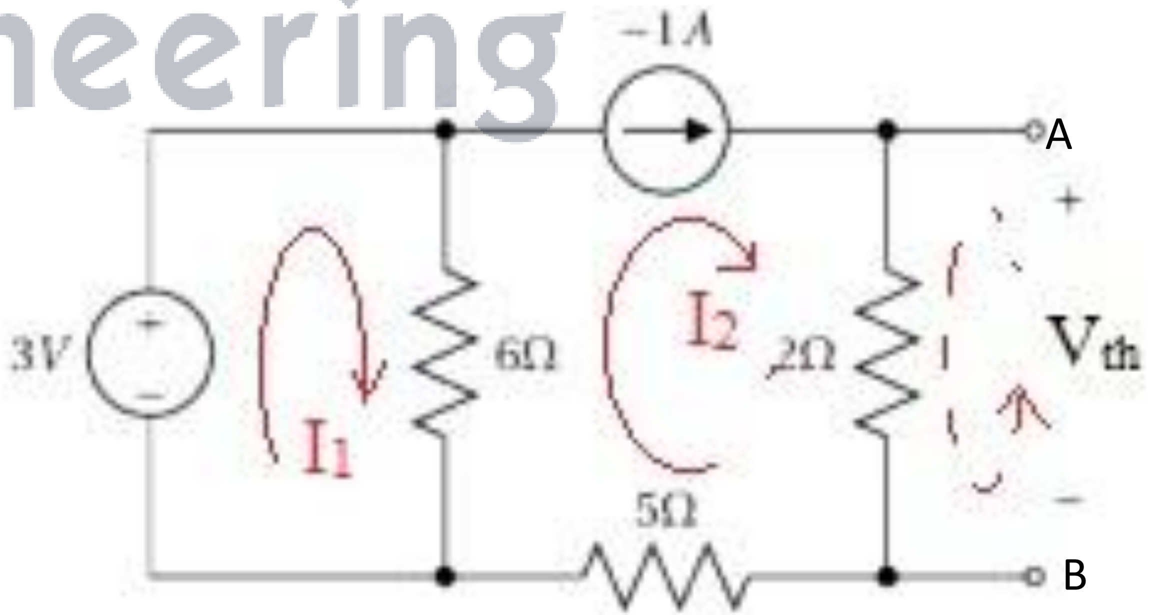
Remove $R_L = 3\Omega$ and find the open circuited voltage across open circuited load terminals. This will be V_{th}

From mesh 2, $I_2 = -1A$

Applying KVL for V_{th} ,

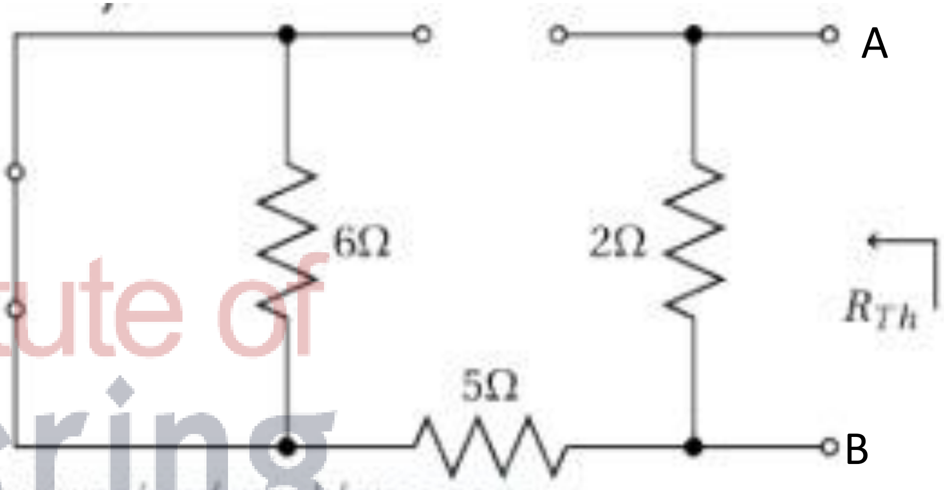
$V_{th} - 2I_2 = 0$

$V_{th} = 2I_2 = 2*(-1) = -2V$



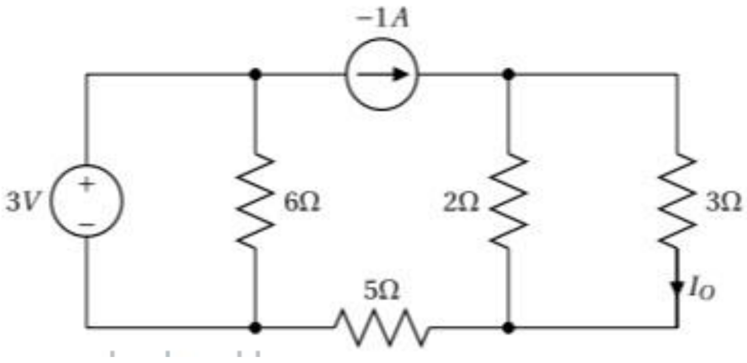
- Now find the equivalent resistance at the load terminals by replacing all the sources by their internal resistance (i.e. short circuit the voltage source and open circuit the current source).

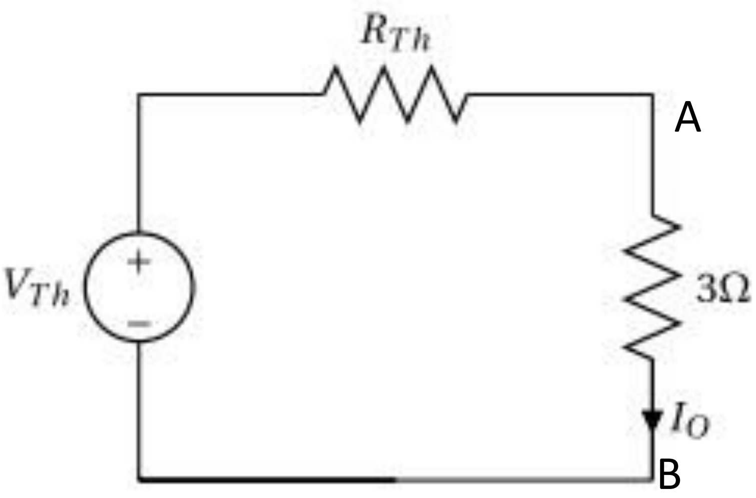
$R_{th} = 2\Omega$



Draw the Thevenin's equivalent circuit with source V_{th} , equivalent resistance R_{th} in series with V_{th} and connect the load resistance.

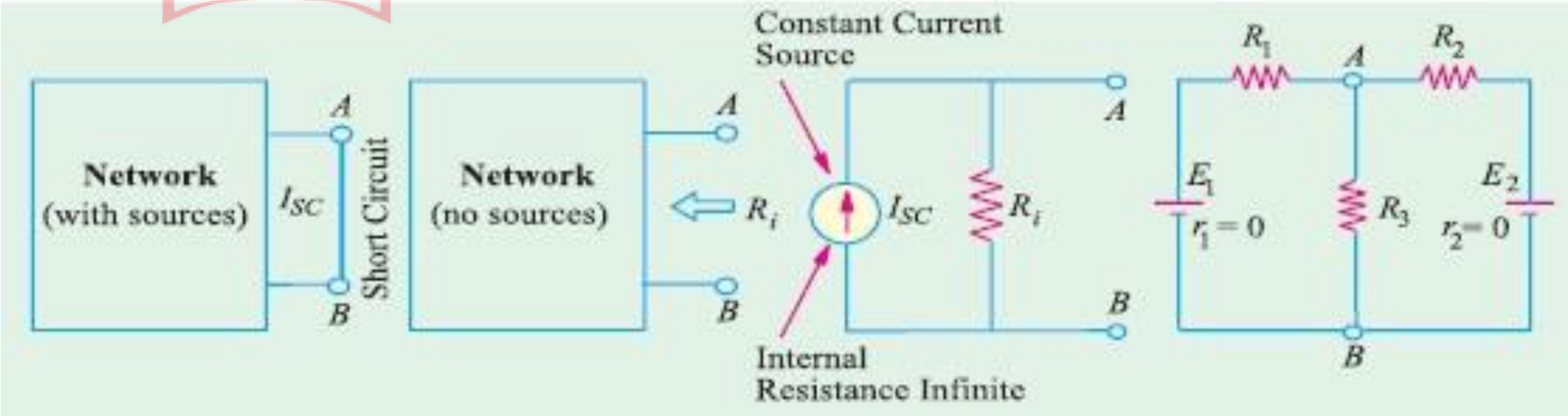
$I_0 = \frac{VTh}{RTh+RL} = \frac{-2}{2+3} = \frac{-2}{5}A$





Norton's Theorem

- Any linear active bilateral network can be replaced by an equivalent circuit consisting of a Current Source in Parallel with a resistance. The Current source is a short circuit Current through the short circuited load terminals and the resistance being the internal resistance of the source is the equivalent resistance of the network looking from the open circuited load terminals.



Steps for Solving Norton's Theorem

Step 1 – First of all remove the load resistance $\mathbf{r_L}$ of the given circuit.

Step 2 – Determine short circuit load currrent, by applying any of the network simplification technique. This will be I_N .

Step 3 – Find the equivalent resistance at the load terminals by replacing all the sources by their internal resistance (If sources are ideal then short circuit the voltage source and open circuit the current source). This will be R_N.

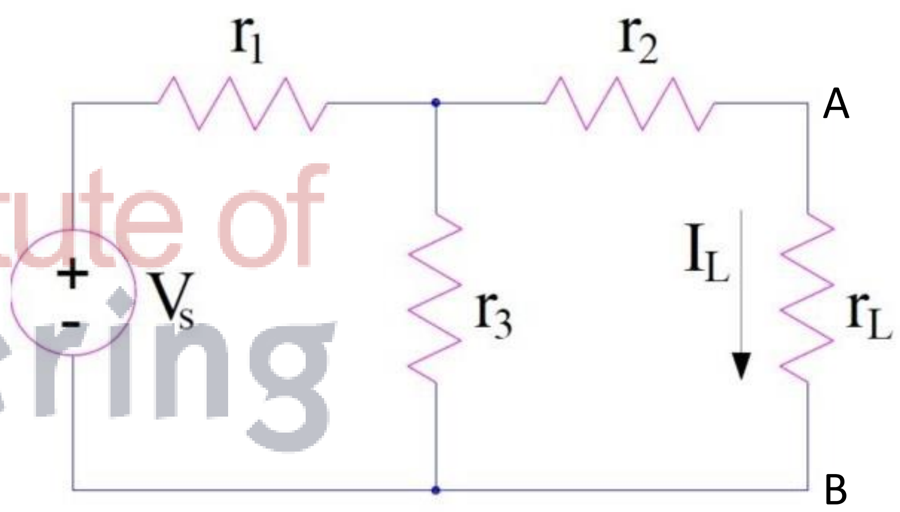
Step 4 - Draw the Norton's equivalent circuit with source I_N , equivalent resistance R_N in parallel with I_N, across terminals of branch of interest.

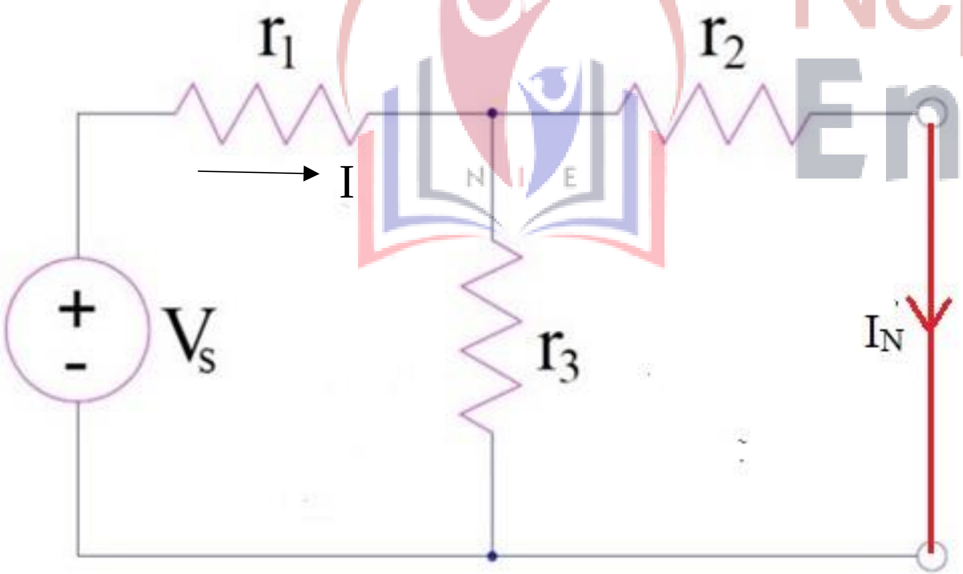
Step 5 – Draw the Norton's equivalent circuit by connecting the load resistance and after that determine the desired response.

Explanation :

Let us consider a circuit as shown in figure. Let current flowing through r_L is to be determine using Norton's theorem, then

Remove r_L and find the short circuited current through short circuited load terminals.





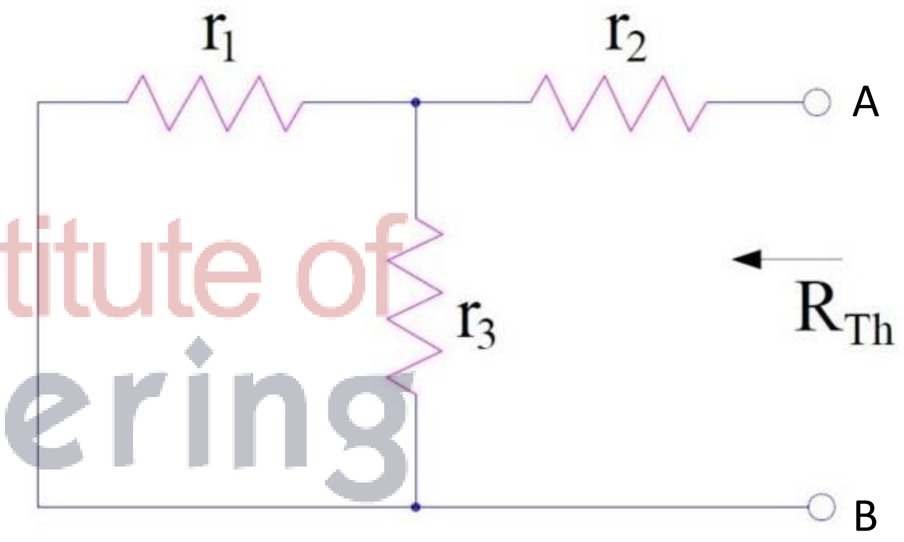
$$R_{eq} = \frac{r2*r3}{r2+r3} + r_1$$

$$I = \frac{Vs}{Req}$$

$$I_N = \frac{I*r3}{r2+r3}$$

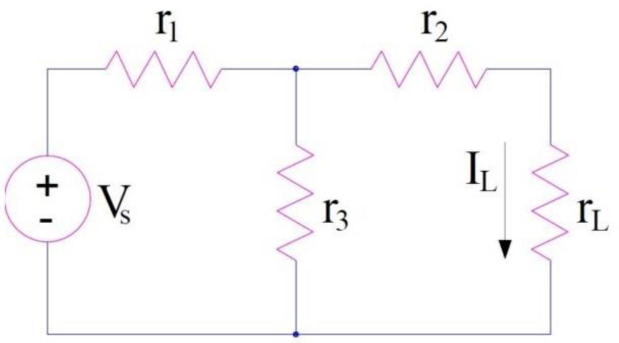
Now to find the equivalent resistance at the load terminals, replacing all the sources by their internal resistance i.e. short circuit the voltage source.

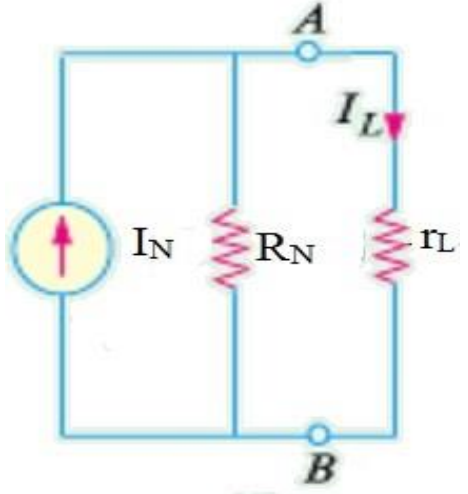
$$R_N = r_2 + \frac{r1 * r3}{r1 + r3}$$



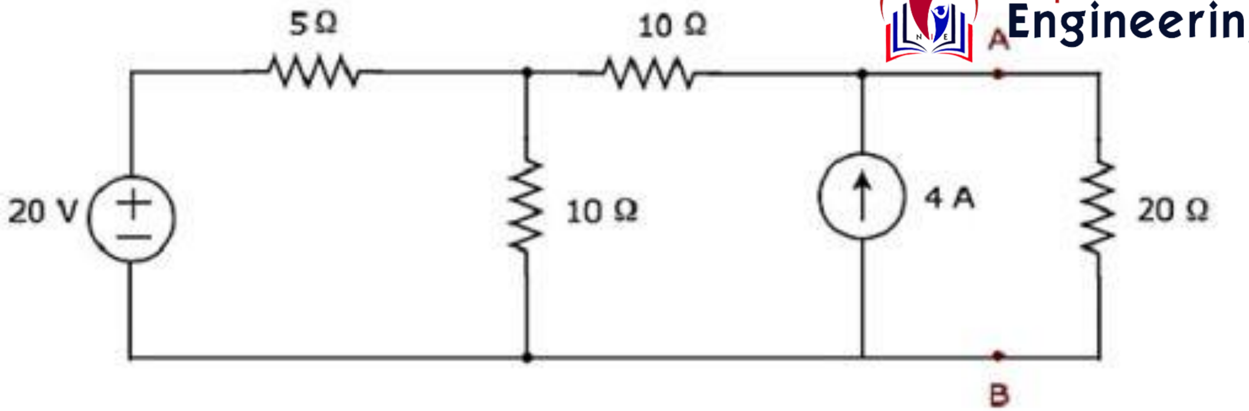
Draw the Norton's equivalent circuit with source I_N , equivalent resistance R_N in parallel with I_N and connect the load resistance.

$$I_L = \frac{IN * RN}{RN + rL}$$





Find current through 20 Ω



Remove R_L = 20 Ω and find the current through short circuited load terminals. This will be I_N

Applying KVL at mesh 1, $20 - 5I_1 - 10I_1 - 10I_2 = 0$

$-15I_1 - 10I_2 + 0I_3 = -20$ …….(i)

From mesh 2 and mesh 2

$I_2 + I_3 = 4$

$0I_1 + I_2 + I_3 = 4$ ……….(ii)

Applying KVL at supermesh

$10I_2 + 10I_2 + 10I_1 = 0$

$10I_1 + 20I_2 + 0I_3 = 0$ ……….(iii)

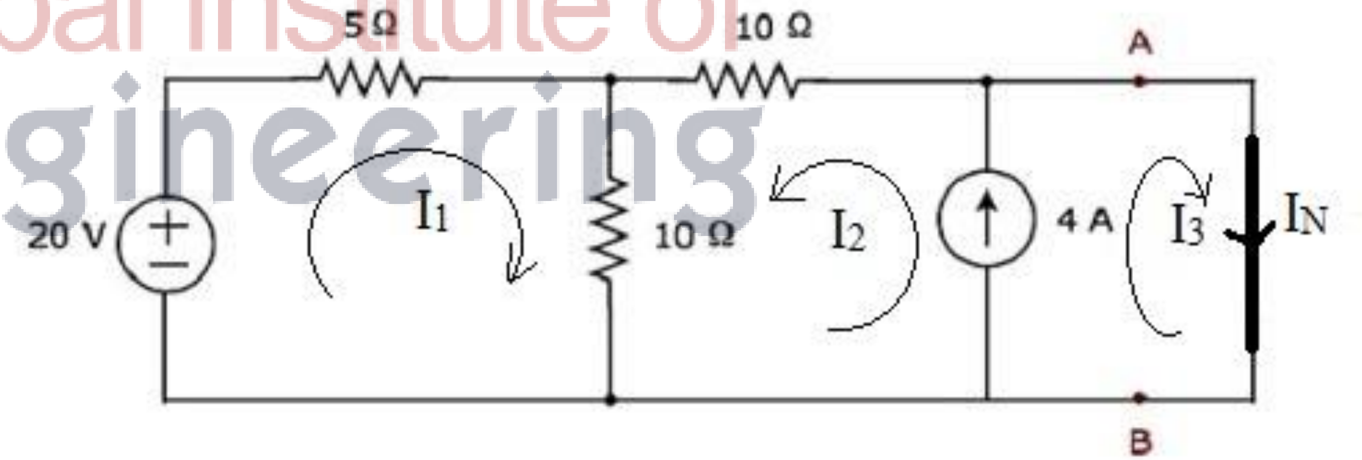
On solving equation (i), (ii) and (iii)

We get,

$I_1 = 2A$

$I_2 = -1\ A$

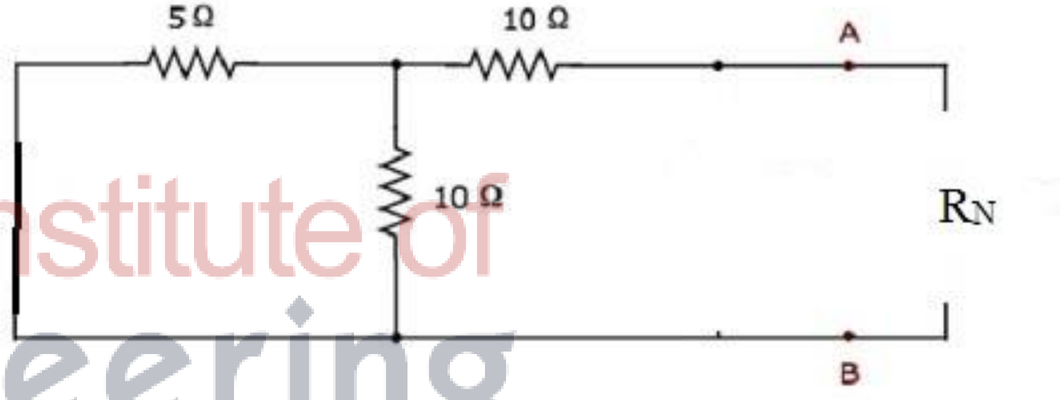
$I_3 = 5A$



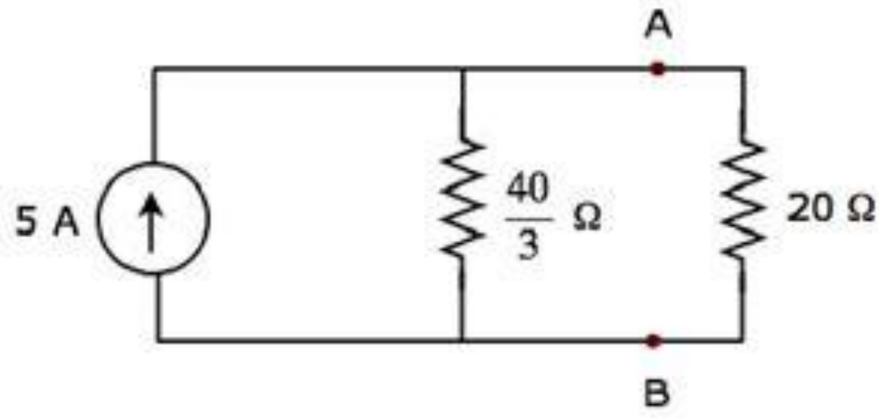
So, $I_N = I_3 = 5A$

- Now find the equivalent resistance at the load terminals by replacing all the sources by their internal resistance (i.e. short circuit the voltage source and open circuit the current source)

$$R_N = \frac{5*10}{5+10} + 10 \qquad = 40/3\ \Omega$$



Draw the Norton's equivalent circuit with source I_N , equivalent resistance R_N in parallel with I_N, across terminals of branch of interest.



$$I_L = \frac{IN*RN}{RN+RL} \qquad = \frac{5*\frac{40}{3}}{\frac{40}{3}+20} \qquad = 2A$$

- Thevenin's Theorem provides an easy method for analyzing power circuits, which typically have a load that changes value during the analysis process. This theorem provides an efficient way to calculate the voltage and current flowing across a load without having to recalculate your entire circuit over again.
- Thevenin's theorem is used to replace a multiple element circuit with a single voltage source and resistor while Norton's theorem is used to replace a multiple element circuit with a single current source and resistor.
- Thevenin and Norton theorems have several advantages for circuit analysis, such as reducing the complexity and size of the network, allowing you to focus on the behavior of the load without worrying about the details, and being applicable to any linear network.

Maximum Power Transfer Theorem

- The Maximum Power Transfer Theorem states that in a linear, bilateral DC network, Maximum Power is delivered to the load when the load resistance is equal to the internal resistance of the source.

Proof:

Let us consider a Thevenin's equivalent circuit of a network as shown in figure. Here the value of R_L is to be determine

So that the network will deliver or the load resistor

R_L will receive maximum power.

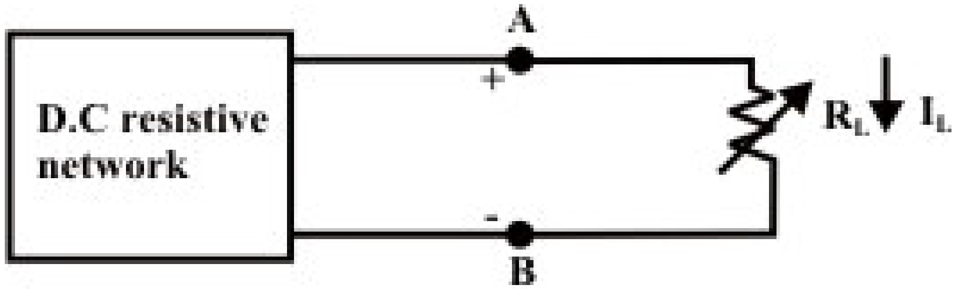
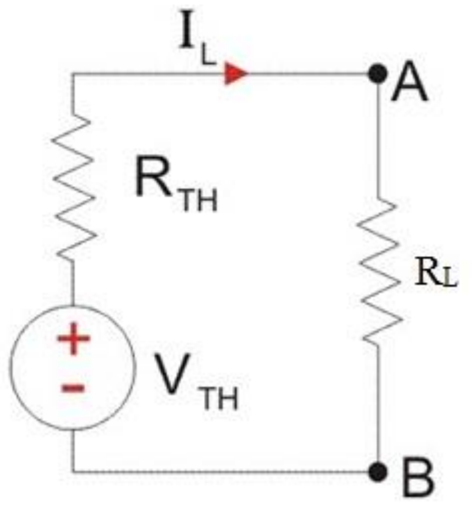
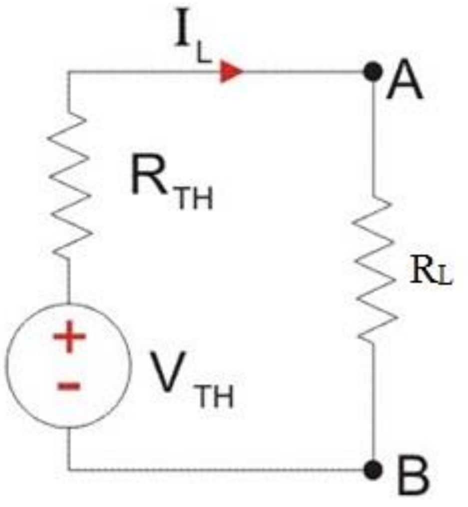


Fig. 8.6(a)



Current through R_L is given by,



$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Then, the power delivered to the load is

$$P_L = I_L^2 \times R_L = \left[\frac{V_{Th}}{R_{Th} + R_L}\right]^2 \times R_L$$

The load power depends on both R_{Th} and R_L ; however, R_{Th} is constant for the equivalent Thevenin network. So power delivered by the equivalent Thevenin network to the load resistor is entirely depends on the value of R_L . To find the value of R_L that absorbs a maximum power from the Thevenin circuit, we differentiate P_L with respect to R_L .

$$\frac{dP(R_L)}{dR_L} = V_{Th}^2\left[\frac{(R_{Th} + R_L)^2 - 2R_L \times (R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

Note : d(U/V) = (VdU –UdV) / V^2

$$\frac{dPL}{dRL} = Vth^2[\frac{(RTh + RL)^2 \frac{dRL}{dRL} - RL\frac{d(RTh + RL)^2}{dRL}}{((RTh + RL)^2)^\wedge 2}$$

For maximum or minimum

$$\frac{dP(R_L)}{dR_L} = 0 \longrightarrow V_{Th}^{\ 2}\left[\frac{(R_{Th}+R_L)^2 - 2R_L \times (R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = 0$$

$$(R_{Th}+R_L) - 2R_L = 0$$

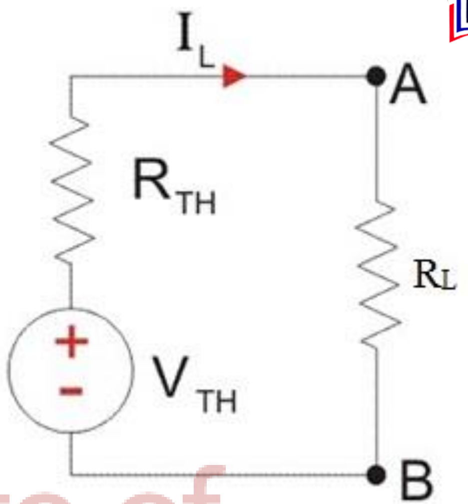
$R_L = R_{Th}$ ……..(*)

For maximum power dissipation in the load, the condition given below must be satisfied

$$\left.\frac{d^2P(R_L)}{dR_L^{\ 2}}\right|_{R_L = R_{Th}} = -\frac{V_{Th}^{\ 2}}{8R_{Th}} < 0$$

Maximum power delivered to load is given by

$$P_{max} = \left[\frac{V_{Th}}{R_{Th}+R_L}\right]^2 \times R_L \Bigg|_{R_L = R_{Th}} = \frac{V_{Th}^{\ 2}}{4\,R_{Th}}$$



The total power delivered by the source

$$P_T = I_L^{\ 2}(R_{Th}+R_L) = 2\times I_L^{\ 2}\,R_L$$

This means that the Thevenin voltage source itself dissipates as much power in its internal resistance R_{Th} as the power absorbed by the load R_L. Efficiency under maximum power transfer condition is given by

$$Efficiency = \frac{I_L^{\ 2}\,R_L}{2\,I_L^{\ 2}\,R_L}\times 100 = 50\,\%$$

For a given circuit, V_{Th} *and* R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as shown in fig.

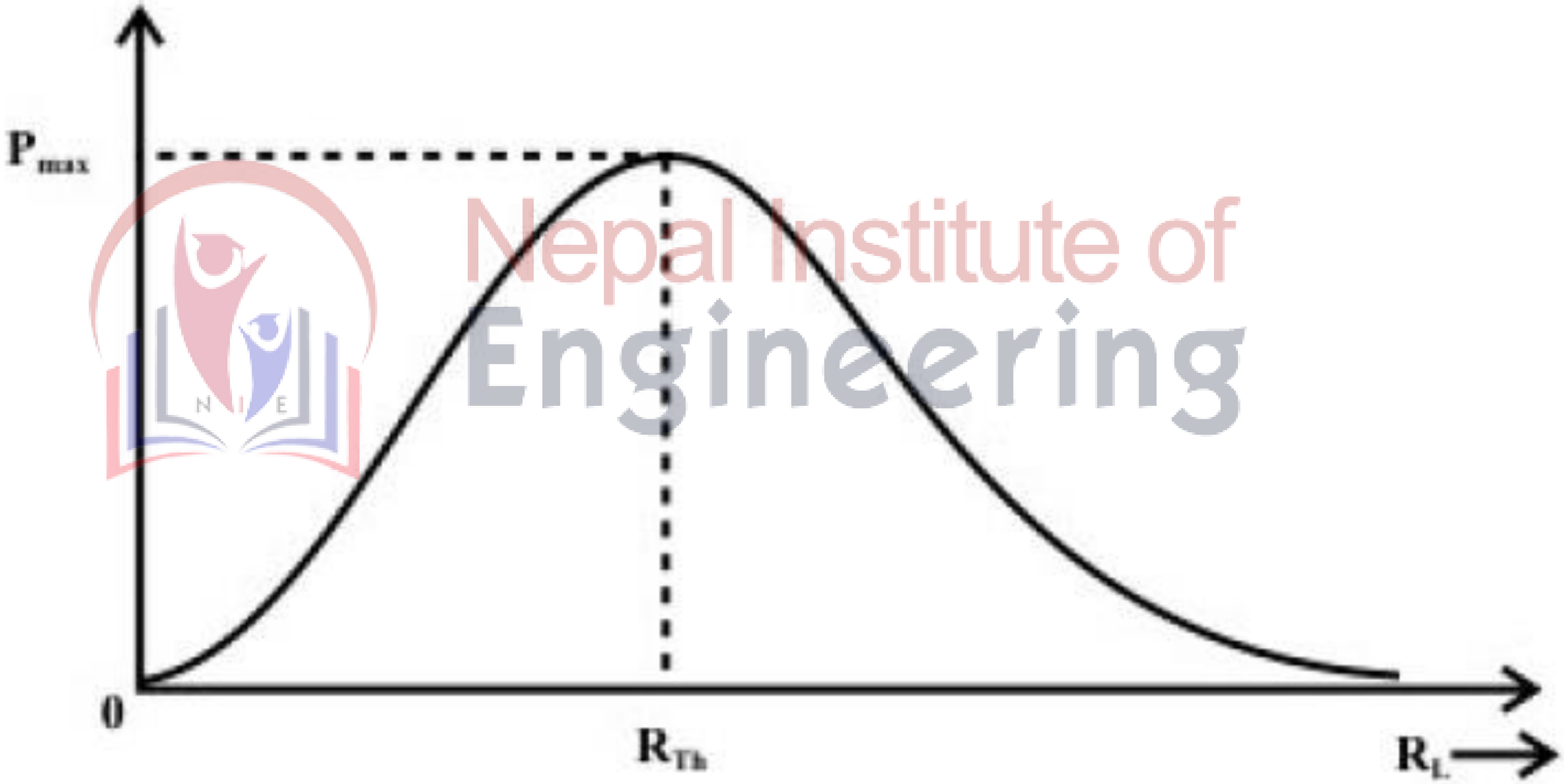


Fig. : Power dissipated to the load as a function of R_L

Efficiency (η) = Output/Input

$$\eta = I^2R_L / (I^2R_L + I^2R_i)$$

$$\eta = I^2R_L / I^2R_L (1 + R_i / R_L)$$

$$\eta = 1 / (1 + R_i / R_L)$$