Nepal Institute of Engineering

Nepal Engineering Council Registration Examination Preparation Class

Computer Engineering

AC fundamentals

1. Concept of Basic Electrical and Electronics Engineering (AExE01)

1.1 Basic concept: Ohm´s law, electric voltage current, power and energy, conducting and insulating materials. Series and parallel electric circuits, start-delta and delta-star conversion, Kirchhoff's law, linear and non-linear circuit, bilateral and unilateral circuits, active and passive circuits.

1.2 Network theorems: concept of superposition theorem, Thevenin`s theorem, Norton`s theorem, maximum power transfer theorem. R-L, R-C, R-L-C circuits, resonance in AC series and parallel circuit, active and reactive power.

1.3 Alternating current fundamentals: Principle of generation of alternating voltages and currents and their equations and waveforms, average, peak and rms values. Three phase system.

1.4 Semiconductor devices: Semiconductor diode and its characteristics, BJT Configuration and biasing, small and large signal model, working principle and application of MOSFET and CMOS.

1.5 Signal generator: Basic Principles of Oscillator, RC, LC and Crystal Oscillators Circuits. Waveform generators.

1.6 Amplifiers: Classification of Output Stages, Class A Output Stage, Class B Output Stage, Class AB Output Stage, Biasing the Class AB Stage, Power BJTs, Transformer-Coupled Push-Pull Stages, and Tuned Amplifiers, op-amps.

Faraday's Laws of Electromagnetic Induction

Faraday summed up the above facts into two laws known as Faraday's Laws of Electromagnetic Induction.

First Law. It states :

Whenever the magnetic flux linked with a circuit changes, an e.m.f. is always induced in it.

or

Whenever a conductor cuts magnetic flux, an e.m.f. is induced in that conductor.

Second Law. It states :

The magnitude of the induced e.m.f. is equal to the rate of change of flux-linkages.

Explanation. Suppose a coil has N turns and flux through it changes from an initial value of Φ_1 webers to the final value of Φ_2 webers in time t seconds. Then, remembering that by flux-linkages mean the product of number of turns and the flux linked with the coil, we have

Initial flux linkages $= N\Phi_1$, Final flux linkages $= N\Phi_2$

$$\therefore \quad \text{induced } e.m.f. \quad e = \frac{N\Phi_2 - N\Phi_1}{t} \text{ Wb/s or volt or } e = N\frac{\Phi_2 - \Phi_1}{t} \text{ volt}$$

Putting the above expression in its differential form, we get

$$e = \frac{d}{dt}(N\,\Phi) = N\,\frac{d\Phi}{dt} \text{ volt}$$

Usually, a minus sign is given to the right-hand side expression to signify the fact that the induced e.m.f. sets up current in such a direction that magnetic effect produced by it opposes the very cause producing it

$$e = -N\frac{d\,\Phi}{dt} \text{ volt}$$

Generation of Alternating Voltages and Currents

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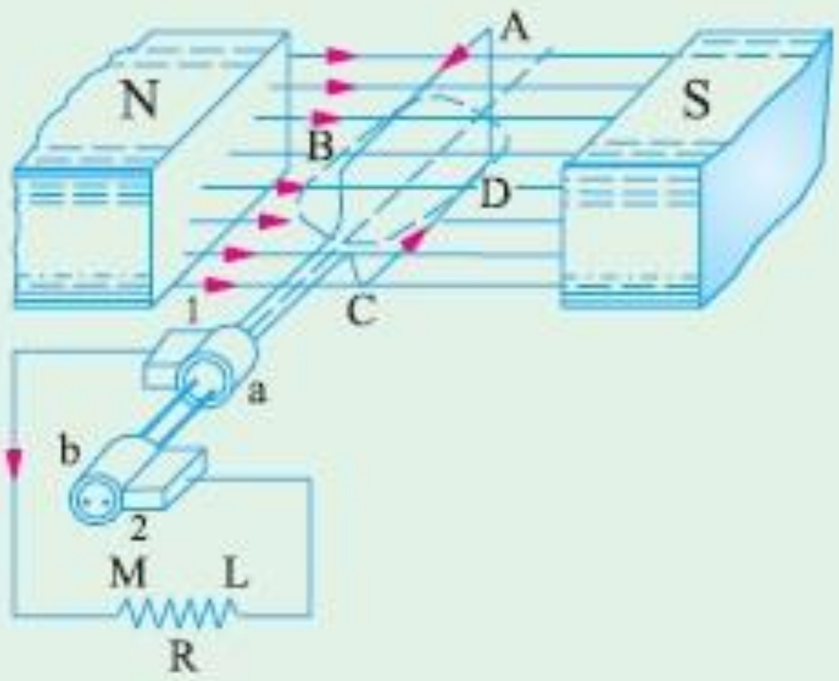
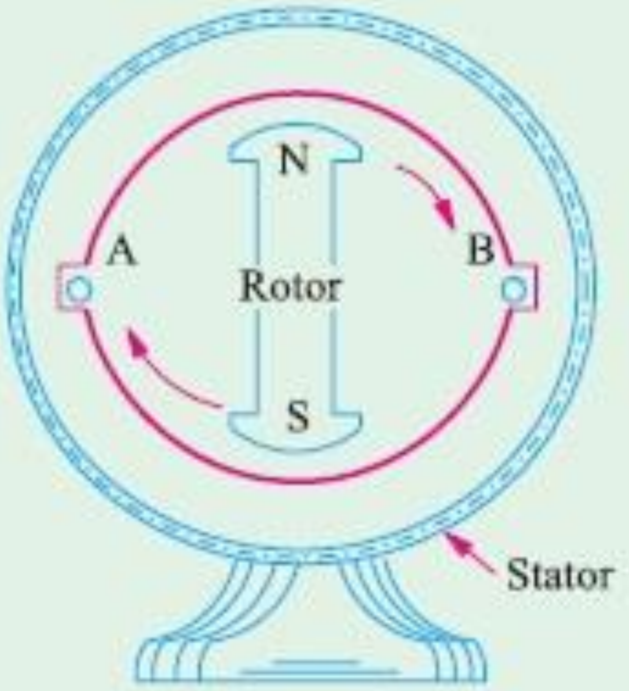
Alternating voltage may be generated by rotating a coil in a magnetic field, as shown in Figure (*a*) or by rotating a magnetic field within a stationary coil, as shown in Figure (*b*).



(*a*)



(*b*)

The value of the voltage generated depends, in each case, upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates. Alternating voltage may be generated in either of the two ways shown above, but rotating-field method is the one which is mostly used in practice.

Equations of the Alternating Voltages and Currents

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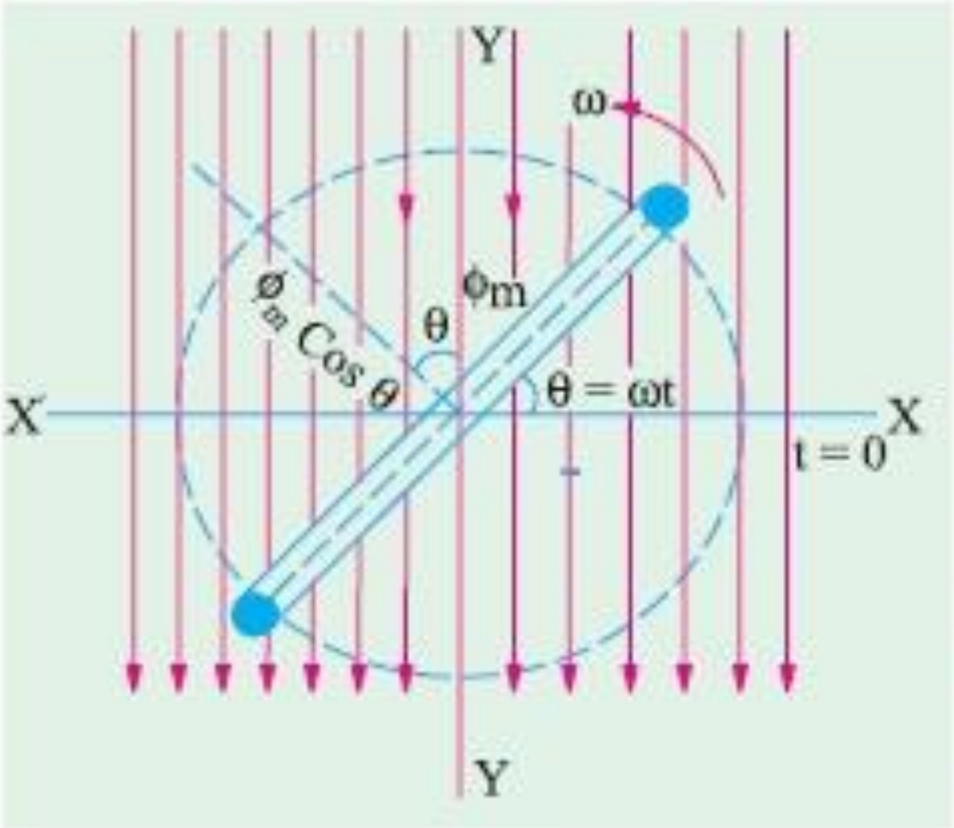
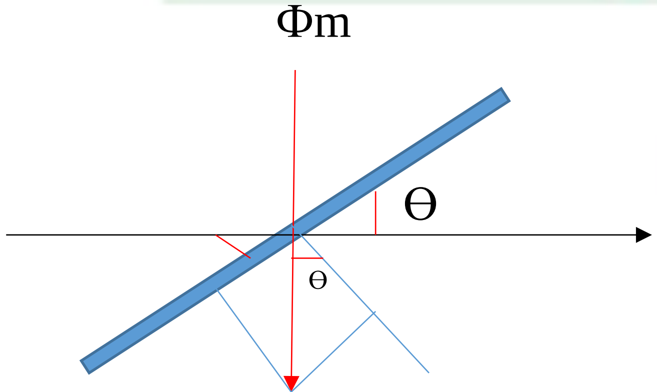
Consider a rectangular coil, having N turns and rotating in a uniform magnetic field, with an angular velocity of ω radian/second, as shown in Fig.



Let time be measured from the X-axis. Maximum flux Φ_m is linked with the coil, when its plane coincides with the X-axis. In time t seconds, this coil rotates through an angle $\theta = \omega t$. In this deflected position, the component of the flux which is perpendicular to the plane of the coil, is $\Phi = \Phi_m \cos \omega t$. Hence, *flux linkages* of the coil at any time are $N\Phi = N\Phi_m \cos \omega t$.

According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil. Hence, the value of the induced e.m.f. at this instant (*i.e.* when $\theta = \omega t$) or the instantaneous value of the induced e.m.f. is



$$e = -\frac{d}{dt}(N\Phi) \text{ volt} = -N.\frac{d}{dt}(\Phi_m \cos \omega t) \text{ volt} = -N\Phi_m\, \omega(-\sin \omega t) \text{ volt}$$

$$= \omega N \Phi_m \sin \omega t \text{ volt} = \omega N \Phi_m \sin \theta \text{ volt} \qquad ...(i)$$

When the coil has turned through 90° *i.e.* when $\theta = 90°$, then $\sin\theta = 1$, hence e has maximum value, say E_m. Therefore, from Eq. (*i*) we get

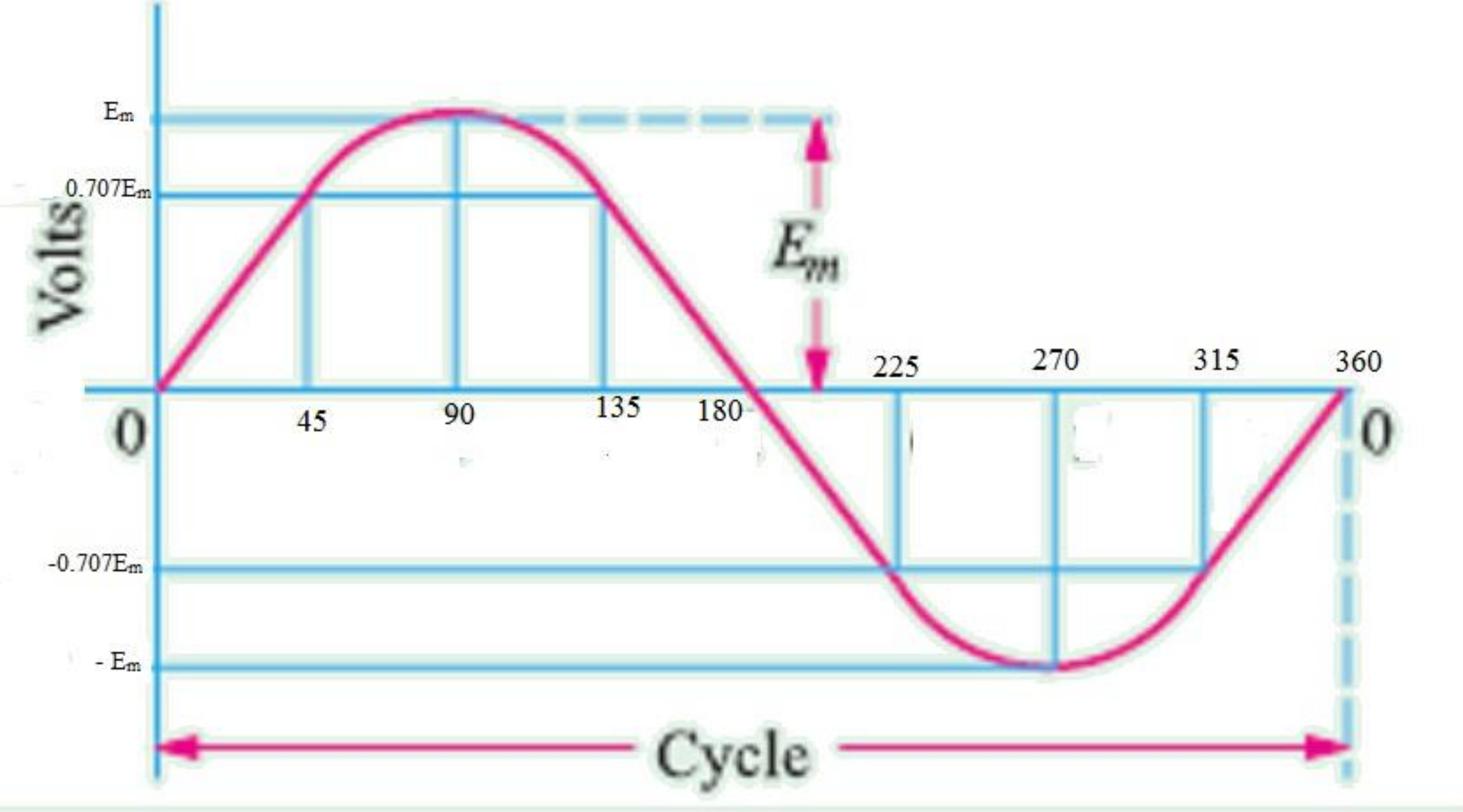
$$E_m = \omega N \Phi_m = \omega N B_m A = 2\pi f N B_m A \text{ volt} \qquad \ldots(ii)$$

where B_m = maximum flux density in Wb/m^2 ; A = area of the coil in m^2

f = frequency of rotation of the coil in rev/second

Substituting this value of E_m in Eq. (*i*), we get $e = E_m \sin\theta = E_m \sin\omega t$ ...(*iii*)

ωt	$e = E_m$ Sinωt
0	0
45	0.707 E_m
90	E_m
135	0.707 E_m
180	0
225	- 0.707 E_m
270	- E_m
315	- 0.707 E_m
360	0



Some Terminologies

Cycle

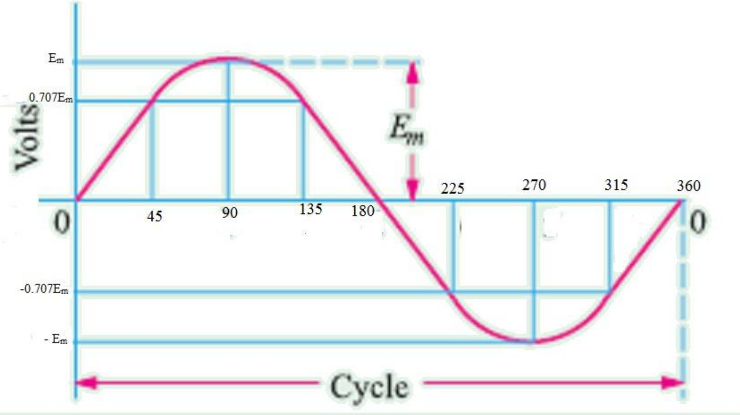
One complete set of positive and negative values of alternating quantity is known as cycle.

Time Period

The time taken by an alternating quantity to complete one cycle is called its time period T. For example, a 50-Hz alternating current has a time period of 1/50 second.

Frequency

The number of cycles/second is called the frequency of the alternating quantity. Its unit is hertz (Hz).



Instantaneous Value:

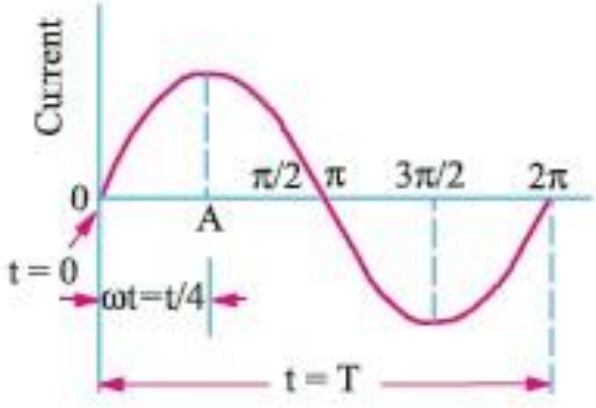
The value of an alternating quantity at particular instant(time) in a cycle is called instantaneous value.

Peak to Peak Value :

The full voltage between positive peak and negative peak of wave form is called peak to peak value.

Phasors:

A phasor is an complex number that represents the amplitude and phase of a sinusoid.



Amplitude

The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already given in Art. 11.2, is

$$e = E_m \sin\theta = E_m \sin\omega t = E_m \sin 2\pi f t = E_m \sin\frac{2\pi}{T}t$$

By closely looking at the above equations, we find that

(i) the maximum value or peak value or amplitude of an *alternating voltage is given by the coefficient of the sine of the time angle.*

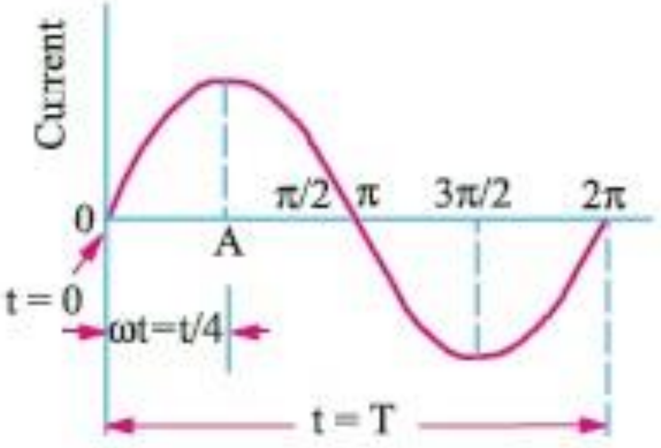
(ii) *the frequency f is given by the coefficient of time divided by* **2π**.

For example, if the equation of an alternating voltage is given by $e = 50 \sin 314t$ then its maximum value of 50 V and its frequency is $f = 314/2\pi = 50$ Hz.

Similarly, if the equation is of the form $e = I_m\sqrt{(R^2 + 4\omega^2L^2)}\sin 2\omega t$, then its maximum value is $E_m = I_m\sqrt{(R^2 + 4\omega^2L^2)}$ and the frequency is $2\omega/2\pi$ or ω/π Hz.

Phase

By phase of an alternating current is meant the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of reference. For example, the phase of current at point *A* is *T*/4 second, where *T* is time period or expressed in terms of angle, it is π/2 radians

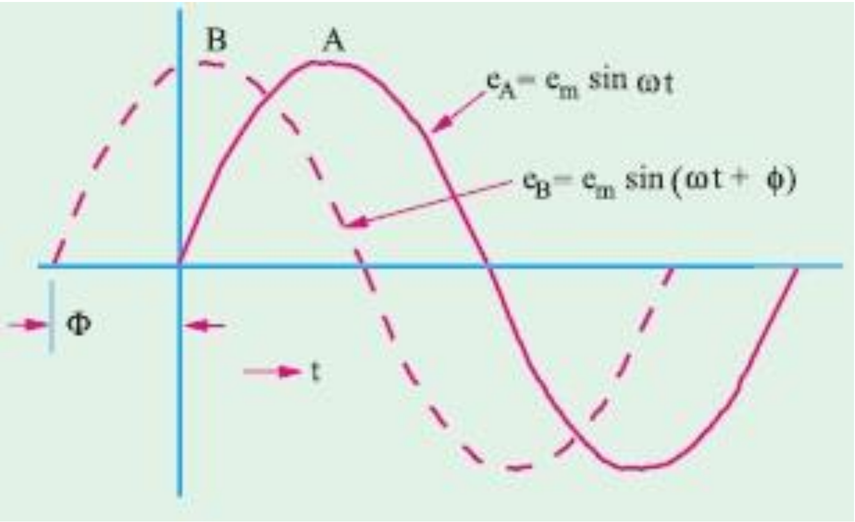


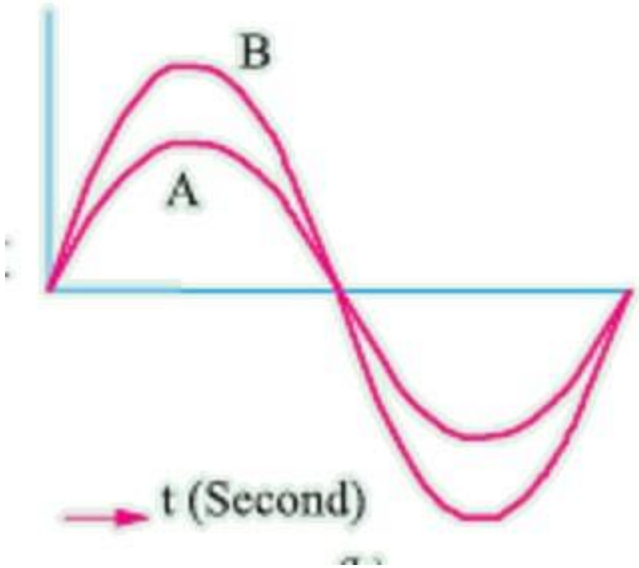
Phase Difference

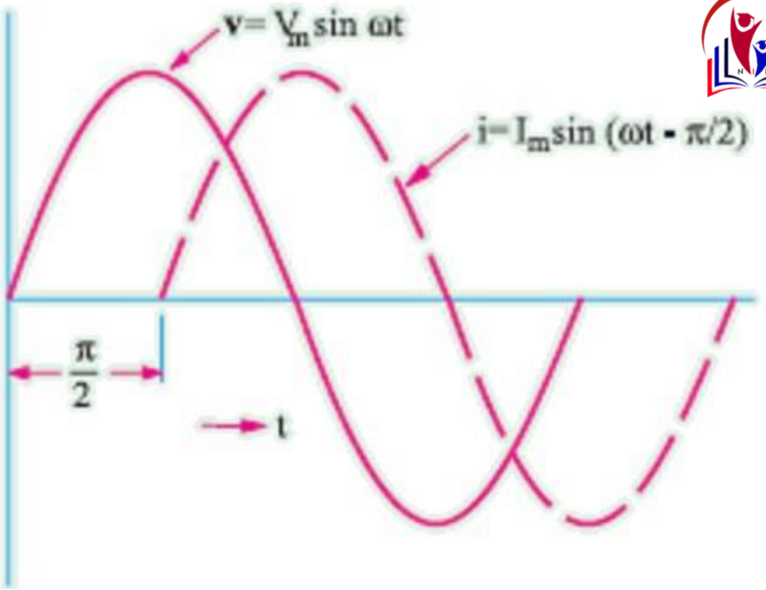
It is the time difference or angle difference between two waves since they last passed through their zero values.

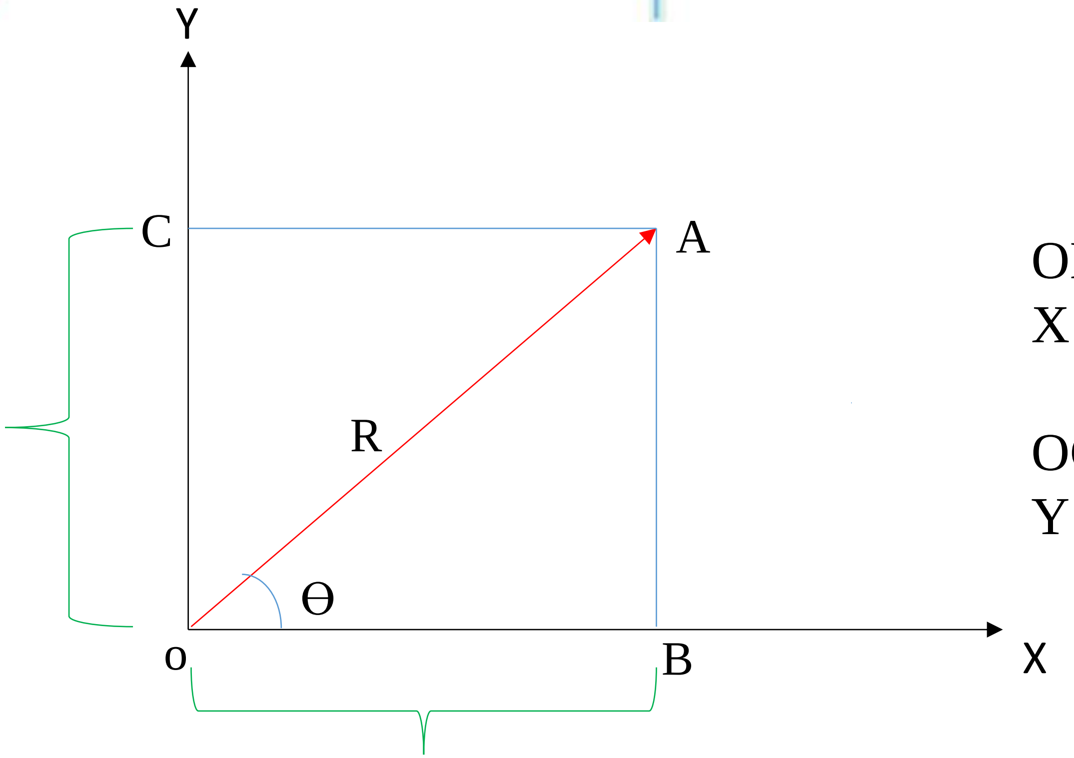
Two quantities are out of phase if they reach their zero , maximum and minimum value at different time but always has equal phase angle between them.

Two quantities are in phase if they reach their zero , maximum and minimum value at same time.









OB = OA CosΘ

X Component of R, = R CosΘ

OC= AB = OA SinΘ

Y Component of R, = R SinΘ

- Add the following currents $i_1 = 7 \sin \omega t$ and $i_2 = 10 \sin(\omega t + \pi/3)$

Maximum value of $i_1 = 7$ and phase angle $\Phi_1 = 0$

Maximum value of $i_2 = 10$ and phase angle $\Phi_2 = 60$

X- Component of $i_1 = 7$

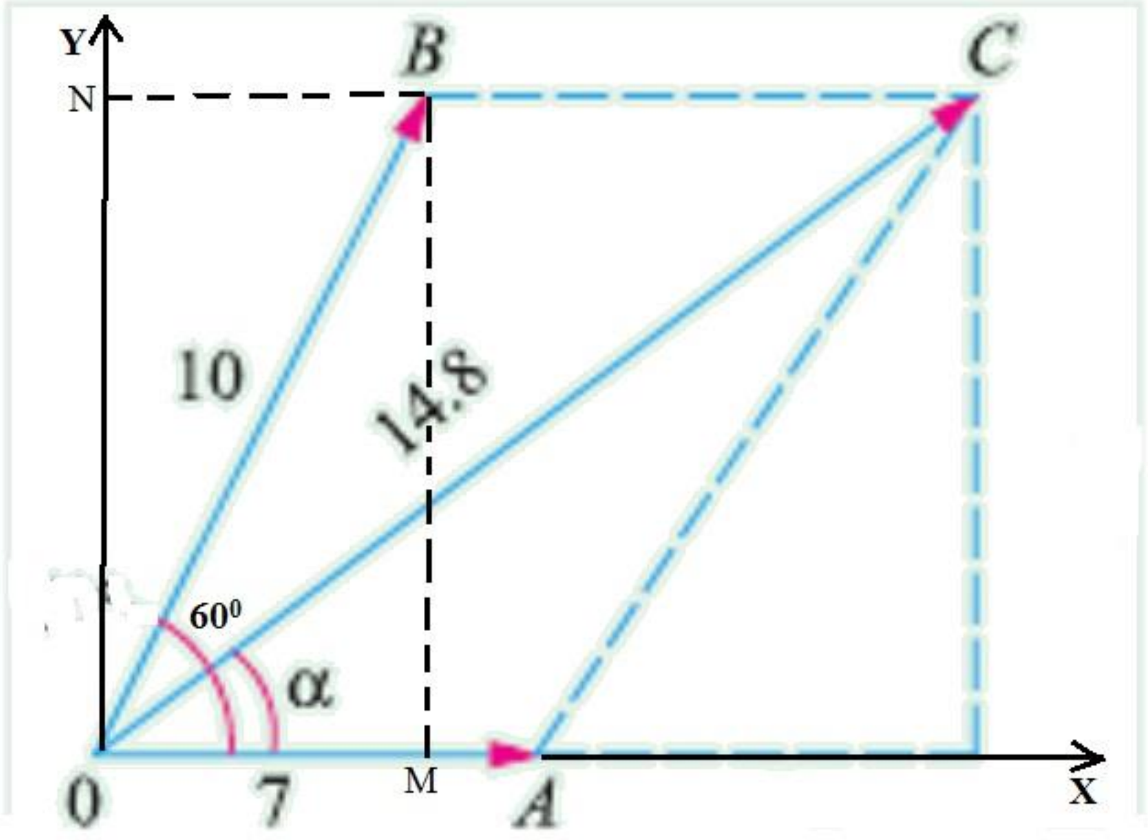
Y- Component of $i_1 = 0$

X- Component of $i_2 = 10 \text{ Cos } 60$

$= 5$

Y- Component of $i_2 = 10 \text{ Sin } 60$

$= 8.66$



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$$\text{Net } X\text{-component} = 7 + 10\cos 60° = 12$$

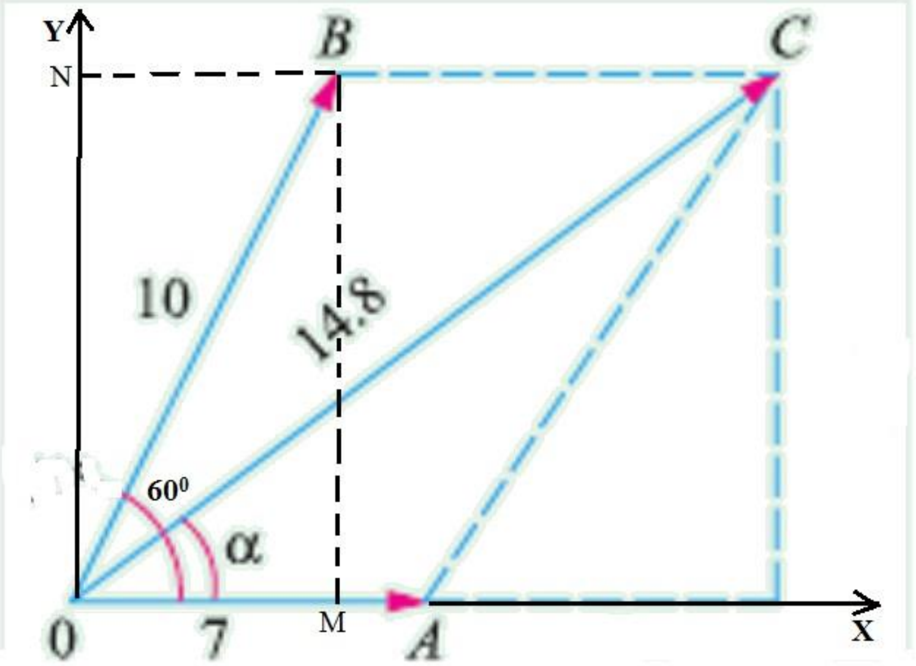
$$\text{Net } Y\text{-component} = 0 + 10\sin 60° = 8.66$$

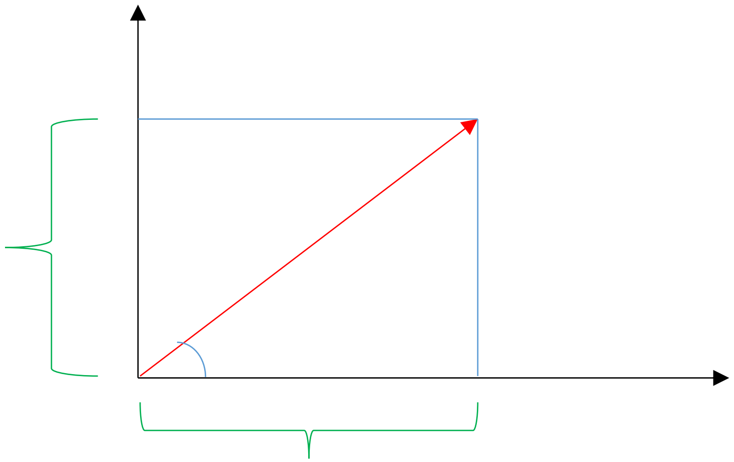
$$\text{Resultant} = \sqrt{(12^2 + 8.66^2)} = 14.8 \text{ A}$$

and $\alpha = \tan^{-1}(8.66/12) = 35.8°$

Hence, the resultant equation can be written as

$$i_r = 14.8 \sin(\omega t + 35.8°)$$





Example *Three voltages represented by*

$$e_1 = 20 \sin \omega t;\ e_2 = 30 \sin (\omega t - \pi/4) \text{ and } e_3 = 40 \cos (\omega t + \pi/6)$$

act together in a circuit. Find an expression for the resultant voltage. Represent them by appropriate vectors.

Solution. First, let us draw the three vectors representing the maximum values of the given alternating voltages.

$e_1 = 20$ sing ωt –here phase angle with X-axis is zero, hence the vector will be drawn parallel to the X-axis

$e_2 = 30 \sin (\omega t - \pi/4)$ –its vector will be below OX by 45°

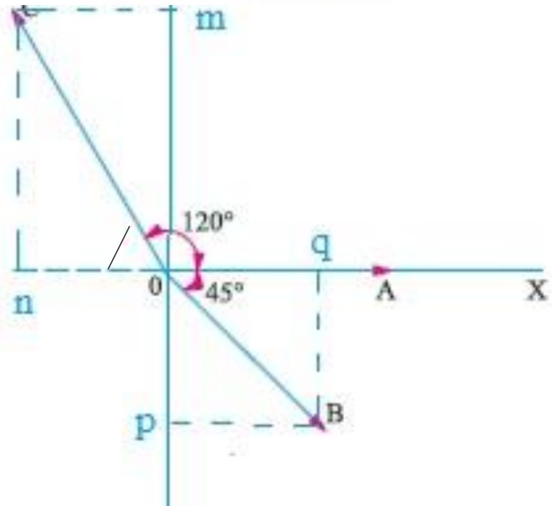
$e_3 = 40 \cos (\omega t + \pi/6) = 40 \sin (90° + \omega t + \pi/6)$*

$= 40 \sin (\omega t + 120°)$ –its vector will be at 120° with respect to OX in counter clockwise direction.

X- Component of e_1 = 20 Y- Component of e_1 = 0

X- Component of e_2 = 30 Cos 45 Y- Component of e_2 = - 30 Sin 45

X- Component of e_3 = - 40 Cos 60 Y- Component of e_3 = 40 Sin 60



These vectors are shown in Fig. Resolving them into X-and Y-components, we get

X - component = 20 + 30 cos 45° – 40 cos 60°= 21.2 V

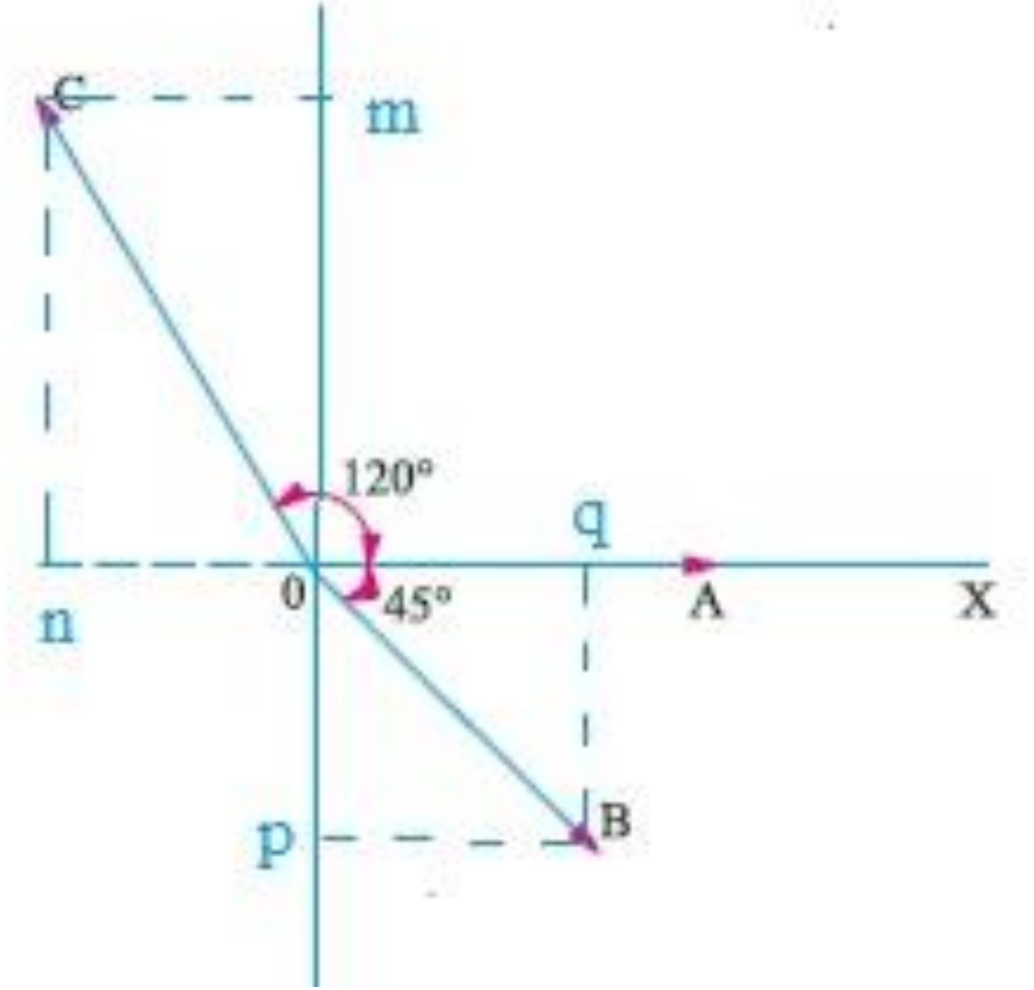
Y - component = 40 sin 60° –30 sin 45° = 13.4 V

Magnitude of resultant Voltage ,

$$R = \sqrt{(net\ X\ component)^2 + (net\ Y\ component)^2}$$

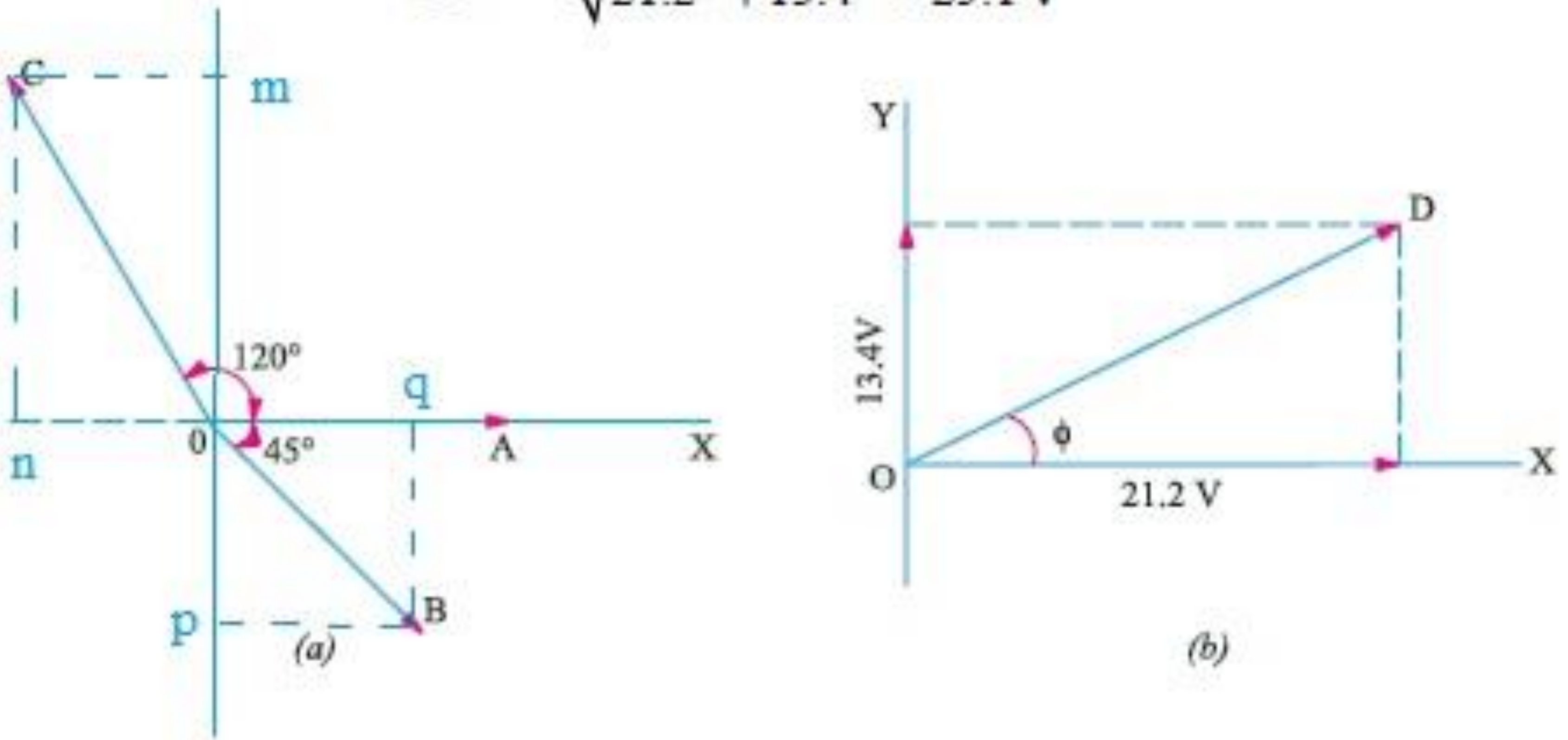
Phase angle of resultant Voltage,

$$\tan\Phi = \frac{net\ Y\ component}{net\ X\ component}$$



As seen from Fig. the maximum value of the resultant voltage is

$$OD = \sqrt{21.2^2 + 13.4^2} = 25.1 \text{ V}$$



The phase angle of the resultant voltage is given by $\tan\phi = \dfrac{\text{Y-component}}{\text{X-component}} \quad \dfrac{13.4}{27.2} \quad$ **0.632**

$\therefore \quad \phi = \tan^{-1} 0.632 = 32.3° =$ **0.564 radian**

The equation of the resultant voltage wave is $e = 25.1 \sin(\omega t + 32.3°)$ or $e = 25.1 \sin(\omega t + 0.564)$

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Example *The maximum values of the alternating voltage and current are 400 V and 20 A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of the voltage and current are 283 V and 10 A respectively at t = 0 both increasing positively.*

Write down the expression for voltage and current at time t.

Solution. In general, the expression for an a.c. voltage is $v = V_m \sin(\omega t + \phi)$ where ϕ is the phase difference with respect to the point where $t = 0$.

$v = V_m$ Sin $(\omega t + \Phi)$ and $i = I_m$ Sin $(\omega t + \Phi)$

$\omega = 2\Pi f$

Maximum value of voltage ,V_m = 400 V

Maximmum value of current ,I_m = 20 A

Frequency , f = 50 Hz

Instantaneous value of voltage at t=o ,v = 283 V

Instantaneous value of current at t=o ,i = 10 A

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Now, $v = 283$ V ; $V_m = 400$ V. Substituting $t = 0$ in the above equation, we get

$283 = 400\,(\sin \omega \times 0 + \phi)$ $\therefore \sin\phi = 283/400 = 0.707$; $\therefore$ $\phi = 45°$ or $\pi/4$ radian.

Hence, general expression for voltage is

$$v = 400\,(\sin 2\pi \times 50 \times t + \pi/4)$$

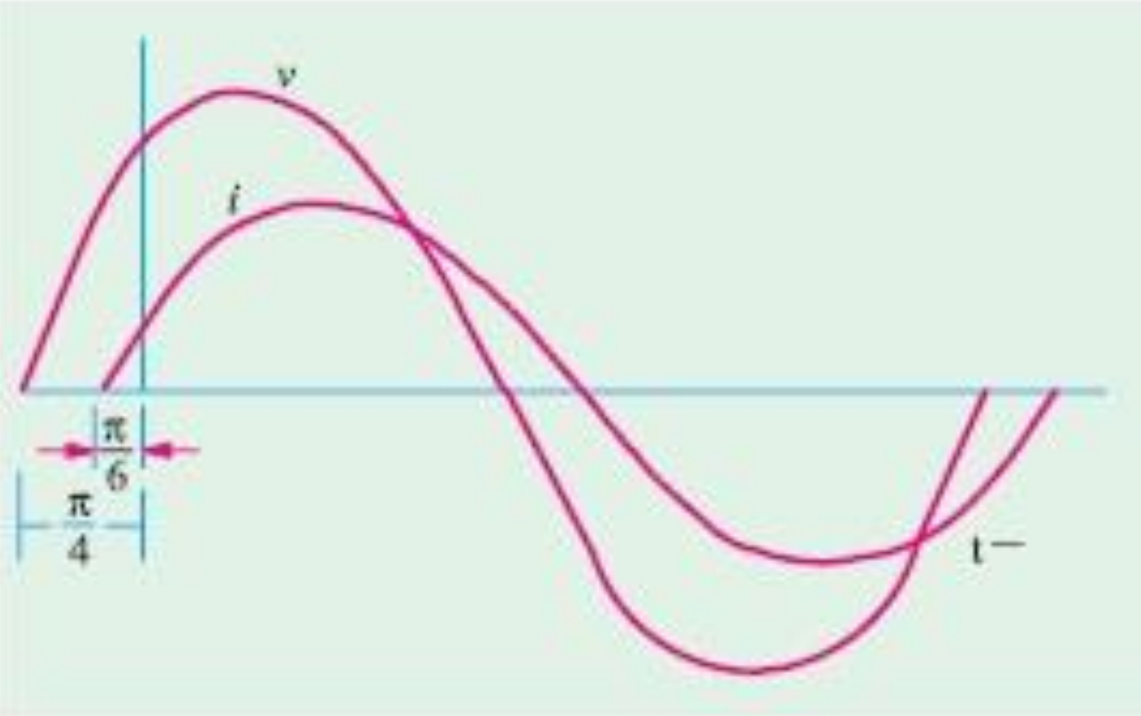
$$= 400 \sin (100\,\pi\, t + \pi/4)$$

Similarly, at $t = 0$, $10 = 20 \sin (\omega \times 0 + \phi)$

$\therefore$ $\sin\phi = 0.5$ $\therefore$ $\phi = 30°$ or $\pi/6$ radian

Hence, the general expression for the current is

$$i = 20\,(\sin 100\,\pi\, t + 30°) = 20 \sin (100\,\pi\, t + \pi/6)$$

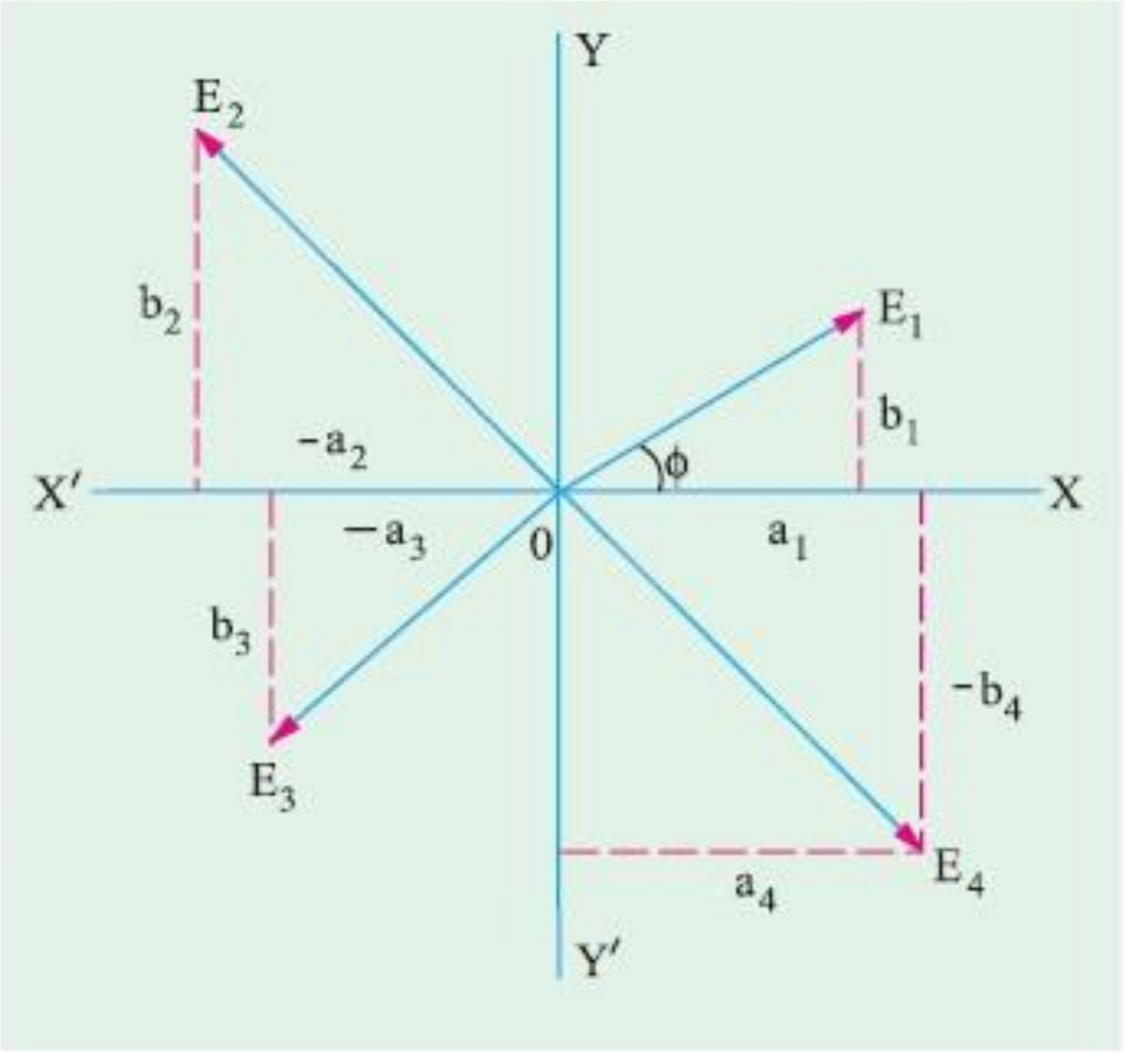


Symbolic Notation

A vector can be specified in terms of its *X*-component and *Y*-component. For example, the vector OE_1 (Figure) may be completely described by stating that its horizontal component is a_1 and vertical component is b_1. But instead of stating this verbally, we may express symbolically

$$\mathbf{E}_1 = a_1 + jb_1$$

where symbol *j*, known as an operator, indicates that component b_1 is perpendicular to component a_1 and that the two terms are ***not*** to be treated like terms in any algebraic expression. The vector written in this way is said to be written in '***complex form***'. In Mathematics, a_1 is known as real component and b_1 as imaginary component but in electrical engineering, these are known as ***in phase*** (or active) and ***quadrature*** (or reactive) components respectively.



The other vectors OE_2, OE_3 and OE_4 can similarly, be expressed in this form.

$$\mathbf{E}_2 = -a_2 + jb_2 \; ; \; \mathbf{E}_3 = -a_3 - jb_3 \; ; \; \mathbf{E}_4 = +a_4 - jb_4$$

The numerical value of vector E_1 is $\sqrt{a_1^2 + b_1^2}$. Its angle with X-axis is given by $\phi = \tan^{-1}(b_1/a_1)$.

Significance of Operator *j*

The letter j used in the above expression is a symbol of an operation. Just as symbols $\times$, $+$, $\sqrt{\ }$, $\int$ etc. are used with numbers for indicating certain operations to be performed on those numbers, similarly, symbol j is used to indicate the counter-clockwise rotation of a vector through 90°. It is assigned a value of $\sqrt{(-1)}$. The double operation of j on a vector rotates it counter-clockwise through 180° and hence reverses its sense

$$jj = j^2 = \sqrt{(-1)^2} = -1$$

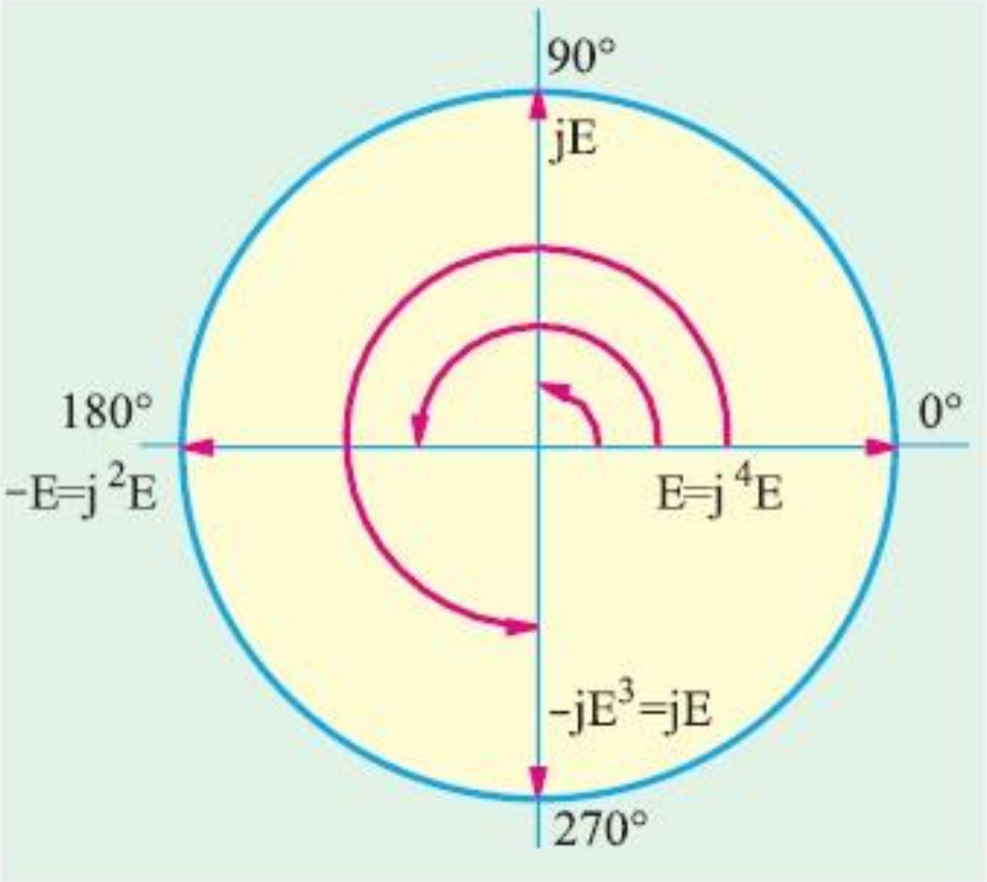
When operator j is operated on vector **E**, we get the new vector j**E** which is displaced by 90° in counter-clockwise direction from **E** (Figure). Further application of j will give j^2**E** = −**E** as shown.

If the operator j is applied to the vector j^2**E**, the result is j^3**E** = −j**E**. The vector j^3**E** is 270° counter-clockwise from the reference axis and is directly opposite to j**E**. If the vector j^3**E** is, turn, operated on by j, the result will be

$$j^4\mathbf{E} = \left[\sqrt{(-1)}\right]^4 \mathbf{E} = \mathbf{E}$$

Hence, it is seen that successive applications of the operator j to the vector **E** produce successive 90° steps of rotation of the vector in the counter-clockwise direction without in anyway affecting the magnitude of the vector.

It will also be seen from Figure that the application of −j to **E** yields −j**E** which is a vector of identical magnitude but rotated 90° *clockwise* from **E**.



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Summarising the above, we have

$$j = 90^\circ \text{ ccw rotation} = \sqrt{(-1)}$$

$$j^2 = 180^\circ \text{ ccw rotation} = [\sqrt{(-1)}]^2 = -1;$$

$$j^3 = 270^\circ \text{ ccw rotation} = [\sqrt{(-1)}]^3 = -\sqrt{(-1)} = -j$$

$$j^4 = 360^\circ \text{ ccw rotation} = [\sqrt{(-1)}]^4 = +1;$$

$$j^5 = 450^\circ \text{ ccw rotation} = [\sqrt{(-1)}]^5 = -\sqrt{(-1)} = j$$

It should also be noted that $\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

Mathematical Representation of Vectors

There are various forms or methods of representing vector quantities, all of which enable those operations which are carried out graphically in a phasor diagram, to be performed analytically. The various methods are :

(i) ***Symbolic Notation.*** According to this method, a vector quantity is expressed algebraically in terms of its rectangular components. Hence, this form of representation is also known as Rectangular or Cartesian form of notation or representation.

(ii) ***Trigonometrical Form*** *(iii)* ***Exponential Form*** *(iv)* ***Polar Form.***

Summarizing, we have the following alternate ways of representing vector quantities

(i) Rectangular form (or complex form) $\mathbf{E} = a + jb$

(ii) Trigonometrical form $\mathbf{E} = E(\cos\theta \pm j\sin\theta)$

(iii) Exponential form $\mathbf{E} = Ee^{\pm j\theta}$

(iv) Polar form (conventional) $\mathbf{E} = E\angle \pm\theta$.

Addition and Subtraction of Vector Quantities

Rectangular form is best suited for addition and subtraction of vector quantities. Suppose we are given two vector quantities $\mathbf{E}_1 = a_1 + jb_1$ and $\mathbf{E}_2 = a_2 + jb_2$ and it is required to find their sum and difference.

Addition. $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = a_1 + jb_1 + a_2 + jb_2 = (a_1 + a_2) + j(b_1 + b_2)$

The magnitude of resultant vector $\mathbf{E}$ is $\sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$

The position of $\mathbf{E}$ with respect to X-axis is $\theta = \tan^{-1}\left(\dfrac{b_1 + b_2}{a_1 + a_2}\right)$

Multiplication and Division of Vector Quantities

Multiplication and division of vectors becomes very simple and easy ***if they are represented in the polar or exponential form.*** As will be shown below, the rectangular form of representation is not well-suited for this process.

Let $\mathbf{A} = a_1 + jb_1 = A\angle\alpha = A\,e^{j\alpha}$ where $\alpha = \tan^{-1}(b_1/a_1)$

$\mathbf{B} = a_2 + jb_2 = B\angle\beta = B\,e^{j\beta}$ where $\beta = \tan^{-1}(b_2/a_2)$

$\therefore\quad \mathbf{AB} = A\angle\alpha \times B\angle\beta = AB\angle(\alpha+\beta)$* or $AB = Ae^{j\alpha} \times Be^{j\beta} = ABe^{j(\alpha+\beta)}$

Hence, product of any two vector **A** and **B** is given by another vector equal in length to **A** × **B** and having a phase angle equal to the sum of the angles of **A** and **B**.

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{A\angle\alpha}{B\angle\beta} = \frac{A}{B}\angle(\alpha-\beta)$$

A.C. Through Pure Ohmic Resistance Alone

The circuit is shown in Fig. below. Let the applied voltage be given by the equation.

$$v = V_m \sin\theta = V_m \sin\omega t \qquad \text{...(i)}$$

Let R = ohmic resistance ; i = instantaneous current

Obviously, the applied voltage has to supply ohmic voltage drop only. Hence for equilibrium

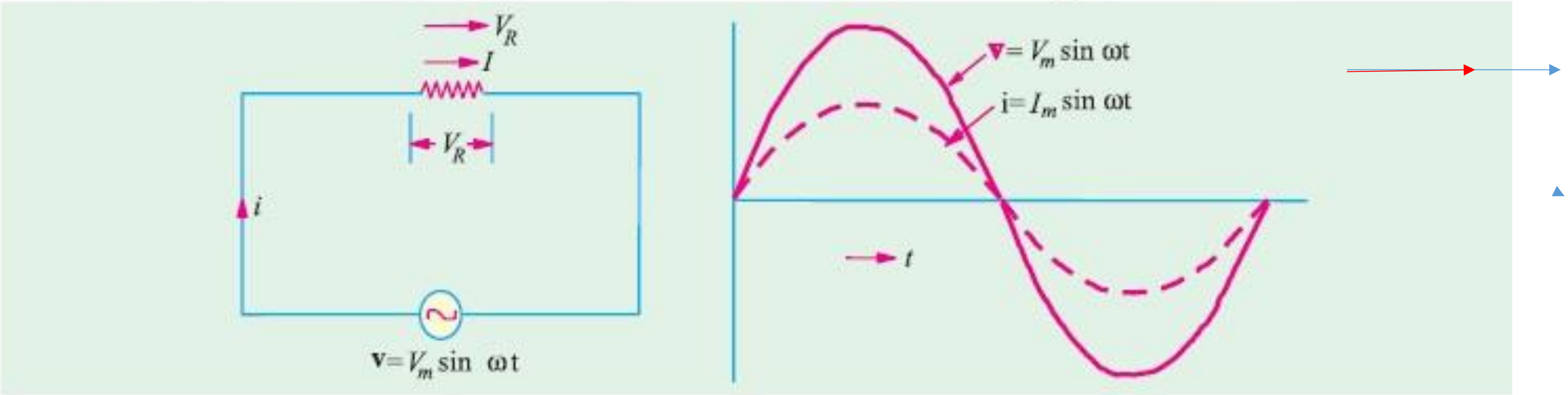
$$v = iR;$$

Putting the value of 'v' from above, we get $V_m \sin\omega t = iR$; $i = \dfrac{V_m}{R}\sin\omega t$ ...(ii)

Current 'i' is maximum when $\sin\omega t$ is unity $\therefore I_m = V_m/R$ Hence, equation (ii) becomes,

$$i = I_m \sin\omega t \qquad \text{...(iii)}$$

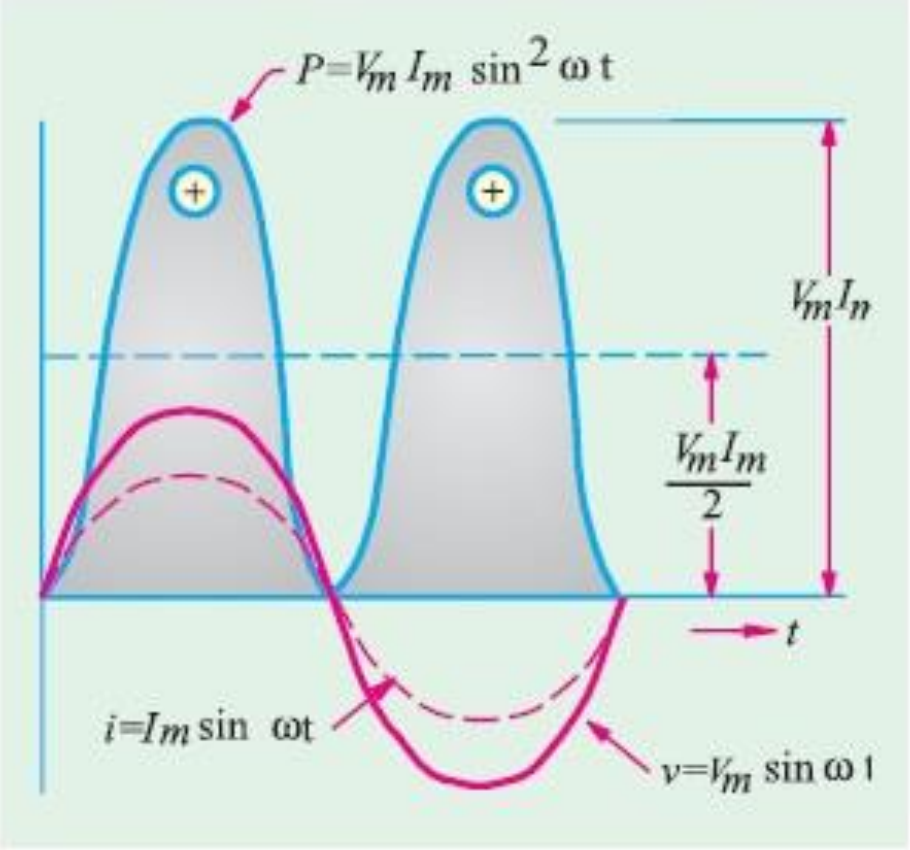
Comparing (i) and (ii), we find that the alternating voltage and current are in phase with each other as shown in Fig. It is also shown vectorially by vectors V_R and I in Fig.



Power. Instantaneous power, $p = vi = V_m I_m \sin^2 \omega t$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t) = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\,\omega t$$

Power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}\cos 2\,\omega t$ of frequency double that of voltage and current waves. For a complete cycle. the average value of $\frac{V_m I_m}{2}\cos 2\omega t$ is zero.



Hence, power for the whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $\quad P = V \times I$ watt

where $\quad V$ = r.m.s. value of applied voltage.

$\qquad I$ = r.m.s. value of the current.

It is seen from Fig. that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.

AC through Pure Inductance alone

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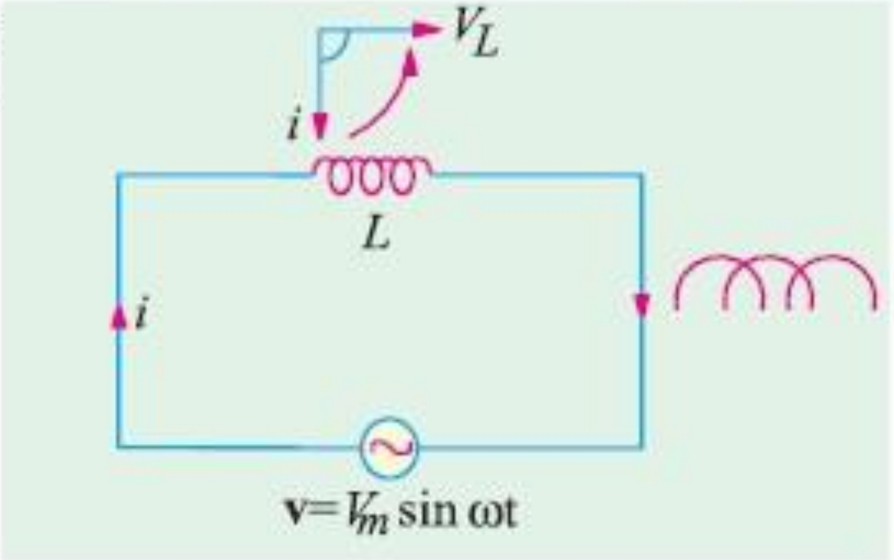
Whenever an alternating voltage is applied to a purely inductive coil, a back e.m.f. is produced due to the self-inductance of the coil. The back e.m.f., at every step, opposes the rise or fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced e.m.f. only. So at every step



$$v = L\frac{di}{dt}$$

Now $\quad v = V_m \sin \omega t$

$$\therefore \quad V_m \sin \omega t = L\frac{di}{dt} \quad \therefore \quad di = \frac{V_m}{L} \sin \omega t \, dt$$

Integrating both sides, we get $i = \frac{V_m}{L} \int \sin \omega t \, dt$

$$\frac{V_m}{wL} (- \text{coswt})$$

Sin (-x) = - sin x
Sin (90-x) = Cos x
Sin (-(x-90)) = Cos x
- Sin (x-90) = Cos x
Sin (x-90) = - Cos x

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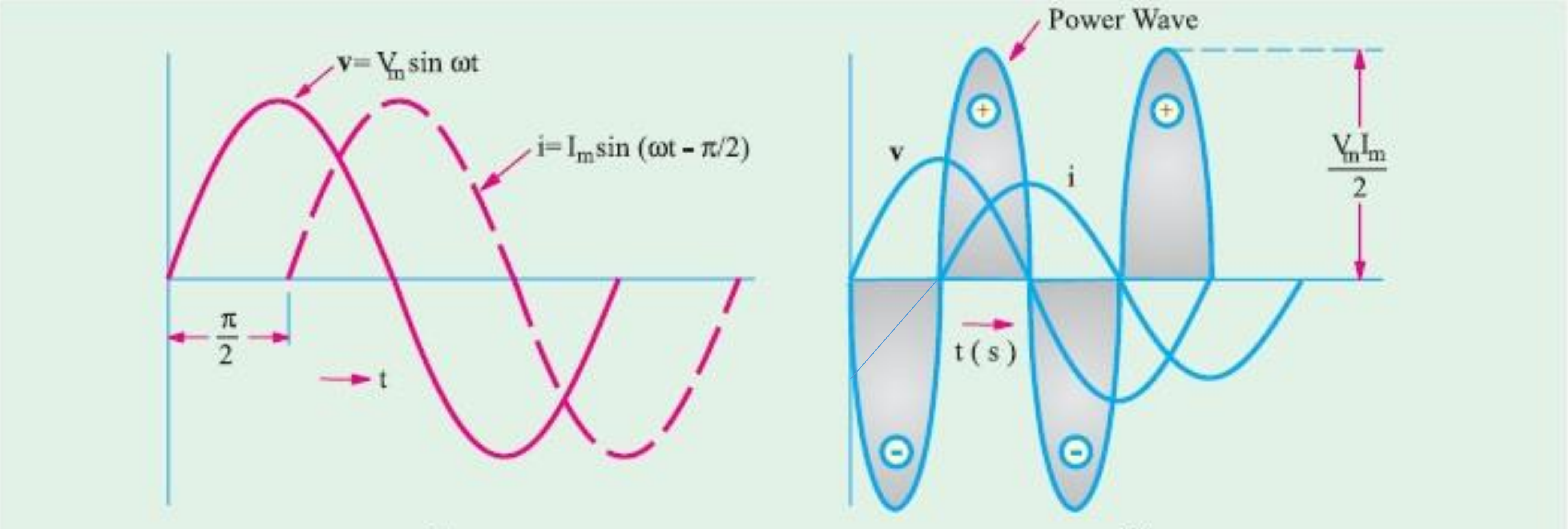
$$\therefore \quad = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) = \frac{V_m}{X_L} \sin(\omega t - \pi/2) \qquad \ldots(ii)$$

Max. value of i is $I_m = \dfrac{V_m}{\omega L}$ when $\sin\left(\omega t - \dfrac{\pi}{2}\right)$ is unity.

Hence, the equation of the current becomes $i = I_m \sin(\omega t - \pi/2)$.

So, we find that if applied voltage is represented by $v = V_m \sin \omega t$, then current flowing in a *purely* inductive circuit is given by $i = I_m \sin\left(\omega t - \dfrac{\pi}{2}\right)$



(a) (b)

Clearly, the current lags behind the applied voltage by a quarter cycle (Fig. above) or the phase difference between the two is $\pi/2$ with voltage leading. Vectors are shown in Fig. where voltage has been taken along the reference axis. We have seen that $I_m = V_m/\omega L = V_m/X_L$. Here '$\omega L$' plays the part of 'resistance'. It is called the (inductive) ***reactance*** X_L of the coil and is given in ohms if L is in henry and ω is in radian/second.

Now, $X_L = \omega L = 2\pi f L$ ohm. It is seen that X_L depends directly on frequency of the voltage. Higher the value of f, greater the reactance offered and ***vice-versa***.

Power

Instantaneous power $= v_i = V_m I_m \sin \omega t \sin \left[\omega t - \frac{\Pi}{2}\right] = - V_m I_m \sin \omega t . \cos \omega t^* = - \frac{V_m I_m}{2} \sin 2 \omega t$

Power for whole cycle is $P = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$

It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. Here again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $V_m I_m/2$.

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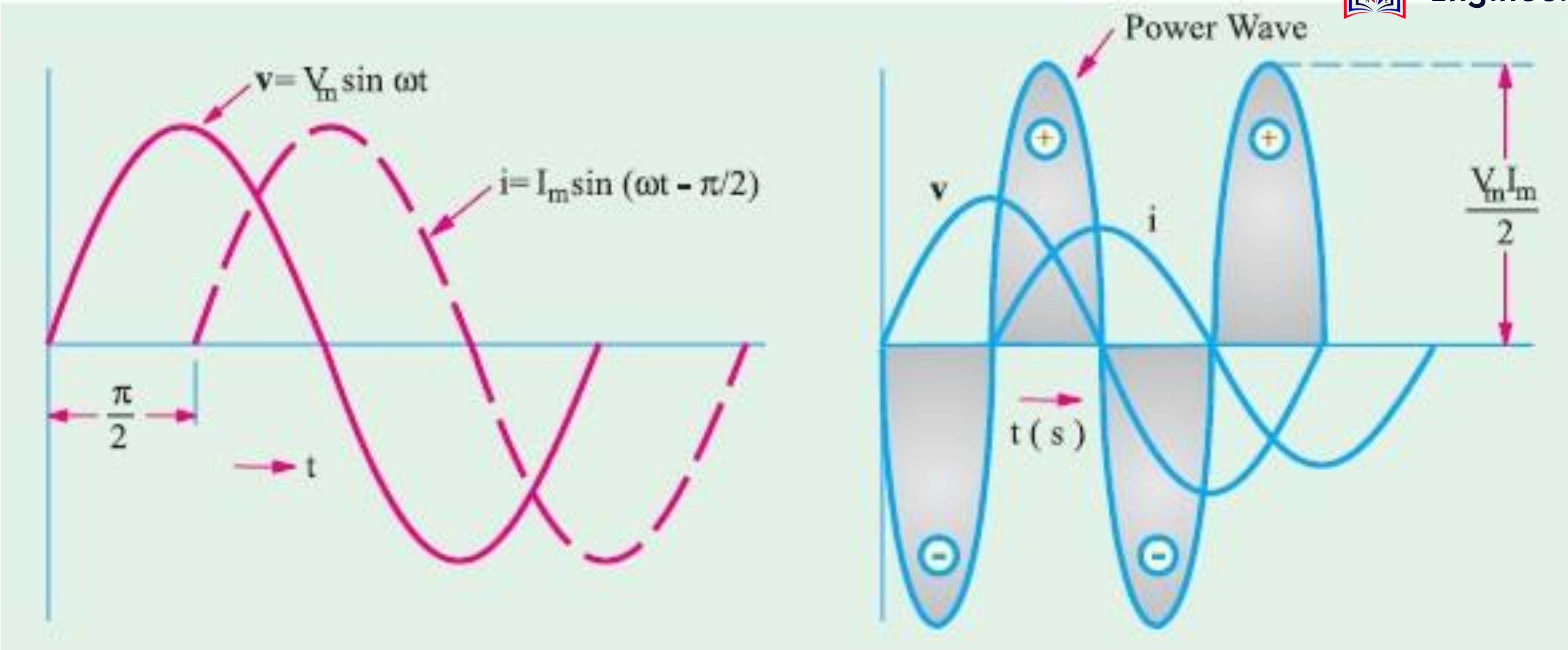


A.C. Through Pure Capacitance Alone

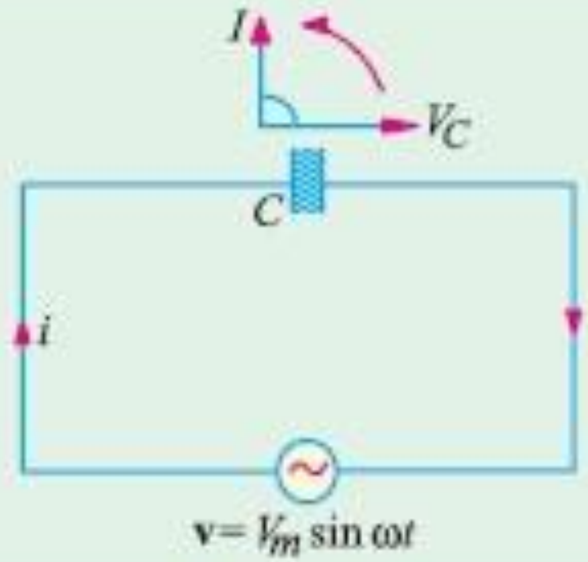
When an alternating voltage is applied to the plates of a capacitor, the capacitor is charged first in one direction and then in the opposite direction. When reference to Fig. , let

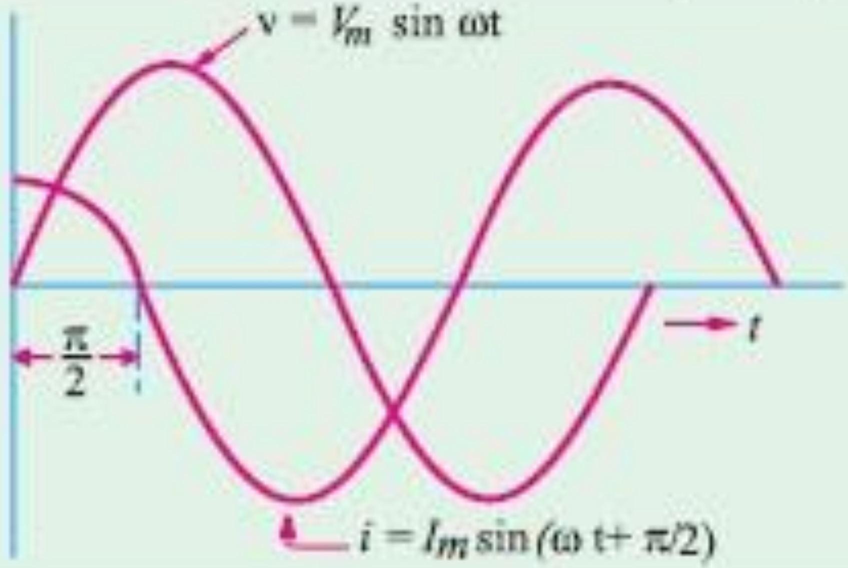
v = p.d. developed between plates at any instant

q = Charge on plates at that instant.

Then $q = Cv$ ...where C is the capacitance

$= C V_m \sin \omega t$ ...putting the value of v.

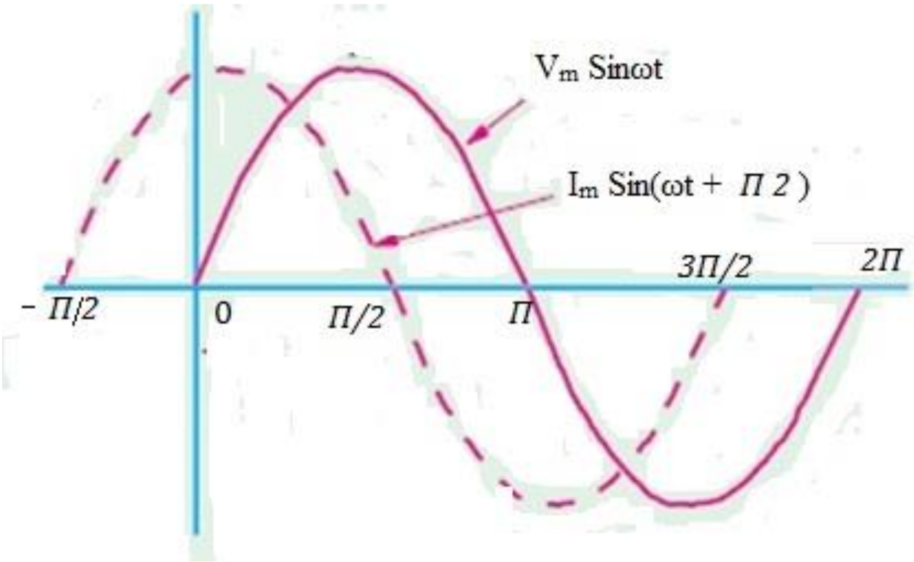




$$i = \frac{dq}{dt} \qquad = \frac{d(Cv)}{dt} \qquad = CV_m \frac{d\,\text{Sin}\omega t}{dt} \qquad = CV_m\omega\,\text{Cos}\,\omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}}\text{Sin}\,(90+\omega t) \qquad = \frac{V_m}{X_c}\,\text{Sin}(\omega t + \frac{\Pi}{2})$$

$$i = I_m\,\text{Sin}(\omega t + \frac{\Pi}{2})$$



Where, X_c = Capacitive reactance = $\frac{1}{\omega C} = \frac{1}{2\Pi f C}$

f = Supply frequency in hertz

C= capacitance in Farad

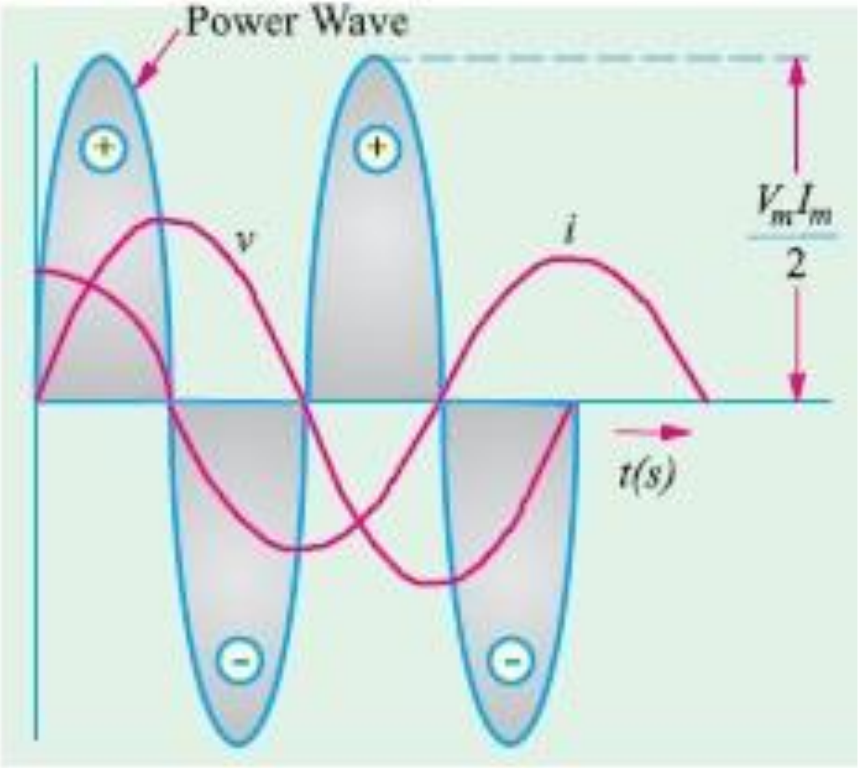
Unit of X_c is ohm.

Instantaneous Power (p) = vi

$$= V_m \operatorname{Sin}\omega t * I_m \operatorname{Sin}(\omega t + \frac{\Pi}{2})$$

$$= V_m \operatorname{Sin}\omega t * I_m \operatorname{Cos}\omega t$$

$$= \frac{V_m\, Im}{2} \operatorname{Sin} 2\omega t$$

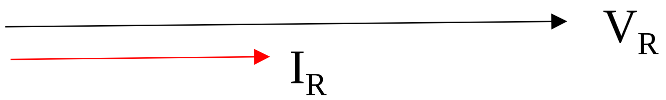


Power for the whole cycle

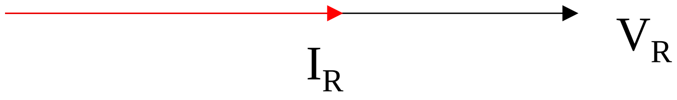
$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

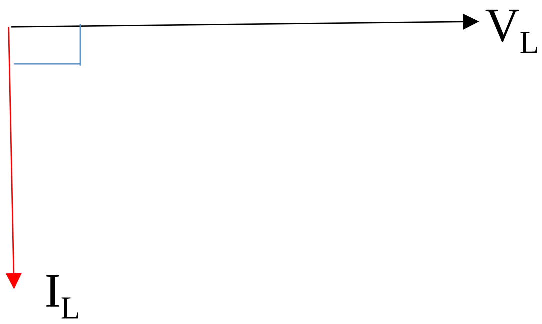
This fact is graphically illustrated in Fig. . We find that in a purely capacitive circuit , the average demand of power from supply is zero (as in a purely inductive circuit). Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $V_m I_m/2$.

Note : CIVIL

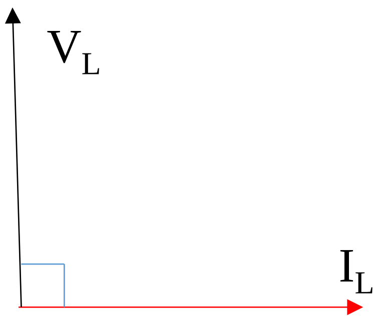


Pure Resistance

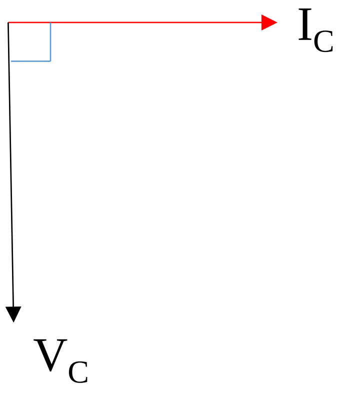




Pure Inductance



Pure Capacitance

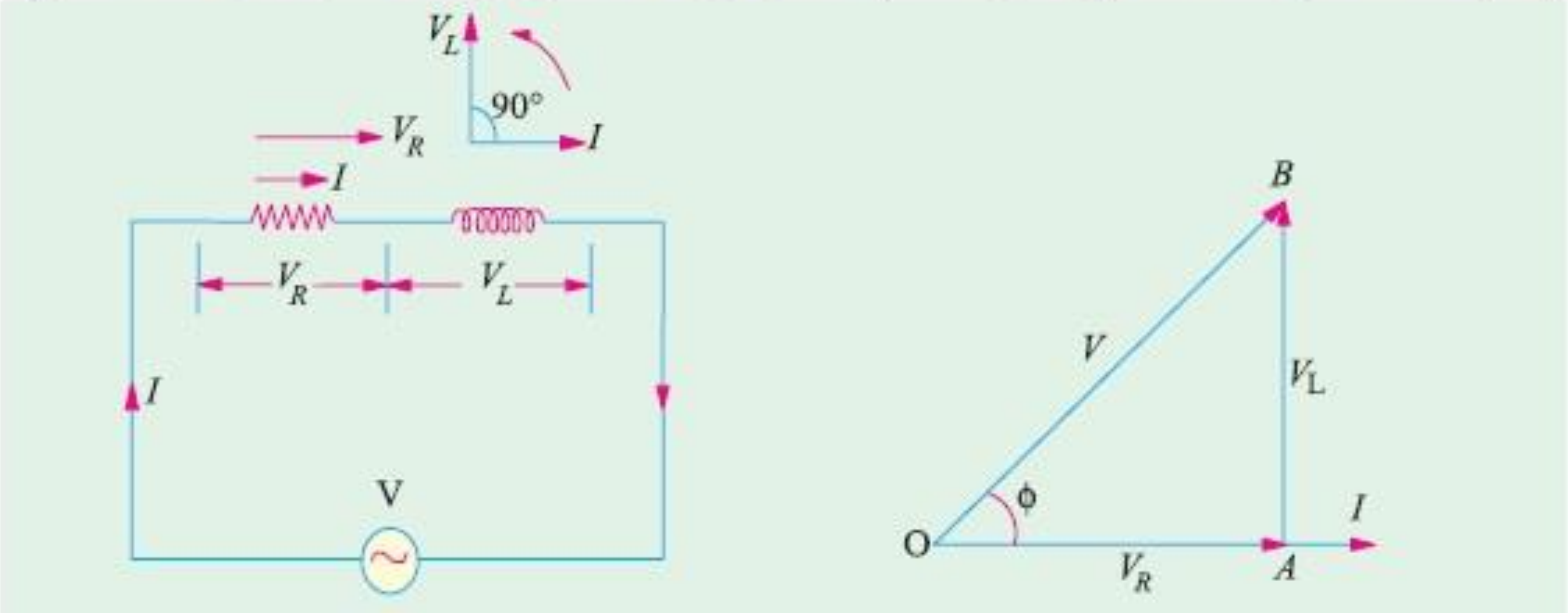


A.C. Through Resistance and Inductance

A pure resistance R and a pure inductive coil of inductance L are shown connected in series in Fig.

Let V = r.m.s. value of the applied voltage, I = r.m.s. value of the resultant current

$V_R = IR$ –voltage drop across R (in phase with I), $V_L = I \,.\, X_L$ –voltage drop across coil (ahead of I by 90°)



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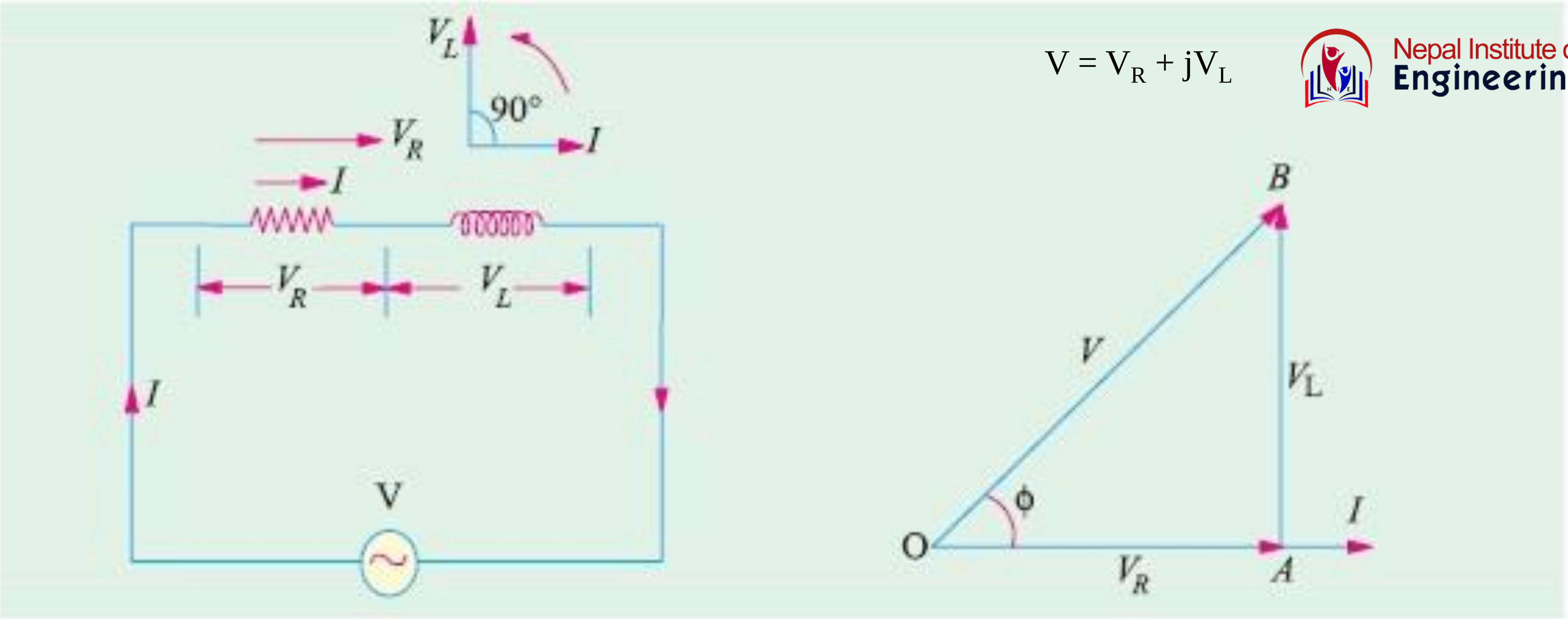
$V = V_R + jV_L$



These voltage drops are shown in voltage triangle *OAB* in Fig. Vector *OA* represents ohmic drop V_R and *AB* represents inductive drop V_L. The applied voltage V is the vector sum of the two *i.e.* *OB*.

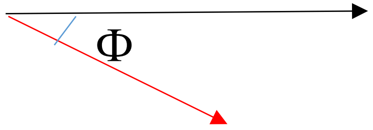
$$\therefore \quad V = \sqrt{(V_R^2 + V_L^2)} = \sqrt{[(IR)^2 + (I \cdot X_L)^2]} = I\sqrt{R^2 + X_L^2}, \quad \frac{V}{\sqrt{(R^2 + X_L^2)}} = I$$

The quantity $\sqrt{(R^2 + X_L^2)}$ is known as the *impedance* (*Z*) of the circuit.

$Z = R + jX_L$

From Fig. it is clear that the applied voltage V leads the current I by an angle ϕ such that

$$\tan\phi = \frac{V_L}{V_R} = \frac{I \cdot X_L}{I \cdot R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{reactance}} \quad \therefore \phi = \tan^{-1}\frac{X_L}{R}$$



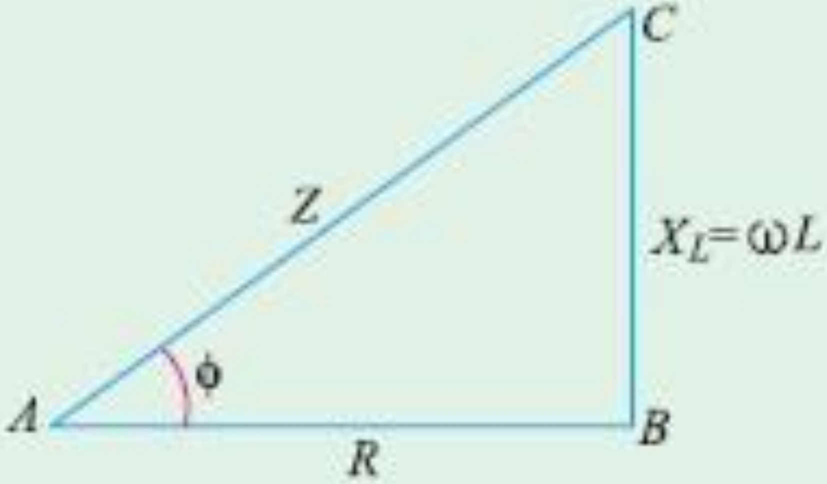
The same fact is illustrated graphically in Fig.

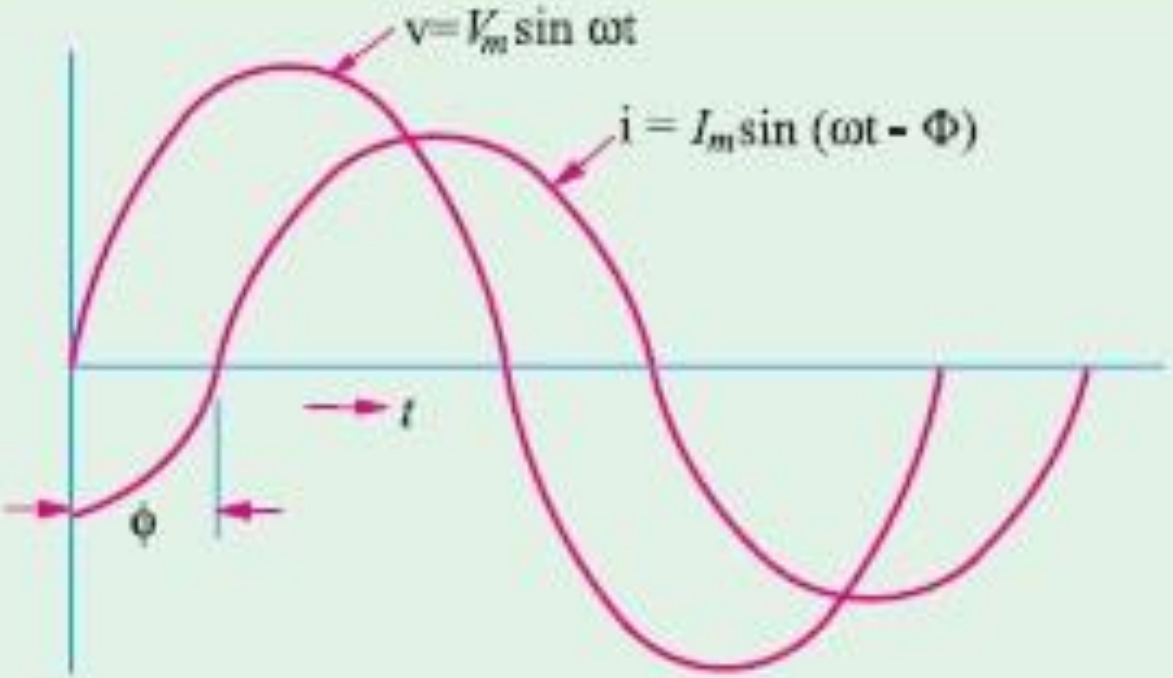
In other words, current I lags behind the applied voltage V by an angle ϕ.

Hence, if applied voltage is given by $v = V_m \sin \omega t$, then current equation is

$$i = I_m \sin(\omega t - \phi) \text{ where } I_m = V_m / Z$$

$Z = R + jX_L$



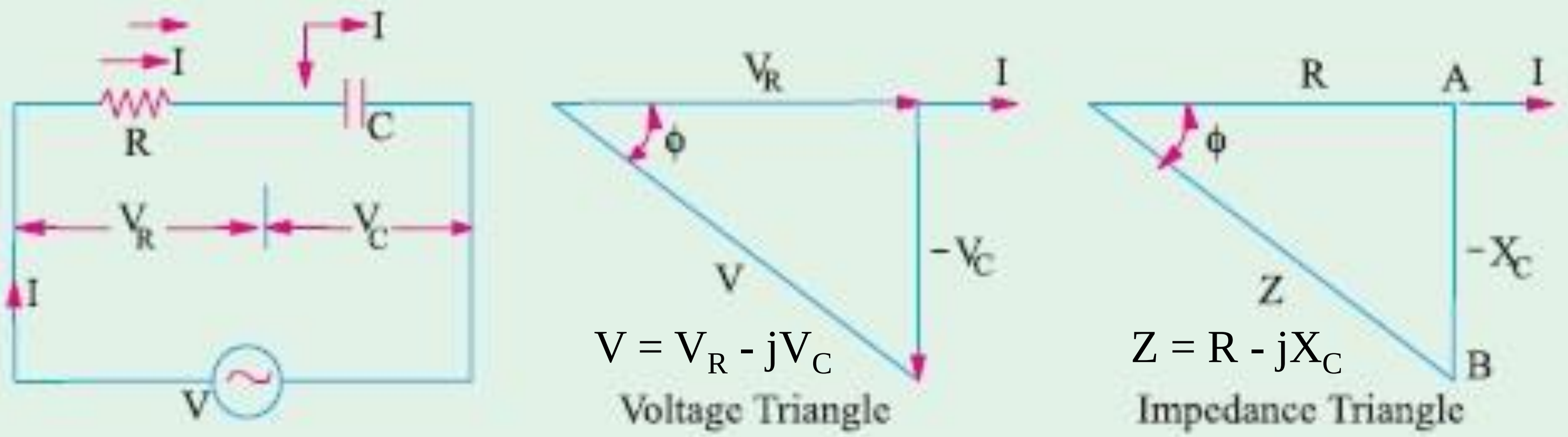


A.C. Through Resistance and Capacitance

The circuit is shown in Fig. (*a*). Here $V_R = IR$ = drop across R in phase with I.

$V_C = IX_C$ = drop across capacitor –lagging I by $\pi/2$

As capacitive reactance X_C is taken negative, V_C is shown along negative direction of Y-axis in the voltage triangle



$V = V_R - jV_C$

Voltage Triangle

$Z = R - jX_C$

Impedance Triangle

(*a*) (*b*) (*c*)

Now $V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2} = I\sqrt{R^2 + X_C^2}$ or $I = \dfrac{V}{\sqrt{R^2 + X_C^2}} = \dfrac{V}{Z}$

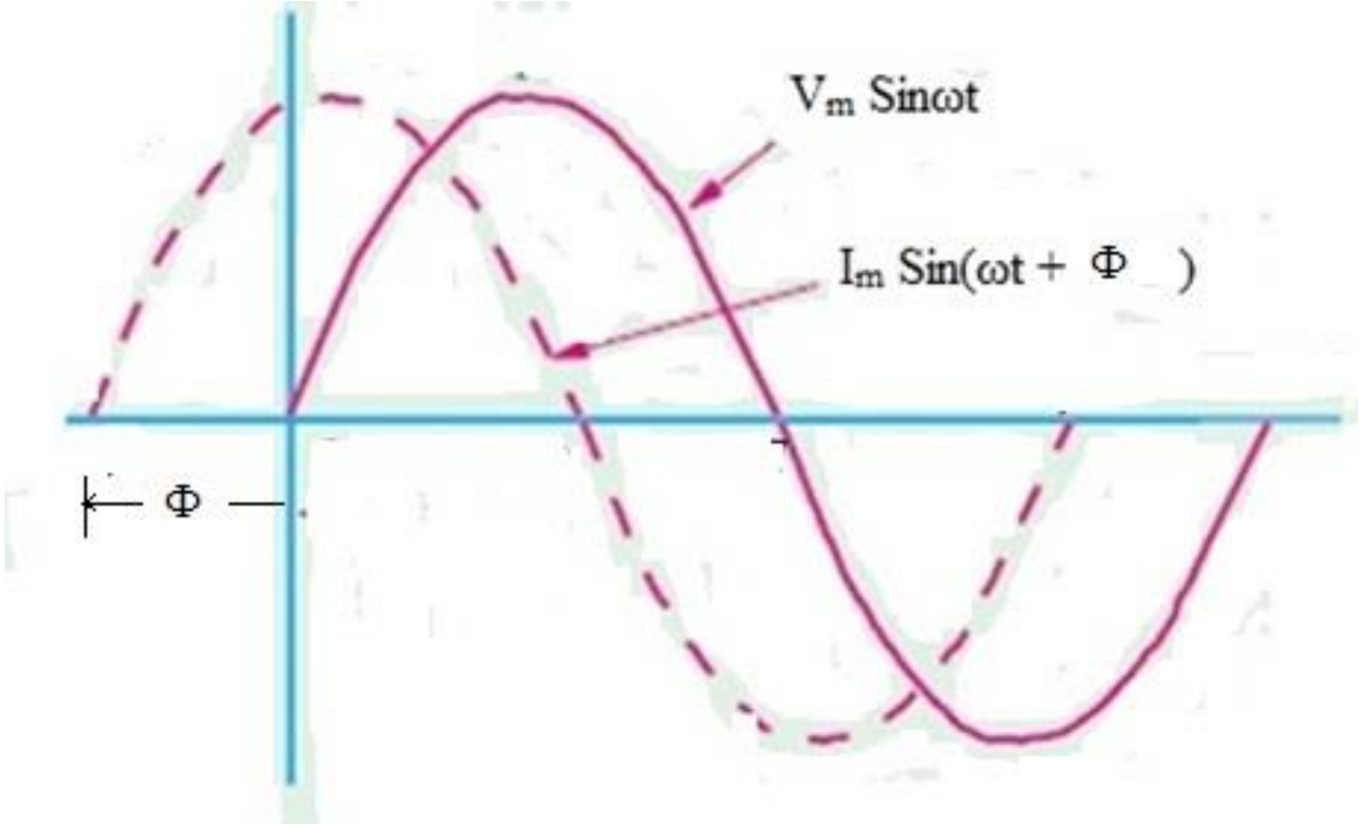
The denominator is called the *impedance* of the circuit. So, $Z = \sqrt{R^2 + X_C^2}$

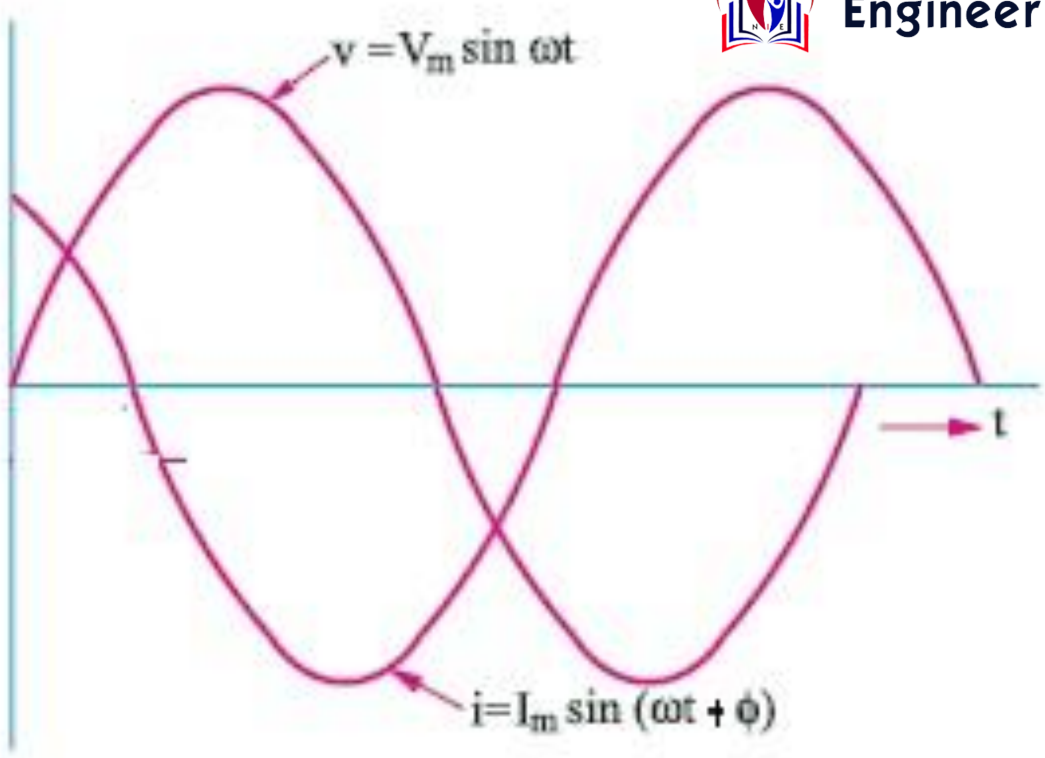
$Z = R - jX_C$

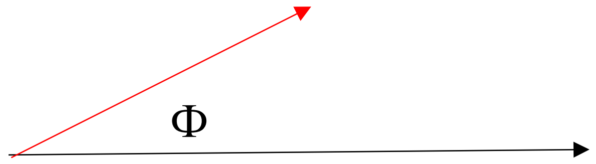
Impedance triangle is shown in Fig. (*c*)

From Fig. (*b*) it is found that I leads V by angle ϕ such that $\tan\phi = -X_C/R$

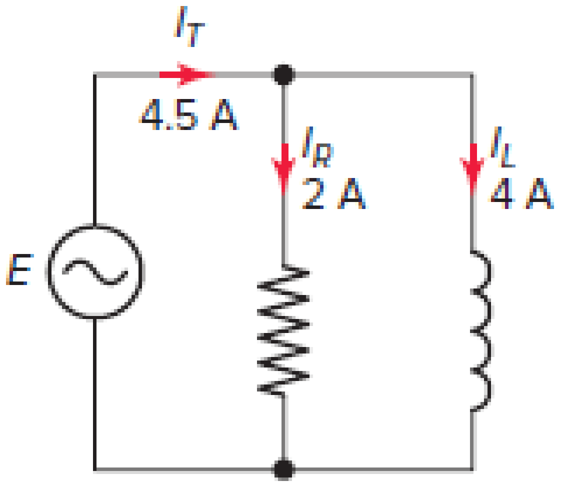
Hence, it means that if the equation of the applied alternating voltage is $v = V_m \sin \omega t$, the equation of the resultant current in the *R-C* circuit is $i = I_m \sin(\omega t + \phi)$ so that current *leads* the applied voltage by an angle ϕ. This fact is shown graphically in Fig

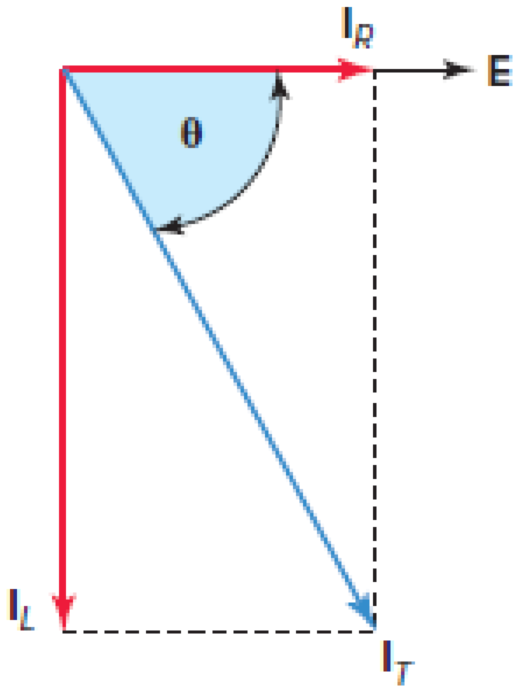






Parallel RL Circuit





$$E_T = E_R = E_L$$

$$I_R = \frac{E}{R}$$

$$I_L = \frac{E}{X_L}$$

$$I_T = \sqrt{I_R^2 + I_L^2}$$

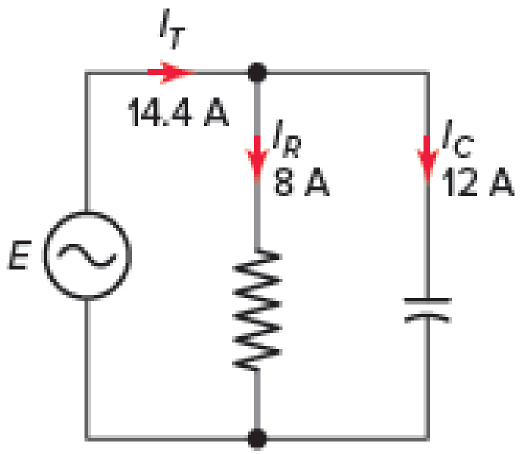
$$Z = \frac{RX_L}{\sqrt{R^2 + X_L^2}}$$

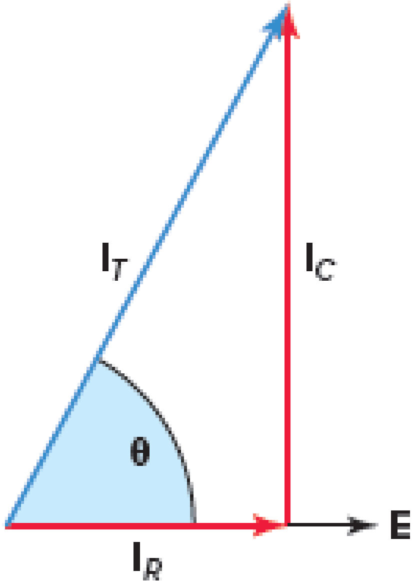
$$\theta = \tan^{-1}\frac{I_L}{I_R}$$

$$\text{PF} = \frac{I_R}{I_T}$$

- when a resistor and inductor are connected in parallel, the two currents will be out of phase with each other. In this case, the total current is equal to the vector sum rather than the arithmetic sum of the currents.
- In all parallel RL circuits, the phase angle theta (θ) by which the total current lags the voltage is somewhere between 0 and 90 degrees. The size of the angle is determined by whether there is more inductive current or resistive current.
- If there is more inductive current, the phase angle will be closer to 90 degrees. It will be closer to 0 degrees if there is more resistive current.
- The impedance (Z) of a parallel RL circuit is the total opposition to the flow of current. It includes the opposition (R) offered by the resistive branch and the inductive reactance (XL) offered by the inductive branch.
- The impedance of a parallel RL circuit is calculated similarly to a parallel resistive circuit. However, since XL and R are vector quantities, they must be added vectorially.
- The impedance of a parallel RL circuit is always less than the resistance or inductive reactance of any one branch. This is because each branch creates a separate path for current flow, thus reducing the overall or total circuit opposition to the current flow.

Parallel RC Circuit





$$\begin{aligned} I_T &= \sqrt{I_R^2 + I_C^2} \\ &= \sqrt{8^2 + 12^2} \\ &= \sqrt{208} \\ &= 14.4 \text{ A} \end{aligned}$$

$$Z = \frac{RX_C}{\sqrt{R^2 + X_C^2}}$$

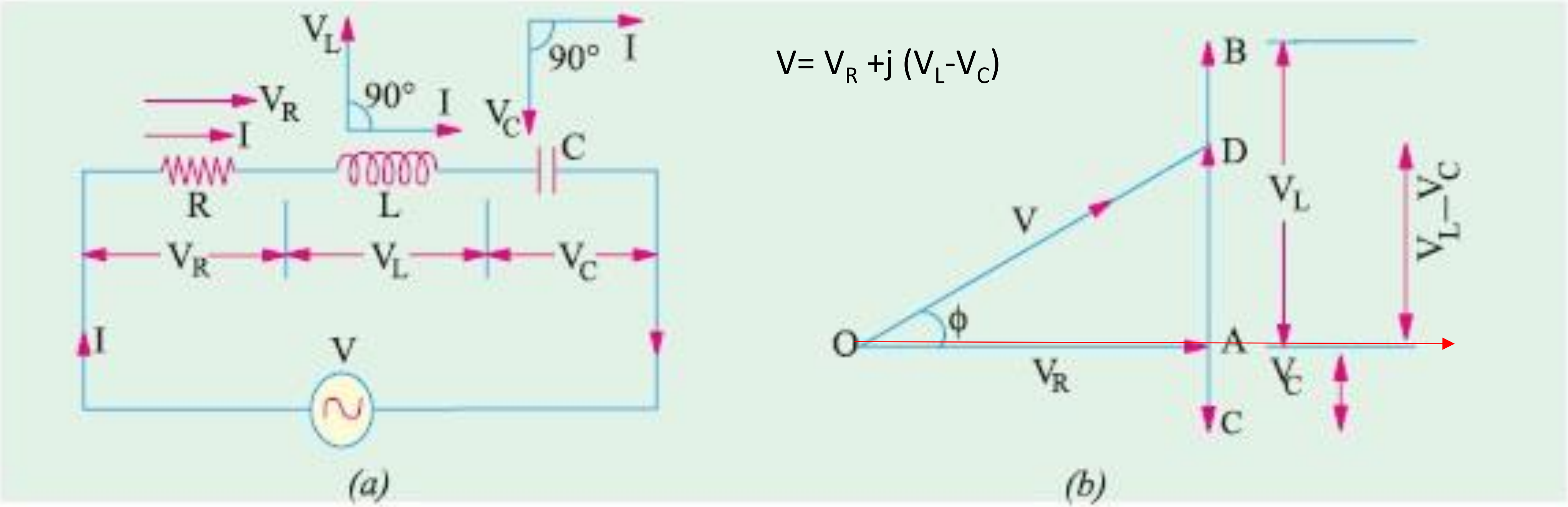
$$\theta = \tan^{-1} \frac{I_C}{I_R}$$

$$\text{Power factor} = \frac{I_R}{I_T}$$

$$\text{Power factor} = \frac{Z}{R}$$

- In a parallel RC circuit, the line current leads the applied voltage by some phase angle less than 90 degrees but greater than 0 degrees. The exact angle depends on whether the capacitive current or resistive current is greater. If there is more capacitive current, the angle will be closer to 90 degrees, while if the resistive current is greater, the angle is closer to 0 degrees.
- The impedance of a parallel RC circuit is always less than the resistance or capacitive reactance of the individual branches.
- The power factor of a parallel RC circuit is always leading. Any time the branch resistance increases, less current flows through it and the circuit becomes more capacitive, resulting in a lower power factor. The reverse is true if the resistance decreases.

Resistance, Inductance and Capacitance in Series



Let $V_R = IR$ = voltage drop across R —in phase with I

$V_L = I.X_L$ = voltage drop across L —leading I by $\pi/2$

$V_C = I.X_C$ = voltage drop across C —lagging I by $\pi/2$

In voltage triangle of Fig. (b), OA represents V_R, AB and AC represent the inductive and capacitive drops respectively. It will be seen that V_L and V_C are 180° out of phase with each other *i.e.* they are in direct opposition to each other.

Subtracting BD (= AC) from AB, we get the net reactive drop $AD = I(X_L - X_C)$

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$$\therefore \quad OD = \sqrt{OA^2 + AD^2} \quad \text{or} \quad V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\text{or} \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The term $\sqrt{R^2 + (X_L - X_C)^2}$ is known as the impedance of the circuit. Obviously,

$$(impedance)^2 = (resistance)^2 + (net\ reactance)^2$$

$$\text{or} \quad Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$$

Or, $Z = R + j(X_L - X_C)$

Type of Impedance	*Value of Impedance*	*Phase angle for current*	*Power factor*
Resistance only	R	0°	1
Inductance only	ωL	90° lag	0
Capacitance only	$1/\omega C$	90° lead	0
Resistance and Inductance	$\sqrt{[R^2 + (\omega L)^2]}$	$0 < \phi < 90°$ lag	$1 > \text{p.f.} > 0$ lag
Resistance and Capacitance	$\sqrt{[R^2 + (-1/\omega C)^2]}$	$0 < \phi < 90°$ lead	$1 > \text{p.f.} > 0$ lead
R-L-C	$\sqrt{[R^2 + (\omega L \sim 1/\omega C)^2]}$	between 0° and 90° lag or lead	between 0 and unity lag or lead

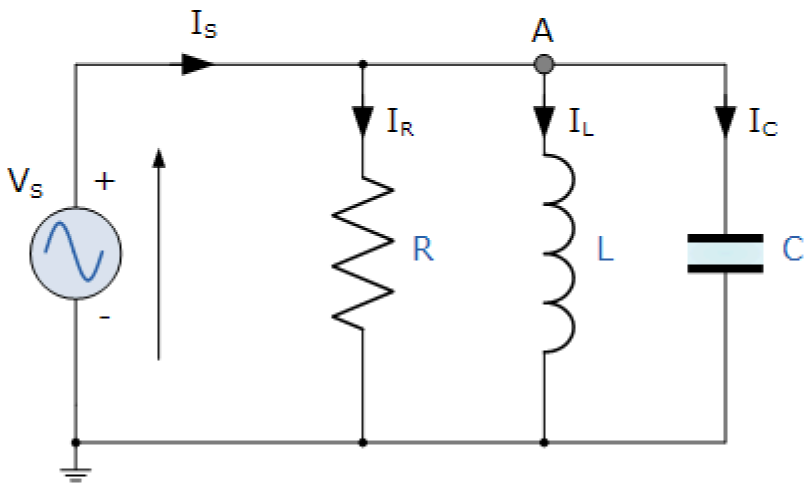
Parallel R-L-C Circuit

$$I_S^2 = I_R^2 + \left(I_L - I_C\right)^2$$

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2}$$

$$\therefore \; I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

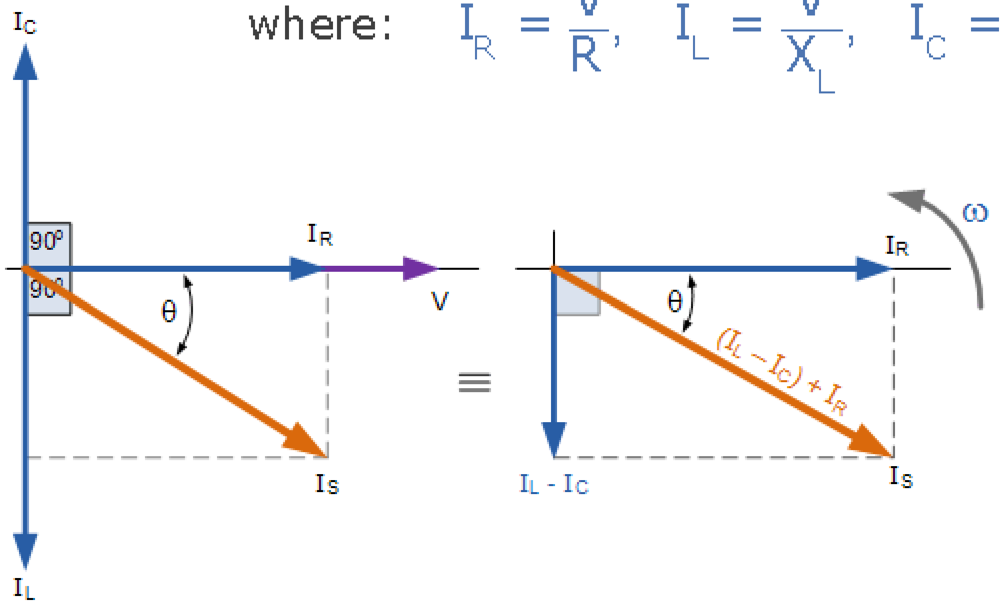
where: $I_R = \frac{V}{R}, \quad I_L = \frac{V}{X_L}, \quad I_C = \frac{V}{X_C}$



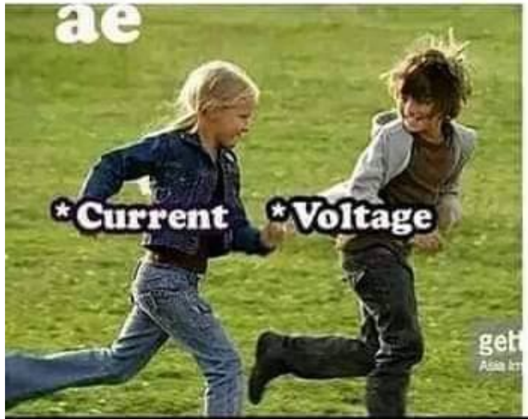
$$R = \frac{V}{I_R} \quad X_L = \frac{V}{I_L} \quad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

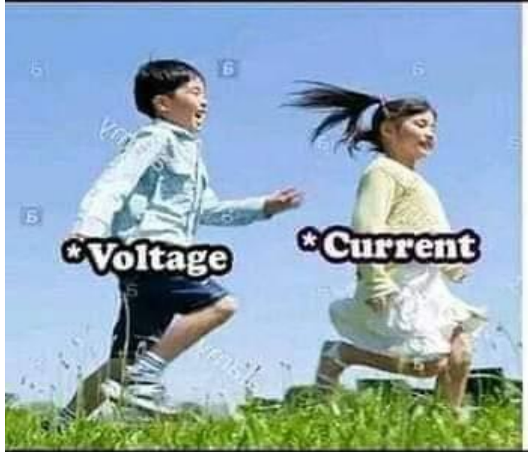
$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$



Current- Voltage Relationship for different types of load



Inductive Load



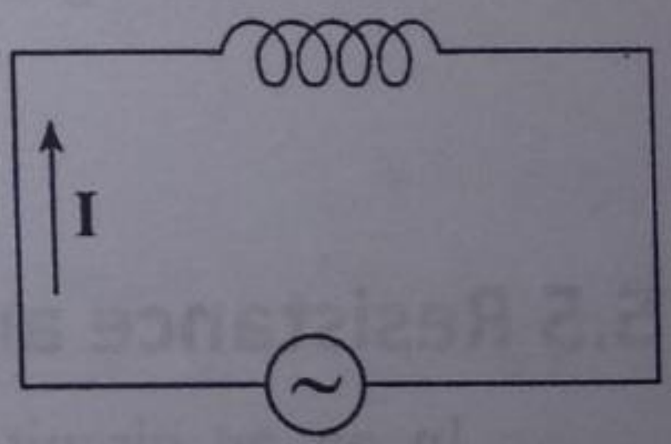
Capacitive Load

Resistive Load

An emf of 200 V at a frequency of 2 kHz is applied to a coil of pure inductance 50 mH. Determine a) the reactance of the coil, b) the current flowing in the coil.

Solution:

Given, supply voltage $V_s = 200$ V



Frequency $f = 2$ kHz $= 2 \times 10^3$ Hz

Inductance of coil $L = 50$ mH $= 50 \times 10^{-3}$ H

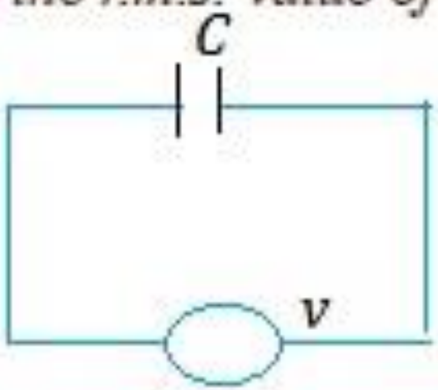
Inductive reactance $X_L = 2\pi fL$

$$= 2\pi \times 2 \times 10^3 \times 50 \times 10^{-3}$$

$$= 628\ \Omega$$

Current $I = \dfrac{V}{X_L} = \dfrac{200}{628} = 0.3185$ A

Example *A 50-μF capacitor is connected across a 230-V, 50-Hz supply. Calculate (a) the reactance offered by the capacitor (b) the maximum current and (c) the r.m.s. value of the current drawn by the capacitor.*



Solution. (*a*) $X_C = 1/2\Pi fC = \dfrac{1}{2\Pi * 50 * 50 * 10^{-6}} = \mathbf{63.6\ \Omega}$

(*c*) Since 230 V represents the r.m.s. value,

$\therefore \quad I_{r.m.s.} = 230/X_c = 230/63.6 = \mathbf{3.62\ A}$ (*b*) $I_m = I_{r.m.s.} \times \sqrt{2} = 3.62 \times \sqrt{2} = \mathbf{5.11\ A}$

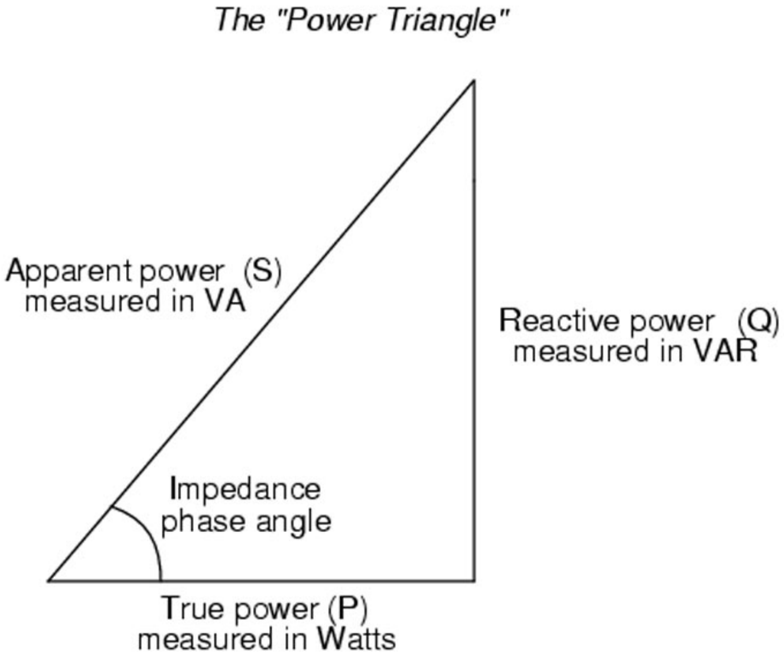
Power in AC Circuits

- Active Power (P) : It is the power which is actually dissipated in the circuit resistance. $P = I^2R = VI\cos\Theta$. Its unit is Watt.
- Reactive Power (Q) = It is the power developed in the reactance of the circuit. $Q= I^2X_L$ or $Q= I^2X_C$ or $Q= VI\sin\Theta$. Its unit is VAR.
- Apparent Power (S) = It is given by the product of rms value of applied voltage and circuit current. $S= VI$. Its unit is VA.

$$S^2 = P^2 + Q^2$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{(VI\cos\Theta)^2 + (VI\sin\Theta)2}$$

$$S= VI$$



- Reactive power does not do any work, so it is represented as the imaginary axis of the vector diagram. Active power does do work, so it is the real axis.
- The power which is actually consumed or utilised in an AC Circuit is called **True power** or **Active power** or **Real power**. It is measured in kilowatt (kW) or MW. It is the actual outcomes of the electrical system which runs the electric circuits or load.

Beer Analogy of Active, Reactive & Apparent Power

Power factor and its significance

- Power factor is the cosine of angle of lead or lag. It is given by the cosine of angle between voltage and current. i.e. Power factor = CosΘ

Where, Θ = Angle between voltage and current

- Power factor is also given by the ratio of active power to apparent power.

i.e. p.f. = $\frac{Active\ Power}{Apparent\ power}$

- Power factor is also equal to ratio of resistance(R) to the impedance (Z) of the circuit.

i.e. p.f. = $\frac{Resistance(R)}{Impedance\ (Z)}$

significance

- As We know, $P = VI \cos\Theta$

$$I = \frac{P}{V \cos\Theta}$$

From above equation, it is seen that, for constant P and Constant V, current varies inversely as the load power factor. Thus a given load takes more current at low power factor than it does at higher power factor.

Higher the power factor, lower the current and hence

- Size of conductor required for transmission and distribution will be small.
- Line losses (I^2R) will be less and so efficiency of the system will be high.
- Voltage drop in line will decreases and so voltage regulation will increases.

The power factor of an ac system should be as high as possible so that current and applied voltage are bought as near as possible.

Resonance is series RLC circuit

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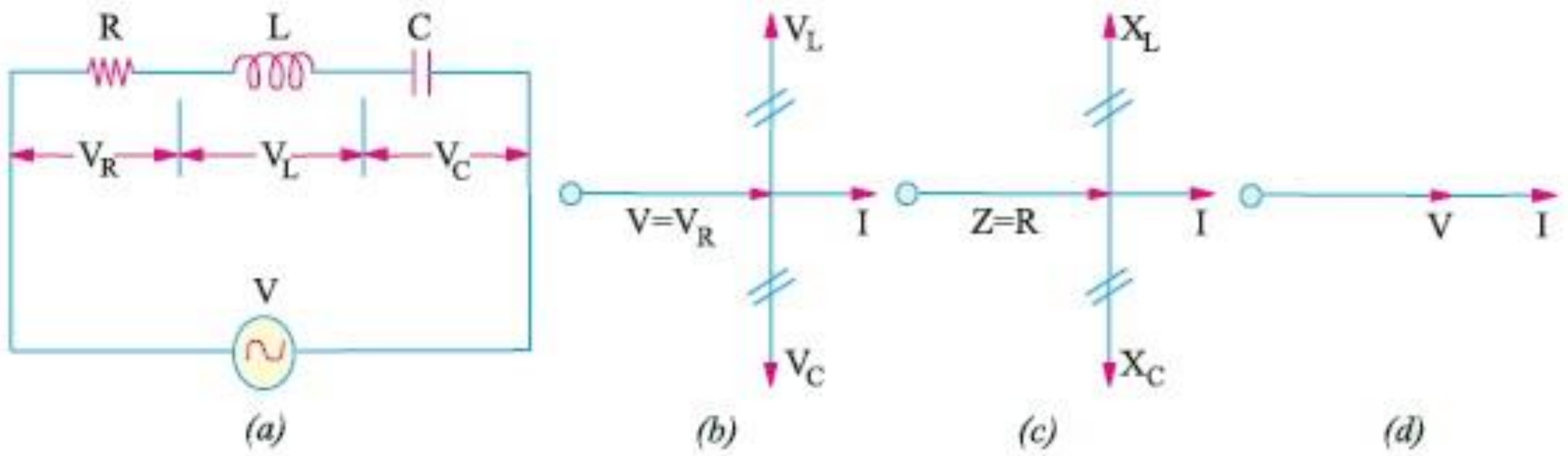
- In series RLC Circuit, Resistance is independent of frequency but inductive reactance and capacitive reactance is frequency dependent. As the frequency increases, inductive reactance ($X_L = 2\,\Pi fL$) will increases whereas capacitive reactance ($X_C = \frac{1}{2\Pi fC}$) will decreases. At a certain frequency the magnitude of X_L becomes equal to X_C . That condition at which X_L is equal to X_C is called electrical resonance and the corresponding frequency at which X_L is equal to X_C is called resonant frequency.
- At resonance, X_L=X_C

$$2\,\Pi f_0 L = \frac{1}{2\Pi f_0\, C}$$

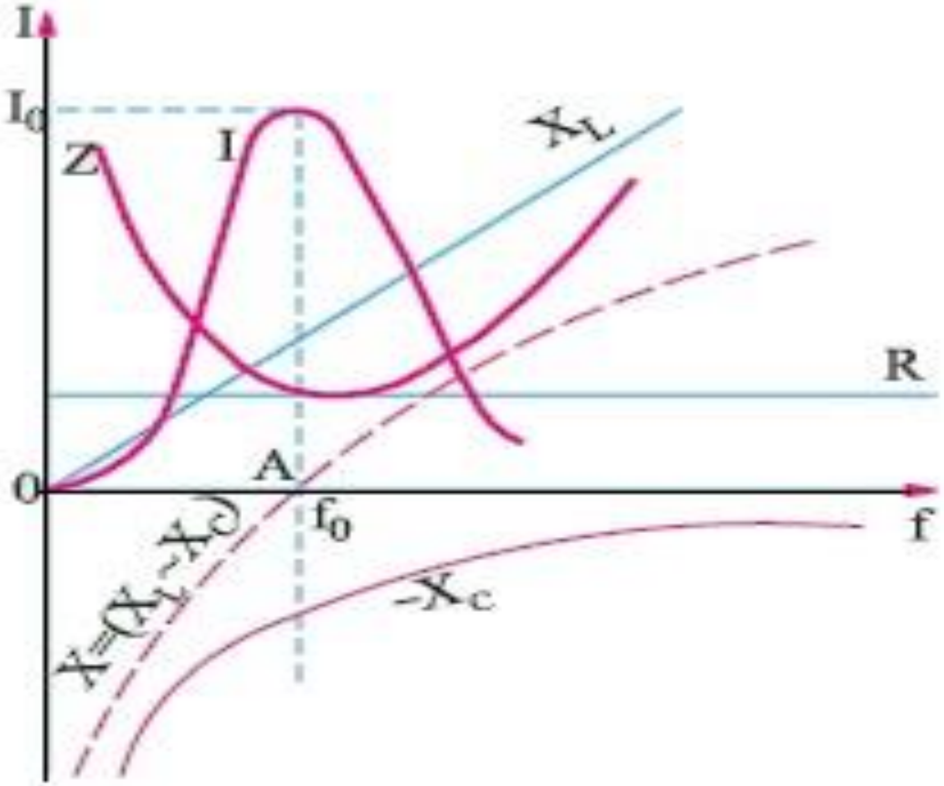
Resonant frequency,

$$f_0 = \frac{1}{2\Pi\sqrt{LC}}$$



When an *R-L-C* circuit is in resonance

1. net reactance of the circuit is zero *i.e.* $(X_L - X_C) = 0$. or $X = 0$.
2. circuit impedance is minimum *i.e.* $Z = R$. Consequently, circuit admittance is maximum.
3. circuit current is maximum and is given by $I_0 = V/Z_0 = V/R$.
4. power dissipated is maximum *i.e.* $P_0 = I_0^2 R = V^2/R$.
5. circuit power factor angle $\theta = 0$. Hence, power factor $\cos\theta = 1$.

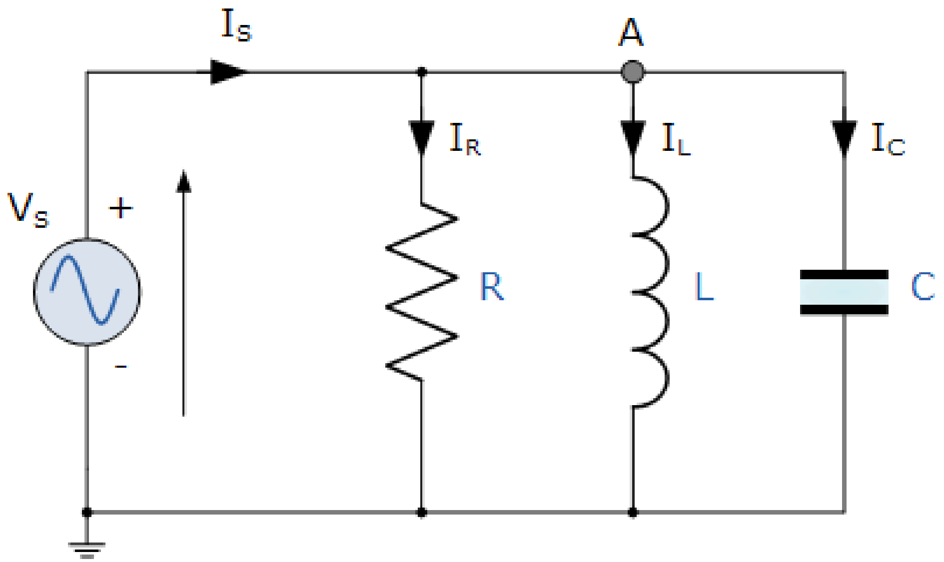


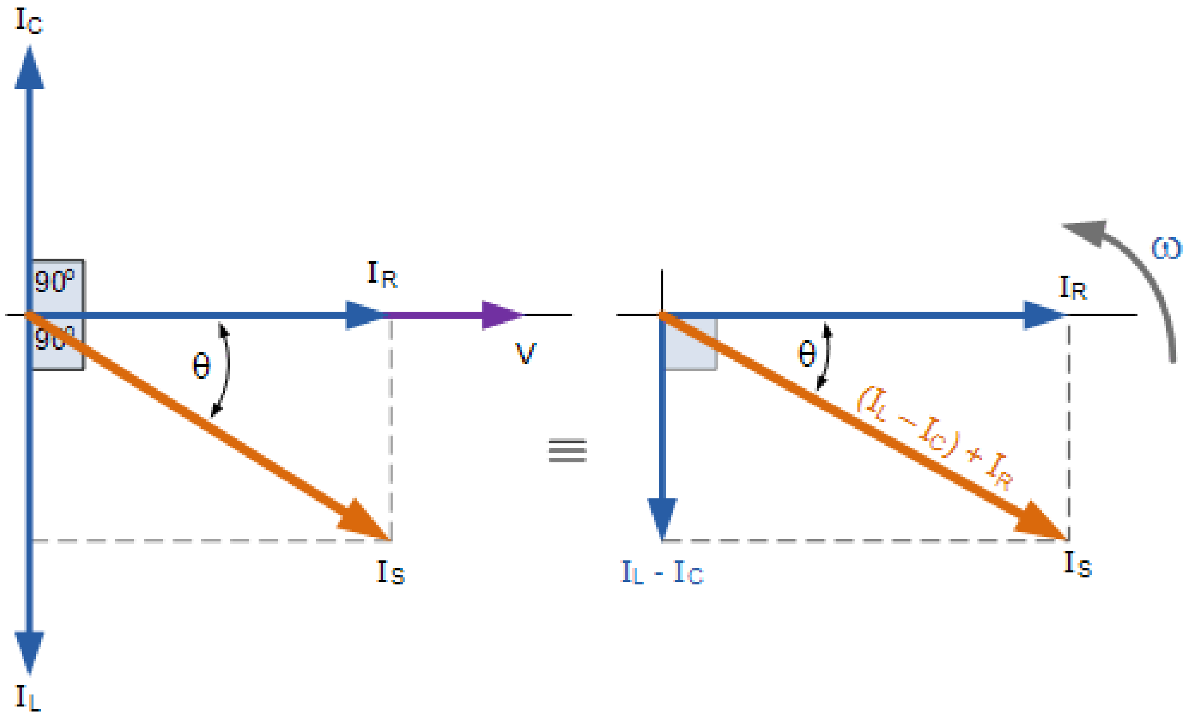
$$Z = R + j(X_L - X_C)$$

$$I = V/Z$$

Resonance in Parallel R-L-C

- A parallel RLC circuit contains a resistor (R), an inductor (L), and a capacitor (C) connected in parallel.
- Resonance in a parallel RLC circuit occurs when the reactive effects of the inductor and capacitor cancel each other out, resulting in a purely resistive circuit.





- At resonance there will be a large circulating current between the inductor and the capacitor due to the energy of the oscillations, then parallel circuits produce current resonance.
- IL and IC will always be equal and opposite and therefore the current drawn from the supply is the vector addition of these two currents and the current flowing in IR.
- at resonance the parallel LC tank circuit acts like an open circuit with the circuit current being determined by the resistor, R only.
- So the total impedance of a parallel resonance circuit at resonance becomes just the value of the resistance in the circuit and Z = R.
- the impedance of the parallel circuit is at its maximum value and equal to the resistance of the circuit creating a circuit condition of high resistance and low current.

- the total circuit current, I will be “in-phase” with the supply voltage, VS.
- IL and IC will always be equal and opposite and therefore the current drawn from the supply is the vector addition of these two currents and the current flowing in IR.
- at resonance the parallel LC tank circuit acts like an open circuit with the circuit current being determined by the resistor, R only.
- So the total impedance of a parallel resonance circuit at resonance becomes just the value of the resistance in the circuit and Z = R.
- the impedance of the parallel circuit is at its maximum value and equal to the resistance of the circuit creating a circuit condition of high resistance and low current.

Root-Mean-Square (R.M.S.) Value

The r.m.s. value of an alternating current *is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same* heat *as produced by the alternating current when flowing through the same circuit for the same time.*

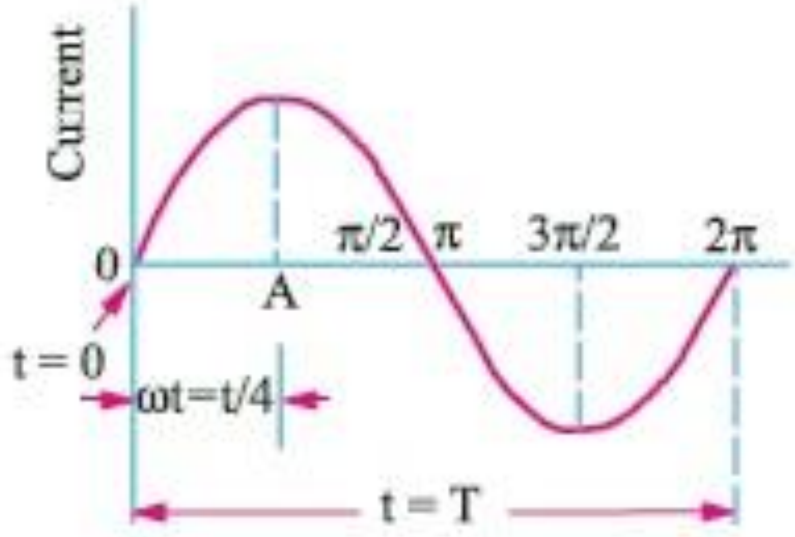
It is also known as the *effective* or *virtual* value of the alternating current.

The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$.

The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do).

$$= \int_0^{2\pi} \frac{i^2 \, d\theta}{(2\pi - 0)}$$

The square root of this value is $= \sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)}$



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Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta\right)} \qquad (\text{put } i = I_m \sin\theta)$$

Now, $\cos 2\theta = 1 - 2\sin^2\theta \quad \therefore \sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$\therefore \qquad I = \sqrt{\left(\frac{I_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\, d\theta\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{2\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{4} \quad 2} \qquad \sqrt{\frac{I_m^2}{2}} \qquad \therefore \boxed{\mathrm{I} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m}$$

Hence, we find that for a symmetrical sinusoidal current

r.m.s. value of current = 0.707 × max. value of current

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively.

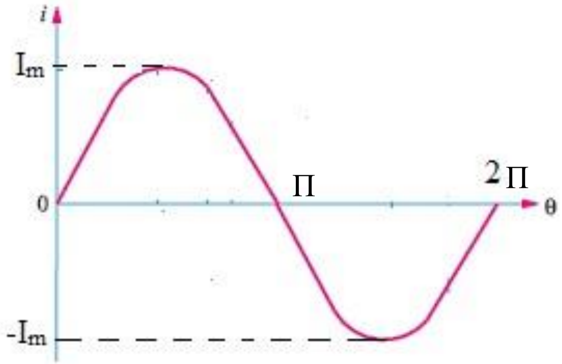
Average Value

The average value I_a of an alternating current is expressed *by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.*

In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. *But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.*

The standard equation of an alternating current is, $i = I_m \sin\theta$

$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$



(putting value of i)

$$= \frac{I_m}{\pi}\left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{\pi}\left| +1-(-1) \right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$$\therefore \qquad I_{av} = I_m / \tfrac{1}{2}\pi = 0.637\, I_m \quad \therefore \textbf{ average value of current} = \mathbf{0.637 \times} \textbf{ maximum value}$$

Note. R.M.S. value is always greater than average value except in the case of a rectangular wave when both are equal.

Form Factor

It is defined as the ratio, $K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{0.707\, I_m}{0.637\, I_m} = 1.1.$ (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = \frac{0.707\, E_m}{0.637\, E_m} = 1.11$

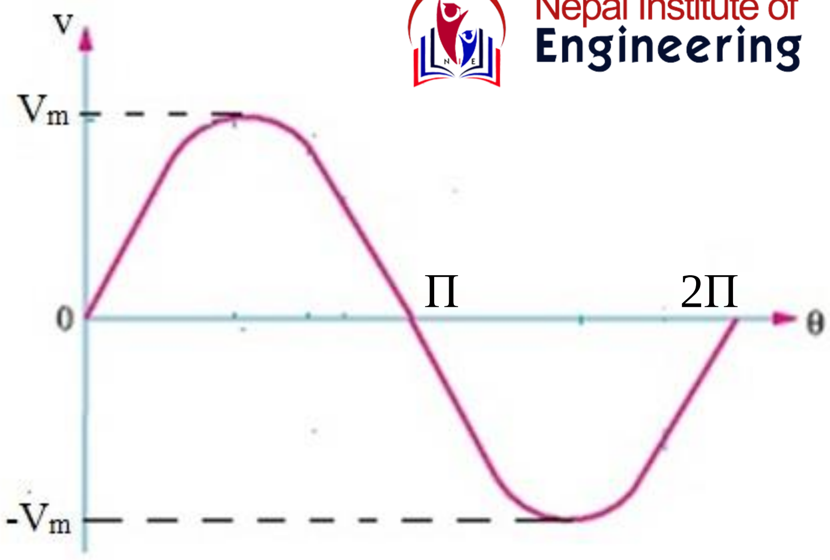
As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and *vice-versa.*

Crest or Peak or Amplitude Factor

It is defined as the ratio $K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$ (for sinusoidal a.c. only)

For sinusoidal alternating voltage also, $K_a = \frac{E_m}{E_m/\sqrt{2}} = 1.414$

Find average value, RMS value, form factor and peak factor of sinusoidal voltage wave as shown in figure.



Solution,

As sine wave is a symmetrical alternating quantity and its average value over a complete cycle is zero. Therefore inorder to find the average value we have to take the half cycle.

Time period for half cycle, (T) = Π

Expression for alternating voltage $v(\Theta) = V_m \, \mathrm{Sin}\Theta$

Now, Average value $(V_{avg}) = \frac{1}{T}\int_0^T v(\Theta)d\Theta$

$= \frac{1}{\Pi}\int_0^{\Pi} V_m \, \mathrm{Sin}\Theta \, d\Theta$

$= \frac{V_m}{\Pi}\int_0^{\Pi} \mathrm{Sin}\Theta \, d\Theta$ $\quad = \frac{V_m}{\Pi}\{-cos\Theta\}^{\Pi}{}_{0}$

$$= \frac{V_m}{\Pi}\{-cos\Pi - (-coso)\} \qquad = \frac{V_m}{\Pi}\{-(-1) - (-1)\}$$

$$V_{avg} = \frac{2Vm}{\Pi}$$

$$\text{RMS value } (V_{RMS}) = \sqrt{\frac{1}{T}\int_0^T v^2(\Theta)d\Theta}$$

$$= \sqrt{\frac{1}{\Pi}\int_0^{\Pi}(V_m Sin\Theta)^2\, d\Theta} \qquad = \sqrt{\frac{{V_m}^2}{\Pi}\int_0^{\Pi}(\,Sin^2\Theta d\Theta)}$$

$$= \sqrt{\frac{{V_m}^2}{2\Pi}\int_0^{\Pi}(\,1-Cos2\Theta)d\Theta} \qquad = \sqrt{\frac{{V_m}^2}{2\Pi}\{\Theta - \frac{sin2\Theta}{2}\}^{\Pi}{}_0}$$

$$V_{RMS} = \sqrt{\frac{V_m^{\ 2}}{2\Pi}\left\{\Pi - \frac{\sin 2\Pi}{2} - 0 + \frac{\sin 0}{2}\right\}}$$

$$V_{RMS} = \sqrt{\frac{V_m^{\ 2}}{2}} \qquad = \frac{V_m}{\sqrt{2}}$$

Next Method:

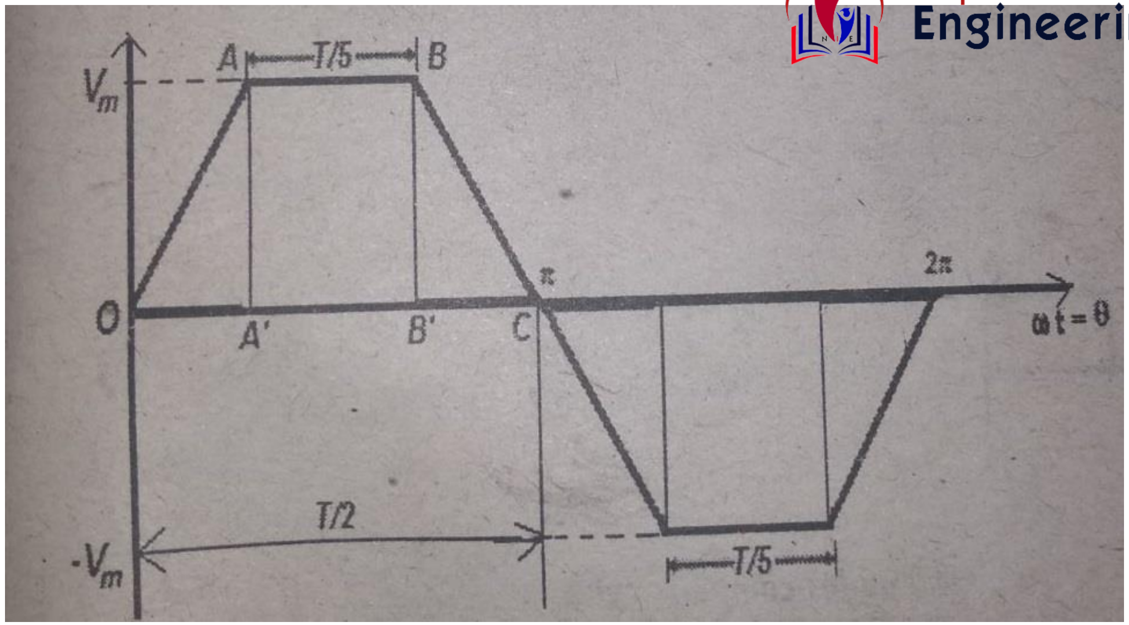
$$\text{Average Value} = \frac{\textit{Area under the wave under consideration}}{\textit{Time considered}}$$

$$\text{RMS Value} = \sqrt{\frac{\textit{Area under the squared wave under consideration}}{\textit{Time considered}}}$$

$$\text{Area under squared wave of triangle} = \frac{1}{3} * \text{base} * \text{height}^2$$

$$\text{Area under squared wave of rectangle} = \text{base} * \text{height}^2$$

Find effective value, and mean value of given wave



From figure, A'B' = T/5 OC = T/2

B'C= OA' = 3T/20

Area under triangle OAA' = $\frac{1}{2} * Base * Height$

$$= \frac{1}{2} * \frac{3T}{20} * Vm$$

Area under rectangle ABB'A' = $Base * Height$

$$= \frac{T}{5} * Vm$$

Area under squared wave of triangle OAA' = $\frac{1}{3}$ * base * height^2 $= \frac{1}{3} * \frac{3T}{20} * V_m{}^2$

Area under squared wave of rectangle = base * height^2 $= \frac{T}{5} * V_m{}^2$

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$$\text{Now, } V_{av} = \frac{\text{Area under the curve OABC}}{\text{Half-period}}$$

$$= \frac{\left(\frac{1}{2}\times\frac{3T}{20}\times V_m\right)+\left(\frac{T}{5}\times V_m\right)+\left(\frac{1}{2}\times\frac{3T}{20}\times V_m\right)}{\frac{T}{2}} = \frac{V_m \times T\left(\frac{3}{40}+\frac{1}{5}+\frac{3}{40}\right)}{\frac{T}{2}}$$

$$\therefore V_{av} = 0.7V_m$$

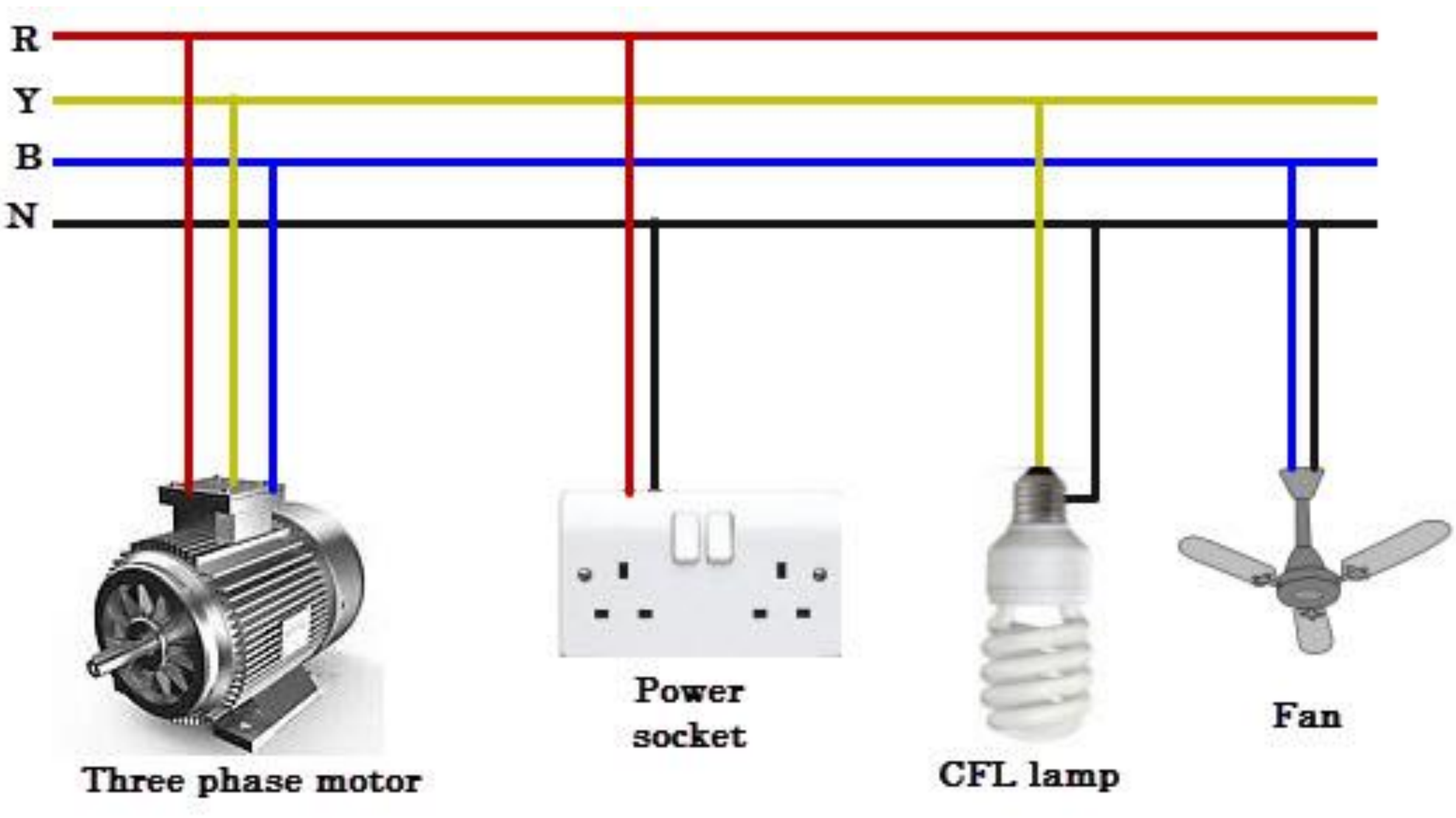
$$V_{rms} = \sqrt{\frac{\text{Area under the squared curve of half cycle}}{\text{Half}-\text{period}}}$$

$$= \sqrt{\frac{\left(\frac{1}{3}\times\frac{3T}{20}\times {V_m}^2\right)+\left(\frac{T}{5}\times {V_m}^2\right)+\left(\frac{1}{3}\times\frac{3T}{20}\times {V_m}^2\right)}{\frac{T}{2}}}$$

$$= \sqrt{2{V_m}^2\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{20}\right)} = 0.7746V_m$$

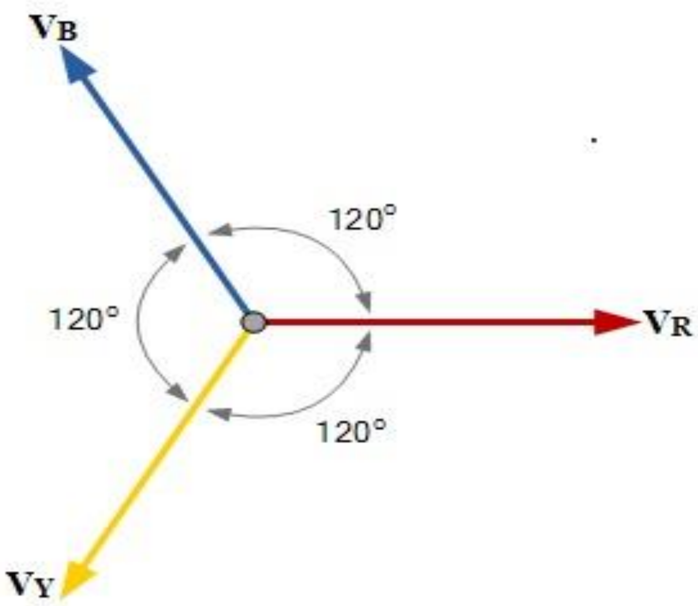
WAVEFORM	MAX. VALUE	AVERAGE VALUE	RMS VALUE	FORM FACTOR	CREST FACTOR
SINUSOIDAL WAVE	A_m	$\frac{2A_m}{\pi}$	$\frac{A_m}{\sqrt{2}}$	$\frac{\frac{A_m}{\sqrt{2}}}{\frac{2A_m}{\pi}} = 1.11$	$\frac{A_m}{\frac{A_m}{\sqrt{2}}} = \sqrt{2}$
SQUARE WAVE	A_m	A_m	A_m	$\frac{A_m}{A_m} = 1$	$\frac{A_m}{A_m} = 1$
TRIANGULAR WAVE	A_m	$\frac{A_m}{2}$	$\frac{A_m}{\sqrt{3}}$	$\frac{\frac{A_m}{\sqrt{3}}}{\frac{A_m}{2}} = \frac{2}{\sqrt{3}}$	$\frac{A_m}{\frac{A_m}{\sqrt{3}}} = \sqrt{3}$
HALF-WAVE RECTIFIED WAVE	A_m	$\frac{A_m}{\pi}$	$\frac{A_m}{2}$	$\frac{\frac{A_m}{2}}{\frac{A_m}{\pi}} = \frac{\pi}{2}$	2

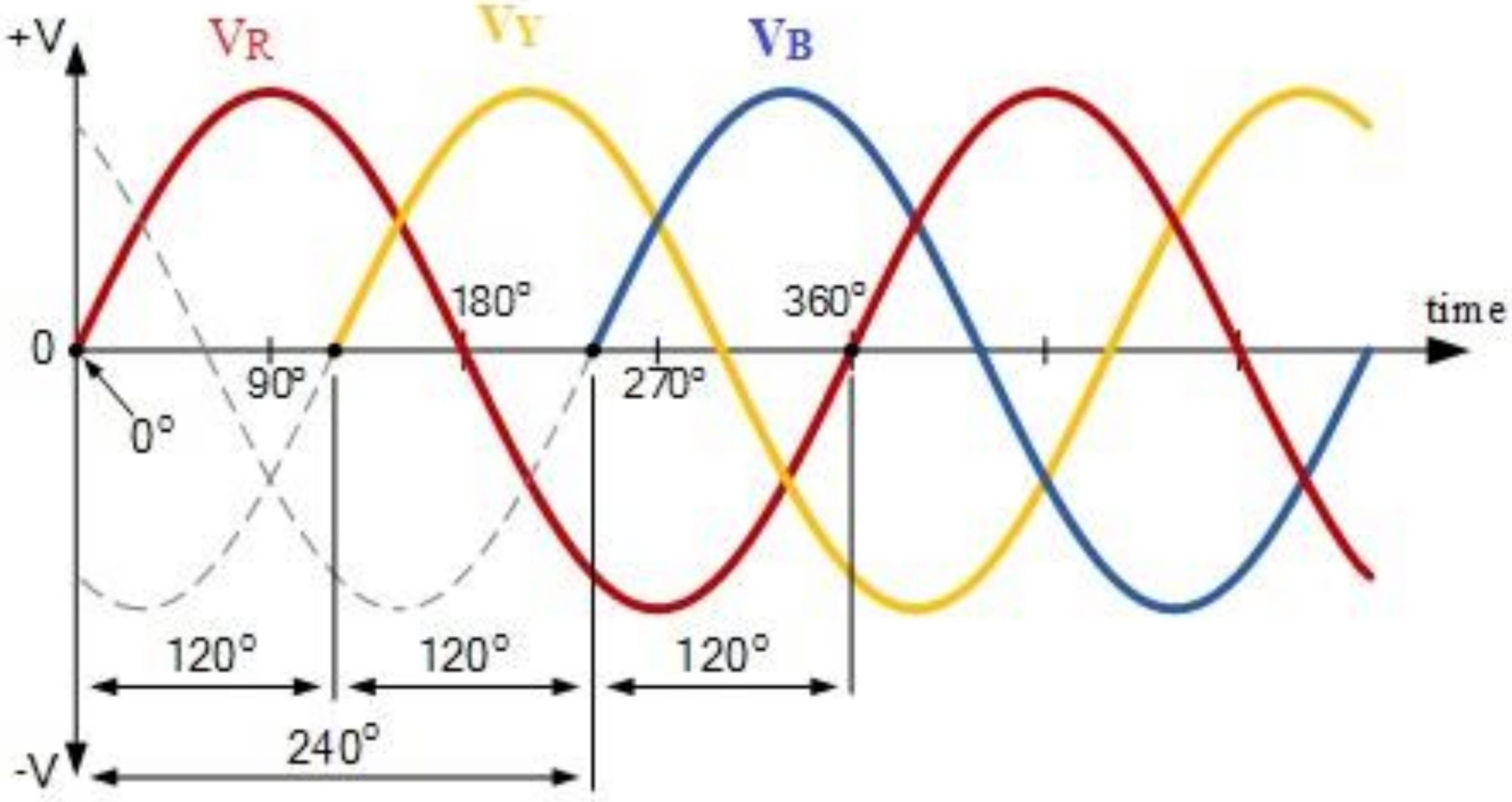
Three Phase System



The voltages generated by a three-phase alternator is shown in Figure (d). The Three Phase Voltage are of the same magnitude and frequency, but are displaced from one another by 120°. Assuming the voltages to be sinusoidal, we can write the equations for the instantaneous values of the voltages of the three phases. Counting the time from the instant when the voltage in phase R is zero. The equations are

$$v_{RR'} = V_m \sin \omega t$$
$$v_{YY'} = V_m \sin (\omega t - 120°)$$
$$v_{BB'} = V_m \sin (\omega t - 240°)$$

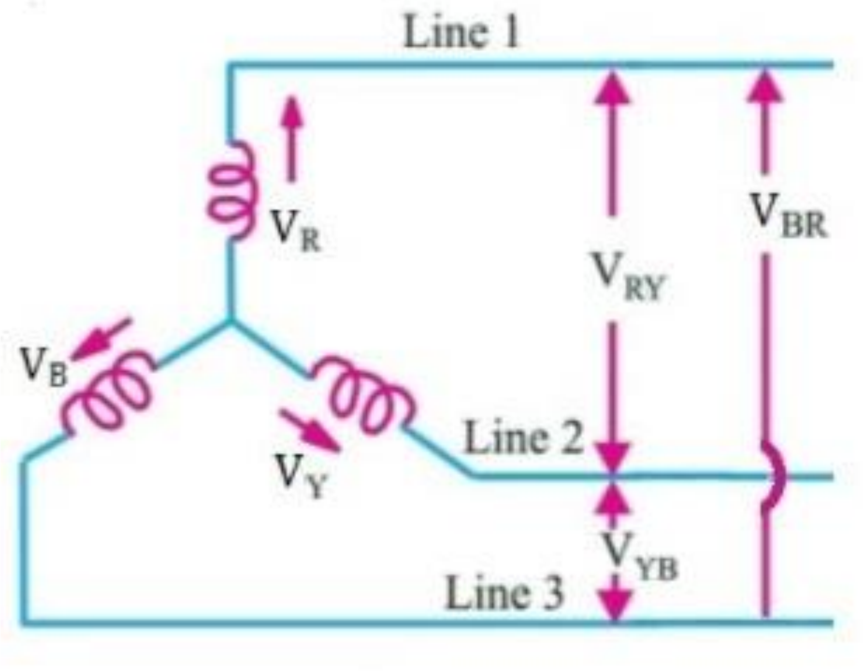


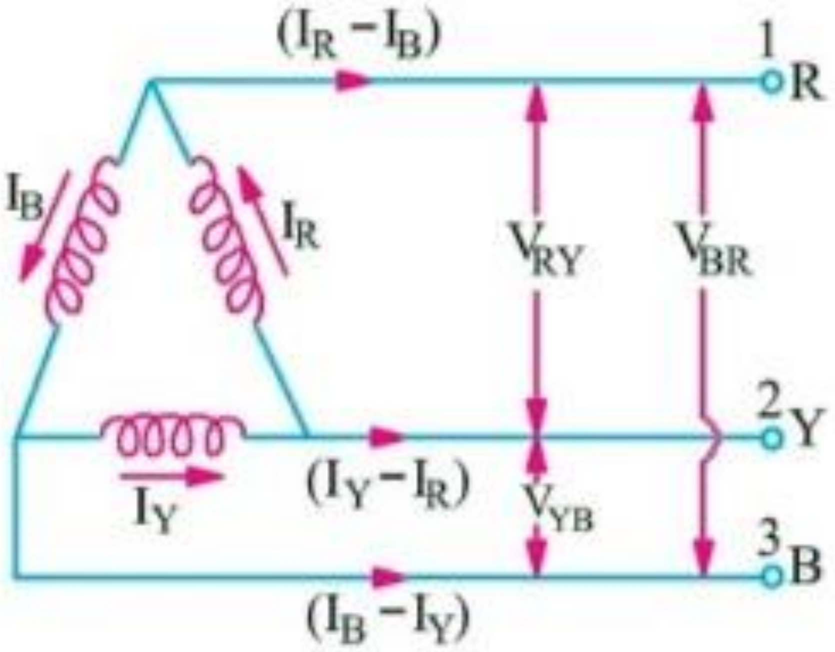


Star Connection and Delta Connection:

when either terminal of three branches is connected to a common point to form a Y like pattern is known as star connection.

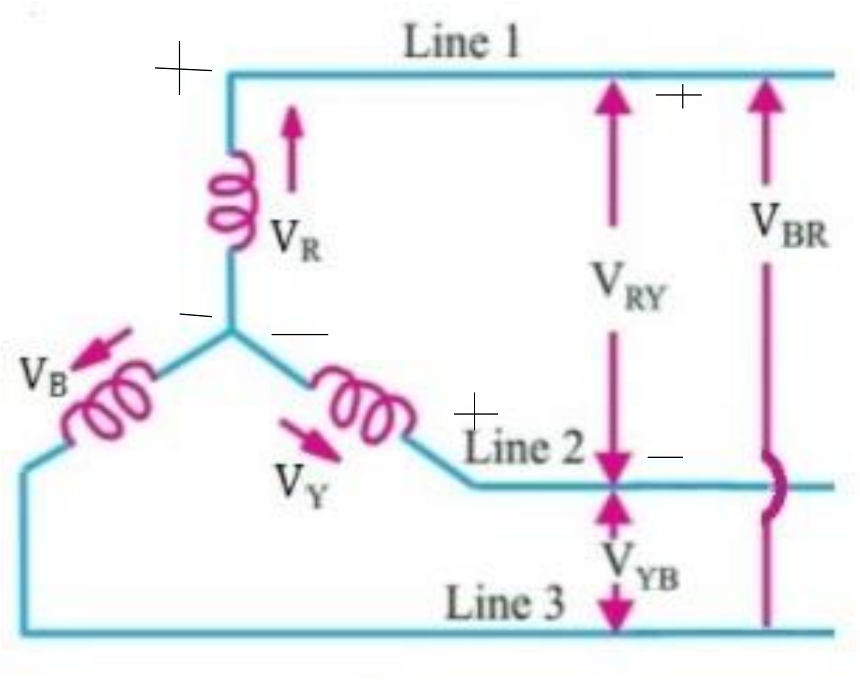
When three branches are so connected that they form a closed loop is known as delta connection. In delta connection three branches are connected nose to tail, and they form a closed loop.

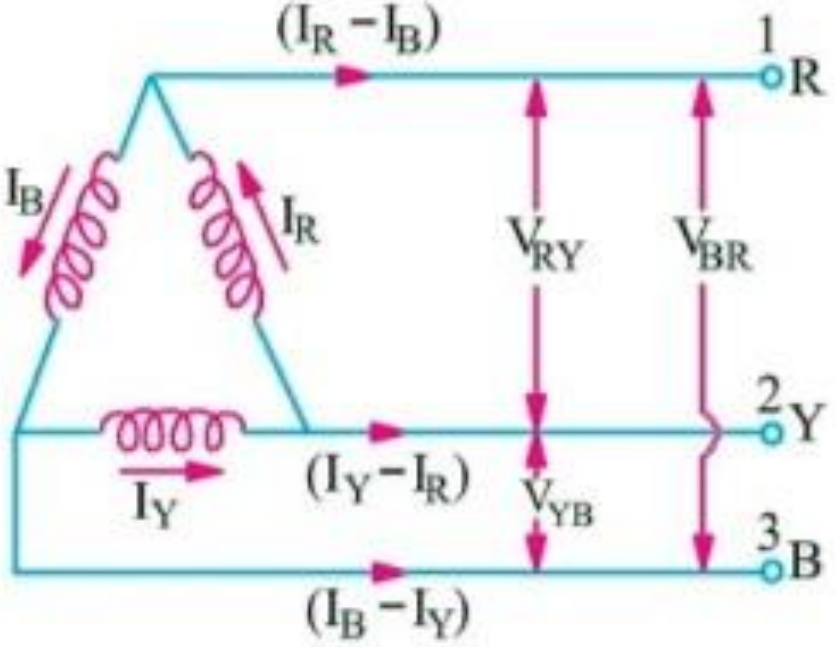




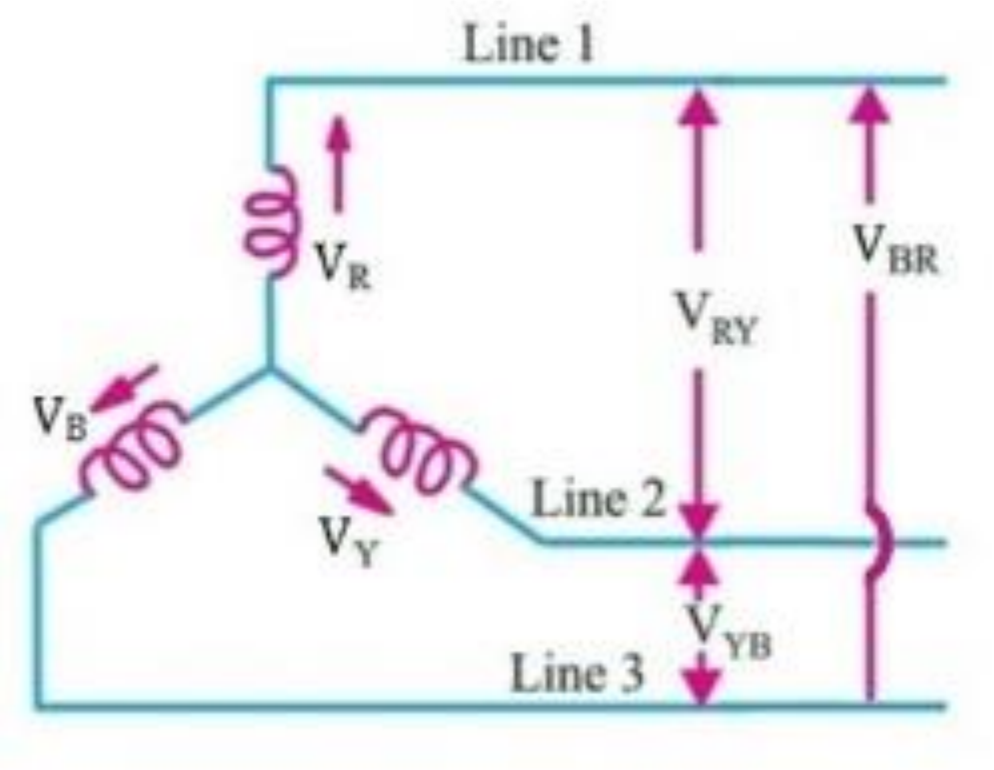
Phase Voltage, Phase current and Line voltage, Line current

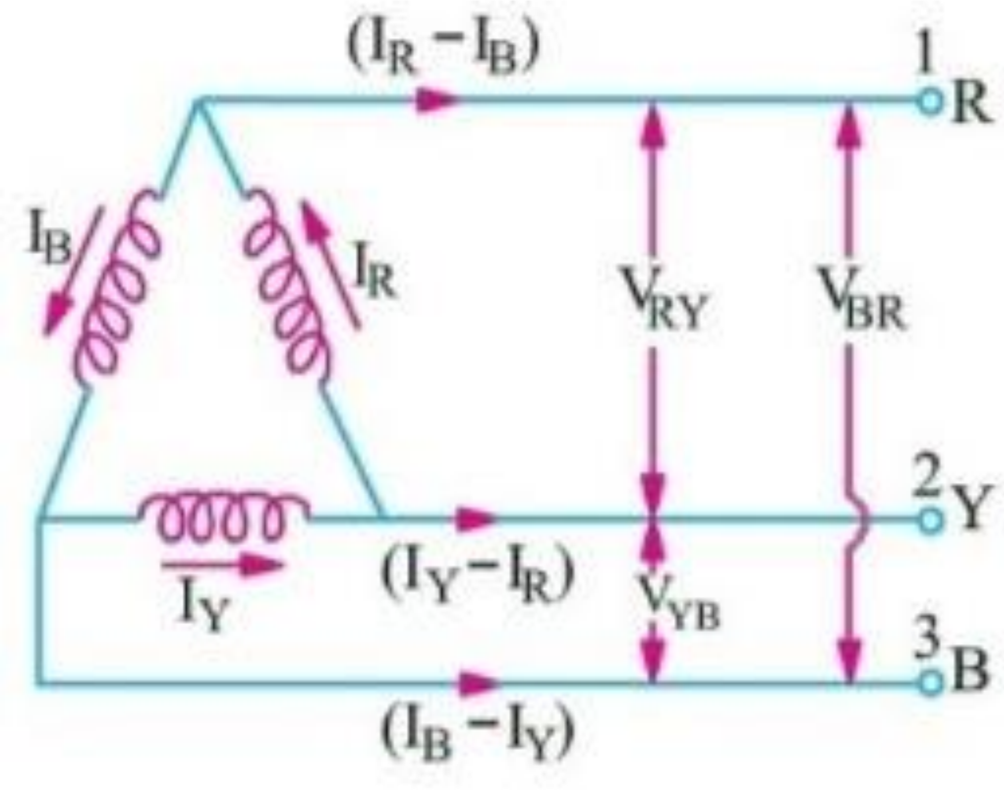
- The voltage induced in each winding is called phase voltage.
- The current in each winding is called phase current.
- The voltage between any pair of terminals (outer line) is called line voltage.
- The current flowing through each line is called line current.



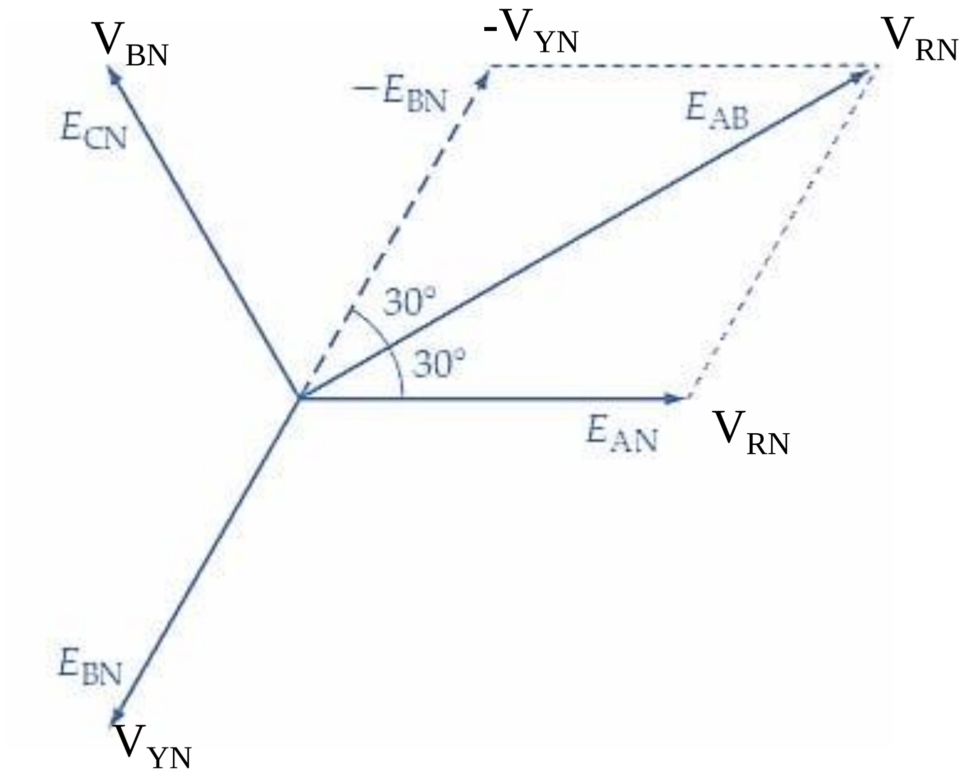


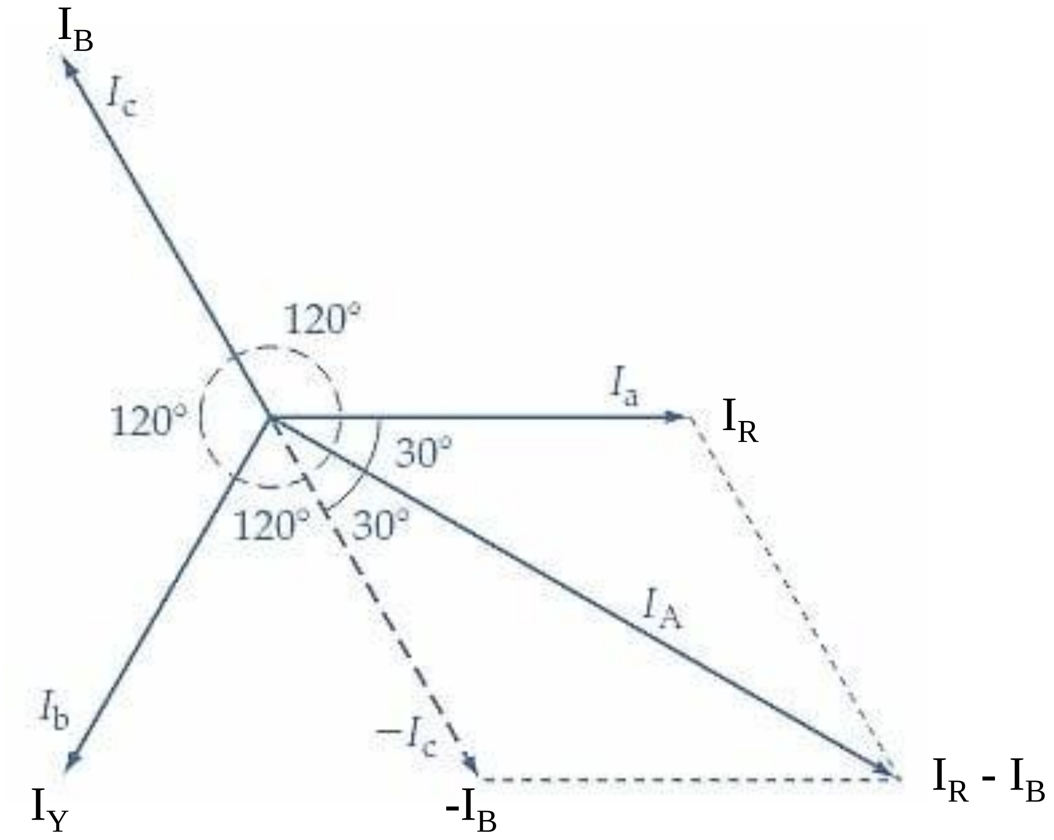
Star Connection	Delta Connection
Line Voltage = √3 * Phase Voltage	Line Voltage = Phase Voltage
Line Current = Phase Current	Line Current = √3 * Phase Current





- In Star connection, Line voltages are 30^0 ahead of their corresponding phase voltage.
- In Delta connection, Line current are 30^0 behind of their corresponding phase current.





Three phase Power

Active Power (P) $= 3\ V_{ph}\ I_{ph}\ \mathrm{Cos}\Theta$

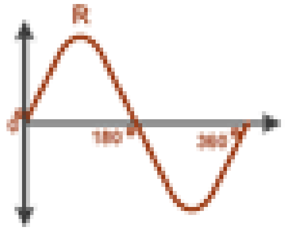
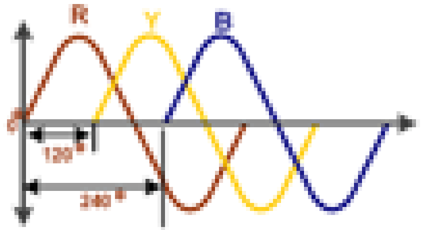
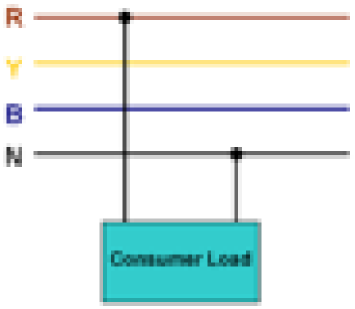
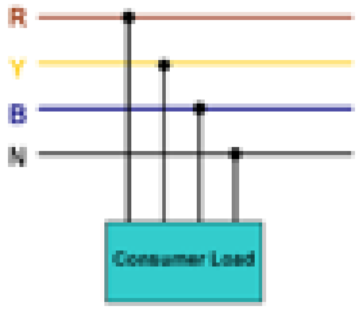
$= \sqrt{3} * V_L\ I_L\ \mathrm{Cos}\Theta$

Reactive Power (Q) $= 3\ V_{ph}\ I_{ph}\ \mathrm{Sin}\Theta$

$= \sqrt{3} * V_L\ I_L\ \mathrm{Sin}\Theta$

Apparent Power (S) $= 3\ V_{ph}\ I_{ph}$

$= \sqrt{3} * V_L\ I_L$

Basis for Comparison	Single Phase	Three Phase
Definition	The power supply through one conductor.	The power supply through three conductors.
Wave Shape		
Number of wire	Require two wires for completing the circuit	Requires four wires for completing the circuit
Voltage	Carry 230V	Carry 415V
Phase Name	Split phase	No other name
Network	Simple	Complicated
Loss	Maximum	Minimum
Power Supply Connection		
Efficiency	Less	High
Economical	Less	More
Uses	For home appliances.	In large industries and for running heavy loads.