

Bubble Sort

- //1. compare all pairs of consecutive elements
- //2. if left element is greater than right element
- //2.1 swap both elements
- //3. repeat above two step till array is not sorted

No. of element = n

No. of passes = $n-1$

pass 1 $\rightarrow n-1$

pass 2 $\rightarrow n-2$

pass 3 $\rightarrow n-3$

\vdots

pass $n-2 \rightarrow 2$

pass $n-1 \rightarrow 1$

$$\begin{aligned} \text{Total comps} &= 1 + 2 + 3 + \dots + \frac{(n-1)}{1} \\ &= \frac{n(n+1)}{2} \end{aligned}$$

$N=6$

$i < N$	$j < N-1$	$j < N-i$	
$i < 5$	$j < 5$		
0	$j=0-4$	$j < 5$	$j=0-4$
1	$j=0-4$	$j < 4$	$j=0-3$
2	$j=0-4$	$j < 3$	$j=0-2$
3	$j=0-4$	$j < 2$	$j=0-1$
4	$j=0-4$	$j < 1$	$j=0-0$
5			

$$\text{Time} \propto \frac{1}{2}(n^2 + n)$$

$$T(n) = O(n^2) \quad \text{Avg Worst}$$

$$T(n) = O(n) \quad \text{Best}$$

Insertion Sort

- //1. pick one element (start from 2nd index) of the array
- //2. compare picked element with all its left neighbours one by one
- //3. if left neighbour is greater than picked element
- //4. move left neighbour one position ahead
- //5. insert picked element at its appropriate position
- //6. repeat above steps till array is not sorted

```

public static void insertionSort(int arr[], int N) {
    for(int i = 1 ; i < N ; i++ ) {
        int temp = arr[i];
        int j = i - 1;
        while(j >= 0 && arr[j] > temp) {
            arr[j + 1] = arr[j];
            j--;
        }
        arr[j + 1] = temp;
    }
}
    
```

$arr = 55, 44, 22, 66, 11, 33$
 (5) $i < N$ temp $j \geq 0 \&\& arr[j] > temp$

1	T	44	0, -1	T, F
2	T	22	1, 0, -1	T, T, F
3	T	66	2	F

No. of elements = N
 No. of passes = $N - 1$
 Pass 1 $\rightarrow 1$
 Pass 2 $\rightarrow 2$
 \vdots
 Pass $N - 1 \rightarrow N - 1$

$$\text{Total comps} = 1 + 2 + 3 + \dots + (n-1) \\
 = \frac{n(n+1)}{2}$$

$$\text{Time} \propto \frac{1}{2}(n^2 + n)$$

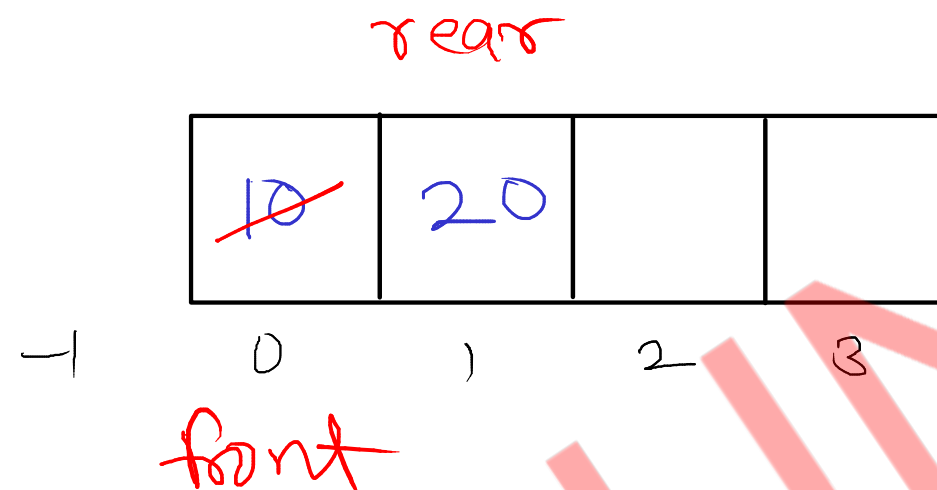
Avg. worst $T(n) = O(n^2)$

Best $T(n) = O(n)$

Linear Queue

- it is a linear data structure which stores similar type of data
- it has two end for data insertion and deletion
 1. rear - data is inserted
 2. front - data is deleted
- queue works on the principle of "First In First Out" / "FIFO"

size=4



Operations

1. Add/Insert/Enqueue/Push:

- a. reposition rear (inc)
- b. add value/data at rear index

2. Delete/Remove/Dequeue/Pop:

- a. reposition front (inc)

3. Peek:

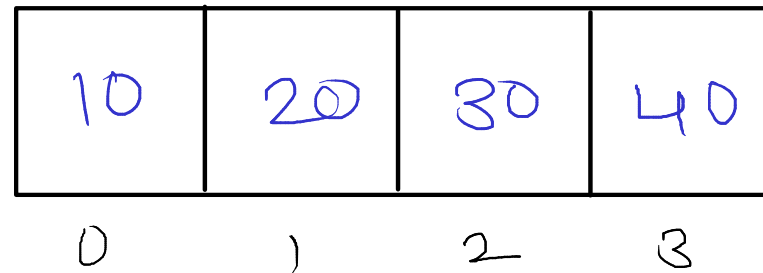
- a. read data/value from front end

- All operations of queue data structure are performed in $O(1)$ time

Linear Queue

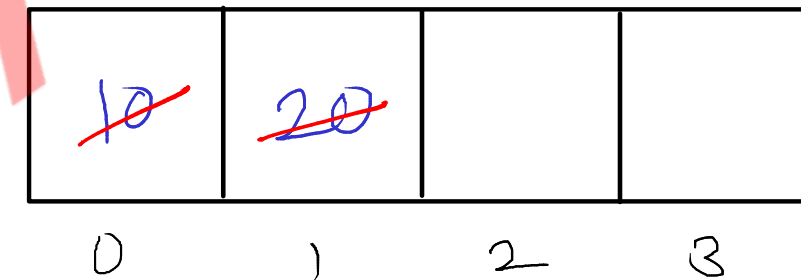
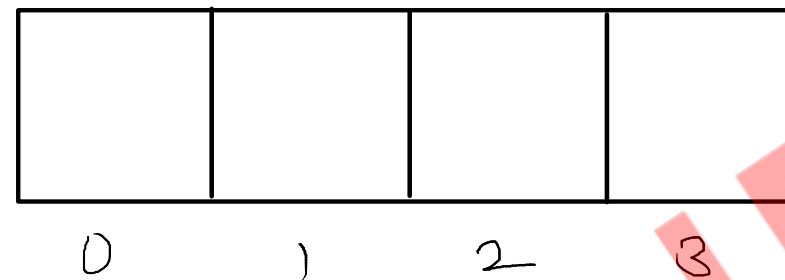
conditions

Full:



$\text{rear} == \text{size} - 1$

Empty:



$\text{front} == \text{rear}$

- in linear queue, once rear is reached to last index and initial few locations are vacant then also queue is full. Means we are not utilising those empty locations again
- this will lead to poor memory utilization
- solution for this is "circular queue"

Circular Queue



1. Add/Insert/Enqueue/Push:

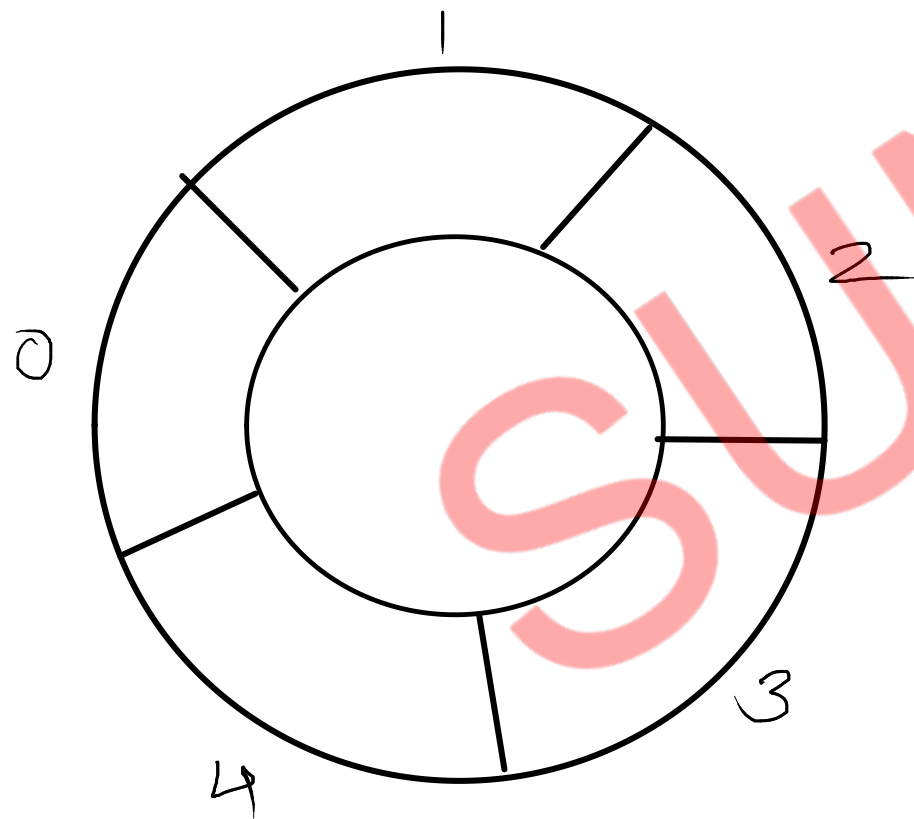
- reposition rear (inc)
- add value/data at rear index

2. Delete/Remove/Dequeue/Pop:

- reposition front (inc)

3. Peek:

- read data/value from front end

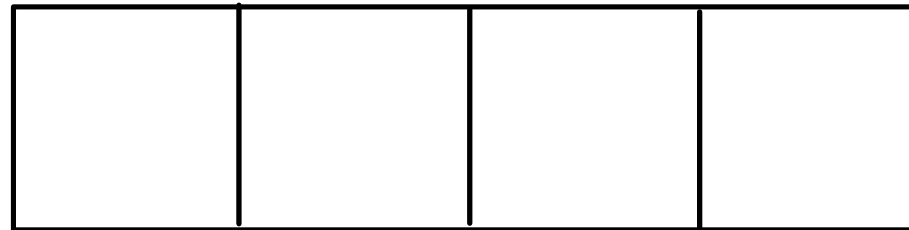


$$\begin{aligned} & \text{; size} = 4 \\ \text{front} &= (\text{front} + 1) \% \text{size} \\ \text{rear} &= (\text{rear} + 1) \% \text{size} \\ \text{front} = \text{rear} &= -1 \\ &= (-1 + 1) \% 4 = 0 \\ &= (0 + 1) \% 4 = 1 \\ &= (1 + 1) \% 4 = 2 \\ &= (2 + 1) \% 4 = 3 \\ &= (3 + 1) \% 4 = 0 \end{aligned}$$

Circular Queue

Conditions

r



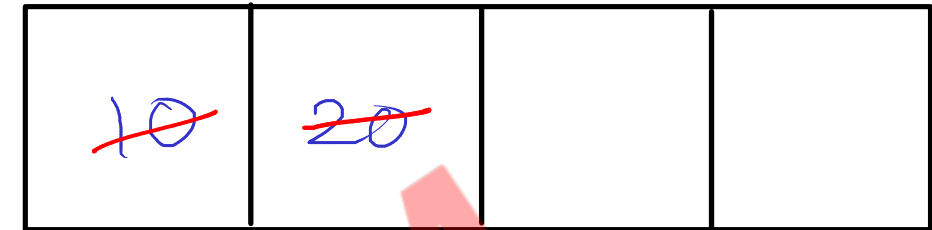
—| 0 1 2 3

f

Empty:

front == rear && rear == -1

r



—| 0 1 2 3

f

Pop:

front = (front + 1) % SIZE;
if(front == rear)
front = rear = -1;

r



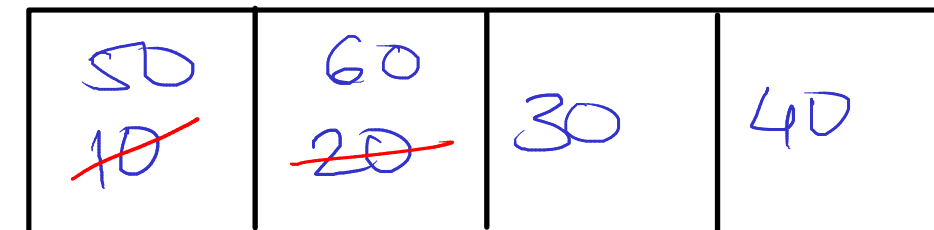
—| 0 1 2 3

f

Full:

front == -1 && rear == SIZE-1

r



—| 0 1 2 3

f

front == rear && rear != -1

Stack

- it is a linear data structure which stores similar type of data
- it has only one end (top) to insert and delete data
- stack works on principle of "Last In First Out" / "LIFO"
- top always points to last inserted data

Operations:

1. Add/Insert/Push:

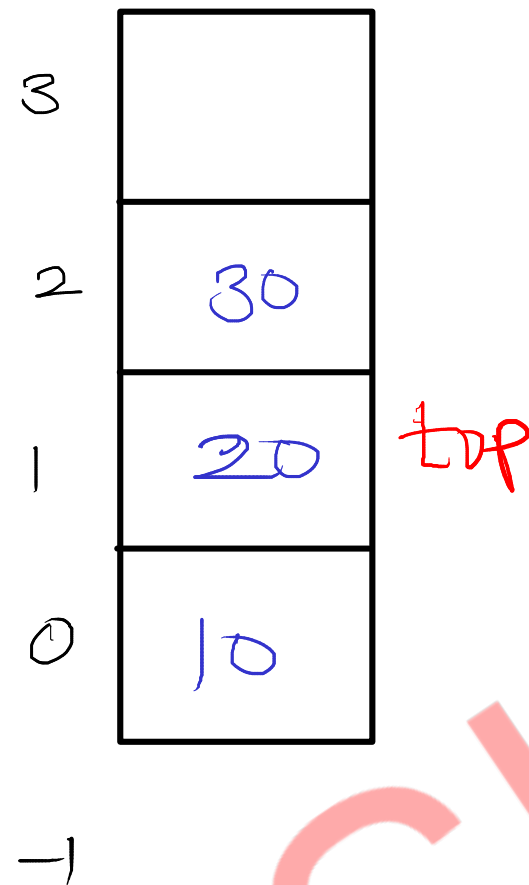
1. reposition top (inc)
2. add value/data at top index

2. Delete/Remove/Pop:

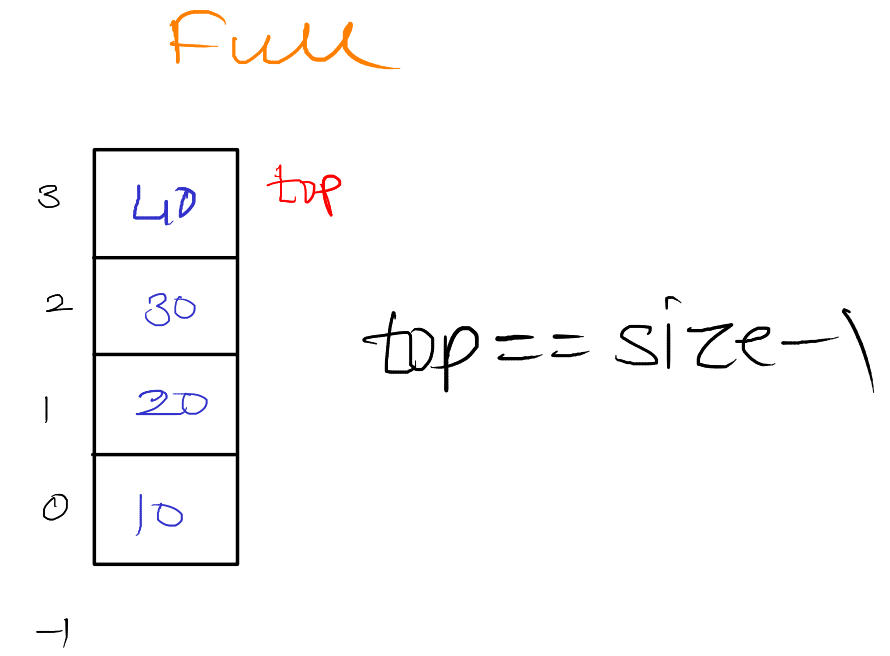
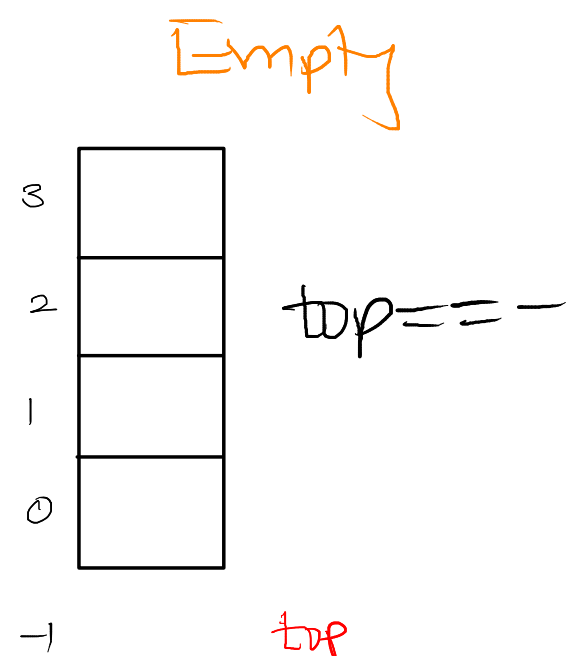
1. reposition top (dec)

3. Peek:

1. read / return data of top end(index)



- all operations of stack, can be performed in $O(1)$ time complexity



Stack and Queue Time Complexity Analysis (Array Implementation)

	Stack	Linear Queue	Circular Queue
Push	$O(1)$	$O(1)$	$O(1)$
Pop	$O(1)$	$O(1)$	$O(1)$
Peek	$O(1)$	$O(1)$	$O(1)$

Stack Application

Expression Evaluation and Conversion

1. Postfix Evaluation
2. Prefix Evaluation
3. Infix to Postfix Conversion
4. Infix to Prefix Conversion

Expression:

- set/combination of operands and operators

operands - values/variables

operators - mathematical symbols (+, -, /, *, %)

e.g. $a + b$, $4 * 2 - 3$

Types:

- | | | |
|------------|---------|----------|
| 1. Infix | $a + b$ | human |
| 2. Prefix | $+ a b$ | computer |
| 3. Postfix | $a b +$ | computer |

Operators:

()
power
* / %
+ -



Postfix Evaluation

Postfix : 4 5 6 * 3 / + 9 + 7 -

left → right

$23 - 7 = 16$

$14 + 9 = 23$

$4 + 10 = 14$

$30 / 3 = 10$

$5 * 6 = 30$

Result = 16

Stack

16
7
23
9
14
10
3
30
6
5
4

Prefix Evaluation

Prefix : - + + 4 / * 5 6 3 9 7

left ← right $25 - 7 = 16$

Result = 16

$$14 + 9 = 23$$

$$4 + 10 = 14$$

$$30 / 3 = 10$$

$$5 * 6 = 30$$

16
23
14
4
10
30
5
6
3
9
7