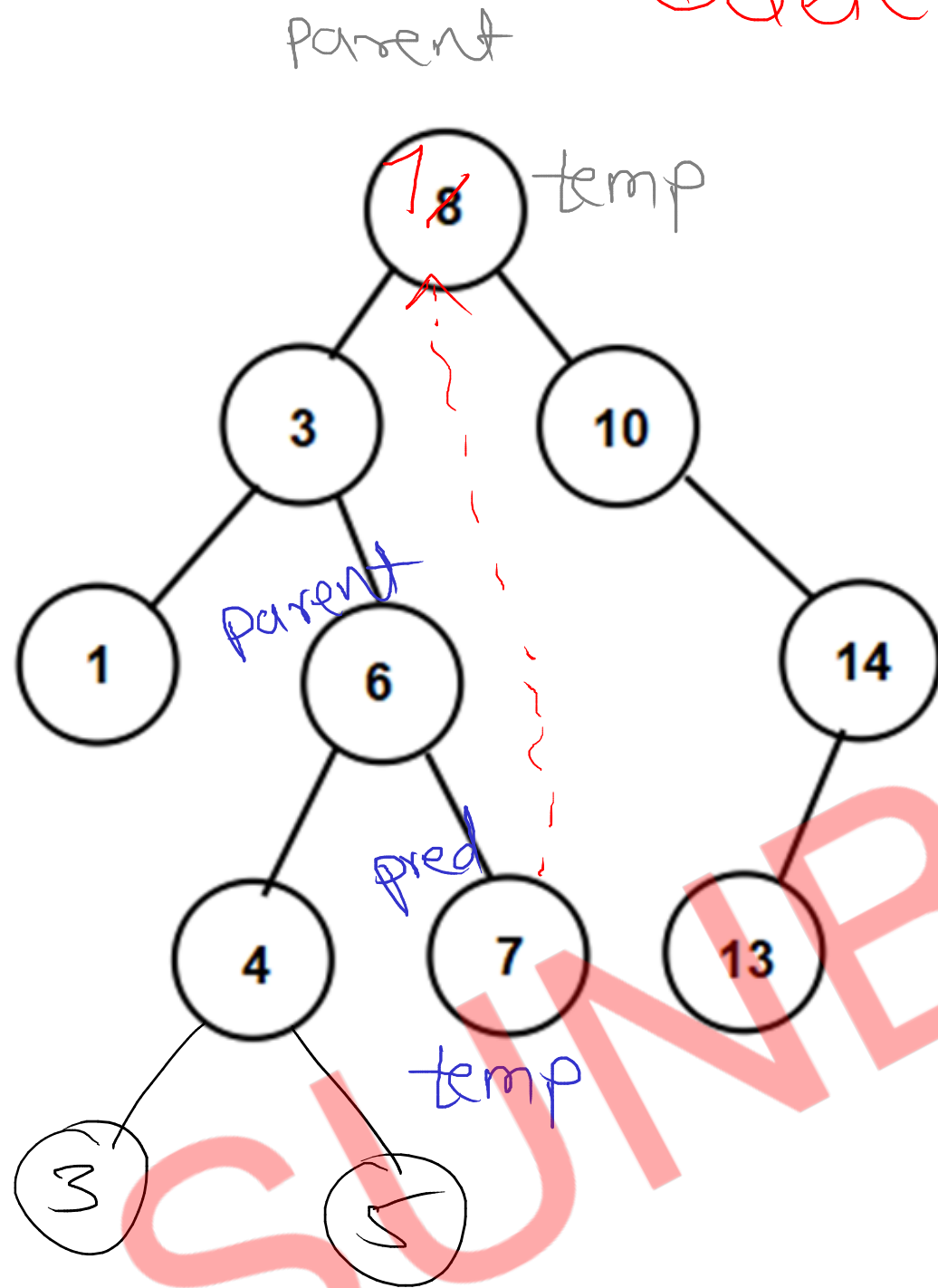


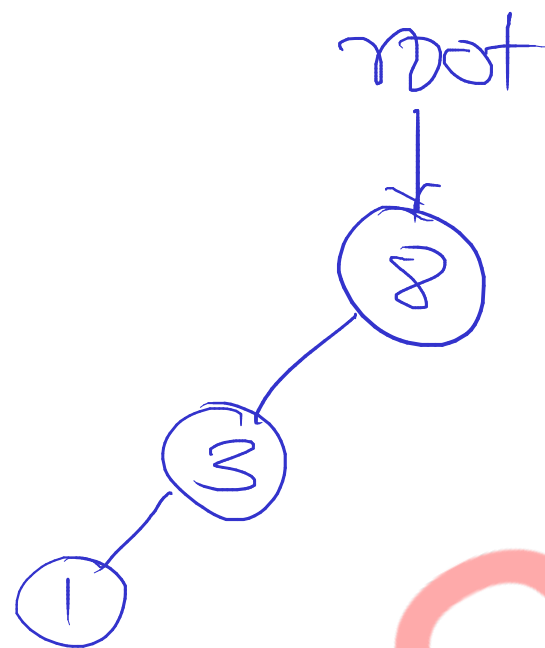
Delete Node - BST



```

void addNode (int value) {
    if (root == null)
        root = new Node (value)
    else
        addNode (root, value);
}

```



BST - Add Node Recursive

```

void addNode(Node trav, int value)
{
    if (value < trav->data) {
        if (trav->left == null)
            trav->left = new Node(value);
        else
            addNode(trav->left, value);
    } else {
        if (trav->right == null)
            trav->right = new Node(value);
        else
            addNode(trav->right, value);
    }
}

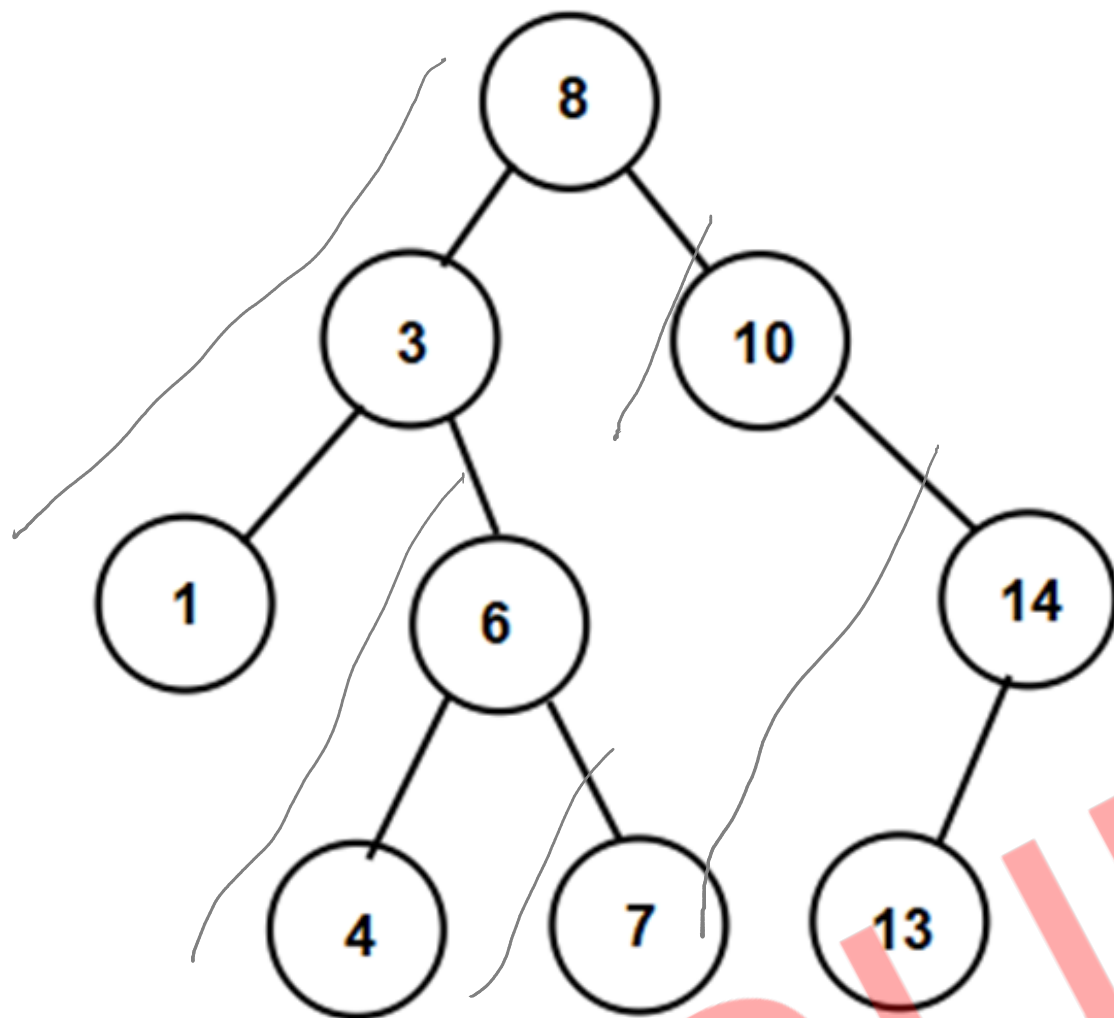
```

8, 1
 3, 1

BST - Binary Search Recursive

```
Node binarySearchRec(int key, Node trav) {  
    if (trav == null)  
        return null;  
    if (key == trav.data)  
        return trav;  
    else if (key < trav.data)  
        return binarySearchRec(key, trav.left);  
    else  
        return binarySearchRec(key, trav.right);  
}
```

BST - DFS (Depth First Search)



Stack

8/3
8/4
8/6
8/7
8/1
8/10
8/13
8/14

//1. push root on stack

//2. pop one node from stack

//3. visit(print) node

//4. if right exist, push it on stack

//5. if left exist, push it on stack

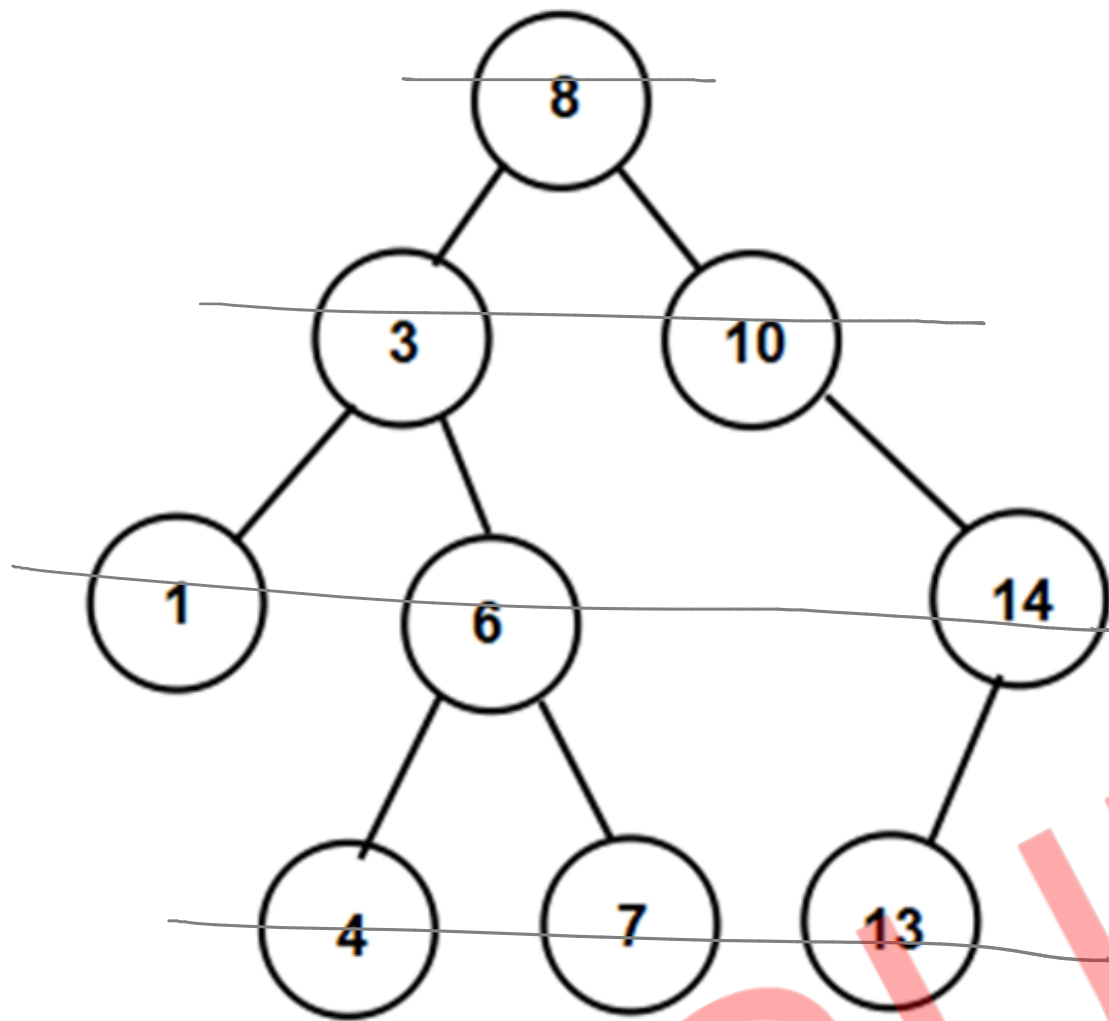
//6. while stack is not empty

//repeat ste 2 to 5

Traversed:

8, 3, 1, 6, 4, 7, 10, 14, 13

BST - BFS (Bredth First Search)



Queue

8.13
8.7
8.4
8.14
8.6
8.1
8.10
8.3
8.8

//1. push root on queue

//2. pop one node from queue

//3. visit(**print**) node

//4. if left exist, push it on queue

//5. if right exist, push it on queue

//6. while queue is not empty

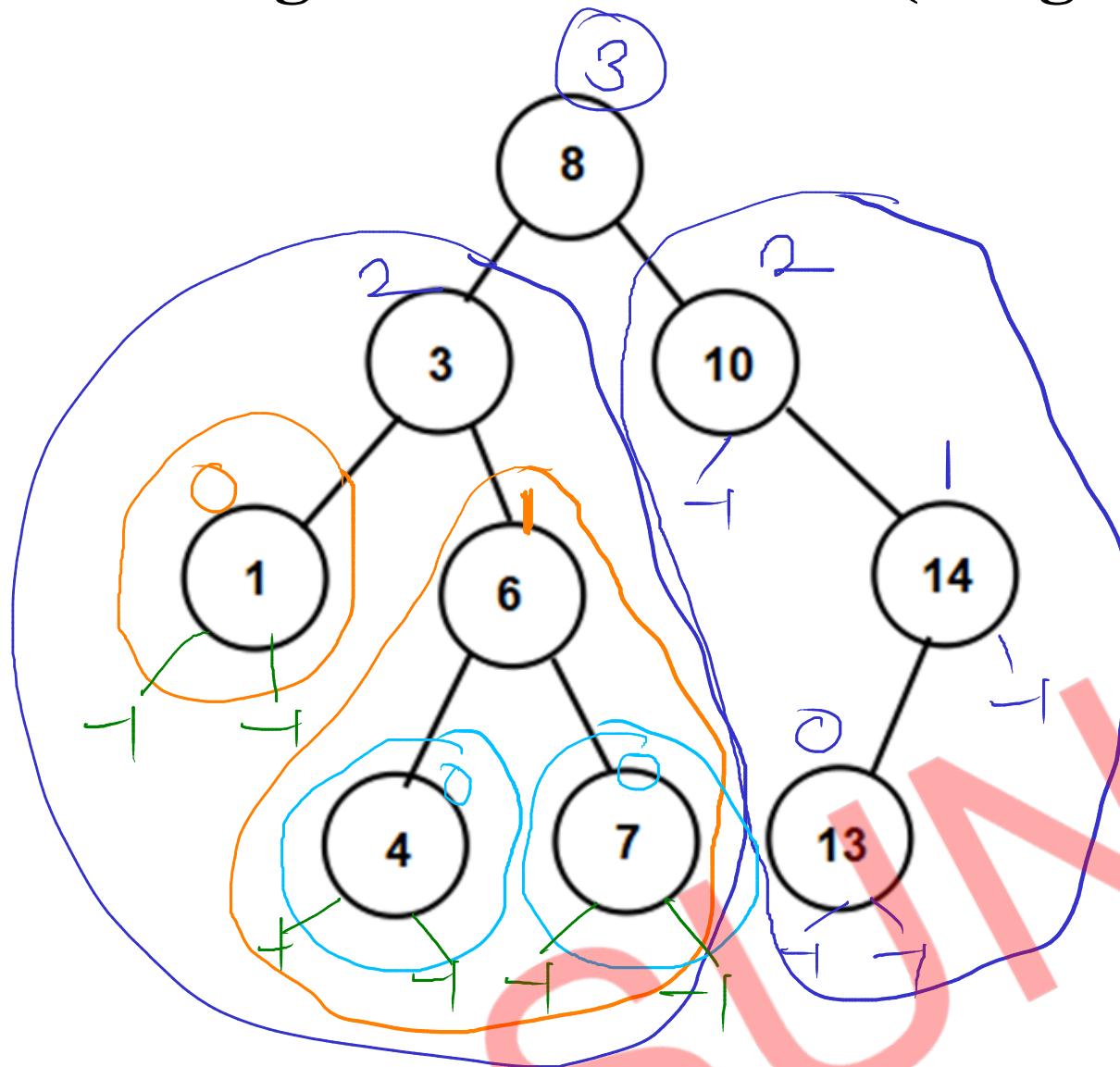
//repeat ste 2 to 5

Traversal :

8, 3, 10, 1, 6, 14, 4, 7, 13

BST - Height

Height of tree = MAX(Height(left sub tree), Height(right sub tree)) + 1



//0. if left or right sub tree is absent

//then return -1

//1. find height of left subtree

//2. find height of right subtree

//3. find max height

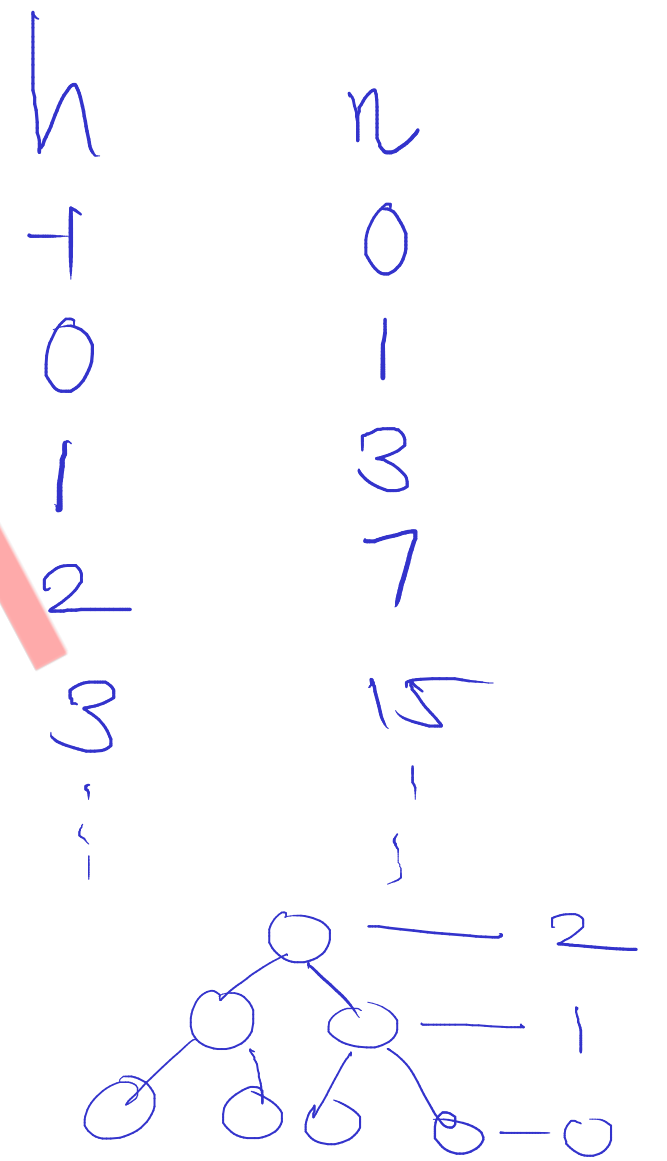
//4. add one into max height(return)

BST - Time complexity of operations

No. of elements = n
height of BST = h

$$n = 2^{h+1} - 1$$

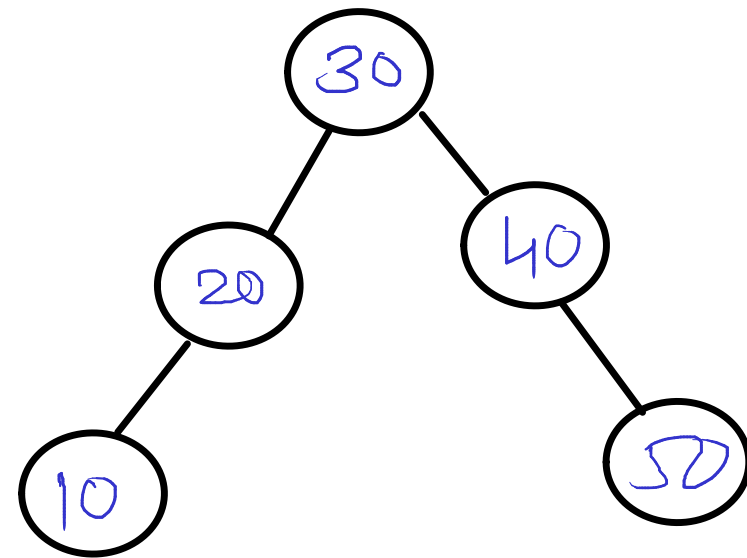
Add : $O(h) / O(\log n)$
search : $O(h) / O(\log n)$
Delete : $O(h) / O(\log n)$
Traversal : $O(n)$



$$2^{h+1} = n+1$$
$$2^h = n$$
$$h = \frac{\log n}{\log 2}$$

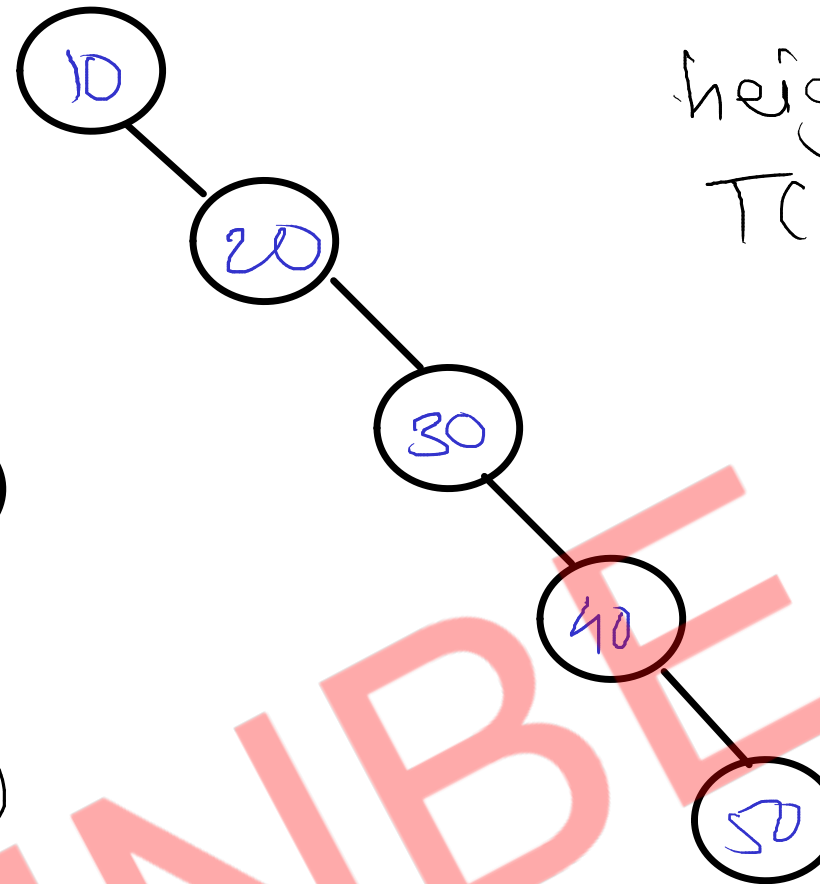
Skewed BST

Keys : 30, 40, 20, 50, 10



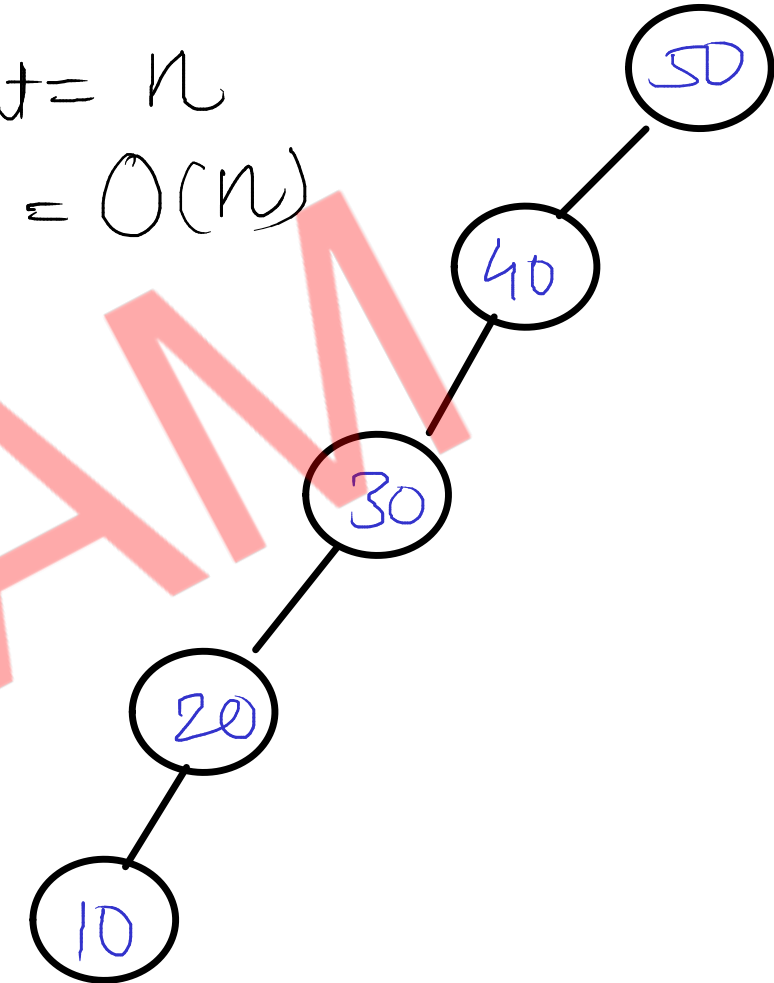
height = $\log n$
 $T(n) = O(\log n)$

Keys : 10, 20, 30, 40, 50



height = n
 $T(n) = O(n)$

Key : 50, 40, 30, 20, 10



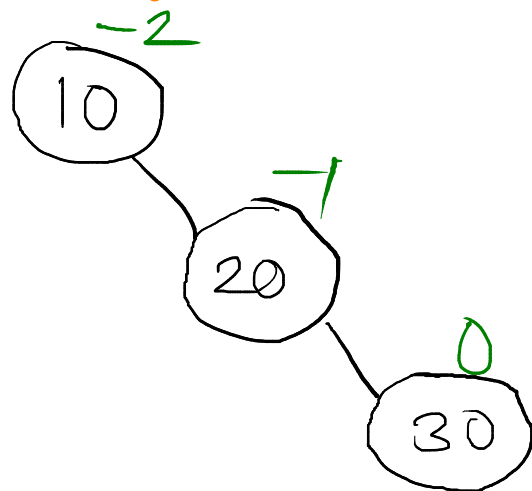
- if tree is growing only in one direction, that tree is skewed BST
- if tree is growing only in left direction, that tree is left skewed BST
- if tree is growing only in right direction, that tree is right skewed BST

Balanced BST

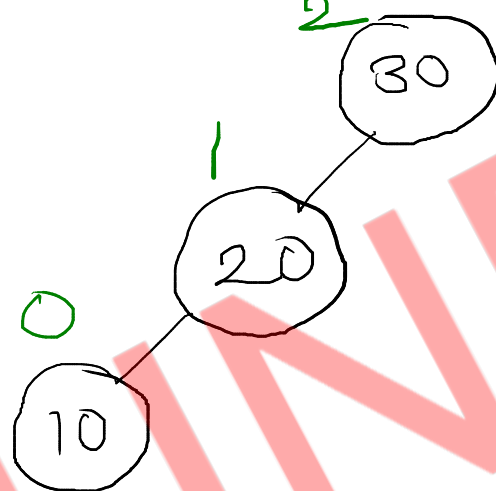
$$\text{Balance Factor} = \text{height}(\text{left sub tree}) - \text{height}(\text{right sub tree})$$

- tree is balanced if balance factor of all the nodes is either -1, 0 or +1
- balance factor = {-1, 0, +1}

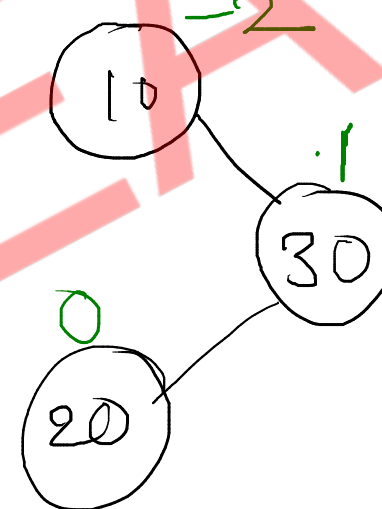
Keys : 10, 20, 30



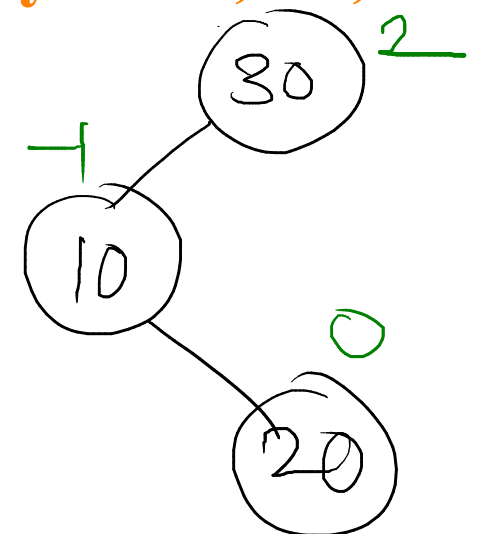
Keys : 30, 20, 10



Keys : 10, 30, 20

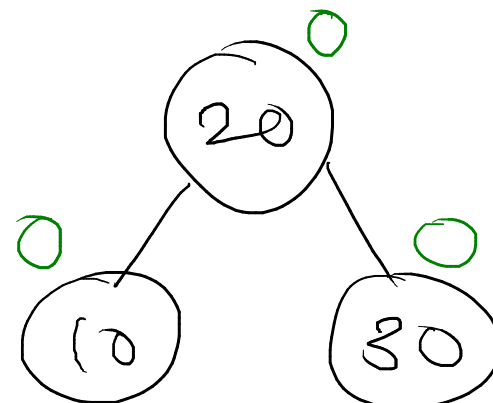


Keys : 30, 10, 20



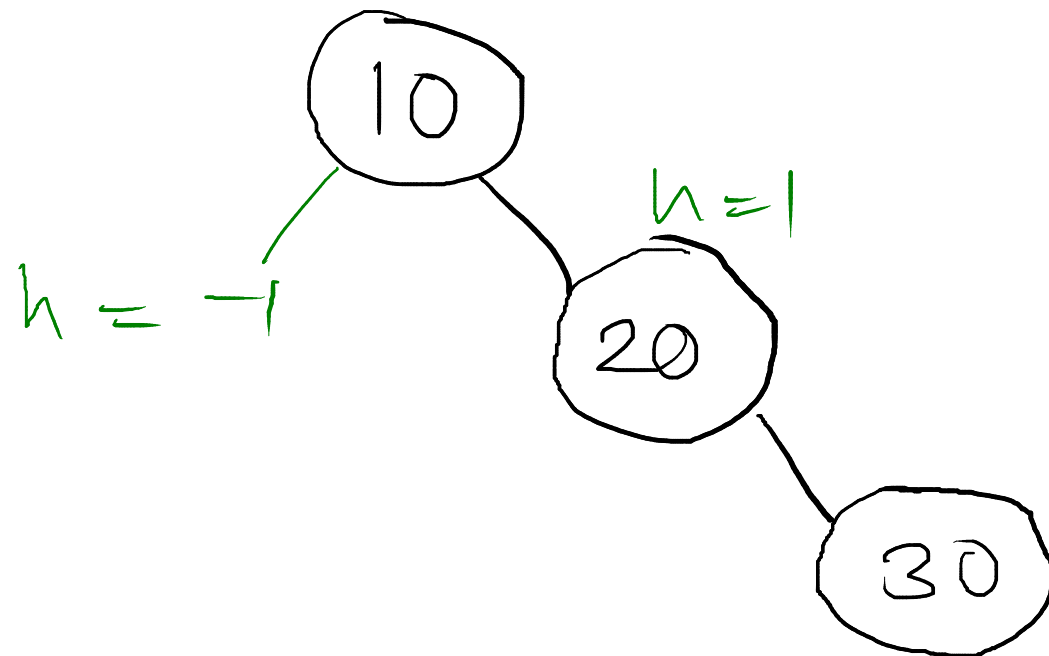
Keys : 20, 10, 30

Keys : 20, 30, 10

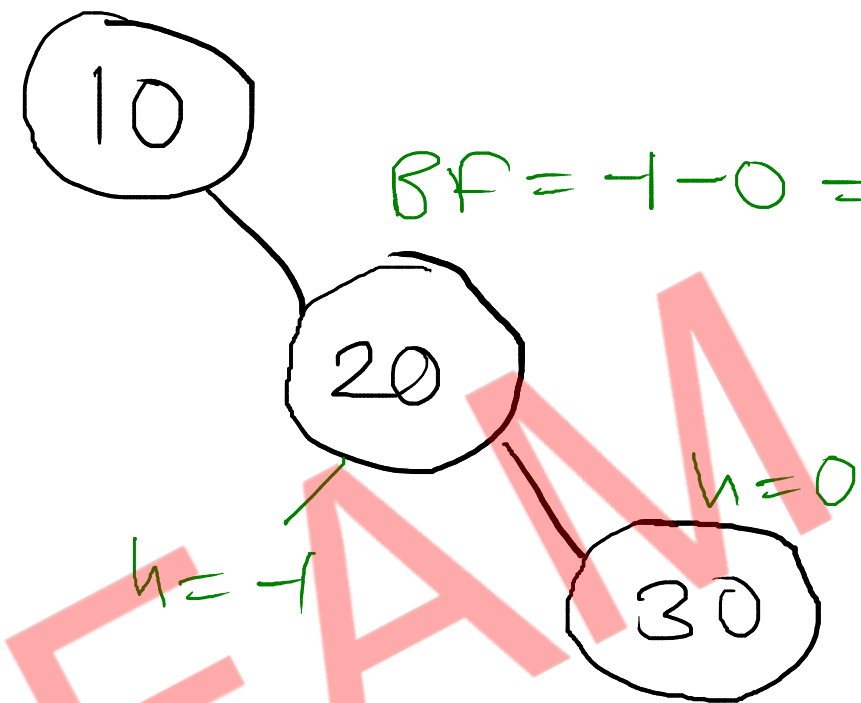


Balanced BST

$$BF = -1 - 1 = -2$$



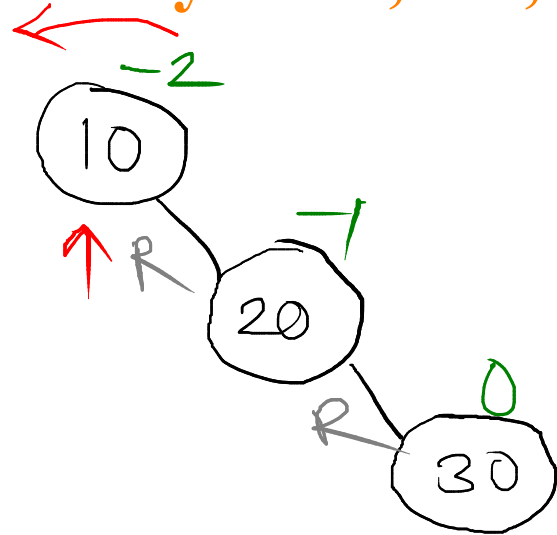
$$BF = -1 - 0 = -1$$



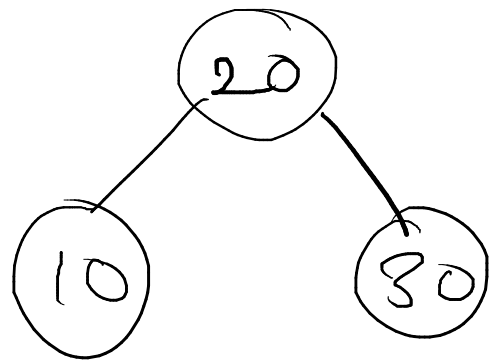
Rotations

RR Imbalance

Keys : 10, 20, 30



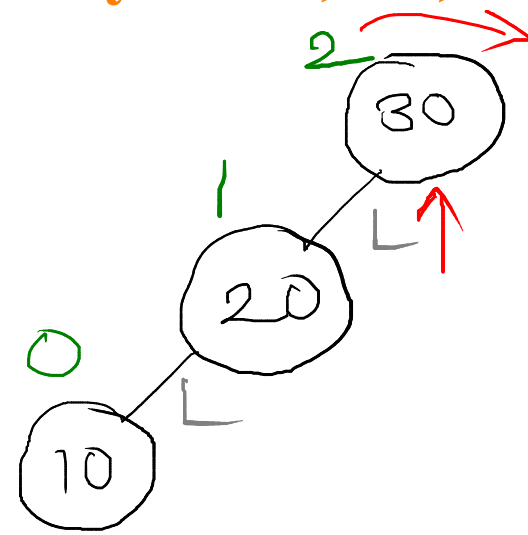
Left Rotation



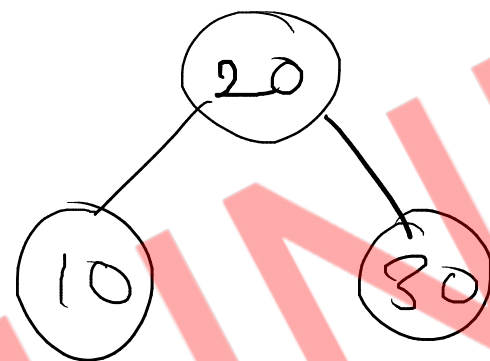
Single Rotation

LL Imbalance

Keys : 30, 20, 10

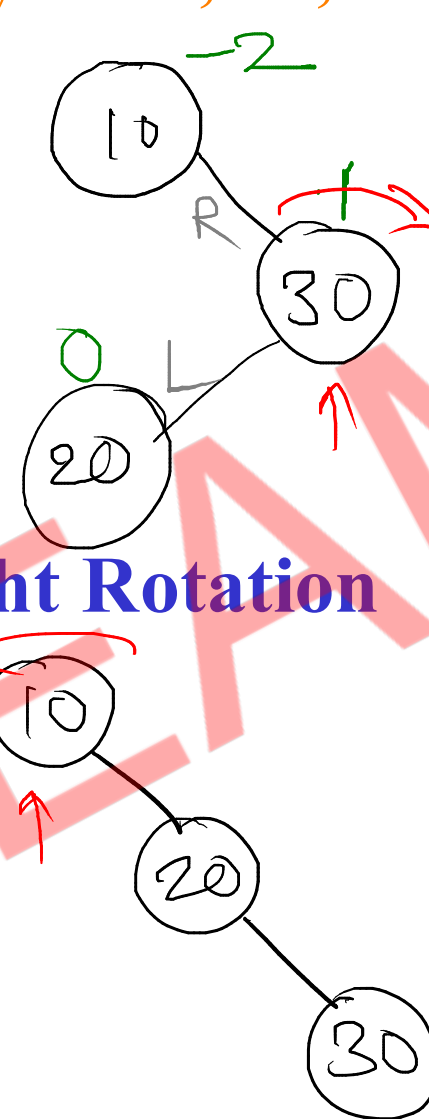


Right Rotation

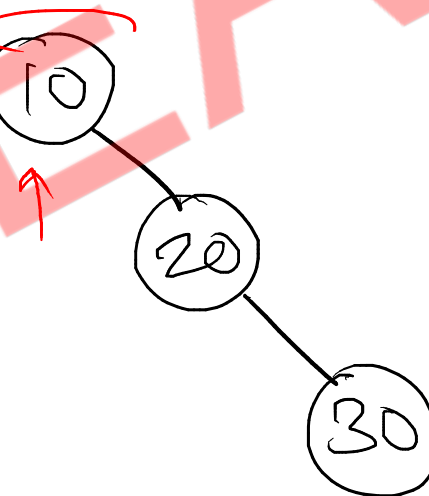


RL Imbalance

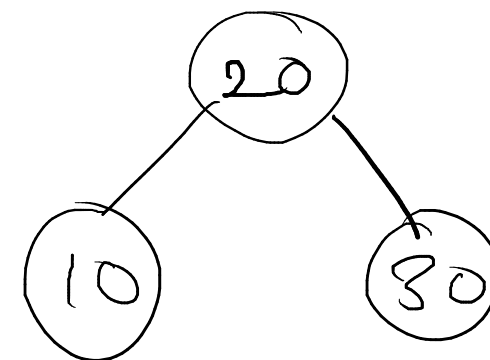
Keys : 10, 30, 20



Right Rotation



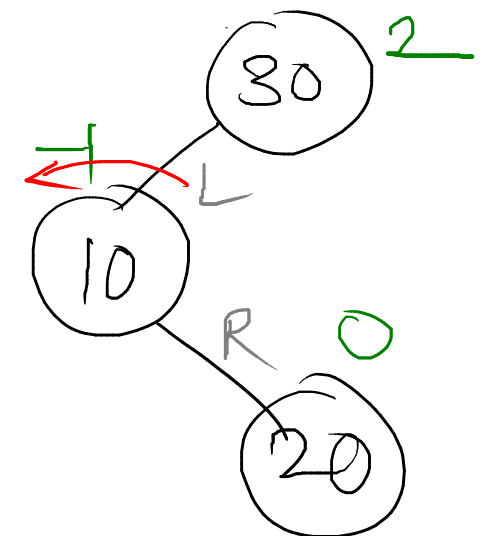
Left Rotation



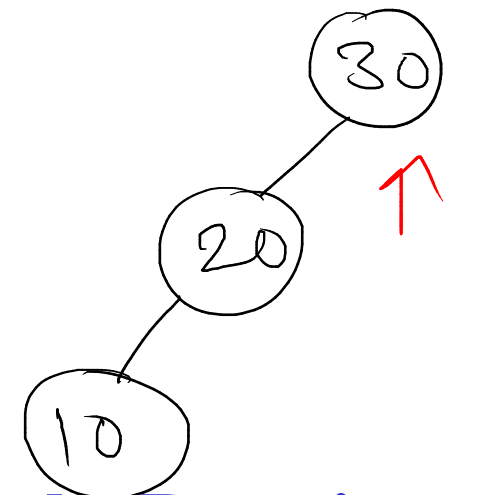
Double Rotation

LR Imbalance

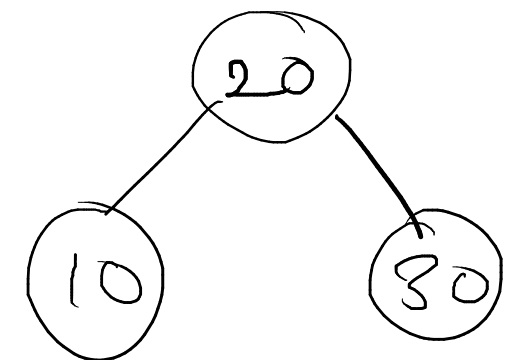
Keys : 30, 10, 20



Left Rotation

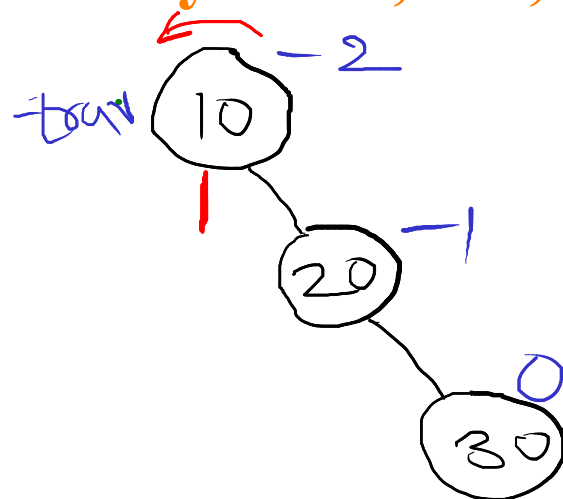


Right Rotation



~~RR~~ Imbalance

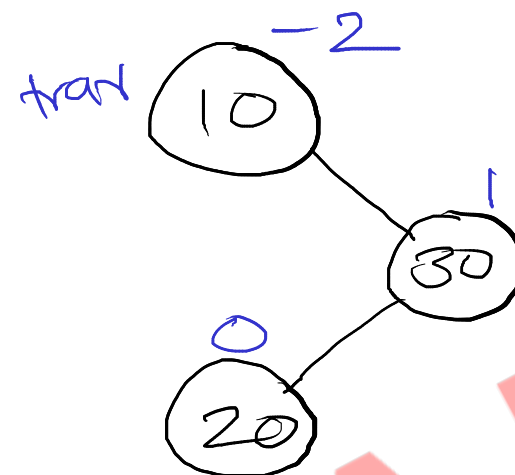
Keys : 10, 20, 30



$bf < -1$
value > trav.right.data

~~RL~~ Imbalance

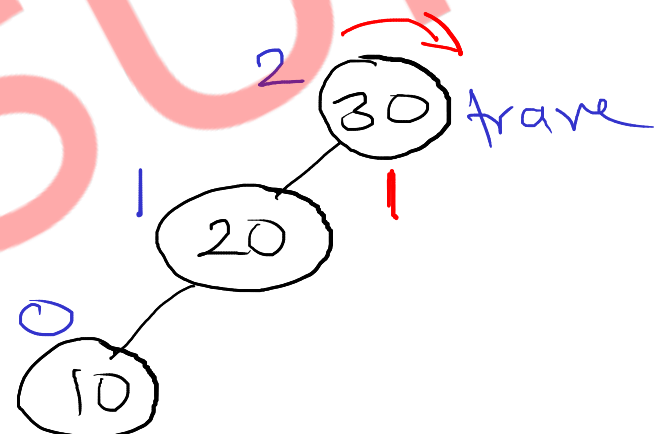
Keys : 10, 30, 20



$bf < -1$
value < trav.right.data

~~LL~~ Imbalance

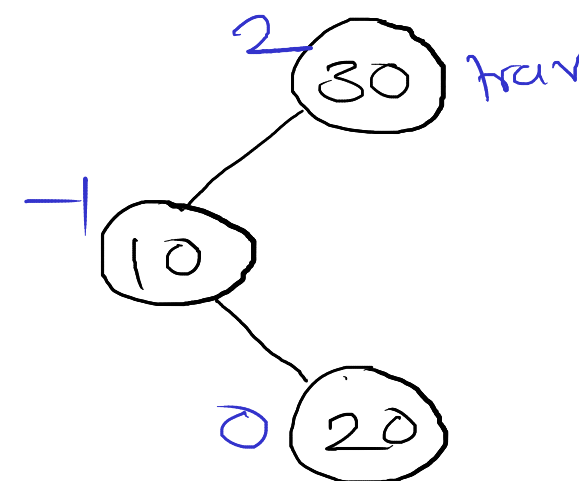
Keys : 30, 20, 10



$bf > 1$
value < trav.left.data

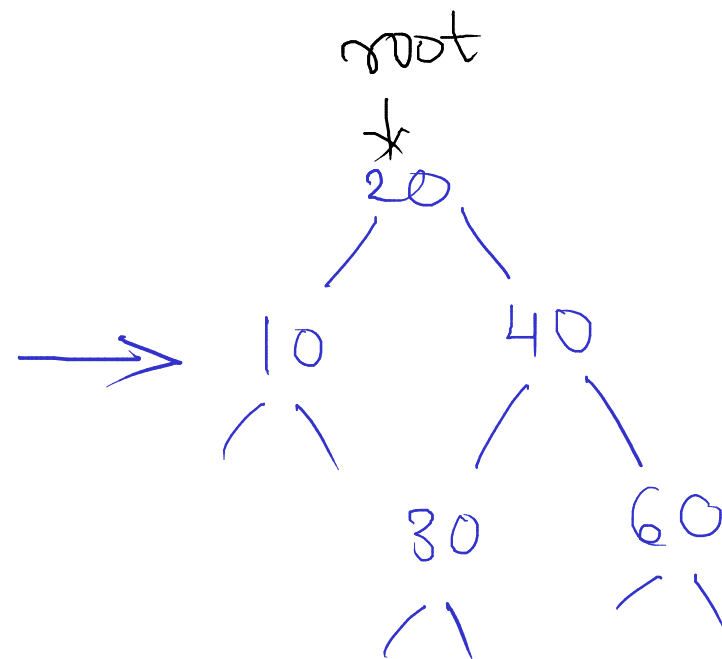
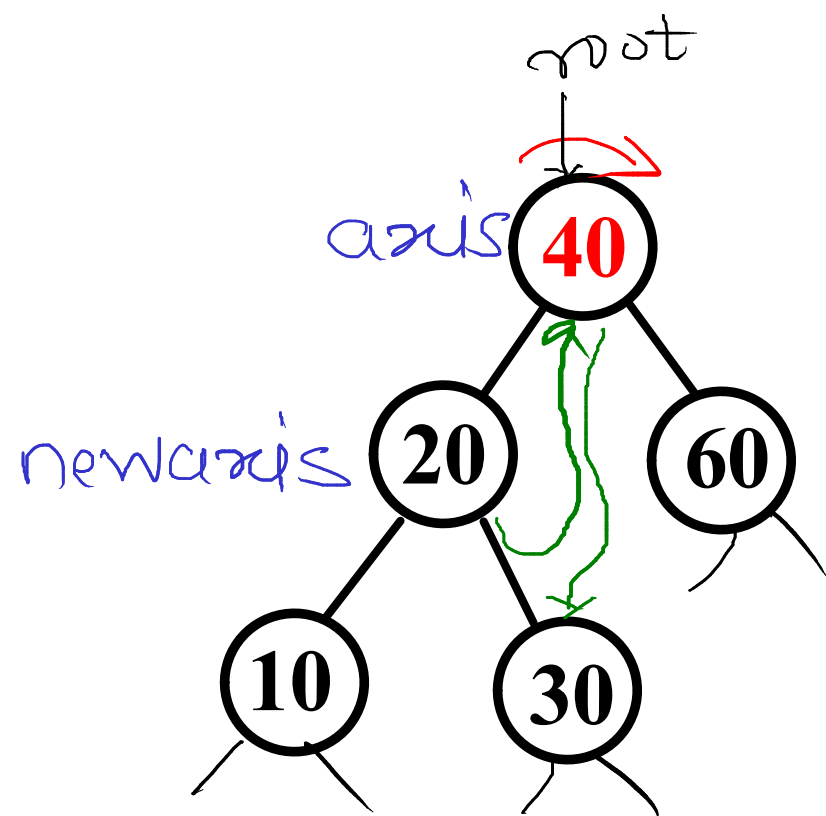
~~LR~~ Imbalance

Keys : 30, 10, 20



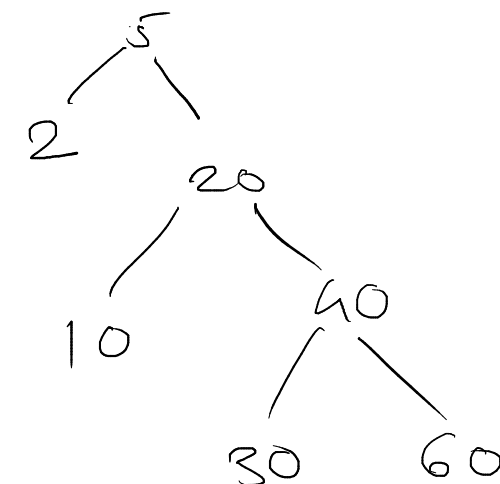
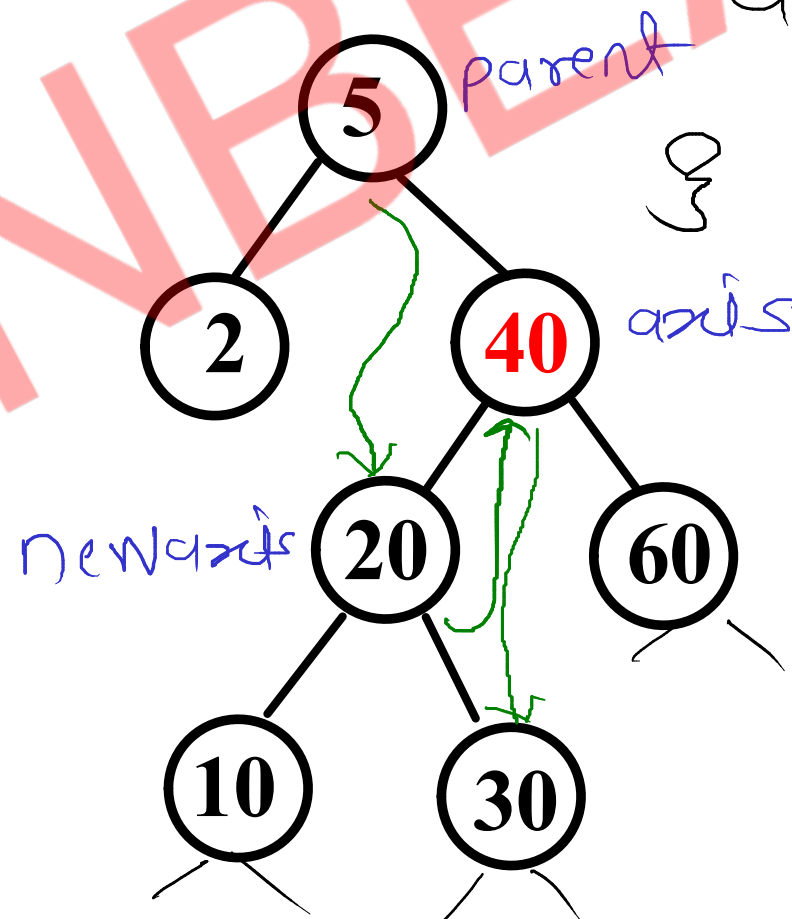
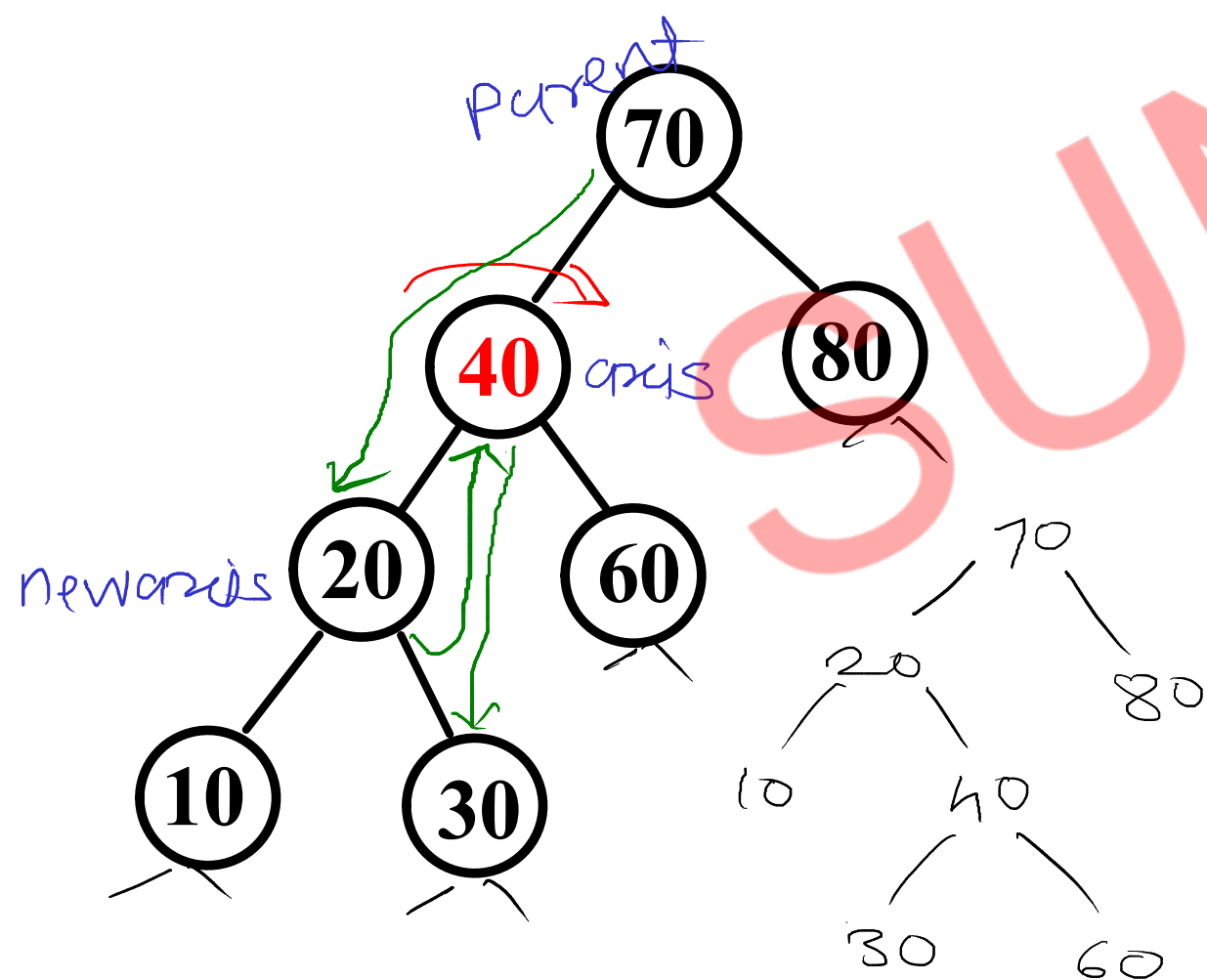
$bf > 1$
value > trav.left.data

Right Rotation

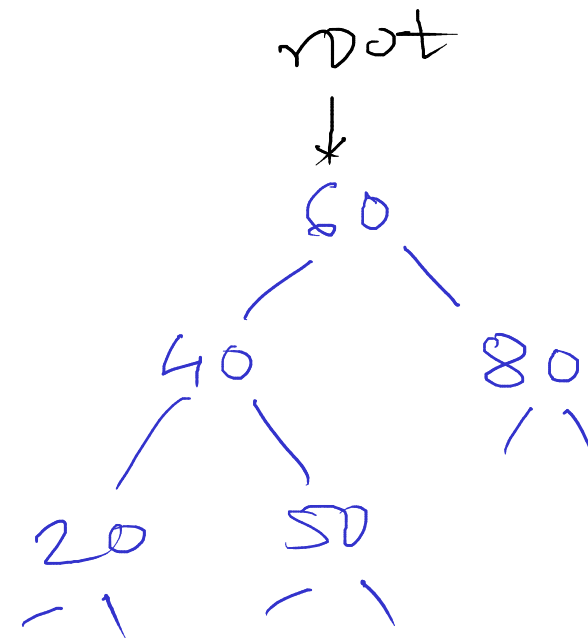
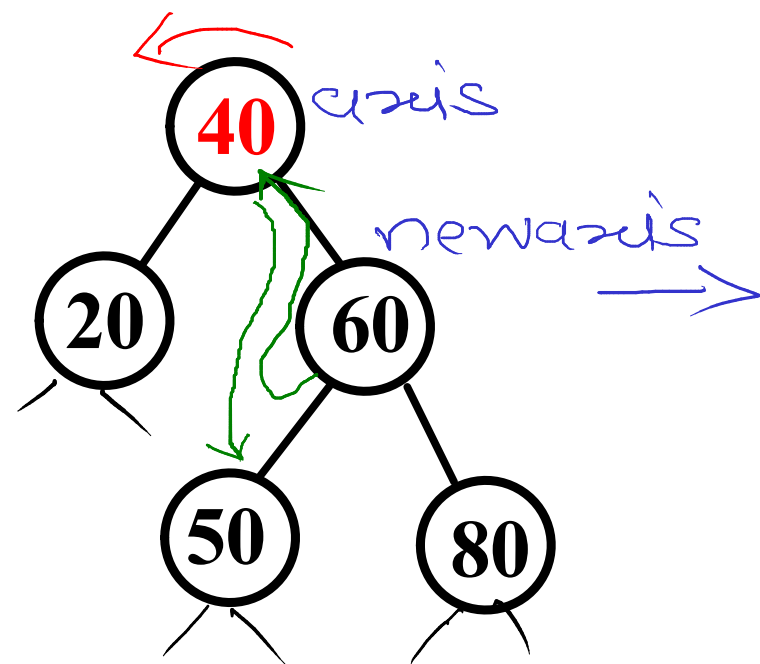


```

rightRotation(axis, parent) {
    newaxis = axis.left
    axis.left = newaxis.right
    newaxis.right = axis
    if (axis == root)
        root = newaxis;
    else if (axis == parent.left)
        parent.left = newaxis;
    else if (axis == parent.right)
        parent.right = newaxis;
}
    
```



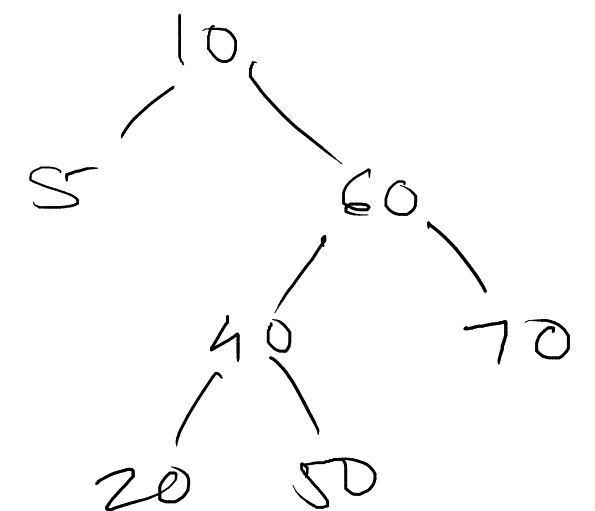
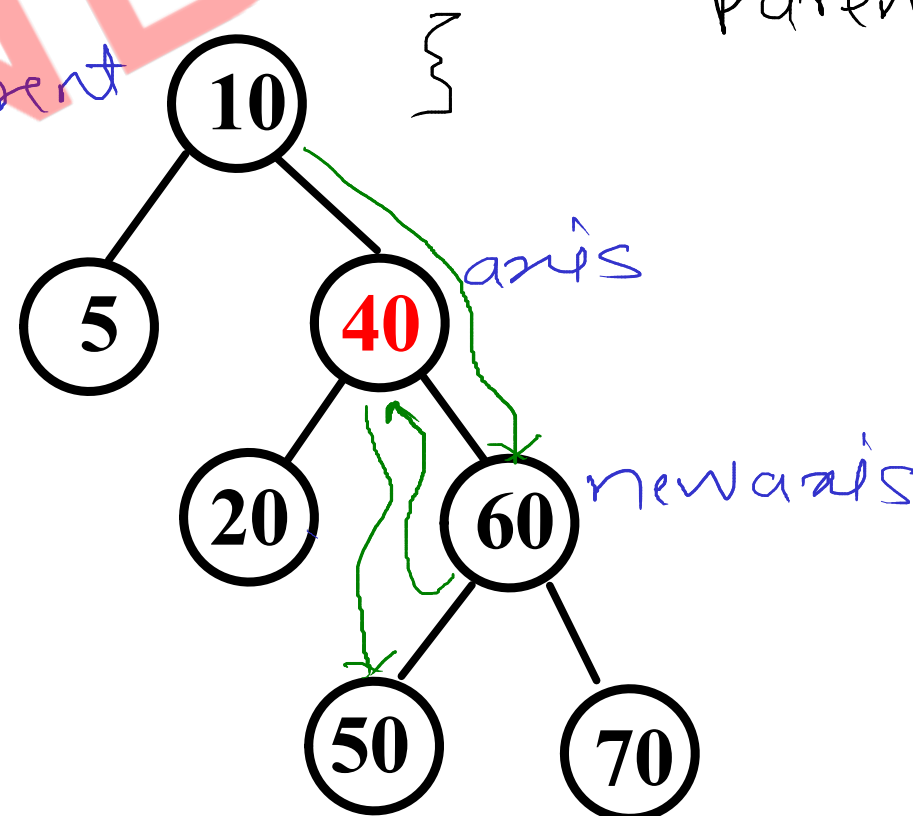
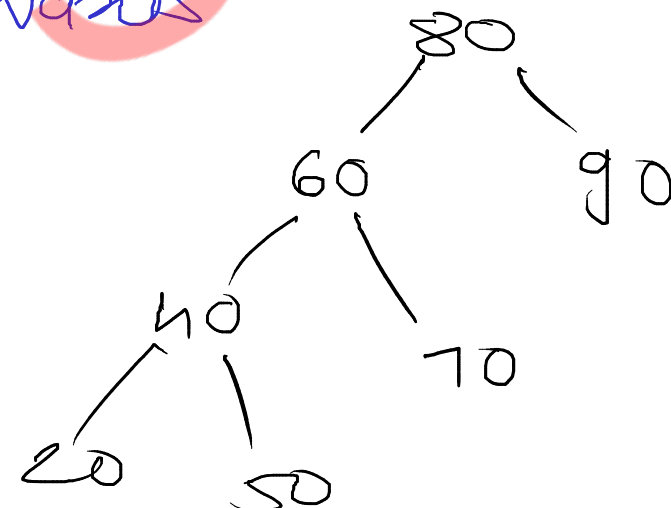
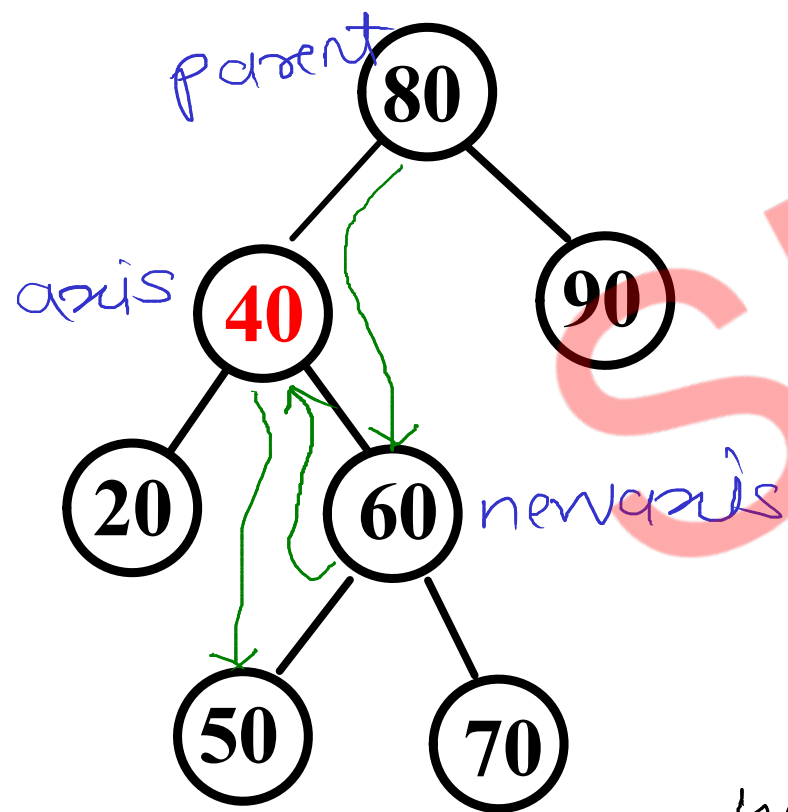
Left Rotation



```

left_rotation(axis, parent) {
    newaxis = axis.right
    axis.right = newaxis.left
    newaxis.left = axis
    if (axis == root)
        root = newaxis;
    else if (axis == parent.left)
        parent.left = newaxis;
    else if (axis == parent.right)
        parent.right = newaxis;
}

```



AVL Tree

- Self balancing binary Search Tree
- on every insertion and deletion of node, tree is balanced
- All operation on AVL tree are performed in $O(\log n)$ time
- Balance factor of all nodes is either -1, 0 or +1

Keys : 40, 20, 10, 25, 30, 22, 50

①

root
↓

②

root
↓
40

③

root
↓
40
↙
20

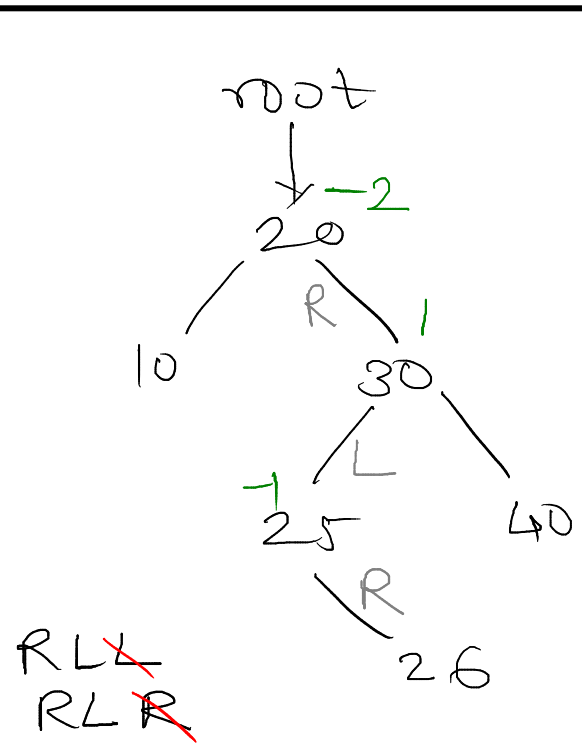
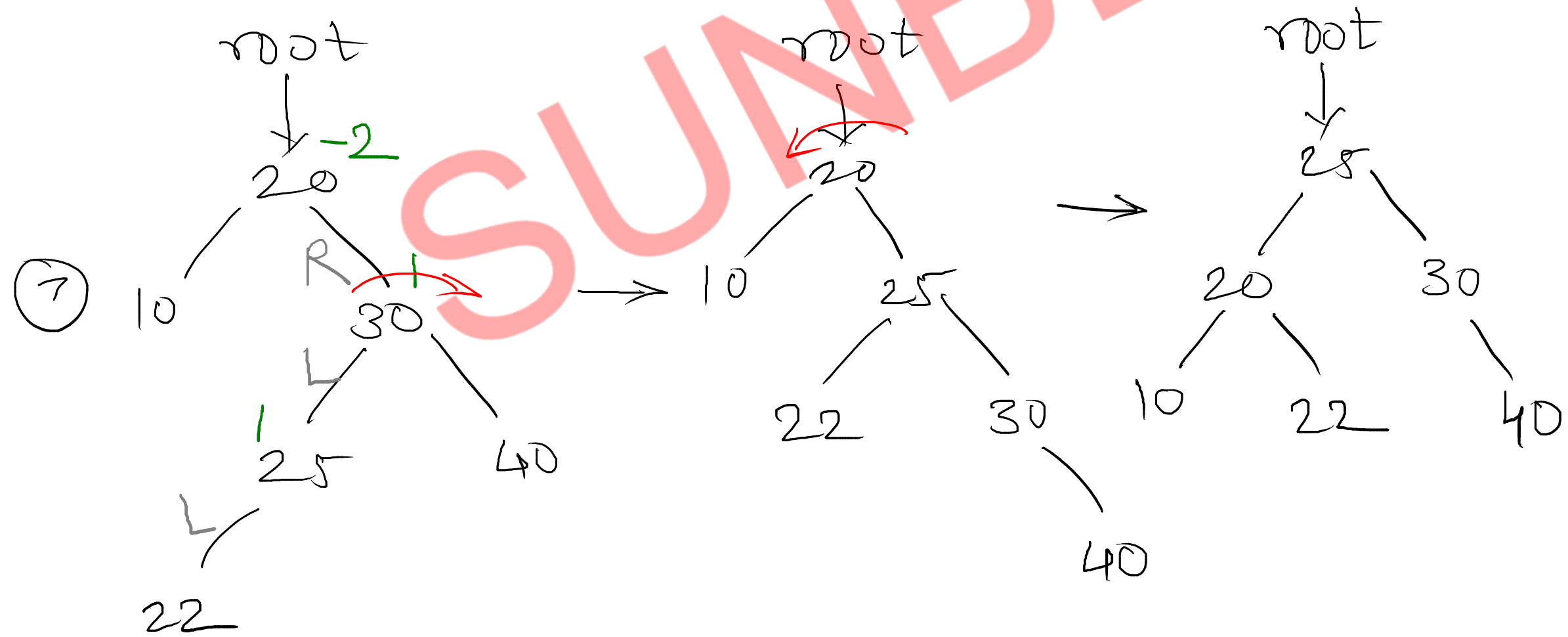
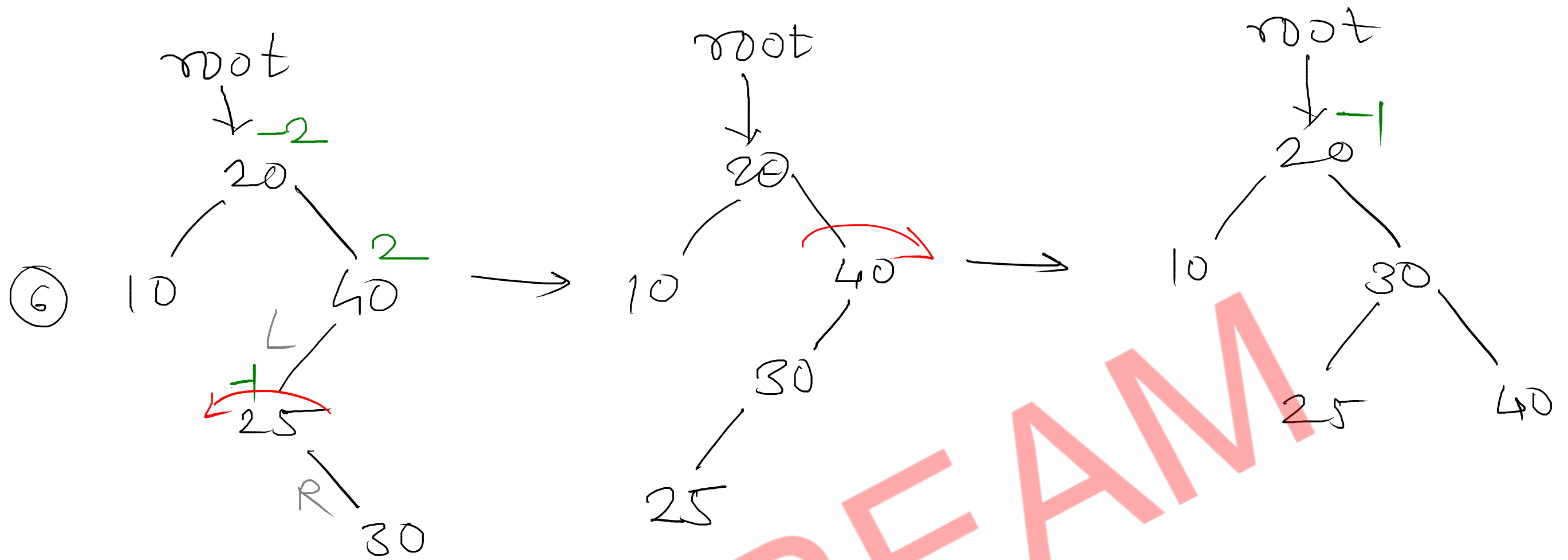
④

root
↓
40
↙ ↘
20 10
↙
10

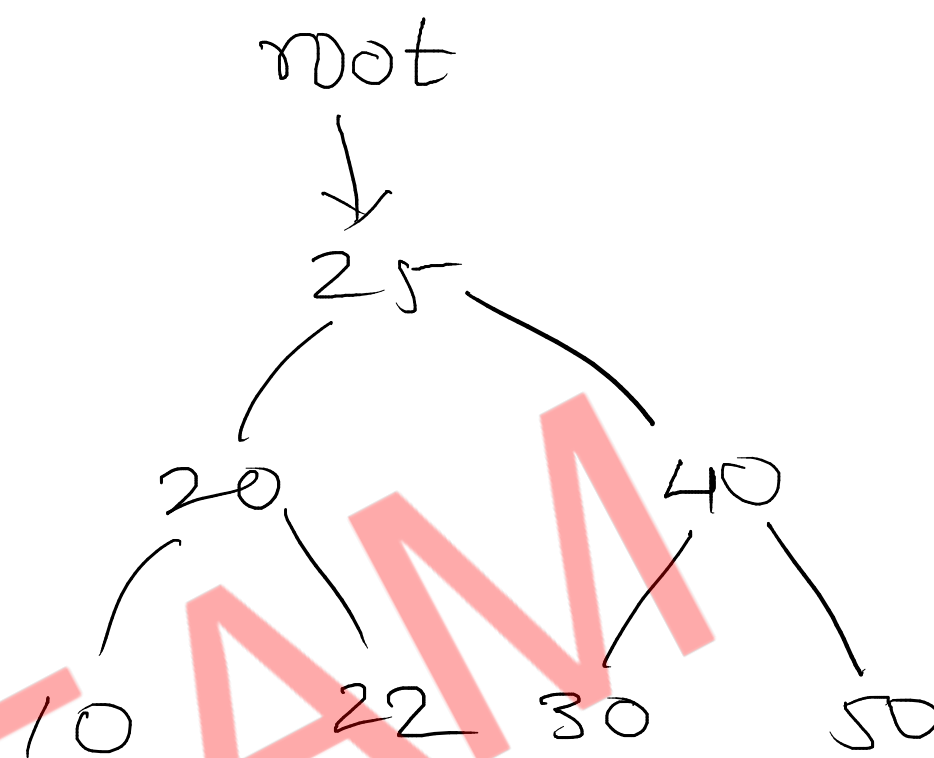
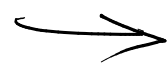
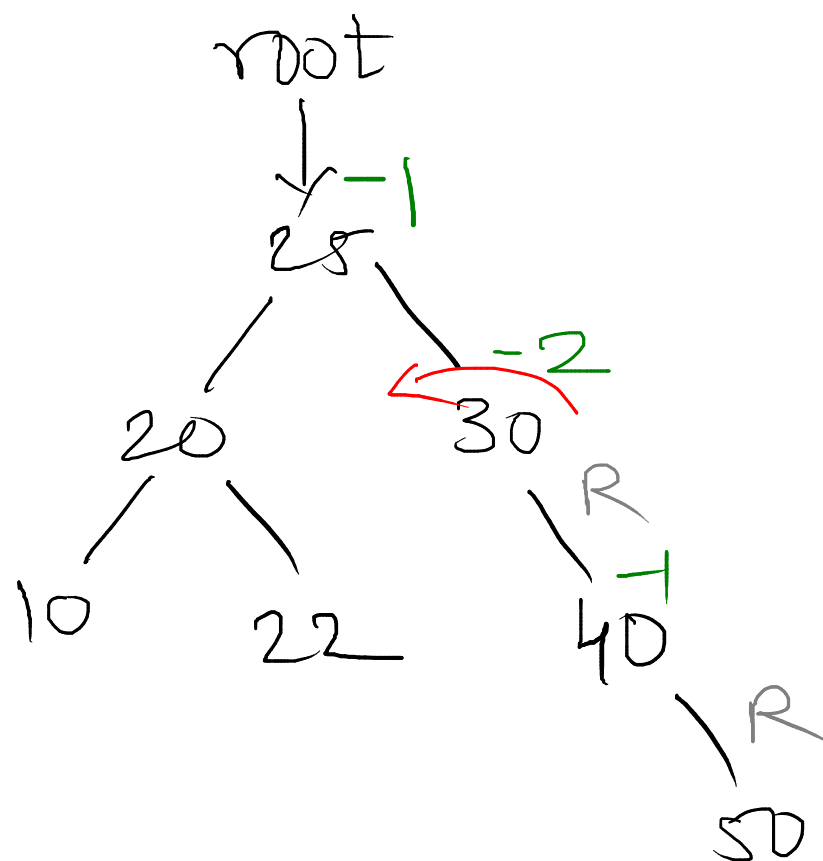
root
↓
20
↙ ↘
10 40

⑤

root
↓
20
↙ ↘
10 40
↘
25

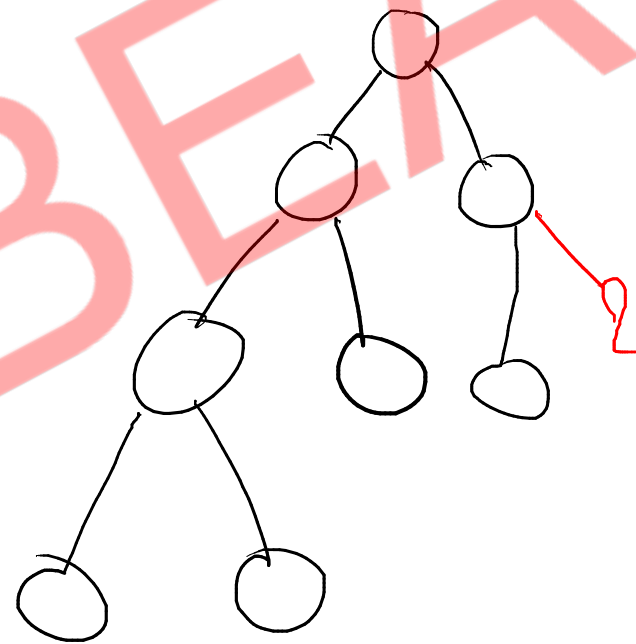
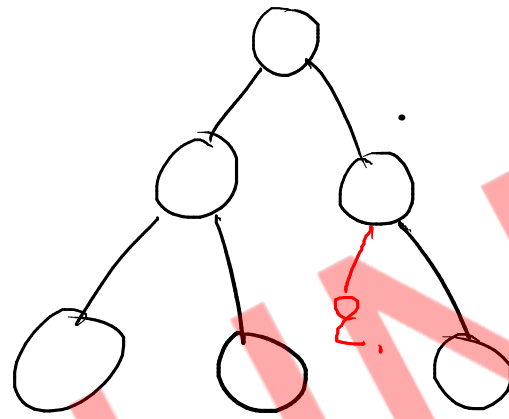
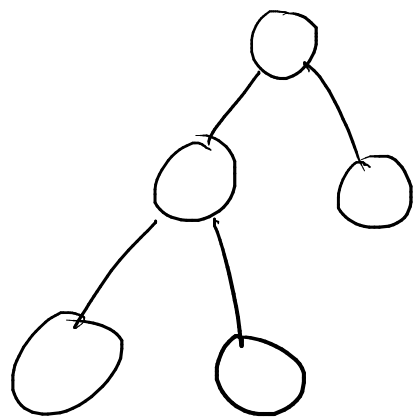


8

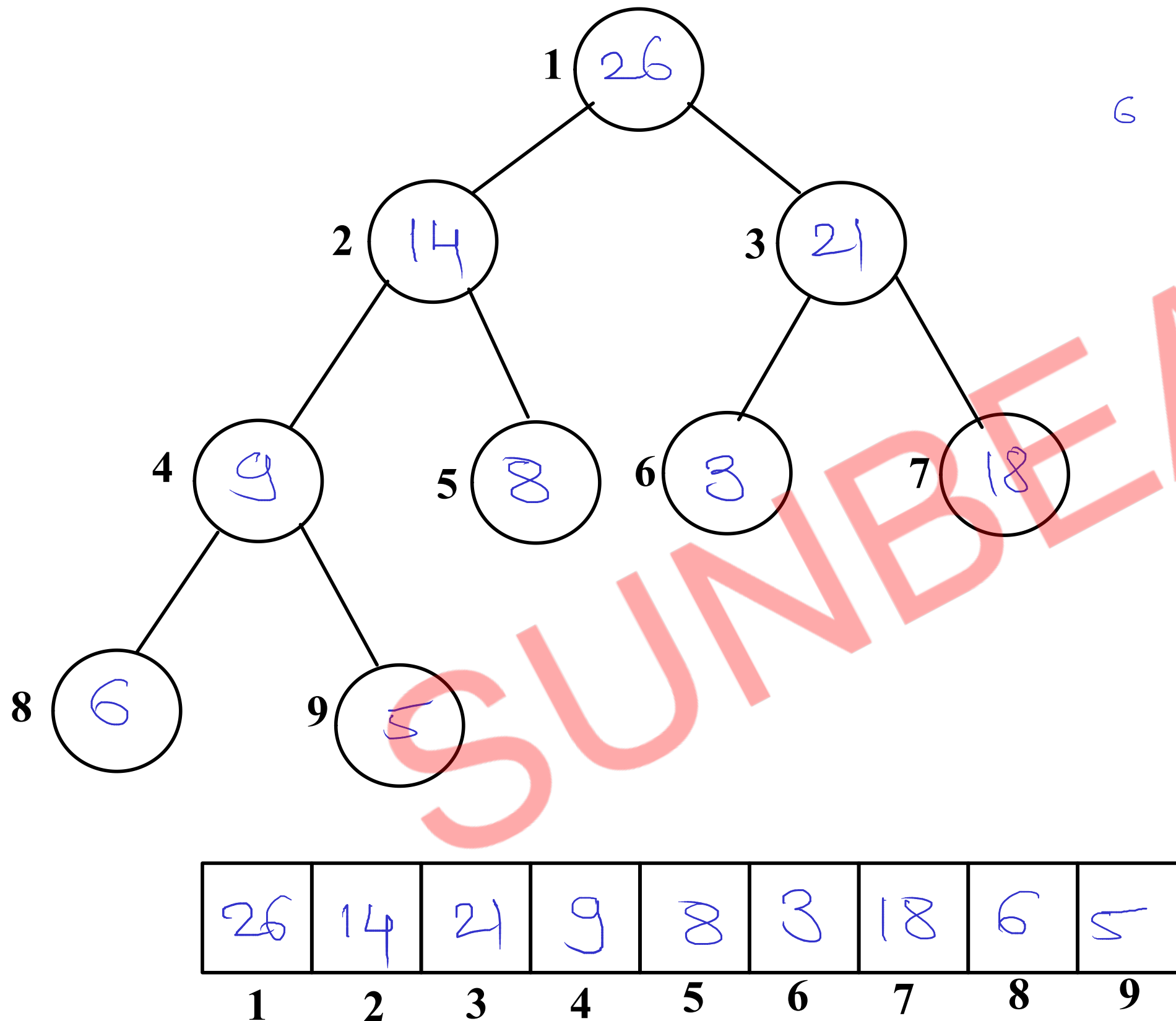


Almost Complete Binary Tree (ACBT)

- this tree is filled level by level (from left to right)
- this tree should satisfy two condition
 1. all leaf nodes must be at level h or $h-1$
 2. nodes of last level should be filled from left to right without keeping any blank (empty)



Heap - Create Heap

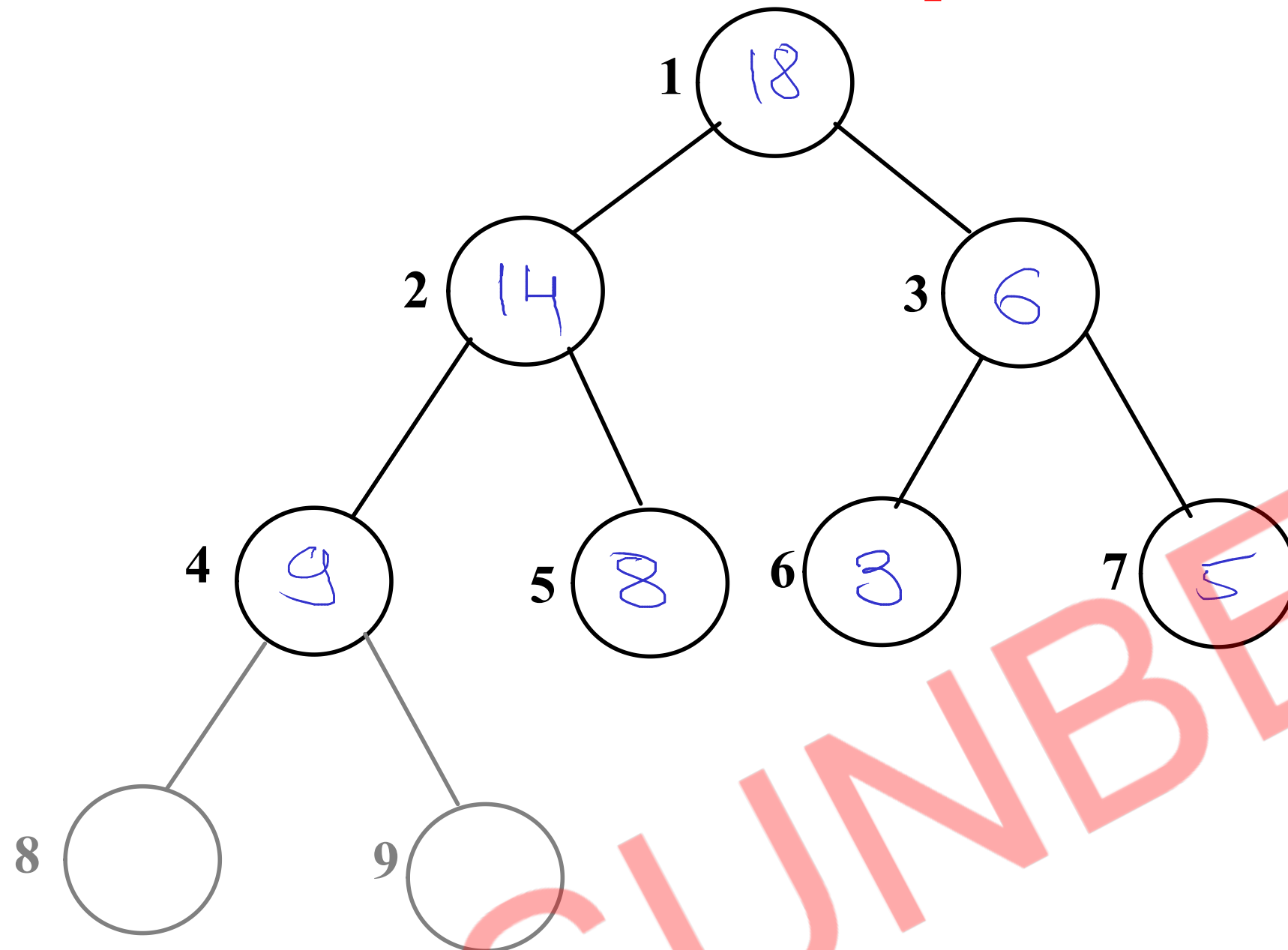


6 14 3 26 8 18 21 9 5

$$T(n) = O(\log n)$$

- 1) add new element at first empty location of the heap
- 2) adjust the position of newly added element by comparing with all its ancestors

Heap - Delete Heap



Max = 28
Max = 21

$$T(n) = O(\log n)$$

- 1) Delete root element
- 2) place last element at root's location
- 3) Adjust the position of it upto leaf nodes.

18	14	6	9	8	3	5		
1	2	3	4	5	6	7	8	9