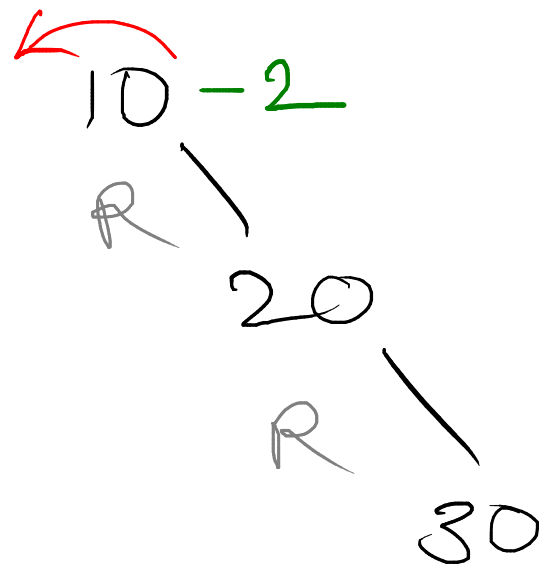
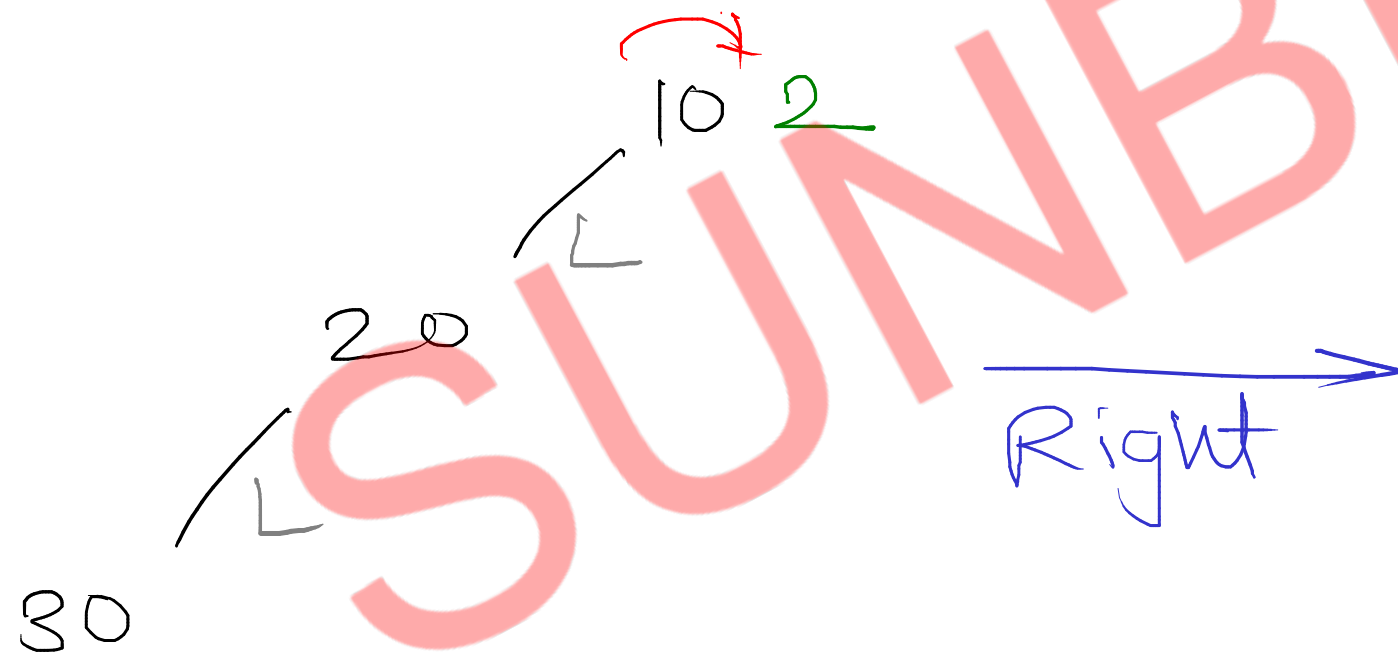


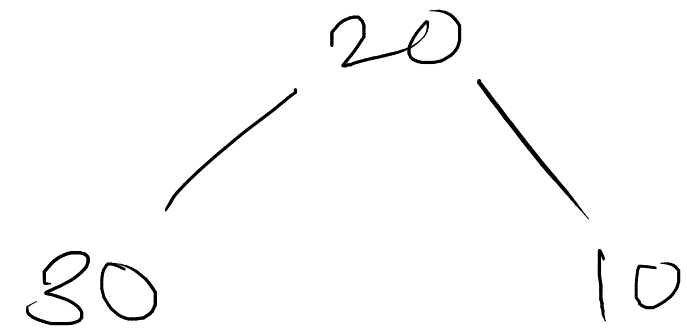
Single Rotations



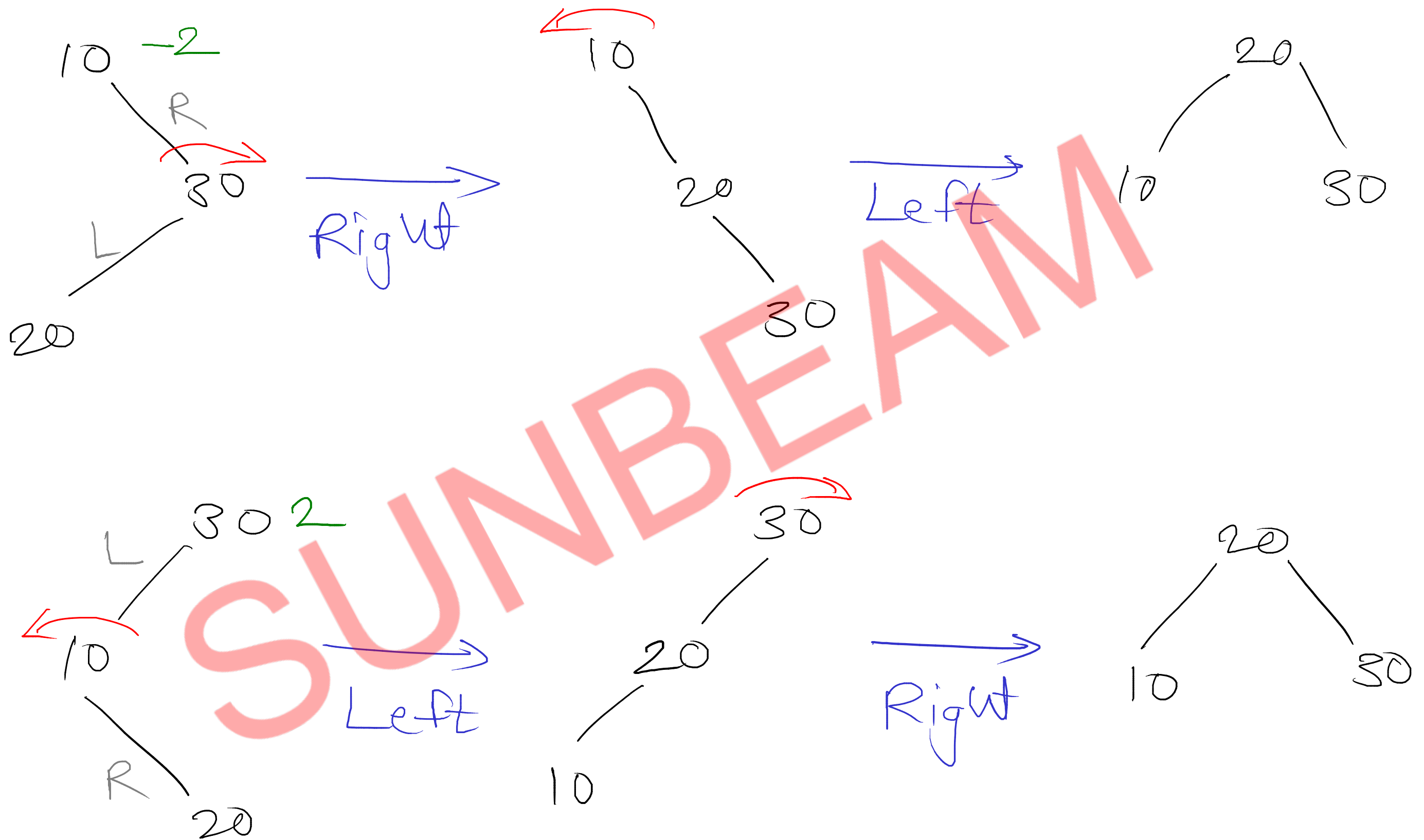
Left



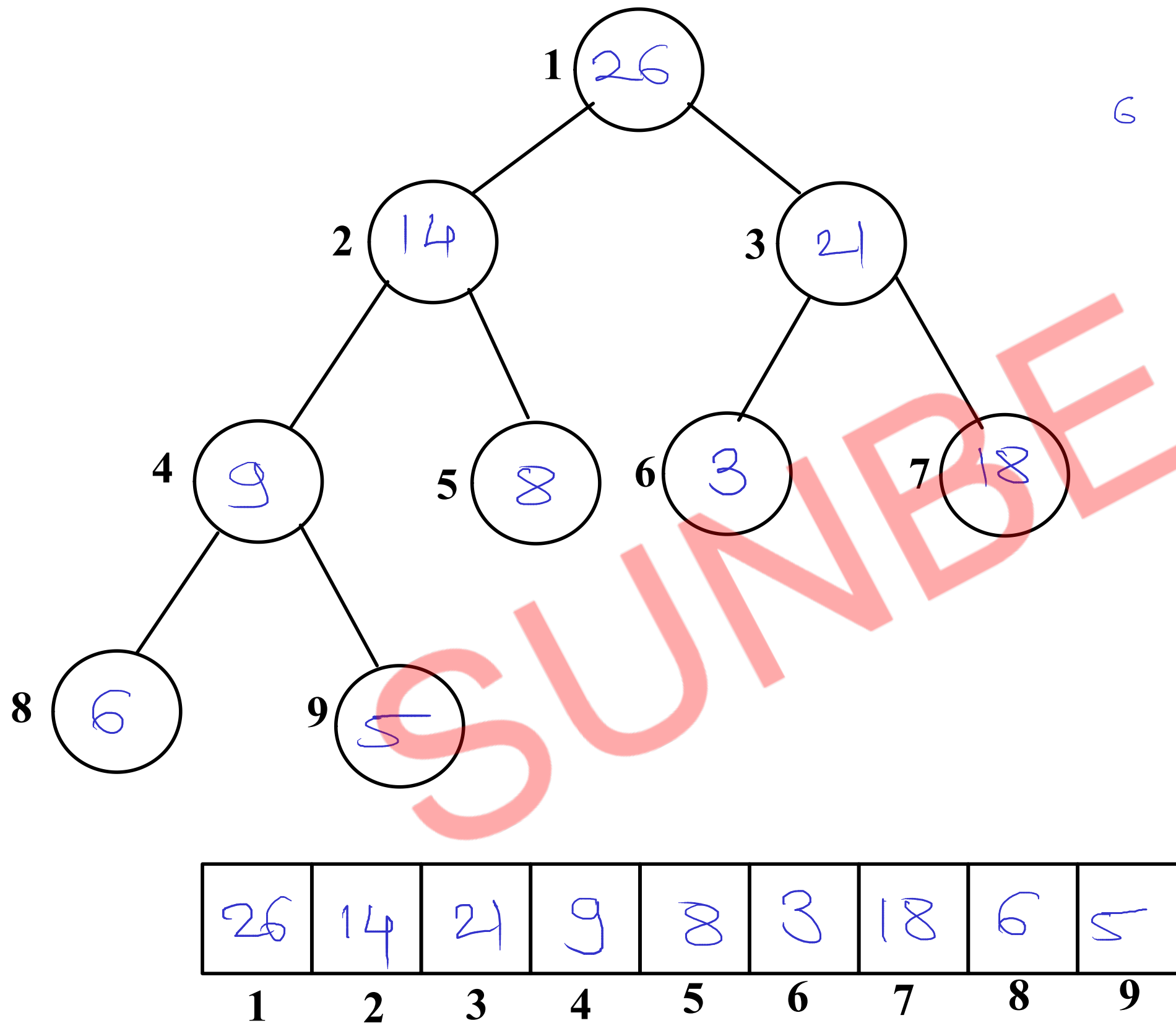
Right



Double Rotations



Heap - Create Heap

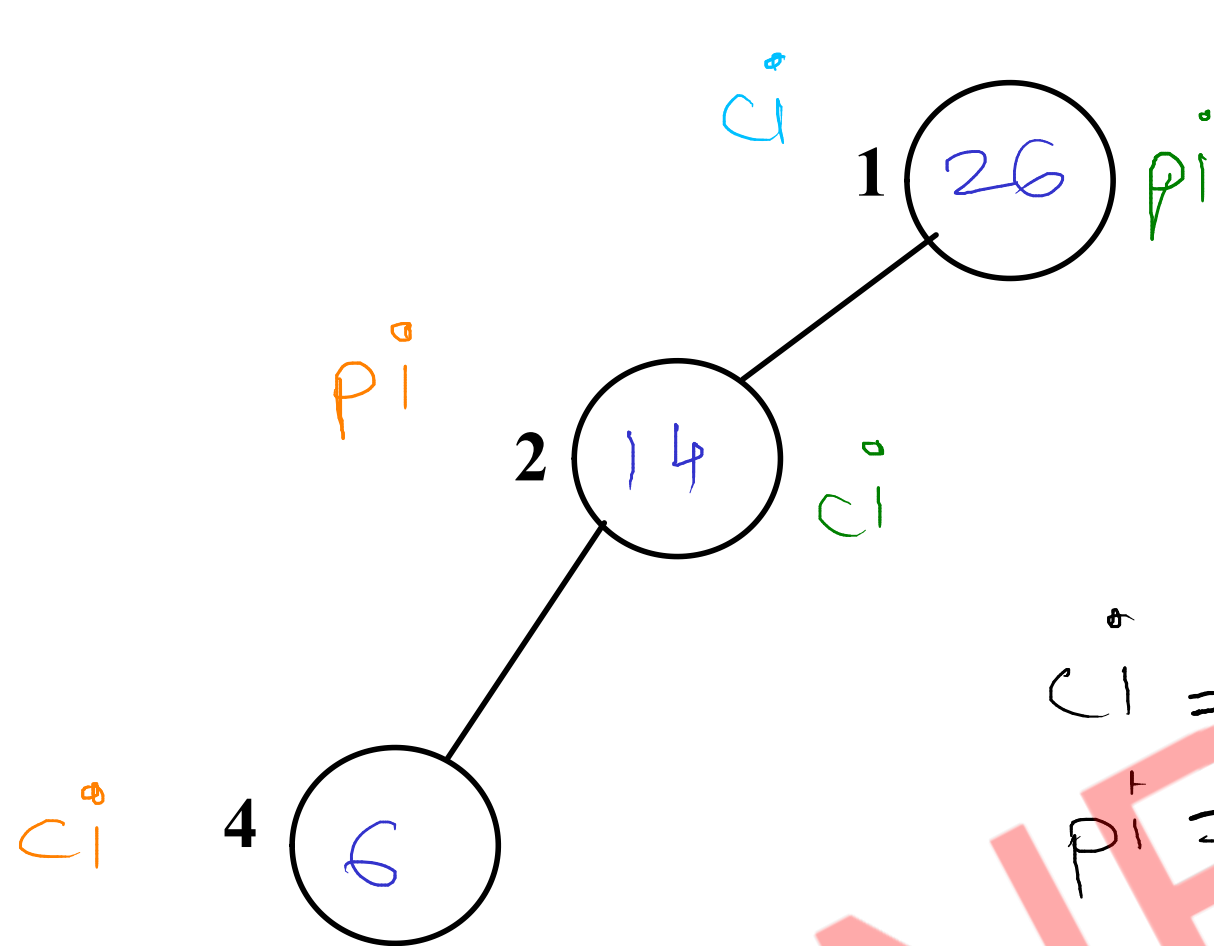


6 14 3 26 8 18 21 9 5

$$T(n) = O(\log n)$$

- 1) add new element at first empty location of the heap
- 2) adjust the position of newly added element by comparing with all its ancestors

$$p_i = 0$$

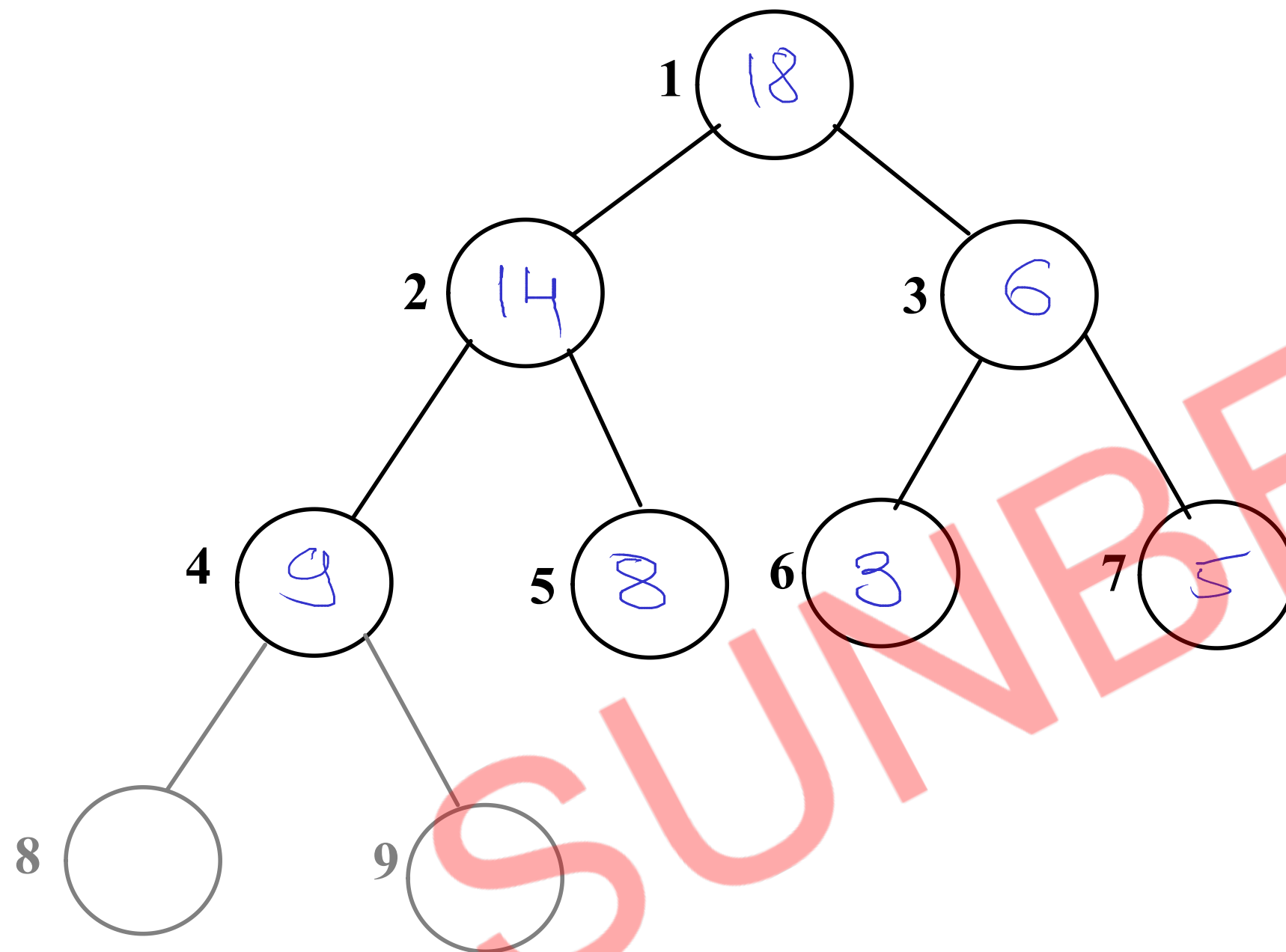


$$c_i = p_i$$

$$p_i = c_i / 2$$

c_i	p_i
4	2
2	1
1	0

Heap - Delete Heap

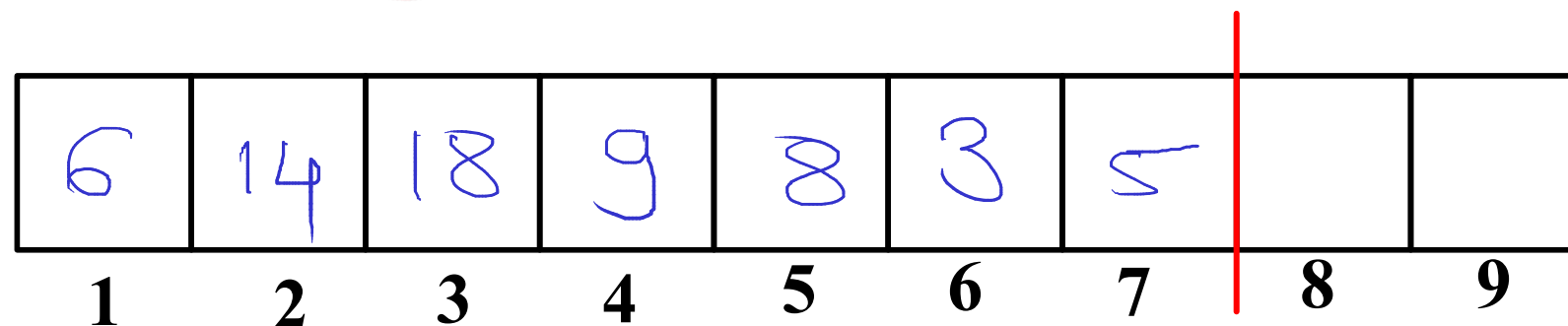


Max = 26

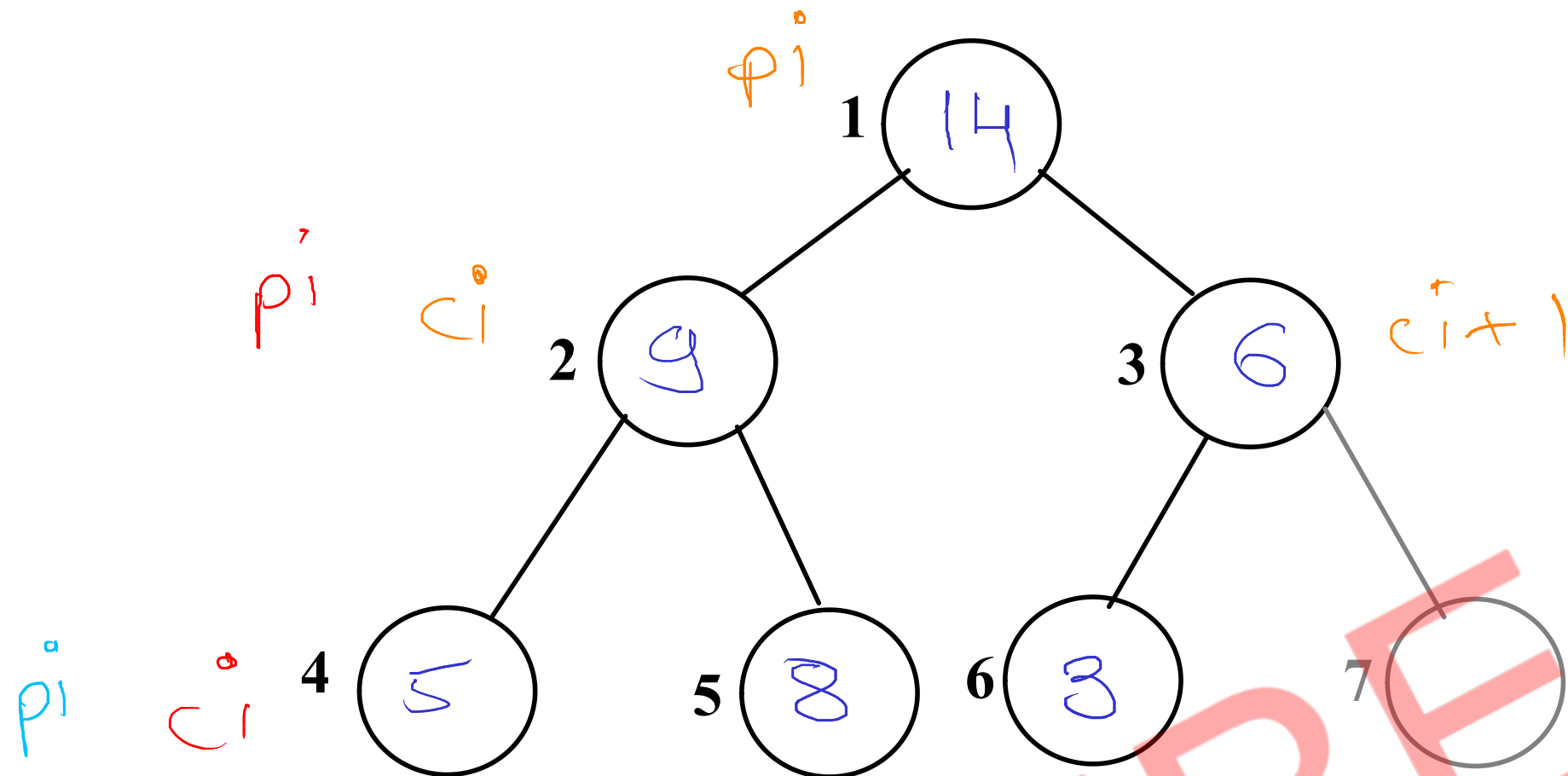
Max = 24

$$T(n) = O(\log n)$$

- 1) Delete root element
- 2) place last element at root's location
- 3) Adjust the position of it upto leaf nodes.



$$\underline{\underline{Max = 18}}$$



pi	ci
1	2
2	4
4	8

$$ci = 8$$

$$pi = ci$$

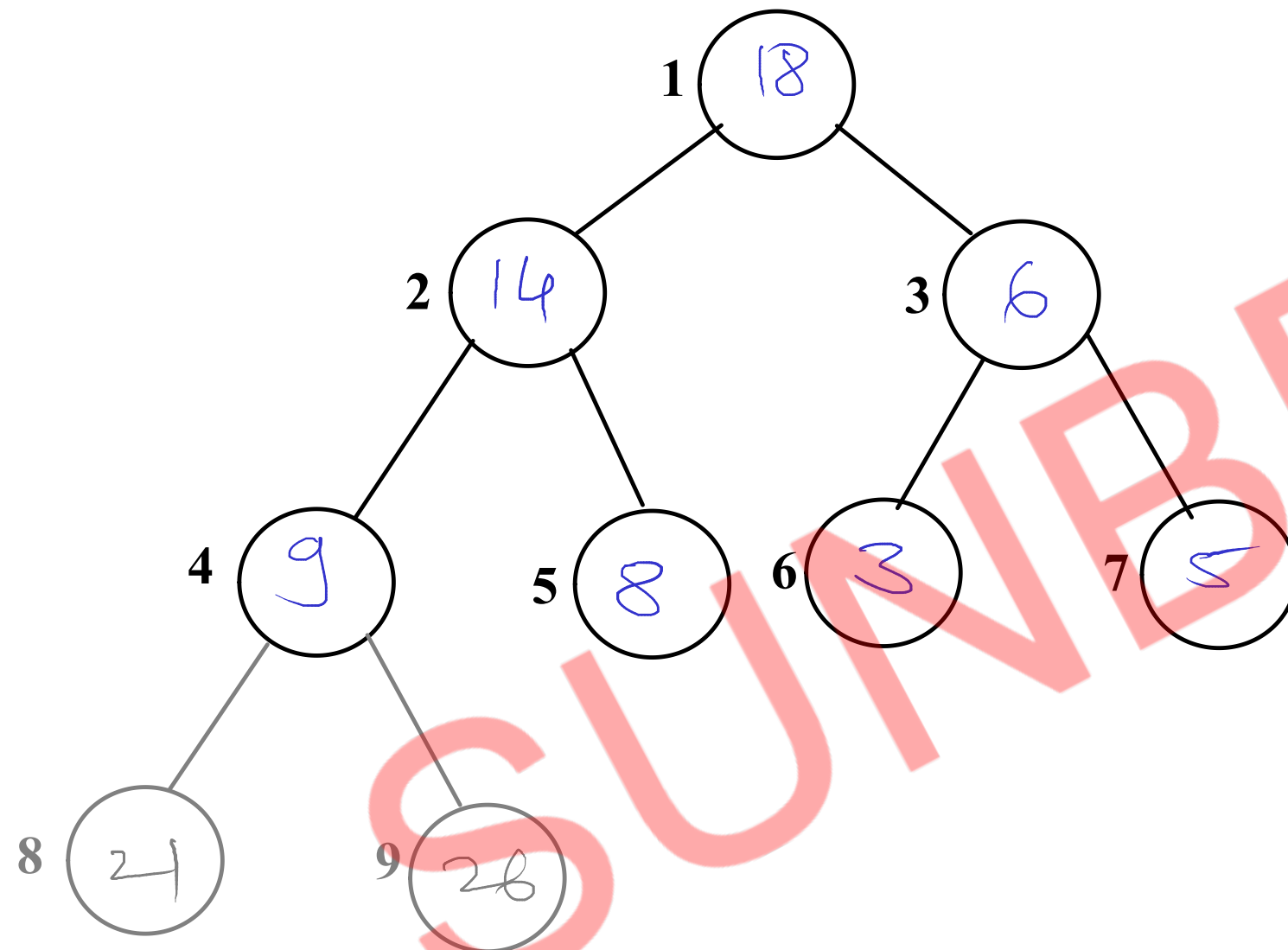
$$ci = pi * 2$$

Heap sort

1. create heap of given array
2. delete all the elements from heap

$$\begin{array}{l} \longrightarrow n \log n \\ \longrightarrow n \log n \\ \hline 2n \log n \end{array}$$

$$T(n) = O(n \log n)$$



18	14	6	9	8	3	5	21	26
1	2	3	4	5	6	7	8	9

Array - linear search - $O(n)$

Binary search - $O(\log n)$

Linked - linear search - $O(n)$

Binary tree - $O(n)$

Binary search tree - $O(\log n)$

Hash Table - $O(1)$

Hashing

- hashing is a technique in which data can be inserted, removed and searched in constant average time ($O(1)$)
- implementation of this technique is known as hash table
- hash table is nothing but a fixed size array in which elements are stored in key-value pairs

Array - hash table

index - slot

- keys are always unique but values can be duplicates
- every key is mapped with one slot of the hash table.
- this mapping is done by a mathematical function known as "hash function"

Hashing

key value



8, v1

3, v2

10, v3

4, v4

6, v5

13, v6

collision →

size = 10

10, v3	0
	1
	2
3, v2	3
4, v4	4
	5
6, v5	6
	7
8, v1	8
	9

Hash Table

$$h(k) = k \% \text{size}$$

$$h(8) = 8 \% 10 = 8$$

$$h(3) = 3 \% 10 = 3$$

$$h(10) = 10 \% 10 = 0$$

$$h(4) = 4 \% 10 = 4$$

$$h(6) = 6 \% 10 = 6$$

$$h(13) = 13 \% 10 = 3$$

Add: $\sim O(1)$

1) find slot

2) $arr[slot] = \{key, value\}$

Search: $\sim O(1)$

1) find slot

2) return $arr[slot]$

Delete: $\sim O(1)$

1) find slot

2) $arr[slot] = \text{null}$

collision:

when multiple keys
yield/give same slot,

collision handling techniques:

1) closed Addressing

2) Open Addressing

Closed Addressing/ Separate Chaining / Chaining (open probing)

size = 10

8, v1
3, v2
10, v3
4, v4
6, v5
13, v6
23, v7
26, v7

0	*	10, v3
1		
2		
3	*	23, v7 — 13, v6 — 3, v2
4	*	4, v4
5		
6	*	26, v7 — 6, v5
7		
8	*	8, v1
9		

Hash Table

$$h(k) = k \% \text{size}$$

$$h(8) = 8 \% 10 = 8$$

$$h(3) = 3 \% 10 = 3$$

$$h(10) = 10 \% 10 = 0$$

$$h(4) = 4 \% 10 = 4$$

$$h(6) = 6 \% 10 = 6$$

$$h(13) = 13 \% 10 = 3$$

$$h(23) = 23 \% 10 = 3$$

$$h(26) = 26 \% 10 = 6$$

bucket

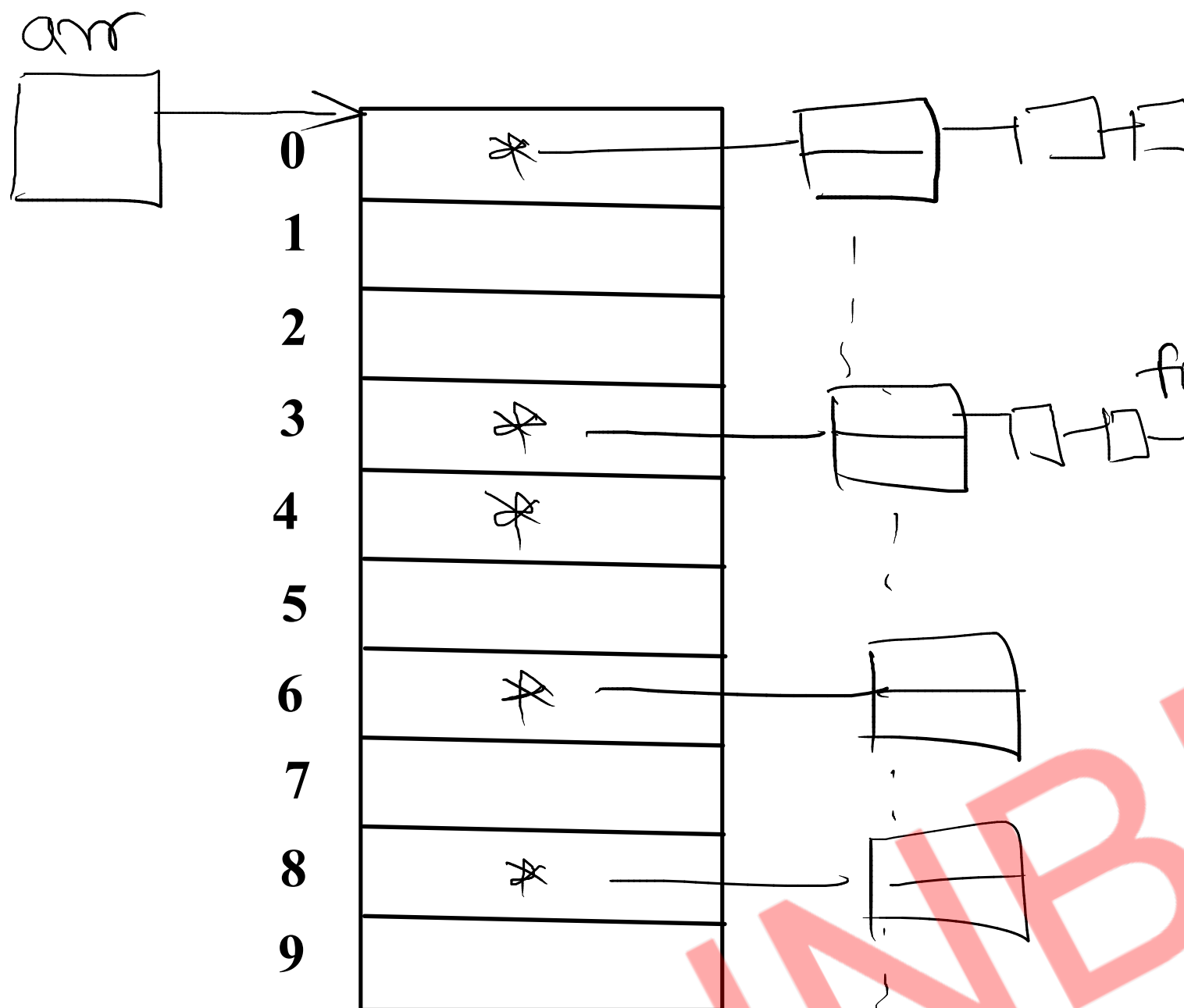
Disadvantage:

- 1) Key, value pairs are stored outside the table.
- 2) memory requirement (space) is more.
- 3) worst case time complexity is $O(n)$ ← when all keys will yield slot.

Advantage:

- multiple key, value pairs can be stored into hash table

arr



0

1

2

3

4

5

6

7

8

9

int arr[];

List arr[];

for(int i=0; i<arr.length; i++)

arr[i] = new LinkedList();

Entry {
int key;
String value;
}

Open Addressing - Linear Probing

closed probing)

size = 10

8, v1		0
3, v2		1
10, v3		2
4, v4		3
6, v5		4
13, v6		5
		6
		7
		8
		9

collision

Hash Table

$$h(k) = \text{key} \% \text{size}$$

$$h(k, i) = [h(k) + f(i)] \% \text{size}$$

$$f(i) = i$$

where $i = 1, 2, 3, \dots$

↑
probe number

$$h(13) = 13 \% 10 = 3 \text{ (c)}$$

$$h(13, 1) = [3 + 1] \% 10 = 4 \text{ (1st probe) (c)}$$

$$h(13, 2) = [3 + 2] \% 10 = 5 \text{ (2nd probe)}$$

Primary clustering:

- needs long run of filled slots to find empty slot "near" key position.

Probing:

finding next free slot of tbl whenever collision will occur.

Open Addressing - Quadratic Probing

size = 10

	10, v3	0
		1
		2
8, v1		
3, v2	3, v2	3
10, v3	4, v4	4
4, v4		5
6, v5	6, v5	6
13, v6	13, v6	7
	8, v1	8
		9

Hash Table

collision →

$$h(k) = \text{key} \% \text{size}$$

$$h(k, i) = [h(k) + f(i)] \% \text{size}$$

$$f(i) = i^2$$

where $i = 1, 2, 3, \dots$

$$h(13) = 13 \% 10 = 3 \quad \textcircled{c}$$

$$h(13, 1) = [3 + 1] \% 10 = 4 \quad (1^{\text{st}} \text{ probe}) \quad \textcircled{c}$$

$$h(13, 2) = [3 + 4] \% 10 = 7 \quad (2^{\text{nd}} \text{ probe})$$

Open Addressing - Quadratic Probing

size = 10

10, v3	0
	1
23, v7	2
3, v2	3
4, v4	4
	5
6, v5	6
13, v6	7
8, v1	8
33, v8	9

Hash Table

$$h(k) = \text{key \% size}$$

$$h(k, i) = [h(k) + f(i)] \% \text{size}$$

$$f(i) = i^2$$

where $i = 1, 2, 3, \dots$

23, v7

33, v8

$$h(23) = 23 \% 10 = 3 \text{ (c)}$$

$$h(23, 1) = [3 + 1] \% 10 = 4 \text{ 1st (c)}$$

$$h(23, 2) = [3 + 4] \% 10 = 7 \text{ 2nd (c)}$$

$$h(23, 3) = [3 + 9] \% 10 = 2 \text{ 3rd}$$

$$h(33) = 33 \% 10 = 3 \text{ (c)}$$

$$h(33, 1) = [3 + 1] \% 10 = 4 \text{ 1st (c)}$$

$$h(33, 2) = [3 + 4] \% 10 = 7 \text{ 2nd (c)}$$

$$h(33, 3) = [3 + 9] \% 10 = 2 \text{ 3rd (c)}$$

$$h(33, 4) = [3 + 16] \% 10 = 9 \text{ 4th}$$

Secondary clustering:
- need long run of filled slots to find empty slot "away" key position

Hashing - Double Hashing

size = 11

$$h_1(k) = \text{key \% size}$$

$$h_2(k) = 7 - (\text{key \% } 7)$$

$$h(k, i) = [h_1(k) + i * h_2(k)] \% \text{size}$$

$$h_1(8) = 8 \% 11 = 8$$

$$h_1(3) = 3 \% 11 = 3$$

$$h_1(10) = 10 \% 11 = 10$$

$$h_1(25) = 25 \% 11 = 3 \text{ (c)}$$

$$h_2(25) = 7 - 4 = 3$$

$$h(25, 1) = [3 + 1 * 3] \% 11 = 6 \text{ 1st}$$

$$h_1(36) = 3 \text{ (c)}$$

$$h_2(36) = 6$$

$$h(36, 1) = [3 + 6] \% 11 = 9$$

8, v1

3, v2

10, v3

25, v6

3, v2

25, v6

8, v1

10, v3

Hash Table

Rehashing

$$\text{Load Factor} = \frac{n}{N} = \frac{6}{10} = 0.6 \rightarrow 60\% \text{ filled}$$

(λ)

n - Number of elements (key value pairs) in hash table — 6

N - Number of slots in hash table — 10

if $n < N$	Load factor < 1	- free slots are available
if $n = N$	Load factor $= 1$	- no free slots
if $n > N$	Load factor > 1	- can not insert at all

- Rehashing is make the hash table size twice of existing size if hash table is 70 or 75 % full

- In rehashing existing key value pairs are again mapped according to new hash table size