1. A manufacturer can manufacture two different types of products, FRP sheets and FRP bath tubs. Each unit of FRP sheets of a particular size needs 5 kg of raw material A and 2 kg of raw material B. Each unit of FRP bath tubs needs 7 kg of raw material A and 1 kg of raw material B. Availability of raw material A in the market is 500 kg and that of raw material B 100 kg. Each FRP sheet contributes profit of Rs 100 and each FRP tub contributes profit of Rs 400. What is the most suitable product mix for the manufacturer to maximize profits? Formulate this problem as LP model and solve using graphical method.
2. A firm can produce three types of cloths A, B and C. Three kinds of wool are required for it, say, red wool, green and blue wool. One unit length of type A cloth needs 2 yards of red wool and 3 yard of blue wool, one unit length of type B cloth needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue wool and one unit of type C cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has a stock of only 8 yards of red wool, 10 yards of green wool and 15 yards of blue wool. It is assumed that the income obtained from one unit of type A cloth is Rs 3, of type B cloth is Rs 5 and that of type C cloth is Rs 4. Formulate the problem as linear programming problem.
3. Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12p./gm. Food Y contains 8 units and 12 units of A and B per gram respectively and costs 20 p./gm. The daily requirements of vitamin A and B are at least 100 units and 12 units respectively. Formulate the problem as LPP to minimize the cost.
4. (i) Express $x = [4,5]$ as a linear combination of vectors $a = [1,3]$, $b = [2,2]$.

   (ii) Find whether the set of vectors are linearly independent $S_1 = \{(1,2,3), (4,-2,7)\}$, $S_2 = \{(1,2,3), (1,0,0), (0,1,0)\}$

   (iii) Show that the following set of vectors forms a basis of $\mathbb{R}^3$:

   $B_1 = \{(2,-1,0), (3,5,1), (1,1,2)\}$, $B_2 = \{(1,1,0), (1,-1,0), (0,0,1)\}$

5. Find the basic solutions of the system

$$\begin{aligned} x + 2z &= 1 \\ y + z &= 4 \end{aligned}.$$

   Which of them are feasible if $x, y, z \geq 0$?

6. Find the basic feasible solutions of the system

$$\begin{aligned} 2x_1 + 6x_2 + 2x_3 + x_4 &= 3 \\ 6x_1 + 4x_2 + 4x_3 + 6x_4 &= 2 \end{aligned}$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

   Show that all the basic solutions are degenerate.

7. Show that although $(2,3,2)$ is a feasible solution to the system of equations

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 9 \\ 3x_1 + 2x_2 + 5x_3 &= 22 \end{aligned}$$

   $x_1, x_2, x_3 \geq 0$, it is not a basic solution. How many basic solutions this system may have? Find all the basic feasible solutions of the given system.

8. Make a graphical representation of the L.P.P

   Maximize $z = x_1 + x_2$

$$\text{Subject to} \quad \begin{aligned} 5x_1 + 9x_2 &\leq 45 \\ x_1 + x_2 &\geq 2 \\ x_2 \leq 4, \quad x_1, x_2 &\geq 0 \end{aligned}$$

   and solve using graphical method.

9. Solve the L.P.P by algebraic method

   Maximize $z = -2x_1 + x_2 + 3x_3$

$$\text{Subject to} \quad \begin{aligned} x_1 - 2x_2 + 3x_3 &= 2 \\ 3x_1 + 2x_2 + 4x_3 &= 1 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

10. Solve the L.P.P by Simplex method

   Maximize $z = 2x_1 + 3x_2 + x_3$

$$\text{Subject to} \quad \begin{aligned} -3x_1 + 2x_2 + 3x_3 &= 8 \\ -3x_1 + 4x_2 + 2x_3 &= 7 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

11. Solve the L.P.P by introducing artificial variables

Maximize $z = -x_1 - x_2 - x_3$

Subject to
$$x_1 - x_2 + 2x_3 = 2$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$x_1, x_2, x_3 \geq 0.$$

12. Solve the following LPP:

Minimize $\quad \mathbf{Z = 4X_1 + 8X_2 + 3X_3}$

Subject to: $\quad X_1 + X_2 \geq 2$

$2X_1 + X_3 \geq 5$

$X_1, X_2, X_3 \geq 0$

13. Solve the following LPP:

Maximize $\quad \mathbf{Z = 5X_1 - 2X_2 + 3X_3}$

Subject to: $\quad 2X_1 + 2X_2 - X_3 \geq 2$

$3X_1 - 4X_2 \leq 3$

$X_2 + 3X_3 \leq 5$

$X_1, X_2, X_3 \geq 0$

14. Put the following problem in the standard form and construct the 1st simplex table

Maximize $z = 4x_1 + 75$

Subject to
$$2x_1 + 3x_2 \leq 8$$
$$x_1 + 4x_2 \leq 10$$

x_1 is unrestricted and $x_2 \geq 0$.

15. Find the dual of the following L.P.P

Maximize $z = 2x_1 + 3x_2 + 4x_3$

Subject to
$$x_1 - 5x_2 + 3x_3 = 7$$
$$2x_1 - 5x_2 \leq 3$$
$$3\,x_2 - x_3 \geq 5$$

x_3 is unrestricted in sign and $x_1, x_2 \geq 0$.

16. Construct the dual of the given LPP

Maximize $z = x_1 - 7x_2$

Subject to
$$2x_1 + x_2 \leq 1$$
$$x_1 - 3x_2 \leq 6$$
$$2x_1 + 4x_2 \leq 7, \;\; x_1, x_2 \geq 0.$$

17. Construct the dual of the given LPP

Maximize $z = 4x_1 + 7x_2$

Subject to
$$2x_1 + x_2 \leq 1000$$
$$10x_1 + 10x_2 \leq 6000$$
$$2x_1 + 4x_2 \leq 2000, \;\; x_1, x_2 \geq 0.$$

18. Use dual simplex method to solve the L.P.P

Maximize $z = -2x_1 - 3x_2 - x_3$

Subject to
$$2x_1 + x_2 + 2x_3 \geq 3$$
$$3x_1 + 2x_2 + x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0.$$

19. Solve the L.P.P by Charne's Big-M method:

Minimize $z = 4x_1 + 2x_2$

Subject to
$$3x_1 + 4x_2 \geq 27$$
$$x_1 + x_2 \geq 21$$
$$x_1 + 2x_2 \geq 30, \;\; x_1, x_2 \geq 0.$$

20. Use dual simplex method to solve the L.P.P

Minimize $z = -2x_1 - x_3$

Subject to
$$x_1 + x_2 - x_3 \geq 5$$
$$x_1 - 2x_2 + 4x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0.$$

21. Use dual simplex method to solve the L.P.P

Minimize $z = x_1 + 2x_2 + 3x_3$

Subject to
$$2x_1 - x_2 + x_3 \geq 4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$x_2 - x_3 \geq 2, \;\; x_1, x_2, x_3 \geq 0.$$

22. The Das company can make three different products X,Y,Z by combining steel and rubber. Product X requires 2 units of steel and 3 units of rubber and can be sold at a contribution margin of Rs. 45 per unit. Y requires 3 units of steel and 3 units of rubber can be sold at a contribution of Rs. 40 per unit. Z requires 1 unit of steel and 2 units of rubber and yields Rs. 25 contribution per unit. There are 100 units of steel and 120 units of rubber available per day. In order to maximize profit the optimal daily mix os X,Y,Z must be determined.
    (i) Pose this as Linear Programming Problem
    (ii) Put the problem in standard form and solve using Simplex.

23. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method.

	D_1	D_2	D_3	D_4	a_i
O_1	5	3	11	7	6
O_2	1	0	8	1	1
O_3	2	8	1	9	10
b_j	7	5	3	2	

24. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method.

	D_1	D_2	D_3	D_4	a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
b_j	20	40	30	10	

25. Identical products are produced in three factories O_1, O_2, O_3 and sent to four warehouses W_1, W_2, W_3, W_4 for delivery to the customers. The cost of transportation, the capacities of the factories and the demands of the warehouses are given in the following matrix:

	W_1	W_2	W_3	W_4	*Capacity*
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
Demands	7	5	3	2	

(i) Find an optimal schedule of delivery for minimization of the cost of transportation.
(ii) Find the unfulfilled demand of the warehouses, if any.
(iii) Is there any alternative optimal solution to the problem?

26. Obtain an optimal basic feasible solution to the transportation problem

	W_1	W_2	W_3	W_4	
F_1	1	3	5	7	**7**
F_2	1	0	6	1	**9**
F_3	5	3	8	9	**18**
	5	**8**	**7**	**14**	

27. Identical products are produced in three factories O_1, O_2, O_3 and sent to four warehouses W_1, W_2, W_3, W_4 for delivery to the customers. The cost of transportation, the capacities of the factories and the demands of the warehouses are given in the following matrix:

	W_1	W_2	W_3	W_4	*Capacity*
O_1	3	8	7	4	**30**
O_2	5	2	9	5	**50**
O_3	4	3	6	2	**80**
Demands	**20**	**60**	**55**	**40**	

(i) Find an optimal schedule of delivery for minimization of the cost of transportation.
(ii) Find the unfulfilled demand of the warehouses, if any.
(iii) Is there any alternative optimal solution to the problem?

28. Three persons A, B, C are being considered for three open positions I, II, III. Each person has been given a rating for each position as shown in the following table:

	I	II	III
A	7	5	6
B	8	4	7
C	9	6	4

Assign each person to one and only one position in such a way that the sum of ratings for all three persons is maximum.

29. Three persons A, B, C are being considered for three open positions I, II, III. Each person has been given a rating for each position as shown in the following table:

	I	II	III
A	32	38	40
B	40	24	28
C	41	27	33

Assign each person to one and only one position in such a way that the sum of ratings for all three persons is maximum.

30. Find the optimal assignment of resource persons A, B, C, D and E to jobs 1, 2, 3, 4 and 5 that will result in maximum profit. What is the maximum profit?

	1	**2**	**3**	**4**	**5**
A	32	38	40	28	40
B	40	24	28	21	36
C	41	27	33	30	37
D	22	38	41	36	36
E	29	33	40	35	39

31. The owner of a small machine shop has four machinists available to assign to jobs for the day. Five jobs are offered with expected profit for each machinist on each job as given in the following table:

	A	B	C	D	E
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

Find the assignment of the machinists to jobs that will result in maximum profit.

32. Solve the following unbalanced assignment problem of minimizing total time for doing all the jobs 1,2,3,4,5 and operators A,B,C,D,E,F.

	1	2	3	4	5
A	8	3	6	3	7
B	3	6	9	8	8
C	9	9	7	9	9
D	7	2	3	5	6
E	10	3	8	9	7
F	5	7	4	7	8

33. Draw a network from the following activities and find a critical path and total project duration.

Activity	Durations (Weeks)	Activity	Durations (Weeks)
1-2	4	4-7	3
1-4	3	5-6	3
1-5	3	6-9	6
2-3	5	7-8	6
3-8	1	8-9	2
4-9	6		

34. A project schedule has the following characteristics as shown in table.
Construct a PERT network and find a critical path and total project duration.

Activity	Name	Time(days)	Activity	Name	Time(days)
1-2	A	4	5-6	G	4
1-4	B	1	5-7	H	8
1-5	C	1	6-8	I	1
2-3	D	1	7-8	J	2
3-8	E	6	8-10	K	5
4-9	F	5	9-10	L	7

35. A project schedule has the following characteristics as shown in table.
Construct a network and find a critical path and total project duration.

Activity	Name	Time(days)	Activity	Name	Time(days)
1-2	A	10	3-7	G	12
1-3	B	4	4-5	H	15
1-4	C	6	5-6	I	6
2-3	D	5	6-7	J	5
2-5	E	12	6-8	K	4
2-6	F	9	7-8	L	7

36. Generate the steady state probabilities of $M/M/1: \infty/\infty$ model.
37. Generate the steady state probabilities of $M/M/1:N/\infty$ model.

38. Generate the steady state probabilities of M/M/1: ∞/∞ model and show that for this model traffic intensity must be less than 1.
39. A bank plans to open a single server drive-in-banking facility at a particular center. It is estimated that 28 customers will arrive each hour on an average. If on an average, it requires 2 minutes to process a customer transaction, determine
    - (i) The probability of time that the system will be idle.
    - (ii) On the average how long the customer will have to wait before receiving the server.
    - (iii) The length of the drive way required to accommodate all the arrivals. On the average 20 feet of drive way is required for each car that is waiting for service.
40. Customers arrive at the executive class air ticketing at the rate of 10 per hour. There is only one airlines clerk serving the customer at the rate of 20 per hour. If the conditions of single channel queuing model apply to this problem i.e arrival rate and service rate probability distribution are approximated to Poisson's and Exponential respectively; determine
    - (i) System being idle probability
    - (ii) The probability that there is no customer waiting to buy the ticket
    - (iii) The probability that the customer is being served and no body is waiting.

41. An electricity bill receiving window in a small town has only one cashier who handles and issues receipts to the customers. He takes on an average 5 minutes per customer. It has been estimated that the persons coming for bill payment have no set pattern but on an average 8 persons come per hour. The management receives a lot of complaints regarding customers waiting for long in the queue and so decided to find out
    - (i) What is the average length of the queue
    - (ii) What time on an average the cashier is idle
    - (iii) What is the average time for which a person has to wait to pay his bill