Tively defined by

$$T(f(\alpha)) = \int f(\alpha)d\alpha$$

in the event of all tentiment funds from the property of the first of all the property of the first of all the property of the prope

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: ≠c st f(c) ≠0. : f(x)=0 & RE R But Eigen vector #0. A ≠ 0 is an eigen value:  $T(f(x)) = \lambda f(x)$ for some non-zero x EV > If (t) dt = \( f(a) \quad \quad \alpha \in \rangle \ f(0) = 0 Francis no elymontus ( change and I + 2)  $f(\alpha) = ce^{\alpha/\lambda}$ #19 (4) = (4) 1/4 = (CA) 1/4 1/3 But f(0)=0 => +(2)=0 But f(x) = 0.  $\# V = M_3(R)$ let A & M3(IR) TOV-V, St U(B) = AB-BA then Tis Show that if A is diagonalisable diagonalisable. o is not rigged violues. Sol: JP St PAP = D. S:V→V S(B) = PBP YBEM3(F) BYBEV BX S(X)=B. X = PXP  $S(B) = 0 \Rightarrow B = 0$ 

since, A is diagonalisable 3P st T(B)=DB-BD PAP = D - diagonal  $U = SOTO_{5}^{1}$ \$ 5'(B) = PBP T(5'(B)) = DPBP'- PBP'P S(T(5'(B))) = P'(DPBP-PBPD) P - PDPBPP .VEXV = P.DPB - BPDP S(B) = PBP1 / 17 (18) of  $S(B) = PBP^{(p)} = \infty$   $T(\bar{s}'(B)) = D(\bar{p}'BP) - (\bar{p}'BP)(D)$ \$ (B) = PBP > P = >  $S(T(S^{\dagger}(B)) = P(DPBP - PDPBP) = PDPBP - BPDP = PDPBP - BPDPBP = PDPBP - BPDPBP = PDPBP = P$ suppose there exists two south BPDP and Pa = AB-BA Vand - alina SOTOS = U [A-12=0] : [P=4-A] B= < Ei, 3 pg Lonolaismit stinitini real sunt spinosion ron S (1 0 0 0 )=

# (v,<,>) - FIP

7(2)) - 6(BP - PEP 9 f:V->F, linear functional

Then there exists a unique vector Bev

st f(x) = <x, B> 4 x EV.

Let La..., and be onthononmal basis.

 $\beta = \sum_{i=1}^{n} \overline{f(\alpha_i)} \, \alpha_i$ 

 $\langle \alpha_i, \beta \rangle = \beta \overline{f(\alpha_i)} = f(\alpha_i)$ 

x = \frac{5}{121} \quad (989) - (989)a = ((a)/3)T

Juny dib d= 1A1

<α, β> = Σς f(α) (1999 - 9890) 9 = (1892) 12

= 8 f( E(xi) = f(x)

suppose there exists two such B. say B, and B2.

 $\langle \alpha, \beta_1 \rangle = \langle \alpha, \beta_2 \rangle \quad \forall \alpha \in V$ 

**∀**∝∈∨ ⇒ <d, β1-β2>=0

 $\beta_1 - \beta_2 = 0$  :  $\beta_1 = \beta_2$ 

not always true for infinite dimensional space.

I Don't I always home I \*  $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$ f(x1, x2, x3) = 2x1+5x2+3x3 1 2 only 1 201 T B=[253]T Aleman I. well Compared insuit he are (V,<,>) IPS acortourige transplan or pino T: V > V linear operator,

A linear operator T: V > V is said to be adjoint of T if <T(d), B>= <x, T, (B)> Y d, B EVITED VER Adjoint of T is denoted by T.\* (3) CALLE OF the is linear functional too each Bev.  $T: \mathbb{R}^2 \to \mathbb{R}$  $T(\chi_1,\chi_2) = (\chi_1 + \chi_2, \chi_1 - \chi_2)$ lap, ev ot Find adj(T). foot 50, by Aden  $< T(\alpha), \beta > = < \alpha, T_i(\beta) >$  $T_1(\beta) = \langle C_1\beta_1 + C_2\beta_2, d_1\beta_1 + d_2\beta_2 \rangle$  touch does verify  $\langle \alpha, T_1(\beta) \rangle = \alpha_1(\alpha_1\beta_1 + \alpha_2\beta_2) + \alpha_2(\alpha_1\beta_1 + \alpha_2\beta_2)$  $\langle \pi(\alpha), \beta \rangle = (\alpha_1 + \alpha_2) \beta_1 + (\alpha_1 - \alpha_2) \beta_2$  $T_1(\beta) = (\beta_1 + \beta_2, \beta_1 - \beta_2)$ vaa doos red gate < q. (a) T = (b) } te very suprime at 100 イント リムコカナをはよる =(1)日 Valle 19-49 III Valle

- 1. Does T always have T\*
- 2. If T has T\*, then is it unique?

T:V->V, linear operation, then Talways have a unique adjoint operation.

May not be tome for infinite dimensional. allow or openedox Tives is said to be

Take BEV define give of COUTIND = < 9,000TD 17 13

fp is linear functional foor each BEV.

13BIEV St

Ti:V->V such that

$$T_i(\beta) = \beta_i$$
  $\forall \beta \in V$ .  $\forall \beta \in$ 

1. Ti is linear

step for each BEV  $f_{\beta}(\alpha) = \langle T(\alpha), \beta \rangle$ 

for fB ∃ unique BIEV st

$$f_{\beta}(x) = \langle x, \beta_1 \rangle$$
 foot all  $x \in V$ .

 $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2)$ 

<TI(1), B> = <4, TI(1)>

(बेन्व, बेनव) = विमा

Find adj(T).

 $\beta', \beta'' \in V$ # TI (XB' + X2B") = X1B' + X2B'' for (x) = <T(x), B'> YXEY f B" (d) = < T(d), B"> YXEV (V, <,>)- FDIPS the per enterings you by the lower of T:V->V linear operation then T has adjoint operator T\* Let BEV may a may not have adjoint.  $f:V \rightarrow F$  define  $f(\beta) = \langle T(\beta), \beta \rangle$ FB'EV St f(K) = < x, B'> YXEV commondation - (n) Tive V Linear operator.  $T_i: V \longrightarrow V$  defined by,  $T_i(\beta) = \beta'$  for all  $\beta \in V$ . we want to show that Ti is adjoint of Time (1001) LASIMAN E =X fB,(x) = < T(x), B,> Yaev  $f_{\beta_2}(\alpha) = \langle T(\alpha), \beta_2 \rangle \quad \forall \alpha \in V$ J unique β'st fβ(d) = <d,β'>  $= \alpha, \beta_2 > \beta_2$  st  $= \alpha, \beta_2 > \beta_$ 二世的 (141) 中国 Ld, T,(A,β,+92β2) 7 a CO T(A), A, A

= 
$$Z_1 < x, \beta_1 > + Z_2 < x, \beta_2 >$$
  
=  $Z_1 < T(x), \beta_1 > + Z_2 < T(x), \beta_2 > = < x, \alpha, \beta_1 + \alpha_2 \beta_2 >$ 

$$(V, \langle >_1)$$
 - FDIPS  
 $(V, \langle >_2)$  - FDIPS  
 $T:V \rightarrow V$ 

T\* is adjoint of T with to S, >

T\* is also adjoint for T cort <, > or not?

一個小月十四二八月十十月十月

No. Adjoint of an operator depends on IP.

May or may not have adjoint.

(di.....dn) - orthonormal basis.

T:V->V linear operator.

$$[T]_{B} = A$$

$$A_{ij} = \langle T(\alpha_j), \alpha_i \rangle$$

$$A_{ij} = \langle T(\alpha_j),$$

$$\alpha = \sum_{i=1}^{n} \langle x, \alpha_i \rangle \alpha_i$$

$$T(\alpha_j) = \sum_{i=1}^{n} A_{ij} \alpha_i$$

198 P.

Tive delined by

1:18) = 10 th port

T has adjoint T\*

T\*: V-> V linear operation.

$$[T]_B = A$$
  $[T*]_B = B$ 

$$Aij = \langle T(\alpha j), \alpha i \rangle$$

$$Bij = \langle T(\alpha j), \alpha i \rangle = \langle \alpha j, T(\alpha i) \rangle = \langle T(\alpha i), \alpha j \rangle$$

$$= \langle T(\alpha j), \alpha i \rangle = \langle \alpha j, T(\alpha i) \rangle = \langle T(\alpha i), \alpha j \rangle$$

$$B=A^*$$
 from Law 4: retrorque protinus belles el : centre

Not true if basis is not orthonormal.

Def? T:V->V

T is called self adjoint if T=T = to unitary to significant to sig

eg; T: R2→R2

 $T(x_1,x_2,x_1-x_2)$ 

Th: (V, <>) FDIPS attended pratiques V =- V is int

T:V->V linear

T is self adjoint iff A\*= A.

T is self adjoint iff A is harmitian.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \xrightarrow{\text{symmetric}} \text{symmetric}$$

$$0,0 \quad (1,0)$$

$$0 \quad \text{orthonormal}$$

$$0 \quad \text{orthonormal}$$

$$A = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} \longrightarrow \text{Not symmetric}$$

कारण में के लें अन्यक्ति है। ज

ATTO COLD IN

The T is unitarry iff A is unitarry.

Def? T:V->V (V,<>) FDIP

T,U:V->V, Linear operator

Def? (V,<,>) FDIP T:V→V LT

Then T is called outitary operator if and only if

T\*T= TT\*= I tomorphise and all stand if and only if

Th: T is unitarry iff A is unitary.

Example of unitary operator: might the bellow at

# Eigenvalues of unitary operator are unit modulus.

Th:  $T: V \rightarrow V$  unitary operator  $\langle T(\alpha), T(y) \rangle = \langle \alpha, y \rangle$   $\langle \alpha, T^*T(y) \rangle = \langle \alpha, y \rangle$ 

unitary operator preserves the inner product.

Def? (V, <, >) FDIP

 $T:V \longrightarrow V$  Lo. Then T is called normal operator if T\*T = TT\*/T.

Th: T is normal iff A is normal.

Normal operator is always diagonalisable. for this will - for Example of one normal operator which self adjoint nor unitary. (a), MBA SH 1. Jordan cannonical formiss per in the first of season 2. Definiteness of Hermitian matrix matrix ردر ۽ دَينَ ) آهِ رها ۽ ندري ۽ الله ۽ مين ۽ المري: ١ المرن = ١  $A \in M^{U}(C)$ a e c<sup>n</sup> 152+137 A (c) + c) = an + 417 + an + an + an 2\*Ax GARGE AT - Yeard 9(21, x2) = 21+ 2, 22+ 22 = 2TAx (1) and all + All = 1507 C> write as a matrix! و المار - المرار = الدما  $(\alpha_1 \ \alpha_2)$  $\begin{pmatrix} 1 \ 0 \\ 1 \ 1 \end{pmatrix}$  $\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \alpha_1^2 + \alpha_1 \alpha_2 + \alpha_2^2$ 167 + 12 = 114 16:45 = id  $(\alpha_1 \ \alpha_2) \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \int$ a= Etil & q(21,..., 2n) = a1121+ a222 + ....+ annan2 + aux12 +.... AIT = AIL CIZI+...+ Cnzn- Diagonal form AEMO(C)  $P^*AP=D=diag(A,... > n)$ withmell - A = "A Definiteness; 2≥0, 2>0, 2<0,2 ≤0. - (9) AT - 5 A Distance Land - A = TA CONTRACT MATERIAL CHARLEST

alderen Birth Winds at refer to A-Hermitian X\*Ax -> Real # A EMn(C) x\*Ax is neal for any recommend then A is Hermitian. (ei+iej)\*A(ei+iej) = aii+aji-iaji+iaij=real (ci+ej)\* A (ci+ej) = aii+ajj+ajj+aji = real ei\*Aei = aii - Yeal (1) - aij+aji = real ( > snite as a matrix ) ← iaij - iaji = real aij = 21 + 141 \$ + 5 k k + 1/2 ( ) ( ) ( ) ( ) ( ) ( ) aji = 22+142 (rd ac) (rd) (rd)  $\Rightarrow y_1 + y_2 = 0$ 

 $2_1 - 2_2 = 0 \Rightarrow 2_1 = 2_2$  $a_{ij} = \overline{a_{ji}}$ i=1,2,3,...

j = 1,2,3,...i ≠ j

 $A \in M_0(\mathbb{C})$ fex.... might enelth  $A^* = A \rightarrow Hermitian$ AEMh(R) and teners good you to get in  $A^T = A \longrightarrow Real$  symmetric A E Mn(c) AT= A -> complex symmetric

Whatever result tome for Hermitian one true for complex symmetric matrix.

def? A-Hermitian

A is said to be positive definite or PS If x\*Ax>0 for all non-zero x∈C?

Thm:

A Hermitian A is PD iff all eigenvalues of A core positive.

A is on iff Lending painting animal. Aft on a A

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·syltizon

The Mattern

SVITTEDE

Fa unitary matrix u such that U\*AU=D.

$$2*Ax = 2*UDU*z$$

$$= (xU)D(xU*)*$$

$$= yDy* = \sum_{i=1}^{n} \lambda_i |y_i|^2 > 0$$

. A is positive definite. A MO IFF all polinague minus, of

λi

 $\alpha_i^* A \alpha_i = \lambda_i \alpha_i^* \alpha_i (\neq 0)$ (20)

(20)

(20)

(20) (>0)

⇒ \(\lambda\_i > 0\) \(\forall i = 1,2,...,n\)

The A-Hermitian, A is PD iff I a non-singular Be Mn(c) such that A=B\*B.

A is PD

A such that A=UDU\*

$$A = UDU^* = B^*B$$
  
=  $UD^{1/2}D^{1/2}U^* = B^*B$ 

$$D = \begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \end{pmatrix} \qquad \nabla D = \begin{pmatrix} \sqrt{d_{11}} & 0 & 0 \\ 0 & \sqrt{d_{22}} & 0 \end{pmatrix}$$

$$= 9*9>0$$

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Th: A-Hermitian

A PD iff all polincipale minors of A is positive.

check definition of perinciple minous!

Th: A-Hermitian

A is PD iff Leading principal minors one positive.

ii

iji

iv

$$\int_{1}^{1} \left( \begin{array}{ccc} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \qquad \left| \begin{array}{cccc} 1 & 2 \\ 2 & 1 \end{array} \right| = -3 < 0$$
Not PD

Def? A-Hermitian

A is said to be positive semidefinite x\*Ax 20

Ynon-zero xech.

PD ⇒ PSP

A-Hermitian

A is PSD iff:

i). all the eigen values of A are non-negative.

ii), A=B\*B foor some Boxo.

111). All possible minor of A are non-negative

iv). All leading principal minors of A are non-negative

Negative definite:

if x\*Ax<0 ¥ non-zero x €C?

A is ND iff

i). All the eigen values of A are negative

ii). is not true for ND.

iii). is not true for NP

iv). is not true for ND