classmate
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Solution of linear BVP by the Spline interpolation technique.

Q.) BVP, $y'' + A(x)y' + B(x)y = C(x)$
$y(a) = y_a$, $y(b) = y_b$, $a < x < b$.

Let $A(x) = 0$,
So, $y'' + B(x)y = C(x)$
i.e. at $x = x_k$

$$M_k + B_k y_k = C_k \quad \text{— (i)}$$
$$M_k = y_k'' \quad k = 0, 1, 2, \dots, n.$$

along with the rel^n for 2^nd derivative in spline interpolation, i.e.

$$M_{k-1} + 4M_k + M_{k+1} = \frac{6}{h^2}\left[y_{k-1} - 2y_k + y_{k+1}\right] \quad \text{— (ii)}$$
$$k = 1, 2, \dots, n-1.$$

$(n+1) + (n-1) = 2n \Rightarrow$ eq^ns. (i) + (ii)

$y_1, y_2, \dots, y_{n-1}$ & $M_0, M_1, \dots, M_n \longrightarrow n-1+n+1 = 2n$ unknowns.

Q.) $y'' - y = 0$, $y(0) = 0$, $y(1) = 1$. $h = 1/2$.

M_0, M_1, M_2.

$M_0 - y_0 = 0. \Rightarrow M_0 = y_0 = 0$
$M_1 - y_1 = 0. \Rightarrow M_1 = y_1$
$M_2 - y_2 = 0. \Rightarrow M_2 = y_2 = 1.$

$y(0.5) = 0.4423$

$$M_0 + 4M_1 + M_2 = \frac{6}{1/4}\left(\cancel{y_0}^{\,0} - 2y_1 + \cancel{y_2}^{\,1}\right)$$

$4y_1 + 1. = -48y_1 + 24.$
$52y_1 = 23$

$y_1 = 0.4423077$

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$A(x) \neq 0.$

$y'' + A(x)y' + B(x)y = C(x)$ $\quad a \leq x \leq b. \quad A(x) \neq 0.$

$y(a) = y_a \quad , \quad y(b) = y_b.$

$y_k'' + A_k y_k' + B_k y_k = C_k$

$A_k y_k' = C_k - M_k - B_k y_k.$

$P_k'(x) = y_k'(x)$ —(i) $\quad P_{k+1}'(x_k) = y_k'$ —(ii) $\quad k = 0, 2, \ldots, n-1$

2 set of alg. eq^{n}s for M_k and y_k.

$A_k P_k'(x_k) = A_k y_k' = C_k - M_k - B_k y_k.$

$A_k P_{k+1}'(x_k) = C_k - M_k - B_k y_k$

P_k $\quad x_{k-1} \to x_k$

<u>Homework</u>:-

$y'' + 2y' + y = 30x$

$y(0) = 0 \quad y(1) = 0 \quad h = 1/2.$

$A(x) \neq 0$

$$y'' + A(x)y' + B(x)y = C(x) \qquad a<x<b$$

$$y_a = y(a) \quad y_b = y(b)$$

at $x = x_k$.

$$y_k'' + A(x_k)y_k' + B(x_k)y_k = C(x_k)$$

$$A_k y_k' = C_k - M_k - B_k y_k$$

$$\left.\begin{array}{l} P_k'(x_k) = y_{k+1}' \\ P_{k+1}'(x_k) = y_{k+1}' \end{array}\right\} \quad k = 1, 2 \ldots$$

Q. $y'' + 2y' + y = 30x$

$y(0) = 0$, $y(1) = 0$, $h = 1/2$

Lab Task, $h = 0.25, 0.1, 0.05$

Sol:

$$M_0 + 4M_1 + M_2 = \frac{6}{h^2}(y_2 + y_0 - 2y_1) = -48y_1$$

$$\Rightarrow \boxed{M_1 = -12y_1}$$

$$M_1 + 2y_1' + y_1 = 15$$

$$y_1' = P_0'(x_1) = \frac{M_1}{6} + \frac{y_1}{1/2}$$

$$y_1' = P_1'(x_1) = -\frac{M_1}{6} - \frac{y_1}{1/2}$$

$$\Rightarrow \quad M + y_1 = 15 \quad \Rightarrow \quad y_1 = -\frac{15}{11}$$

$$M_1 = \frac{180}{11}$$

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$\Rightarrow$ ~~$f(x) = \frac{M_1}{3}(x_1 - x)^3 + (2x-3)(y_1 - \frac{M_1}{m}) + ...$~~

Not flat end condition

$$\frac{M_{k-1}}{12} + \frac{M_k}{6} + 2(y_k - y_{k-1}) = -\frac{M_k}{6} - \frac{M_{k+1}}{12} + 2(y_{k+1} - y_k)$$
$$= y'(x_k)$$

k=1

$$\frac{M_0}{12} + \frac{M_1}{6} + 2(y_1 - 0) = -\frac{M_1}{6} - \frac{M_2}{12} + 2(-y_1) = y'(x_1)$$

and $M_1 + 2y'(1) + y_1 = 15$

k=2

$$\frac{M_1}{12} + \frac{M_2}{6} + 2(y_2 - y_1) = y'(x_2)$$

k=0

$$-\frac{M_0}{6} - \frac{M_1}{12} + 2(y_1 - y_0) = y'(x_0)$$

$M_0 + 2y'(0) = 0 \Rightarrow y'(x_0) = -\frac{M_0}{2}$ $\quad M_1 = 4M_0 + 24y_1$

$\Rightarrow \boxed{\frac{M_0}{3} - \frac{M_1}{12} = -2y_1}$

$M_2 + 2y'(x_2) + 0 = 30 \Rightarrow y'(x_2) = 15 - \frac{M_2}{2}$

$\Rightarrow \boxed{\frac{M_1}{12} + \frac{5M_2}{6} + 2y_2 = 0} \Rightarrow \boxed{M_1 = -10M_2}$

$$M_1 + y_1 + \frac{M_0}{6} + \frac{M_1}{3} + 4y_1 = 15$$

$$4M_0 + 25y_1 + \frac{M_0}{6} + \frac{4M_0}{3} + 12y_1 = 15$$

$$\frac{11M_0}{2} + 37y_1 = 15$$

$\frac{19M_2}{12} - 2y_1 = 15 - y_1 \quad = 10M_2$

$15 - y_1 = \frac{101}{12}M_2 = -\frac{101}{120}M_1$

$15 - y_1 = -\frac{101}{12}\left(\frac{M_0}{3} + 2y_1\right)$

$\frac{101}{36}M_0 + \frac{211}{12}y_1 = -15$

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