Q.) $y^{(4)} + 2y = \frac{x^2}{9} + \frac{2x}{3} + 4$ $\quad y(0) = y'(0) = y(3) = y'(3) = 0$.

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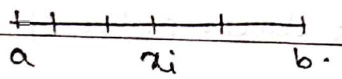
Non-linear BVP :

$f(x, y, y', y'') = 0 \qquad a < x < b$

$y(a) = y_a, \quad y(b) = y_b.$



Let the discretized eq^n at x_i.

$G_i(x_i, y_i, y_i', y_i'') = 0.$

or, $G_i(x_i, y_1, y_2, \ldots, y_{n-1}) = 0 \qquad i = 1, 2, 3, \ldots, n-1.$

which forms $(n-1)$ algebraic eq^{n}s (non-linear) involving $(n-1)$ variables.

Thus a compact system results.

Consider a non-linear ~~system~~ eq^n.

$$\boxed{\phi(x) = 0}$$

Newton-Raphson iterative method :-

Let $x^{(k)}$ be the approximation of the root at any k^{th} iteration level.

Exact root, $\alpha = x^{(k)} + \text{Error}.$

$\alpha = x^{(k)} + \Delta x$

Now, $y(\alpha) = 0 \Rightarrow y(x^{(k)} + \Delta x) = 0$

Expand by Taylor's series

$$\phi(x^{(k)}) + \Delta x\, \phi'(x^{(k)}) + \frac{(\Delta x)^2}{2!} \phi''(x^{(k)}) + \ldots = 0$$

if $\Delta x \ll 1$, then $\phi(x^{(k)}) + \Delta x\, \phi'(x^{(k)}) = 0$

$$\therefore \Delta x = -\frac{\phi(x^{(k)})}{\phi'(x^{(k)})}$$

Next approximation.

$$x^{(k+1)} = x^{(k)} - \frac{\phi(x^{(k)})}{\phi'(x^{(k)})}, \quad k \geq 0$$

Process repeated till,

$$|x^{(k+1)} - x^{(k)}| < \varepsilon \quad \forall\ k \geq K$$

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$G_i(y_1, y_2, \ldots, y_{n-1}, x_i) = 0 \qquad i = 1, 2, \ldots, n-1.$

will be solved iteratively, ——— ①

Let $y_i^{(k+1)} = y_i^{(k)} + \Delta y_i \qquad k \geq 0 \qquad i = 1, 2, \ldots, n-1$

Substitute this to ①, we get,

$$G_i\left(y_1^{(k)} + \Delta y_1,\ y_2^{(k)} + \Delta y_2, \ldots, \Delta y_n^{(k)} + \Delta y_{n-1}, x_i\right) = 0$$

Expand by Taylor series, retain the terms linear order of Δy_i's to get,

$$G_i\left(y_1^{(k)}, y_2^{(k)}, \ldots, y_{n-1}^{(k)}, x_i\right) + \Delta y_1 \left.\frac{\partial G_i}{\partial y_1}\right|^{(k)} + \Delta y_2 \left.\frac{\partial G_i}{\partial y_2}\right|^{(k)} + \cdots + \Delta y_{n-1} \left.\frac{\partial G_i}{\partial y_{n-1}}\right|^{(k)} = 0. \quad \text{———} (*)$$

The system of eqⁿs (*) is a system of $(n-1)$ linear alg. eqⁿs of $(n-1)$ variables $\Delta y_1, \Delta y_2, \ldots, \Delta y_{n-1}$.

Solving the $(n-1)$ linear algebraic eqⁿ (*) for $(n-1)$ variables $\Delta y_1, \Delta y_2, \ldots, \Delta y_{n-1}$, we can obtain the modified solⁿ.

$$y_i^{(k+1)} = y_i^{(k)} + \Delta y_i \qquad k \geq 0, \quad i = 1, 2, \ldots, n-1.$$

Repeat the process till.

~~Itera~~

Iteration starts with an initial guesses for $y_i^{(0)}$'s i.e. $y_1^{(0)}, y_2^{(0)}, \ldots, y_{n-1}^{(0)}$ and $\Delta y_0 = \Delta y_n = 0$ at the boundary.

(Newton's linearization techniques)

$y_i^{(0)}$ for $i = 1, 2, 3, \ldots, n-1$

$$y^{(0)}(x) = f(x) = \frac{(x-a)}{(b-a)} y_b + \frac{(b-x)}{(b-a)} y_a.$$

$y^{(0)}(x) \longrightarrow$ choose s.t. it satisfies the B.C.'s.

Eg. $u'' - (u')^2 - u^2 + u + 1 = 0$, $u(0) = 1/2$, $u(\pi) = 1/2$