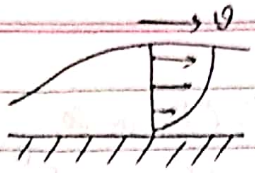
5/2/2020

Flow past a flat plat :-



f' = fluid velocity.

$$f''' + ff'' + 1 - (f')^2 = 0.$$

B.C. $\begin{cases} \eta = 0, & f = 0, \quad f' = 0 \\ \eta >> 1, & f' = 1 \\ \eta = 10, & f' = 1 \end{cases}$

<Newton's linearization technique>

Let, $F = f'$ —— (1)

$$F'' + fF' + 1 - F^2 = 0 \quad \text{(2)} \qquad f(0) = 0, \; F(0) = 0, \; F(10) = 1.$$

$$\frac{F_{i+1} + F_{i-1} - 2F_i}{h^2} + f_i\left(\frac{F_{i+1} - F_{i-1}}{2h}\right) + 1 - F_i^2 = 0. \quad \text{—— (ii)}$$

$$f_i - f_{i-1} - \frac{h}{2}\left[F_i + F_{i-1}\right] = 0 \quad \text{—— (i)}$$

$$\frac{F_{i+1} + F_{i-1} - 2F_i}{h^2} + f_i\left(\frac{F_{i+1} - F_{i-1}}{2h}\right) - F_i^2 = -1 \quad \text{—— (ii)}$$

$i = 1, 2, \ldots, n-1$

At $(k+1)^{th}$ iteration,

$$f_i^{(k+1)} = f_i^{(k)} + \Delta f_i$$
$$F_i^{(k+1)} = F_i^{(k)} + \Delta F_i$$

Substitute in (i) & (ii) and retain terms upto the linear order of $\Delta f_i, \Delta F_i$.

$$-\Delta f_{i-1} + \Delta f_i - \frac{h}{2}\Delta F_{i-1} - \frac{h}{2}\Delta F_i = -f_i^{(k)} - f_{i-1}^{(k)} + \frac{h}{2}\left[F_i^{(k)} + F_{i-1}^{(k)}\right]$$

—— (i)

~~[illegible]~~

$$\frac{f_{i+1}^{(k)} + \Delta f_{i+1} + f_{i-1}^{(k)} + \Delta f_{i-1} - 2f_i^{(k)} - 2\Delta f_i}{h^2} + \left(f_i^{(k)} + \Delta f_i\right)\left(\frac{f_{i+1}^{(k)} + \Delta f_{i+1} - f_{i-1}^{(k)} - \Delta f_{i-1}}{2h}\right) - \underbrace{\left(f_i^{(k)} + \Delta f_i\right)^2}_{\left(f_i^{2(k)} + 2f_i^{(k)}\Delta f_i\right)} = -1. \qquad \text{——(ii)}$$

$$\frac{\Delta f_{i+1} + \Delta f_{i-1} - 2\Delta f_i}{h^2} + f_i^{(k)}\left(\frac{\Delta f_{i+1} - \Delta f_{i-1}}{2h}\right) - 2f_i^{(k)}\Delta f_i + \left(\frac{f_{i+1}^{(k)} - f_{i-1}^{(k)}}{2h}\right)\Delta f_i$$

$$= -\left(\frac{f_{i+1}^{(k)} + f_{i-1}^{(k)} - 2f_i^{(k)}}{h^2}\right) - f_i^{(k)}\left(\frac{f_{i+1}^{(k)} - f_{i-1}^{(k)}}{2h}\right) + \left(f_i^{(k)}\right)^2 - 1. \qquad \text{——(ii')}$$

$k \geq 0$, $i = 1, 2, 3, \dots, n-1$.

$$\Delta f_{i-1}\left(\frac{1}{h^2} - \frac{f_i^{(k)}}{2h}\right) + \Delta f_i\left[-\frac{2}{h^2} - 2f_i^{(k)}\right] + \Delta f_{i+1}\left[\frac{1}{h^2} + \frac{f_i^{(k)}}{2h}\right] + \Delta f_i\left(\frac{f_{i+1}^{(k)} - f_{i-1}^{(k)}}{2h}\right).$$

$X^T = [x_1\, x_2 \cdots x_{n-1}]$, $x_i = \begin{pmatrix} \Delta f_i \\ \Delta f_i \end{pmatrix}$, which ~~[illegible]~~ solves the $\underline{A}\,\underline{x} = \underline{D}$

$$A = \begin{bmatrix} \underline{b_1} & \underline{c_1} & 0 & \cdots & 0 \\ \underline{a_2} & \underline{b_2} & \underline{c_2} & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \underline{a_{n-1}} & \underline{b_{n-1}} \end{bmatrix}$$

$$a_i X_{i-1} + b_i X_i + C_i X_{i+1} = d_i$$

Date
Page

$a_i = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $b_i = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, $C_i = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

Define the coeff matrices

$a_i = \begin{bmatrix} -1 & -h/2 \\ 0 & \frac{1}{h^2} - \frac{f_i^{(k)}}{2h} \end{bmatrix}$ $\quad D = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_{n-1} \end{pmatrix}$

$C_i = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{h^2} + \frac{f_i^{(k)}}{2h} \end{bmatrix}$

$b_i = \begin{bmatrix} 1 & -h/2 \\ \left(\frac{F_{i+1}^{(k)} - F_{i-1}^{(k)}}{2h}\right) & \left[-\frac{2}{h^2} - 2F_i^{(k)}\right] \end{bmatrix}$

$$\Delta f_0 = \Delta F_0 = \Delta f_n = \Delta F_n = 0$$

Compatible scheme, $\longrightarrow$ if the truncation error tends to zero, as step size $\to 0$, then the numerical scheme is said to be compatible with differential eqn.

Vary the step size, and soln should be almost independent of the step size.

Now, how to choose initial condns.

$F^{(0)}(\eta) = \eta/10 \qquad f^{(0)}(\eta) = \frac{\eta^2}{20}$

Choose, $\delta_\eta = 0.05, 0.01, 0.005$

Ex. $f''' + ff'' + f'^2 = 0$

at $\eta = 0$: $f(0) = 0 \quad f''(0) = 0$

$\eta = 10$ = $f' = 0.1$

Q.) $f''' + (2f+4)f' = 0$ $\quad f(0) = 0, \quad f''(0) = -k$

$f'(w) = 0.$

$k = 0.1$

$w = 0.087$

Q.) $y'' = \frac{1}{2}(1+x+y)^3$ $\quad h = 0.25,$

$y(0) = y(1) = 0$

Quasi-linearization technique :-

BVP as,

$$\phi(y'', y', y, x) \equiv y' - F(x, y, y') = 0$$

or, $\phi(y'', y', y, x) = 0, \quad a < x < b$

$y(a) = y_a \quad y(b) = y_b.$

We treat ϕ as a function of the functions y, y', y''.
Let $y^{(k)}(x), y'^{(k)}(x), y''^{(k)}(x)$ $(k \geq 0)$ be an approximate form of $y(x)$, $y'(x)$ & $y''(x)$ for any x, $a < x < b$. Expand ϕ by Taylor series about the known functions $y^{(k)}(x)$, $y'^{(k)}(x)$, $y''^{(k)}(x)$ to get.

$$\phi(y, y', y'', x) = \phi(y^{(k)}, y'^{(k)}, y''^{(k)}, x)$$
$$+ \frac{\partial \phi}{\partial y}\Big|^{(k)} (y(x) - y^{(k)}(x)) + \frac{\partial \phi}{\partial y'}\Big|^{(k)} (y'(x) - y'^{(k)}(x))$$
$$+ \frac{\partial \phi}{\partial y''}\Big|^{(k)} (y''(x) - y''^{(k)}(x)) + \frac{\partial^2 \phi}{\partial y^2}\Big|^{(k)} (y(x) - y^{(k)}(x))^2$$
$$+ \cdots$$

Superscript with k are known functions.

$$0 = \phi(y^{(k)}, y'^{(k)}, y''^{(k)}, x) + \frac{\partial \phi}{\partial y}(\ ) + \frac{\partial \phi}{\partial y'}(\ ) + \frac{\partial \phi}{\partial y''}(\ )$$
$$+ \cdots$$

Scanned by CamScanner

only upto
Taking linear order of the variable, we get.

$$y''\frac{\partial\phi}{\partial y''}\Big|^{(k)} + y'\frac{\partial\phi}{\partial y'}\Big|^{(k)} + y\frac{\partial\phi}{\partial y}\Big|^{(k)}$$

$$= -\phi(y^{(k)}, y'^{(k)}, y''^{(k)}, x) + \frac{\partial\phi}{\partial y}\Big|^{(k)} y(x) + \frac{\partial\phi}{\partial y'}\Big|^{(k)} y'(x) + \frac{\partial\phi}{\partial y''}\Big|^{(k)} y''(x)$$

which is a linear differential eq^n b.c.s. are $y(a) = y_a$, $y(b) = y_b$.

Solution of this reduced linear BVP is designated as $y^{(k+1)}$, $y'^{(k+1)}$, $y''^{(k+1)}$ which is the next approximation for the variable.

Continue for $k \geq 0$ starting with an initial form $y^{(0)}, y'^{(0)}, y''^{(0)}$

Q.) $3yy'' + (y')^2 = 0$ $\quad y(0) = 0$, $y(1) = 1$.

Quasi linear form of the BVP is,

$$\phi(y'', y', y, x) = 3yy'' + (y')^2 = 0.$$

$$\phi(y''^{(k)}, y'^{(k)}, y^{(k)}, x) + (y''^{(k+1)} - y''^{(k)})\frac{\partial\phi}{\partial y''}\Big|^{(k)} + (y'^{(k+1)} - y'^{(k)})\frac{\partial\phi}{\partial y'}\Big|^{(k)} + (y^{(k+1)} - y^{(k)})\frac{\partial\phi}{\partial y}\Big|^{(k)} = 0$$

$y^{(k+1)}(0) = 0$, $y^{(k+1)}(1) = 1$

BVP for $y^{(k+1)}$ is $(k \geq 0)$