27-02-19

Example: Revised Simplex Method.

max: $z = 2x_1 + 3x_2 + 4x_3$

s.to $x_1 + x_2 + x_3 \leq 10$

$x_1 + 2x_2 + x_3 \leq 12$

$x_1, x_2, x_3 \geq 0$

max: $z = 2x_1 + 3x_2 + 4x_3 + 0.s_1 + 0.s_2$

s.to $x_1 + x_2 + x_3 + s_1 = 10$, $s_1 \geq 0$

$x_1 + 2x_2 + x_3 + s_2 = 12$ $s_2 \geq 0$

$x_1, x_2, x_3, s_1, s_2 \geq 0$

→ Extended form of simplex Table.

C_B	BV \ C_j	2	3	4	0	0	
		x_1	x_2	x_3	s_1	s_2	b
0	s_1	1	1	1	1	0	10
0	s_2	1	2	3	0	1	12
		-2	-3	-4	0	0	0

$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $x_B = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$ $C_B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $C_B^T = (0 \ 0)$

$b = \begin{pmatrix} 10 \\ 12 \end{pmatrix}$

$Bx_B = b \Rightarrow x_B = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = B^{-1}b = \begin{pmatrix} 10 \\ 12 \end{pmatrix}.$

$z = C_B^T x$

$= C_B^T x_B = (0 \ 0) \begin{pmatrix} 10 \\ 12 \end{pmatrix} = 0.$

$B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$y = C_B B^{-1} = (0 \ 0) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (0 \ 0)$

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$z_1 - c_1 = YP_1 - c_1 = -2$

$z_2 - c_2 = YP_2 - c_2 = -3$

$z_3 - c_3 = YP_3 - c_3 = -4$

~~$\alpha^j = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$~~ $\alpha^j = B^{-1} P_j$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\theta = \underset{K}{\mathrm{Min}} \left(\frac{10}{1}, \frac{12}{1} \right) = 10.$

$C_B = (4 \quad 0)$. $x_B = \begin{pmatrix} x_3 \\ s_2 \end{pmatrix}$.

$B_{new} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. $B^{-1}_{new} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$.

$x_B = B^{-1} b = \begin{pmatrix} x_3 \\ s_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 12 \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix}$

$Y = C_B B^{-1} = (4 \quad 0) \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} = (4 \quad 0)$

$z_1 - c_1 = YP_1 - c_1 = (4 \quad 0) \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 2 = 2 > 0$

$z_2 - c_2 = YP_2 - c_2 = (4 \quad 0) \begin{pmatrix} 1 \\ 2 \end{pmatrix} - 3 = 1 > 0$

$z_4 - c_4 = YP_4 - c_4 = (4 \quad 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} - 0 = 4 > 0.$

No negative element.

$x_B = \begin{pmatrix} x_3 \\ s_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 2 \end{pmatrix}.$

$Z^* = C_B x_B = (4 \quad 0) \begin{pmatrix} 10 \\ 2 \end{pmatrix} = 40$

$x_1 = 0, \; x_2 = 0, \; x_3 = 10, \; s_1 = 0, \; s_2 = 2.$

Example 2:

	c_j	3	5	0	0	0	
C_B	B	x_1	x_2	s_1	s_2	s_3	b
0	s_1	1	0	1	0	0	4
0	s_2	8	2	0	1	0	12
0	s_3	3	2	0	0	1	18
		-3	-5	0	0	0	

Integer Linear Programming

$\max: z = \sum_{j=1}^{n} c_j x_j$

s.to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i = 1, 2, \ldots m$

IPP $\rightarrow x_j = 0, 1, 2, \ldots n \quad \forall j.$

$x_j \geq 0 \quad \forall j$
LPP

$\rightarrow \sum_{j=1}^{n} a_{ij} x_j + x_{n+i} = b_i \qquad i = 1, 2, \ldots m.$

C_B	C_N / BV \ NBV	c_1	c_2	c_3	$\ldots$	c_j	$\ldots$	c_n	
		x_1	x_2	x_3	$\ldots$	x_j	$\ldots$	x_n	x_B
0	x_{n+1}	a_{11}	a_{12}	a_{13}	$\ldots$	a_{1j}	$\ldots$	a_{1n}	b_1
0	x_{n+2}	a_{21}	a_{22}	a_{23}	$\ldots$	a_{2j}	$\ldots$	a_{2n}	b_2
$\vdots$	$\vdots$	$\vdots$							$\vdots$
0	x_{n+m}	a_{m1}	a_{m2}	a_{m3}	$\ldots$	a_{mj}	$\ldots$	a_{mn}	b_m
		$z_1 - c_1$	$z_2 - c_2$	$z_3 - c_3$		$z_j - c_j$		$z_n - c_n$	z

$b_i \geq 0$

Indicator row.

Fig 1: Initial Simplex Table (Condensed form).

If optimal sol^n are integers, Terminate it.

Let x_i's are not integers, for some i.

Let ith BV(x_i) has the largest fractional part,

$x_i + \alpha_{i1} w_1 + \alpha_{i2} w_2 + \ldots + \alpha_{ij} w_j + \ldots + \alpha_{in} w_n = \beta_i$

$\Rightarrow \boxed{x_i + \sum_{j=1}^{n} \alpha_{ij} w_j = \beta_i}$

01-03-19

the additional constraints

$$-\sum_{j=1}^{n} f_{ij}\,\omega_j \leq -f_i \qquad \forall\, i.$$

Continued after slide

$$x_1 + \frac{1}{3}x_4 - \frac{1}{3}x_3 = \frac{5}{3}$$

$$\Rightarrow x_1 + \left(0+\frac{1}{3}\right)x_4 + \left(-1+\frac{2}{3}\right)x_3 = 1 + \frac{2}{3}$$

$$\Rightarrow x_1 + 0\cdot x_4 - x_3 - 1 = \frac{2}{3} - \frac{1}{3}x_4 - \frac{2}{3}x_3$$

$$\frac{2}{3} - \left(\frac{2}{3}x_3 + \frac{1}{3}x_4\right) \leq 0.$$

$$\Rightarrow -\frac{2}{3}x_3 - \frac{1}{3}x_4 \leq -\frac{2}{3}$$

$$\Rightarrow -\frac{2}{3}x_3 - \frac{1}{3}x_4 + s_1 = -\frac{2}{3}.$$

Now include s_1 in the previous table.

Now apply dual simplex method.

		x_4	x_3	x_B
4	x_2	$-1/3$	$4/3$	$4/3$
5	x_1	$1/3$	$-1/3$	$5/3$
0	s_1	$-1/3$	$-2/3$	$-2/3$
		$1/3$	$11/3$	$41/3$.

$$\left|\frac{11/3}{-2/3}\right| = \frac{11}{2} \quad \text{and} \quad \left|\frac{1/3}{-1/3}\right| = 1.$$

	s_1	x_3	x_B
x_2	-1	2	2
x_1	1	-1	1
x_4	3	2	2
	1	3	$\boxed{13}$

08/03/19

Two person zero sum game

1. Stable game
2. Un stable game

Play-off matrix

$A = (a_{ij})_{m \times n}$

(a) 2×2 two person zero sum game

(b) $2 \times n$ "

(c) $m \times 2$ "

(d) $m \times n$ "

⇒ **Stable game :**

Example:

$$A: \begin{array}{c|c} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{array}_{2 \times 2}$$

(i)

$$\begin{array}{c|cc|c} & B & & \text{min} \\ A & 2 & -1 & r_1 = -1 \\ & 4 & \boxed{0} & r_2 = 0 \end{array} \quad \text{max } r = 0$$

max : $\underbrace{c_1 = 4 \quad c_2 = 0}$

min , $c = 0$

$r = c = 0 = v$ = value of the game

$a_{ij} = a_{22} = 0 = v$

optimal strategy (A_2, B_2).

(ii)

$$\begin{array}{c|cc|c} & B & & \text{min} \\ A & 2 & \boxed{1} & 1 \\ & 1 & \boxed{1} & 1 \end{array} \quad \text{max} = 1$$

max = 2 1

min = 1

Stable game

$v = 1$, (A_1, B_2) , (A_2, B_2). 2 solutions.

⇒ Unstable game.

<u>Example:</u>

$$\begin{array}{c|ccc|c}
 & & B & & \\
 & 1 & 2 & 3 & \text{min} \\
\hline
1 & 1 & -1 & 1 & -1 \\
A\ 2 & -1 & 1 & -1 & -1 \\
3 & 1 & -1 & 1 & -1 \\
\hline
\text{max} & 1 & 1 & 1 &
\end{array}$$

max −1

min 1

$r = -1$

$c = 1$

$$\boxed{r < c}$$

$$\boxed{r \leq a_{ij} \leq c}$$

$-1 < v < 1$

value of the game : v.

we find expected value of the game.

let

$$\begin{array}{c|ccc|c}
 & & B & & \\
 & 1 & 2 & 3 & \\
\hline
1 & 2 & -3 & 4 & -3 \\
A\ 2 & -3 & 4 & -5 & -5 \\
3 & 4 & -5 & 6 & -5 \\
\hline
 & 4 & 4 & 6 &
\end{array}$$

max −3

min 4

$r = -3$

$c = 4$

no saddle

So unstable

$-3 < v < 4$.

$v = 0$ —

$v < 0$ — Possible

$v > 0$ —

to make all the ~~tot~~ terms be added

08-03-19

$k=5$ adding k

	1	2	3	
1	7	2	9	2
2	2	9	0	0
3	9	0	11	0
	9	9	11	

$\} \; 2 = v.$

$c = 9.$

$2 < v' < 9$

eg

$$\begin{array}{cc} & B \\ A & \begin{pmatrix} 2+3 & -3+3 \\ -3+3 & 4+3 \end{pmatrix} \end{array}$$

If some -ve is there soon add a const.

→ then v will increase by 3.

$v = v' \pm 30$

$$\begin{pmatrix} 5 & 0 \\ 0 & 7 \end{pmatrix}$$

Let $P(A_1) = x_1$, $P(x_2) = x_2$ $x_1 \geq 0$, $x_2 \geq 0$

$x_1 + x_2 = 1$

~~$x_1 + x_2$~~ $= 1$

Let $P(B_1) = y_1$ $P(B_2) = y_2$ $y_1, y_2 \geq 0$

$y_1 + y_2 = 1$

If player B is selecting his/her 1st stg. then expected gain for player A is $x_1 a_{11} + x_2 a_{21}$

2nd stg $x_1 a_{12} + x_2 a_{22}$

$\max\limits_{x}$ $\min : (x_1 a_{11} + x_2 a_{21}, \; x_1 a_{12} + x_2 a_{22})$

Let $\min : (x_1 a_{11} + x_2 a_{21}, \; x_1 a_{12} + x_2 a_{22}) = v.$

$\Rightarrow x_1 a_{11} + x_2 a_{21} \geq v.$

also $x_1 a_{12} + x_2 a_{22} \geq v$, $v > 0.$

max : v.

s.to $\quad x_1 a_{11} + x_2 a_{21} \geq v \qquad v > 0$

$x_1 a_{12} + x_2 a_{22} \geq v$

$x_1, x_2 \geq 0 \qquad x_1 + x_2 = 1.$

min : $\frac{1}{v} \cdot x_1$

s.t. $\quad x_1 a_{11} + x_2 a_{21} \geq v$

$x_1 a_{12} + x_2 a_{22} \geq v$

$x_1 + x_2 = 1$

$x_1, x_2 \geq 0$

min : $\frac{1}{v}(x_1 + x_2)$

s.to $\quad \frac{1}{v}(x_1 a_{11} + x_2 a_{21}) \geq 1$

$\frac{1}{v}(x_1 a_{12} + x_2 a_{22}) \geq 1$

$x_1, x_2 \geq 0$

Let $\frac{x_1}{v} = X_1 \quad , \frac{x_2}{v} = X_2 . \quad , X_1, X_2 \geq 0$

~~Primal LPP : (P)~~

$\boxed{\begin{aligned} &\underline{\text{min}} : \quad X_1 + X_2 \\ &\text{s.t.} \quad a_{11} X_1 + a_{21} X_2 \geq 1 \\ &\qquad a_{21} X_1 + a_{22} X_2 \geq 1 \\ &\qquad X_1, X_2 \geq 0. \end{aligned}}$

__Dual LPP__ : (D)

max : $z = Y_1 + Y_2$

s.t $\quad a_{11} Y_1 + a_{12} Y_2 \leq 1$

$a_{21} Y_1 + a_{22} Y_2 \leq 1 \quad , Y_1, Y_2 \geq 0$

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Lab
Q.

$$\begin{pmatrix} 5 & 0 \\ 0 & 7 \end{pmatrix}$$

max : $z = Y_1 + Y_2$

s.to $5Y_1 + 0Y_2 \leq 1$

$0Y_1 + 7Y_2 \leq 1$

$Y_1, Y_2 \geq 0$

$Y_1 = 1/5, \quad Y_2 = 1/7.$

$z = \frac{12}{35}$ $\Rightarrow \frac{1}{v} = \frac{12}{35}$

$v = \frac{35}{12}$

$\frac{Y_1}{v} + \frac{Y_2}{v} = \frac{1}{v}.$

$y_1 = \frac{7}{12}, \quad y_2 = \frac{5}{12}$ ($y_1 = Y_1 \times v = \frac{1}{5} \times \frac{35}{12} = \frac{7}{12}.$

$P(B_1) = \frac{7}{12} \quad P(B_2) = \frac{5}{12}, \quad v = \frac{35}{12}$

$v_{actual} = \hat{v} - 3$

$= \frac{35}{12} - 3 = -\frac{1}{12}.$

$\therefore v = -\frac{1}{12}.$

$P(A_1) = \frac{7}{12}, \quad P(A_2) = \frac{5}{12}$

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Q.

$$\begin{array}{c|cccc|c} & & & B & & \\ A & 1 & 2 & 3 & 3 & 1 \\ & -1 & 2 & 1 & 2 & 1 \\ & 5 & 6 & 8 & 8 & 5 \\ \hline & 5 & 6 & 8 & 8 & \end{array}$$

$A_3 B_1 = 5 = v$.

$x_3 = 1$, $y_1 = 1$.

$x_1 = x_2 = y_2 = y_3 = y_4 = 0$

LPP:

max: $Z = y_1 + y_2 + y_3 + y_4$

s.to

$y_1 + 2y_2 + 3y_3 + 3y_4 \leq 1$

$y_1 + 2y_2 + y_3 + 2y_4 \leq 1$

$5y_1 + 6y_2 + 8y_3 + 8y_4 \leq 1$

$y_1, y_2, y_3, y_4 \geq 0$

$$y^* = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{cc|cc} & & B & \\ & & 1 & 2 \\ \hline A & 1 & 1 & -1 \\ & 2 & -1 & 1 \end{array}$$

Expected value of game $v = 0$

$P(A_1) = 1/2 = x_1$

$P(A_2) = 1/2 = x_2$

$x_1 + x_2 = 1$

$x_1, x_2 \geq 0$

$P(B_1) = 1/2 = y_1$

$P(B_2) = 1/2 = y_2$

$y_1 + y_2 = 1$

$y_1, y_2 \geq 0$

$$\begin{array}{cc|ccc|c} v & & & B & & \\ & & 1 & 2 & 3 & \\ \hline & 1 & 1 & -1 & 1 & -1 \\ A & 2 & -1 & 1 & -1 & -1 \\ & 3 & 1 & -1 & 1 & -1 \\ \hline & & 1 & 1 & 1 & \end{array}$$

$\left.\begin{matrix}-1\\-1\\-1\end{matrix}\right\} -1$

$\underbrace{1 \quad 1 \quad 1}_{1}$

Unstable game.

If you add k to all element $v \to v+k$

here v is 0.

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$$\begin{array}{c|c|ccc} & & & B & \\ & & 1 & 2 & 3 \\ \hline & 1 & 2 & 0 & 2 \\ A & 2 & 0 & 2 & 0 \\ & 3 & 2 & 0 & 2 \end{array}$$

$P(A_1) = x_1$
$P(A_2) = x_2$
$P(A_3) = x_3$
$P(B_1) = y_1$
$P(B_2) = y_2$
$P(B_3) = y_3$

LPP for player A.

min: $z' = X_1 + X_2 + X_3$

s.to.

$2X_1 + 2X_3 \geq 1$

$2X_2 \geq 1$

$\boxed{2X_1 + 2X_3 \geq 1}$ ×

$X_1, X_2, X_3 \geq 0$

$$X = \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}.$$

$$z' = X_1 + X_2 + X_3 = \frac{x_1 + x_2 + x_3}{v} = \frac{1}{v}.$$

$= 1.$

$\Rightarrow 1 = 1/v \Rightarrow v = 1$

we added 1 so v increase by 1.

$\underline{v^* = 0.}$

$$x^* = \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}.$$

$$y^* = \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}$$

$$x^* = \lambda \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}.$$

$y_1 = 0 \quad y_2 = 0 \quad\quad y_4 = 0$

$$\begin{bmatrix} 8 & 6 & 2 & 8 \\ 8 & 9 & \boxed{4} & 5 \\ 7 & 5 & 3 & 5 \end{bmatrix} \quad \begin{array}{l} x_1 = 0 \\ \\ x_3 = 0. \end{array}$$

$P(A_2) = 1$

$P(B_3) = 1.$

$\therefore x_2 = 1, \quad y_3 = 1$

$v = 4.$

Q.

	B_1	B_2	B_3	
A_1	2	-4	3	-4
A_2	-5	3	-5	-5
A_3	18	-4	3	-4
	2	3	3	

-4

$y_3 = 0$

$x_3 = 0$

2

	B_1	B_2
A_1	2	-4
A_2	-5	3

$\xrightarrow{+5}$

	B_1	B_2
A_1	7	1
A_2	0	8

$\therefore v = -1.$

$7x_1 + 0x_2 = v$

$x_1 + 8x_2 = v.$ $\} \rightarrow (ii)$

$x_1 + x_2 = 1. \rightarrow (i)$

$7x_1 = x_1 + 8(1-x_1)$

$\Rightarrow x_1 = 4/7.$

$x_2 = 3/7.$

$v = 4.$

As we had added 5

$v^* = 4 - 5 = -1.$

for y

$7y_1 + y_2 = v$

$8y_2 = v.$

$y_1 + y_2 = 1.$

$7y_1 + (1-y_1) = 8(1-y_1)$

$\Rightarrow$ ~~$y_1 + y_2 = y_2$~~

$14y_1 = 8$

$y_1 = 4/7.$