Compton Scattering

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(Dated: March 21, 2023)

Compton scattering is a fundamental phenomenon in which photons scatter off free electrons, resulting in a shift in wavelength and a change in the direction of propagation of the photons. In this lab report, we investigated the Compton scattering process by measuring the scattered photons' scattering angle and energy. Using a gamma-ray source and a scintillation detector, we collected data on the scattered photons at different angles and computed the corresponding energy shifts. Our results demonstrate the validity of the Compton scattering formula and provide insights into the behavior of photons and electrons in the scattering process. The findings of this phenomenon have implications for a wide range of fields, including nuclear physics, astrophysics, and medical imaging.

I. OBJECTIVES

- Energy calibration of the scintillation detector.
- Determination of change in wavelength of the scattered gamma radiation as a function of the scattering angle.
- Determination of the differential cross-section using the Klein-Nishina formula and calibration factor calculation.

II. THEORY

A. Compton scattering

Compton scattering is a process in which a photon interacts with a free electron, resulting in a change in the photon's energy and direction of propagation. The phenomenon was first described by Arthur Compton in 1923 and has since been a significant topic of research in physics.

The Compton scattering process can be explained using the principles of both classical and quantum mechanics. According to classical mechanics, a photon behaves like a particle with energy

$$E = h\nu \tag{1}$$

where h is Planck's constant and ν is the frequency of the photon.

When a photon interacts with an electron, it can transfer some of its energy to the electron, resulting in the electron being excited to a higher energy level. The photon's energy is then reduced, and it changes its direction of propagation.

However, quantum mechanics provides a more accurate explanation of the Compton scattering process. In this theory, photons are treated as waves with a wavelength

$$\lambda = \frac{c}{\nu} \tag{2}$$

where c is the speed of light.

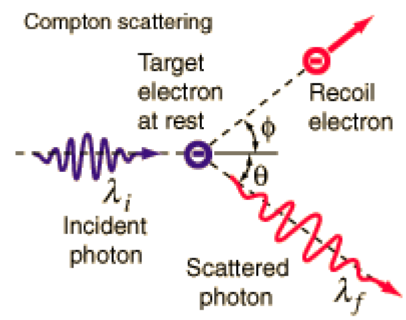


FIG. 1. Compton scattering

When a photon interacts with an electron, the electron's wave function is disturbed, leading to a change in the momentum of both the photon and the electron. The scattered photon has a longer wavelength and lower energy than the incident photon, while the electron is left with some residual momentum.

The Compton scattering formula describes the relationship between the energy and angle of the scattered photon as:

$$\Delta\lambda = \frac{h}{mc}(1 - cos\theta) \tag{3}$$

where $\Delta\lambda$ is the change in wavelength, h is Planck's constant, m is the rest mass of the electron, c is the speed of light, and θ is the scattering angle. This formula is derived from the conservation of energy and momentum in the scattering process.

In this lab report, we will investigate the Compton scattering process by measuring the scattering angle and the energy of the scattered photons. By comparing our results to the Compton scattering formula, we can validate the theory and gain insights into the behavior of photons and electrons in this fundamental process.

Writing eqn. (3) in terms of energy, we get,

$$E_\theta = E_o \frac{1}{1+\gamma(1-cos\theta)} \tag{4}$$

where E_θ and E_o are energy of scattered and incident photon respectively and $\gamma = \frac{E_o}{mc^2}$.
For high energy photons with ($\lambda << 0.02A^o$ or $E >> 511keV$), the wavelength of the scattered radiation is always of the order of the Compton wavelength whereas for low energy photons ($E << 511keV$), the Compton shift is very small. In other words, in a non-relativistic energy regime, Compton scattering results approaches the results predicted by classical Thompson scattering.

B. Differential cross section or Klein-Nishina formula

The differential Compton scattering cross section was correctly formulated by Klein-Nishina in 1928 using quantum mechanical calculations. This formula is famously known as the Klein-Nishina formula which is expressed as follows:

$$\frac{d\sigma}{d\Omega} = r_o^2 \left(\frac{1+cos^2\theta}{2(1+\gamma(1-cos\theta)^2)}\right)\left(1+\frac{\gamma^2(1-cos\theta)^2}{(1+cos^2\theta)(1+\gamma(1-cos\theta))}\right) \tag{5}$$

where $r_o = e/4\pi\epsilon_o mc^2$ is the classical electron radius and has the value $r_o = 2.818 \times 10^{-15}m$. This result is for the cross-section averaged over all incoming photon polarizations.
In our experiment gamma rays from a Cesium-137 source are used as the source of photons that are scattered. The difference in the incident and scattered energy and wavelength of the photons is determined by a calibrated scintillation detector placed at different scattering angles. The relative intensities I_θ of the scatter radiation peaks can be compared with the predictions of the Klein-Nishina formula for the differential effective cross-section by calculating the calibration factor C using the formula below:

$$C = \frac{1}{n}\sum_{\theta=0}^{n} \frac{I_\theta}{\left(\frac{d\sigma}{d\Omega}\right)} \tag{6}$$

III. EXPERIMENTAL SETUP

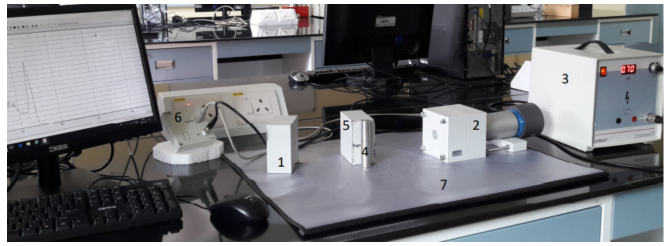


FIG. 2. Experimental setup

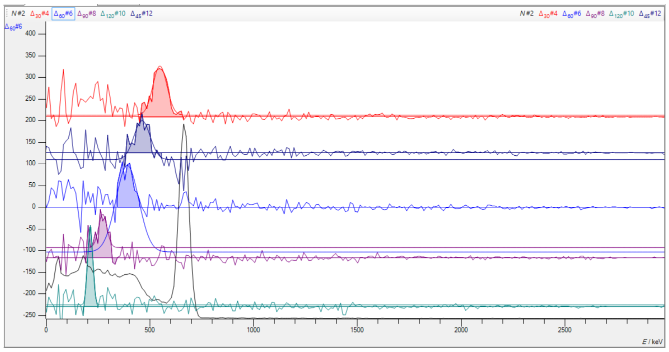
Components required:

- **Mixed sample of Am^{241} and Cs^{137}**: for energy calibration.
- **Cs^{137} sample**: Source of γ rays.
- **Source holder**
- **NaI Scintillation detector with lead shielding**
- **High voltage supply**
- **Brass and Aluminium scatterer**
- **Additional movable shielding**: to reduce the intensity of unscattered γ radiation, particularly for small scattering angles.
- **MCA**: Multi-Channel Analyzer
- **Experimental panel with angular scale**

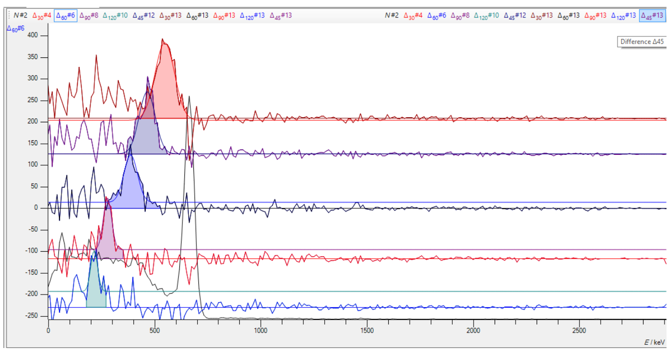
The source produces 662 keV γ rays which can escape the shielded cavity only through a small hole. The beam is collimated and reaches the scatterer. Some parts of the γ rays are scattered by the target and detected by the detector. The detected signal is further processed by MCA and the complete spectrum is obtained in the desktop.

IV. OBSERVATIONS AND DATA

Spectrum 1: Scattering spectra of Cs^{137} using Aluminium scatterer



Spectrum 2: Scattering spectra of Cs^{137} using Brass scatterer



V. PLOTS AND CALCULATIONS

A. Using Al scatterer

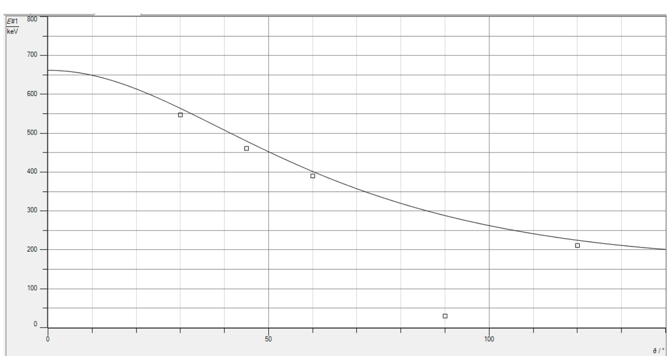


FIG. 3. Plot of E vs $\theta(^{\circ})$

The dataset is fitted for Gaussian in a suitable range around the peak with the function $\frac{661.66}{1+\frac{661.66}{511}(1-cosx)}$.

Change in wavelength as a function of scattering angle:
Change in wavelength is given by:

$$\Delta\lambda = hc\frac{\Delta E}{E_o(E_o+\Delta E)} \tag{7}$$

NOTE:

$$\Delta\lambda \neq \frac{hc}{\Delta E}$$

Table 1: Change in energy and wavelength as a function of scattering angle (for Al scatterer) Mass of electron:

Aluminium		
θ	ΔE (keV)	$\Delta\lambda$ (m)
30	547.4	8.50109E-13
45	461.2	7.71226E-13
60	390.3	6.96654E-13
90	300	5.85757E-13
120	211.5	4.54815E-13

From the fit done in FIG.(3) keeping the mass of the electron as an independent parameter, we get,

$$m_e = 463.68keV$$

$$m_e = 8.266 \times 10^{-31}kg$$

Differential cross section, relative intensities and calibration factor:
Table 2: Differential cross-section and relative intensities to determine the calibration factor

Aluminium				
$\theta(^{\circ})$	$cos\theta$	$\frac{d\sigma}{d\Omega}(m^2)$	$I_\theta(m^{-2})$	$I_\theta/\frac{d\sigma}{d\Omega}$
30	0.866	4.98E-30	888	1.78E+32
45	0.707	3.35E-30	470	1.40E+32
60	0.5	2.20E-30	663	3.01E+32
90	0	1.30E-30	600	4.60E+32
120	-0.5	1.16E-30	466	4.01E+32
Calibration factor				2.96E+32

The differential cross section is calculated using eqn. (5) and hence calibration factor is calculated from it.
Error in calibration factor:
It is obtained by the standard deviation.

$$\delta C_{Al} = 1.23 \times 10^{32}$$

Error in the mass of electron:
Relative error $= \frac{\Delta m_e}{m_e} = 0.101$

B. Using Brass scatterer

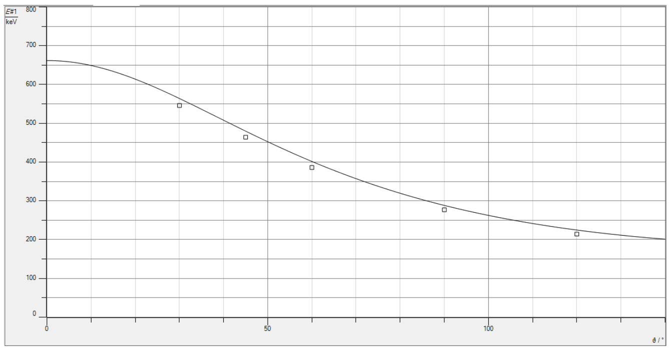


FIG. 4. Plot of E vs $\theta(^\circ)$

The dataset is fitted for Gaussian in a suitable range around the peak with the function $\frac{661.66}{1+\frac{661.66}{511}(1-cosx)}$.

Change in wavelength as a function of scattering angle:
Change in wavelength is given by:

$$\Delta\lambda = hc\frac{\Delta E}{E_o(E_o + \Delta E)} \tag{8}$$

NOTE:

$$\Delta\lambda \neq \frac{hc}{\Delta E}$$

Table 3: Change in energy and wavelength as a function of scattering angle (for Brass scatterer)
Mass of electron:

Brass		
θ	ΔE (keV)	$\Delta\lambda$ (m)
30	546	8.48E-13
45	464.2	7.73E-13
60	386.2	6.91E-13
90	277.1	5.53E-13
120	214.5	4.59E-13

From the fit done in FIG.(4) keeping the mass of the electron as an independent parameter, we get,

$$m_e = 459.97keV$$

$$m_e = 8.14 \times 10^{-31}kg$$

Differential cross section, relative intensities and calibration factor:
Table 4: Differential cross-section and relative intensities to determine the calibration factor
The differential cross section is calculated using eqn. (5) and hence calibration factor is calculated from it.

Brass				
$\theta(^\circ)$	$cos\theta$	$\frac{d\sigma}{d\Omega}(m^2)$	$I_\theta(m^{-2})$	$I_\theta/\frac{d\sigma}{d\Omega}$
30	0.866	5.12E-30	1802	3.52E+32
45	0.707	3.35E-30	1202	3.59E+32
60	0.5	2.20E-30	1088	4.94E+32
90	0	1.30E-30	769	5.89E+32
120	-0.5	1.16E-30	655	5.64E+32
Calibration factor				4.72E+32

Error in calibration factor:
It is obtained by the standard deviation.

$$\delta C_{Al} = 2.26 \times 10^{32}$$

Error in the mass of electron:
Relative error $= \frac{\Delta m_e}{m_e} = 0.118$

VI. RESULTS

- **Using Al scatterer:**
  - **Mass of electron:**
    $$m_e = (8.266 \pm 0.835) \times 10^{-31}kg$$
  - **Calibration factor:**
    $$C = (2.96 \pm 1.23) \times 10^{32}$$
- **Using Brass scatterer:**
  - **Mass of electron:**
    $$m_e = (8.14 \pm 0.96) \times 10^{-31}kg$$
  - **Calibration factor:**
    $$C = (4.72 \pm 2.26) \times 10^{32}$$

VII. CONCLUSIONS AND DISCUSSIONS

- In the lab manual, it was given that $\Delta\lambda$ was calculated as hc/ΔE, but in reality, it is given by Eqn. (7). This was a correction done in the report, and hence the analysis changes. The mass of the electron hence obtained from the Gaussian fit suffices the literature value of $9.1\times10^{-31}kg$ with some error bar, for both the cases (Al and brass scatterer).
- The intensity of the scattered radiation is higher in the case of a brass scatterer as it has a higher density as compared to Al, and hence scatters more.
- The intensity of the scattered radiation decreases with an increase in the scattering angle implying a decrease in differential cross-sectional area.
- The calibration factors obtained are different for different materials of the scatterer due to differences in their densities.

VIII. SOURCES OF ERRORS

- Presence of other radioactive sources in the lab.
- Systemic or Instrumental errors associated with the detector.
- Improper placing of the lead block to prevent direct radiation to fall into the detector.
- While doing peak analysis in the software, sometimes the gaussian fits don't cover the whole peak, which contributes to errors in intensity and ΔE values.

IX. REFERENCES

- NISER Lab Manual
- http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/comptint.html
- https://phys.libretexts.org/Bookshelves/Modern_Physics/Book%3A_Spiral_Modern_Physics_(D'Alessandris)/4%3A_The_Photon/4.2%3A_Compton_Scattering

Brass		Aluminium	
Scattering angle	Intensity	Scattering angle	Intensity
30	1802	30	888
45	1202	45	470
60	1088	60	663
90	769	90	600
120	655	120	466

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FIG. 5. Signed data