Rutherford scattering experiment

Biswaranjan Meher
Integrated M.Sc.
Roll No.-2011050
School of Physical Sciences
National Institute of Science Education and Research, Bhubaneswar
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In atomic and nuclear physics, the discovery of the atomic nucleus by Ernest Rutherford was a major breakthrough, which he did by performing one of the most famous and simple experiments called the α-scattering experiment using a gold foil. In this experiment, we are supposed to repeat the experiment using the gold foil and conclude various results like the relation between the scattering rate and the scattering angle, verifying Rutherford's scattering formula, and measuring the atomic number of Aluminium using various obtained results in the previous objectives.

I. OBJECTIVES

- To record the direct counting rate N_d of α particles scattered by a gold foil as a function of the angle θ.
- To determine the corrected counting rates N with respect to the scattering distribution in space.
- To validate the "Rutherford's scattering formula"

II. THEORY

A. α- scattering experiment

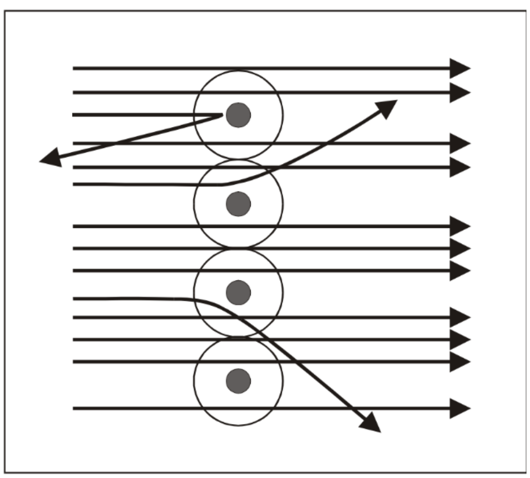

FIG. 1. Scattering of α particles on a monolayer of atoms

Rutherford performed the α-scattering experiment in 1911, in which we observed that many of the incident α particles passed through the gold foil undeflected or get deflected by less than 1°. Very few of the α particles deflected considerably and almost 1 of 10000 α particles got reflected back. From this, he concluded that most of the atomic space is empty with a very small region at the center that has some mass causing the deflection, and thus the nucleus was discovered.

B. Rutherford's scattering formula

If α-particles are allowed to strike a thin gold foil, they are deflected from their path, each by an angle θ. The majority of α-particles is scattered by angles less than 1° as shown in Fig.1. A few particles, however, show substantially large scattering angles θ, in the extreme case up to 180°(called the backscattering).
These initially qualitative observations can only be explained by assuming that the gold atoms have a very small nucleus, containing practically the whole atomic mass, and being positively charged.
On the basis of this idea, Rutherford calculated the angular distribution of the scattering rate N(θ). The scattering rate is the number of particles that are scattered during the time unit in a determined interval dθ around an average angle θ.
The result of this calculation is **Rutherford's scattering formula**:

$$N(\theta) = N_0.c_F.d_F \frac{Z^2 e^4}{(8\pi\epsilon_o E_\alpha)^2 sin^4\left(\frac{\theta}{2}\right)} \tag{1}$$

N_0: particle rate in the foil
c_F: atomic concentration in the foil
d_F: thickness of the foil
Z: nuclear charge number of the scattering material
E_α:energy of the α-particles
e: elementary charge (e = 1.6021×10^{-19} C)
ϵ_o: dielectric constant in vacuum.

C. Scattering rate as a function of angle

From eqn. (1), we can see that keeping every other factor constant, the scattering rate is related to the scattering angle as an angular distribution function defined as:

$$f(\theta) = \frac{1}{sin^4\left(\frac{\theta}{2}\right)} \tag{2}$$

The values of $f(\theta)$ decrease rapidly with increasing scattering angle θ. A singularity spot occurs at $\theta = 0°$.

Therefore we will compare measuring results with the theoretical slope only outside of this region, i.e. for values $|\theta| \geq 5°$. For keeping the total measuring time sustainable the angular range can be restricted to $|\theta| \leq 30°$.

D. Determining the nuclear charge of aluminium

If we compare the scattering rates between two different foil materials (e.g. Au and Al) at the same angle θ, we can derive from the scattering formula:

$$\frac{N_{Au}}{N_{Al}} = \frac{c_{Au} d_{Au} Z_{Au}^2}{c_{Al} d_{Al} Z_{Al}^2} \tag{3}$$

Hence the nuclear charge number of aluminium Z_{Al} can be determined by scattering experiments as:

$$Z_{Al} = \sqrt{\frac{N_{Al}(\theta) c_{Au} d_{Au} Z_{Au}^2}{N_{Au}(\theta) c_{Al} d_{Al}}} \tag{4}$$

III. EXPERIMENTAL SETUP

A. Apparatus required

- **Vacuum chamber:** containing the whole experimental apparatus, the foils, detectors, etc. is connected to a vacuum pump.

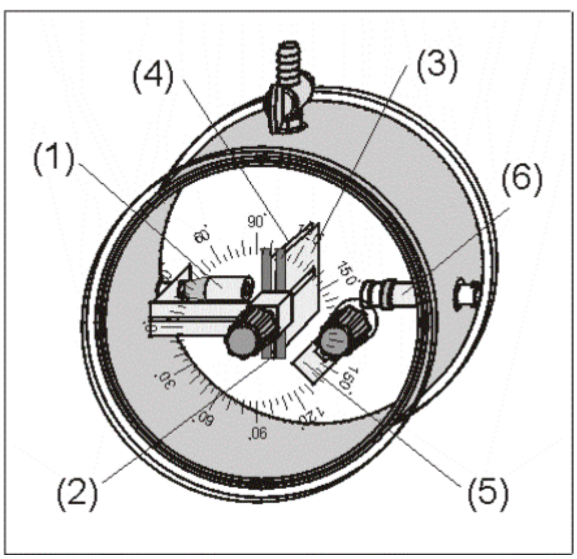

FIG. 2. Vacuum chamber
(1) Preparation: to incident α particles into the foil
(2) Holder: to hold the foils
(3) Foil: sample material into which the α particles are incident: gold and aluminium
(4) Slit: available aperture
(5) Swivel arm
(6) Detector: to detect scattered α particles

- **Discriminator preamplifier:** to distinguish between the signal and noise in which the signal (the scattered α particles) is preferred and the noise is rejected.

FIG. 3. Discriminator preamplifier

- **Counter:** to count the number of scattered α particles detected on the detector inside the vacuum chamber.

All these components are connected using wires, BNC cables, etc. to the main unit.

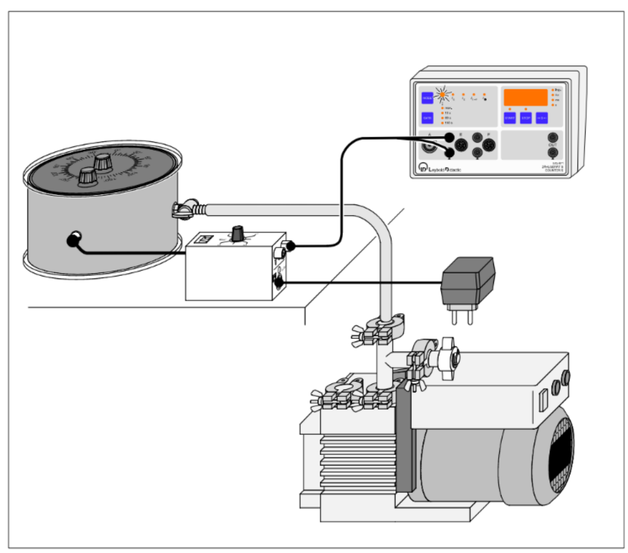

FIG. 4. Experimental setup for the Rutherford scattering experiment

IV. OBSERVATIONS AND DATA

Table 1: Count rates vs Scattering angle in a gold foil with slit width=5mm

Gold foil, slit=5mm				
Angle	Gate time	Pulse counts	Pulse mean value	Counting rate
30	900	44	36.5	0.040555556
		29		
25	600	86	74.5	0.124166667
		63		
20	200	60	65.33333333	0.326666667
		69		
		67		
15	100	202	193	1.93
		189		
		188		
10	100	632	608.6666667	6.086666667
		588		
		606		
5	100	1297	1297	12.97
-5	100	1537	1537	15.37
-10	100	967	940.6666667	9.406666667
		953		
		902		
-15	100	336	328.3333333	3.283333333
		314		
		335		
-20	200	141	122.3333333	0.611666667
		116		
		110		
-25	600	103	93.5	0.155833333
		84		
-30	900	46	43.5	0.048333333
		41		

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Table 2: Count rates vs Scattering angle in gold and aluminium foils with slit width=1mm around scattering angle=15°

Gold foil, slit=1mm				
Angle	Gate time	Pulse counts	Pulse mean value	Counting rate
15	100	24	20	0.2
		16		
-15	100	49	47.5	0.475
		46		

Aluminium foil, slit=1mm				
Angle	Gate time	Pulse counts	Pulse mean value	Counting rate
15	1000	11	9	0.009
		7		
-15	1000	52	47	0.047
		42		

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Table 3: Angle(θ) vs Phase corrected $N(\theta) = 2\pi sin(\theta)N$

Angle(θ) (deg)	Angle(θ) (rad)	Counting rate	Phase corrected $N(\theta)$
30	0.523333333	0.040555	0.12729
25	0.436111111	0.124166	0.32939
20	0.348888889	0.326666	0.7013
15	0.261666667	1.93	3.13544
10	0.174444444	6.08666	6.63424
5	0.087222222	12.97	7.09538
-5	-0.087222222	15.37	8.40833
-10	-0.174444444	9.406666	10.2529
-15	-0.261666667	3.28333	5.33403
-20	-0.348888889	0.611666	1.31315
-25	-0.436111111	0.1558333	0.41339
-30	-0.523333333	0.048333	0.1517

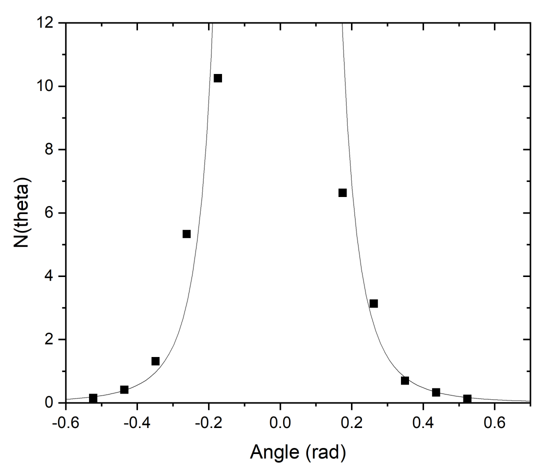

FIG. 5. Fitted plot of Phase corrected N(θ) vs scattering angle θ(radians) against $\frac{A}{sin^4(\frac{X-B}{2})}$ with parameters A=0.0008 and B=-0.008

V. CALCULATIONS

Determination of Atomic number of Aluminium:
We have, The thickness of foils: $d_{Au} = 2\mu m$, $d_{Al} = 8\mu m$
Atomic number of Gold, $Z_{Au} = 79$
From eqn.(4), we have:

$$Z_{Al} = \sqrt{\frac{N_{Al}(\theta) c_{Au} d_{Au} Z_{Au}^2}{N_{Au}(\theta) c_{Al} d_{Al}}}$$

Substituting the given values,

$$Z_{Al}^{(1)} = 8.38$$

$$Z_{Al}^{(2)} = 12.425$$

Mean Atomic number of Aluminium:

$$Z_{Al} = 10.4025$$

Standard deviation:

$$\Delta Z_{Al} = 4.0905$$

VI. RESULTS

- Atomic number of Aluminium as determined by the experimental observation:

$$Z_{Al} = (10.4025 \pm 4.0905)$$

VII. CONCLUSIONS

- While measuring the counts of the scattered α particles, we observed that there was a great increase in the number of counts when the scattering angle is around $\pm 5°$ as theoretically verified. The counting rate is maximum around $0°$ and minimum when we measure for the scattering angle $\approx 30°$.

- The plot of $N(\theta)$ vs θ fits well with the theoretical formula, which validates Rutherford's scattering formula with some constant parameters.

- Keeping the scattering angle constant i.e. $15°$, we compared the count rates between the gold and aluminium foils, in which we observed a sharp decrease in count rate in the case of Al as compared To gold, because of the smaller atomic nucleus of Al as compared to Au, thus resulting in lesser scattered particles.

- We can conclude that the nuclear size of Aluminium is much smaller than that of Gold by observing the count rates at a particular scattering angle, i.e. $Z_{Au} > Z_{Al}$.
- While determining the atomic number of Aluminium, we got a major standard deviation or error, because of ground fluctuations, electrical noises, problems in vacuum creation inside the chamber, thermal noises, etc.

- We could get better results if we perform the experiment over a longer period of time, i.e. more readings will yield better mean values and hence smaller standard deviation (smaller errors).

VIII. SOURCES OF ERRORS

- Time constraints for the experiment.

- Improper vacuum inside the vacuum chamber.

- Ground fluctuations in the room.

- Electromagnetic noises, thermal noises, electrical noises, noise due to the available light in the room, etc.

- The foils should not be touched and should be changed carefully as their thickness is very small.

- Proper calibration of the discriminator preamplifier should be ensured before starting the count rates in the experiment.

IX. REFERENCES

- NISER Lab Manual

- `http://hyperphysics.phy-astr.gsu.edu/hbase/rutsca.html`

- `http://www.uop.edu.pk/ocontents/Ch5%20Lecture%204.pdf`