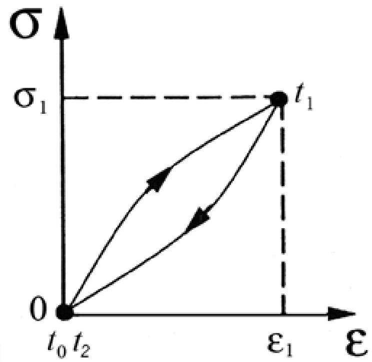
材料科学与工程学院

第6次课

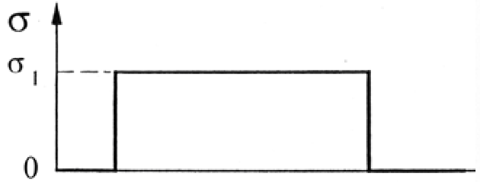
主讲：叶荣昌

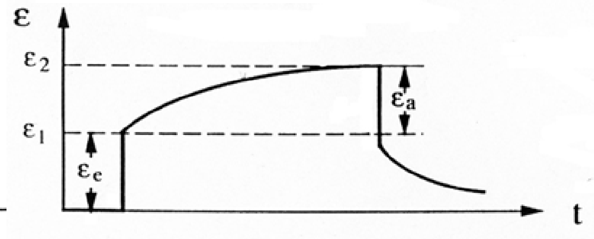
1.8 材料的滞弹性

——材料承受应力作用时，其弹性变形在时间上滞后的现象。

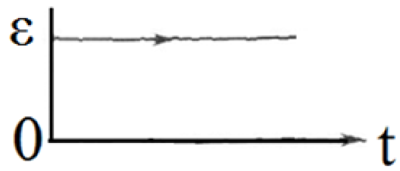


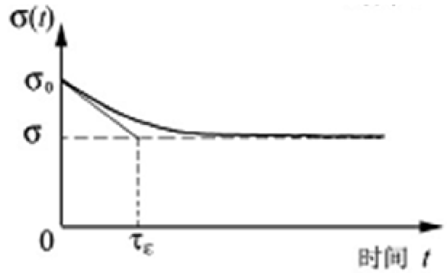
1. 弹性滞后



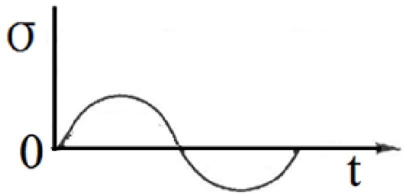


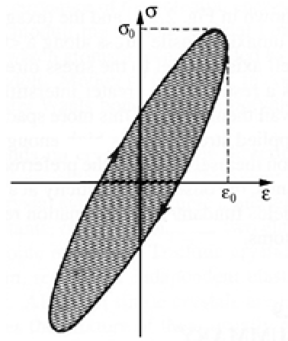
2. 弹性应力松弛





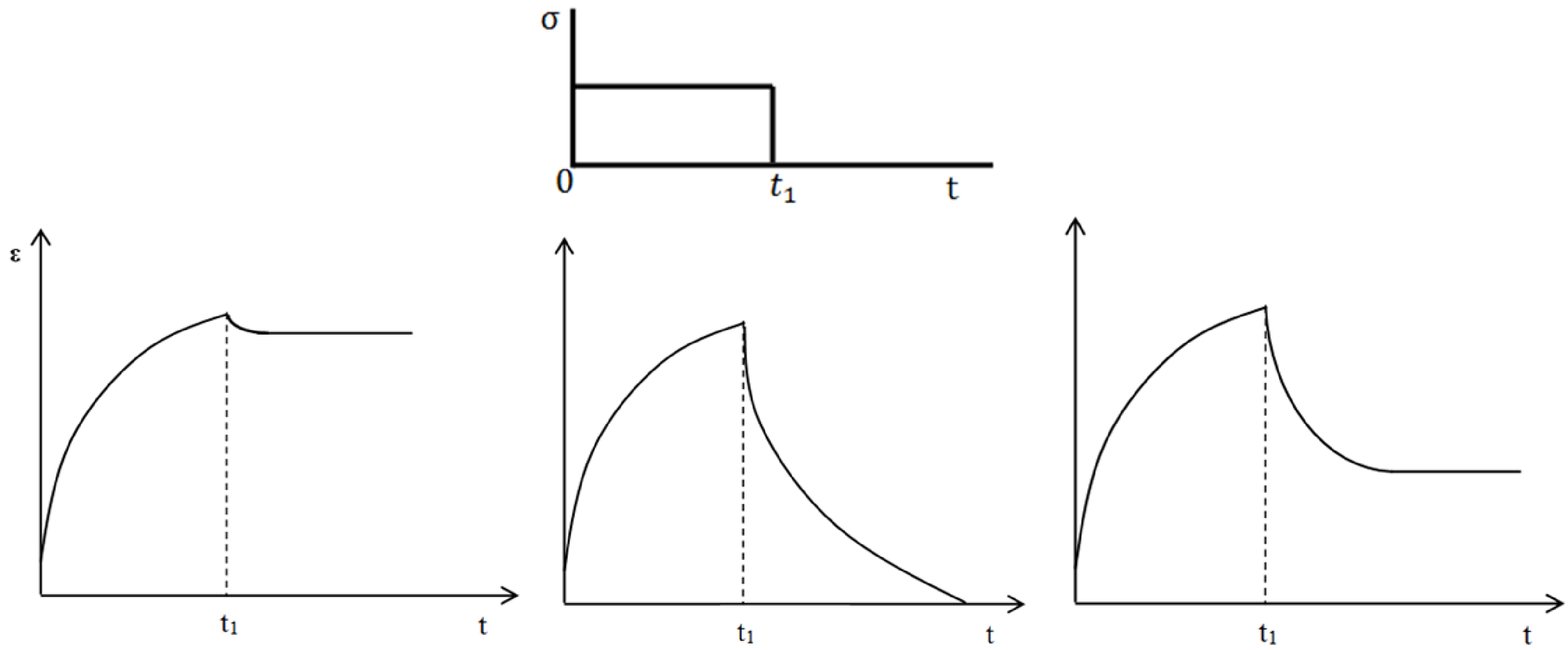
3. 能量损耗





Viscoelasticity is observed in a number of metallic and nonmetallic materials, but is most important in polymers.

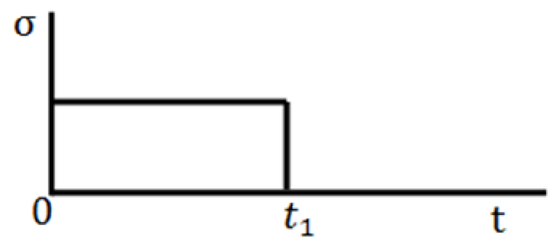
——**Long-chain polymers in the vicinity of their glass transition temperature** are capable of viscoelastic behavior.

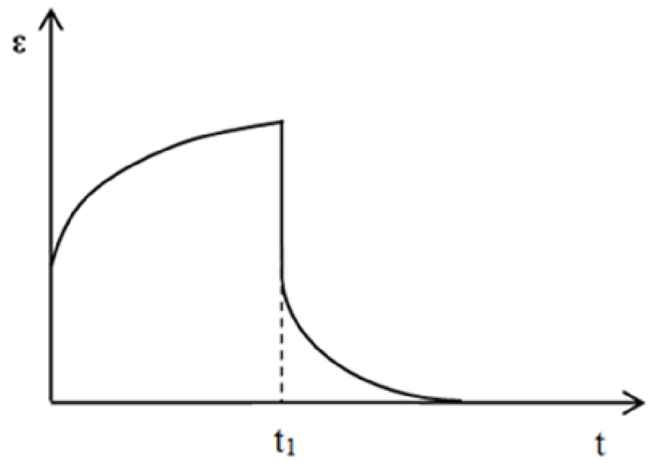


Long-chain polymers in absence of cross-linking

Long-chain polymers with cross-linking

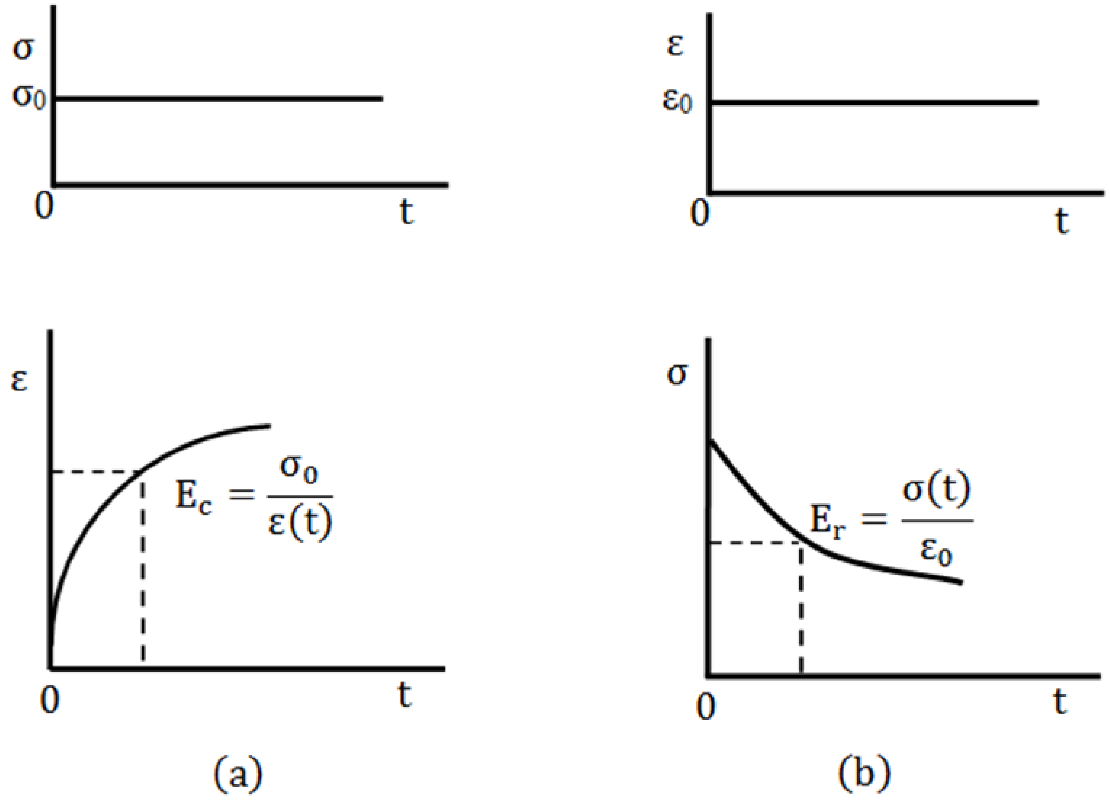
Long-chain polymers with many cross-linkings broken



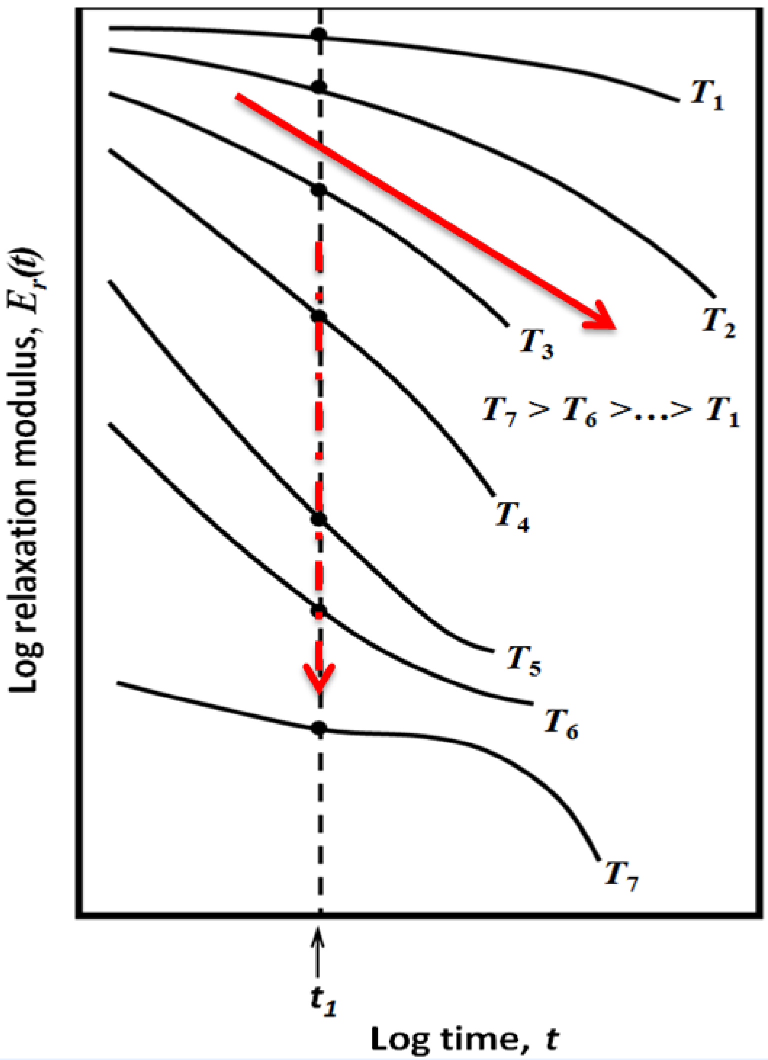


——**Anelasticity is a subset of viscielastity, which is usually observed in metals and alloys.**

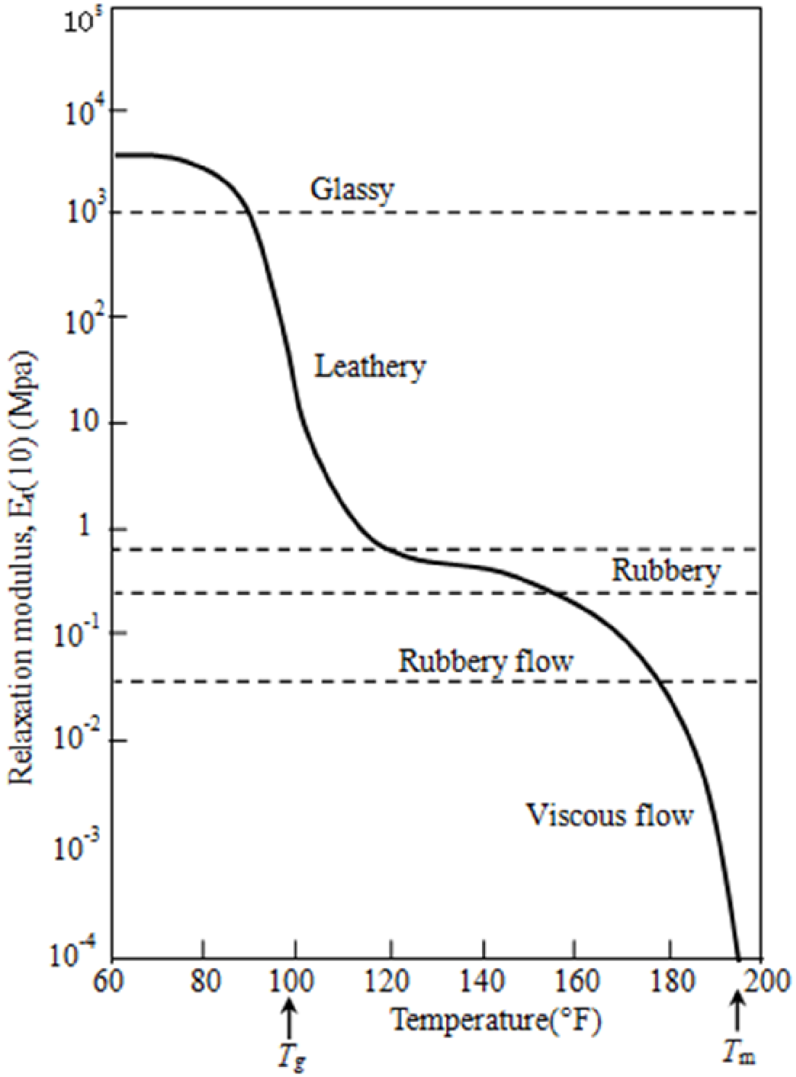
——The strain is not as apparent as that in polymers.



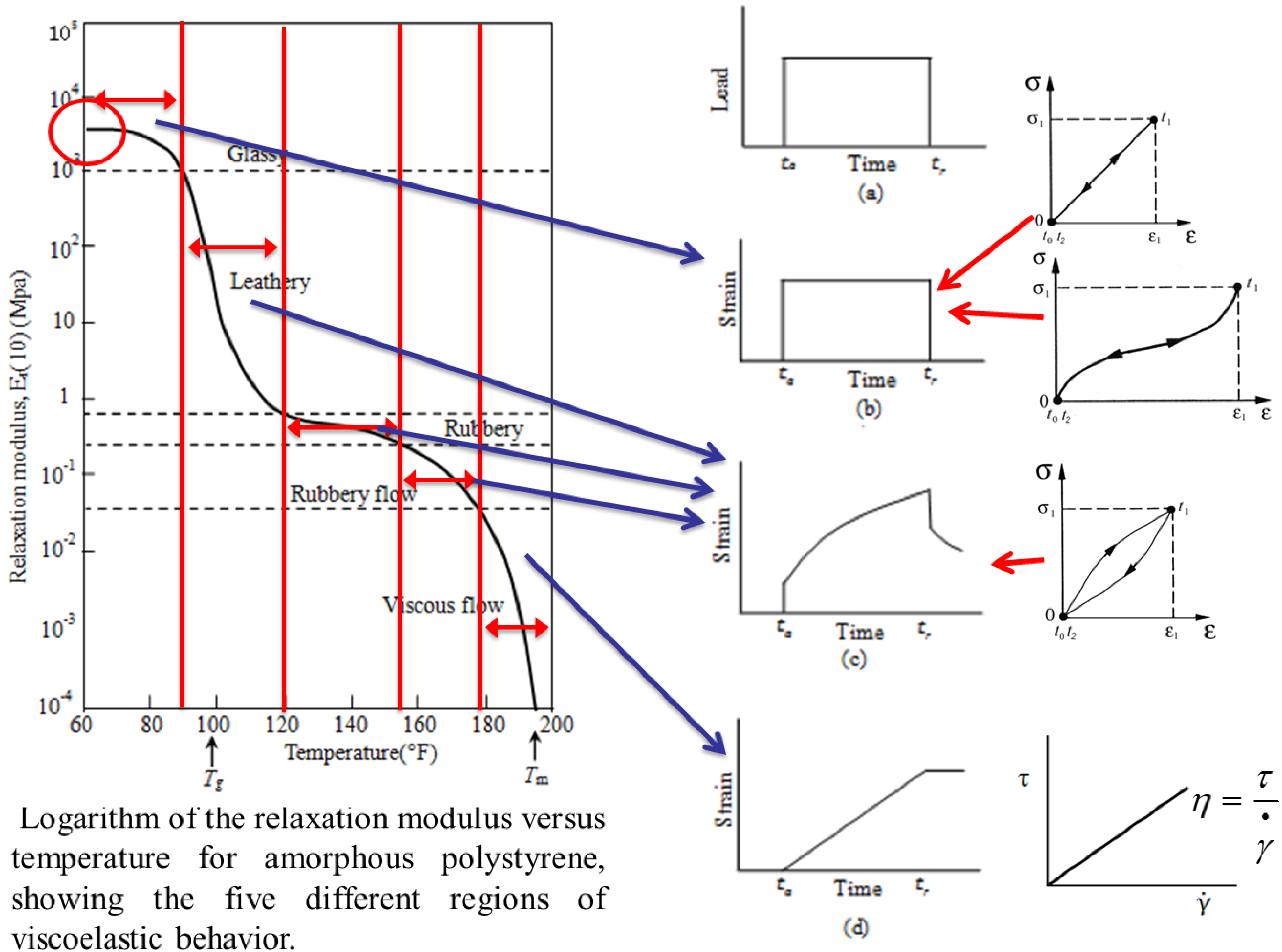
Schematic of methods used to determine the modulus of a viscoelastic solid. In (a) a finite stress is applied at t =0 and held constant. The strain is then measured and a modulus defined as the ratio of the applied stress to the strain. Since the strain is a function of time, so is the modulus. (b) shows how the modulus can be measured by a stress-relaxation test. In the test, a specified strain is applied and the stress required to maintain this strain is monitored as a function of time. This stress decreases with time as does the modulus, which is again determined by the ratio of the stress to the strain.



Schematic plot of logarithm of relaxation modulus versus logarithm of time for a viscoelastic polymer; isothermal curves are generated at temperatures T_1 through T_7.



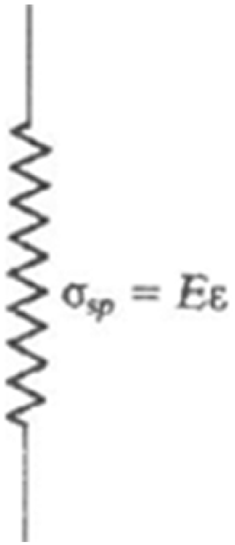
Logarithm of the relaxation modulus versus temperature for amorphous polystyrene, showing the five different regions of viscoelastic behavior.

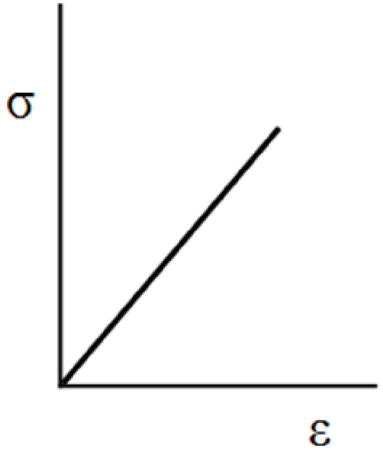


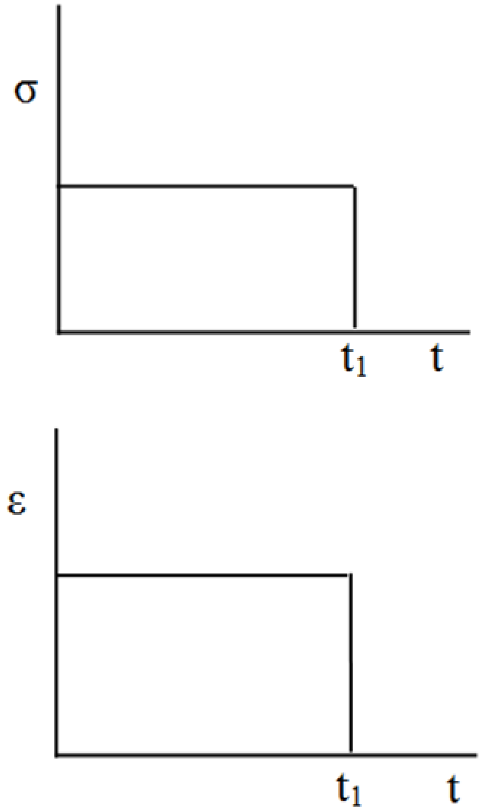
Logarithm of the relaxation modulus versus temperature for amorphous polystyrene, showing the five different regions of viscoelastic behavior.

1.8.1滞弹性的数学建模

➢ **Ideal elastic solid**



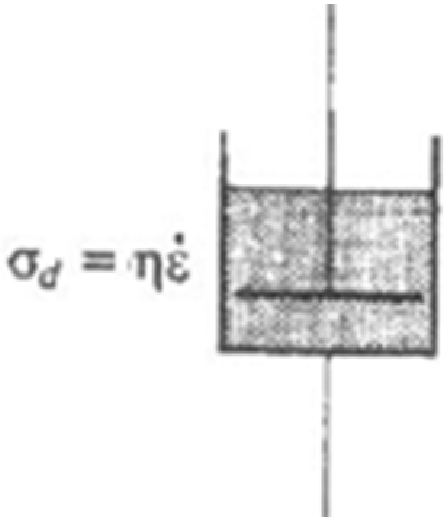


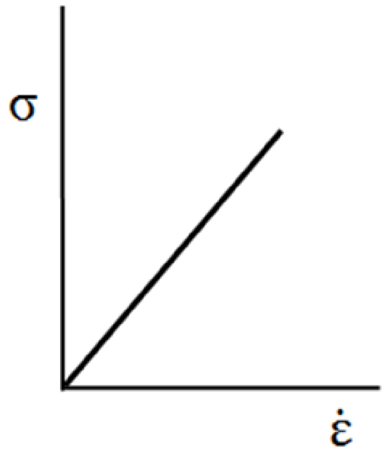


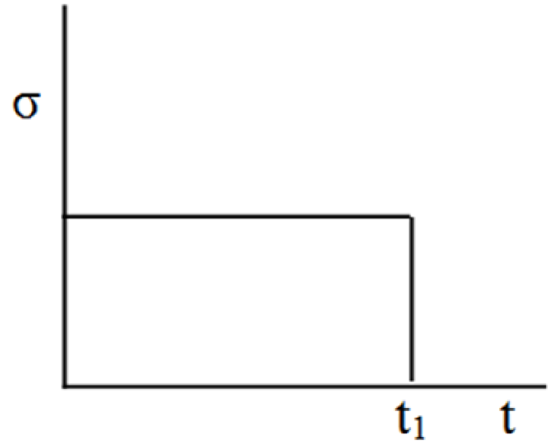
spring

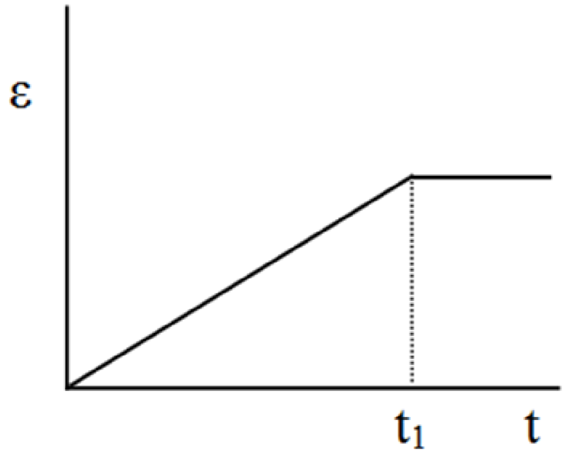
——linear elastic elememt, for which stress is related linearly to strain.

➢ Ideal viscous liquid









dashpot

——viscous element, for which stress is related linearly to strain rate.

——Maxwell model

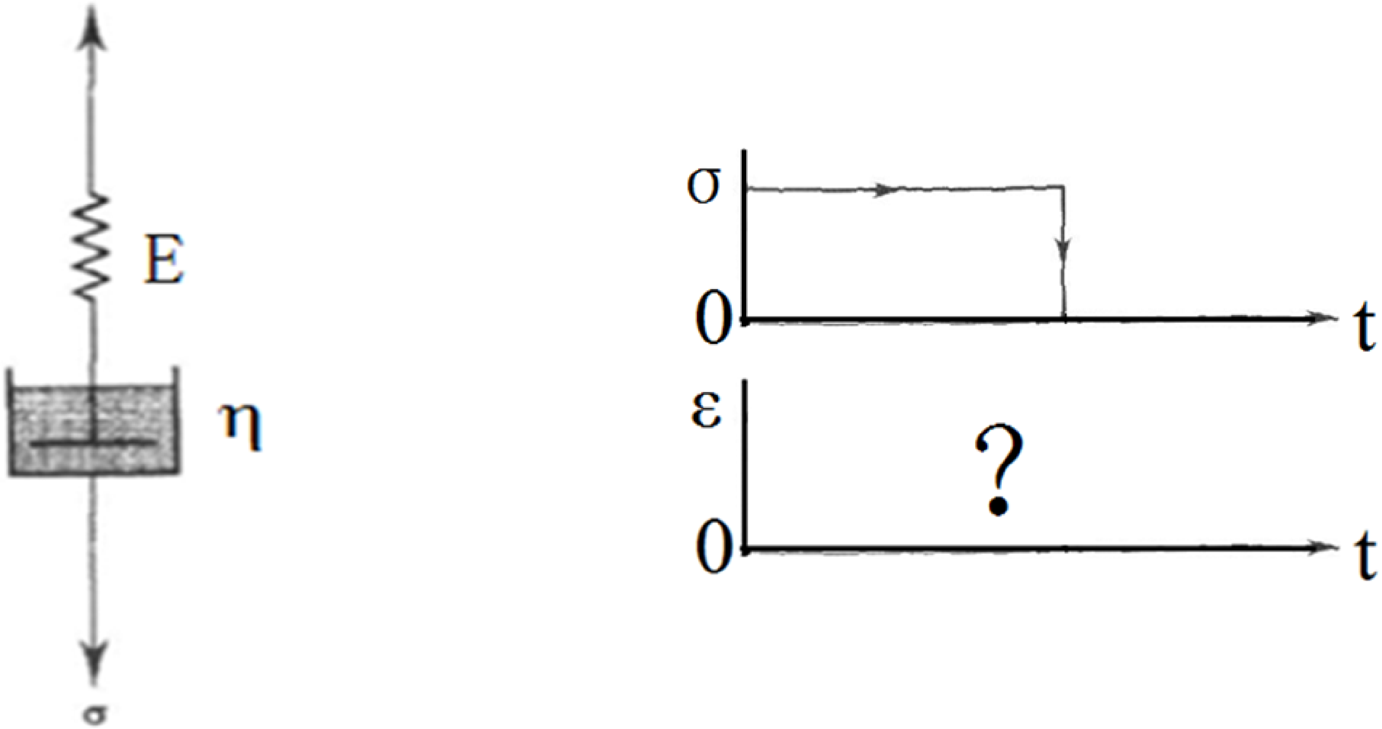
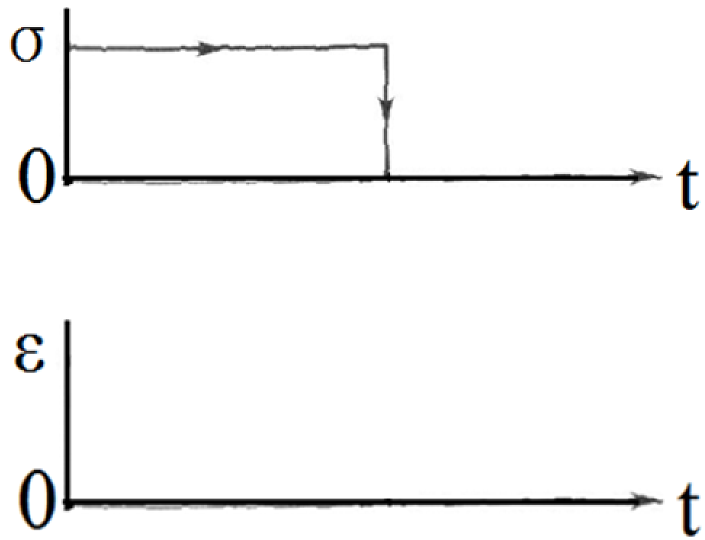
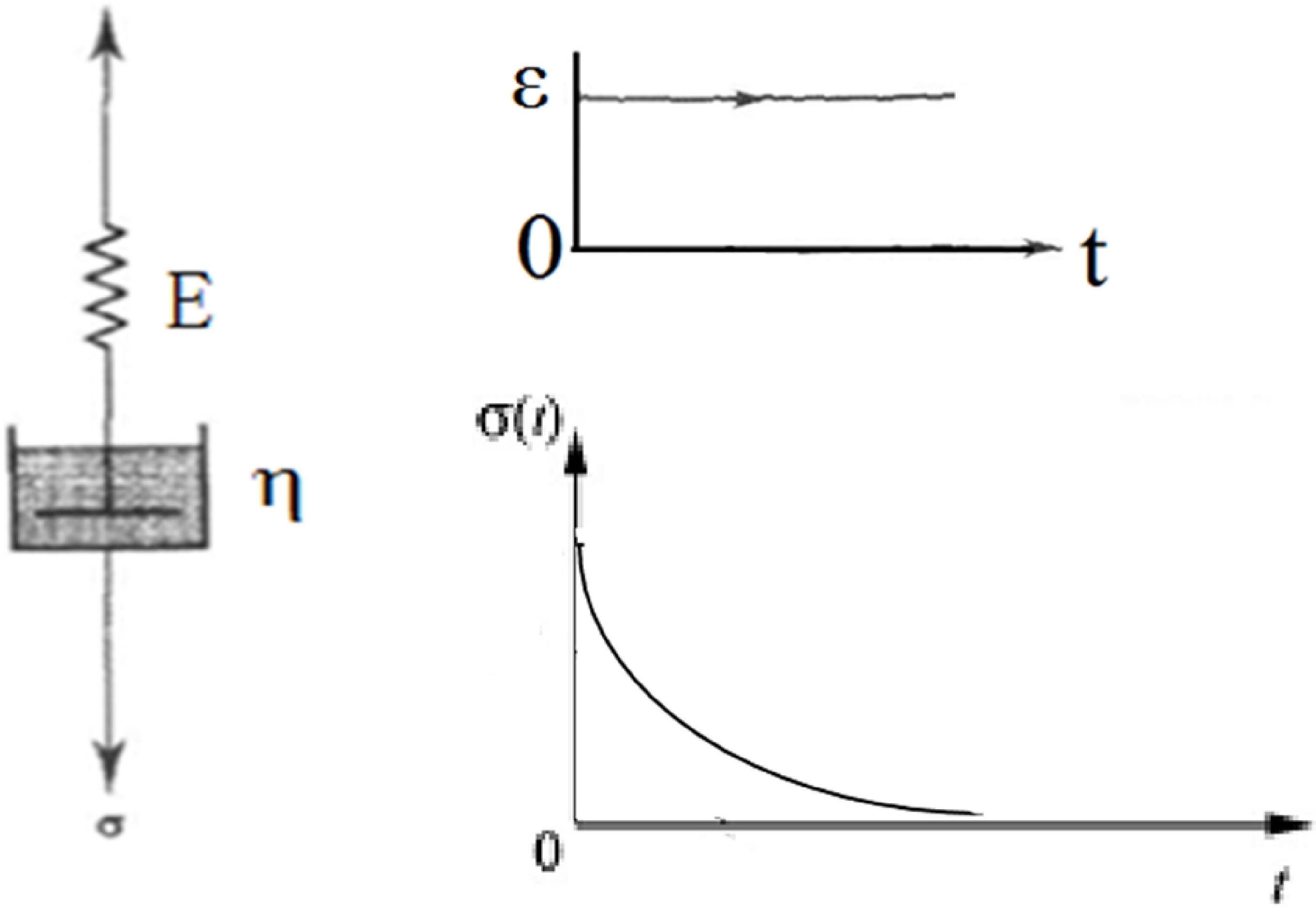


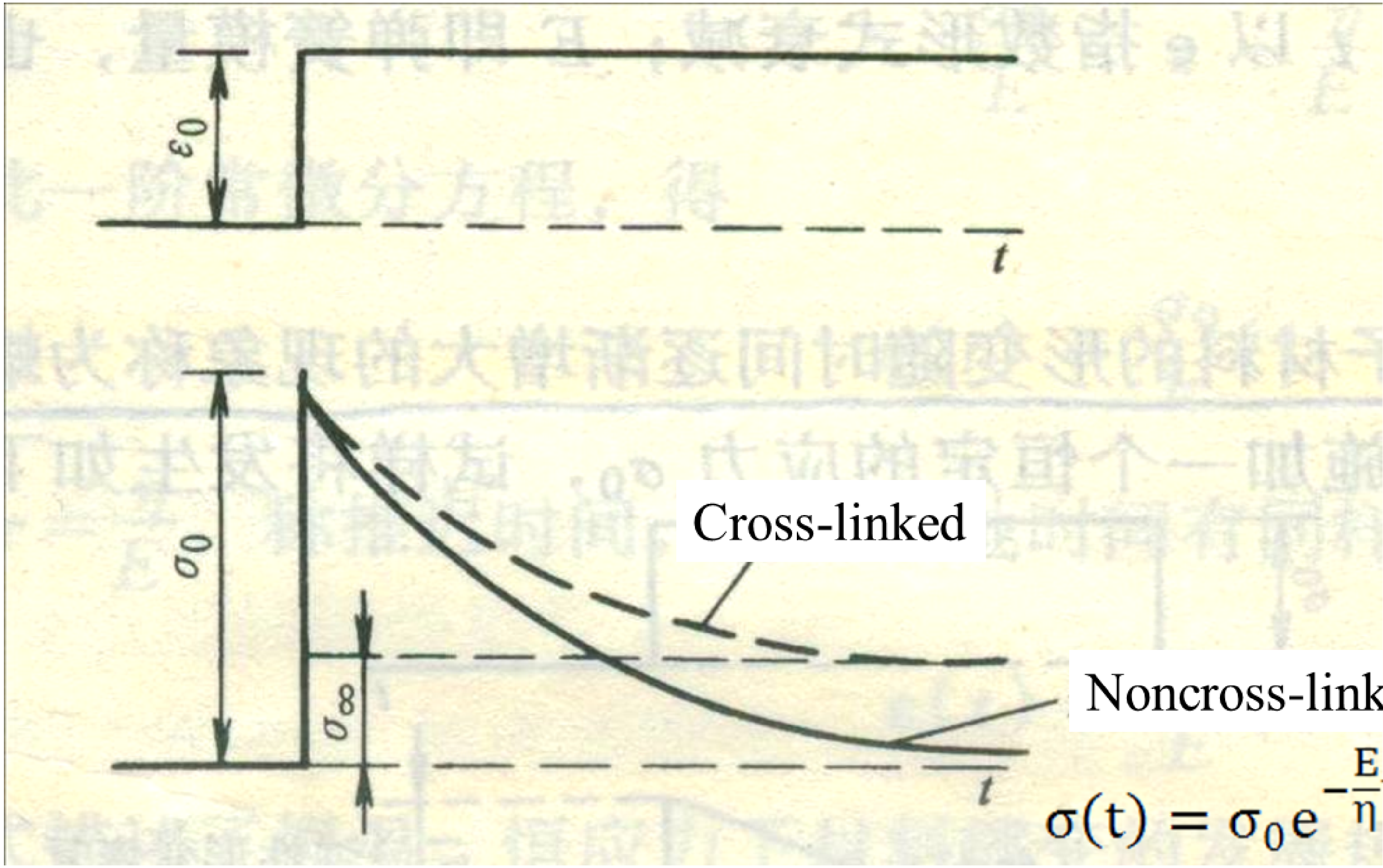
Fig.15 A Maxwell model. The model consists of a linear elastic and a viscoplastic member in series. For the former, strain and stress are linearly related; for the latter, strain rate varies linearly with stress. While the stress experienced by the spring and dashpot are equal, the strain is a sum of the strains of the spring and the dashpot.

主观题 10分

Please show the strain-time relationship for the Maxwell model when the stress is applied as shown in the figure below.



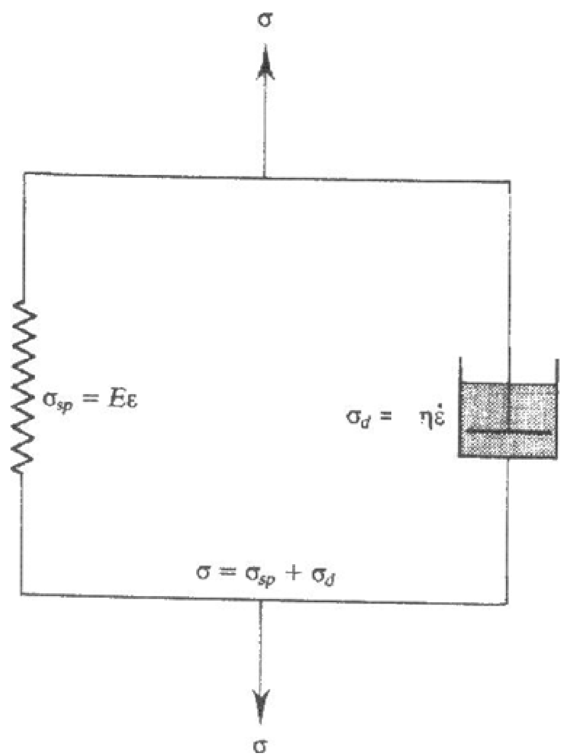

E
η
σ
ε
0
t
σ(t)
0
t



$$\sigma(t) = \sigma_0 e^{-\frac{E}{\eta}t}$$

Stress relaxation curves of long-chain polymers

——Voigt model



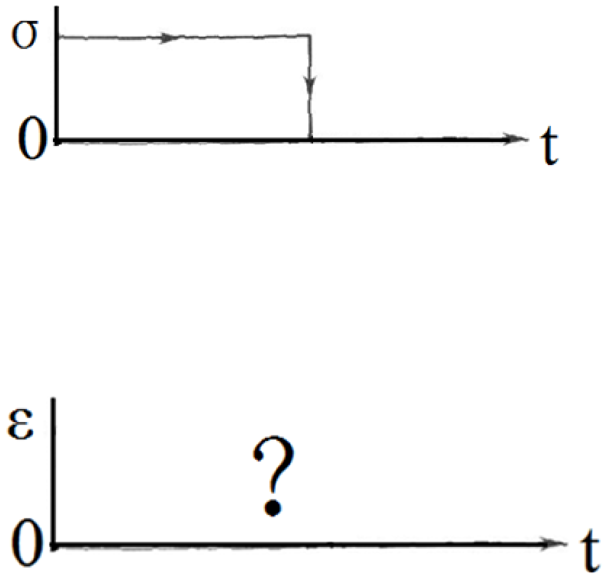
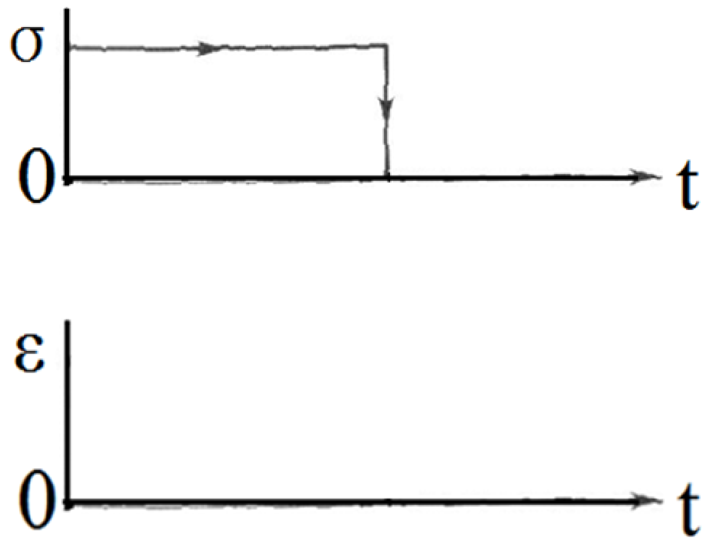
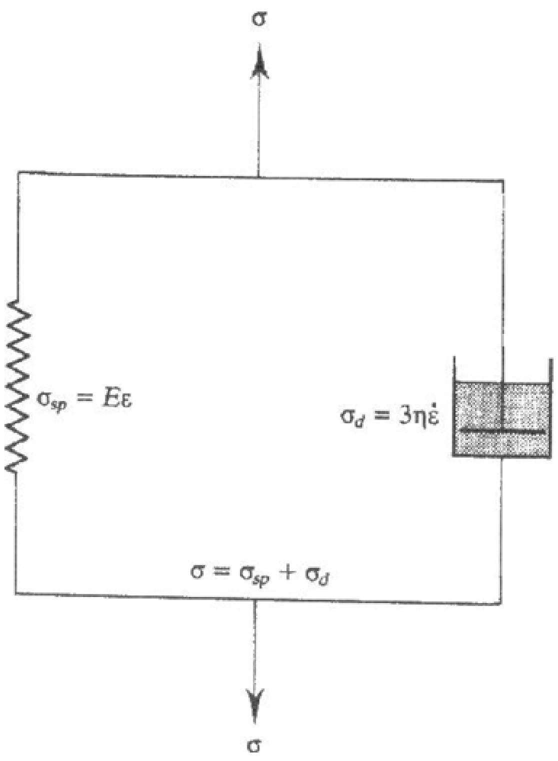


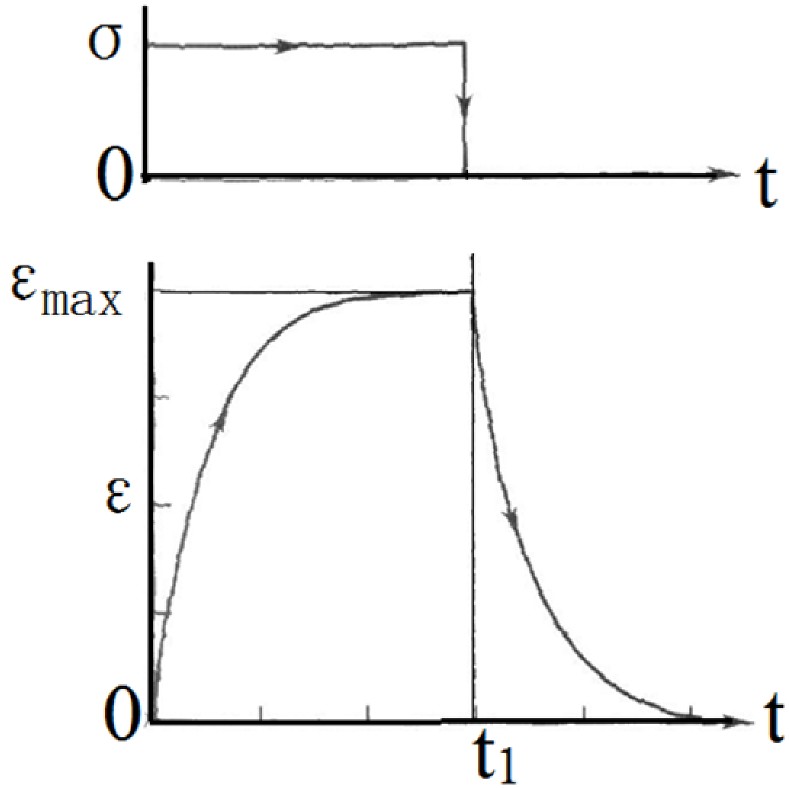
Fig.14 A Voigt model. The model consists of a linear elastic and a viscoplastic member in parallel. For the former, strain and stress are linearly related; for the latter, strain rate varies linearly with stress. While the strains experienced by the spring and dashpot are equal, the stresses are not and vary with time as well.

主观题 10分

Please show the strain-time relationship for the Voigt model when the stress is applied as shown in the figure below.







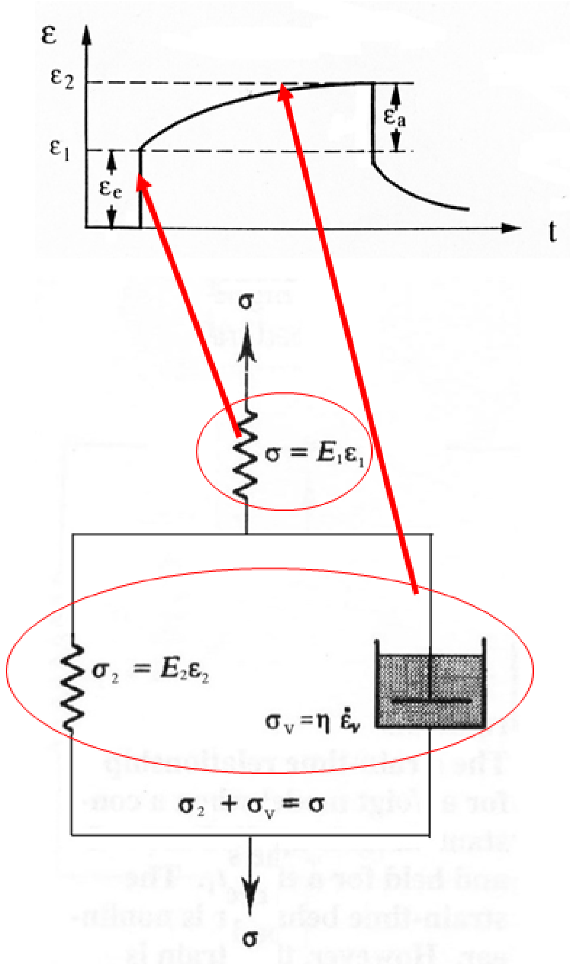
课后作业：

a) Determine the time variation of σ_{sp},σ_d, ε and strain rate of the Voigt model when loading.

b) Assume the Voigt element has been extended to its asymptotic strain. Remove the stress and repeat the exercise of part (a).

滞弹性的标准线性固体模型（岑纳(C. Zener)）

——用两个理想弹簧和一个阻尼器组合起来，模拟具有滞弹性的金属材料的应力-应变关系。



组成单元的基本特性及应力、应变之间的相互关联性：

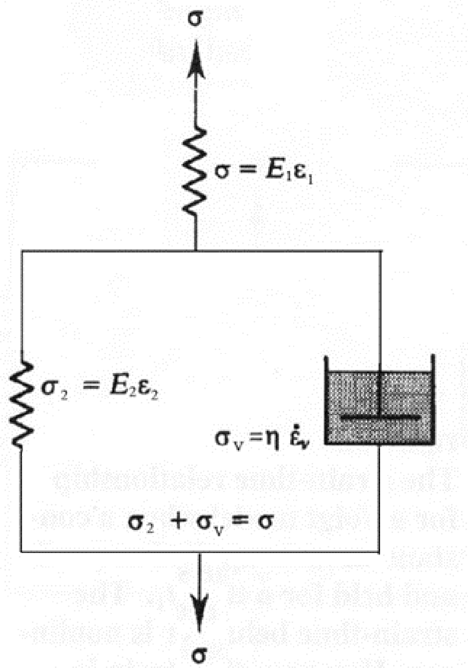
- 理想弹簧特性 $\sigma_1 = E_1\varepsilon_1$ (1) $\sigma_2 = E_2\varepsilon_2$ (2)
- 阻尼器特性 $\sigma_V = \eta\dot{\varepsilon}_V$ (3) $\dot{\varepsilon}_V = \mathrm{d}\varepsilon_V/\mathrm{d}t$
- 相互关联性 $\varepsilon_2 = \varepsilon_V$ (4)

$\varepsilon = \varepsilon_1 + \varepsilon_2$ (5)

$\sigma_1 = \sigma_2 + \sigma_V = \sigma$ (6)

标准线性固体的应力-应变-时间关系 $f(\sigma,\varepsilon,t)=0$

$$\sigma+\dot{\sigma}\tau_{\varepsilon}=E_{\mathrm{R}}\left(\varepsilon+\dot{\varepsilon}\tau_{\sigma}\right)$$



式中，固体的弛豫弹性模量 $E_{\mathrm{R}}=\dfrac{E_1E_2}{E_1+E_2}$

未弛豫弹性模量 $E_{\mathrm{U}}=E_1$

恒应力条件下的弛豫时间 $\tau_{\sigma}=\eta/E_2$

恒应变条件下的弛豫时间 $\tau_{\varepsilon}=\tau_{\sigma}\cdot\dfrac{E_{\mathrm{R}}}{E_{\mathrm{U}}}=\dfrac{\eta}{E_1+E_2}$

弛豫时间的几何平均值 $\bar{\tau}=\sqrt{\tau_{\sigma}\cdot\tau_{\varepsilon}}$

➢ 标准线性固体的应力松弛

——在开始瞬间施加一定量弹性变形 ε_0，之后维持弹性变形量恒定不变，固体材料中弹性应力随着时间发生变化。

恒应变条件 $\varepsilon(t) \equiv \varepsilon_0$

$$\Rightarrow \quad \dot{\varepsilon} = 0$$

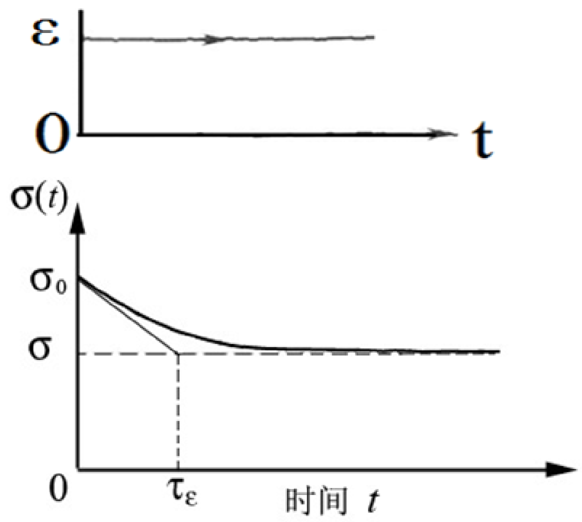


图1-26 应力松弛曲线

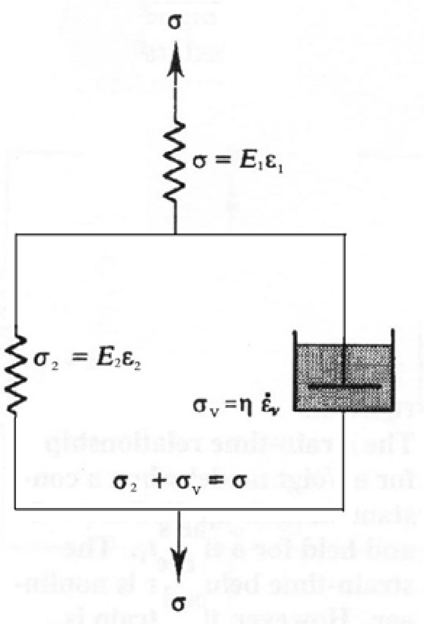
代入标准线性固体的应力-应变-时间关系式

$$\sigma + \dot{\sigma}\tau_{\varepsilon} = E_{R}\left(\varepsilon + \dot{\varepsilon}\tau_{\sigma}\right)$$

$$\Rightarrow \quad \sigma + \dot{\sigma}\tau_{\varepsilon} = E_{R}\varepsilon_{0}$$

通解为 $\sigma(t) = C_1 \exp(-t/\tau_\varepsilon) + C_2$

根据“边界条件”确定常数



•初始时刻 $\varepsilon(t=0) = \varepsilon_0 = \dfrac{\sigma(t=0)}{E_1}$

$$\sigma_0 \equiv \sigma(t=0) = C_1 + C_2 = E_1\varepsilon_0 = E_U\varepsilon_0$$

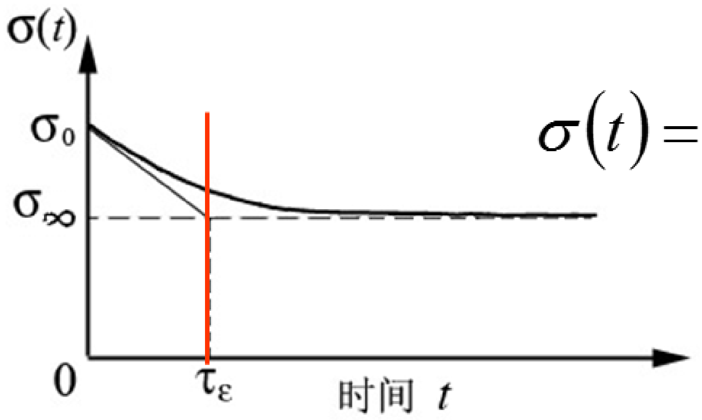
•时间无限长 $\varepsilon(t \to \infty) = \varepsilon_0 = \sigma(t \to \infty) \cdot \dfrac{E_1 + E_2}{E_1 E_2}$

$$\sigma_\infty \equiv \sigma(t \to \infty) = C_2 = \frac{E_1 E_2}{E_1 + E_2}\varepsilon_0 = E_R\varepsilon_0$$

$$C_1 = E_U\varepsilon_0 - C_2 = (E_U - E_R)\varepsilon_0$$

标准线性固体的应力松弛规律为

$$\sigma(t) = E_U\varepsilon_0 - (E_U - E_R)\varepsilon_0[1 - \exp(-t/\tau_\varepsilon)]$$



$$\sigma(t)=E_U\varepsilon_0-(E_U-E_R)\varepsilon_0[1-\exp(-t/\tau_\varepsilon)]$$

图1-26 应力松弛曲线

应力松弛的特征：

① 最大弹性应力出现在加载最初瞬间，σ_0等于$E_U\varepsilon_0$

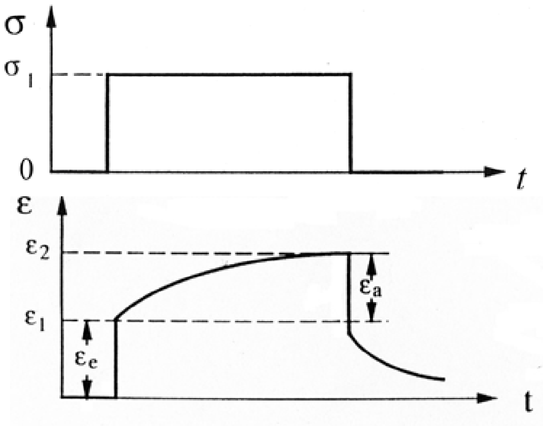
② 最低弹性应力出现在时间无限长时，σ_∞等于$E_R\varepsilon_0$

③ 随着时间的延长，应力呈指数规律降低，降低速度由时间常数τ_ε决定。

——时间常数τ_ε的含义：应力松弛完成弹性应力的总衰减量$|\sigma_0-\sigma_\infty|$的63.2%所需要时间；

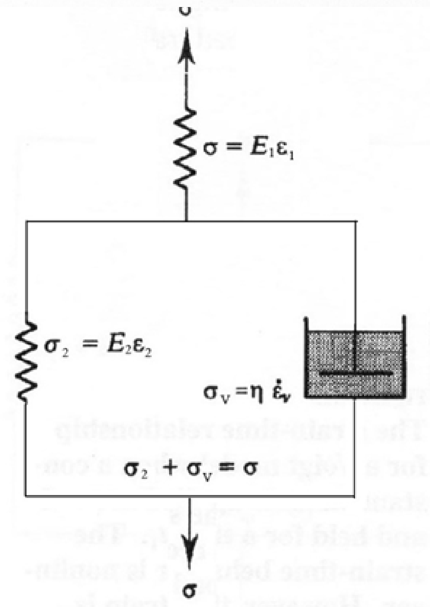
➤ 标准线性固体的弹性后效

——在恒定应力情况下，弹性变形量随着应力作用时间的延长而逐渐增加，卸除应力后，弹性变形随着时间延长逐渐衰减。



恒应力条件 $\sigma(t) \equiv \sigma_0$

代入标准线性固体的应力-应变-时间关系式

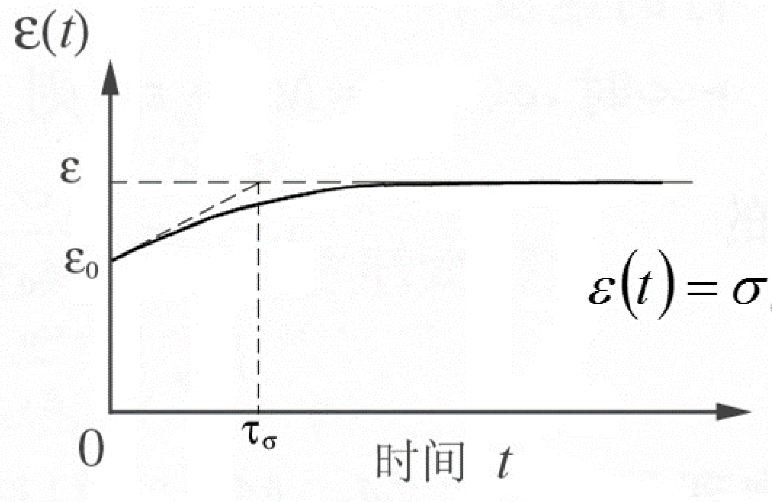


$$\sigma + \dot{\sigma}\tau_\varepsilon = E_R\left(\varepsilon + \dot{\varepsilon}\tau_\sigma\right)$$

$$\sigma_0 = E_R\left(\varepsilon + \dot{\varepsilon}\tau_\sigma\right)$$

标准线性固体的弹性后效规律

$$\varepsilon(t) = \sigma_0/E_U + \sigma_0\left(\frac{1}{E_R} - \frac{1}{E_U}\right)\left[1 - \exp(-t/\tau_\sigma)\right]$$



$$\varepsilon(t)=\sigma_0/E_U+\sigma_0\left(\frac{1}{E_R}-\frac{1}{E_U}\right)\left[1-\exp(-t/\tau_\sigma)\right]$$

图1-27 标准线性固体的弹性后效曲线

弹性后效的特征：

① 存在初始瞬间弹性应变， $\varepsilon_0=\sigma_0/E_U$；

② 弹性应变极限值， $\varepsilon_\infty=\sigma_0/E_R$；

③ 随着时间的延长，滞弹性变形量按指数规律趋近于其极限值；变化的速度由弛豫时间常数 τ_σ 决定，

——弛豫时间常数 τ_σ 是滞弹性变形完成其极限值变形量($\varepsilon_\infty-\varepsilon_0$)的63.2%所需要的时间。