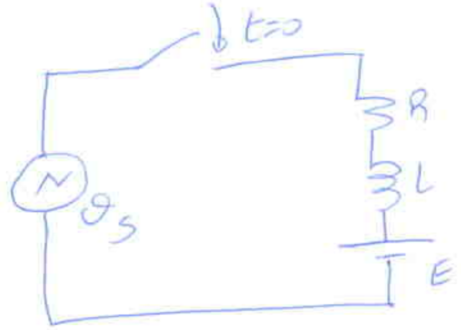
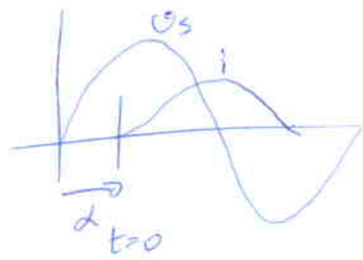
FORMULÁRIO

Circuito RLE



o interruptor é fechado em ~~t=0~~ t=0 quando v_s apresenta uma fase com ângulo α

Condições iniciais (t=0)



$$\begin{cases} i(0) = 0 \\ v_s = V_{max} \operatorname{sen}(wt + \alpha) \end{cases}$$

Equação do circuito:

$$V_{max} \operatorname{sen}(wt + \alpha) = R\, i(t) + L \frac{di(t)}{dt} + E$$

Solução da equação diferencial:

$$i(t) = \left[\frac{E}{R} - \frac{V_{max}}{|Z|} \operatorname{sen}(\alpha - \phi) \right] \cdot e^{-\frac{R}{L}t} - \frac{E}{R} + \frac{V_{max}}{|Z|} \operatorname{sen}(wt + \alpha - \phi)$$

com $|Z| = \sqrt{R^2 + (wL)^2}$

$$\phi = \operatorname{arctg}\left(\frac{wL}{R}\right)$$

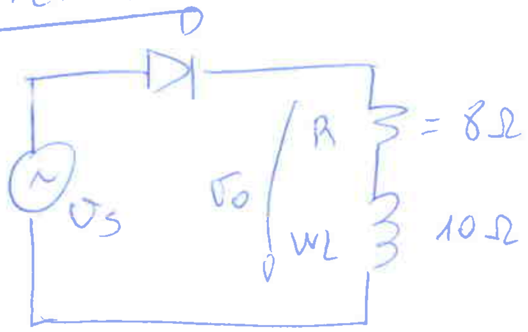
$\boxed{\text{OU}}$

$\theta = wt$

$$i(\theta) = \left[\frac{E}{R} - \frac{V_{max}}{|Z|} \operatorname{sen}(\alpha - \phi) \right] \cdot e^{-\theta/Q} - \frac{E}{R} + \frac{V_{max}}{|Z|} \operatorname{sen}(\theta + \alpha - \phi)$$

com $Q = \frac{wL}{R}$

Exercício 1



$v_s = 230\sqrt{2}\,\text{sen}(100\pi t)$

D ideal

a) Determine durante quanto tempo conduz o díodo

Carga RL (E=0)

Díodo conduz quando $v_s = 0 \Rightarrow \alpha = 0$

$$V_M \,\text{sen}(\omega t) = R\,i(t) + L\frac{di(t)}{dt}$$

$\Downarrow$

$$\boxed{i(t) = \frac{V_M}{|Z|}\,\text{sen}(\omega t - \phi) - \frac{V_M}{|Z|}\,\text{sen}(-\phi)\cdot e^{-\frac{R}{L}t}}$$

$V_M = 230\sqrt{2}$

$|Z| = \sqrt{8^2 + 10^2} = 12,8$

$\phi = \text{arctg}\left(\frac{10}{8}\right) = 51,34° = 0,896 \text{ rad}$

$\omega L = 2\pi f L \Leftrightarrow 10 = 2\pi \cdot 50 \cdot L$

$\Leftrightarrow L = 31,83 \text{ mH}$

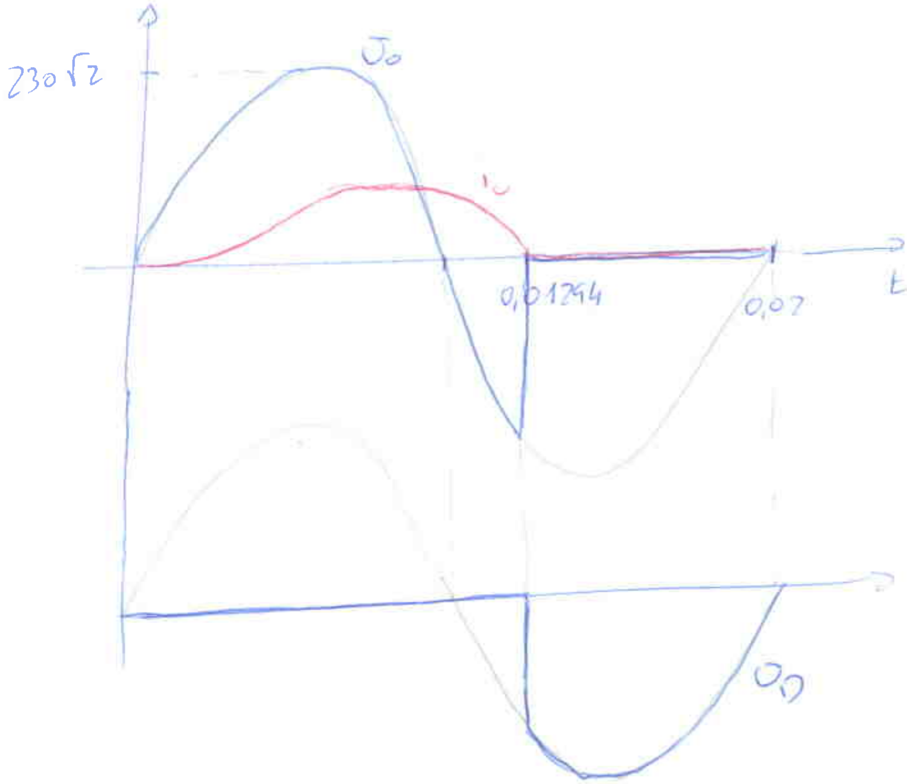
$$\boxed{i(t) = \frac{230\sqrt{2}}{12,8}\left(\text{sen}(100\pi t - 0,896)\right) - \frac{230\sqrt{2}}{12,8}\,\text{sen}(-0,896)\cdot e^{-\frac{8\cdot t}{0,03183}}}$$

o díodo sai de condução quando $i(t) = 0 \Rightarrow t = 12,94 \text{ ms}$

$\Updownarrow$

$\theta = 4,066 \text{ rad}$

b) Esboce as formas de onda de $v_o(t)$, $i(t)$ e $v_D(t)$



c) Calcule os valores médio e eficaz da tensão e corrente na carga

$t = 12{,}94\ ms$

$\Updownarrow$

$\theta = 4{,}066\ rad$

$$v_{o_{médio}} = \frac{1}{T}\int_0^T v_o(t)\,dt = \frac{1}{0{,}02}\int_0^{0{,}01294} 230\sqrt{2}\,\text{sen}(100\pi t)\,dt$$

ou $$\frac{1}{2\pi}\int_0^{4{,}066} 230\sqrt{2}\,\text{sen}(\theta)\,d\theta = \frac{230\sqrt{2}}{2\pi}\Big[-\cos\theta\Big]_0^{4{,}066} = 87{,}9\,V$$

$$v_{o_{RMS}} = \sqrt{\frac{1}{T}\int_0^T v_o^2(t)\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{4{,}066}(230\sqrt{2}\,\text{sen}\,\theta)^2\,d\theta} = 173{,}8\,V$$

$$i(\theta) = \frac{230\sqrt{2}}{12,8} \operatorname{sen}(\theta - 0,896) - \frac{230\sqrt{2}}{12,8} \operatorname{sen}(-0,896) \times e^{-\theta \cdot 8/10}$$

$$i_{o\,medio} = \frac{1}{2\pi}\int_0^{2\pi} i(\theta)\,d\theta = \frac{1}{2\pi}\int_0^{4,066} i(\theta)\,d\theta = 10,4\ A$$

$$i_{o\,RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i(\theta)^2\,d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{4,066} i(\theta)^2\,d\theta} = 15\ A$$

Nota: $i_{o\,medio} = \dfrac{v_{o\,medio}}{R} = \dfrac{82,9}{8} = 10,4\ A$ ✓

---

d) Calcule a potência dissipada na carga

$$P_{act} = R \cdot i_{o\,RMS}^2 = 8 \times 15^2 = 1800\ W$$

ou

$$P_{act} = \frac{1}{T}\int_0^T v_o(t) \times i_o(t)\,dt = 1800\ W$$

---

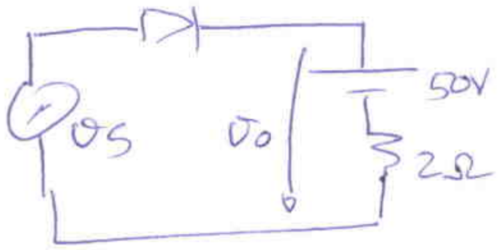
e) Determine o fator de potência na fonte

$$P_{fonte} = \frac{1}{2\pi}\int_0^{4,066} v_s(\theta) \cdot i_s(\theta)\,d\theta = \frac{1}{2\pi}\int_0^{4,066} 230\sqrt{2} \operatorname{sen}\theta \cdot i_o(\theta)\,d\theta = 1801\ W$$

$$S_{fonte} = v_{s\,RMS} \times i_{s\,RMS} = 230 \times 15 = 3450\ VA$$

$$FP_{fonte} = \frac{P_{fonte}}{S_{fonte}} = \frac{1801}{3450} = 0,522$$

Exercício 2



$v_S = 230\sqrt{2}\, \text{sen}(\theta)$

a) Calcule o valor da fase da tensão de entrada que coloca o diodo em condução

D ON se $v_S > 50$

$50 = 230\sqrt{2}\, \text{sen}(\theta) \Rightarrow \theta = \text{arcsen}\left(\dfrac{50}{230\sqrt{2}}\right)$

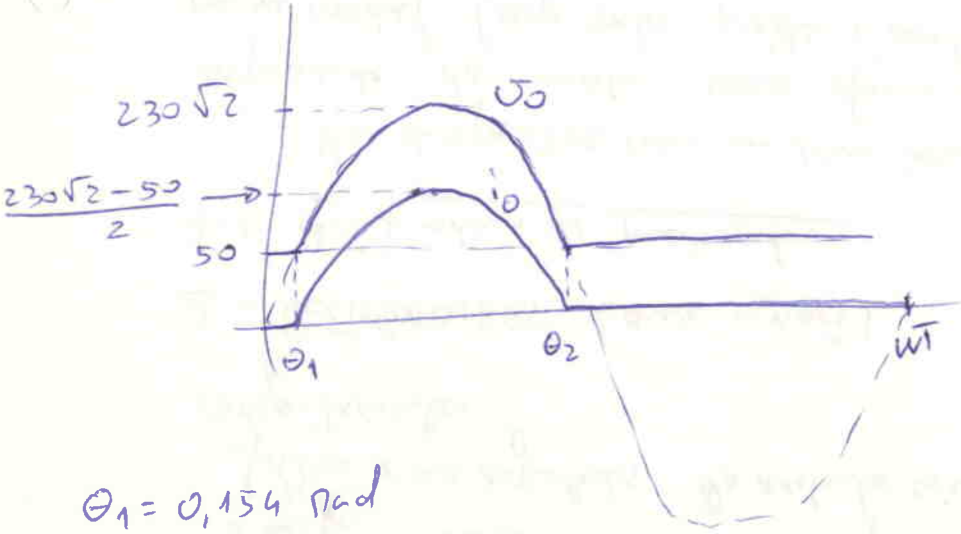
$= 0{,}154$ rad

$= 8{,}8^{\circ}$

b) Represente $v_0(t)$ e $i(t)$

D ON $\Rightarrow$ $v_S = Ri + 50 \Leftrightarrow i(t) = \dfrac{230\sqrt{2}\,\text{sen}\theta - 50}{2}$

D ON (para $v_S > 50$ V) $v_0 = v_S$



$\theta_1 = 0{,}154$ rad

$\theta_2 = \pi - 0{,}154$ rad

c) Determine os valores médio e eficaz da corrente na carga

$$i_{O_{AV}} = \frac{1}{T}\int_0^T i_O(t)\,dt = \frac{1}{2\pi}\int_{0,154}^{\pi-0,154} \frac{230\sqrt{2}\,\operatorname{sen}\theta - 50}{2}\,d\theta = 39,9\ A$$

$$i_{O_{RMS}} = \left[\frac{1}{2\pi}\int_{0,154}^{\pi-0,154}\left(\frac{230\sqrt{2}\,\operatorname{sen}\theta - 50}{2}\right)^2 d\theta\right]^{1/2} = 65,8\ A$$

d) Determine a potência média na carga

$$P = R\, i_{O_{RMS}}^2 + E\, i_{O_{AV}}$$

$$= 2 \times 65,8^2 + 50 \times 39,9$$

$$= 10654\ W$$

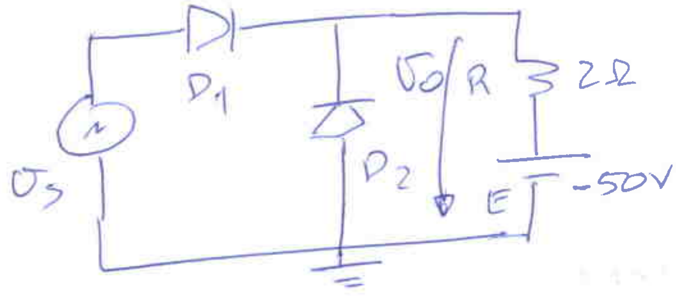
ou

$$P = \frac{1}{T}\int_0^T v_O(t)\, i_O(t)\,dt$$

$$= \frac{1}{2\pi}\int_{0,154}^{\pi-0,154} 230\sqrt{2}\,\operatorname{sen}\theta \times \left(\frac{230\sqrt{2}\,\operatorname{sen}\theta - 50}{2}\right) d\theta$$

$$= \underline{10647\ W}$$

Exercício 3



$v_s = 230\sqrt{2}\, \text{sen}(100\pi t)$

Díodos ideais

a) Calcule o valor da fase da tensão de entrada que coloca o díodo D_1 em condução

Análise do circuito

Se D_1 OFF, E polariza diretamente D_2
D_2 ON $\Rightarrow v_o = 0$

Portanto, D_1 só conduz quando $v_s > 0$

Conclusão: D_1 ON se $\underline{\theta = 0}$

b) Represente as formas de onda da tensão e da corrente na carga

$v_o = -v_{D_2}$ $\quad$ D_2 ON $\Rightarrow v_o = 0\,V$

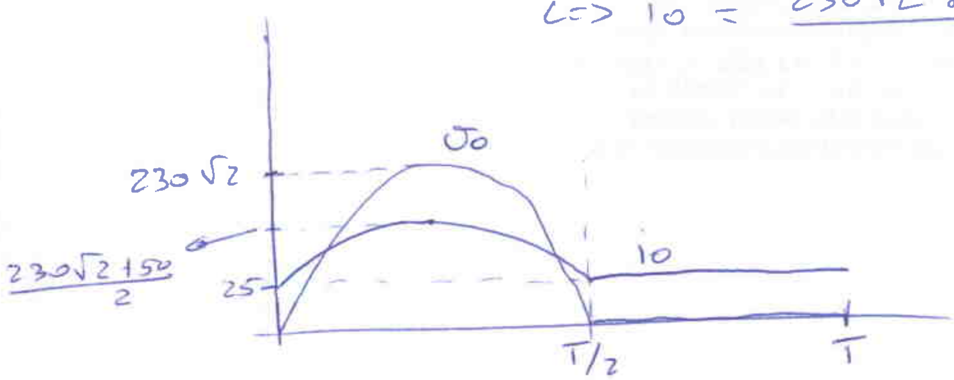
D_2 OFF $\Rightarrow v_o = v_s$

Corrente

D_2 ON $\quad 0 = R i_o - 50 \iff i_o = \frac{50}{2} = 25\,A$

D_2 OFF $\quad v_s = R i_o - 50 \iff 230\sqrt{2}\,\text{sen}(wt) = 2 i_o - 50$

$\iff i_o = \frac{230\sqrt{2}\,\text{sen}(wt) + 50}{2}$



c) Calcule os valores médio e eficaz da corrente carga

$$i_{OAV} = \frac{1}{T}\int_0^T i_o(t)\,dt = \frac{1}{2\pi}\left[\int_0^{\pi} \frac{230\sqrt{2}\,sen(\theta)+50}{2}\,d\theta + \int_{\pi}^{2\pi} 25\,d\theta\right]$$

$$= 76,8\ A$$

$$i_{ORms} = \frac{1}{T}\int_0^T i_o^2(t)\,dt$$

$$i_{ORms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi}\left(\frac{230\sqrt{2}\,sen\,\theta+50}{2}\right)^2 d\theta + \int_{\pi}^{2\pi} 25^2\,d\theta\right] = 9825,9\ A^2$$

$$i_{ORms} = 99,1\ A$$

---

d) Calcule a potência média na carga

$$P = R\, i_{ORms}^2 + E \times i_{OAV}$$

$$= 2 \times 99,1 - 50 \times 76,8 = 15802\ W$$

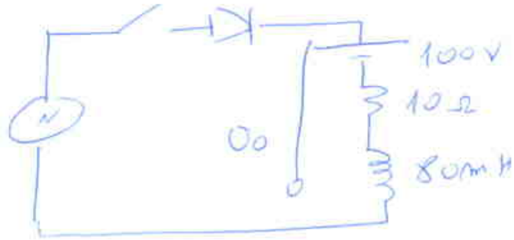
ou

$$P = \frac{1}{T}\int_0^T v_o(t)\, i_o(t)\,dt = \frac{1}{2\pi}\int_0^{\pi} 230\sqrt{2}\,sen\,\theta \times \left(\frac{230\sqrt{2}\,sen\,\theta+50}{2}\right) d\theta$$

$$= \underline{15813\ W}$$

Exercício 4

O interruptor é fechado em $t=0$ quando $v_s = 230\sqrt{2}\ \text{sen}\ \alpha$



$v_s = 230\sqrt{2}\ \text{sen}(\omega t)$

a) Determine α para o qual existe corrente no circuito com $i=0$, $v_o = 100\,V$

D conduz se $v_s > 100\,V$ $\quad 230\sqrt{2}\ \text{sen}\ \alpha > 100$

$$\Leftrightarrow \alpha = \text{arc sen}\left(\frac{100}{230\sqrt{2}}\right) =$$

$$= 17{,}9° = 0{,}312\ \text{rad}$$

b) Para $\alpha = 90°$

i) Esboce as formas de onda de $v_o(t)$ e $i(t)$

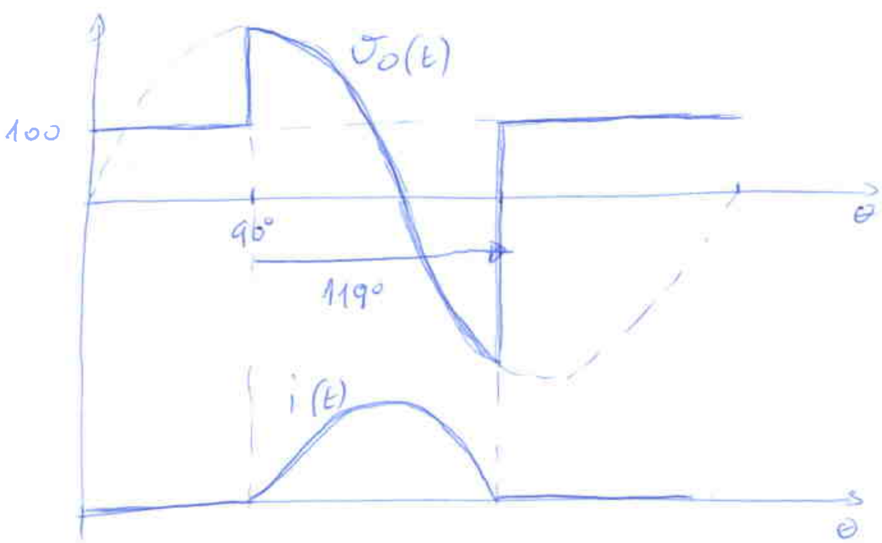
Durante quanto tempo conduz o díodo? Até $i(t)=0$

Calcular o ângulo de condução γ

$$|Z| = \sqrt{10^2 + 0{,}08^2} = 27\,\Omega \qquad \phi = \text{arctg}\left(\frac{2\pi 50 \cdot 80 \cdot 10^{-3}}{10}\right) = 1{,}19\ \text{rad}$$

$$i(\theta) = \left[\frac{100}{10} - \frac{230\sqrt{2}}{27}\ \text{sen}\left(\frac{\pi}{2} - 1{,}19\right)\right] \cdot e^{-\frac{10}{0{,}08}t} - \frac{100}{10} + \frac{230\sqrt{2}}{27}\ \text{sen}\left(\theta + \frac{\pi}{2} - 1{,}19\right)$$

$$i(\theta) = 0 \Rightarrow \theta = 2{,}08\ \text{rad} = 119°$$



b) ii) Determinar v_{oAV}, v_{orms}, i_{oAV}, i_{orms}

$$v_{oAV} = \frac{1}{2\pi}\left[\int_0^{\alpha} 100\,d\theta + \int_{\alpha}^{\alpha+\gamma} 230\sqrt{2}\sin\theta\,d\theta + \int_{\alpha+\gamma}^{2\pi} 100\,d\theta\right] = 112,0\ V$$

com $\boxed{\alpha = \frac{\pi}{2} \qquad \gamma = 2,08}$

$$v_{orms} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\alpha} 100^2\,d\theta + \int_{\alpha}^{\alpha+\gamma} (230\sqrt{2}\sin\theta)^2\,d\theta + \int_{\alpha+\gamma}^{2\pi} 100^2\,d\theta\right]} = 143,5\ V$$

$$i_{oAV} = \frac{v_{oAV} - E}{R} = \frac{112 - 100}{10} = 1,2\ A$$

ou

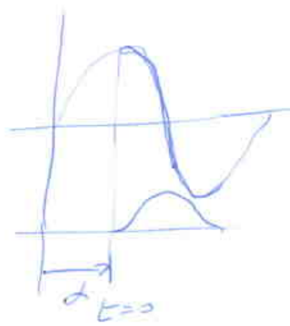
$$i_{oAV} = \frac{1}{2\pi}\int_0^{\gamma = 2,08} i(\theta)\,d\theta = 1,2\ A$$

$$i_{orms} = \sqrt{\frac{1}{2\pi}\int_0^{2,08} i(\theta)^2\,d\theta} = 2,3\ A$$

b) iii) Potência ativa na fonte

$$P = \frac{1}{T}\int_0^T v_s(t)\,i(t)\,dt = \frac{1}{2\pi}\int_0^{\gamma} 230\sqrt{2}\sin(\theta+\alpha)\,i(\theta)\,d\theta$$

$$= 173,4\ W$$



b iv) Fator de potência na fonte

$$S = V_{srms} \times i_{rms} = 230 \times 2,3 = 529\ VA$$

$$FP = \frac{P}{S} = \frac{173,4}{529} = 0,328$$