3/1/2020 EXAM 25 de Janeiro 2019

1ª parte

X - " espessura de uma chapa (mm) "

$X \sim U_n [5, 6]$

a) $\bar{X}$ - " Espessura média de 40 chapas (mm) "

$\bar{X} = \sum_{i=1}^{40} \frac{X_i}{40} \underset{\substack{TLC \\ n \geq 30}}{\sim} N\left(\mu ; \frac{\sigma^2}{n}\right)$

X_i = " espessura da i-ésima chapa (mm) "

$i = 1, \ldots, 40$

$\mu = E(X_i) = \frac{a+b}{2} = \frac{5+6}{2} = 5{,}5$

$\sigma^2 = V(X_i) = \frac{(b-a)^2}{12} = \frac{6-5}{12} = \frac{1}{12}$

$\bar{X} \sim N\left(5{,}5 ; \frac{\frac{1}{12}}{40}\right)$

R: $P(5.4 \leq \bar{X} \leq 5{,}5)$ = normcdf $(5.4, 5.5, 5{,}5, \sqrt{1/12/40})$

$= 0{,}486$

$\sigma^2 = \int_{-\infty}^{+\infty} x^2 f(x)\, dx - \mu^2$

exemplo sugerido

a$_1$) calcule a probabilidade da % de chapas com espessura superior a 5,5 (mm) numa amostra de 40 chapas exceder 52%

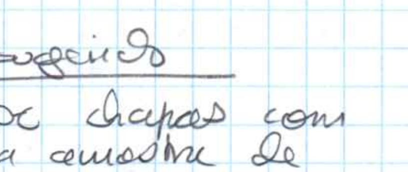
$\hat{p}$ - " proporção de chapas c/ + de 5.5 (mm), em 40 "

$\hat{p} = \frac{X}{40} \sim N\left(p ; \frac{pq}{40}\right)$ $p = p(X > 5.5)$

X - " nº de chapas com espessura > 5.5 mm, em 40 "

$X \sim Bi(40, p)$

$P(X > 5.5) = \frac{1}{2}$



$\hat{p} \sim N\left(0{,}5 ; \frac{0{,}5 \times 0{,}5}{40}\right)$

R: $P(\hat{p} > 0{,}52)$ = normcdf $\left(0{,}52, 9999, 0{,}5, \sqrt{\frac{0{,}5 \times 0{,}5}{40}}\right)$

$= 0{,}1889$

b) $\begin{cases} \bar{x}_B = 5,6 \\ s_B = 0,5 \end{cases}$ $\quad n_B = 30$

$\begin{cases} \bar{x}_C = 5.4 \\ s_C = 0,3 \end{cases}$ $\quad n_C = 30$

$H_0 : \mu_B - \mu_C = 0$

$H_1 : \mu_B - \mu_C \neq 0$

$\bar{X}_B$ - " E.M de 30 chapas B (mm)"

X " " " C " "

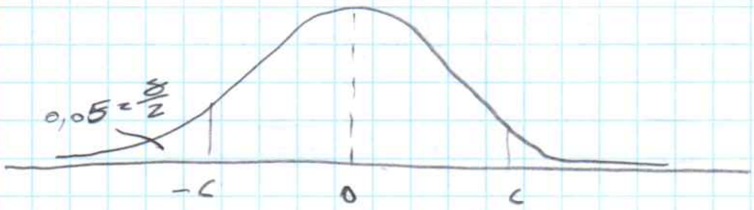
$\bar{X}_B \underset{TLC}{\sim} N\left(\mu_B ; \frac{0,5^2}{30}\right) \quad s_B = s_B$

$\bar{X}_C \underset{TLC}{\sim} N\left(\mu_C ; \frac{0,3^2}{30}\right) \quad s_C = s_C$

TLL ou AN $\Updownarrow$ Dist normal

$\bar{X}_B - \bar{X}_C \underset{AN}{\sim} N\left(\mu_B - \mu_C , \frac{0,5^2}{30} + \frac{0,3^2}{30}\right)$

$\bar{W}|_{H_0} = \bar{X}_B - \bar{X}_C|_{H_0} \sim N(0; 0,106^2)$



$P(\bar{W}|_{H_0} \leq -c) = 0.05$

$F_W(-c) = 0,05$

$-c = F_W^{-1}(0.05) = \text{Invnorm}(0.05, 0, 0.106)$

$\begin{cases} \Phi\left(\frac{-c-0}{0,104}\right) = 0.05 \\ \frac{-c}{0,104} = \Phi^{-1}(0.05) \end{cases}$ $\quad RC_W =]-\infty, -0.17] \cup [0.17, +\infty[$

Decisão

$\bar{X}_B - \bar{X}_C = 5.6 - 5.4 = 0,2 \in RC_W$

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como $\bar{X}_D - \bar{X}_C = 5.6 - 5.4 = 0.2 \in RC_w$

Rejeita-se H_0 ao nível de significância de 10 %.

logo pode afirmar-se que existe dif significativa.

"Dir ou Esq"

$\hat{p}$ - "

$$\hat{p}_0 = \frac{34}{200} = 0{,}17$$

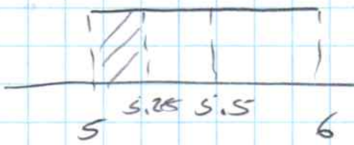
$$IC_p = [\hat{p}_0 - \Delta \; ; \; \hat{p}_0 + \Delta])$$

$$\Delta = Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_0 \hat{q}_0}{n}} = 1.645 \times \sqrt{\frac{0{,}17 \times 0.83}{200}}$$

$1 - \alpha = 0{,}9 \quad Z_{1-\frac{\alpha}{2}} = 1.645$

$$Z_{1-\frac{\alpha}{2}} = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) = \Phi^{-1}(0{,}95)$$

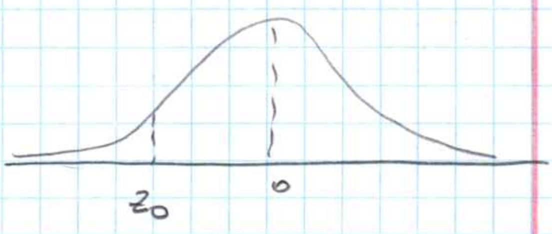
c2) $\begin{cases} H_0: p = 0{,}25 \\ H_1: p < 0{,}25 \end{cases}$



$P = P(X < 5{,}25) = 0{,}25$

$$\hat{P}_{H_0} \sim N\left(0{,}25 \; ; \; \frac{0.25 \times 0.75}{200}\right)$$

$$Z = \frac{\hat{p}_0 - 0{,}25}{0{,}0306} = \frac{0.17 - 0.25}{0.0306} = -2{,}61$$



$p_{value} = P(Z < -2{,}61)$

$= \text{normcdf}(-99999, -2{,}61, 0, 1)$

$= 0{,}0045$

R: como $p_{value} = 0.0045 < \alpha = 0.01$

Rejeita-se H_0 ao nível de 1% logo pode afirmar-se que as provocou uma diminuição na % de chapas enviadas para recalibragem.

$H_0 : \mu = 5.5$

$H_1 : \mu = ?$

$\beta = 50\%$