8.1

$\hat{p}_{D_0} = \frac{890}{9900}$ $n = 890$

$\hat{p}_{p_0} = \frac{1588}{9900}$ $n = 1588$

$\hat{p}_D$ - "v.a representa numero de diabeticos numa população de 9900"

$\hat{p}_p$ - " v.a. " " " gordos

$$\hat{p}_D = \frac{X}{n} \sim N\left(\hat{p}_p , \frac{\frac{890}{9900} \times \frac{9010}{9900}}{9900}\right)$$

$$\hat{p}_p = \frac{X}{n} \underset{\substack{TLC \\ n \geq 30}}{\sim} N\left(\hat{p}_p , \frac{\frac{1588}{9900} * \frac{8312}{9900}}{9900}\right)$$

$$IC_{D\,95\%} = [\hat{p}_D - \Delta ; \hat{p}_D + \Delta]$$

$\alpha = 0{,}05$
$$\Delta_D = Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\frac{890}{9900} \times \frac{9010}{9900}}{9900}}$$
$$= 1.96 \times \text{"}$$
$$= 0{,}01969$$

$$IC_{D95\%} = [0{,}0702 ; 0.109]$$

$$IC_{p95\%} = [0.140 ; 0.180]$$

8.2

$\begin{cases} N = 40 \\ n = 20 \end{cases}$ $\hat{p}_0 = 0.5$

$$\hat{p} = \frac{X}{N} \underset{\substack{TLC \\ n \geq 30}}{\sim} N\left(\hat{p} , \frac{\frac{n}{N} \cdot \frac{n-N}{N}}{N}\right)$$

$$IC_{95\%} = [\hat{p}_0 - \Delta ; \hat{p}_0 + \Delta]$$

$\alpha = 0.05$ $= [0.34505 ; 0.6549]$

8.3

$\begin{cases} N = 320 \\ n = 18 \end{cases}$ $\hat{p}_0 = \frac{n}{N} = \frac{18}{320}$

a) $IC_{95\%} = [\frac{n}{N} - \Delta ; \frac{n}{N} + \Delta]$

$\alpha = 0.05$ $= [0.031 ; 0.081]$

b) $IC_{98\%} = [0.02628 ; 0.08621]$

$\alpha = 0{,}02$

8.4 $\begin{cases} N = 450 \\ n = 108 \end{cases}$ $\hat{p}_{f_0} = \frac{108}{450} = 0.24$

$\hat{p}_f$ "Via proporcas de fumadores de 450"

$\hat{p}_f = \frac{X}{M} \sim N\left(\hat{p}; \frac{\frac{108}{450}\times\left(1-\frac{108}{450}\right)}{450}\right)$

a) $IC_{95\%} = [\hat{p}_{f_0} - \Delta ; \hat{p}_{f_0} + \Delta]$

$\alpha = 0.05$ $= [0.200 ; 0.279]$ $\Delta \cong 0.03946$

b) $N = ?$ $\Delta = \frac{0.039}{3}$

$$\Delta = Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\frac{108}{450}\times\left(1-\frac{108}{450}\right)}{N}}$$

$$N = \frac{1}{\left(\frac{\Delta}{Z_{1-\frac{\alpha}{2}}}\right)^2} \times \frac{108}{450}\times\left(1-\frac{108}{450}\right)$$

$= 4146.047$

$\cong 4146$ utentes

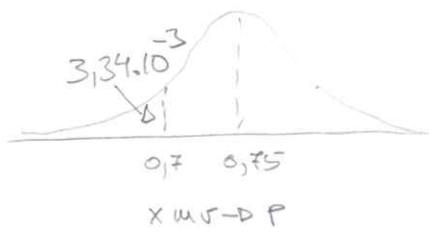
c) $IC_? = [0.70 ; 0.80]$ $N = 600$

$\Delta = \frac{0.8 - 0.7}{2}$ $p \times q = 0.1824$

$= 0.05$

$\delta^2 = 3.04 \times 10^{-4}$

$$\Delta = Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{0.1824}{600}}$$



$$Z_{1-\frac{\alpha}{2}} = \frac{\Delta}{\sqrt{\frac{0.1824}{600}}}$$

$$= \frac{0.05}{\sqrt{\frac{0.1824}{600}}}$$

$$1 - \frac{\alpha}{2} = \Phi\left(\frac{0.05}{\sqrt{\frac{0.1824}{600}}}\right)$$

$= 0.9979$

$1 - \alpha = 99,56\%$

8.5.

a) $N = ?$ $\quad A = 2,8\%$ $\quad 1-\alpha = 95\%$

$0.028 = 2\Delta$

$\Delta = 0.014$

$\hat{p}$ - "a proporção de respostas de "marca nova" ... "

$\hat{p} = \frac{X}{n} \sim N(\hat{p}, \delta^2)$

$IC_{1-\alpha} = [\bar{x}_0 - \Delta ; \bar{x}_0 + \Delta]$

$\Delta = Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{p \times q}{N}}$

$0.014 = 1.96$

$p = ?$

$se = 0.5 \quad n = 4900$

$se = \frac{123}{860} \quad n \cong 2402,33$

b) $\begin{cases} \hat{p}_0 = \frac{123}{860} = 0.143 \\ 1-\alpha = 95\% \end{cases}$

$p \times q = 0.12256$

$Z_{1-\frac{\alpha}{2}} \cong 1,96$

$\delta \cong 0.0119382$

$\Delta \cong 0.0234$

$IC_{95\%} = [0.1196 ; 0.1664]$

c) $\begin{cases} n = 30 \\ \bar{x}_0 = 143,5 \quad \delta^2 = 900 \end{cases}$

$\bar{X} = \frac{1}{30} \sum_{i=1}^{30} x_i \sim N(\mu, \frac{900}{30})$

$IC_{96\%} = [132,25 ; 154.74]$

$\alpha = 0.04$

$Z_{1-\frac{\alpha}{2}} = 2.05$

8.6.

$\begin{cases} n_p = 600 \\ 350 \end{cases}$ $\begin{cases} n_L = 800 \\ 280 \end{cases}$

$\hat{p}_{p_0} = \frac{350}{600} \cong 0.583$ $\hat{p}_L = \frac{280}{800} = 0.35$

a)

$\hat{p}_p = \frac{X}{n} \sim N\left(\hat{p}_p ; \frac{\frac{350}{600} \times \left(1 - \frac{350}{600}\right)}{600}\right)$ $\cong 4.051 \times 10^{-4}$

$\hat{p}_L = \frac{X}{n} \sim N\left(\hat{p}_L ; \frac{\frac{280}{800} \times \left(1 - \frac{280}{800}\right)}{800}\right)$ $\cong 2.84 \times 10^{-4}$

$\hat{p}_p - \hat{p}_L \sim N\left(\hat{p}_{pL} ; 4.05 \times 10^{-4} + 2.84 \times 10^{-4}\right)$

$\hat{p}_p - \hat{p}_L = 0.233$

$IC_{95\%, p_{dif}} = [0.181, 0.2844]$

$\alpha = 0.05$

$\Delta = 0.05144$

b) Sim o índice de confiança dem os sinais iguais não engo o ponto ϕ.

8.7. X_i - "durabilidade vedantes em 1000×horas"

$\left\{ n = 150 \right.$

$$\bar{X} = \frac{1}{150} \sum_{i=1}^{150} x_i$$

$$= \frac{1}{150} \left(1.5 \times 29 + 2.5 \times 43 + 3.5 \times 57 + 4.5 \times 21 \right)$$

$$\simeq 2.97$$

$$s^2 = \frac{1}{150} \sum_{i=1}^{4} (x_i - \bar{x})^2 \times n_i$$

$$\simeq 0{,}96^2$$

a) $$P(\bar{X} > 4) = 1 - P(\bar{X} < 4)$$

$$= 1 - \Phi\left(\frac{4 - 2.97}{0.96}\right)$$

$$= 0.1416$$

14,16% duram pelo menos 4000 horas

$\hat{p}$ - "proporção na qual duram mais que 4000 horas"

$$\hat{p} = \frac{X}{n} \sim N\left(\hat{p}, \frac{0.1416 \times (1 - 0.1416)}{150}\right) \quad \simeq 8.1 \cdot 10^{-4}$$

$$IC_{95\%} = [0.1416 - \Delta ; 0.1416 + \Delta]$$

$\Delta = 0.0557$ $$= [0.086 ; 0.1973]$$

b) $n = ?$ Δ_A $P(X < 2) = 0.15614$

$0.05 \times 0.15614 = 7.8 \cdot 10^{-3}$ $$\hat{p} = \frac{X}{n} \sim N\left(\mu, \frac{0.15614 \times (1 - 0.15614)}{n}\right) \quad z = 1.0104$$

$$\Delta = Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{0.15614 \times (1 - 0.15614)}{n}}$$

$$7.8 \times 10^{-3} = 1.96 \qquad n = 8319$$

TP8.

1. $N_M = 9900$ $\quad n_D = 890$
$n_P = 1588$

$IC_{D\,95\%} = ?$
$IC_{P\,95\%} = ?$

$\hat{p}_D$ - "proporção de pessoas com diabetes"

$\hat{p}_P$ - "proporção de pessoas com excesso de peso"

$$\hat{p}_{D,P} = \frac{X}{n} \sim N\left(p_{D,P}, \frac{p_{D,P}\,q_{D,P}}{n}\right)$$

$$\hat{p}_D = \frac{890}{9900} = 0{,}08989$$

$$z_{1-\frac{\alpha}{2}} = 1.96$$

$$\hat{p}_P = \frac{1588}{9900} = 0{,}1604$$

$$\sigma_D = \sqrt{\frac{\frac{890}{9900} \times \left(1 - \frac{890}{9900}\right)}{9900}} \simeq 0.00287478$$

$$\Delta = 1{,}96 \times 0{,}00287478 = 0{,}00563$$

$$IC_{D\,95\%} = [\hat{p}_D - \Delta, \hat{p}_D + \Delta] = [0.0842, 0.0955]$$

$$IC_{P\,95\%} = [0.153, 0.1676]$$

2. $\hat{p}$ - "proporção de coroas em 40 lançamentos"
$\hat{p}_0 = \frac{1}{2}$

$$IC_{p\,95\%} = [\hat{p}_0 - \Delta, \hat{p}_0 + \Delta]$$

$$\Delta \simeq 1.96 \cdot \sqrt{\frac{0.5 \cdot 0.5}{40}} = 0{,}15494$$

$$IC_{p\,95\%} = [0.345, 0.6549]$$

work directly with formula sheet.

3. $N = 320$ $n = 18$

$\hat{p}$ - " Artigos defeituosos "

$\hat{p} = \frac{18}{320} = 0.05625$

$\hat{P}_{320} \sim N(p; \frac{pq}{320})$

a) $IC_{p\,95} = [\hat{p} - \Delta, \hat{p} + \Delta]$

$Z_{1-\frac{\alpha}{2}} \cong 1.96$ $S = \sqrt{\frac{0.05625 \times (1-0.5625)}{320}}$

$\Delta = Z_{1-\frac{\alpha}{2}} \times S$ $\cong 0.01288$

$= 1.96 \times 0.01288$

$= 0.02524$

$IC_{p\,95\%} = [0.05625 - 0.02524, 0.05625 + 0.02524]$

$= [0.03101, 0.08149]$

b) $IC_{p\,98\%} = [0.02628, 0.08621]$

$Z_{1-\frac{\alpha}{2}} \cong 2.33$

$\Delta \cong 0.0299\cancel{7}$

4. $N = 450$ $n_f = 108$

$\hat{p}$ - " proporcão de fumadores de 450 "

$\hat{p} = 0.24$ $\alpha = 0.05$

$Z_{1-\frac{\alpha}{2}} \cong 1.96$ $S \cong 0.020133$

$\Delta \cong 0.03946$

a) $IC_{p\,95\%} = [0.2, 0.28]$

b) 2Δ = Amplitude

0.07892 = Amplitude

$\frac{\text{Amplitude}}{3} \cong 0.02631$

$\Delta = \frac{0.02631}{2}$

$\cong 0.1315$

TP8

4. b) $\Delta = 0.01315 = 1.96 \cdot \sqrt{\frac{0.1824}{n}}$

$\hat{p} \times \hat{q} = 0.1824$
$\hat{p} = 0.24$ show all steps

$$n = \frac{0.1824}{\left(\frac{0.01315}{1.96}\right)^2}$$
$$= 4052$$

c) $IC_? \; [0.70 , 0.80] = $ $A = 0.1$ $\Delta = 0.05$
$n = 600$

$\hat{p}_1$ - " proporção de não fumadores de 450 inquiridos"

$\hat{p}_1 = 0,76$ $\Delta = ?$

$$s = \sqrt{\frac{0.1824}{600}}$$
$$\simeq 0.017436$$

$\Delta = z_{1-\frac{\alpha}{2}} \times 0.017436$

$0.05 = z_{1-\frac{\alpha}{2}} \times 0.017436$

$z_{1-\frac{\alpha}{2}} = \frac{0.05}{0.017436} = 2,8676$

$\therefore$ NormCD $(-2,8676, 2,8676) = 99,57\%$

TP8

5. $\hat{p}$ - "v.a representa a proporção de refrigerantes vendidos num café"

$$\hat{p} = \frac{X}{n} \sim N\left(p; \frac{pq}{n}\right)$$

a) $n = ?$ $A = 2{,}8\%$ $IC_{95\%}$

$$\Delta = \frac{0.028}{2} = 0.014$$

$$z_{1-\frac{\alpha}{2}} \cong 1.96$$

$$0.014 = 1{,}96 \times \sqrt{\frac{\frac{123}{860} \times \left(1-\frac{123}{860}\right)}{n}}$$

Não é dado!

$$n = \frac{0.12257}{\left(\frac{0.014}{1.96}\right)^2} \cong 2402$$

$\boxed{p = \frac{1}{2}}$

$IC_{p\,95\%}$ $[\hat{p} - 0.014 , \hat{p} + 0.014)$

b) ICp calculadora

$\hat{p} = 0{,}1430$

$\hat{p} . \hat{q} \cong 0{,}1225$

$\alpha = 0.05$

$z_{1-\frac{\alpha}{2}} \cong 1{,}9599$ $\sigma \cong 0.01194$

$\Delta \cong 0.02339$

$IC_{95\%}$ $[0.119624, 0.166421]$

c)

X_i - "v.a representa total de vendas diárias"

$\bar{x}_0 = 143{,}5$

$\sigma^2 \cong s^2 = 900$

$$\bar{X} = \frac{1}{30} \sum_{i=1}^{30} X_i \sim N(\mu, 900)$$

$IC_{\bar{X}\,96\%} = [132{,}25 , 154.748]$

$\alpha = 0.04$ $z_c = 2.053$ $\Delta = 11{,}24$

TP8

6. $n_{porto} = 600$ $n_{Lisboa} = 800$

$\hat{P}_p$ - " v.a representa proporcão de pessoas que tomam medicamentos"

P_L - " " "

$P_p = \frac{350}{600} = 0.5833$ $P_p \times q_p = \frac{35}{144}$

$P_L = 0.35$ $P_L \times q_L = \frac{91}{400}$

a) $IC_{P_p - P_L} = [\hat{P}_p - \hat{P}_2 - \Delta , \hat{P}_p - \hat{P}_L + \Delta]$

$Z_{1-\frac{\alpha}{2}} \approx 1.96$

$$\hat{P}_p - \hat{P}_p \underset{\substack{AN \\ n>>30}}{\sim} N\left(0.233 , \frac{\frac{35}{144}}{600} + \frac{\frac{91}{400}}{800}\right)$$

$$\sim N(0.233 ; 0.0004051)$$

$\Delta = 0.03944$

$IC_{P_p - P_L} = [0.1935 , 0.2724]$

b) sim pois os limits do IC têm o mesmo sinal.

☆ os professores são para acelerar o processo de aprendizagem, senão ninguem ia estudar, todos ser vivos aprendem faz parte de sua naturega.

TP 8

7. $\bar{x} = ?$
$s = ?$ $\quad \sigma = s$

a) $P(X > 4000) = \hat{p}$ - "v.a proporção de durações de velantes que duram pelo menos 4000 horas"

X_i - "v.a durações velantes"

$\bar{x} \cong 2.97$
$s \cong 0.96 \cong \sigma$ $\quad n \geq 30$ TLC

$H_0: p = p_0$
$H_1: p \neq p_0$

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{150} X_i \left(\mu, \frac{0.96^2}{150}\right)$$

$P(X \geq 4) = \text{NormCD}(4, 9999, 0.96, 2,97)$
$= 0,141654$

$p_0 \cong 0,141654 = \frac{x}{150}$ $\quad pq \cong 0.121588$

$\alpha = 0.05$ $\quad Z_{1-\frac{\alpha}{2}} \cong 1,9599$ $\quad \sigma = 0.028470$

$\Delta \cong 0.0558$

$IC_p = [\, 0,08585,\ 0,1974 \,]$

b)

Estatística 29/11/2019

TP 20845

1. Estimação da Proporção populacional TP8

$IC(p) = [A, B]$

$(1-\alpha) \times 100\%$

$n = 9900$
nº de diabetes $= 890$
nº de excesso de peso $= 1588$

$IC(pd) = [A, B] = ?$
95%

Pd : proporção de pessoas com diabetes em toda a população, do sexo masculino

Amostra
V.A principal

$\hat{P}_{obs} = \frac{890}{9900} = 0{,}0898989$

$\hat{P}_{9900}$: v.a que representa a proporção de pessoas com diabetes quando considerada uma amostra aleatória de 9900 pessoas, do sexo masculino.

$0{,}081817$

$\hat{P}_{9900} \sim N\left(pd ; \frac{pd(1-pd)}{9900}\right)$

$n >> 30$

Nível de confiança: $1-\alpha$

$1-\alpha = 0{,}95$
$\alpha = 0{,}05$

$Z_{crítico}$: $Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0{,}05}{2}}$
$= Z_{0{,}975} = \Phi^{-1}(0{,}975)$
$\approx 1{,}96$

Erro do Intervalo: Δ

$\Delta = Z_{1-\frac{\alpha}{2}} \sqrt{\frac{pd(1-pd)}{9900}}$

c.a) $\frac{pd(1-pd)}{9900} \approx \frac{\hat{P}_{obs}(1-\hat{P}_{obs})}{9900}$
$= \frac{\frac{890}{9900} \times \left(1 - \frac{890}{9900}\right)}{9900}$

$= \frac{\frac{890}{9900} \times \left(1 - \frac{890}{9900}\right)}{9900}$

$$\Delta = 1{,}96 \times \sqrt{\frac{\frac{890}{9900} \times \left(1 - \frac{890}{9900}\right)}{9900}}$$

Depois da amostragem

$$I.C\,(P_d) = [\hat{P}d_{obs} - \Delta\ ;\ \hat{P}d_{obs} + \Delta]$$

$$= \frac{890}{9900} - 1{,}96 \times \sqrt{\frac{\frac{890}{9900} \times \left(1 - \frac{890}{9900}\right)}{9900}}$$

$$= \frac{890}{9900} + 1{,}96 \sqrt{\frac{\frac{890}{9900} \times \left(1 - \frac{890}{9900}\right)}{9900}}$$

$$IC_{95\%}\,(P_d) = [0{,}084\ ,\ 0{,}096]$$

trocar tudo Pd para Pe

890 para 1588

e da:

$$IC_{95\%}\,(P_e) = [0{,}153\ ,\ 0{,}167]$$

$\boxed{\text{Binomial para Normal. grandes amostras.}}$

2. $n = 40$
nº de caras

$$IC_{95\%}\,(P_c) = [A\ ,\ B] = ?$$

P_c: proporcão de caras

V.A principal

$\hat{P}c_{40}$ proporcão de caras num lancamento de 40 moedas.

$\hat{P}_{c_{40}} \sim N\left(P_c ; \frac{P_c(1-P_c)}{40}\right)$

$n = 40 >> 30$

Nível de confiança: $1-\alpha$

$\alpha = 0,05$

Z crítico: $Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0,05}{2}}$

$= Z_{0,975} = \Phi^{-1}(0,975)$

$= 1,96$

Erro do Intervalo: Δ

$\Delta = Z_{1-\frac{\alpha}{2}} \sqrt{\frac{P_c(1-P_c)}{40}}$

$= \frac{\frac{20}{40} \times \left(1 - \frac{20}{40}\right)}{40}$

$\Delta = 0,1549$

Depois da amostragem

$IC(P_c) = [\hat{P}_{c_{obs}} - \Delta , \hat{P}_{c_{obs}} + \Delta]$

$= \left[\frac{20}{40} - 0,1549 , \frac{20}{40} + 0,1549\right]$

$[0,345 , 0,65495]$

3) Amostra

$n = 320$

a) $\hat{P}_{obs} = \frac{18}{320}$ $\quad Z_{1-\frac{\alpha}{2}} = Z_{0,975} = 1,96$

~~igual~~ anterior

$\Delta = 1,96 \times \sqrt{\frac{\frac{18}{320} \times 1 - \frac{18}{320}}{9900}}$

$IC = \frac{18}{320} - 1,96 \sqrt{\frac{\frac{18}{320} \times \left(1 - \frac{18}{320}\right)}{320}}$; " "

$IC\ P_c\ [0,031 , 0,081]$

b) $P_{obs} = \frac{18}{320}$ $\quad \Delta =$ $\quad$ same but with $Z_{1-\frac{\alpha}{2}} = 2,33$

IC

4.

a) $\Sigma n_i = 450$
nº de fumadores

$\hat{p}_{obs} = \frac{108}{450}$ $\hat{p}_{ob}$ - proporção de fumadores numa população de 450

$z_{1-\frac{\alpha}{2}} = 1,96$

$IC_{95\%}(p_f) = [A, B]$

$$\Delta = 1.96 \times \sqrt{\frac{\frac{108}{450} \times \left(1 - \frac{108}{450}\right)}{450}}$$

$$= 0,03946$$

$IC(p_f) = [\ 0,24 - 0,03946\ ,\ 0,24 + 0,03946\ ]$
$= [\ 0,2005\ ,\ 0,2794\ ]$

b) Amplitude: $0,279 - 0,201 = 0,078$.

$\frac{1}{3}$ AMPLITUDE: $\frac{0,078}{3} = 0,026$

$$2 \times z_{1-\frac{\alpha}{2}} \sqrt{\frac{pq}{n}} = 0,026$$

$$z_{1-\frac{\alpha}{2}} \sqrt{\frac{pq}{n}} = 0,013$$

$$1,96 \times \sqrt{\frac{\frac{108}{450} \times \left(1 - \frac{108}{450}\right)}{n}} = 0,013$$

$$n = \frac{1,96 \times \sqrt{\frac{108}{450} \times \left(1 - \frac{108}{450}\right)}}{0,013} \cong 4146$$

b) $\hat{p}_{obs\ n.f} = \frac{x}{600}$

$IC_{?}(p_{nf}) = [\ 0,70 ; 0,80\ ]$

$[\ \hat{p}_{obs} - \Delta\ ,\ \hat{p}_{obs} + \Delta\ ]$ $\bar{x} = 0,75$

$2\Delta = 0,1 \Leftrightarrow \Delta = 0,05$ $\Delta = 0,05$

$\hat{p}_{obs} = 0,7 + 0,05 = 0,75$

Estadística

29/11/2019

22:00

TP ___ / L ___

$$Z_{1-\frac{\alpha}{2}} \sqrt{\frac{0,75 \times 0,25}{600}} = 0,05$$

$$Z_{1-\frac{\alpha}{2}} = \frac{0,05}{\sqrt{\frac{0,75 \times 0,25}{600}}}$$

$$1-\frac{\alpha}{2} = \Phi\left(\frac{0,05}{\sqrt{\frac{0,75 \times 0,25}{600}}}\right)$$

$$\alpha = 2 \times \left[1 - \Phi\left(\underbrace{\frac{0,05}{\sqrt{\frac{0,75 \times 0,25}{600}}}}_{}\right) \right] \quad \boxed{2.828}$$

$$= 2 \times (1 - 0,99765)$$

$$= 0,0047$$

$$1-\alpha = 1 - 0,0047$$
$$= 0,9953$$