

Distribuição Exponencial

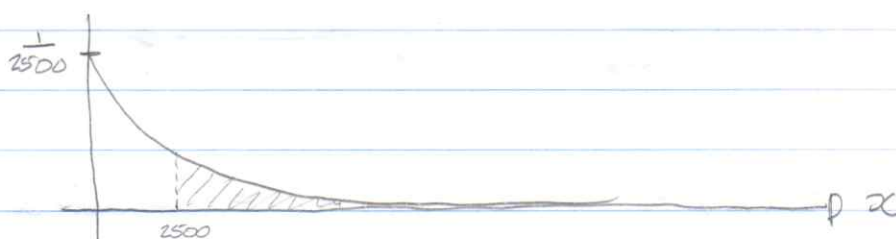
O tempo de duração de um componente electrónico A tem distribuição exponencial de média 2500 horas.

a) X - "tempo de duração de um componente electrónico (horas)"

$$X \sim E_x(\lambda)$$

$$E(X) = 2500 = \frac{1}{\lambda} \quad \text{zp (domulcerio)}$$

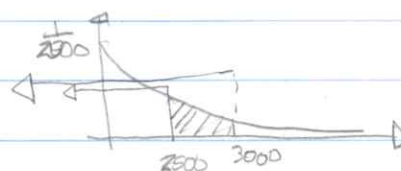
$$\lambda = \frac{1}{2500}$$



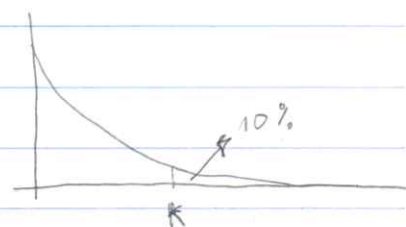
$$\begin{aligned} \text{a)} \quad P(X > 2500) &= 1 - P(X \leq 2500) \\ &= 1 - F(2500) \\ &= 1 - (1 - e^{-\frac{1}{2500} \times 2500}) \\ &= e^{-1} \approx 0,3679 \end{aligned}$$

$$\text{b)} \quad P(2500 < X < 3000)$$

$$\begin{aligned} &= P(X < 3000) - P(X \leq 2500) \\ &= F(3000) - F(2500) \\ &= 1 - e^{-\frac{3000}{2500}} - (1 - e^{-\frac{2500}{2500}}) \\ &= e^{-1} - e^{-\frac{30}{25}} \approx 0,0667 \end{aligned}$$



$$\begin{aligned} \text{c)} \quad \text{Det } k : P(X > k) &= 0,1 \\ 1 - P(X \leq k) &= 0,1 \\ P(X \leq k) &= 0,9 \end{aligned}$$

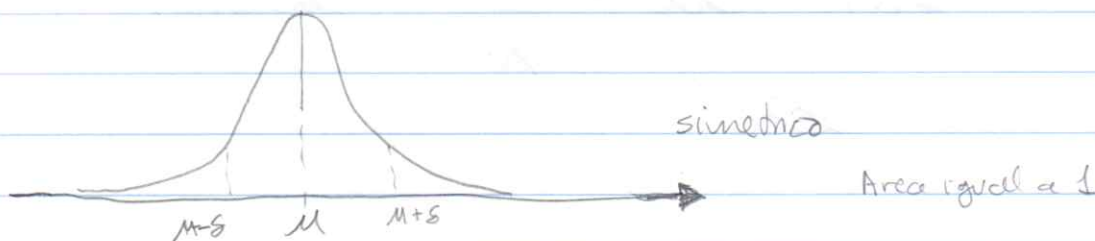


$$\begin{aligned}
 F(k) &= 0,9 \\
 1 - e^{-\frac{k}{2500}} &= 0,9 \\
 e^{-\frac{k}{2500}} &= 0,1 \\
 -\frac{k}{2500} &= \ln(0,1) \Rightarrow k = -2500 \ln(0,1) \\
 &= 5756 \text{ horas.}
 \end{aligned}$$

-6/-

Distribuição Normal é mais importante.

$$X \sim N(\mu, \sigma^2) \Leftrightarrow f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2}, x \in \mathbb{R}$$



$$E(X) = \int_{-\infty}^{+\infty} x \cdot f(x) dx = \text{conduz-se} = \mu.$$

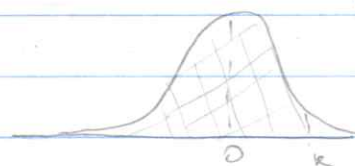
$$V(X) = \int_{-\infty}^{+\infty} (x-\mu)^2 \cdot f(x) dx = \dots = \sigma^2$$

Distribución Normal Reducida

$$Z \sim N(0, 1)$$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \quad z \in \mathbb{R}$$

$$Z \sim N(0, 1)$$

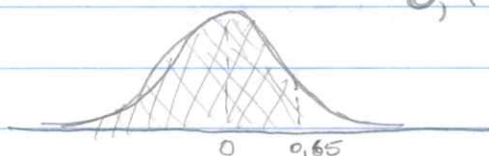


$$P(Z < k) = \int_{-\infty}^k f(z) dz$$

$$= \int_{-\infty}^k \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \approx \Phi(k)$$

↓
valores obtenidos
numéricamente

a) $P(Z < 0,65) = \Phi(0,65)$
 $= 0,7422$



casio
dx-st05p

BC Normal
Inf -99999
Sup 0,65
δ 1
μ 0

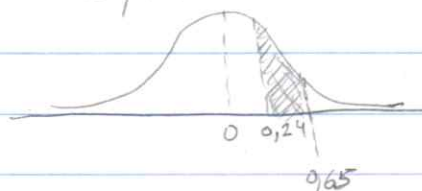
b)

$$P(Z > 0,24) = 1 - P(Z \leq 0,24)$$

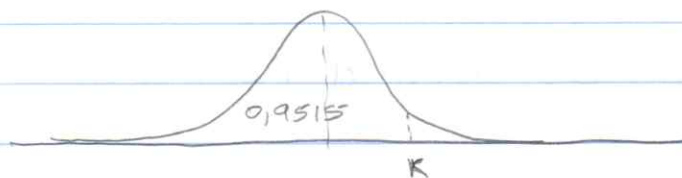
 $= 1 - \Phi(0,24)$
 $= 1 - 0,594$

c) $P(0,24 < Z < 0,65) = \Phi(0,65) - \Phi(0,24)$
 $= 0,1473$

Inf 0,24
Sup 0,65
δ 1
μ 0



d) def $K: P(Z < K) = 0,9515$



$$\Phi^{-1}(\Phi(K)) = \Phi^{-1}(0,9515)$$

$$K = \Phi^{-1}(0,9515)$$

$$= 1,6595$$

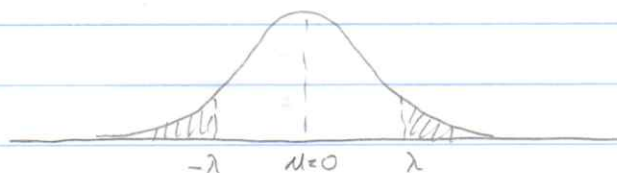
Normal inverso

caso

$$\left\{ \begin{array}{l} \text{Area : } 0,9515 \\ \sigma : 1 \\ \mu : 0 \end{array} \right.$$

casos previous explorados.

$$P(Z < -\lambda) = P(Z > \lambda)$$



$$\Phi(-\lambda) = 1 - P(Z \leq \lambda)$$

$$\Phi(-\lambda) = 1 - \Phi(\lambda)$$

ex: $\Phi(-1,31) = 1 - \Phi(1,31)$