Estatística

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Prova modelo 1

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1.

$$\begin{cases} A+B=1 \\ A=3B \end{cases} \quad \begin{cases} 3B+B=1 \\ A=\frac{3}{4} \end{cases} = B=\frac{1}{4}$$

X_A - "v.a tempo de espera para iniciar o carregamento" [μ.t]

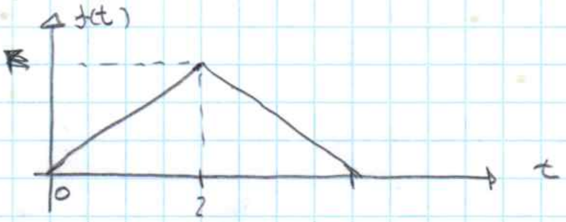
$X_A \sim E_x(\lambda)$

$\mu = E(X) = \frac{1}{\lambda}$ $\delta^2 = V(X) = \frac{1}{\lambda^2}$ $\delta = \mu$

$\mu = 5 \Leftrightarrow \lambda = \frac{1}{5}$

$X_A \sim E_x(\frac{1}{5})$ $\delta = 5$, $\mu = 5$; $\delta^2 = 25$

X_B - "v.a tempo de espera para iniciar o carregamento" [μ.t]



a) $P(A) = \frac{3}{4}$ $P(B) = \frac{1}{4}$ distribuição discreta

0	0	0	
0	0	1	-
0	1	0	-
0	1	1	
1	0	0	-
1	0	1	
1	1	0	
1	1	1	

$2^3 = 8$

$\frac{3}{8} = 0,375$

Distribuição Binomial

$P(B) = \frac{1}{4}$

X - "ser carregador B" $X \sim B_i(3, \frac{1}{4})$ $\mu = \frac{9}{4}$ $\delta = 0,75$

i) Bernoulli
ii) independente
iii) probabilidade constante

$P(X=1) = $ BinomialPD$(2, 3, \frac{1}{4})$

$= 0.4218$

?

b) i.a) $f(t) = \frac{K}{2}t$

$f(t) = \frac{0-K}{4-2}t - 4$

$4 \times K = 2$

$K = \frac{1}{2}$

$$f(t) \begin{cases} 0 & x < 0 \\ \frac{K}{2}t & 0 < x < 2 \\ -\frac{K}{2}t + 1 & 2 < x < 4 \end{cases}$$

$f(x)$ e f.d.p sse $\begin{cases} f(x) \geq 0 & x \in \mathbb{R} \\ \int_{-\infty}^{+\infty} f(x)\,dx = 1 \end{cases}$

$$f(x) \begin{cases} 0 & , x < 0 \\ \frac{1}{2}kt & , 0 < t < 2 \\ -\frac{1}{2}kt+1 & , 2 < t < 4 \\ 0 & , \text{o.r} \end{cases}$$

$P_{0}(0, 0)$
$P_{1}(2, k)$

$P_{0}(2, k)$
$P_{1}(4, 0)$

$$\int_0^2 \frac{1}{2}kt\,dt + \int_2^4 -\frac{1}{2}kt + 1\,dt = 1$$

$$\frac{1}{2}k \left|_0^2 \frac{t^2}{2} - \frac{1}{2}k \right|_2^4 \frac{t^2}{2} + 1 \left|_2^4 t \right. = 1$$

$$\frac{1}{2}k\left(\frac{2^2}{2} - \frac{0^2}{2}\right) - \frac{1}{2}k\left(\frac{4^2}{2} - \frac{2^2}{2}\right) + 1(4-2) = 1$$

$$\frac{1}{2}k(2) - \frac{1}{2}k(6) + 1.(2) = 1$$

$$k - 3k + 2 = 1$$

$$-2k + 2 = 1$$

$$k = \frac{-1}{-2} = \frac{1}{2}$$

c.a pela Area:

$$\frac{4 \times k}{2} = 1$$

$$4 \times k = 2$$

$$k = \frac{1}{2}$$

$$f(x) \begin{cases} 0 & , x < 0 \\ \frac{1}{4}t & , 0 < t < 2 \\ -\frac{1}{4}t + 1 & , 2 < t < 4 \\ 0 & , x > 4 \end{cases}$$

$$P(3{,}5 < t < 1) = \int_0^1 \frac{1}{4}t\,dt + \int_{3,5}^4 -\frac{1}{4}t + 1\,dt$$

$$= \frac{1}{4}\left[\frac{t^2}{2}\right]_0^1 - \frac{1}{4}\left[\frac{t^2}{2}\right]_{3,5}^4 + \left[t\right]_{3,5}^4$$

$$= \frac{1}{4}\left(\frac{1^2}{2} - \frac{0}{2}\right) - \frac{1}{4}\left(\frac{4^2}{2} - \frac{3{,}5^2}{2}\right) + (4 - 3{,}5)$$

$$= \frac{5}{32}$$

$\frac{1}{2}$ — h_1 $\qquad$ $\frac{1}{2}$ — h_2

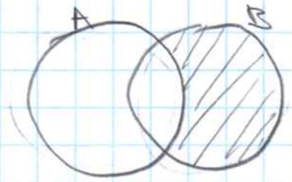
2 $\quad$ 1 $\qquad$ 2 $\quad$ $0{,}5$

$h_1 = \frac{1}{4}$ $\qquad$ $h_2 = \frac{1}{8}$

1.
c) $X_A \sim Ex\left(\frac{1}{5}\right)$ $\quad \delta = 5,\ \mu = 5,\ \delta^2 = 25$

$$P(X_A < 2) = F(2) = 1 - e^{-\frac{1}{5}\cdot 2} = 0.32967$$

$$P(X_B < 2) = \int_0^2 \frac{1}{4}\, t \, dt = \frac{1}{4}\left[\frac{t^2}{2}\right]_0^2 = \frac{1}{2}$$



experiencias <u>independentes</u>

$$P(B \cap \bar{A}) = P(A \cup B) - P(A)$$

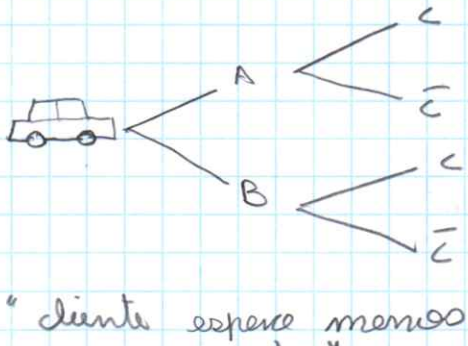
$$\Downarrow$$

$$P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}_{P(A) \times P(B)}$$

$$= 0.32967 + \frac{1}{2} - 0.32967 \times 0.5$$

$$= 0.664835$$

$$= P(A \cup B) - P(A)$$
$$= 0.335165$$



C: "cliente espere menos de 2 u.t para iniciar o carregamento."

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{P(C|B)\,P(B)}{P(C \cap A) + P(C \cap B)}$$

$$= \frac{0.5 \times 0.25}{0{,}32968 \times 0.75 + 0.5 \times 0.25}$$

2. X :- " duração das baterias do tipo 1" [anos]

$X \sim N(\mu, \delta^2) \quad \delta = 1$

$P(X > 4.3) = 0.5 \Rightarrow$ Media = 4.3 anos
$\Rightarrow P(X < 4.3) = 0.5$

$X \sim N(4.3, 1^2)$

a) $\sum_{i=1}^{3} X_B \sim N(12.9, \sqrt{3}^2)$ pelo teorema da aditividade de Dist Normal.

$P(X_B < 12) = 0,30166$

b)

$P(Y > 7\cancel{4} \mid P($utilizador atribuir 2 estrelas$)$

E :- " Numero estrelas atribuidas por um utilizador "

$E = e_i$	0	1	2
$P(E = e_i)$	0,0107	0,0861	0,9032

$P(X=0) = P(X<2) = $ normCD(2 , 1, 4.3)

$P(2 < X < 3) = $ normCD(2, 3, 1, 4.3)

$P(X > 3) = 1 - $ normCD(-9999, 4, 1, 4.3)

$E(E) = 1.89$

$V(E) = 0.12$

$\sum_{i=1}^{40} E_i \rightarrow$ v.a que representa o nº de estrelas total atribuido por 39 utilizadores.

E_i - nº estrelas atribuidas pelo utilizador i, $i = 1 \ldots 40$

$\sum_{i=1}^{40} E_i = \sum_{i=1}^{39} E_i + 2$ como $n = 39 \geq 30$ e Eis

v.a independentes podemos aplicar o teorema do Limite central

$$\sum_{i=1}^{39} E_i \underset{\text{aprox}}{\sim} N(39 \times 1.89 ; 39 \times 0.13)$$

Probabilidade pedida

$$P\left(\sum_{i=1}^{39} E_i > 74\right) \cong \text{normCD}(74, +\infty, 39 \times 1.89, \sqrt{39 \times 0.13})$$

$$P\left(\sum_{i=1}^{39} E_i > 74.5\right) =$$