7.1

$\delta = 0,6$ gramas

X_i "v.a peso dos pacotinhos açúcar em gramas"

$P(X > 8) = 0,5$

$P(X < 6,7)$ percentagem sacos mal cheios.

$P(X > 8) = 0,5$; $\delta = 0,6$

$P(X < 8) = 0,5$

$\Phi\left(\frac{8-\mu}{0,6}\right) = 0,5$

$\frac{8-\mu}{0,6} = \Phi^{-1}(0,5)$

$= 0$

$\frac{8-\mu}{0,6} = 0 \Leftrightarrow 8-\mu = 0$

$\mu = 8$

$X_i \sim (8, 0,6^2)$

a) $P(X < 6,7) = \Phi\left(\frac{6.7-8}{0,6}\right)$

$= 0,01513$

1,513% pacotinhos mal cheios

b) 80 sacos → 648 gramas

$\begin{cases} n = 80 \\ \bar{x}_0 = \frac{648}{80} \end{cases}$ $\bar{X} = \frac{1}{80}\sum_{i=1}^{80} X_i \sim \left(\mu, \frac{0,6^2}{80}\right)$

AN

$n \geq 30$ $\mu = ?$

$IC_{98\%} = [8.1 - \Delta, 8.1 + \Delta]$

$\alpha = 0.02$ $\Delta = \Phi^{-1}\left(1 - \frac{0.02}{2}\right) \times \frac{0.6}{\sqrt{80}}$

$= \text{InvNormRD}\left(1 - \frac{0.02}{2}\right) \times \frac{0,6}{\sqrt{80}}$

$\simeq 0,156$

$IC_{98\%} = [7.944 , 8.256]$

c) Não se deve rejeitar a média anterior pois está dentro do intervalo de confiança: $7.944 < 8 < 8.256$

7.2 $n_A = 100$

X_{Ai} - "v.a duração de uma lâmpada em u.t"

a)

$$\overline{X}_{A_0} = \frac{1}{100} \sum_{i=1}^{100} x_i$$

$$= \frac{1}{100} \sum_{i=1}^{4} x_i \cdot n_i$$

$$= \frac{1}{100} \times (50 \times 14 + 35 \times 150 + 45 \times 350 + 6 \times 450) = 244$$

$$s_A^2 = \frac{1}{100-1} \sum_{i=1}^{4} (x_i - \overline{x})^2 \times n_i$$

$$= \frac{1}{100-1} \left((50-244)^2 \cdot 14 + (150-244)^2 \cdot 35 + (350-244)^2 \cdot 45 + (450-244)^2 \cdot 6 \right)$$

$$= 16125$$

$$s_A = \sqrt{16125} \simeq 126{,}985$$

$$\therefore \overline{X}_{A_{100}} \sim N\left(\mu, \frac{126{,}985^2}{100}\right)$$

$$\begin{cases} \overline{x}_{A_0} = 244 \\ n_A = 100 \end{cases}$$

b)

$$IC_{95\%} = [\overline{x}_{A_0} - \Delta, \overline{x}_{A_0} + \Delta]$$

$\alpha = 0{,}05$

$$\Delta = z_{1-\frac{\alpha}{2}} \times \frac{126{,}985}{\sqrt{100}}$$

$$= \Phi^{-1}\left(1-\frac{\alpha}{2}\right) \times \frac{126{,}985}{\sqrt{100}} = 24{,}888$$

$$IC_{95\%} = [219{,}112, 268{,}888]$$

7.2

c) $\bar{x}_F = 350$

Nao esta dentro do intervalo de confiança com alfa de 0.055, logo pode-se por em causa a afirmação como falso.

7.3

X_i - "v.a peso de um ~~recado~~ "

$\begin{cases} n = 86 & s^2 = 230{,}2 \\ \bar{x}_0 = 186{,}2 \end{cases}$

$n >> 30 \Rightarrow S^2 \simeq s^2$

a) $IC_{96\%}$

$\bar{X} = \frac{1}{86} \sum_{i=1}^{86} x_i \sim N\left(\mu, \frac{230{,}2^2}{86}\right)$

$\alpha = 0{,}04$

$IC_{96\%} = [\bar{x}_0 - \Delta, \bar{x}_0 + \Delta]$

$\Delta = Z_{1-\frac{\alpha}{2}} \times \frac{S}{\sqrt{n}}$

$= \Phi^{-1}\left(1 - \frac{0{,}04}{2}\right) \times \frac{\sqrt{230{,}2}}{\sqrt{86}}$

$= 2{,}0537 \times 2{,}6767$

$= 3{,}36$

$IC_{96\%} \simeq [182{,}839, 189{,}56]$

b) $IC_{99\%}$

$\alpha = 0{,}01$

$IC_{99\%} \simeq [181{,}985, 190.414]$

c) $IC_{95\%}$ se $n = 120$, $S^2 = 320{,}2$, $\bar{x}_0 = 186{,}2$

$IC_{95\%} = [182.99, 189.40]$

7.4

X_i - v.a distância entre os tijolos em cm "

$\sigma^2 = 1\ cm^2$

$\begin{cases} n = 50 \\ \bar{x}_0 = 2,2\ cm \end{cases}$

a) $IC_{95\%}$

$IC_{95\%} = [\bar{x}_0 - \Delta ; \bar{x}_0 + \Delta]$

$\alpha = 0,05$

$$\Delta = z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$$

$$= \Phi^{-1}\left(1 - \frac{0,05}{2}\right) \times \frac{1}{\sqrt{50}}$$

$$= 1,96 \times \frac{1}{\sqrt{50}}$$

$$= 0,27718$$

$$IC_{95\%} = [1,9228, 2,4771]$$

b) $IC_{98\%} = [1,871 , 2,528]$

$\alpha = 0,02$

$z_0 = 2,326$

probable wrong solution

c) $\cancel{A = 2\Delta}$ $\Rightarrow$ $\Delta = \frac{0,27718}{2}$ $n = ?$

$$\cancel{\frac{0,27718}{2} = 1,96 \times \frac{1}{\sqrt{n}}}$$

$$\cancel{\sqrt{n} = \frac{1,96}{\frac{0,27718}{2}}}$$

$$\cancel{n = \left(\frac{1,96}{\frac{0,27718}{2}}\right)^2 \simeq 200}$$

X MP→V

$s^2 = 5,323 \cdot 10^{-3} = \frac{1}{n}$

$n = 187,86$

d) $IC_? = [2,2 - 0,06 ; 2,2 + 0,06]$

$1 - \alpha = 48,89\%$

$\Delta = 0,06$

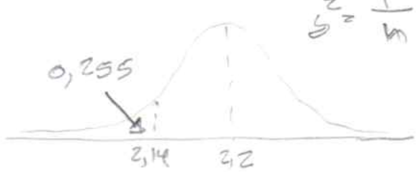
$$= z_{1-\frac{\alpha}{2}} \cdot \frac{1}{\sqrt{120}}$$

$$z_{1-\frac{\alpha}{2}} = 0.06 \times \sqrt{120}$$
$$= 0,6572$$

$1 - \alpha = 0,4889$

$1 - \alpha = 49\%$

$\sigma^2 = \frac{1}{n}$



X MV→P

$\alpha = 2 \times 0,255$

$$\Phi(0,6572) = 0,74447$$

$$1 - \frac{\alpha}{2} = 0,74447 \Rightarrow \alpha = 0,5110$$

7.5

$$\begin{cases} \bar{x}_{50} = 14{,}8 \\ s^2 = 1{,}2^2 \end{cases}$$

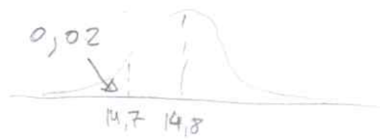
a)

$$\bar{X} = \frac{1}{50} \sum_{i=1}^{50} X_i \sim N\left(\mu, \frac{1{,}2^2}{50}\right)$$

$n = ?$ para $IC_{96\%}$ $\Delta = \frac{0{,}2}{2} = 0{,}1$

$\alpha = 0{,}04$

$$0.1 = Z_{1-\frac{\alpha}{2}} \times \frac{1.2}{\sqrt{n}}$$



$$0.1 = \Phi^{-1}\left(1 - \frac{0{,}04}{2}\right) \times \frac{1.2}{\sqrt{n}}$$

$$\sqrt{n} = \frac{\Phi^{-1}\left(1-\frac{0{,}04}{2}\right) \times 1.2}{0{,}1}$$

X M P ⇒ r

$S^2 = 2.37 \cdot 10^{-3}$

$\Rightarrow 607{,}375$

$$n = \left(\frac{\Phi^{-1}\left(1-\frac{0{,}04}{2}\right) \times 1.2}{0.1}\right)^2 = 607.375$$

b) $IC_{99\%} = [\, 14.36 \,,\, 15.23 \,]$

$\alpha = 0{,}01$

$Z = 2.575$

$\Delta = 0.4371$

c) $\Delta = 0{,}1$ $n = 250$ $IC_?$

$1 - \alpha = ?$

$$0{,}1 = Z_{1-\frac{\alpha}{2}} \times \frac{1.2}{\sqrt{250}}$$

$$Z_{1-\frac{\alpha}{2}} = 0.1 \times \frac{\sqrt{250}}{1.2}$$

$$\Phi\left(0.1 \times \frac{\sqrt{250}}{1.2}\right) = 1 - \frac{\alpha}{2}$$

$$= 1 - \frac{\alpha}{2}$$

$\alpha = 0{,}1876$

$\therefore$

$1 - 0{,}1876 = 0{,}8123$

$81{,}23\%$

7.6.

X_{Ai} "v.a duração vida útil em 1000×horas"

$\{ n = 150$

$\overline{X}_A = \frac{1}{150} \sum_{i=1}^{150} X_i \underset{\substack{T.L.C \\ n \geq 30}}{\sim} N\left(\mu, \frac{0{,}96^2}{150}\right)$

$= \frac{1}{150}\left(1.5 \times 29 + 2.5 \times 43 + 3.5 \times 57 + 4.5 \times 21\right)$

$\overline{x}_{A0} = 2{,}96$

$s_A^2 = \frac{1}{150-1}\left((1.5-2.96)^2 \times 29 + (2.5-2.96)^2 \times 43 \ldots\right.$

$\simeq 0.96^2$

a) $IC_{95\%} = [2{,}806, 3.1136]$

$z \simeq 1{,}96$

$\Delta \simeq 0{,}15363$

b) $\begin{cases} n_B = 100 \\ \overline{x}_{B0} = 3{,}150 \quad s = 0{,}85 \end{cases}$ $\overline{X}_B = \frac{1}{100} \sum_{i=1}^{100} X_{Bi} \sim N\left(\mu, \frac{0{,}85^2}{100}\right)$

$\overline{X}_A \sim N\left(\mu, \frac{0{,}96^2}{150}\right)$ & $\overline{X}_B \sim N\left(\mu, \frac{0{,}85^2}{100}\right)$

$\overline{X}_A - \overline{X}_B \underset{A.N}{\sim} N\left(\mu, \frac{0{,}85^2}{100} + \frac{0.96^2}{150}\right)$ $= 0{,}013369$

$IC_{90\%\ \overline{X}_A - \overline{X}_B} = [-1{,}9 - \Delta ; -1{,}9 + \Delta]$

$= [-0{,}380 ; 1{,}85 \cdot 10^{-4}]$

$\alpha = 0{,}1$

$z \simeq 1{,}645$

$\Delta \simeq 0{,}1902$

Não se pode dizer que há grande diferença têm sinais opostos.

c) $n = ?$ $\Delta_A = 0{,}1$ $IC_{95\%}$

$\alpha = 0{,}05$

$$\Delta_A = Z_{1-\frac{\alpha}{2}} \times \frac{\cancel{s}}{\sqrt{n}}$$

$$0{,}1 = Z_{1-\frac{\alpha}{2}} \times \frac{0{,}96}{\sqrt{n}}$$

$$n = \left(\frac{\Phi^{-1}\left(1 - \frac{0{,}05}{2}\right) \times 0.96}{0{,}1} \right)^2 = 354.02$$

TP 7

6. X_{Ai} - " v.a Durabilidade de vedantes industriais em x 1000 horas." [~~h~~]

$n_A = 150$

$$\bar{X}_A = \frac{1}{150} \sum_{i=1}^{150} X_i \sim N\left(\mu, \frac{\sigma}{\sqrt{150}}\right)$$

Duracao	[1-2[	[2-3[	[3-4[	[4-5[
marca	1.5	2.5	3.5	4.5
nº	29	43	57	21

$n \geqslant 30 \therefore \sigma \simeq s.$

$$\bar{x}_A = \frac{1}{n} \sum x_i \, n_i$$

$$= \frac{1}{150} (1.5 \times 29 + 2.5 \times 43 + 3.5 \times 57 + 4.5 \times 21)$$

$$\simeq 2,97 \times 1000 \text{ hours.}$$

$$s^2 = \frac{1}{150-1} \sum_{i=1}^{150} (x_i - \bar{x}_A)^2 \cdot n_i$$

$$= \frac{1}{149} \left((1.5 - 2.97)^2 \times 29 + (2.5 - 2.97)^2 \times 43 + (3.5 - 2.97)^2 \times 57 + (4.5 - 2.97)^2 \times 21 \right)$$

$$= 0,9217$$

$$s = \sqrt{0,9217} = 0,960 \quad ; \quad z_{1-\frac{\alpha}{2}} \simeq 1,96$$

a) $IC_{\mu;95\%} = [2.816, 3.124]$ $\quad \Delta = 0,15363$

b) $n = ?$ $\quad \Delta = 0,1$ $\quad IC_{95\%} \rightarrow z_{1-\frac{\alpha}{2}} = 1,96$

$$\Delta = 1.96 \, \frac{0.960}{\sqrt{n}}$$

$$0.1 = 1.96 \, \frac{0.960}{\sqrt{n}}$$

$x_{MP} \to \sigma = 2,60 \cdot 10^{-3}$

$$n = \left(\frac{1.96 \times 0.960}{0.1} \right)^2 = 354.04$$

$= \frac{0,9217}{n}$

$\hookrightarrow 354,06$

TP7

6.

c) $n_B = 100$ $\bar{x}_B = 3.15$ $s = 0.85$

$n_A = 150$ $\bar{x}_A = 2.97$ $s = 0.96$

$$\bar{Y} = \bar{X}_A - \bar{X}_B \sim \left(\mu_A - \mu_B \,,\, \frac{0.85^2}{100} + \frac{0.96^2}{150} \right)$$

$$\bar{x}_A - \bar{x}_B = -0.18$$

$$IC_{90\%} \to z_{1-\frac{\alpha}{2}} \tilde{=} 1{,}645$$

$$\Delta = z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{0.85^2}{100} + \frac{0.96^2}{150}}$$

$$= 1{,}645 \times 0.115624$$

$$\tilde{=} 0.190$$

$$IC_{\mu A - \mu B} \left[-0.18 - 0.19 \,,\, -0.18 + 0.19 \right]$$

$$\left[-0{,}37 \,,\, 0{,}01 \right]$$

Nao ha grande diferenca, os sinais são contrarios.