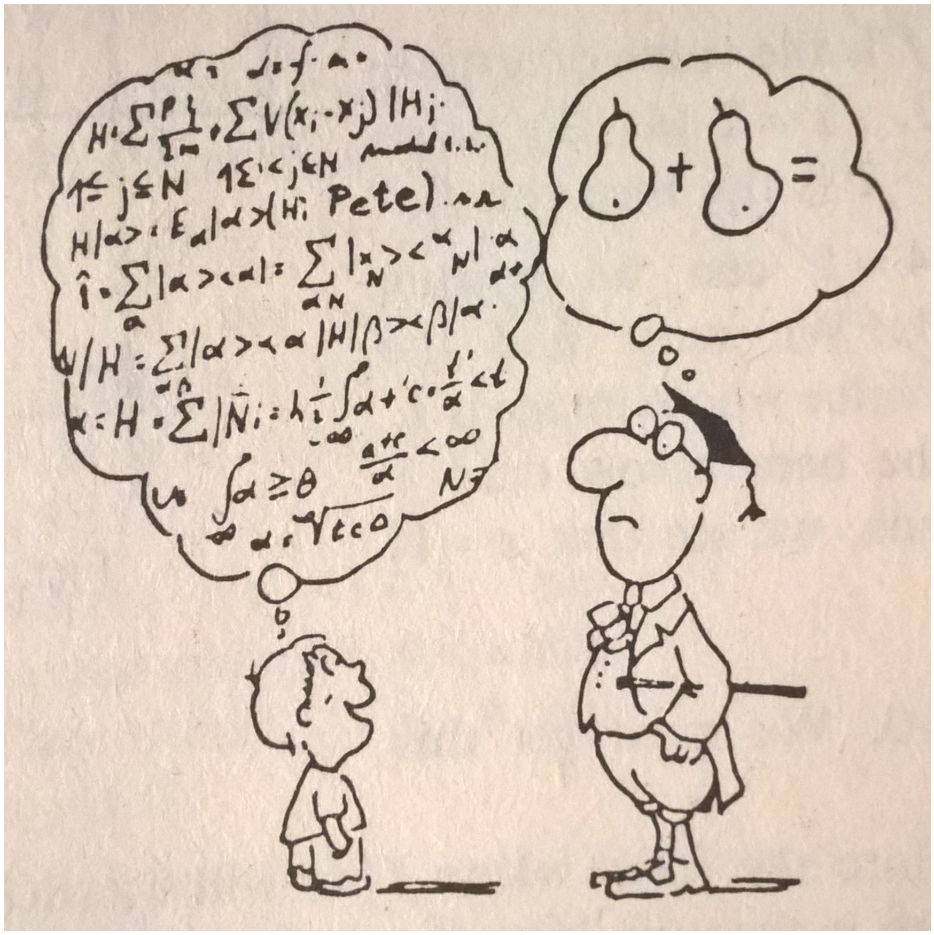
# MATH BASICS

**Autor** :
*Sérgio Santos*

**Mathematic**



# Conteúdo

# 1 Definition

## 1.1 Exponents

1.

$$a^n = a \times a \times a \times a \ldots . n \quad factors \qquad (n \in \mathbb{N},\ a \in \mathbb{R})$$

2.

$$a^{-m} = \frac{1}{a^m} \qquad (m \in \mathbb{Z}^+,\ a \in \mathbb{R},\ a \neq 0)$$

$$and:$$

$$\frac{1}{a^{-m}} = a^m$$

3.

$$a^0 = 1 \qquad (a \in \mathbb{R},\ a \neq 0)$$

## 1.2 Rational Exponents:

1.

$$\sqrt[n]{a} = r \qquad (a > 0,\ n \in \mathbb{N},\ n \geqslant 2,\ r > 0), \iff r^n = a$$

2.

$$a^{\frac{1}{n}} = \sqrt[n]{a}; \qquad (a > 0,\ n \geqslant 2,\ n \in \mathbb{N})$$

3.

$$a^{\frac{-1}{n}} = \sqrt[n]{a^{-1}}; \qquad (a > 0,\ n > 0,\ n \in \mathbb{N})$$

4.

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}; \qquad (a > 0;\ m,\ n \in \mathbb{Z},\ n \geqslant 2)$$

# 2 Law

## 2.1 Exponents

1.

$$a^m \times a^n = a^{m+n} \qquad (m,\ n \in \mathbb{N})$$

$$a^m \times a^n = a^{m+n} \qquad (m,\ n \in \mathbb{Z};\ a \neq 0,\ if \quad m \quad or \quad n < 0)$$

2.

$$\frac{a^m}{a^n} = a^{m-n} \qquad (m,\ n \in \mathbb{Z};\ a \in \mathbb{R};\ a \neq 0)$$

3.

$$(ab)^m = a^m\, b^m \qquad (m \in \mathbb{Z})$$

4.

$$(a^m)^n = a^{mn} \qquad (m,\ n \in \mathbb{Z})$$

## 2.2 Rational Exponents

1.

$$a^r \times a^t = a^{r+t} \qquad (a > 0; \quad r,\ t \in \mathbb{Q})$$

2.

$$\frac{a^r}{a^t} = a^{r-t} \qquad (a > 0; \quad r,\ t \in \mathbb{Q})$$

3.

$$(a^t)^r = a^{tr} \qquad (a > 0, \quad t,\ r \in \mathbb{Q})$$

4.

$$(ab)^t = a^t\, b^t\ ; \quad \left(\frac{a}{b}\right)^t = \frac{a^t}{b^t}\ ; \qquad (a,\ b > 0, \quad t \in \mathbb{Q})$$

$$and:$$

$$a^t\, b^t = (ab)^t \qquad and \qquad \frac{a^t}{b^t} = \left(\frac{a}{b}\right)^t$$

### 2.3 Distributive law

$$a(b + c) = ab + ac$$

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= ac + bc + ad + bd\end{aligned}$$

$$A^2 - B^2 = (A - B)(A + B)$$

### 2.4 Commutative law

$$ab = ba$$

# 3 Properties

## 3.1 Addition

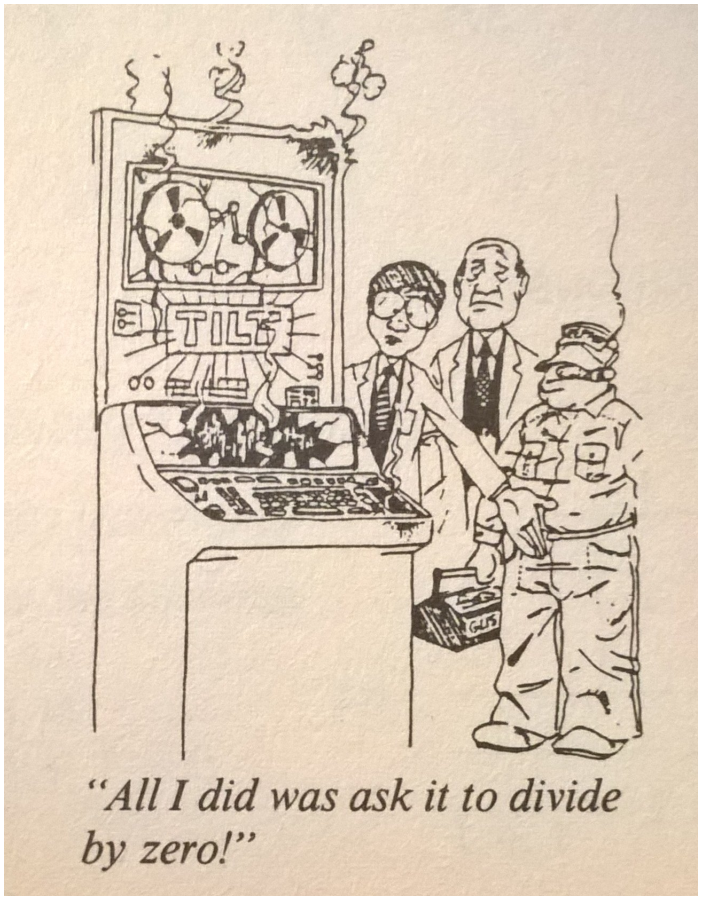
$$0 + a = a$$
$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \qquad (b \neq 0)$$
$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \qquad (b \neq 0)$$

## 3.2 Multiplication

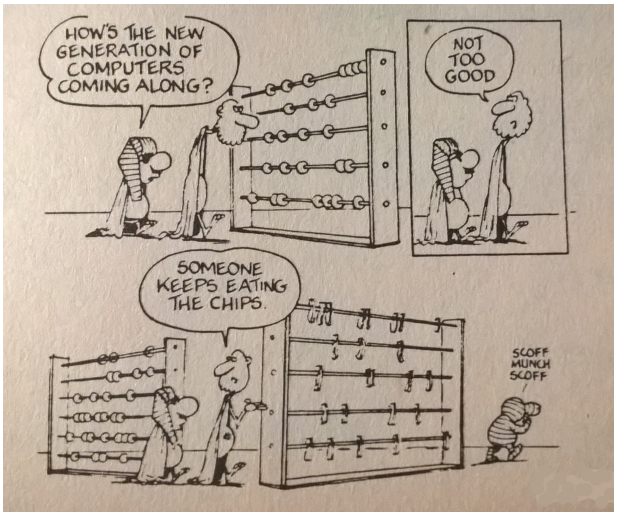
$$0 \times a = 0$$
$$\frac{0}{a} = 0 \times \frac{1}{a} = 0$$
$$1 \times a = a$$
$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \qquad (b \neq 0;\ d \neq 0)$$

## 3.3 Division

$$\frac{a}{0} = undefined$$
$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr} \qquad (q \neq 0;\ r \neq 0;\ s \neq 0)$$




*"All I did was ask it to divide by zero!"*



# 4 Examples


$$\sqrt{a^2} = a \qquad (a > 0) \qquad\qquad \sqrt{a^n} = a^{\frac{n}{2}}$$

$$\sqrt{\frac{1}{a}} = \frac{1}{\sqrt{a}}$$

$$a^{\frac{m}{1}} = a^m \qquad\qquad \frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a \angle \alpha^\circ \; b \angle \beta^\circ}{c \angle \gamma^\circ} = \frac{a \times b}{c} \angle (\alpha^\circ + \beta^\circ - \gamma^\circ)$$

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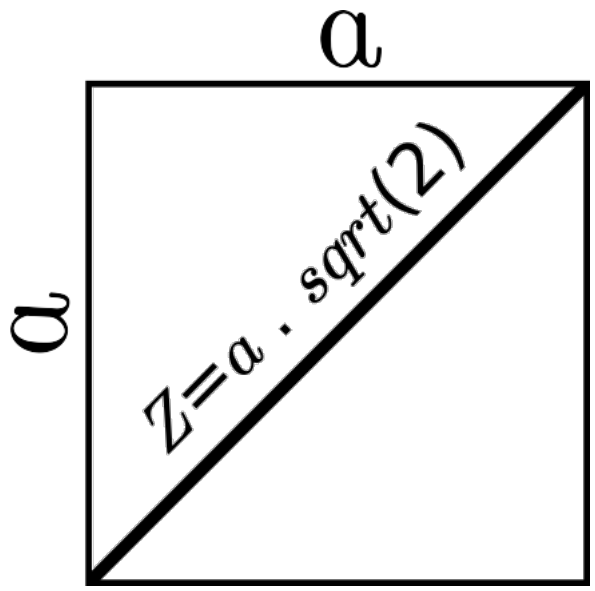
$$If \quad a \,.\, b \;=\; 0, \quad then \quad a \;=\; 0 \quad or \quad b \;=\; 0$$




**Square Property**

$$Z = \sqrt{a^2 \;+\; a^2}$$
$$Z = \sqrt{2\; a^2}$$
$$Z = \sqrt{2} \quad \sqrt{a^2}$$
$$Z = \sqrt{2} \quad a$$

$$a \angle\alpha \;+\; b \angle\beta \;=\; \sqrt{(a\, \sin\alpha \;+\; b\, \sin\beta)^2 \;+\; (a\, \cos\alpha \;+\; b\, \cos\beta)^2}$$

and

$$\angle \arctan(\frac{(a\, \sin\alpha \;+\; b\, \sin\beta)}{(a\, \cos\alpha \;+\; b\, \cos\beta)})$$

$$\lim_{s\to 0} GH(s) = \lim_{s\to 0} \quad 7\, \frac{4+3s}{5+2s+6s^2}$$

$$= \lim_{s\to 0} \quad 7\, \frac{4(1+\frac{3}{4}s)}{5(1+\frac{2}{5}s+\frac{6}{5}s^2)}$$

$$= 7\, \frac{4}{5}$$

$$y'(x) = \mathrm{e}^{4x+5}$$
$$= 4\,\mathrm{e}^{4x+5}$$

Derivatives made easy.

$y = \cos^3(\operatorname{sen}(x^2+x))$

$y' = x^{3\prime}\Big|_{x=\cos(\operatorname{sen}(x^2+x))} \cdot \cos(x)'\Big|_{x=\operatorname{sen}(x^2+x)} \cdot \operatorname{sen}(x)'\Big|_{x=x^2+x} \cdot (x^2+x)'$

$= 3x^2\Big|_{x=\cos(\operatorname{sen}(x^2+x))} \cdot (-1).\operatorname{sen}(x)\Big|_{x=\operatorname{sen}(x^2+x)} \cdot \cos(x)\Big|_{x=x^2+x} \cdot (2x+1)$

$= 3\cos^2(\operatorname{sen}(x^2+x)) \cdot (-1) \cdot \operatorname{sen}(\operatorname{sen}(x^2+x)) \cdot \cos(x^2+x) \cdot (2x+1)$

$= -(6x+3) \cdot \cos^2(\operatorname{sen}(x^2+x)) \cdot \operatorname{sen}(\operatorname{sen}(x^2+x)) \cdot \cos(x^2+x)$

$$y = \frac{(x+3)^3}{x^{3/2}}$$

$$y' = \frac{(x+1)^3 \cdot x^{3/2} - x^{3/2} \cdot (x+1)^3}{x^3}$$

$$= \frac{3(x+1)^2 \cdot x^{3/2} - \frac{3}{2} \cdot x^{1/2} \cdot (x+1)^3}{x^3} \quad \text{HCF}$$

$$= \frac{[\,3(x+1)^2 \cdot x - \frac{3}{2}(x+1)^3\,]\, x^{\frac{1}{2}}}{x^3}$$

$$= \frac{3(x+1)^2 \cdot x - \frac{3}{2}(x+1)^3}{x^{5/2}} \quad \text{HCF}$$

$$= \frac{(x+1)^2 \cdot (3x - \frac{3}{2}(x+1))}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot (3x - \frac{3}{2}x - \frac{3}{2})}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot (\frac{3}{2}x - \frac{3}{2})}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot \frac{3}{2}(x-1)}{x^{5/2}}$$

$$= \frac{3}{2} \cdot \frac{(x+1)^2 \cdot (x-1)}{x^{5/2}}$$

(ou) $\frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

$x^{\frac{3}{2}} = x + x^{\frac{1}{2}}$

$\frac{1}{2} - \frac{6}{2} = -\frac{5}{2}$

$\frac{6}{2} - \frac{3}{2} = \frac{3}{2}$

$$\int x = \frac{x^2}{2}$$

$$\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}} = \pi$$

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + \ldots$$

# 5 Methods

HCF - highest common factor (pôr variável em evidência)

*Factorisation*

LCD or LCM - Lowest common denominator or lowest common multiple

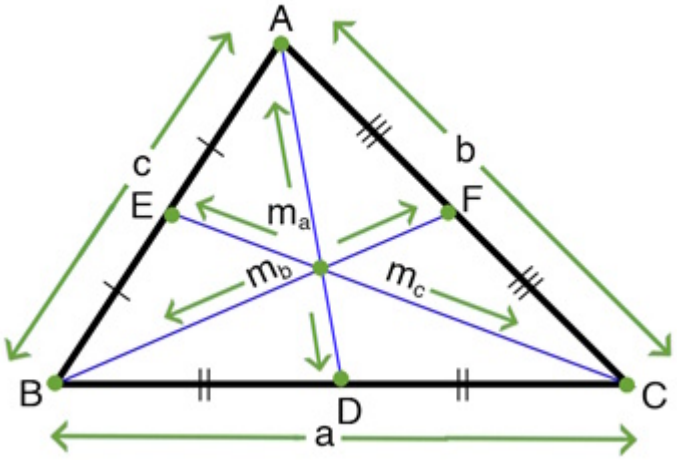
# Bibliografia

[1] Machado, V. A. Tenreiro: *Teoria Dos Sistemas de Controlo*. Instituto Superior de Engenharia do Porto, 1999. 6

[2] TIPLER, PAUL A. e GENE MOSCA: *PHYSICS FOR SCIENTISTS AND ENGINEERS, Extended Version fifth edition*. W. H. Freeman and Company, 1999. 19

# Apêndice A

# Triangle Theorems and Formulas




## 1. Basic Angle Rules

$$A + B + C = 180°$$

Exterior Angle = Sum of two opposite interior angles

## 2. Side and Angle Relationships

### Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

### Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

## 3. Area Formulas

### Basic

$$\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$

### Heron's Formula

Let $s = \frac{a+b+c}{2}$:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

### Using Sine

$$\text{Area} = \frac{1}{2}ab\sin C$$

## 4. Right Triangle Formulas

### Pythagorean Theorem

$$a^2 + b^2 = c^2$$

**Trigonometric Ratios**

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

## 5. Medians (Mid-Height)

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$
$$m_b = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$$
$$m_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$$

## 6. Heights (Altitudes)

$$h_a = \frac{2 \cdot \text{Area}}{a}$$
$$h_b = \frac{2 \cdot \text{Area}}{b}$$
$$h_c = \frac{2 \cdot \text{Area}}{c}$$

## 7. Equilateral Triangle (side $= a$)

$$\text{Each angle} = 60^\circ$$
$$\text{Height } h = \frac{\sqrt{3}}{2}a$$
$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

## 8. Coordinate Geometry

### Distance between points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### Area from coordinates

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

### Centroid

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

## 9. Inradius and Circumradius

### Inradius

$$r = \frac{\text{Area}}{s} = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

### Circumradius

$$R = \frac{abc}{4 \cdot \text{Area}} = \frac{a}{2\sin A}$$

# Apêndice B

# Electric Definition

**Definição 1** *Capacitância*

$$\begin{aligned} Q_c(t) &= \int^t i(t) \quad dt \\ &= Q_c(0^-) + \int_{0^-}^t i(t) \quad dt \\ V_c(t) &= \frac{Q_c(t)}{C} \\ &= \frac{1}{C} \quad \int^t i_c(t) \quad dt \\ &= \frac{Q_c(0^-)}{C} + \frac{1}{C} \quad \int_0^t i_c(t) \quad dt \\ &= V(0^-) + \frac{1}{C} \quad \int_0^t i_c(t) \quad dt \\ i_c(t) &= C \quad \frac{dV_c(t)}{dt} \\ W &= \frac{1}{2}\, C\, V^2 \end{aligned}$$

**Definição 2** *Indutância*

$$\begin{aligned} \psi_L(t) &= \int^t V_L(t) \quad dt \\ &= \psi_L(0^-) + \int_{0^-}^t V_L(t) \quad dt \\ V_L(t) &= L \quad \frac{di_L(t)}{dt} \\ i_L(t) &= \frac{\psi_L(t)}{L} \\ &= \frac{1}{L} \quad \int^t V_L(t) \quad dt \\ &= \frac{\psi_L(0^-)}{L} + \frac{1}{L} \quad \int_0^t V_L(t) \quad dt \\ &= i_L(0^-) + \frac{1}{L} \quad \int_0^t V_L(t) \quad dt \\ W &= \frac{1}{2}\, L\, i^2 \end{aligned}$$

**Definição 3** *Resistência*

$$\begin{aligned} V_R(t) &= R \quad i_R(t) \\ i_R(t) &= \frac{V_R(t)}{R} \\ P &= Ri^2 \\ P &= \frac{U^2}{R} \\ W &= P\, \Delta t \end{aligned}$$

**Definição 4** *Valor Médio*

$$X_{av} = \frac{1}{T} \int_0^T X(t)dt$$

**Definição 5** *Valor Eficaz*

$$X_{ef} = \sqrt{\frac{1}{T} \int_0^T \overset{2}{X}(t)dt}$$

# Apêndice C

# Physics

## Motion

$$x_{(t)} = x_o + v_o\,t + \tfrac{1}{2}\,a\,t^2$$

$$x_{(t)} \quad \underset{\int dt}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftarrows}} \quad v_{(t)} \quad \underset{\int dt}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftarrows}} \quad a_{(t)}$$

## Força [N] [Kgf]

$$\sum F_{(t)} = M\,a_{(t)} = M\,\ddot{x}_{(t)}$$

$$\sum F_R = \sum F_{action} - \sum F_{reaction}$$

$$f_{(t)} = -K\,x_{(t)}$$
$$f_{(t)} = -B\,\dot{x}_{(t)}$$

## Torque [N.m]

$$\sum T_{(t)} = J\,\gamma_{(t)} = M\,\ddot{\theta}_{(t)}$$

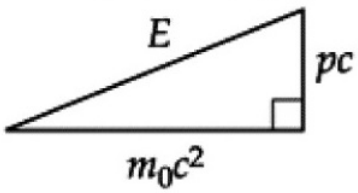
$$\sum T_R = \sum T_{action} - \sum T_{reaction}$$

$$T_{(t)} = -K\,\theta_{(t)}$$
$$T_{(t)} = -B\,\dot{\theta}_{(t)}$$
$$T = F \times r$$

## Energia [Joule]

$$W = F\,d$$
$$W = P\,\Delta t$$
$$E = M\,C^2$$

$$E^2 = (pc)^2 + (m_0c^2)^2$$

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## Energia Cinética [Joule]

$$E_c = \tfrac{1}{2}\,m\,v^2$$

## Energia Potencial [Joule]

$$E_p = m\,g\,h$$

## Energia Térmica

$Q - Heat \quad energy$
$Q_{(t)} - temperature$
$R - heat \quad resistance$

$$Q = \frac{Q_{1(t)} - Q_{2(t)}}{R}$$

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[1] Apontamento