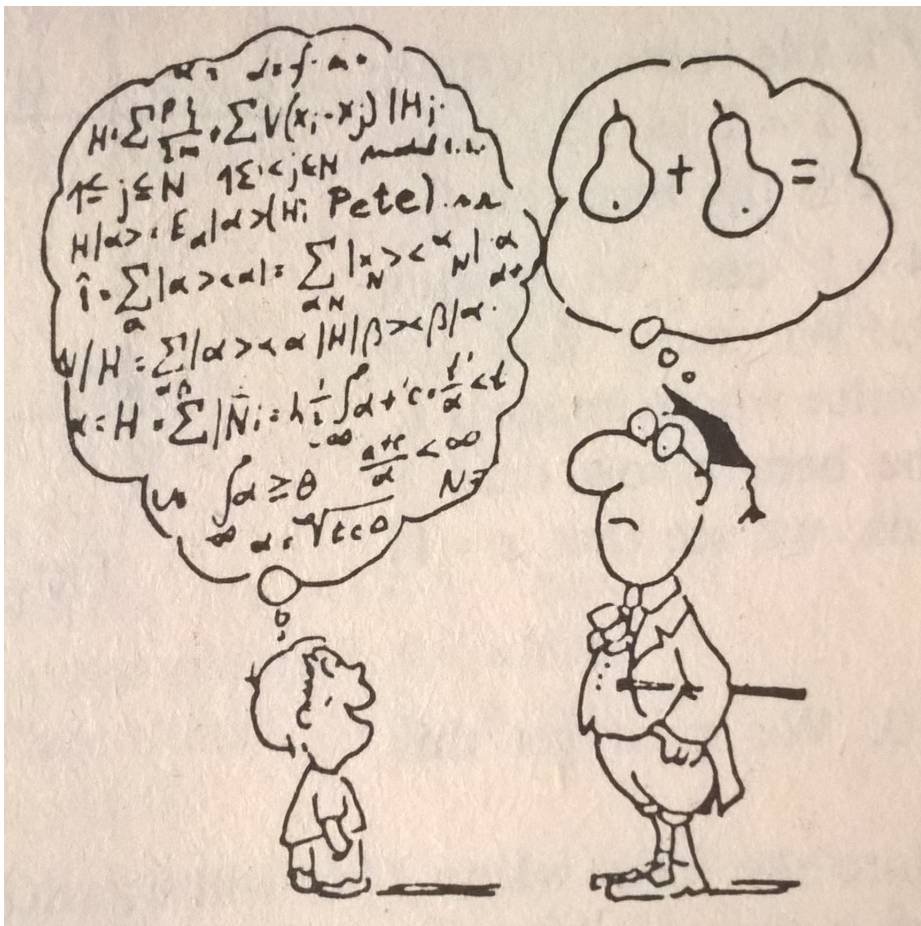


MATH BASICS

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Mathematic



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Siglas

FACTORISATION *Factorisation*

HCF *Highest Common Factor*

LCD *Lowest Common Denominator*

LCM *Least Common Multiple*

1 Definition

1.1 Exponents

1.

$$a^n = a \times a \times a \times a \dots n \text{ factors} \quad (n \in \mathbb{N}, a \in \mathbb{R})$$

2.

$$a^{-m} = \frac{1}{a^m} \quad (m \in \mathbb{Z}^+, a \in \mathbb{R}, a \neq 0)$$

and:

$$\frac{1}{a^{-m}} = a^m$$

3.

$$a^0 = 1 \quad (a \in \mathbb{R}, a \neq 0)$$

1.2 Rational Exponents:

1.

$$\sqrt[n]{a} = r \quad (a > 0, n \in \mathbb{N}, n \geq 2, r > 0), \iff r^n = a$$

2.

$$a^{\frac{1}{n}} = \sqrt[n]{a}; \quad (a > 0, n \geq 2, n \in \mathbb{N})$$

3.

$$a^{\frac{-1}{n}} = \sqrt[n]{a^{-1}}; \quad (a > 0, n > 0, n \in \mathbb{N})$$

4.

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}; \quad (a > 0; m, n \in \mathbb{Z}, n \geq 2)$$

2 Law

2.1 Exponents

1.

$$a^m \times a^n = a^{m+n} \quad (m, n \in \mathbb{N})$$

$$a^m \times a^n = a^{m+n} \quad (m, n \in \mathbb{Z}; a \neq 0, \text{ if } m \text{ or } n < 0)$$

2.

$$\frac{a^m}{a^n} = a^{m-n} \quad (m, n \in \mathbb{Z}; a \in \mathbb{R}; a \neq 0)$$

3.

$$(ab)^m = a^m b^m \quad (m \in \mathbb{Z})$$

4.

$$(a^m)^n = a^{mn} \quad (m, n \in \mathbb{Z})$$

2.2 Rational Exponents

1.

$$a^r \times a^t = a^{r+t} \quad (a > 0; r, t \in \mathbb{Q})$$

2.

$$\frac{a^r}{a^t} = a^{r-t} \quad (a > 0; r, t \in \mathbb{Q})$$

3.

$$(a^t)^r = a^{tr} \quad (a > 0, t, r \in \mathbb{Q})$$

4.

$$(ab)^t = a^t b^t; \quad \left(\frac{a}{b}\right)^t = \frac{a^t}{b^t}; \quad (a, b > 0, t \in \mathbb{Q})$$

and:

$$a^t b^t = (ab)^t \quad \text{and} \quad \frac{a^t}{b^t} = \left(\frac{a}{b}\right)^t$$

2.3 Distributive law

$$a(b + c) = ab + ac$$

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= ac + bc + ad + bd\end{aligned}$$

$$A^2 - B^2 = (A - B)(A + B)$$

2.4 Commutative law

$$ab = ba$$

3 Properties

3.1 Addition

$$0 + a = a$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \quad (b \neq 0)$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \quad (b \neq 0)$$

3.2 Multiplication

$$0 \times a = 0$$

$$\frac{0}{a} = 0 \times \frac{1}{a} = 0$$

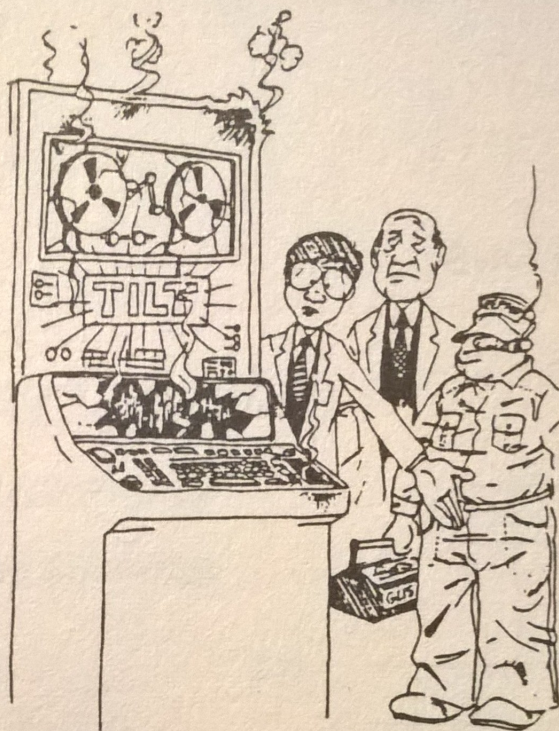
$$1 \times a = a$$

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \quad (b \neq 0; d \neq 0)$$

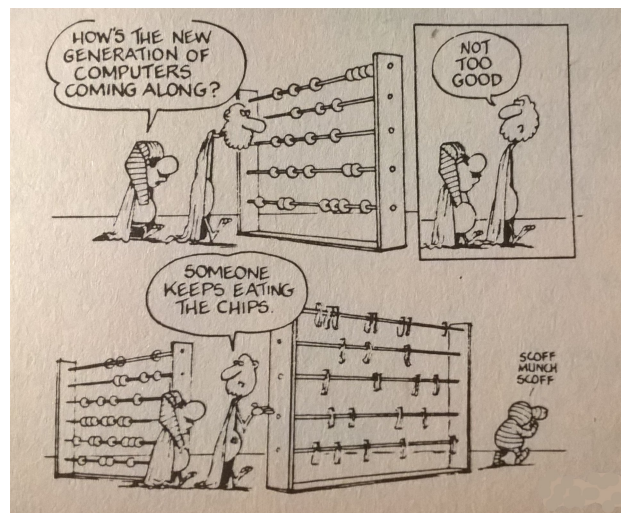
3.3 Division

$$\frac{a}{0} = \textit{undefined}$$

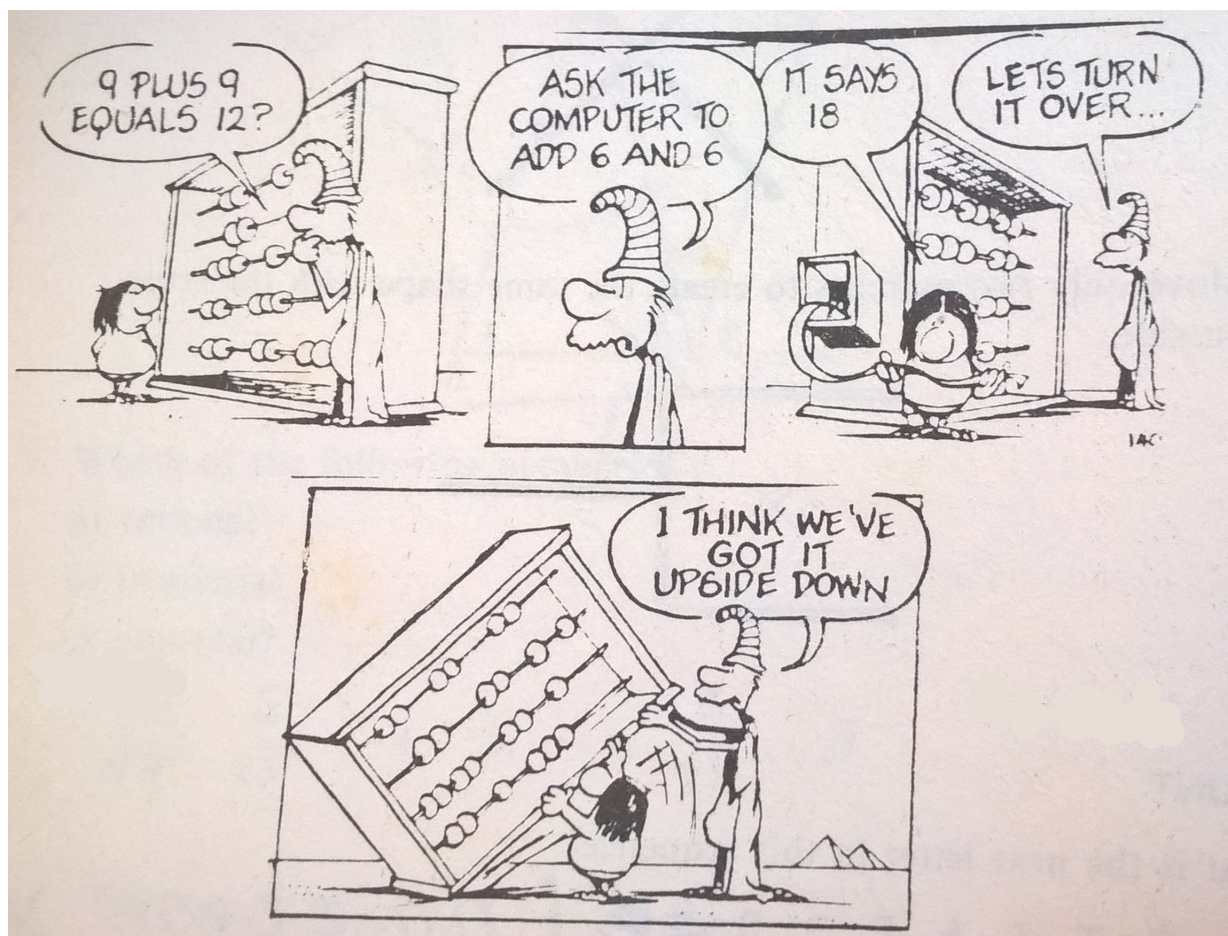
$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr} \quad (q \neq 0; r \neq 0; s \neq 0)$$



"All I did was ask it to divide by zero!"



4 Examples



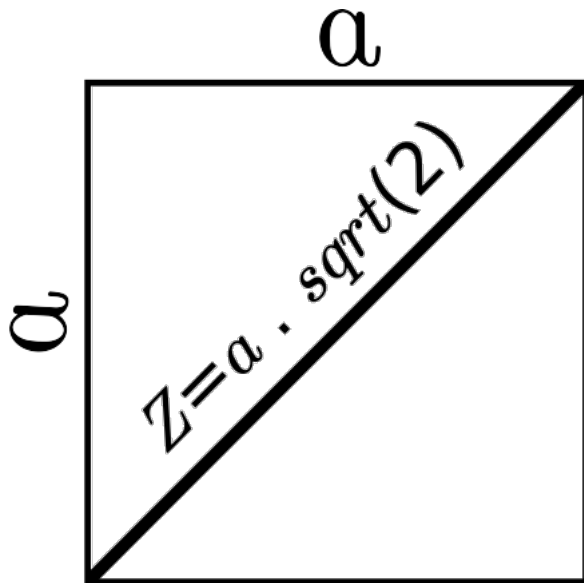
$$\sqrt{a^2} = a \quad (a > 0) \quad \sqrt{a^n} = a^{\frac{n}{2}}$$

$$\sqrt{\frac{1}{a}} = \frac{1}{\sqrt{a}}$$

$$a^{\frac{m}{1}} = a^m \quad \frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a \angle \alpha^\circ \quad b \angle \beta^\circ}{c \angle \gamma^\circ} = \frac{a \times b}{c} \angle (\alpha^\circ + \beta^\circ - \gamma^\circ)$$

If $a \cdot b = 0$, then $a = 0$ or $b = 0$



Square Property

$$Z = \sqrt{a^2 + a^2}$$

$$Z = \sqrt{2} a^2$$

$$Z = \sqrt{2} \sqrt{a^2}$$

$$Z = \sqrt{2} a$$

$$a \angle \alpha + b \angle \beta = \sqrt{(a \sin \alpha + b \sin \beta)^2 + (a \cos \alpha + b \cos \beta)^2}$$

and

$$\angle \arctan\left(\frac{(a \sin \alpha + b \sin \beta)}{(a \cos \alpha + b \cos \beta)}\right)$$

$$\lim_{s \rightarrow 0} GH(s) = \lim_{s \rightarrow 0} 7 \frac{4 + 3s}{5 + 2s + 6s^2}$$

$$= \lim_{s \rightarrow 0} 7 \frac{4(1 + \frac{3}{4}s)}{5(1 + \frac{2}{5}s + \frac{6}{5}s^2)}$$

$$= 7 \frac{4}{5}$$

$$y'(x) = e^{4x+5}$$

$$= 4 e^{4x+5}$$

Derivatives made easy.

$$y = \cos^3(\sin(x^2+x))$$

$$y' = x^3 \cdot \cos(x) \cdot \sin(x) \cdot (x^2+x)'$$

$$\left| \begin{array}{l} x = \cos(\sin(x^2+x)) \\ x = \sin(x^2+x) \\ x = x^2+x \end{array} \right.$$

$$= 3x^2 \cdot (-1) \cdot \sin(x) \cdot \cos(x) \cdot (2x+1)$$

$$\left| \begin{array}{l} x = \cos(\sin(x^2+x)) \\ x = \sin(x^2+x) \\ x = x^2+x \end{array} \right.$$

$$= 3 \cos^2(\sin(x^2+x)) \cdot (-1) \cdot \sin(\sin(x^2+x)) \cdot \cos(x^2+x) \cdot (2x+1)$$

$$= -(6x+3) \cdot \cos^2(\sin(x^2+x)) \cdot \sin(\sin(x^2+x)) \cdot \cos(x^2+x)$$

$$y = \frac{(x+1)^3}{x^{3/2}}$$

$$y' = \frac{(x+1)^3 \cdot x^{3/2} - x^{3/2} \cdot (x+1)^3}{x^3}$$

$$= \frac{3(x+1)^2 \cdot x^{3/2} - \frac{3}{2} x^{1/2} \cdot (x+1)^3}{x^3} \quad \text{HCF}$$

$$= \frac{[3(x+1)^2 \cdot x - \frac{3}{2}(x+1)^3] x^{1/2}}{x^3}$$

$$= \frac{3(x+1)^2 \cdot x - \frac{3}{2}(x+1)^3}{x^{5/2}} \quad \text{HCF}$$

$$= \frac{(x+1)^2 \cdot (3x - \frac{3}{2}(x+1))}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot (3x - \frac{3}{2}x - \frac{3}{2})}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot (\frac{3}{2}x - \frac{3}{2})}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot \frac{3}{2}(x-1)}{x^{5/2}}$$

$$= \frac{3}{2} \cdot \frac{(x+1)^2 \cdot (x-1)}{x^{5/2}}$$

$$\therefore \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$$

$$x^{3/2} = x + x^{1/2}$$

$$\frac{1}{2} - \frac{6}{2} = -\frac{5}{2}$$

$$\frac{6}{2} - \frac{3}{2} = \frac{3}{2}$$

$$\int x = \frac{x^2}{2}$$

$$\int_{-1}^1 \sqrt{\frac{1+x}{1-x}} = \pi$$

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} + \dots$$

5 Methods

HCF - highest common factor (pôr variável em evidência)

Factorisation

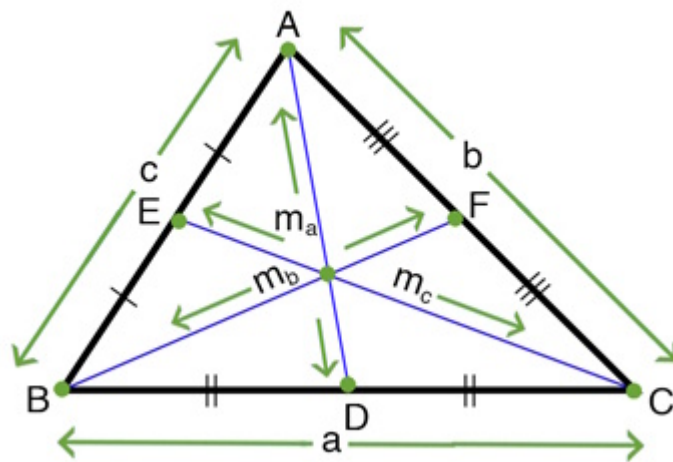
LCD or LCM - Lowest common denominator or lowest common multiple

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- [1] Machado, V. A. Tenreiro: *Teoria Dos Sistemas de Controlo*. Instituto Superior de Engenharia do Porto, 1999. 6
- [2] TIPLER, PAUL A. e GENE MOSCA: *PHYSICS FOR SCIENTISTS AND ENGINEERS, Extended Version fifth edition*. W. H. Freeman and Company, 1999. 19

Apêndice A

Triangle Theorems and Formulas



1. Basic Angle Rules

$$A + B + C = 180^\circ$$

Exterior Angle = Sum of two opposite interior angles

2. Side and Angle Relationships

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

3. Area Formulas

Basic

$$\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$

Heron's Formula

Let $s = \frac{a+b+c}{2}$:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Using Sine

$$\text{Area} = \frac{1}{2}ab \sin C$$

4. Right Triangle Formulas

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Trigonometric Ratios

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

5. Medians (Mid-Height)

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

$$m_b = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$$

$$m_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$$

6. Heights (Altitudes)

$$h_a = \frac{2 \cdot \text{Area}}{a}$$

$$h_b = \frac{2 \cdot \text{Area}}{b}$$

$$h_c = \frac{2 \cdot \text{Area}}{c}$$

7. Equilateral Triangle (side = a)

Each angle = 60°

$$\text{Height } h = \frac{\sqrt{3}}{2}a$$

$$\text{Area} = \frac{\sqrt{3}}{4}a^2$$

8. Coordinate Geometry

Distance between points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Area from coordinates

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Centroid

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

9. Inradius and Circumradius

Inradius

$$r = \frac{\text{Area}}{s} = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

Circumradius

$$R = \frac{abc}{4 \cdot \text{Area}} = \frac{a}{2 \sin A}$$

Apêndice B

Electric Definition

Definição 1 Capacitância

$$\begin{aligned}Q_c(t) &= \int^t i(t) \, dt \\&= Q_c(0^-) + \int_{0^-}^t i(t) \, dt \\V_c(t) &= \frac{Q_c(t)}{C} \\&= \frac{1}{C} \int^t i_c(t) \, dt \\&= \frac{Q_c(0^-)}{C} + \frac{1}{C} \int_0^t i_c(t) \, dt \\&= V(0^-) + \frac{1}{C} \int_0^t i_c(t) \, dt \\i_c(t) &= C \frac{dV_c(t)}{dt} \\W &= \frac{1}{2} C V^2\end{aligned}$$

Definição 3 Resistência

$$\begin{aligned}V_R(t) &= R \, i_R(t) \\i_R(t) &= \frac{V_R(t)}{R} \\P &= R i^2 \\P &= \frac{U^2}{R} \\W &= P \, \Delta t\end{aligned}$$

Definição 2 Indutância

$$\begin{aligned}\psi_L(t) &= \int^t V_L(t) \, dt \\&= \psi_L(0^-) + \int_{0^-}^t V_L(t) \, dt \\V_L(t) &= L \frac{di_L(t)}{dt} \\i_L(t) &= \frac{\psi_L(t)}{L} \\&= \frac{1}{L} \int^t V_L(t) \, dt \\&= \frac{\psi_L(0^-)}{L} + \frac{1}{L} \int_0^t V_L(t) \, dt \\&= i_L(0^-) + \frac{1}{L} \int_0^t V_L(t) \, dt \\W &= \frac{1}{2} L i^2\end{aligned}$$

Definição 4 Valor Médio

$$X_{av} = \frac{1}{T} \int_0^T X(t) dt$$

Definição 5 Valor Eficaz

$$X_{ef} = \sqrt{\frac{1}{T} \int_0^T X^2(t) dt}$$

Apêndice C

Physics

Motion

$$x_{(t)} = x_o + v_o t + \frac{1}{2} a t^2$$

$$x_{(t)} \begin{matrix} \xrightarrow{\frac{d}{dt}} \\ \xleftarrow{\int dt} \end{matrix} v_{(t)} \begin{matrix} \xrightarrow{\frac{d}{dt}} \\ \xleftarrow{\int dt} \end{matrix} a_{(t)}$$

Força [N] [Kgf]

$$\Sigma F_{(t)} = M a_{(t)} = M \ddot{x}_{(t)}$$

$$\Sigma F_R = \Sigma F_{action} - \Sigma F_{reaction}$$

$$f_{(t)} = -K x_{(t)}$$

$$f_{(t)} = -B \dot{x}_{(t)}$$

Torque [N.m]

$$\Sigma T_{(t)} = J \gamma_{(t)} = M \ddot{\theta}_{(t)}$$

$$\Sigma T_R = \Sigma T_{action} - \Sigma T_{reaction}$$

$$T_{(t)} = -K \theta_{(t)}$$

$$T_{(t)} = -B \dot{\theta}_{(t)}$$

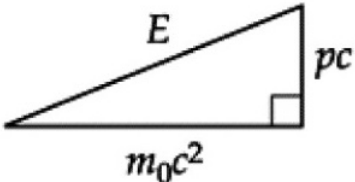
$$T = F \times r$$

Energia [Joule]

$$W = F d$$

$$W = P \Delta t$$

$$E = M C^2$$

$$E^2 = (pc)^2 + (m_0 c^2)^2$$


[2]

Energia Cinética [Joule]

$$E_c = \frac{1}{2} m v^2$$

Energia Potencial [Joule]

$$E_p = m g h$$

Energia Térmica

Q – *Heat energy*

$Q_{(t)}$ – *temperature*

R – *heat resistance*

$$Q = \frac{Q_{1(t)} - Q_{2(t)}}{R}$$

¹Apontamento