MATH BASICS

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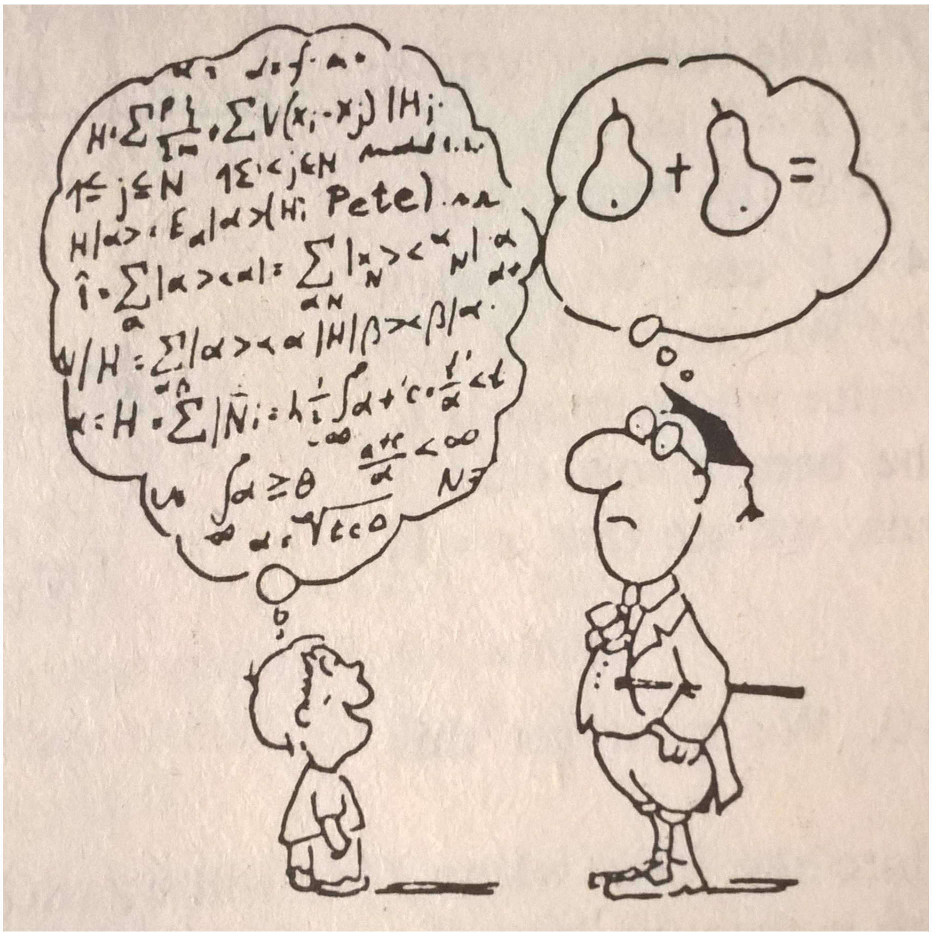
Mathematic



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Most up to date model 16052025

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/**************************************************************************
  LCD
Author: Sergio Santos
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License: GNU General Public License
Hardware: all
Date: 12112022
Comment:
  Tested Atemga128 16Mhz and Atmega328 8Mhz
*************************************************************************/
```

1 Definition

1.1 Exponents

1.

$$a^n = a \times a \times a \times a \; .... \; n \quad factors \qquad (n \in \mathbb{N},\ a \in \mathbb{R})$$

2.

$$a^{-m} = \frac{1}{a^m} \qquad (m \in \mathbb{Z}^+,\ a \in \mathbb{R},\ a \neq 0)$$

$$and:$$

$$\frac{1}{a^{-m}} = a^m$$

3.

$$a^0 = 1 \qquad (a \in \mathbb{R},\ a \neq 0)$$

1.2 Rational Exponents:

1.

$$\sqrt[n]{a} = r \qquad (a > 0,\ n \in \mathbb{N},\ n \geqslant 2,\ r > 0), \iff r^n = a$$

2.

$$a^{\frac{1}{n}} = \sqrt[n]{a}; \qquad (a > 0,\ n \geqslant 2,\ n \in \mathbb{N})$$

3.

$$a^{\frac{-1}{n}} = \sqrt[n]{a^{-1}}; \qquad (a > 0,\ n > 0,\ n \in \mathbb{N})$$

4.

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}; \qquad (a > 0;\ m,\ n \in \mathbb{Z},\ n \geqslant 2)$$

2 Law

2.1 Exponents

1.

$$a^m \times a^n = a^{m+n} \qquad (m,\ n \in \mathbb{N})$$

$$a^m \times a^n = a^{m+n} \qquad (m,\ n \in \mathbb{Z};\ a \neq 0,\ if \quad m \quad or \quad n < 0)$$

2.

$$\frac{a^m}{a^n} = a^{m-n} \qquad (m,\ n \in \mathbb{Z};\ a \in \mathbb{R};\ a \neq 0)$$

3.

$$(ab)^m = a^m\, b^m \qquad (m \in \mathbb{Z})$$

4.

$$(a^m)^n = a^{mn} \qquad (m,\ n \in \mathbb{Z})$$

2.2 Rational Exponents

1.

$$a^r \times a^t = a^{r+t} \qquad (a > 0; \quad r,\ t \in \mathbb{Q})$$

2.

$$\frac{a^r}{a^t} = a^{r-t} \qquad (a > 0; \quad r,\ t \in \mathbb{Q})$$

3.

$$(a^t)^r = a^{tr} \qquad (a > 0, \quad t,\ r \in \mathbb{Q})$$

4.

$$(ab)^t = a^t\, b^t\ ; \quad \left(\frac{a}{b}\right)^t = \frac{a^t}{b^t}\ ; \qquad (a,\ b > 0, \quad t \in \mathbb{Q})$$

$$and:$$

$$a^t\, b^t = (ab)^t \qquad and \qquad \frac{a^t}{b^t} = \left(\frac{a}{b}\right)^t$$

2.3 Distributive law

$$a(b + c) = ab + ac$$

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= ac + bc + ad + bd\end{aligned}$$

$$A^2 - B^2 = (A - B)(A + B)$$

2.4 Commutative law

$$ab = ba$$

3 Properties

3.1 Addition

$$0 + a = a$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \qquad (b \neq 0)$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \qquad (b \neq 0)$$

3.2 Multiplication

$$0 \times a = 0$$

$$\frac{0}{a} = 0 \times \frac{1}{a} = 0$$

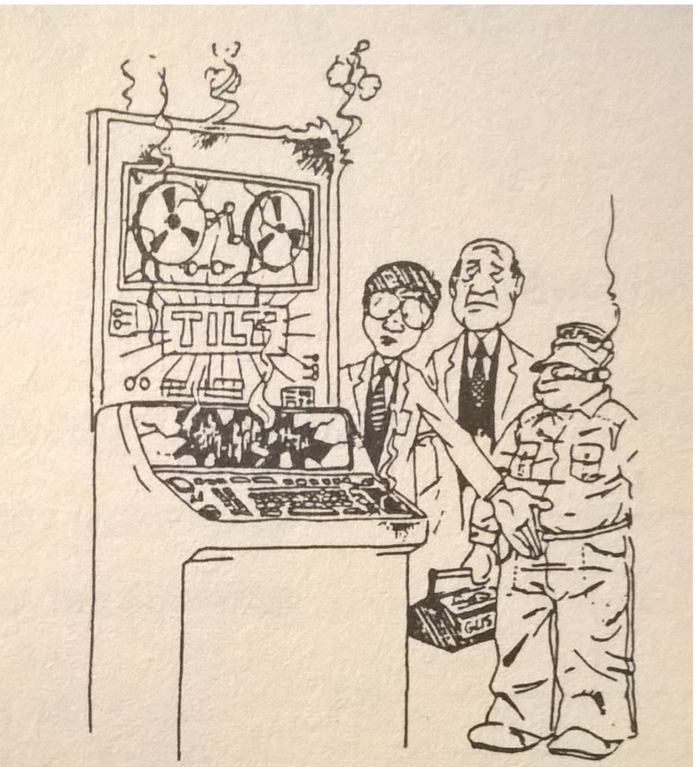
$$1 \times a = a$$

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \qquad (b \neq 0;\ d \neq 0)$$

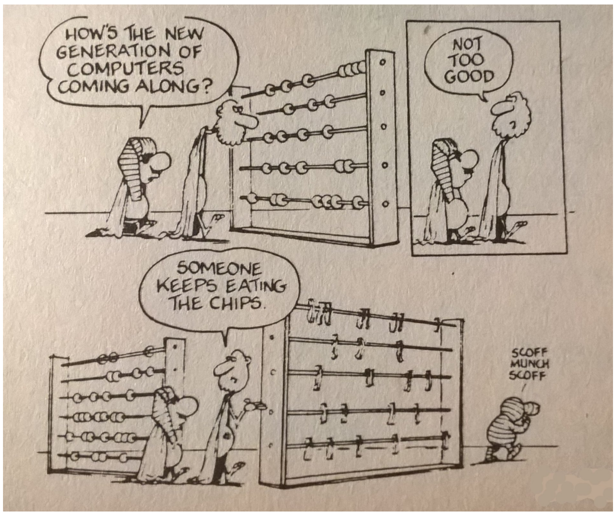
3.3 Division

$$\frac{a}{0} = undefined$$

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr} \qquad (q \neq 0;\ r \neq 0;\ s \neq 0)$$



"All I did was ask it to divide by zero!"



4 Examples

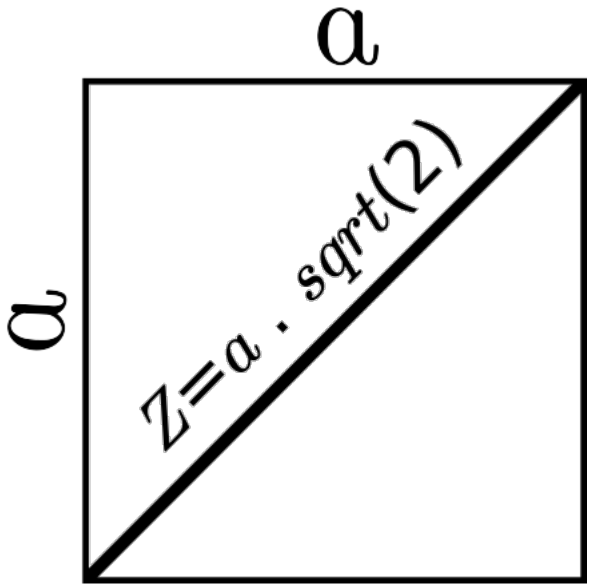
$$\sqrt{a^2} = a \qquad (a > 0) \qquad\qquad \sqrt{a^n} = a^{\frac{n}{2}}$$

$$\sqrt{\frac{1}{a}} = \frac{1}{\sqrt{a}}$$

$$a^{\frac{m}{1}} = a^m \qquad\qquad \frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a \angle \alpha^\circ \; b \angle \beta^\circ}{c \angle \gamma^\circ} = \frac{a \times b}{c} \quad \angle \left(\alpha^\circ + \beta^\circ - \gamma^\circ\right)$$

$$If \quad a \,.\, b \;=\; 0, \quad then \quad a \;=\; 0 \quad or \quad b \;=\; 0$$



Square Property

$$Z = \sqrt{a^2 \;+\; a^2}$$
$$Z = \sqrt{2\, a^2}$$
$$Z = \sqrt{2} \quad \sqrt{a^2}$$
$$Z = \sqrt{2} \quad a$$

$$a \,\angle\alpha \;+\; b\,\angle\beta \;=\; \sqrt{(a\;\sin\alpha \;+\; b\;\sin\beta)^2 \;+\; (a\;\cos\alpha \;+\; b\;\cos\beta)^2}$$

and

$$\angle\,\arctan(\frac{(a\;\sin\alpha \;+\; b\;\sin\beta)}{(a\;\cos\alpha \;+\; b\;\cos\beta)})$$

$$\lim_{s\to 0} GH(s) = \lim_{s\to 0} \quad 7\,\frac{4+3s}{5+2s+6s^2}$$

$$= \lim_{s\to 0} \quad 7\,\frac{4(1+\frac{3}{4}s)}{5(1+\frac{2}{5}s+\frac{6}{5}s^2)}$$

$$= 7\,\frac{4}{5}$$

$$y'(x) = e^{4x+5}$$
$$= 4\, e^{4x+5}$$

Derivatives made easy.

$y = \cos^3(\operatorname{sen}(x^2+x))$

$$y' = x^{3\,'}\Big|_{x=\cos(\operatorname{sen}(x^2+x))} \cdot \cos(x)'\Big|_{x=\operatorname{sen}(x^2+x)} \cdot \operatorname{sen}(x)'\Big|_{x=x^2+x} \cdot (x^2+x)'$$

$$= 3\,x^2\Big|_{x=\cos(\operatorname{sen}(x^2+x))} \cdot (-1)\cdot\operatorname{sen}(x)\Big|_{x=\operatorname{sen}(x^2+x)} \cdot \cos(x)\Big|_{x=x^2+x} \cdot (2x+1)$$

$$= 3\cos^2(\operatorname{sen}(x^2+x)) \cdot (-1) \cdot \operatorname{sen}(\operatorname{sen}(x^2+x)) \cdot \cos(x^2+x) \cdot (2x+1)$$

$$= -(6x+3) \cdot \cos^2(\operatorname{sen}(x^2+x)) \cdot \operatorname{sen}(\operatorname{sen}(x^2+x)) \cdot \cos(x^2+x)$$

$$y = \frac{(x+3)^3}{x^{3/2}}$$

$$y' = \frac{\left((x+1)^3\right)' \cdot x^{3/2} - \left(x^{3/2}\right)' \cdot (x+1)^3}{x^3}$$

$$= \frac{3(x+1)^2 \cdot x^{3/2} - \frac{3}{2} \cdot x^{1/2} \cdot (x+1)^3}{x^3} \qquad \text{HCF}$$

$$= \frac{\left[3(x+1)^2 \cdot x - \frac{3}{2}(x+1)^3\right] x^{\frac{1}{2}}}{x^3}$$

$$= \frac{3(x+1)^2 \cdot x - \frac{3}{2}(x+1)^3}{x^{5/2}} \qquad \text{HCF}$$

$$= \frac{(x+1)^2 \cdot \left(3x - \frac{3}{2}(x+1)\right)}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot \left(3x - \frac{3}{2}x - \frac{3}{2}\right)}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot \left(\frac{3}{2}x - \frac{3}{2}\right)}{x^{5/2}}$$

i.e) $\frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

$x^{\frac{3}{2}} = x + x^{\frac{1}{2}}$

$$= \frac{(x+1)^2 \cdot \frac{3}{2}(x-1)}{x^{5/2}}$$

$\frac{1}{2} - \frac{6}{2} = -\frac{5}{2}$

$\frac{6}{2} - \frac{3}{2} = \frac{3}{2}$

$$= \frac{3}{2} \cdot \frac{(x+1)^2 \cdot (x-1)}{x^{5/2}}$$

$$\int x = \frac{x^2}{2}$$

$$\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}} = \pi$$

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + \ldots$$

5 Methods

HCF - highest common factor (pôr variável em evidência)

Factorisation

LCD or LCM - Lowest common denominator or lowest common multiple

Bibliografia

[1] TIPLER, PAUL A. e GENE MOSCA: *PHYSICS FOR SCIENTISTS AND ENGINEERS, Extended Version fifth edition.* W. H. Freeman and Company, 1999. 14

Apêndice A

Anexos

Definição 1 *Capacitância*

$$
\begin{aligned}
Q_c(t) &= \int^t i(t) \quad dt \\
&= Q_c(0^-) + \int_{0^-}^t i(t) \quad dt \\
V_c(t) &= \frac{Q_c(t)}{C} \\
&= \frac{1}{C} \quad \int^t i_c(t) \quad dt \\
&= \frac{Q_c(0^-)}{C} + \frac{1}{C} \quad \int_0^t i_c(t) \quad dt \\
&= V(0^-) + \frac{1}{C} \quad \int_0^t i_c(t) \quad dt \\
i_c(t) &= C \quad \frac{dV_c(t)}{dt} \\
W &= \frac{1}{2}\, C\, V^2
\end{aligned}
$$

Definição 2 *Indutância*

$$
\begin{aligned}
\psi_L(t) &= \int^t V_L(t) \quad dt \\
&= \psi_L(0^-) + \int_{0^-}^t V_L(t) \quad dt \\
V_L(t) &= L \quad \frac{di_L(t)}{dt} \\
i_L(t) &= \frac{\psi_L(t)}{L} \\
&= \frac{1}{L} \quad \int^t V_L(t) \quad dt \\
&= \frac{\psi_L(0^-)}{L} + \frac{1}{L} \quad \int_0^t V_L(t) \quad dt \\
&= i_L(0^-) + \frac{1}{L} \quad \int_0^t V_L(t) \quad dt \\
W &= \frac{1}{2}\, L\, i^2
\end{aligned}
$$

Definição 3 *Resistência*

$$
\begin{aligned}
V_R(t) &= R \quad i_R(t) \\
i_R(t) &= \frac{V_R(t)}{R} \\
P &= Ri^2 \\
P &= \frac{U^2}{R} \\
W &= P\, \Delta t
\end{aligned}
$$

Definição 4 *Valor Médio*

$$X_{av} = \frac{1}{T} \int_0^T X(t)dt$$

Definição 5 *Valor Eficaz*

$$X_{ef} = \sqrt{\frac{1}{T} \int_0^T X^2(t)dt}$$

Motion

$$x_{(t)} = x_o + v_o t + \frac{1}{2} a t^2$$

$$x_{(t)} \quad \underset{\int\, dt}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftarrows}} \quad v_{(t)} \quad \underset{\int\, dt}{\overset{\frac{\mathrm{d}}{\mathrm{d}t}}{\rightleftarrows}} \quad a_{(t)}$$

Força [N] [Kgf]

$$\sum F_{(t)} = M\, a_{(t)} = M\, \ddot{x}_{(t)}$$

$$\sum F_R = \sum F_{action} - \sum F_{reaction}$$

$$\begin{aligned} f_{(t)} &= -K\, x_{(t)} \\ f_{(t)} &= -B\, \dot{x}_{(t)} \end{aligned}$$

Torque [N.m]

$$\sum T_{(t)} = J\, \gamma_{(t)} = M\, \ddot{\theta}_{(t)}$$

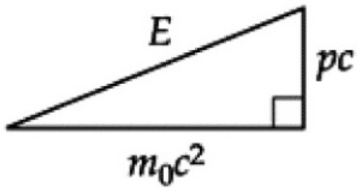
$$\sum T_R = \sum T_{action} - \sum T_{reaction}$$

$$\begin{aligned} T_{(t)} &= -K\, \theta_{(t)} \\ T_{(t)} &= -B\, \dot{\theta}_{(t)} \\ T &= F \times r \end{aligned}$$

Energia [Joule]

$$\begin{aligned} W &= F\, d \\ W &= P\, \Delta t \\ E &= M\, C^2 \end{aligned}$$

$$E^2 = (pc)^2 + (m_0 c^2)^2$$



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Energia Cinética [Joule]

$$E_c = \frac{1}{2} m\, v^2$$

Energia Potencial [Joule]

$$E_p = m\, g\, h$$

Energia Térmica

$$\begin{aligned} &Q - Heat \quad energy \\ &Q_{(t)} - temperature \\ &R - heat \quad resistance \end{aligned}$$

$$Q = \frac{Q_{1(t)} - Q_{2(t)}}{R}$$

[1]

[1] Apontamento