example.

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$F(s) = \frac{s+3}{(s+1)(s+2)}$$

$$F(s) = \frac{s+3}{(s+1)(s+2)} = \frac{a1}{s+1} + \frac{a2}{s+2}$$

method 1:

$$s+3 = a_1(s+2) + a_2(s+1)$$
$$= a_1 s + a_1 2 + a_2 s + a_2$$
$$= (a_1 + a_2)s + 2a_1 + a_2$$

$$a_1 + a_2 = 1 \quad \& \quad 2a_1 + a_2 = 3$$
$$a_2 = 1 - a_1$$
$$\Rightarrow 2a_1 + (1 - a_1) = 3$$
$$2a_1 - a_1 + 1 = 3$$
$$a_1 = 2$$
$$\therefore a_2 = 1 - a_1$$
$$= 1 - 2$$
$$= -1$$

method 2:

$$a_1 = \left[\cancel{(s+1)} \frac{s+3}{\cancel{(s+1)}(s+2)}\right]\Bigg|_{s=-1} = 2$$

$$a_2 = \left[\cancel{(s+2)} \frac{s+3}{(s+1)\cancel{(s+2)}}\right]\Bigg|_{s=-2} = -1$$

$$F(s) = \frac{2}{s+1} + \frac{-1}{s+2}$$

$$G(s) = \frac{s^3 + 5s^2 + 9s + 7}{(s+1)(s+2)}$$

$$= s + 2 + \frac{s+3}{(s+1)(s+2)}$$

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$$F(s) = \frac{2s + 12}{s^2 + 2s + 5}$$

(c.a)

$$s^2 + 2s + 5 = (s + 1 + j2)(s + 1 - j2)$$
$$= ((s+1) + j2)((s+1) - j2)$$
$$= (s+1)^2 + 2^2$$

$$F(s) = \frac{10 + 2(s+1)}{(s+1)^2 + 2^2}$$
$$= 5 \cdot \frac{2}{(s+1)^2 + 2^2} + 2 \cdot \frac{s+1}{(s+1)^2 + 2^2}$$

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$$F(s) = \frac{s^2 + 2s + 3}{(s+1)^3}$$

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$$\ddot{x} + 2\dot{x} + 5x = 3 \; ; \; x(0) = 0, \; \dot{x}(0) = 0$$

$$s^2 X(s) + 2s X(s) + 5 X(s) = \frac{3}{s}$$

$$X(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{3}{5} \cdot \frac{1}{s} - \frac{3}{5} \cdot \frac{(s+2)}{s^2 + 2s + 5}$$

$$= \frac{3}{5} \cdot \frac{1}{s} - \frac{3}{10} \cdot \frac{2}{(s+1)^2 + 2^2} - \frac{3}{5} \cdot \frac{s+1}{(s+1)^2 + 2^2}$$

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