6)

$$\frac{\Theta(s)}{E_f(s)} = \frac{1,25}{s\left(\frac{s}{20}+1\right)(s+1)}$$

$$E_f(s) = 24 \operatorname{sen}(\underbrace{100\pi}, t)$$

$$\Theta(t) = 24 \times \left| \frac{\Theta(j\omega)}{E_f(j\omega)} \Big|_{\omega = 100\pi} \right| \times \operatorname{sen}\Big(100\pi t +$$

$$\arg\left[\frac{\Theta(j\omega)}{E_f(j\omega)} \Big|_{\omega = 100\pi} \right]\Big)$$

$$\boxed{\text{modulo} = \sqrt{x^2}}$$

$$\left| \frac{\Theta(j\omega)}{E_f(j\omega)} \right|_{\omega=100\pi} = \frac{1,25}{100\pi \times \sqrt{\left(\frac{100\pi}{20}\right)^2+1} \times \sqrt{(100\pi)^2+1}}$$

$$= 8,04 \cdot 10^{-7}$$

$$\frac{1}{100\pi} \quad \frac{1}{\sqrt{\left(\frac{100\pi}{20}\right)^2+1}}$$

$$\boxed{\frac{\Theta(j\omega)}{E_f(j\omega)} \Big|_{100\pi}} = -90° - \operatorname{arctg}\left(\frac{100\pi}{20}\right) - \operatorname{arctg}(100\pi)$$

$$= -266,18°$$

$$= -4,65 \text{ rad}$$

$$\Theta(t) = 24 \times 8 \times 10^{-7} \operatorname{sen}(100\pi t - 4,65)$$

6a)

$$GH(s) = \frac{K e^{-Ts}}{s(s+1)(s+2)}$$

$$T = 0$$

$$K = ?$$

$$\begin{cases} i) = MG = 15,6 \text{ dB} \\ ii) = MF = 25° \end{cases}$$

tesis.

6 a)

$$GH(s) = \frac{K}{s(s+1)(s+2)}$$

calcular Margem ganho.

ceniome. $\left\{ \arg \left[\frac{K}{s(s+1)(s+2)} \right] = -\pi = -180^\circ \right.$

$\Leftrightarrow \quad -90^\circ - \text{arctg}(\omega_\pi) - \text{arctg}\left(\frac{\omega_\pi}{2}\right) = -\pi$

$$\underbrace{\text{arctg}(\omega_\pi)}_{\alpha} + \underbrace{\text{arctg}\left(\frac{\omega_\pi}{2}\right)}_{\beta} = 90^\circ$$

$$\text{tg}(\alpha \pm \beta) = \frac{\text{tg}\,\alpha \pm \text{tg}\,\beta}{1 \mp \text{tg}\,\alpha\,\text{tg}\,\beta}$$

$$\frac{\omega_\pi + \frac{\omega_\pi}{2}}{1 - \omega_\pi \cdot \frac{\omega_\pi}{2}} = \infty$$

$\Leftrightarrow \quad \frac{\omega_\pi^2}{2} = 1 \quad \Leftrightarrow \quad \omega_\pi = \pm\sqrt{2}$ rad/s.

$\omega_\pi = \sqrt{2}$ rad/seg.

$$MG_{dB} = 20 \log_{10} \frac{1}{|GH(j\omega_\pi)|}$$

$$MG = \left[\frac{1}{|GH(j\omega_\pi)|} \right]$$

$$15{,}6 = 20 \log_{10} \left[\frac{1}{\underbrace{|GH(j\omega_\pi)|}_{\text{modulo.}}} \right]$$

$$\therefore \quad MG = 10^{\frac{15{,}6}{20}}$$

$$= 6{,}02$$

tesis

$$6{,}02 = \frac{1}{|GH(j\omega_{\pi})|}$$

$$= \frac{1}{\dfrac{K}{\omega_{\pi}\sqrt{\omega_{\pi}^{2}+1}\cdot\sqrt{\omega_{\pi}^{2}+4}}}$$

$$6{,}02\,K = \sqrt{2}\cdot\sqrt{2+1}\cdot\sqrt{2+4}$$

$$K = \frac{\sqrt{2}\cdot\sqrt{3}\cdot\sqrt{6}}{6{,}02}$$

$$= 0{,}9958$$

6a)

$$GH(s) = \frac{K e^{-Ts}}{s(s+1)(s+2)}$$

Evaluate K when $T = \phi$ for $\boxed{GM = 15,6\ dB \quad PM = 25^{\circ}}$

sei exam!

If $T=0$ then:

$$GH(j\omega) = \frac{K}{j\omega \cdot (j\omega+1)(j\omega+2)}$$

- GAIN MARGIN (GM):

when $\Rightarrow$ $\arg[GH(j\omega)] = -\pi$

then $\Rightarrow$ $MG = \dfrac{1}{|GH(j\omega)|}$

Note:

$MG_{dB} = 20 \log MG$

Find for what values of "ω" $\arg[GH(j\omega)] = -\pi$

$$\arg[GH(j\omega)] = -\pi$$

$$\arg\left[\frac{K}{j\omega(j\omega+1)(j\omega+2)}\right] = -180^{\circ}$$

$$\arg(K) - [\arg(j\omega) + \arg(j\omega+1) + \arg(j\omega+2)] = -180^{\circ}$$

$$\phi - [90^{\circ} + tg^{-1}(\omega) + tg^{-1}(\tfrac{\omega}{2})] = -180^{\circ}$$

$$tg^{-1}(\omega) + tg^{-1}\left(\frac{\omega}{2}\right) = 90^{\circ}$$

$$tg^{-1}\left(\frac{\omega+\frac{\omega}{2}}{1-\frac{\omega^2}{2}}\right) = 90^{\circ}$$

$$\frac{\omega+\frac{\omega}{2}}{1-\frac{\omega^2}{2}} = \underbrace{tg\,90^{\circ}}_{\infty}$$

Note:

arctangent addition formula:

$$tg^{-1}(\mu) \pm tg^{-1}(v) = tg^{-1}\left(\frac{\mu+v}{1-\mu.v}\right)$$

$$\therefore\ 1-\frac{\omega^2}{2} = \phi \Rightarrow \frac{\omega^2}{2} = 1 \Rightarrow \omega = \sqrt{2}\ rad/sec$$

Replace value of "ω" in the MG equation in order to find "K":

$$|GH(j\omega)| = \left|\frac{K}{j\omega(j\omega+1)(j\omega+2)}\right|$$

6 a) continuação

$$|G.H(j\omega)| = \frac{K}{\omega.\sqrt{\omega^2+1}.\sqrt{\omega^2+2^2}} \quad |\omega=\sqrt{2}$$

$$= \frac{K}{\sqrt{2}.\sqrt{\sqrt{2}^2+1^2}.\sqrt{\sqrt{2}^2+2^2}} = \frac{K}{\sqrt{2}.\sqrt{3}.\sqrt{6}}$$

$$= \frac{K}{6}$$

$$MG_{dB} = 20 \, Log \frac{1}{|GH(j\omega)|}$$

$$15{,}6 dB = 20 \, Log \frac{1}{\frac{K}{6}} \Rightarrow K = 0{,}9958$$

- Phase Margin (PM):

when $\Rightarrow |GH(j\omega)| = 1$ then $\Rightarrow MF = 180° + arg[GH(j\omega)]$

Find for what values of "ω" $180° + arg[GH(j\omega)] = 25°$

$$180° + arg\left[\frac{K}{j\omega.(j\omega+1)(j\omega+2)}\right] = 25°$$

$$\phi - 90° - tg^{-1}(\omega) - tg^{-1}\left(\frac{\omega}{2}\right) = -155$$

$$tg^{-1}(\omega) + tg^{-1}\left(\frac{\omega}{2}\right) = 65°$$

$$tg^{-1}\left[\frac{\omega+\frac{\omega}{2}}{1-\frac{\omega^2}{2}}\right] = 65°$$

$$\frac{\omega+\frac{\omega}{2}}{1-\frac{\omega^2}{2}} = tg(65°) \Rightarrow \omega + \frac{\omega}{2} = tg(65°).\left(1-\frac{\omega^2}{2}\right)$$

$$\frac{tg(65°)}{2}\omega^2 + \left(1+\frac{1}{2}\right).\omega - tg(65°) = 0$$

$\boxed{\text{the results always has to be} > \phi \,!!}$

$$\begin{cases} \omega = -2{,}2805 \; \times \\ \omega = 0{,}8781 \; \checkmark \end{cases}$$

6 a) Replace the value of "w" in the MG equation in order to find the value of "K":

$$|GH(j\omega)| = 1 \iff \frac{K}{\omega \cdot \sqrt{\omega^2+1} \cdot \sqrt{\omega^2+4}} \Bigg|_{\substack{EQ.A \\ \omega = 0{,}8781}}$$

$$\boxed{K = 2{,}55}$$

b) In order to have phase margin equals to 25° the value of K must be 2,55 at freq. $\omega = 0{,}8781$.

$$\angle G(j\omega) = -\omega T \text{ (rad)}$$

$$\frac{-25° \cdot \pi}{180°} = -0{,}8781 \cdot T$$

$$T = 0{,}497 \text{ sec.}$$

c) check the solutions!

6a) $$|G(j\omega)| = \frac{K}{\omega\sqrt{\omega^2+1} \cdot \sqrt{\omega^2+2^2}} = \frac{K}{\omega\sqrt{(\omega^2+1)(\omega^2+4)}}$$

$$\angle G(j\omega) = \overset{=0}{Arg(K)} + arg(e^{-Ts}) - arg(s) - arg(s+1) - arg(s+2)$$

$$= 0 + -T\omega - \frac{\pi}{2} - \arctan(\omega) - \arctan\left(\frac{\omega}{2}\right)$$

$$-T\omega - \frac{\pi}{2} - \arctan(\omega) - \arctan\left(\frac{\omega}{2}\right) = 0 \Big|_{\omega=0{,}878}$$

$$-T\omega = \frac{\pi}{2} - \arctan(\omega) - \arctan\left(\frac{\omega}{2}\right) \Big|_{\omega=0{,}878}$$

$$T = 0{,}497$$