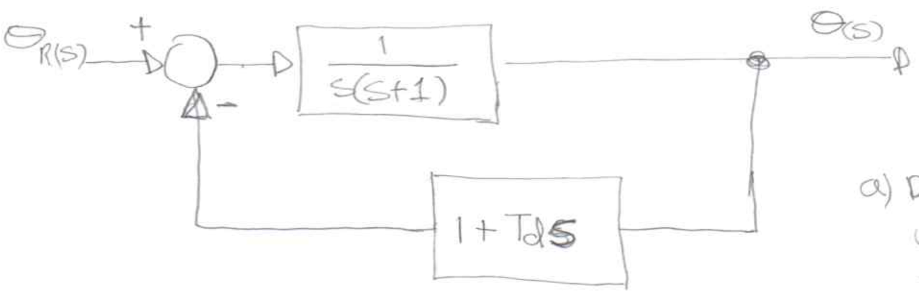
3.



caso especial

a) Draw root locus in function of Td:

R. - obtain the characteristic equation in the form: $\boxed{GH_{(s)} = -1}$

$$FTMF = \frac{\frac{1}{s(s+1)}}{1+\frac{1+T_d s}{s(s+1)}} = \frac{\frac{1}{\cancel{s(s+1)}}}{\frac{s(s+1)+(1+T_d s)}{\cancel{s(s+1)}}}$$

$$= \frac{1}{s(s+1)+(1+T_d s)}$$

$$\boxed{s(s+1)+(1+T_d s) = \phi}$$

characteristic equation

Funcao Caracteristica

$$1+GH_{(s)} \Rightarrow 1+\frac{1+T_d s}{s(s+1)} = \phi$$

$$= \frac{s(s+1)+(1+T_d s)}{\cancel{s(s+1)}} = \phi$$

$$= \boxed{s(s+1)+(1+T_d s) = \phi}$$

$$P(s) = s(s+1)+1+T_d s = 0$$

$$= s(s+1)+1 = -T_d s$$

$$= 1 = \frac{-T_d s}{s(s+1)+1} \Rightarrow$$

Funcao characteristica

$$\boxed{\frac{T_d s}{s(s+1)+1}} = -1$$

Sai exacam!

when the variable parameter "Td" does not appear as a multiplying factor of G.H(s).

$$s(s+1)+(1+T_d.s) = \phi$$

$$s(s+1)+1+T_d s = \phi$$

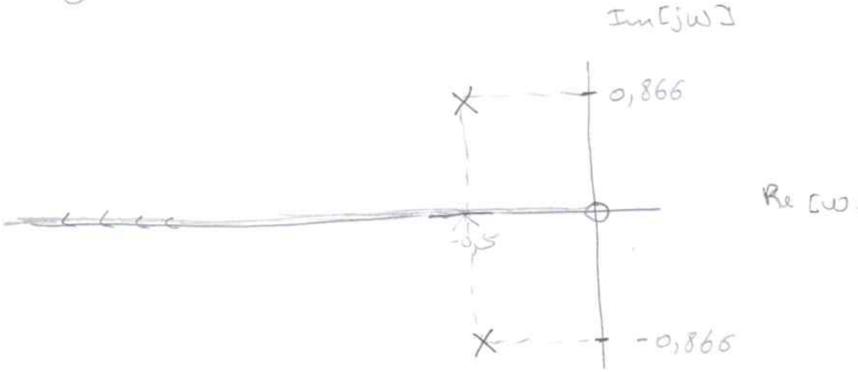
$$\boxed{\frac{T_d.s}{s(s+1)+1} = -1}$$

characteristic fraction where poles and zeros are adquired.

$$1+\frac{T_d s}{s(s+1)+1} = \phi$$

3. continuaces

R_3 Are branches of the root-locus on the Real Axis if the initial number of Real Poles and Real zeros to the right is ODD.



$$\frac{Td.S}{s(s+1)+1} \Rightarrow \begin{array}{l} \text{zeros} = \phi \\ \text{poles} = s^2+s+1 \Rightarrow -0,5 \pm 0,866j \end{array}$$

R_4 Determine de Asymptotes of Root-Locus.

- Number of Restinct asymptotes is $d-n = 2-1 = 1$
- Angles of Assymptotes $= \dfrac{(1+2h) \cdot 180°}{d-n}$

$\measuredangle = \pi$, in this case the asymptote is the Real axis, it does not make sense to calculate the Real axis intersection.

R_5 Find the Break-away and Break-in point:

$$\frac{d}{ds} Td = \phi$$

characteristic equation:

$$Td \times \frac{s}{s^2+s+1} = -1$$

$$Td = -\frac{s^2+s+1}{s}$$

$$\frac{d}{ds}\left[-\frac{s^2+s+1}{s}\right] = \phi \Leftrightarrow -\frac{s^2+1}{s^2} = \phi$$

$$-s^2+1 = \phi \Rightarrow \begin{cases} \boxed{s_1 = -1} \\ s_2 = 1 \times \end{cases}$$

Dont belong to Root Locus

3. continuaces

R6 Determine the angle of departure (angle of arrival) of the root locus from the complex pole:

$$\phi = 180° - \left(\sum_{i=1}^{d-1} \arg(s-p_i) \right) + \left(\sum_{i=1}^{n} \arg(s-z_i) \right)$$

Pole : $-0{,}5 + 0{,}866j$

$$\phi = 180° - [\arg(s+0{,}5+0{,}866j) - \arg(s)]\Big|_{s=-0{,}5+0{,}866j}$$

$$= 180° - \arg(1{,}732j) + \arg(-0{,}5+0{,}866j)$$

$$= 180° - 90 + 120°$$

$$= 210°$$

R7 Find the point where the root locus may cross the imaginary axis:

characteristic equation: $1 + G\,H(s)\Big|_{s=j\omega} = \phi$

$$1 + Td\,\frac{s}{s^2+s+1}\Bigg|_{s=j\omega} = \phi$$

$$\frac{s^2+s+1}{s^2+s+1} + \frac{Td.s}{s^2+s+1} = \frac{s^2+s+1+Td.s}{s^2+s+1} = \phi$$

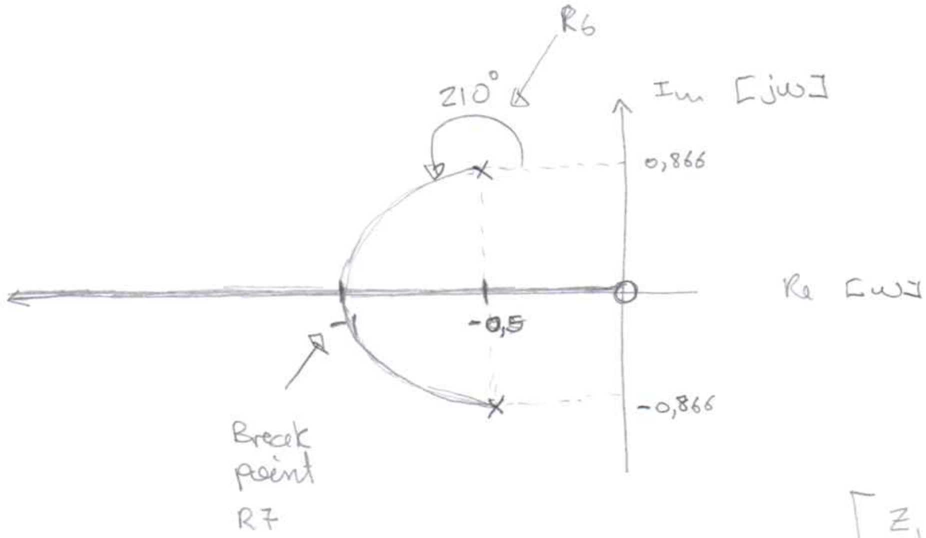
$$\Rightarrow \quad s^2+s+1+Td\,s\Big|_{s=j\omega} = \phi$$

$$-\omega^2 + j\omega + 1 + Td.j\omega = \phi$$

$$\begin{array}{l} \text{Real} \\ \text{Im} \end{array} \begin{cases} -\omega^2+1 = \phi \\ \cancel{j}\omega + Td\cancel{j}\omega \neq 0 \end{cases} \quad \begin{cases} \omega^2 = 1 \\ \omega = 0 \end{cases} \quad \vee \quad \begin{cases} \omega^2 = 1 \Leftrightarrow \omega = 1 \\ \underbrace{1+Td. = 0 \Leftrightarrow Td = -1}_{\text{Impossible}} \end{cases}$$

We can conclude that there are no interceptions with the imaginary axis.

3. continuação



Parameters.

$Z_1 = \phi$

$P_1 = -0,5 + 0,866j$

$P_2 = -0,5 - 0,866j$

Break in ~~P~~ $S = -1$

Angle departure of pole $-0,5 + 0,866j$

$\measuredangle$ 210°

b) Values of Td that the system does not oscilate

- Poles in the real axis does not oscilate
- the Break in point in $s = -1$ indicates when the system stops to oscilate

$$1 + GH(s) \Big|_{s=-1} = 0$$

calculador

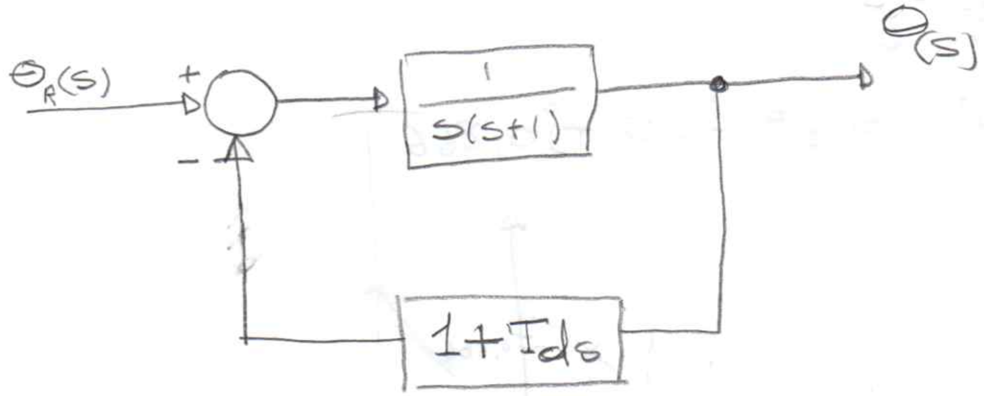
$-1 \to s$ ↵

$\text{solve}\left(1 + \frac{(x \cdot s)}{s^2+s+1} = \phi, x\right)$ ↵

$\boxed{x = Td = 1}$

$\therefore$ Td > 1 the system does not oscilate tesis PL 27/5/2009

3)



$$\frac{\frac{1}{s(s+1)}}{1+\frac{1+T_d s}{s(s+1)}}$$

$$\frac{1+T_d s}{s(s+1)} = -1$$

$$1+T_d s = -s(s+1)$$

$$\frac{1}{s(s+1)+1+T_d s} = \frac{1}{s^2+s(1+T_d)+1}$$

$$s^2+s+\overset{K}{1+T_d s} = 0$$

$$\boxed{s(s+1)+T_d s = -1}$$

$$\underbrace{s^2+s+1}_{1}+T_d s = 0$$

$$1+\boxed{\frac{T_d s}{s^2+s+1}} = 0$$

↳ FTLG

$$s^2+s+T_d s = -1$$

$$T_d s = -s^2-s-1$$

$$\frac{T_d s}{(s^2+s+1)} = \frac{-(s^2+s+1)}{(s^2+s+1)}$$

zeros: não há zeros

polos:

$$s(s+1) = -1-T_d s$$

$$1+T_d s = -s(s+1)$$

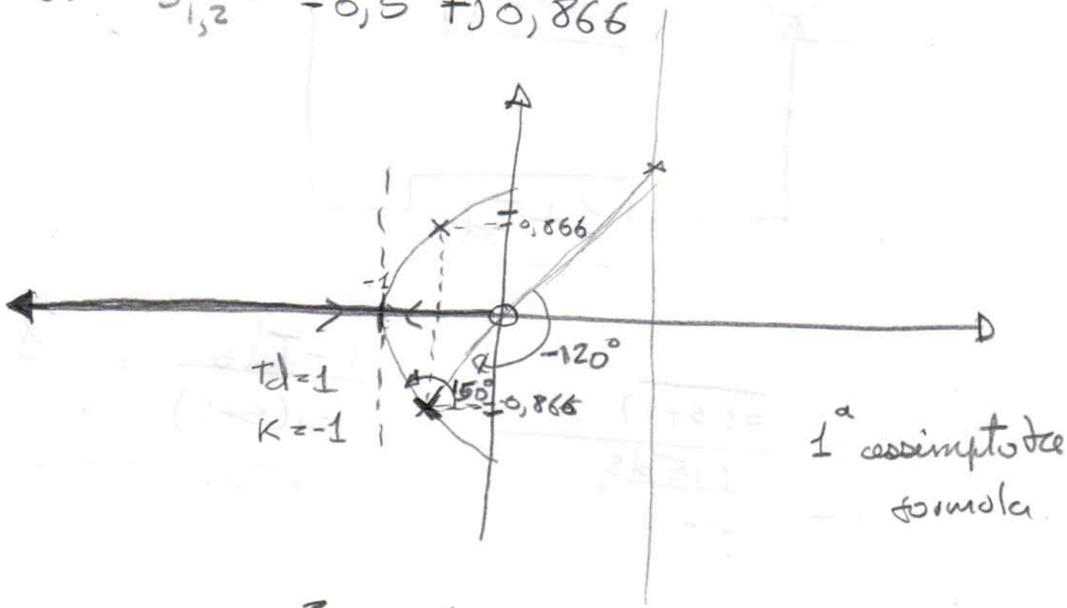
$$T_d s = -s^2-s-1$$

$$\boxed{T_d \cdot \frac{s}{s^2+s+1} = -1}$$

formato desejado

$$Td\,\frac{s}{s^2+s+1} = -1$$

$P = 2$
$N = 1$
zeros: 0
polos: $s_{1,2} = -0,5 \pm j0,866$



$$Td = \frac{s^2+s+1}{s}$$

$$\frac{d}{ds}Td = \frac{(2s+1).s-(s^2+s+1).1}{s^2} = 0$$

$$-s^2+1=0 \Leftrightarrow s = \pm 1.$$

$$Td = \left.\frac{s^2+s+1}{s}\right|_{s=-1} = 1$$

$$\phi = 180° - \sum_{i=1}^{d-1} \arg(s-p_i) + \sum_{i=1}^{\infty} \arg(s-z)\Bigg|_{s=-0,5-j0,866} =$$

$$\phi = 180° - \arg(s-(-0,5+j0,866)) + \arg(s+0)\Big|_{s=-0,5-j0,866}$$

$$\phi = 180° - \arg(-0,5-j0,866+0,5-j0,866) + \arg(-0,5-j0,866)$$

3o

$$FTMF = \frac{\frac{1}{(s+1)s}}{1 + \frac{1}{(s+1)s} \times (Tds+1)}$$

$$= \frac{1}{(Td\,s+1) + s(s+1)}$$

$D_{(S)}\big|_{FTMF} \Rightarrow (TdS+1) + s(s+1) = \phi$

$$Td\,s + 1 + s(s+1) = \phi \qquad \frac{1}{1+s(s+1)}$$

$$\frac{Td\,s+1+s(s+1)}{1+s(s+1)} = 0$$

$$\frac{Td\,s}{1+s(s+1)} + 1 = \phi$$

$$\boxed{\frac{Td\,s}{1+s^2+s} = -1}$$

$\Downarrow$

FTLG

Decompose
separate Numerador
Divide by rest

$$Td\,s + \underbrace{1 + s^2 + s}\ = 0 \qquad \frac{1}{1+s^2+s}$$

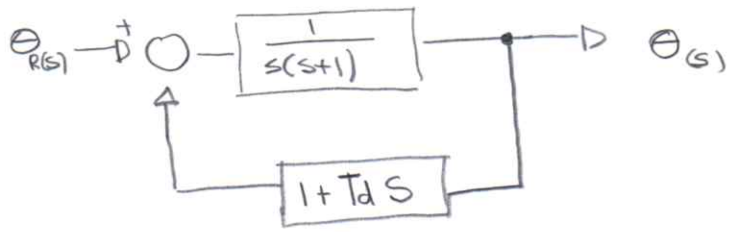
$$\boxed{\frac{Td\,s}{1+s^2+s} + 1 \neq 0}$$

$$\boxed{\frac{\square}{\square} = 1}$$

$\Downarrow$

FTLG

3.



$$GH_{(s)} = \frac{Td\,S+1}{s(s+1)}$$

special case.

c.a) $s(s+1)+1$

s^2+1s+1

$-0,5\pm 8,66j$

$$s(s+1)+Td\,S+1=\phi$$

$$\underbrace{s(s+1)+1}+Td\,s \neq 0$$

$$1+\boxed{\frac{Td\,s}{s(s+1)+1}}=0$$

FTLG – zeros: ϕ
– polos: 0; $-0,5\pm 8,66j$

$$s(s+1)+Td\,S+1=\phi \iff s^2+s+1+Td\,s=0 \Big|_{s=j\omega}$$

intercepcao
not needed.
$\omega<0$
$K<0$

$$td\,s = -(s^2+s+1)$$

$$Td\,\cancel{s} = -\frac{(s^2+s+1)}{s}$$

$$\frac{d\,Td}{d(s)}=0 \Rightarrow \text{Quebras}$$

$$\phi = \frac{(2s+1).s-s^2+s+1}{\cancel{s}^2}$$

$$\phi = (2s+1)s-s^2+s+1$$

$$= 2\cancel{s^2}+\cancel{s}-\cancel{s^2}-\cancel{s}-1$$

$$= s^2-1$$

$$\phi = s^2-1 \qquad s=\pm\sqrt{1}$$

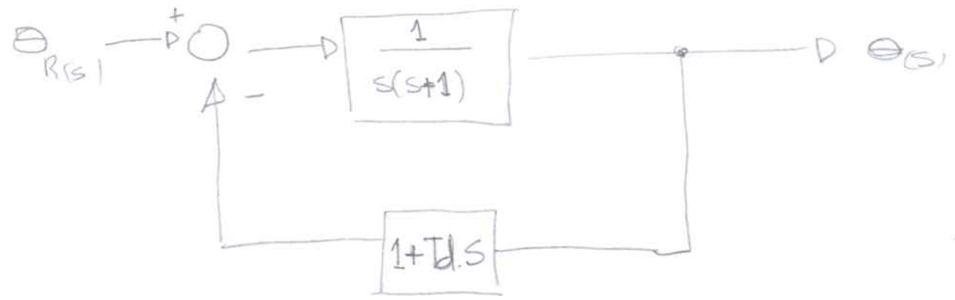
$\therefore$ $-\sqrt{1}$ pertence LGR

-1

tesis PL 27/5/2009.

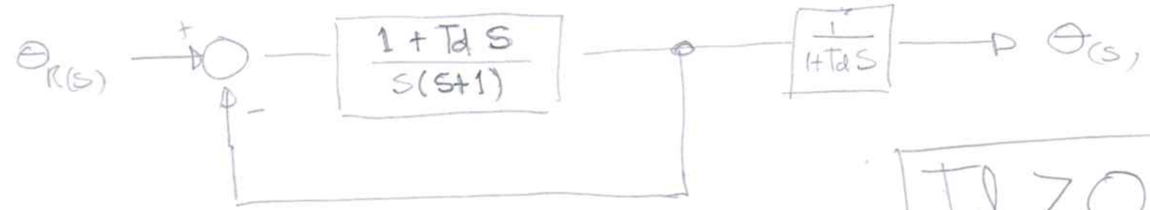
$\phi = 180^\circ + 90^\circ - 120^\circ = 150^\circ$

3.



ratoeira

special case



$\boxed{Td > 0}$ sempre

$$FTMA = \frac{1+Td\,s}{s(s+1)}$$

$$D_{(s)} = \boxed{s(s+1)} + \boxed{1+Td\,s} \rightarrow \frac{1+Td\,s}{s(s+1)}$$

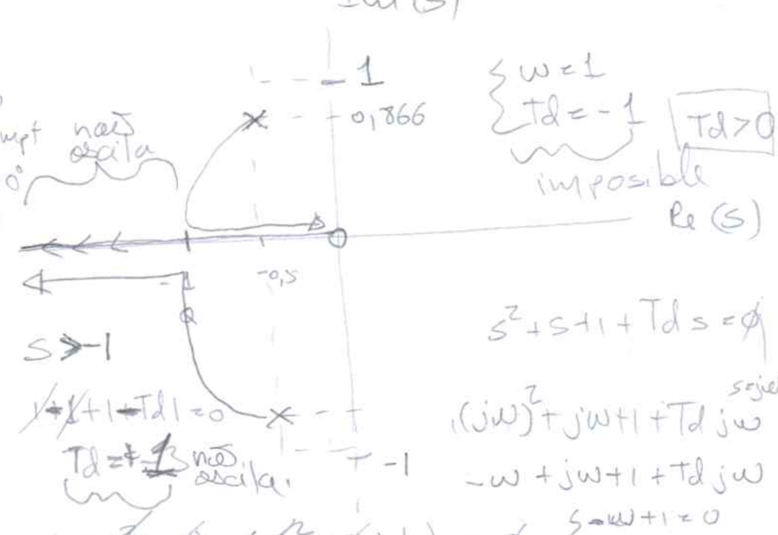
$$= \boxed{s^2+s+1} + \boxed{Td\,s} \rightarrow \frac{Td\,s}{s^2+s+1}$$

$$FTMA_{1°} = \frac{Td.s}{s^2+s+1} = FTMF \rightarrow D_{(s)} = s^2+s+1+Td\,s$$

LGR

- Zeros - 2 ramos
- Polos - 1 Assympt
- centroide +180°
- Ang Assymp → -1

não oscila



$\begin{cases} w=1 \\ Td=-1 \end{cases}$ $\boxed{Td>0}$

impossible

$$s^2+s+1+Td\,s = \phi$$

seja

$$(jw)^2 + jw + 1 + Td\,jw$$

$$-w + jw + 1 + Td\,jw$$

$$\begin{cases} -w+1=0 \\ w(Td+1)=0 \end{cases}$$

$s > -1$

$\cancel{1}\cancel{+1}+1-Td\cdot 1 = 0$

$Td = \underbrace{+1}$ não oscila

$$Td = -\frac{(s^2+s+1)}{s}$$

$$\frac{d\,td}{ds} = -\frac{(2s+1).s - s^2+s+1}{s^2}$$

$$\phi = \cancel{-}\frac{(2s^2+s)-(s^2+s+1)}{\cancel{s^2}}$$

$$(2s^2+\cancel{s}) - (\cancel{s^2}+\cancel{s}+1) = \phi$$

$$s^2-1=0 \quad s = \pm\sqrt{1} = -1$$