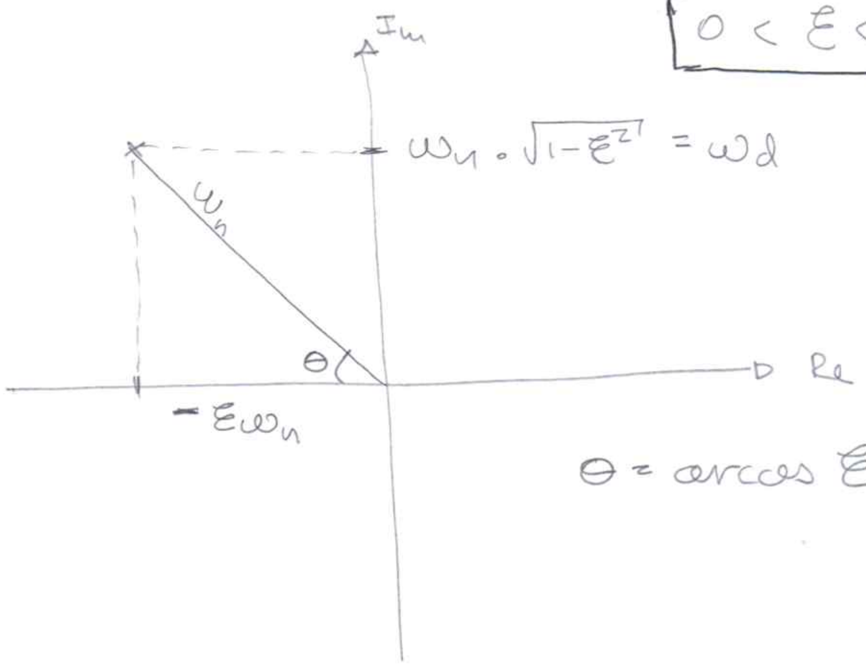
tips & tricks

$$\boxed{0 < \varepsilon < 1}$$



$\theta = \arccos \varepsilon$

$$p_1, p_2 = -\varepsilon\omega_n \pm j\,\omega_n\sqrt{1-\varepsilon^2}$$

ω_n - natural frequenci

ω_d → Damp frequency

tips & tricks.

$$Y(s) = \frac{s^2+2s+2}{(s+1)(s+2)}$$

NOTA: grau numerador tem que ser inferior ao do Dominador para aplicar as fracções parciais!

choice:

$$\begin{cases} (s+1)(s+1) = s^2+2s+1 \\ (s+1)^2+1 = s^2+2s+2 \end{cases}$$

$$\begin{cases} (s+1)(s+2) = s^2+3s+2 \\ (s+1)(s+2)-s = s^2+2s+2 \end{cases}$$

$$Y(s) = \frac{(s+1)^2+1}{(s+1)(s+2)} = \frac{(s+1)^{\cancel{2}}}{\cancel{(s+1)}(s+2)} + \frac{1}{(s+1)(s+2)}$$

$$= \frac{s+1}{s+2} + \frac{1}{(s+1)(s+2)}$$

$$= \frac{(s+2)-1}{s+2} + \frac{1}{(s+1)(s+2)}$$

$$= \frac{s+2}{s+2} - \frac{1}{s+2} + \frac{1}{(s+1)(s+2)}$$

$$= 1 - \frac{1}{s+2} + \frac{1}{(s+1)(s+2)}$$

c.a) $$\frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$A = \left.\frac{1}{s+2}\right|_{s=-1} = 1$$

$$B = \left.\frac{1}{s+1}\right|_{s=-2} = -1$$

$$\frac{1}{(s+1)(s+2)} = \frac{1}{(s+1)} + \frac{-1}{s+2}$$

$$Y(s) = 1 - \frac{1}{s+2} + \frac{1}{s+1} - \frac{1}{s+2}$$

$$= 1 - \frac{2}{s+2} + \frac{1}{s+1}$$

$$Y(s) = \frac{s^2+2s+2}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{s+2}$$

$$A = \left.\frac{s^2+2s+2}{(s+2)}\right|_{s=-1} = \frac{1}{1} = 1$$

$$B = \left.\frac{s^2+2s+2}{(s+1)}\right|_{s=-2} = \frac{2}{-1}$$

$$Y(s) = \frac{1}{(s+1)} - \frac{2}{(s+2)}$$

note this does not work

tips & tricks

$$s^2 + 2\mathcal{E}\omega_n s + \omega_n = \phi$$

if $\mathcal{E} = 1$ or $\mathcal{E} > 1$ critically or over damped.

$$s = -\mathcal{E}\omega_n \pm \omega_n\sqrt{\mathcal{E}^2 - 1} \quad \mathrm{Re}$$

if $\phi < \mathcal{E} < 1$ under damped

$$s = -\mathcal{E}\omega_n \pm j\omega_n\sqrt{1-\mathcal{E}^2} \quad \text{complex.}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\mathcal{E}^2}} \;;\; y(t_p) = 1 + e^{\frac{-\mathcal{E}\omega_n}{\sqrt{1-\mathcal{E}^2}}}$$

$$M_p = e^{\frac{\mathcal{E}\pi}{\sqrt{1-\mathcal{E}^2}}}$$

$$t_s = \frac{4}{\mathcal{E}\omega_n}$$

$$t_r \cong \frac{e^{\theta/\tan(\theta)}}{\omega_n} \;,\; \theta = \arccos(\mathcal{E})$$

$$\mathcal{E} = \sqrt{\frac{1}{1+\left(\frac{\pi t_s}{4t_p}\right)^2}}$$

$$\omega_n = \frac{4}{\mathcal{E} t_s}$$

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \times 100\%$$

$$y(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s \times Y(s)$$

$$\boxed{1 + \frac{a}{b} = \frac{b}{b} + \frac{a}{b} = \frac{b+a}{b}} \qquad \boxed{1 = \frac{\square}{\square}}$$

$$x(\phi) = \lim_{t\to 0} x(t) = \lim_{s\to\infty} s \cdot X(s)$$

$$x(\infty) = \lim_{t\to\infty} x(t) = \lim_{s\to\phi} s \cdot X(s)$$

tips & tricks

ex:

$$\frac{4+3s}{5+2s+6s} = \frac{4\left(1+\frac{3}{4}s\right)}{5\left(1+\frac{2}{5}s+\frac{6}{5}s\right)}$$

ex:

$$\frac{2\angle 5^\circ \cdot 3\angle 40^\circ}{7\angle 10^\circ} = \frac{2\times 3}{7}\angle(5+40-10)$$

ex:

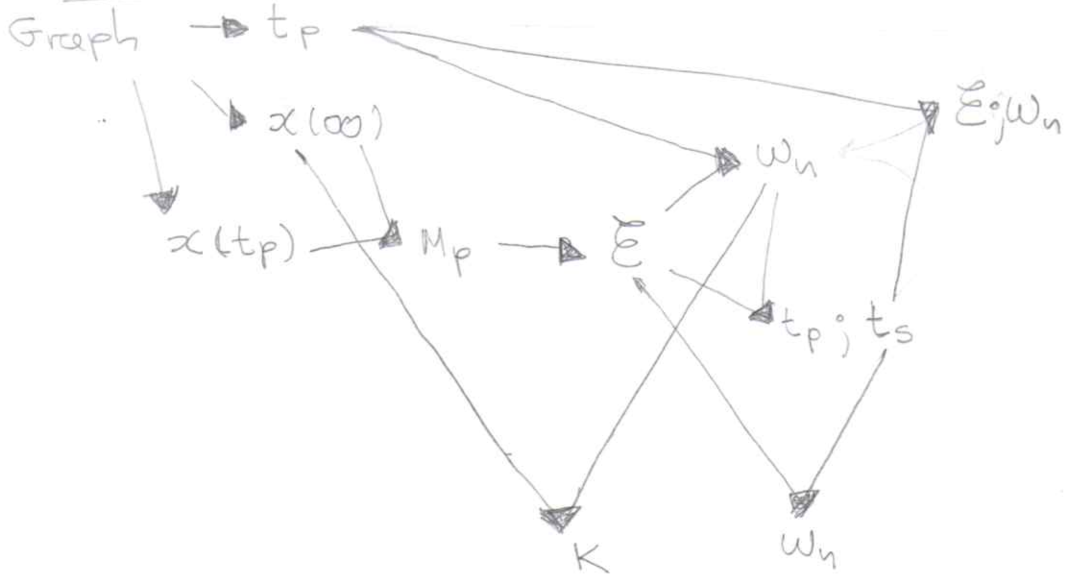
$$\frac{\frac{a}{b}}{\frac{a}{b}+1} = \frac{\frac{a}{b}\times\frac{b}{a}}{\left(\frac{a}{b}+1\right)\times\frac{b}{a}} \Rightarrow \frac{1}{1+\frac{b}{a}}$$

$$\boxed{1=\frac{a}{a}} \qquad = \frac{1}{\frac{a}{a}+\frac{b}{a}} = \frac{1}{\frac{a+b}{a}} \Rightarrow \frac{a}{a+b}$$

or

$$\frac{\frac{a}{b}}{\frac{a}{b}+1} = \frac{\frac{a}{b}}{\frac{a}{b}+\frac{b}{b}} = \frac{\frac{a}{\cancel{b}}}{\frac{a+b}{\cancel{b}}} = \frac{a}{a+b}$$

all possible cases!



$$\frac{K(x+a)}{x+b} = K \times \frac{x+a}{x+b}$$

$$= K \times \frac{x+b+z}{x+b} \qquad [b+z=a]$$

$$= K \times \left[\frac{x+b}{x+b} + \frac{z}{x+b}\right]$$

$$= K \times \left[1 + \frac{z}{x+b}\right]$$

$$= K + \frac{Kz}{x+b}$$

two Fondamental
Rules of Math

- Anything multiplied by $\underline{1}$ is itself.
- Anything added by $\emptyset$ is itself.

$\square \times 1 = \square$

$\square + \emptyset = \square$

what is $\boxed{1}$ $\boxed{\frac{a}{a} = a^0}$

what is $\boxed{\emptyset}$ $\boxed{a - a = \emptyset}$

ex:

$$\boxed{\frac{3600 \text{ sec}}{1 \text{ hour}} = 1}$$

$$\boxed{7 = 7 + 3 - 3}$$

Analise

1º law Inercia $I_{Disc} = \frac{1}{2} m r^2$

2º law $\boxed{\sum F_{R(t)} = m\, a_{(t)}}$

3º law $\sum Action - \sum Reaction = \sum F_R$

$$\sum T = F \times r$$

momentum

$p = m . v$

tips & tricks

1º processo

$$\frac{\frac{a}{b}}{\frac{a}{b}+1} = \frac{\frac{a}{b}\times\frac{b}{a}}{\frac{a}{b}+1\times\frac{b}{a}} \Rightarrow \frac{1}{1+\frac{b}{a}}$$

$$= \frac{1}{1+\frac{b}{a}} \quad ; \quad 1 = \frac{a}{a} = a^{1-1} = a^0 = 1$$

$$= \frac{1}{\frac{a}{a}+\frac{b}{a}} \quad ; \quad \frac{a}{a}+\frac{b}{a} = \frac{a+b}{a}$$, common denominator

$$= \frac{1}{\frac{a+b}{a}} \quad ; \quad \frac{1}{\frac{X}{Y}} = \frac{Y}{X}$$, inverse

$$= \frac{a}{a+b}$$

$$(a+b)(c+D) = a(c+D) + b(c+D)$$
$$= ac + ac + bc + bD$$

$$aX + bX = (a+b)X$$

2º processo

$$\frac{\frac{a}{b}}{\frac{a}{b}+1} = \frac{\frac{a}{b}}{\frac{a+b}{b}} = \frac{a}{\cancel{b}} \times \frac{\cancel{b}}{a+b}$$
$$= \frac{a}{a+b}$$

$$\boxed{1 = \frac{b}{b}}$$

$$\underset{\text{redliuni}}{FTMA} = \frac{X}{Y} \Rightarrow D_{(s)}\Big|_{FTMF} = Y + X$$

tips & tricks.

$$\lim_{s \to 0} 7 \frac{4+3s}{5+2s+6s^2} = 7 \frac{4+0}{5+0+0} = 7 \underbrace{\frac{4}{5}}_{\substack{\text{Board Gain} \\ K}}$$

better

$$\lim_{s \to 0} 7 \frac{4\left(1+\frac{3}{4}s\right)}{5\left(1+\frac{2}{5}s+\frac{6}{5}s^2\right)} = 7 \frac{4}{5}$$

$$\frac{1}{\sqrt{\frac{a}{b}}} = \sqrt{\frac{b}{a}}$$

$$20 \log_{10} A \Rightarrow y$$

$$10^{\frac{y}{20}} = A$$

tips & tricks

50 km/h convert to m/s

$$\frac{1000\,m}{km} = 1 \qquad \frac{1\,hour}{3600\,sec} = 1$$

$\therefore$

$$50 \times \overset{=1}{\frac{1000\,m}{\cancel{km}}} \times \overset{=1}{\frac{1\,\cancel{hour}}{3600\,sec}} \times \frac{\cancel{km}}{\cancel{h}}$$

$$\frac{50 \times 1000}{3600} \frac{m}{sec}$$

Note: these are pre-established unities by laws of physics but some can be defined subjectively.

tips & tricks.

$$\frac{C_{(s)}}{R_{(s)}} = \frac{\frac{K}{s(s+1)}}{1+\frac{K}{s(s+1)}} = \frac{\frac{K}{s\cancel{(s+1)}}}{\frac{s(s+1)+K}{s\cancel{(s+1)}}}$$

$$= \frac{K}{s(s+1)+K}$$

$$\frac{C_{(s)}}{R_{(s)}} = FTMF = \frac{K}{s^2+s+K}$$

$$G_{(s)} = \frac{K}{s(s+1)} \quad ; \quad H_{(s)} = 1 \quad ; \quad N_{(s)} = K \quad ; \quad D_{(s)} = s(s+1)$$

$$G_{(s)}H_{(s)} = \frac{K}{s(s+1)} \quad \Rightarrow \quad G_{(s)} = G_{(s)} \cdot H_{(s)} \quad ; \quad G_{(s)} = \frac{N_{(s)}}{D_{(s)}}$$

$$\frac{C_{(s)}}{R_{(s)}} = FTMF = \frac{G_{(s)}}{1+G_{(s)}H_{(s)}} = \frac{G_{(s)}}{1+G_{(s)}} \quad ; \quad H_{(s)} = 1$$

$$= \frac{\frac{N_{(s)}}{D_{(s)}}}{1+\frac{N_{(s)}}{D_{(s)}}}$$

$$= \frac{\frac{N_{(s)}}{\cancel{D_{(s)}}}}{\frac{D_{(s)}+N_{(s)}}{\cancel{D_{(s)}}}} = \frac{N_{(s)}}{D_{(s)}+N_{(s)}}$$

$$P_1, P_2 = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4a\omega_n^2}}{2 \cdot a}$$

$$\boxed{D_{(s)} = s^2+s+K} \quad \Rightarrow \quad P_1, P_2 = \frac{-1 \pm \sqrt{1-4K}}{2}$$

$$1-4K \geq 0 \quad \Rightarrow -4K \geq -1$$

$$\Rightarrow 4K \leq 1 \quad \Rightarrow K \leq \frac{1}{4}$$

$$\frac{d}{ds}K = \frac{d}{ds}\left(-s^3 - 3s^2 - 2s\right) = 0$$

$$= -3s^2 - 6s - 2 = 0 \quad \boxed{\times -\frac{1}{3}}$$

$$= s^2 + 2s + \frac{2}{3} = 0$$

tips & tricks

$$\frac{C(s)}{R(s)} = FTMF = \frac{G(s)}{1+G(s)\cdot H(s)} = \frac{G(s)}{1+\frac{N(s)}{D(s)}}$$

1º

$$\boxed{G(s)\cdot H(s) = \frac{N(s)}{D(s)}}$$

$$= \frac{G(s)}{\frac{D(s)+N(s)}{D(s)}}$$

2º

$$= \boxed{\frac{G(s)\cdot D(s)}{D(s)+N(s)}}$$

$$\frac{C(s)}{R(s)} = FTMF = \frac{K\,G(s)}{1+G(s)\,H(s)}$$

$$\boxed{G(s)\,H(s) = \frac{K\,N(s)}{D(s)}}$$

$$= \frac{K\,G(s)}{1+K\frac{N(s)}{D(s)}}$$

$$= \frac{K\,G(s)}{\frac{D(s)+K\,N(s)}{D(s)}}$$

$$= \frac{K\,G(s)\,D(s)}{D(s)+K\,N(s)}$$

P_i: $D(s)+K\,N(s) = 0$

1º Numero de ramos = numero polos

2º Ramos são curvas continuas

$$D(s) = -K\,N(s)$$

$$\frac{N(s)}{D(s)} = -\frac{1}{K} \Rightarrow K = -\frac{D(s)}{N(s)}$$

3º Lugar de raizes começa nos polos e acaba nos zeros ou o infinito.

4º Lugar de raizes é simétrico ao eixo real.

5º Desenhar e Determinar valores de K intercepções

$$\frac{d}{d(s)}K = \frac{d}{d(s)}\left[-\frac{D(s)}{N(s)}\right] = \emptyset$$ Ponto de Quebra

tips & tricks

$$(x-1)(0{,}1x+1) = (x-1)(x+10) \times 0.1$$

$$= (0.1x - 0.1)(x+10)$$

$$= \frac{1}{10}(x-1)(x+10)$$

$$= \frac{1}{10}(x^2+9x-10)$$

Rules

$$K = \lim_{s \to 0} \frac{Y(s)}{R(s)}$$

or put in form

$(1 + \dots)$ and K pops out

— // —

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{\omega_n^2}{\omega_n^2\left(\frac{s^2}{\omega_n^2} + 2\xi\frac{1}{\omega_n}s + 1\right)}$$

$$\boxed{K = \frac{\omega_n^2}{\omega_n^2} = 1}$$

because if $s \to 0$ implies $\frac{\omega_n^2}{\omega_n^2} = K$

$\therefore$ always when limit $s \to 0$ put in form of $K.\left(1 + \frac{s}{x} + \frac{s^2}{xy} + e\frac{s^4}{x^4} \text{ etc}\right) = K$

— // —

$$(s^2 - a^2) = (s+a)(s-a)$$
$$= s^2 + \cancel{as} - \cancel{as} - a^2$$
$$= s^2 - a^2$$

erro regime permanente

theory.
Rules

$R(s) \rightarrow$ entrada r

$$R(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$$

$R(s) = \frac{1}{s}$ degrau uni

$$e_{ss} = \frac{1}{1+K_p} \iff l = 0$$

$R(s) = \frac{1}{s^2}$ rampa.

$$e_{ss} = \frac{1}{K_r} \iff l = 1$$

$R(s) = \frac{1}{s^3}$ parabola

$$e_{ss} = \frac{1}{K_a} \iff l = 2$$

$$G(s) \rightarrow \lim_{s \to 0} R(s) \cdot s \cdot G(s)$$

$$\downarrow$$

$$K_p$$

$$\downarrow$$

$$e_{ss} = \frac{1}{1+K_p}$$