

Análise no domínio dos tempos.

13.



$$h=1 \Rightarrow \lim_{s \rightarrow 0} \phi \cdot \frac{K}{\phi(s+a)} \Rightarrow \lim_{s \rightarrow 0} \frac{K}{a(\frac{s}{a}+1)}$$

$$K_v = \frac{K}{a}$$

$$e_{ss} = \frac{a}{K} = 0,01$$

sobrealongo, 0 máxi - $M_p = 0,1$

$$\xi = 0,591$$

$$FTMF = \frac{K}{s(s+a)+K} = \frac{K}{s^2+as+K}$$

$$a = 2\xi\omega_n$$

$$K = \omega_n^2$$

$$\begin{cases} a = 0,01K \\ 2 \times 0,591 \times \sqrt{K} = a \end{cases} \Rightarrow \begin{cases} K = 13971 \\ a = 139,7 \end{cases}$$

13)



$$G(s) = \frac{K}{s(s+a)}$$

↳ type 1 system
(one pole in origin)

$$K = ?$$

$$\alpha = ?$$

$$e_{ss} = 1\%$$

$$M_p = 10\%$$

⇒ Since the system is a type 1 system the steady-state error has finite values for an unit Ramp input:

$$e_{ss} = \frac{1}{K_r}$$

$$K_r = \lim_{s \rightarrow 0} s G(s) = \lim_{s \rightarrow 0} s \cdot \frac{K}{s(s+a)} = \lim_{s \rightarrow 0} \frac{K}{s+a} = \frac{K}{a}$$

$$e_{ss} = \frac{a}{K} \quad (1)$$

⇒ closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{\frac{K}{s(s+a)}}{1 + \frac{K}{s(s+a)}} = \frac{K}{s(s+a) + K}$$

$$\frac{Y(s)}{R(s)} = \frac{K}{s^2 + as + K}$$

$$\begin{cases} K = \omega_n^2 \\ a = 2\zeta\omega_n \end{cases} \Leftrightarrow \begin{cases} \omega_n = \sqrt{K} \\ a = 2\zeta\sqrt{K} \end{cases} \quad (2)$$

Note:

T.F of 2nd order system

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

• From the specification of M_p :

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \Leftrightarrow 0,1 = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \Leftrightarrow \zeta = 0,591$$

Equation (1) and (2):

$$\begin{cases} a = 0,01K \\ a = 2\zeta\sqrt{K} \end{cases} \Leftrightarrow \begin{cases} K = 13971 \\ a = 139,7 \end{cases}$$