theory

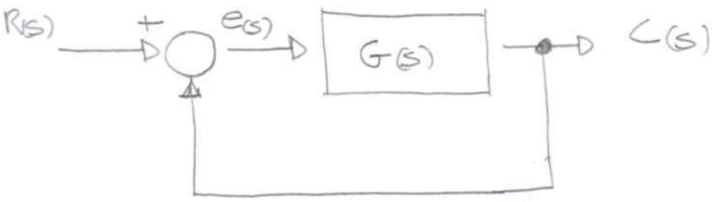
steady-state Errors in Unity-Feedback control systems

- classification of control systems:

A control system may be classified by its ability to follow:

- step inputs:
- Ramp inputs
- Parabolic inputs
- etc

- Consider:



G(s) - open-loop transfer function

$$G(s) = \frac{K\,(T_a \cdot S+1)(T_b S+1)\ldots(T_m S+1)}{S^N\,(T_1 S+1)(T_2 S+1)\ldots(T_p S+1)}$$

S^N – Represents a pole in the origem of Multiplicity "N"

A system is called type ϕ, type 1, type 2 if $N=\phi$, $N=1$, $N=2, \ldots$, respectivily.

type Number	Accuracy	stability
↑	↑	↓

A compromise must be found.

- steady-state errors:
  - the closed loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

  - Error signal $e(t)$:

$$E(s) = R(s) - C(s)$$

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$$\frac{E(s)}{R(s)} = \left[1 - \frac{C(s)}{R(s)}\right] = 1 - \frac{G(s)}{1+G(s)}$$

$$= \frac{1}{1 + G(s)}$$

$$\boxed{E(s) = \frac{1}{1+G(s)} \times R(s)}$$

- Applying the final-value theorem:

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to \emptyset} s \cdot E(s)} = \lim_{s \to \emptyset} \frac{s \cdot R(s)}{1+G(s)}$$

- static position error constant $\boxed{K_p}$ – the steady state error of the system for a Unit step input

$$\boxed{R(s) = \frac{1}{s}}$$

$$e_{ss} = \lim_{s \to \emptyset} \frac{s}{1+G(s)} \cdot \frac{1}{s}$$

$$e_{ss} = \frac{1}{1 + G(\emptyset)}$$

$$\boxed{K_p = \lim_{s \to \emptyset} G(s) = G(\emptyset)}$$ very important

$$\boxed{e_{ss} = \frac{1}{1+K_p}}$$ steady state error in terms of K_p

- For a type $\emptyset$ system:

$$K_p = \lim_{s \to \emptyset} \frac{K(T_a s+1)\dots}{(T_1 s+1)\dots} = K$$

- For a type 1 system:

$$K_p = \lim_{s \to \emptyset} \frac{K(T_a s+1)\dots}{s \cdot (T_b s+1)\dots} = \infty$$

For A Unit Step input signal:

- $e_{ss} = \frac{1}{1+K}$, for type $\emptyset$ systems
- $e_{ss} = \emptyset$, for type 1 or higher systems.

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- static velocity error constant $\boxed{K_v}$ – the steady state Error of the system Ramp input

$$R(s) = \frac{1}{S^2}$$

$$e_{ss} = \lim_{s \to \phi} \frac{s}{1 + G(s)} \cdot \frac{1}{S^2}$$

$$e_{ss} = \lim_{s \to \phi} \frac{1}{s\, G(s)}$$

$$\boxed{K_v = \lim_{s \to \phi} s \cdot G(s)} \Rightarrow \boxed{e_{ss} = \frac{1}{K_v}}$$

steady-state error in terms of K_v

– For a type ϕ system:

$$K_v = \lim_{s \to \phi} \frac{s\, K\, (T_a s + 1) \cdots}{(T_1 s + 1) \cdots} = \phi$$

– For a type 1 system:

$$K_v = \lim_{s \to 0} \frac{\cancel{s}\, K\, (T_a s + 1) \cdots}{\cancel{s}\, (T_1 s + 1) \cdots} = K$$

– For a type 2 or higher system:

$$K_v = \lim_{s \to 0} \frac{s\, K\, (T_a s + 1) \cdots}{s^N\, (T_1 s + 1) \cdots} = \infty, \quad N \geq 2$$

For a Unit Ramp input:

- $e_{ss} = \frac{1}{K_v} = \infty$, for type ϕ systems
- $e_{ss} = \frac{1}{K_v} = \frac{1}{K}$, for type 1 systems
- $e_{ss} = \frac{1}{K_v} = \phi$, for type 2 or higher systems

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- static acceleration error constant $\boxed{K_a}$ — the steady-state error of the system with a Unit <u>Parabolic</u> input

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \to \phi} \frac{s}{1 + G(s)} \cdot \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s^2 \cdot G(s)}$$

$$\boxed{K_a = \lim_{s \to 0} s^2 G(s)} \Rightarrow \boxed{e_{ss} = \frac{1}{K_a}}$$

- For a type ϕ system:

$$K_a = \lim_{s \to \phi} \frac{s^2 K (T_a s + 1) \dots}{(T_1 + 1) \dots} = \phi$$

- For a type 1 system:

$$K_a = \lim_{s \to \phi} \frac{s^2 K (T_a s + 1) \dots}{s (T_1 s + 1) \dots} = \phi$$

- For a type 2 system:

$$K_a = \lim_{s \to \phi} \frac{s^2 K (T_a + 1) \dots}{s^2 \cdot (T_1 + 1) \dots} = K$$

- For a type 3 or higher system:

$$K_a = \lim_{s \to \phi} \frac{s^2 K (T_a s + 1) \dots}{s^N (T_1 s + 1) \dots} = \infty, \text{ for } N \geq 3$$

For a Unit-Parabolic input:

- $e_{ss} = \frac{1}{K_a} = \infty$, for type ϕ and type 1 systems
- $e_{ss} = \frac{1}{K_a} = \frac{1}{K}$, for type 2 systems
- $e_{ss} = \frac{1}{K_a} = 0$, for type 3 or higher systems

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steady - state error in terms of Gain K:

	type ∅ system	type 1 system	type 2 system
step input $R(s) = \frac{1}{s}$	$\frac{1}{1+K_p}$	∅	∅
Ramp input $R(s) = \frac{1}{s^2}$	∞	$\frac{1}{K_v}$	∅
Parabolic input $R(s) = \frac{1}{s^3}$	∞	∞	$\frac{1}{K_a}$

diagonal line present the finite values

$K_p = \lim_{s \to 0} s^0 \cdot G(s)$ $\qquad$ $b = 0 \wedge R(s) = \frac{1}{s}$

$\Rightarrow e_{ss} = \frac{1}{1+K_p}$

$K_v = \lim_{s \to 0} s^1 G(s)$ $\qquad$ $b = 1 \wedge R(s) = \frac{1}{s^2}$

$\Rightarrow e_{ss} = \frac{1}{K_v}$

$K_a = \lim_{s \to 0} s^2 G(s)$ $\qquad$ $b = 2 \wedge R(s) = \frac{1}{s^3}$

$\Rightarrow e_{ss} = \frac{1}{K_a}$

$$\boxed{\begin{gathered} y = \log_a K \\ K = a^y \end{gathered}}$$