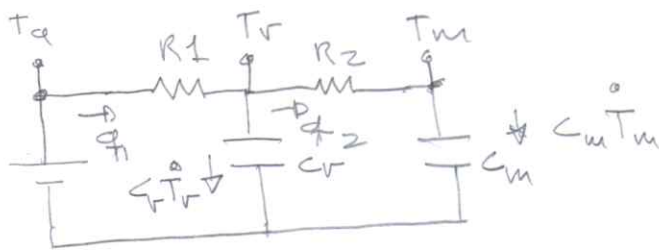


thermometer

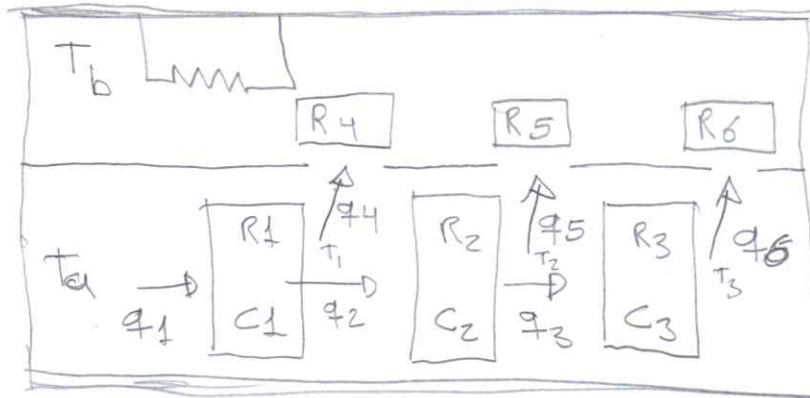
easy



$$q_1 = q_2 + C_v T_a ; \quad q_2 = \frac{T_r - T_m}{R_2} ; \quad q_1 = \frac{T_a - T_r}{R_1}$$

$$q_2 = C_m T_m$$

2.



C - capacidade térmica

R - resistências térmicas

q - fluxo de calor

T - temperatura.

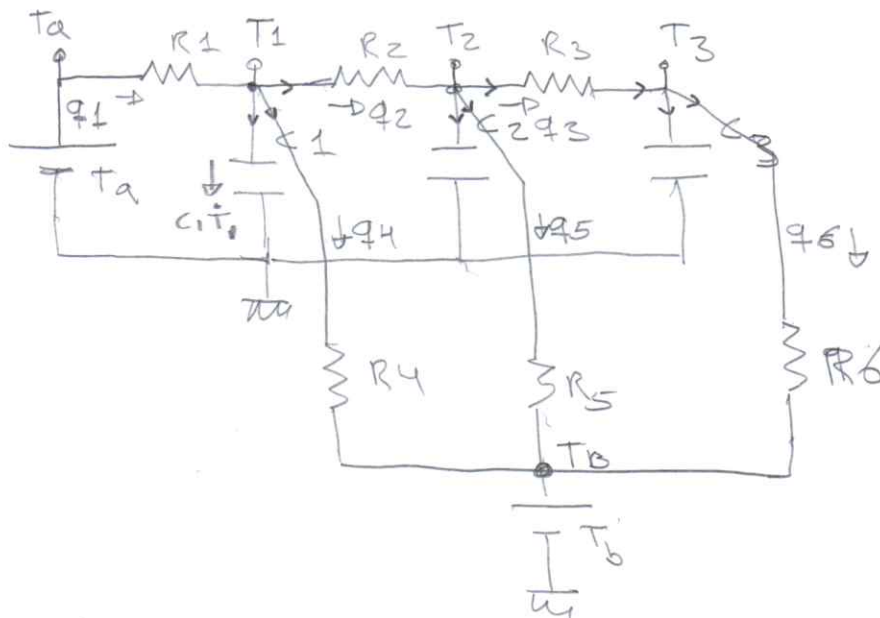
A)  $q_1 + q_2 + q_3 = 0$

B)  $q_1 = C_1 \dot{T}_1 - q_2 + q_3$

C)  $q_1 = C_1 \dot{T}_1 + q_2$

D) outro resultado ✓ ✓

D



$$q_1 = C_1 \dot{T}_1 + q_2 + q_4, \quad q_2 = \frac{T_1 - T_2}{R_2}, \quad q_1 = \frac{T_a - T_1}{R_1}$$

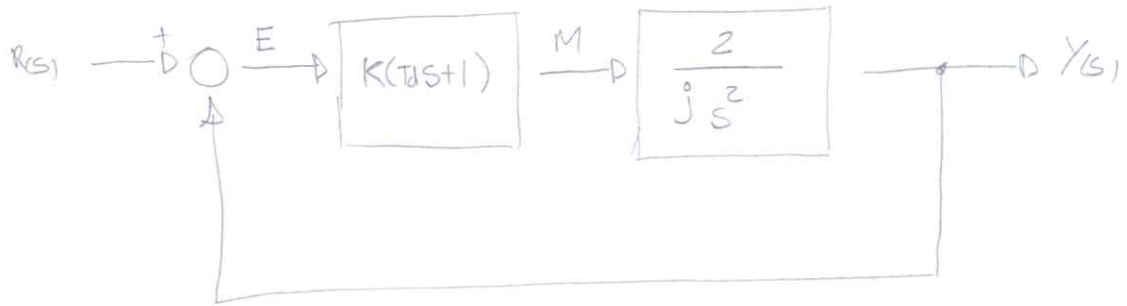
$$q_2 = C_2 \dot{T}_2 + q_3 + q_6, \quad q_3 = \frac{T_2 - T_3}{R_3}$$

$$q_3 = C_3 \dot{T}_3 + q_6, \quad q_4 = \frac{T_1 - T_b}{R_4}, \quad q_5 = \frac{T_2 - T_b}{R_5}$$

$$q_6 = \frac{T_3 - T_b}{R_6}$$

5.

3 Feb 2004



$T_d = ?$  de modo  $\xi = 0,6$

A)  $T_d = 2 \sqrt{\frac{J}{2K}}$

B)  $T_d = 1,2 \sqrt{\frac{2K}{J}}$

C)  $T_d = 2 \sqrt{\frac{2K}{J}}$

D)  $T_d = 1,2 \sqrt{\frac{J}{2K}} \checkmark$

$$G_H(s) = \frac{K(T_d s + 1) \cdot 2}{j s^2}$$

$$D(s) = j s^2 + 2K(T_d s + 1)$$

RTMP

$$j s^2 + 2K T_d s + 2K = 0$$

$\times \frac{1}{2K}$

$$j s^2 + 2K + 2K T_d s = 0$$

$$1 + \frac{2K T_d s}{j s^2 + 2K} = 0$$

$$j s^2 + 2K T_d s + 2K = 0$$

$$s^2 + \frac{2K}{j} T_d s + \frac{2K}{j} = 0$$

$$\omega_n^2 = \frac{2K}{j}$$

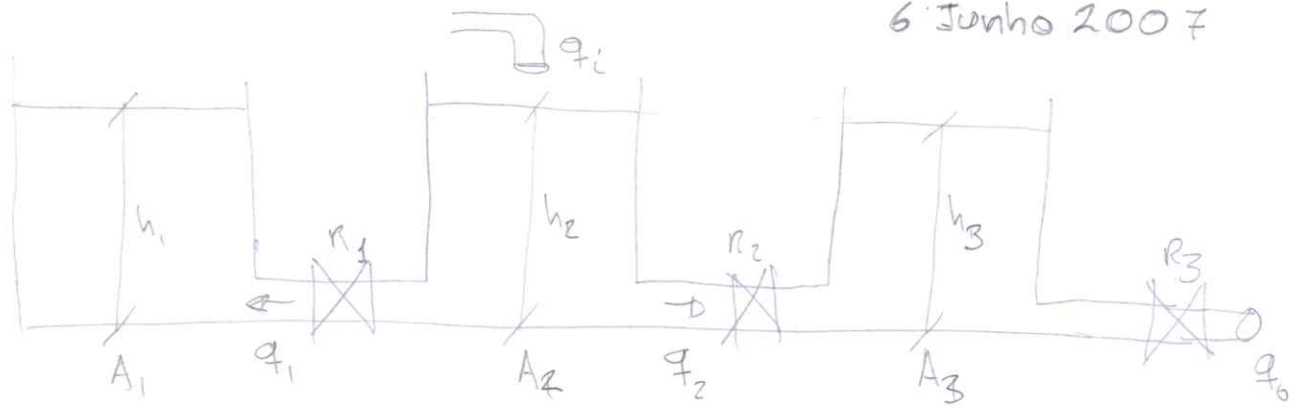
$$1,2 \omega_n = \omega_n^2 T_d$$

$$1,2 \frac{1}{\omega_n} = T_d$$

$$1,2 \sqrt{\frac{j}{2K}} = T_d$$

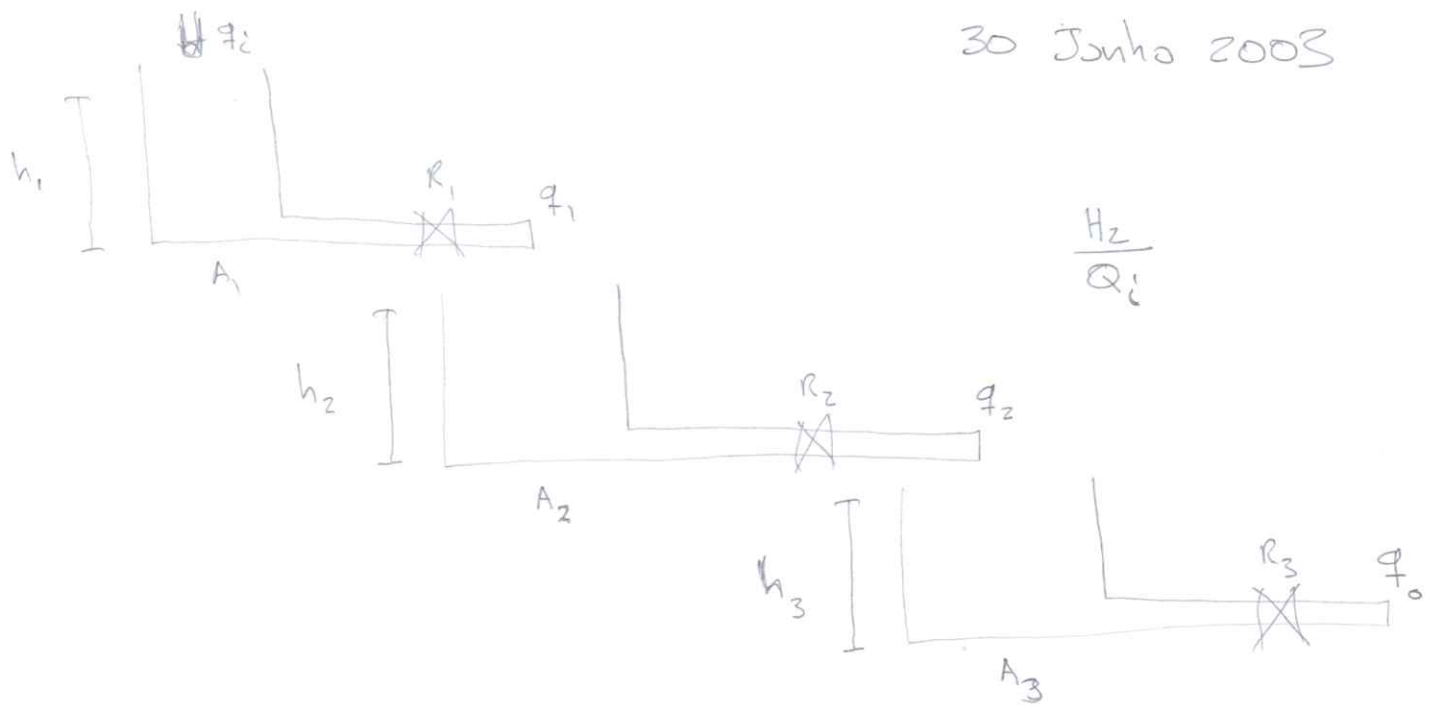
$$1,2 \sqrt{\frac{j}{2K}} = T_d$$

6 Junho 2007



$$\left\{ \begin{array}{l} Q_i = SA_2 H_2 + Q_1 + Q_2 \\ Q_1 = SA_1 H_1 \\ Q_2 = SA_3 H_3 + Q_0 \end{array} \right. \quad \left\{ \begin{array}{l} Q_1 = \frac{H_2 - H_1}{R_1} \\ Q_2 = \frac{H_2 - H_3}{R_2} \\ Q_0 = \frac{H_3}{R_3} \end{array} \right.$$

30 Junho 2003

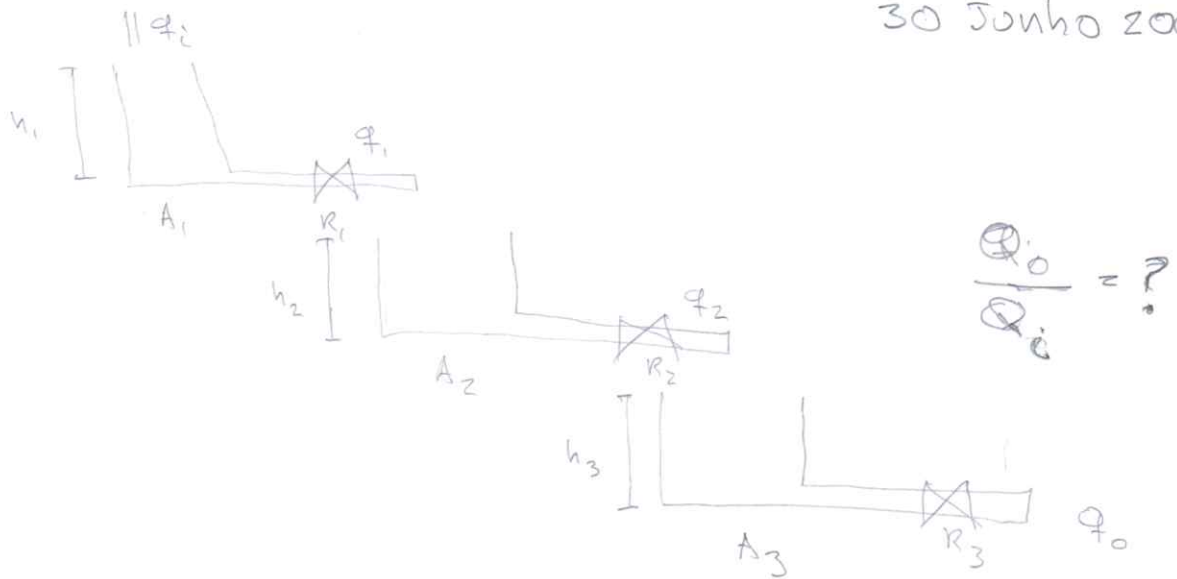


$$\left\{ \begin{array}{l} Q_1 = S H_1 A_1 + \frac{H_1}{R_1} \\ Q_1 = S H_2 A_2 + \frac{H_2}{R_2} \end{array} \right. \quad \left\{ \begin{array}{l} Q_1 = (S A_1 + \frac{1}{R_1}) H_1 \\ \frac{H_1}{R_1} = (S A_2 + \frac{1}{R_2}) H_2 \end{array} \right.$$

$$Q_1 = (S A_1 + \frac{1}{R_1}) (S A_2 + \frac{1}{R_2}) H_2 \cdot R_1$$

$$\frac{H_2}{Q_1} = \frac{1}{(S A_1 + \frac{1}{R_1}) (S A_2 + \frac{1}{R_2}) \cdot R_1}$$

30 Junho 2003



$$\frac{Q_0}{Q_i} = ?$$

$$\left\{ \begin{array}{l} q_i = A_1 \frac{dh_1}{dt} + q_1 \\ q_1 = A_2 \frac{dh_2}{dt} + q_2 \\ q_2 = A_3 \frac{dh_3}{dt} + q_0 \end{array} \right. \quad \begin{array}{l} q_1 = \frac{h_1}{R_1} \\ q_2 = \frac{h_2}{R_2} \\ q_0 = \frac{h_3}{R_3} \end{array}$$

Por em evidência

$$\left\{ \begin{array}{l} Q_i = S H_1 A_1 + Q_1 ; Q_1 = \frac{H_1}{R_1} \Rightarrow Q_1 R_1 = H_1 \\ Q_1 = S H_2 A_2 + Q_2 ; Q_2 = \frac{H_2}{R_2} \Rightarrow Q_2 R_2 = H_2 \\ Q_2 = S H_3 A_3 + Q_0 ; Q_0 = \frac{H_3}{R_3} \Rightarrow Q_0 R_3 = H_3 \end{array} \right.$$

$$\left\{ \begin{array}{l} Q_i = S Q_1 R_1 A_1 + Q_1 \\ Q_1 = S Q_2 R_2 A_2 + Q_2 \\ Q_2 = S Q_0 R_3 A_3 + Q_0 \end{array} \right. \quad \left\{ \begin{array}{l} Q_i = (S R_1 A_1 + 1) Q_1 \\ Q_1 = (S R_2 A_2 + 1) Q_2 \\ Q_2 = (S R_3 A_3 + 1) Q_0 \end{array} \right.$$

$$Q_i = (S R_1 A_1 + 1) (S R_2 A_2 + 1) (S R_3 A_3 + 1) Q_0$$

$$\frac{Q_0}{Q_i} = \frac{1}{(S R_1 A_1 + 1) (S R_2 A_2 + 1) (S R_3 A_3 + 1)}$$

3.

2 Feb 1999

4	1	3	K
3	2	4	0
2	1	K	0
1	<del>4-2K</del>	0	
0	K		

$$4 - 2K > 0$$

$$-2K > -4$$

$$K < 2$$

$$K > 0$$

$$0 < K < 2$$

$$\frac{6 - 4}{2 \times 3 - 4} = 1$$

$$\frac{2K - 0}{2} = K$$

$$4 - 2K$$

$$\frac{(\cancel{4-2K})K - 0}{4 - 2K}$$

4.

$$\text{calc MP } (y(t_p) = 1,163; y(\infty) = 1) = 0,163$$

$$M_p = 0,163 \Rightarrow \varepsilon \approx 0,5 \Rightarrow \omega_n = 9.$$

5.

$$K_p = \frac{10 \times 2}{3 \times 5} = 1,33^\circ$$

$G(s) \rightarrow K_p$  tipo sistema  
e sinal

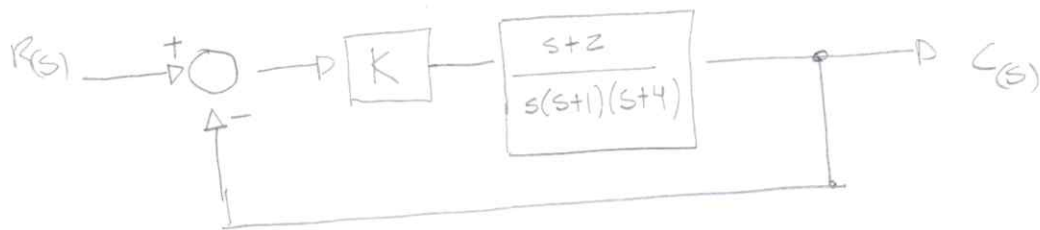
$$e_{ss} = \frac{1}{1 + K_p} = 0,428$$



6.

2 Feb 1999

2ª parte



- 1a)  $K > 0$  =? estável?
- 1b)  $K = ?$  se  $e_{ss} = 0,1$  e  $R(s) = \frac{1}{s^2}$
- 1c) lugar geométrico de raízes
- 1d) diagrama de Bode de Amplitude

$K > 0$

$$FTMF = 1 + K \frac{s+2}{s(s+1)(s+4)}$$

↓  
FTLG

a)

$$s(s+1)(s+4) + K(s+2)$$

$$(s^2+s)(s+4) + K(s+2)$$

$$s^3 + 4s^2 + s^2 + 4s + Ks + 2K$$

$$s^3 + 5s^2 + (K+4)s + 2K$$

FTTMF → D → Prov. Amplitude

3	1	K+4
2	5	2K
1	$\frac{3K+20}{5}$	0
0	2K	

$$\frac{5K+20 - 2K}{5} = \frac{3K+20}{5}$$

b)

FTMA →  $b = 1$  e  $e_{ss}$

$$\left\{ \begin{array}{l} \frac{3K+20}{5} > 0 \\ 2K > 0 \end{array} \right\} \left\{ \begin{array}{l} 3K+20 > 0 \\ K > 0 \end{array} \right\} \left\{ \begin{array}{l} 3K > -20 \\ K > -\frac{20}{3} \end{array} \right\}$$

$b = 1 \Rightarrow \lim_{s \rightarrow 0} \frac{K(s+2)}{s(s+1)(s+4)} = \lim_{s \rightarrow 0} \frac{K \cdot 2 \left(\frac{s}{2} + 1\right)}{(s+1)4 \left(\frac{s}{4} + 1\right)}$

$$K_r = \frac{1}{2} K$$

$$e_{ss} = \frac{1}{K_r} = \frac{1}{\frac{K}{2}} = \left[ \frac{2}{K} \right] = 0,1$$

$$K = \frac{2}{0,1} = 20$$



c) PTMA  $\rightarrow$  LGR

$\left\{ \begin{array}{l} \text{Zeros} \\ \text{Poles} \\ \text{Angle} \end{array} \right.$

$$K \cdot \frac{s+2}{s(s+1)(s+4)}$$

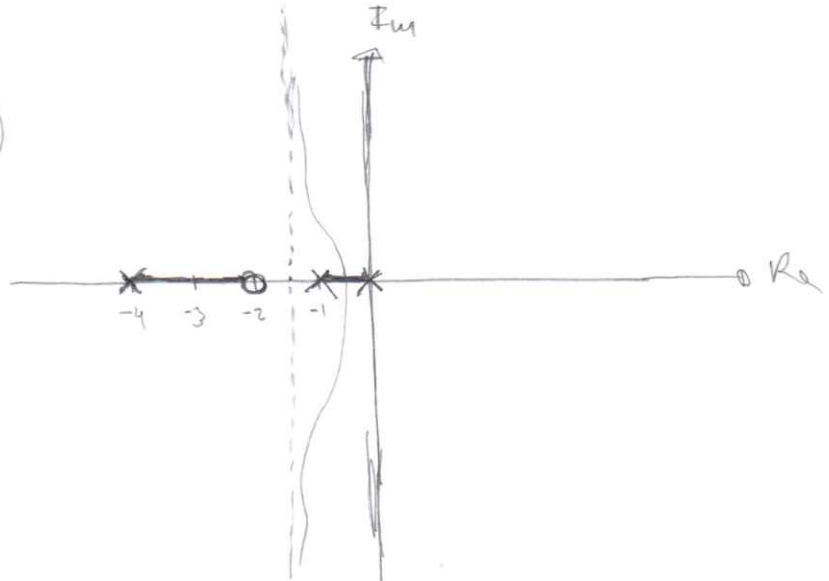
3 Branches

2 Asymptotes

$90^\circ$

$270^\circ$

$-1,5$



$$s^3 + 5s^2 + (K+4)s + 2K = 0$$

$$s^3 + 5s^2 + Ks + 4s + 2K = 0$$

$$s^3 + 5s^2 + (s+2)K + 4s = 0$$

$$(s+2)K = -(s^3 + 5s^2 + 4s)$$

$$K = \frac{-(s^3 + 5s^2 + 4s)}{(s+2)}$$

$$\frac{d}{dt} K = 0 \quad 0 = - \left[ (3s^2 + 10s + 4)(s+2) - (s^3 + 5s^2 + 4s) \right]$$

$$3s^3 + 6s^2 + 10s^2 + 8s + 20s + 4s - s^3 - 5s^2 - 4s = 0$$

$$2s^3 + 11s^2 + 20s + 8 = 0$$

$$x = -0,549$$

d)

# Exame

19 Julho 2016

consider a system with the T.F  $G(s)$  the closed loop T.F include P.I.D controller and a unity feed back loop. tune the controller by the method ziegler-nichols open-loop.

$$G(s) = \frac{5 \cdot e^{-1,5s}}{s}$$

$$w_2 = \frac{K'_p \cdot e^{-st}}{s}$$

$$\begin{bmatrix} K'_p = 5 \\ T = 1,5 \end{bmatrix}$$

Applying the formulas from the table:

$$K = \frac{1,2}{T \cdot K'_p} \Leftrightarrow K = 0,16$$

$$T_i = 2T \Leftrightarrow T_i = 3$$

$$T_d = 0,5T \Leftrightarrow T_d = 0,75$$

— // —

2 Julho 2012

consider a open loop transfer function  $G(s)$  tune the P.I.D controller by the method cohen-coon.

cohen-coon use Model 1

$$w_1 = \frac{K_p \cdot e^{-st}}{s\tau + 1}$$

$$G(s) = \frac{7 e^{-3s}}{5s + 1}$$

$$K_p = 7$$

$$T = 3$$

$$\tau = 5$$

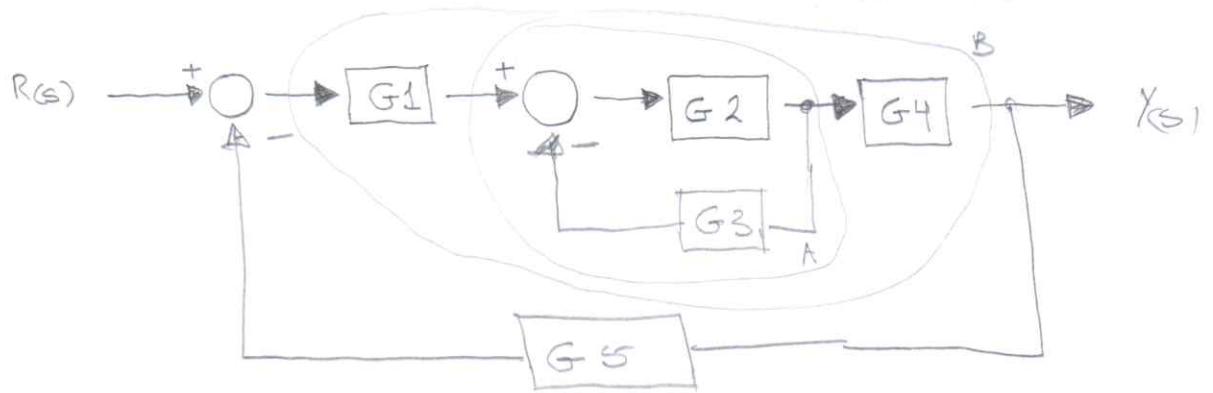
Applying the formulas from the table:

$$K \rightarrow \frac{T}{T K_p} (1,35 + 0,27 \frac{T}{\tau}) \Rightarrow K = 0,36$$

$$T_i \rightarrow T \left( \frac{2,5 + 0,5 T/\tau}{1 + 0,6 T/\tau} \right) \Rightarrow T_i = 6,176$$

$$T_d \rightarrow T \left( \frac{0,37}{1 + 0,2 T/\tau} \right) \Rightarrow T_d = 0,991$$

1.



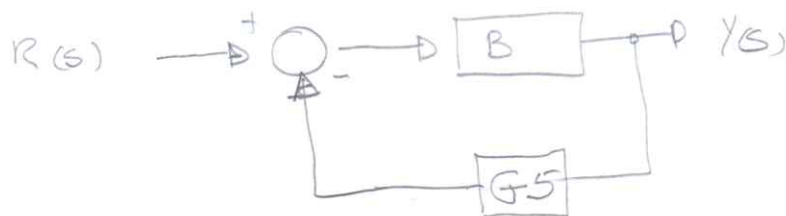
1. simplify feedback loop:

$$A = \frac{G2}{1 + G2G3}$$

2. simplify block in cascade:

$$B = G1 \cdot \frac{G2}{1 + G2G3} \cdot G4 = \frac{G1G2G4}{1 + G2G3}$$

3. simplify feedback loop:



$$\frac{Y(s)}{R(s)} = \frac{B}{1 + B \cdot G5} = \frac{\frac{G1G2G4}{1 + G2G3}}{1 + \frac{G1G2G4}{1 + G2G3} \cdot G5}$$

$$= \frac{G1G2G4}{1 + G2G3 + G1G2G4G5} = \frac{G1 \cdot G2G4}{1 + G2(G3 + G1 \cdot G4G5)}$$

C

2.

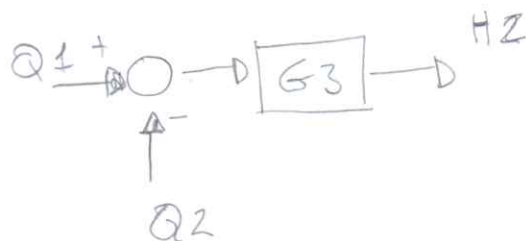
$$\text{TANK 1} \quad \textcircled{1} \quad \left\{ \begin{array}{l} q_1 = q_0 + A_1 \frac{d h_1}{dt} \\ q_1 = \frac{h_1 - h_2}{R_1} \\ q_0 = \frac{h_1}{R_0} \end{array} \right.$$

$$\text{TANK 2} \quad \textcircled{2} \quad \left\{ \begin{array}{l} q_1 = q_2 + A_2 \frac{d h_2}{dt} \\ q_2 = \frac{h_2 - h_3}{R_2} \end{array} \right.$$

$$\text{TANK 3} \quad \textcircled{3} \quad \left\{ \begin{array}{l} q_2 = q_3 + A_3 \frac{d h_3}{dt} \\ q_3 = \frac{h_3}{R_3} \end{array} \right.$$

D

3.



$$G_3 = \frac{H_2(s)}{Q_1(s) - Q_2(s)}$$

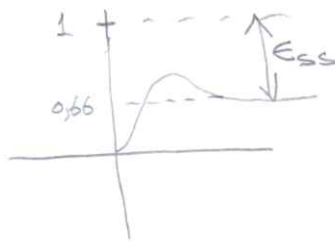
$$\therefore q_1 - q_2 = A_2 \frac{d h_2}{dt}$$

$$Q_1(s) - Q_2(s) = A_2 s H_2(s)$$

$$\boxed{\frac{1}{A_2 s}} = \frac{H_2(s)}{Q_1(s) - Q_2(s)}$$

A

4.



$$E_{ss} = 1 - \frac{2}{3} = \frac{1}{3}$$

estável em 0,66

sistema tipo 1 e Diagrama unitário

$E_{ss} = 0 \Rightarrow$  ~~8~~ não pode ser  
ver tabela

sistema tipo 0 e Diagrama unitário

caso **A**  $K_p = \frac{4}{(s+1)(s+2)} = 2 \Rightarrow \frac{1}{1+2} = \frac{1}{3}$

caso **B**  $K_p = \frac{2}{(s+1)(s+3)} = 0,5 \Rightarrow \frac{1}{1+0,5} = \frac{2}{3}$

logo é a **A**

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s}{1 + G(s)} = \frac{1}{\cancel{s}}$$

$$= \frac{1}{1 + \underbrace{\lim_{s \rightarrow 0} G(s)}_{K_p}}$$

$$\frac{4}{(s+1)(s+2)} = \frac{4}{(s+1)(s+2) + 4} = \frac{4}{s^2 + 3s + 6}$$

$$0,0877$$

$$\frac{4}{(s+1)(s+2) + 4} = \frac{\frac{4}{6} \times 6}{s^2 + 2s + s + 2 + 4} = \frac{\frac{4}{6} \times 6}{s^2 + 3s + 6}$$

2.  $\frac{4}{6} \times 6 = 4$

$$4 \times x = 6 \Rightarrow x = 1,5$$

$$\frac{4}{x} = 6 \Rightarrow x = \frac{4}{6}$$

$$s = 1,2\%$$

5)  $t_p = 1,6223 \text{ seg}$

$x(t_p) = 0,1444$

$$x(t_p) = K \left( 1 + e^{-\pi \xi / \sqrt{1 - \xi^2}} \right)$$

↓

$0,1 \Rightarrow \xi = 0,25 \Rightarrow \omega_n = 2$

↓

$0,15 \Rightarrow \xi = 0,35 \Rightarrow \omega_n = 1$

↓

$0,2 \Rightarrow \xi = 0,15 \Rightarrow \omega_n = 3$

$$t_p = \frac{\pi}{\omega_n \sqrt{1 + \xi^2}}$$

$$0,1444 = 0,1 \left( 1 + e^{-\pi \xi / \sqrt{1 - \xi^2}} \right)$$

$$1,444 = 1 + e^{-\pi \xi / \sqrt{1 - \xi^2}}$$

$$0,444 = e^{-\pi \xi / \sqrt{1 - \xi^2}}$$

↓

$M_p \Rightarrow \xi = 0,25 \Rightarrow \omega_n = 2$

logo  $0,1 \Rightarrow 0,25 \Rightarrow 2 \checkmark$

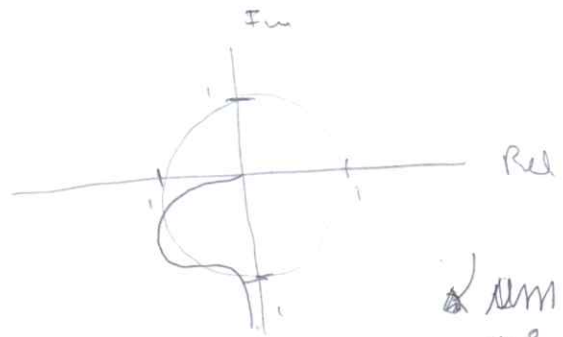
[A]

test [B]

[Mp da negativo] X

6.

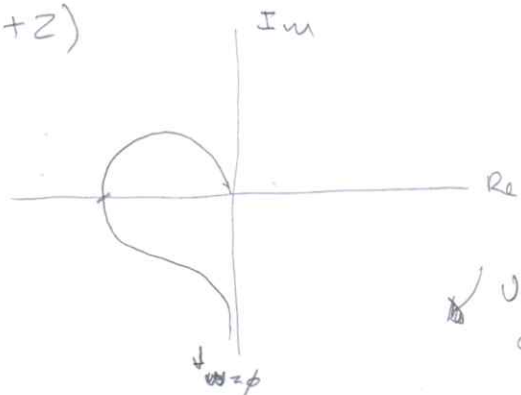
caso A  $\frac{1}{s(s+1)}$



Não tem Margem  
de fase logo não  
pode ser

MM S  
90°  
Um(s+1)  
90° curvatura

caso B  $\frac{1}{s(s+1)(s+2)}$



dezev contos.  
MF calc

$s = x$

Um S  
90°

Dois  
(s+1) e (s+2)  
Duas curvaturas  
90°

$|G(j\omega)| = 1$   
LW

Solve  $\left( \frac{1}{x \cdot \sqrt{x^2+1} \cdot \sqrt{x^2+2}} \right) = 1, x$

$\Rightarrow 0,4457 \checkmark$   $0,4457 \rightarrow \text{LW}$

$MF = 180^\circ + \text{Arg} \left( \frac{1}{Wj \times (Wj+1)(Wj+2)} \right) \Big|_{W=0,4457}$   
 $= 53,410^\circ \checkmark$

MG calc

$\text{Arg}[G(j\omega)] = -\pi$   
LW

**B** ✓

Solve  $(-90 - \tan^{-1}(W) - \tan^{-1}(\frac{W}{2}) = -180^\circ) \Rightarrow W = 1,414$

$GM = \frac{1}{|G(j\omega)|}$

$\omega \cdot \sqrt{\omega^2+1} \cdot \sqrt{\omega^2+2} = 5,998$

$GM_{dB} = 20 \log GM$   
 $= 20 \log 5,998$   
 $= 15,56 \text{ dB}$