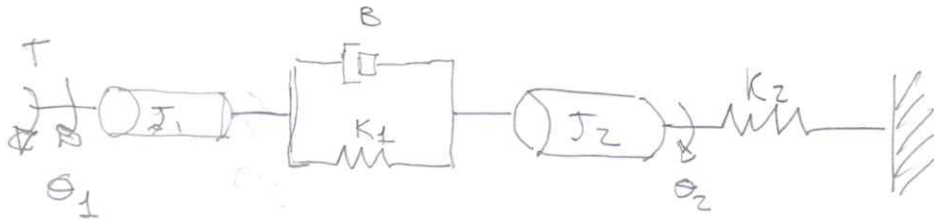


7a)



$$G(s) = \frac{\theta_2(s)}{T(s)} = ?$$

$$\sum T = J \ddot{\theta}(t)$$

$$\begin{cases} \theta_1: T - B(\dot{\theta}_1 - \dot{\theta}_2) - K_1(\theta_1 - \theta_2) = J_1 \ddot{\theta}_1 \\ \theta_2: -B(\dot{\theta}_2 - \dot{\theta}_1) - K_1(\theta_2 - \theta_1) - K_2\theta_2 = J_2 \ddot{\theta}_2 \end{cases}$$

\mathcal{L} ; condições iniciais nulas.

$$\begin{cases} T(s) - BS\theta_1(s) + BS\theta_2(s) - K_1\theta_1(s) + K_1\theta_2(s) = s^2 J_1 \theta_1(s) \\ -BS\theta_2(s) + BS\theta_1(s) - K_1\theta_2(s) + K_1\theta_1(s) - K_2\theta_2(s) = s^2 J_2 \theta_2(s) \end{cases}$$

substituir $\theta_1(s)$ primeira equação

$$-(BS + K_2 + K_1 + s^2 J_2)\theta_2(s) + (BS + K_1)\theta_1(s) = 0$$

$$(SB + K_1)\theta_1(s) = (s^2 J_2 + SB + K_1 + K_2)\theta_2(s)$$

$$\theta_1(s) = \frac{(s^2 J_2 + SB + K_1 + K_2)}{SB + K_1} \theta_2(s)$$

$$T(s) + (SB + K_1)\theta_2(s) = (s^2 J_1 + SB + K_1)\theta_1(s)$$

$$= \left(\frac{s^2 J_1 + SB + K_1}{SB + K_1} \cdot (s^2 J_2 + SB + K_1 + K_2) \right) \theta_2(s)$$

$$T(s) = \left(\frac{(s^2 J_1 + SB + K_1)(s^2 J_2 + SB + K_1 + K_2)}{(SB + K_1)} - \frac{(SB + K_1)^2}{(SB + K_1)} \right) \theta_2(s)$$

7a) continuada

$$\Theta_1(s) = \frac{s^2 J_2 + s B + K_1 K_2}{(s B + K_1)} \Theta_2(s)$$

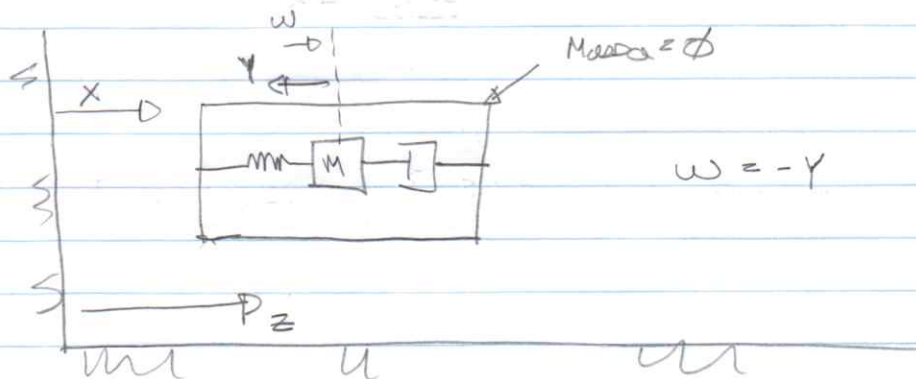
simplificar en adelante para substituir

$$T(s) - K_1 \Theta_1(s) + K_1 \Theta_2(s) - s B \Theta_1(s) + s B \Theta_2(s) = s^2 J_1 \Theta_1(s)$$

$$T(s) + \Theta_1(s) (-K_1 - s B - s^2 J_1) + K_1 \Theta_2(s) + s B \Theta_2(s) = 0$$

acelerómetro mecánico.

NOTA.



$$X - Y = Z \Rightarrow \ddot{X} - \ddot{Y} = \ddot{Z}$$

$$f_M = M \ddot{Z}$$

masa

$$f_K = K W$$

mola

$$f_B = B \dot{W}$$

amortiguador

$$f_K + f_B + f_M = 0$$

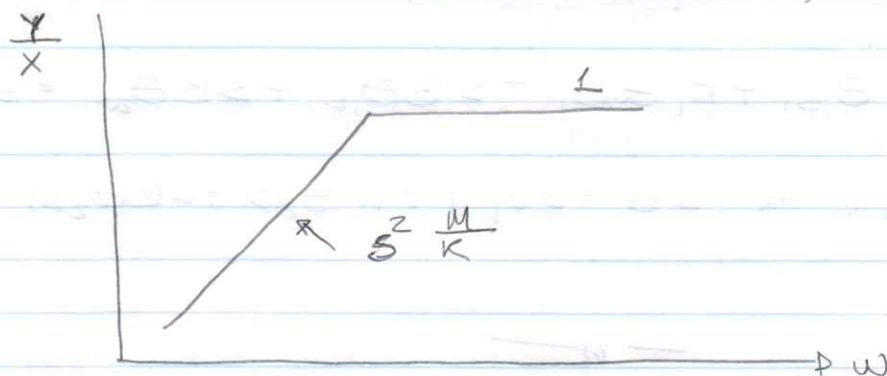
$$K W + B \dot{W} + M \ddot{Z} = 0$$

$$-KY - B\dot{Y} + M(\ddot{x} - \ddot{Y}) = 0 \Rightarrow M\ddot{x} = M\ddot{Y} + B\dot{Y} + KY$$

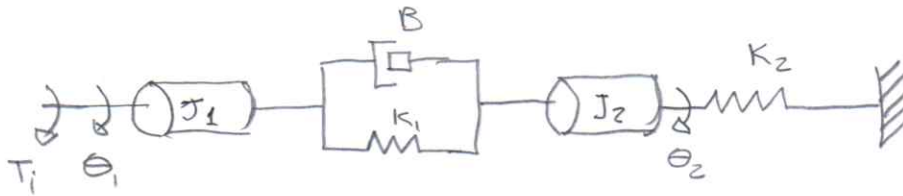
$$\boxed{\frac{Y(s)}{X(s)} = \frac{s^2 M}{s^2 M + sB + K}}$$

$$s \rightarrow 0 \quad \frac{Y(s)}{X(s)} = \frac{s^2 M}{K}$$

$$s \rightarrow \infty \quad \frac{Y(s)}{X(s)} = 1$$



7a)



$$\frac{\theta_2}{T}$$

$$\theta_1: \sum FR = J_1 \ddot{\theta}_1$$

$$\theta_2: \sum FR = J_2 \ddot{\theta}_2$$

$$\theta_1 \left\{ T_1 - B(\dot{\theta}_1 - \dot{\theta}_2) - K_1(\theta_1 - \theta_2) = J_1 \ddot{\theta}_1 \right.$$

$$\theta_2 \left\{ -K_1(\theta_2 - \theta_1) - B(\dot{\theta}_2 - \dot{\theta}_1) - K_2(\theta_2) = J_2 \ddot{\theta}_2 \right.$$

$$\left\{ T_1 = s^2 J_1 \theta_1 + s B \theta_1 - s B \theta_2 + K_1 \theta_1 - K_1 \theta_2 \right.$$

$$\left\{ 0 = s^2 J_2 \theta_2 + K_1 \theta_2 - K_1 \theta_1 + s B \theta_2 - s B \theta_1 + K_2 \theta_2 \right.$$

$$\left\{ T_1 = (s^2 J_1 + s B + K_1) \theta_1 - (s B + K_1) \theta_2 \right.$$

$$\left\{ 0 = -(s B + K_1) \theta_1 + (s^2 J_2 + s B + K_1) \theta_2 \right.$$

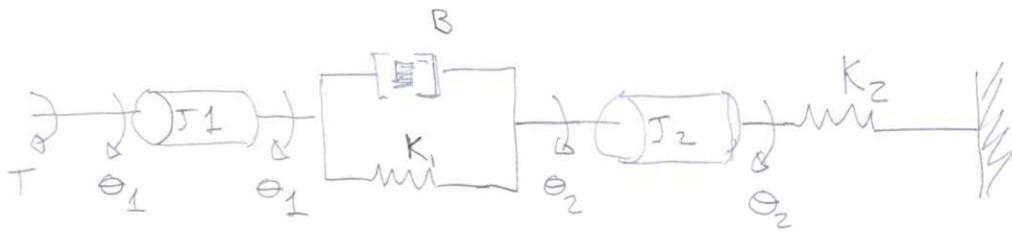
$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \begin{vmatrix} s^2 J_1 + s B + K_1 & -(s B + K_1) \\ -(s B + K_1) & s^2 J_2 + s B + K_1 \end{vmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

$$\theta_2 = \frac{\begin{vmatrix} s^2 J_1 + s B + K_1 & T \\ -(s B + K_1) & 0 \end{vmatrix}}{\begin{vmatrix} s^2 J_1 + s B + K_1 & -(s B + K_1) \\ -(s B + K_1) & s^2 J_2 + s B + K_1 \end{vmatrix}}$$

$$= \frac{T (s B + K_1)}{(s^2 J_1 + s B + K_1)(s^2 J_2 + s B + K_1) - (s B + K_1)^2}$$

$$\frac{\theta_2}{T} = \frac{s B + K_1}{s^4 J_1 J_2 + s^2 J_1 (s B + K_1) + s^2 J_2 (s B + K_1) + (s B + K_1)^2 - (s B + K_1)^2}$$

7a)



$$\frac{\theta_2}{T} = ?$$

$$\Sigma T = J \ddot{\theta}$$

Funções

$$\begin{cases} \theta_1 - \theta_1(t) \\ \theta_2 - \theta_2(t) \\ T - T(t) \\ \theta_1 - \theta_1(s) \\ \theta_2 - \theta_2(s) \\ T - T(s) \end{cases}$$

$$\begin{cases} \theta_1: J_1 \ddot{\theta}_1 = T - K_1(\theta_1 - \theta_2) - B(\dot{\theta}_1 - \dot{\theta}_2) \\ \theta_2: J_2 \ddot{\theta}_2 = -K_1(\theta_2 - \theta_1) - B(\dot{\theta}_2 - \dot{\theta}_1) - K_2 \theta_2 \end{cases}$$

$$\begin{cases} T = (s^2 J_1 + K_1 + sB) \theta_1 - (K_1 + sB) \theta_2 \\ 0 = (s^2 J_2 + K_1 + sB + K_2) \theta_2 - (K_1 + sB) \theta_1 \end{cases}$$

$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \begin{bmatrix} s^2 J_1 + sB + K_1 & -(K_1 + sB) \\ -(sB + K_1) & s^2 J_2 + sB + K_1 + K_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\begin{aligned} \frac{\theta_2}{T} &= \frac{sB + K_1}{(s^2 J_2 + sB + K_1 + K_2)(s^2 J_1 + sB + K_1) - (sB + K_1)^2} \\ &= \frac{sB + K_1}{s^4 J_2 J_1 + s^2 J_2 (sB + K_1) + s^2 J_1 (sB + K_1) + (sB + K_1)^2 + s^2 J_1 K_2 + K_2 (sB + K_1) - (sB + K_1)^2} \end{aligned}$$

$$\frac{\theta_2}{T} = \frac{sB + K_1}{s^4 J_1 J_2 + (s^2 J_2 + s^2 J_1 + K_2)(sB + K_1) + s^2 J_1 K_2} \quad \checkmark$$