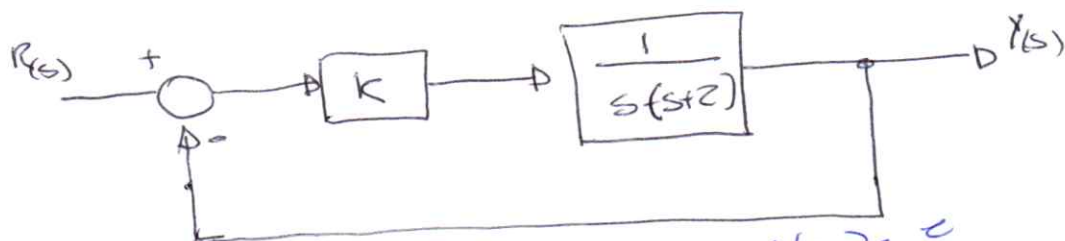


7)

 $K = ?$ $M_p \leq 10\%$

17



$$FTMF \approx \frac{Y(s)}{R(s)} \approx$$

$$\frac{Y(s)}{R(s)} = \frac{K}{s(s+2)} \cdot \frac{1}{1 + \frac{K}{s(s+2)}} =$$

$$= \frac{K}{s(s+2)} \cdot \frac{1}{1 + \frac{K}{s(s+2)}} =$$

$$= \frac{K}{s(s+2) + K}$$

$$= \frac{K}{s^2 + 2s + K}$$

$$M_p = e^{-\frac{\pi}{\sqrt{1-\xi^2}}}$$

$$\begin{aligned} \phi &= e^{-\frac{\pi}{\sqrt{1-\xi^2}}} \\ &= e^{-\frac{\pi}{\sqrt{K} \cdot \sqrt{1-1/K}}} \end{aligned}$$

$$\ln(10) = \phi \cdot \frac{\pi}{\sqrt{K-1}}$$

$$\begin{aligned} \sqrt{K-1} &= \frac{\pi}{\ln(10)} \quad (\Rightarrow) \quad K-1 = \left(\frac{\pi}{\ln(10)} \right)^2 + 1 \\ &= 2,8615 \end{aligned}$$

$$\xi = 0,59$$

$$FTMA \approx 6H \cdot \omega_n = 1,69$$

$$K = 0,89$$

$$M_p = 0,1 \quad -\pi \xi / \sqrt{1-\xi^2}$$

$$\approx 0$$

$$\Rightarrow \xi = 0,591$$

$$\omega_n = \sqrt{K}$$

$$\omega_n^2 = K$$

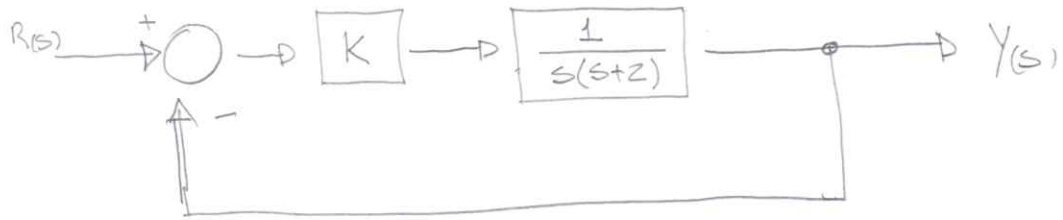
$$2\xi\omega_n = 2$$

$$\left. \begin{aligned} \omega_n^2 &= K \\ 2\xi\omega_n &= 2 \end{aligned} \right\} \begin{aligned} \xi \sqrt{K} &= 1 \\ \xi &= \frac{1}{\sqrt{K}} \end{aligned}$$

$$\left\{ \begin{aligned} 2\xi\omega_n &= 2 \\ \omega_n^2 &= K \end{aligned} \right\} \begin{aligned} \omega_n &= 1,692 \text{ rad/s} \\ K &= 2,86 \end{aligned}$$

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$$

7.)

 $K = ?$

$$R(s) = \frac{1}{s} \wedge M_p < 10\%$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\frac{K}{s(s+2)}}{1 + \frac{K}{s(s+2)}} = \frac{K}{s(s+2) + K} = \frac{K}{s^2 + 2s + K}$$

- From the specification of M_p (Maximum overshoot):

$$M_p = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}}$$

$$0,1 = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = 0,591$$

- Standard form of a second order system:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

(closed loop)

$$G(s) = \frac{K}{s^2 + 2s + K}$$

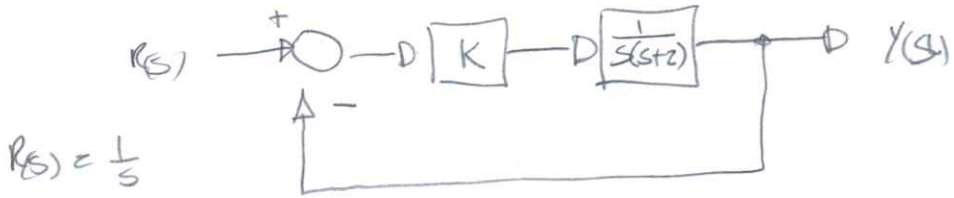
comparing the two equations

$$\begin{cases} 2\xi\omega_n = 2 \\ \omega_n^2 = K \end{cases} \Leftrightarrow \begin{cases} \omega_n = 1,692 \text{ rad/sec} \\ K = 2,86 \end{cases} \quad h_1$$

c.o.c

$$\frac{\frac{K}{s(s+2)}}{1 + \frac{K}{s(s+2)}} = \frac{\frac{K}{s(s+2)}}{\frac{s(s+2) + K}{s(s+2)}} = \frac{K}{s(s+2) + K} = \frac{K}{s^2 + 2s + K}$$

7.



$$R(s) = \frac{1}{s}$$

$$M_p < 10\%,$$

$$\xi \approx 0,6$$

$$D(s) = s^2 + 2s + K \quad \frac{\frac{K}{s(s+2)}}{1 + \frac{K}{s(s+2)}} = \frac{K}{s(s+2) + K}$$

$$2\xi\omega_n = 2$$

$$2 \cdot 0,6 \omega_n = 2$$

$$\omega_n = \frac{1}{0,6} = \frac{5}{3} \quad K = \omega_n^2 = 2,777$$