transient Response Analysis of second-order systems

- standard Equation:

$$\boxed{\frac{d^2}{dt} y(t) + 2\,\mathcal{E}\omega_n \frac{d}{dt} y(t) + \omega_n^2 y(t) = \omega_n^2 u(t)}$$

ω_n - Undamped natural Frequency

$\mathcal{E}$ - damping Ratio

- transfer Function:

$$\boxed{\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\,\mathcal{E}\omega_n s + \omega_n^2}}$$

- characteristic equation:

$$s^2 + 2\,\mathcal{E}\omega_n s + \omega_n = 0$$

- the roots of characteristic equation:

$$s = -\mathcal{E}.\omega_n \pm \omega_n \sqrt{\mathcal{E}^2 - 1}$$

$\mathcal{E} = 1$ - Response of the system is critically damped

- two roots are real and equal

$\mathcal{E} > 1$ - Response of the system is over damped

- two roots are real but not equal

$0 < \mathcal{E} < 1$ - Response of the system is under damped

- two roots are complex conjugate

- If under Damped:

$$\text{Poles} = -\mathcal{E}\times\omega_n \pm j\,\omega_n \sqrt{1 - \mathcal{E}^2}$$

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- these systems are described by the linear differentical equation: 1

$$\tau \frac{d}{dt} y(t) + y(t) = K\,\mu(t)$$

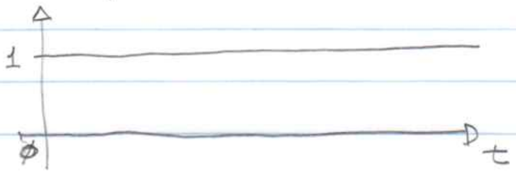
; $\mu(t) \rightarrow$ input signal
$y(t) \rightarrow$ output signal
$\tau \rightarrow$ system time constante

- In Laplace Domain with the transfer function:

$$\boxed{\frac{Y(s)}{U(s)} = \frac{K}{(\tau s + 1)}}$$

<u>special case 1</u>
I $R(s) = \frac{1}{s}$ and is FTMF

- Unit step:



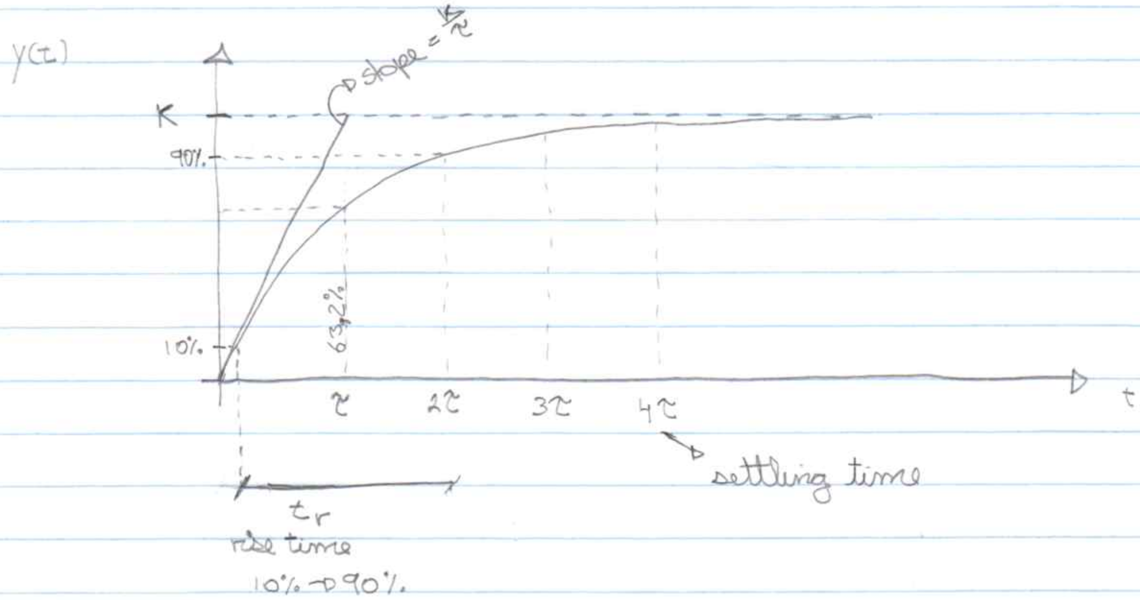
$$\mu(t) \overset{\mathcal{L}}{\Rightarrow} \frac{1}{s}$$

- $U(s) = \frac{1}{s}$

$$\boxed{Y(s) = \frac{K}{(\tau s + 1)} \times \frac{1}{s}}$$

- Expanding into parcial fractions gives:

$$Y(s) = K\left(\frac{1}{s} - \frac{\tau}{\tau s + 1}\right) \overset{\mathcal{L}^{-1}}{\Rightarrow} y(t) = K \cdot (1 - e^{-t/\tau})$$



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Formulas

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \quad , \quad y(t_p) = 1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$M_p = e^{\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$t_s = \frac{4}{\xi\omega_n}$$

$$t_r \cong \frac{e^{\cancel{\theta}\tan(\theta)}}{\omega_n} \quad , \quad \theta = \arccos(\xi)$$

$$\boxed{\xi = \sqrt{\frac{1}{1+\left(\frac{\pi . t_s}{4 . t_p}\right)^2}}}$$

$$\boxed{\omega_n = \frac{4}{\xi t_s}}$$

- time Domain specification:
- Maximum overshoot (%):

$$\boxed{M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \times 100\,\%}$$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\xi = \sqrt{\frac{1}{1+\left(\frac{\pi t_s}{4t_p}\right)^2}}$$

- Peak time:

$$\boxed{t_p = \frac{\pi}{\omega_n \cdot \sqrt{1-\xi^2}}} \quad ; \quad \boxed{y(t_p) = 1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}$$

- Setling time ($\pm$ 1,8% of the Final value):

$$\boxed{t_s = \frac{4}{\xi \times \omega_n}}$$

- Final Value $y(\infty)$:

$$\boxed{y(\infty) = \lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot Y(s)}$$

- Rise time:

$$t_r \cong \frac{e^{\theta/\tan(\theta)}}{\omega_n} \quad , \quad \theta = \arccos(\xi)$$

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Second order system

system Equation

$$\boxed{\frac{d^2}{dt} y(t) + 2 \cdot \varepsilon \omega_n \frac{d}{dt} y(t) + \omega_n^2 y(t) = \omega_n^2 \mu(t)}$$

ω_n - undamped natural frequência
ε - damping rate

$$\frac{Y(s)}{R(s)} = \frac{K \omega_n^2}{s^2 + 2 \varepsilon \omega_n s + \omega_n^2} \quad ; \quad K = 1$$

formula canónica

- Response of the system depending on the value of ε :

$0 \leq \varepsilon < 1 \rightarrow$ Under Damped
$\varepsilon = 1 \rightarrow$ Criticcally damped
$\varepsilon > 1 \rightarrow$ Over damped.

- Oscillatory Response based on the poles of the transferfunction [T.F]:

  - Pair of conjugate Poles $\rightarrow$ Under Damped
  - Double pole on the Real axis $\rightarrow$ Critically damped
  - two distinct Real poles $\rightarrow$ over damped.

(27.55) over damped $\rightarrow$ the response increases monotonicly

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sistema de 2ª ordem

Forma canónica dos sistemas de 2ª ordem:

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

- Se $\xi > 1 \Rightarrow$ sistema sobreamortecido
  $\Rightarrow$ polos reais distintos
- Se $\xi = 1 \Rightarrow$ sistema criticamente Amortecido
  $\Rightarrow$ polos reais duplos (iguais)
- Se $0 < \xi < 1 \Rightarrow$ sistemo subamortecido
  $\Rightarrow$ polos complexos conjugados.

special case 2.

if $R(s) = \frac{1}{s}$ and is FTMF then time Response can be deduced by parameters without doing $\mathcal{L}^{-1}$, or in other words obtain the time domain equation.