

↓

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if $R(s) = \frac{1}{s}$:

$$\frac{K}{\gamma s + 1} \quad | K=1$$

$$K \left(1 - e^{-\frac{t}{\tau}} \right)$$

$$K = \lim_{s \rightarrow 0} s \cdot \text{TFMF} \times R(s)$$

if $R(s) = \frac{1}{s}$:

$$\frac{K \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \bigg| \quad K=1$$

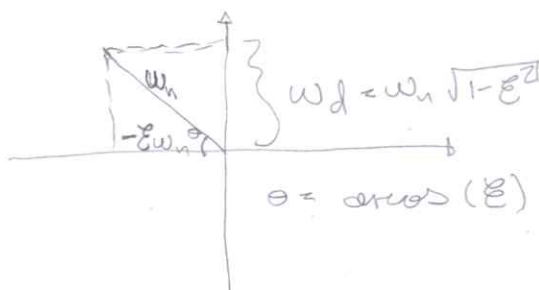
$$K = \lim_{s \rightarrow 0} s \times \text{FTMP}_x(R(s))$$

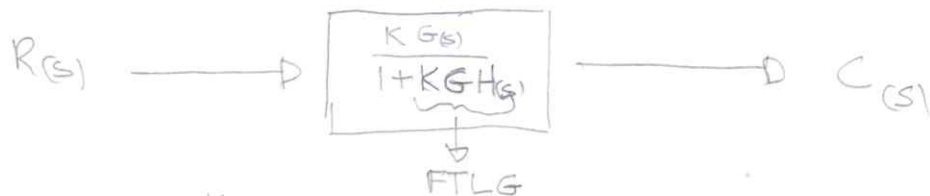
$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} \times 100\% = e^{-\frac{\pi}{\sqrt{1-\zeta^2}}}$$

$$t_s = \frac{4}{E\omega_n}$$

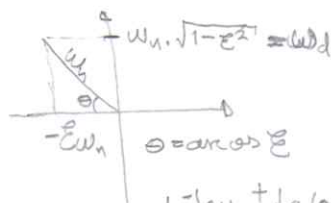
$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}}$$

$$Y(\infty) = \lim_{t \rightarrow \infty} Y(t) = \lim_{s \rightarrow 0} Y(s)$$





$0 < \xi < 1 \quad G(s) = \frac{N(s)}{D(s)}$



$\frac{d}{ds} \left(\frac{1}{K} \right) = 0 \quad K=0$
 $\frac{d}{ds} \left(\frac{1}{K} \right) = \frac{tg'(a) \pm tg'(b)}{1 \mp tg(a) \cdot tg(b)}$

Domínio do tempo

$R(s)$, FTMF

$KGH(s) = FTLG$
 $= \frac{N(s)/FTLG}{D(s)/FTLG} \rightarrow \text{Body / Zeros / Poles}$
 $\rightarrow \text{LGR / CSS}$

$\omega > K > D(s) = 0 \quad (LGR)$

$\frac{D(s)}{FTMF} = \frac{D(s)}{FTLG} + \frac{N(s)}{FTLG}$

FTLG
MG; MF
Body; Zeros
Poles; LGR
CSS

Routh Hurwitz

estabilidade $\rightarrow K$

stabilizado $-K \Rightarrow 0 < \xi < 1$
Zag MF $-T_i$
 $-T_d$

$b=0 \Rightarrow R(s) = \frac{1}{s} \Rightarrow K_p = \lim_{s \rightarrow 0} FTLG$
 $b=1 \Rightarrow R(s) = \frac{1}{s^2} \Rightarrow K_v = \lim_{s \rightarrow 0} s FTLG$
 $b < 2 \Rightarrow R(s) = \frac{1}{s^3} \Rightarrow K_a = \lim_{s \rightarrow 0} s^2 FTLG$

$y = \log K$
 $K = 10^y \rightarrow e_{ss}$

$s = -\xi \cdot \omega_n \pm \omega_n \sqrt{\xi^2 - 1}$

$\xi \gg 1$

$t(s) = \frac{4}{\xi \omega_n}$

$s = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2}$

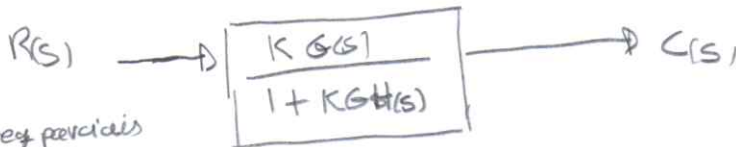
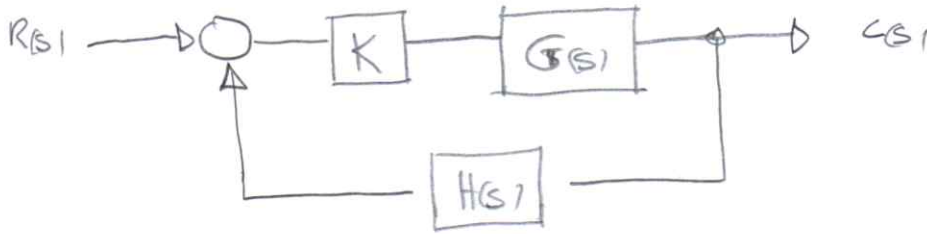
$t_p = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}}; \quad y(t_p) = 1 + e^{-\frac{\xi \pi}{\sqrt{1 - \xi^2}}}$

$M_p = \frac{e^{-\frac{\xi \pi}{\sqrt{1 - \xi^2}}}}{e^{\frac{\xi \pi}{\sqrt{1 - \xi^2}}}}; \quad t(s) = \frac{4}{\xi \omega_n}$

$z = \frac{y(t_p) - y(\infty)}{y(\infty)}; \quad t(r) \approx \frac{e^{\theta / \tan \theta}}{\omega_n}; \quad \theta = \arccos(\xi)$

$\xi = \sqrt{\frac{1}{1 + (\frac{\pi t_p}{4 t_p})^2}}$

Resumo I



$$1 + \frac{q}{b} = 0$$

$$b + a = 0$$

$R(s)$ - FTMF \rightarrow eq parciais

domínio dos tempos
 $\Re(s) = \frac{1}{s}$
casos especiais

$$FTMF = \frac{KG(s)}{1 + KGH(s)}$$

$$1 + \underbrace{KGH(s)}_{FTLG}$$

$$KGH(s) = \frac{N}{D}$$

$$D(s) = D + N$$

$$P(s) = 0$$

$KGH(s) \rightarrow MF_{(K)} \cdot MG_{(K)}$
[FTLG] \rightarrow Bode; Nyquist
 \rightarrow LGR - zeros - polos
 \rightarrow ESS - $\delta; K$ - polo

$D(s)$
[$D(s)$ FTMF]
- Quebra de $\frac{dK}{ds} = 0 \rightarrow s = 0$
- interseção eixo im
- Routh Hurwitz
- estabilidade
- simulação
- Zeig MF
- casos especiais
- PID

Quebra: $D(s) \rightarrow \frac{dK}{ds} = 0 \rightarrow s = 0$

interseção eixo imaginário: $D(s) \rightarrow P(s) = 0 \mid s = j\omega \leftarrow K_w$

Domínio dos tempos

FTMF

$$R(s) = \frac{1}{s} \Rightarrow \frac{K}{s+a} \Rightarrow \frac{K}{a} \cdot \frac{1}{\frac{s}{a} + 1} \quad \gamma = \frac{1}{a}$$

$$y(t) = \frac{K}{a} (1 - e^{-\frac{t}{\tau}})$$

$$R(s) = \frac{1}{s} \Rightarrow \frac{K \cdot \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad | \quad K=1$$

$$\xi \geq 1$$

$$s = -\xi \omega_n \pm \omega_n \sqrt{\xi^2 - 1}$$

$$0 < \xi < 1$$

$$s = -\xi\omega_n \pm j\omega_n \sqrt{1 - \xi^2}$$

—||—

Frequency

$$r_{ct} = \sin(\omega t) \quad t > 0$$

$$y(t) = |G(j\omega)| \cdot \sin[\omega t + \angle G(j\omega)]$$

- Diagrama de Blocos Maténcia

- Modelação de sistemas

- por variável em evidência
- denominador comum.

FTMA
FTMF
 $G(s) \leftarrow$
 $G H(s) \rightarrow$ PFTLG
 \downarrow
 $D(s)$

$$F_r = m a$$

$$T_r = J \ddot{\theta}$$

térmico

Hidráulico

- tirar equações

- Matrices.

- acelerometria

$$\frac{Y(s)}{X(s)} = \frac{s^2 M}{s^2 M + s B + K}$$

$$\lim_{s \rightarrow 0} \frac{Y(s)}{X(s)} = \frac{s^2 M}{K}$$

$$\lim_{s \rightarrow \infty} \frac{Y(s)}{X(s)} = 1 \cdot \frac{Y}{X}$$



- Análise de sistemas no domínio dos tempos [FTMF]

$$R(s) = \frac{1}{s} \Rightarrow \frac{K}{s+a} \Rightarrow \frac{K}{a} \frac{1}{(\frac{s}{a} + 1)} \quad \tau = \frac{1}{a}$$

$$\Rightarrow \frac{K}{a} (1 - e^{-\frac{t}{\tau}})$$

$$R(s) = \frac{1}{s} \Rightarrow \frac{K \cdot \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \quad | K=1$$

fracção porções $\Rightarrow \rho^{-1}$

Formulário

- tipo de sistema $\begin{matrix} FT \\ LG \end{matrix}$
- Ess

- Crítérios de Estabilidade de Routh-Hurwitz

$G H(s) \rightarrow D(s)$ - Routh-Hurwitz.

special cases

- Lugar Geométrico de Raíces [Muito Importante]

$$G H(s) \rightarrow D(s) \rightarrow \frac{d}{ds} K = 0 \rightarrow s \rightarrow \text{algebra}$$

\downarrow
 s, p
 \downarrow
 p, z
 \downarrow
 zeros

$D(s) = 0 \quad | \quad s = j\omega < \omega$ intercept
nota special
 ω imaginário

- Dominant des Frequências

$GH(s) \rightarrow$ Bode
 \rightarrow Nyquist
 \rightarrow LGR

Limit format for graphs
 $\hookrightarrow K \rightarrow 20 \log(K)$ -inc point

$$\frac{1}{j\omega} = \frac{1}{\omega} \cdot \angle -90^\circ$$

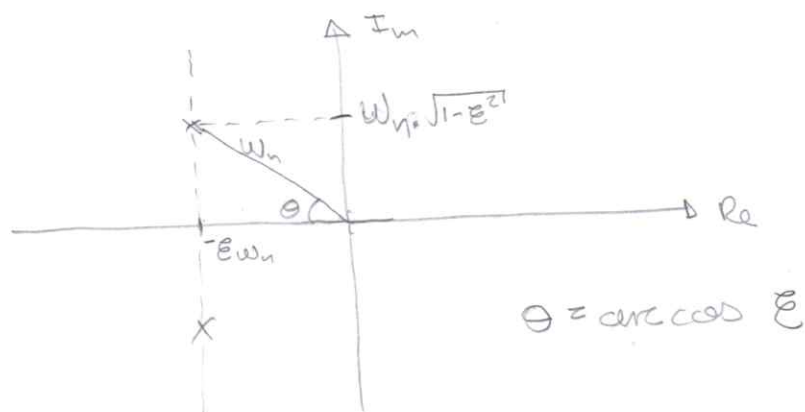
$$(j\omega + 1) = \sqrt{\omega^2 + 1} \angle \arctan\left(\frac{\omega}{1}\right)$$

PID

FTMF $\rightarrow D(s) \rightarrow$ Ziegler
- Nichols closed loop
 \downarrow
2nd order
Epsilon
cases
 \downarrow
 $c_{n-1} = 0$
 $\hookrightarrow K$
 $b_{n-1} + k_{n-2} \mid K$
 \downarrow
 s

$$0 \leq \xi < 1$$

$$p_1, p_2 = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2}$$



$$\omega_d = \omega_n \cdot \sqrt{1 - \xi^2}$$

Nota $\omega_d \leq \omega_n$ e que $\omega_d = \omega_n$ para $\xi = 0$

$\omega_d \rightarrow$ frequência amortecida

$\omega_n \rightarrow$ frequência natural.

$$0 \leq \xi < 1$$

$$t_p = \frac{\pi}{\omega_n \cdot \sqrt{1 - \xi^2}}, \quad y(t_p) = 1 + e^{-\frac{\xi \pi}{\sqrt{1 - \xi^2}}}$$

$$M_p = e^{-\frac{\xi \pi}{\sqrt{1 - \xi^2}}}$$

$$t_s = \frac{4}{\xi \cdot \omega_n}$$

$$t_r \approx \frac{e^{(\theta / \tan \theta)}}{\omega_n}, \quad \theta = \cos^{-1}(\xi)$$

$$\tan^{-1}(\alpha \pm \beta) = \frac{\tan^{-1}(\alpha) \pm \tan^{-1}(\beta)}{1 \mp \tan^{-1}(\alpha) \cdot \tan^{-1}(\beta)}$$

2nd order system

if the response to a unit step input is known, then it is mathematically possible to compute the response to any input.

[control system fundamentals CRC]

1. Rise time (t_r)
2. Percent overshoot (PO)
3. Peak time (t_p)
4. Settling time (t_s)
5. Delay time (t_d)

time rise for overdamped $[0\% \rightarrow 100\%]$
 " " underdamped $[10\% \rightarrow 90\%]$

settling time is when it is within $[2\% \rightarrow 5\%]$ of final value.

Delay time is the time to reach half of its final value.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$PO = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} ; t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$t_{s_{2\%}} = \frac{4}{\omega_n \zeta} ; t_{s_{5\%}} = \frac{3}{\zeta\omega_n}$$

$$t_d \approx \frac{1 + 0.125\zeta + 0.469\zeta^2}{\omega_n}$$

$$t_r \approx \frac{1 - 0.4167\zeta + 2.917\zeta^2}{\omega_n}$$

if $\zeta < 1$

$$p_1 = -\zeta\omega_n + j\omega_n \sqrt{1-\zeta^2} ; p_2 = -\zeta\omega_n - j\omega_n \sqrt{1-\zeta^2}$$

Notas

2019 → 2020

Sistemas eléctricos, electrónicos con operaciones,
diferencias, engranajes redes distadas, não sei em
tesis vai ser dado em si sel.

"Motores não sei"

usar eléctrico para freinar


Matéria.

tanques, técnico, mecânica

$$\begin{aligned} y &= 20 \log_a K \\ K &= a^{\frac{y}{20}} \end{aligned}$$

$$\text{Arg} = \arctan \left(\frac{J}{R} \right) = \frac{\sqrt{J^2 + R^2}}{R}$$

$$\arctan(\alpha \pm \beta) = \frac{\arctan(\alpha) \pm \arctan(\beta)}{1 \mp \arctan(\alpha) \cdot \arctan(\beta)}$$


$$\frac{2}{3} \pi r^2$$

23/4/2020

review



Modelos de Sistemas
exercícios de técnico
Agudo

9) 10)

Análise de Sistemas no
Domínio dos Tempos



$$0 < \xi < 1$$

subamortecido

e depois
ordens ω ; $N=1, N=2, 3$

9) 13)

— melhora algoritmos + completos e abrangente.

Basics

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \quad \sqrt{1} = 1$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$1^0 = 1$$

$$1^n = 1$$

$$\sqrt{5} \cdot \sqrt{5} = (\sqrt{5})^2 = \sqrt{25}$$

$$\frac{1}{\sqrt{\frac{a}{b}}} = \sqrt{\frac{b}{a}}$$

$$(\sqrt{a})^2 = a$$

$$\sqrt{a^2} = a$$

$$\frac{1}{0} = \infty$$

$$\frac{1}{\infty} = 0$$

$$\boxed{\frac{a}{a} = 1}$$

$$a \cdot 1 = a$$

$$\boxed{\frac{\Delta}{\Delta} = 1}$$

$$\frac{a}{1} = a$$

$$\frac{1}{a} = a^{-1}$$

$$(a^n)^m = a^{m \cdot n}$$

$$5 \neq 5 + 3 - 3$$

$$a^n \cdot b^m = b^m \cdot a^n$$

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$a+b+c$$

$$= (a+b)+c$$

$$= a+(b+c)$$

$$= b+(a+c)$$

$$a \cdot a = a^2$$

$$\frac{a}{a} = a^0 = 1$$

$$\sqrt[n]{a^n} = a^{\frac{n}{n}}$$

$$\frac{1}{1+\frac{a}{b}} = \frac{1}{\frac{b+a}{b}} = \frac{b}{b+a}$$

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$(-a) \cdot (-a) = a^2$$

$$a+a = 2a \quad a \cdot a = a^2$$

$$a-a = 0 \quad a \div a = 1$$

$$ab+ac = (b+c) \cdot a$$

$$a+b-c = a+(b-c)$$

$$= b-(c-a)$$

$$\underbrace{ab+ac}_{1} = 0$$

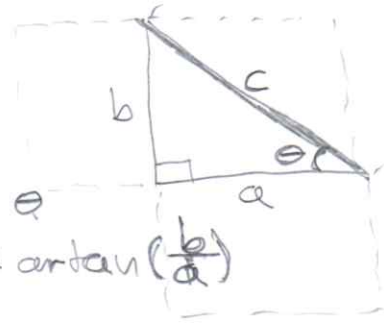
$$1 + \frac{ac}{ab} = 0 \Leftrightarrow 1 + \frac{c}{b}$$

$$a+bi = c \quad |c| = \text{Abs}(a+bi) = \sqrt{a^2+b^2} = \sqrt{a^2+b^2} = \sqrt{a^2+b^2}$$

$$a+ai = c \quad |c| = a\sqrt{2}$$

$$a+bi = c \quad \angle = \text{Arg}(a+bi) = \theta = \arctan\left(\frac{b}{a}\right)$$

$$a+ai = c \quad \angle = 45^\circ$$



$$(a+bi)(c+di) = e \quad |e| = \text{Abs}(a+bi) \cdot \text{Abs}(c+di)$$

$$\underline{e = |e| \cdot \angle e}$$

$$\underline{\angle e} = \text{Arg}(a+bi) + \text{Arg}(c+di)$$

$$X_C = \frac{1}{j\omega C} = \frac{1}{\omega C} \angle -90^\circ$$

$$X_L = j\omega L = \omega L \angle 90^\circ$$

$$\log_a x = z$$

$$x = a^z$$

$$\arctan(\alpha \pm \beta) = \frac{\arctan(\alpha) \pm \arctan(\beta)}{1 \mp \arctan(\alpha) \cdot \arctan(\beta)}$$

1.

transitorio

Rules

$$y(t) = 1 - e^{-st} = 1 - e^{-\frac{1}{\tau}t}$$

$$\tau = RC$$

$$t_r = t_{90} - t_{10}$$

$$t_s = 4\tau$$

$$\frac{\frac{K}{s(s+2)}}{1 + \frac{K}{s(s+2)}}$$

$$\begin{aligned} \frac{K}{s(s+2)} \cdot \frac{1}{1 + \frac{K}{s(s+2)}} &= \frac{K}{s(s+2) + K} \\ &= \frac{K}{s^2 + 2s + K} \end{aligned}$$

$$\begin{aligned} 2\xi\omega_n &= 2 \\ K &= \omega_n^2 \end{aligned}$$

$$\begin{aligned} 2\xi\sqrt{K} &= 2 \\ \xi &= \frac{1}{\sqrt{K}} \end{aligned}$$

NOTA

$$G(s) = \frac{s+2}{s(s+4)}$$

$$K_p = \lim_{s \rightarrow 0} G(s) \quad \text{constante de erro posico}$$

$$K_v = \lim_{s \rightarrow 0} s \cdot G(s) \quad \text{constante de erro velocidade}$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s) \quad \text{constante de erro aceleracao}$$

em regime permanente

SISEL

$$G(j\omega) = \frac{j\omega B + K}{(j\omega)^2 M + j\omega B + K} \rightarrow \left(j \frac{\omega}{T} + 1 \right)$$

$$\rightarrow \left(1 + 2 \zeta \left(j \frac{\omega}{\omega_n} \right) + \left(j \frac{\omega}{\omega_n} \right)^2 \right)$$

$$G(j\omega) = \frac{j\omega \frac{B}{K} + 1}{(j\omega)^2 \frac{M}{K} + j\omega \frac{B}{K} + 1}$$

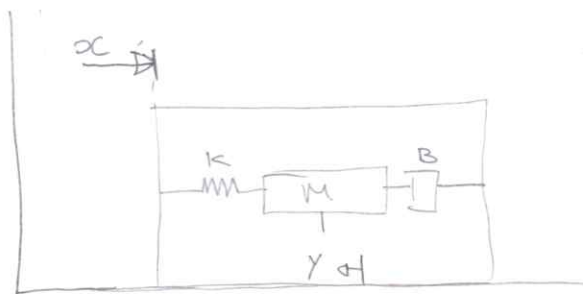
$$= \frac{j \frac{\omega}{250} + 1}{\frac{(j\omega)^2}{25} + j \frac{\omega}{250} + 1}$$

$$\log \left(\frac{a}{b} \right)$$

$$= \log(a) - \log(b)$$

$$= 20 \log \sqrt{1 + \left(\frac{\omega}{250} \right)^2} - 20 \log \sqrt{\left(1 - \frac{\omega^2}{25} \right)^2 + \left(\frac{\omega}{250} \right)^2}$$

Accelerometro



$$\sum F_R = M a(t)$$

$$-k y(t) - B \frac{dy(t)}{dt} = M \frac{d^2}{dt^2} (y - x)$$

$$\Rightarrow \frac{dx(t)}{dt} = v$$

$$\Rightarrow y(t) = \left(\frac{M}{K} \right) \frac{d^2}{dt^2} x(t)$$

$$2^a \text{ Loi de Newton } \sum F(t) = M a(t)$$

$$2^a \text{ Loi de Newton } \sum T(t) = J \alpha(t)$$

$$\begin{cases} q = \frac{\theta_1(t) - \theta_2(t)}{R} \\ q = C \frac{d}{dt} [\theta_1(t) - \theta_2(t)] \end{cases}$$

$$\begin{cases} C = A \frac{dh(t)}{dt} = \frac{d}{dt} V(t) \\ q_0 = \frac{h}{R} \end{cases}$$

Control system Fundamentals

apontamentos

$$F(s) = \frac{c}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n > 0 \wedge 0 < \zeta < 1$$

$$p_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

$$\omega_n > 0 \wedge \zeta > 1, \text{ poles reais diferentes}$$

$$F(s) = \frac{c}{(s-p_1)(s-p_2)} = \frac{c}{p_1-p_2} \left[\frac{1}{s-p_1} - \frac{1}{s-p_2} \right]$$

$$\mathcal{L}^{-1}$$
$$f(t) = \frac{c}{p_1-p_2} [e^{p_1 t} - e^{p_2 t}]$$

$$\omega_n > 0 \wedge \zeta = 1, p_1 = p_2 = -\omega_n$$

$$F(s) = \frac{c}{(s+\omega_n)^2} \quad ; p_1 = p_2 = -\omega_n$$
$$\mathcal{L}^{-1}$$

$$f(t) = c.t.e^{-\omega_n t}$$

Proof.

$$s^2 + 2\epsilon\omega_n s + \omega_n^2$$

$$s = \frac{-2\epsilon\omega_n \pm \sqrt{(2\epsilon\omega_n)^2 - 4\omega_n^2}}{2}$$

$$s = -\epsilon\omega_n \pm \frac{\sqrt{(2\epsilon\omega_n)^2 - 4\omega_n^2}}{2}$$

$$s = -\epsilon\omega_n \pm \sqrt{\frac{(2\epsilon\omega_n)^2 - 4\omega_n^2}{4}}$$

$$s = -\epsilon\omega_n \pm \sqrt{(\epsilon\omega_n)^2 - \cancel{\frac{4\omega_n^2}{4}}}$$

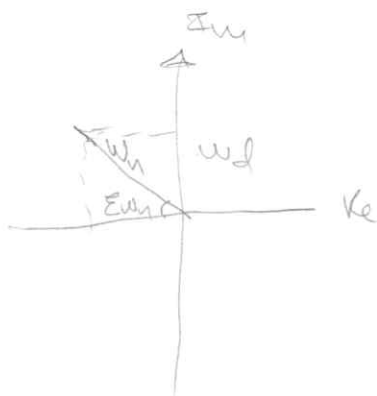
$$s = -\epsilon\omega_n \pm \sqrt{\epsilon^2\omega_n^2 - \omega_n^2}$$

$$s = -\epsilon\omega_n \pm \omega_n \sqrt{\epsilon^2 - 1}$$

$$s = \omega_n (\epsilon \pm \sqrt{\epsilon^2 - 1})$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\frac{\epsilon^2\omega_n^2 - \omega_n^2}{\sqrt{(\epsilon^2 - 1)\omega_n^2}} = \sqrt{\epsilon^2 - 1} \sqrt{\omega_n^2}$$



$$\epsilon^2 - 1 > 0$$

$$\epsilon^2 > 1$$

$$\epsilon > 1$$

$$\epsilon^2 - 1 = 0$$

$$\epsilon = 1$$

$$\epsilon^2 - 1 < 0$$

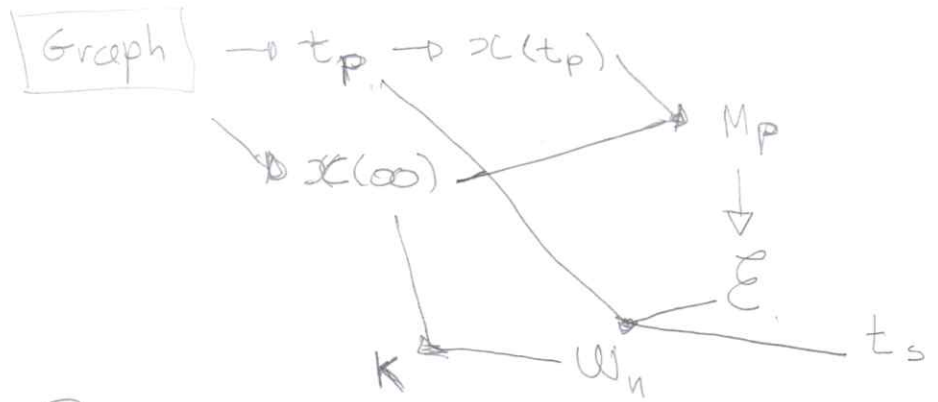
$$\epsilon^2 < 1$$

$$\epsilon < 1$$

NOTA

Analysis

17/4/2020



$$\xi > 1$$

$$\xi = 1$$

$$0 < \xi < 1$$

Diferentes dampening
different equations

$$0 < \xi < 1 \Rightarrow \text{Poles} = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2}$$



NOTA

$$G(s) = \frac{(s+z)}{(s+p_1)(s+p_2)}$$

$|G(s)| \rightarrow$ modulo (Abs)

$\arg[G(j\omega)] \rightarrow$ Fase

$$|G(s)| = \frac{\overline{BD}}{\overline{CD} \cdot \overline{AD}}$$

$$\arg[G(j\omega)] = \beta - \gamma - \alpha \quad (\text{Arg})$$

Definições

$$\arg[G_H(j\omega_\pi)] = -\pi \Rightarrow MG = \frac{1}{|G_H(j\omega_\pi)|}$$

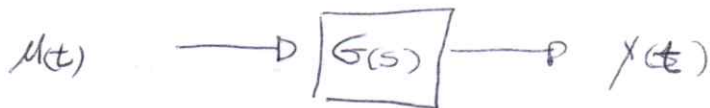
$$\text{ou } MG_{dB} = 20 \log_{10} \frac{1}{|G_H(j\omega_\pi)|}$$

Definições

$$|G_H(j\omega_i)| = 1 \Rightarrow MF = 180^\circ + \arg[G_H(j\omega_i)]$$

Análise de sistemas no Domínio das Frequências.

- FTMA
- Gráfico em frequências
- Diagrama polares
- Diagrama de Bode
- FTMF
- Estabilidade → MG + margem ganho
 - MF → margem de fase



$$U(t) = \sin(2t) = \frac{\omega}{s^2 + \omega^2}$$

$$G(s) = \frac{1}{s+1}$$

$$s = j2$$

$$Y(s) = G(s) \cdot U(s)$$
$$= \frac{1}{s+1} \Big|_{s=j2} \cdot \frac{2}{s^2 + 2^2}$$

$$= \frac{1}{j2+1} \cdot \frac{2}{s^2 + 2^2}$$

$$= \frac{1}{\sqrt{5}} \cdot e^{-j1,1} \cdot \frac{2}{s^2 + 2^2} = \frac{1}{\sqrt{5}} \cdot \sin(2t - 1,1)$$