8. considere:

$$G(s) = \frac{1}{s(1+\frac{s}{2})^2} = \frac{4}{s(s+2)(s+2)}$$

a) Efectue a representação assimptota de Bode.

b) calcule MG e MF.

a) • Rewrite the T.F as product of basic factors:

$$G(s) = \frac{1}{s} \cdot \frac{1}{(1+\frac{s}{2})} \cdot \frac{1}{(1+\frac{s}{2})}$$

• Replace "s" by "jw":

$$G(jw) = \frac{1}{jw} \cdot \frac{1}{(1+\frac{jw}{2})} \cdot \frac{1}{(1+\frac{jw}{2})}$$

Pole at origin

Double pole at frequency 2

• $|G(jw)|$:

$$20 \operatorname{Log}|G(jw)| = -20 \operatorname{Log}(w) - 20 \operatorname{Log}\left(\left|\cancel{1}+\frac{jw}{2}\right|\right) - 20 \operatorname{Log}\left(\left|1+\frac{jw}{2}\right|\right)$$

• $|G(jw)|$:

$$20 \operatorname{Log}|G(jw)| = -20 \operatorname{Log}(w) - \underbrace{20 \operatorname{Log}\left(\left|1+\frac{jw}{2}\right|\right)}_{A} - 20 \operatorname{Log}\left(\left|1+\frac{jw}{2}\right|\right)$$

Evaluate A for possible values of "w":

$w >> 2$: $A = +\infty$

$w = 2$: $A = 20 \operatorname{Log} \sqrt{2} = 3dB$

$w << 2$: $A = 0$ $[w = \phi]$

8. continua

- $\underline{|G(j\omega)}$:

$$\underline{|G(j\omega)} = -\arg(j\omega) - \arg\left(1+\frac{j\omega}{2}\right) - \arg\left(1+\frac{j\omega}{2}\right)$$
$$= -90^\circ - \underbrace{\arg\left(1+\frac{j\omega}{2}\right)}_{B} - \arg\left(1+\frac{j\omega}{2}\right)$$

Evaluate "B" for possible values of "ω":

$\omega >> 2$: $B = 90^\circ$
$\omega = 2$: $B = 45^\circ$
$\omega << 2$: $B = \phi$

b) $G(j\omega) = \frac{1}{j\omega} \cdot \frac{1}{\left(1+\frac{j\omega}{2}\right)} \cdot \frac{1}{\left(1+\frac{j\omega}{2}\right)}$

- GM and PM analitically:

when $\Rightarrow \arg[G(j\omega)] = -\pi$
then $\Rightarrow MG = \frac{1}{|G(j\omega)|}$

Note:
$MG_{dB} = 20\log MG$

Find for what values of "ω" $\arg[G(j\omega)] = -\pi$:

$\arg\left[\frac{1}{j\omega} \cdot \frac{1}{\left(1+\frac{j\omega}{2}\right)} \cdot \frac{1}{\left(1+\frac{j\omega}{2}\right)}\right] = -180^\circ$

$\phi - 90^\circ - tg^{-1}\left(\frac{\omega}{2}\right) - tg^{-1}\left(\frac{\omega}{2}\right) = -180^\circ$

$2\,tg^{-1}\left(\frac{\omega}{2}\right) = 90^\circ \Rightarrow tg^{-1}\left(\frac{\omega}{2}\right) = 45^\circ \Leftrightarrow \omega = \underline{2(rad/sec)}$

Replace the value of "ω" in the equation in order to find the GM:

$$MG = \frac{1}{\left|\frac{1}{j\omega\left(1+\frac{j\omega}{2}\right)^2}\right|}\Bigg|_{\omega=2 rad/sec}$$

$$= \frac{1}{\frac{1}{\omega\cdot\left(\sqrt{1^2+\left(\frac{\omega}{2}\right)^2}\right)^2}}\Bigg|_{\omega=2} = \frac{1}{\frac{1}{2\cdot 2}} = 4$$

8. continuação

$$MG_{dB} = 20 \log 4 = 12{,}04 \text{ dB}$$

Phase Margin:

when $\Rightarrow |G(j\omega)| = 1$

then $\Rightarrow MF = 180° + \arg[G(j\omega)]$

Find for what value of "ω" $|G(j\omega)| = 1$:

$$\frac{1}{\left|j\omega\cdot\left(1+\frac{j\omega}{2}\right)^2\right|} = 1 \Leftrightarrow \omega\cdot\left(\sqrt{1^2+\left(\frac{\omega}{2}\right)^2}\right)^2 = 1$$

$$\Leftrightarrow \omega\cdot\left(1+\frac{\omega^2}{4}\right) = 1 \Leftrightarrow \omega + \frac{\omega^3}{4} = 1$$

$$\Leftrightarrow \frac{4\omega + \omega^3}{4} = 1$$

must always be positive!

$$\Leftrightarrow \omega^3 + 4\omega = 4 \quad \begin{cases} \omega = 0{,}848 & \checkmark \\ \omega = -0{,}424 + 2{,}130j & \times \\ \omega = -0{,}424 - 2{,}130j & \times \end{cases}$$

Replace the value of "ω" in MG equation in order to obtain PM:

$$PM = 180° + \arg\left[\frac{1}{j\omega}\cdot\frac{1}{\left(1+\frac{j\omega}{2}\right)^2}\right]\Bigg|_{\omega = 0{,}848 \text{ (rad/sec)}}$$

$$= 180° - 90° - 2\,tg^{-1}\left(\frac{\omega}{2}\right)\Bigg|_{\omega=0{,}848}$$

$$= 180° - 90° - 45{,}95°$$

$$= \boxed{44{,}06°}$$

$\therefore GM_{dB} > \emptyset \wedge MF > \emptyset$

$\Rightarrow$ system is stable.

graficos

8a) $G(s) = \dfrac{1}{s.(1+\frac{s}{2})^2}$

- Rewrite the T.F as product of basic factors

$$G(s) = \frac{1}{s} \cdot \frac{1}{(1+\frac{s}{2})} \cdot \frac{1}{(1+\frac{s}{2})}$$

Nota: exercicio dos gráficos e perguntas feitos? sai exam.

Modern control Engineering.

- Replace "s" by "jw"

$$G(s) = \frac{1}{s} \cdot \frac{1}{(1+\frac{jw}{2})} \cdot \frac{1}{(1+\frac{jw}{2})}$$

- convert into standard time constant form:

$$G(s) = \frac{1}{s} \cdot \frac{1}{1+\frac{s}{2}} \cdot \frac{1}{1+\frac{s}{2}}$$

- Replace "s" by "jw":

$$G(jw) = \frac{1}{jw} \cdot \frac{1}{(1+\frac{jw}{2})} \cdot \frac{1}{(1+\frac{jw}{2})}$$

Pole at origin

Double pole at frequency 2

- $|G(jw)|$:

$$20 \log |G(jw)| = -20 \log(w) \underbrace{-20 \log\left(\left|1+\frac{jw}{2}\right|\right)}_{A} -20 \log\left(\left|1+\frac{jw}{2}\right|\right)$$

Evaluate "i" for possible values of "w":

$w >> 2$: $A = +\infty$

$w = 2$: $A = 20 \log \sqrt{2} = 3\,dB$

$w << 2$: $A = 0$

- $\underline{/G(jw)}$

$$\underline{/G(jw)} = -\arg(jw) - \arg\left(1+\frac{jw}{2}\right) - \arg\left(1+\frac{jw}{2}\right)$$

$$= -90^\circ \underbrace{- \arg\left(1+\frac{jw}{2}\right)}_{B} - \arg\left(1+\frac{jw}{2}\right)$$

Evaluate "B" for possible values of "w":

$w >> 2$: $B = 90^\circ$

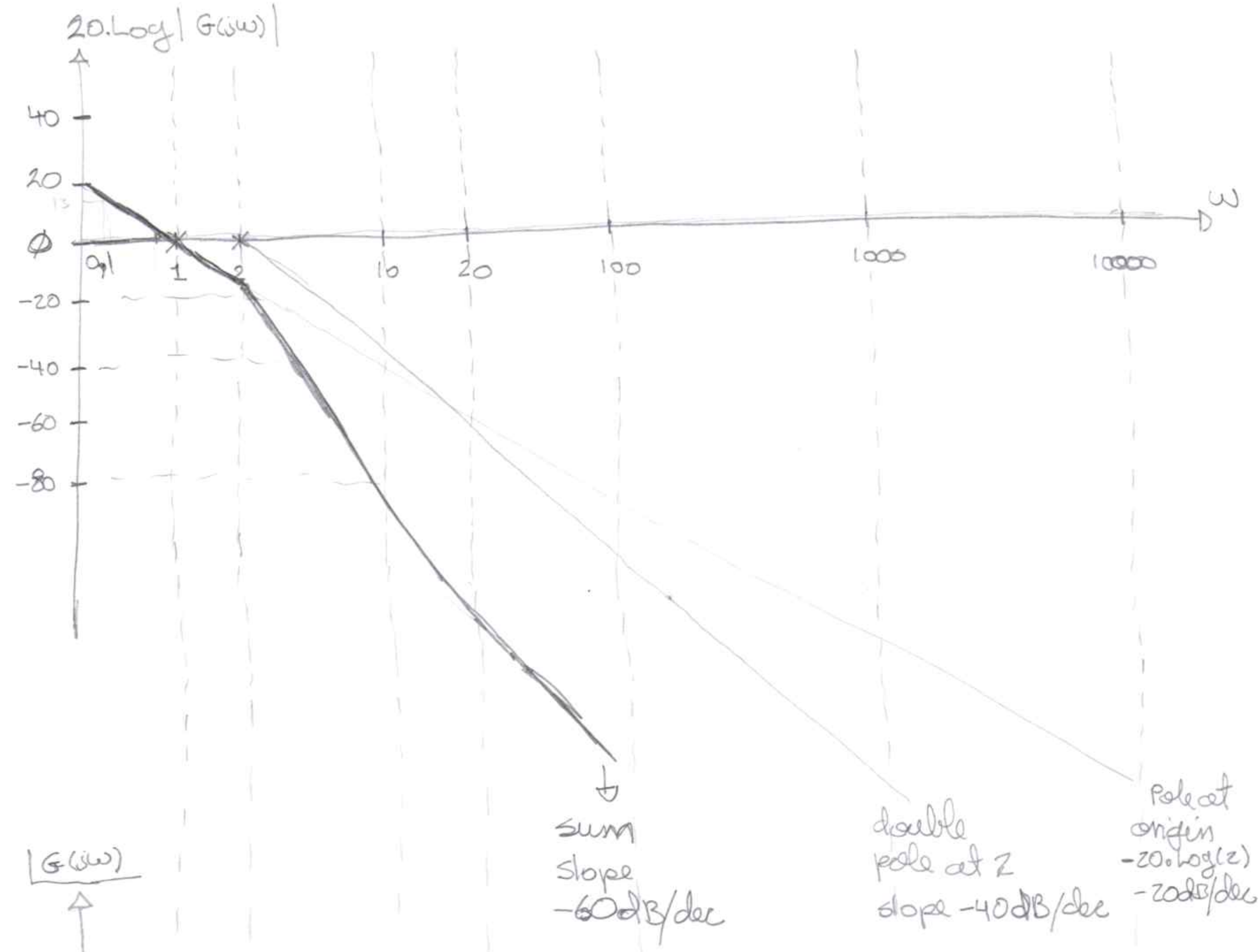
$w = 2$: $B = 45^\circ$

$w << 2$: $B = \emptyset^\circ$

Frequency Response

practica

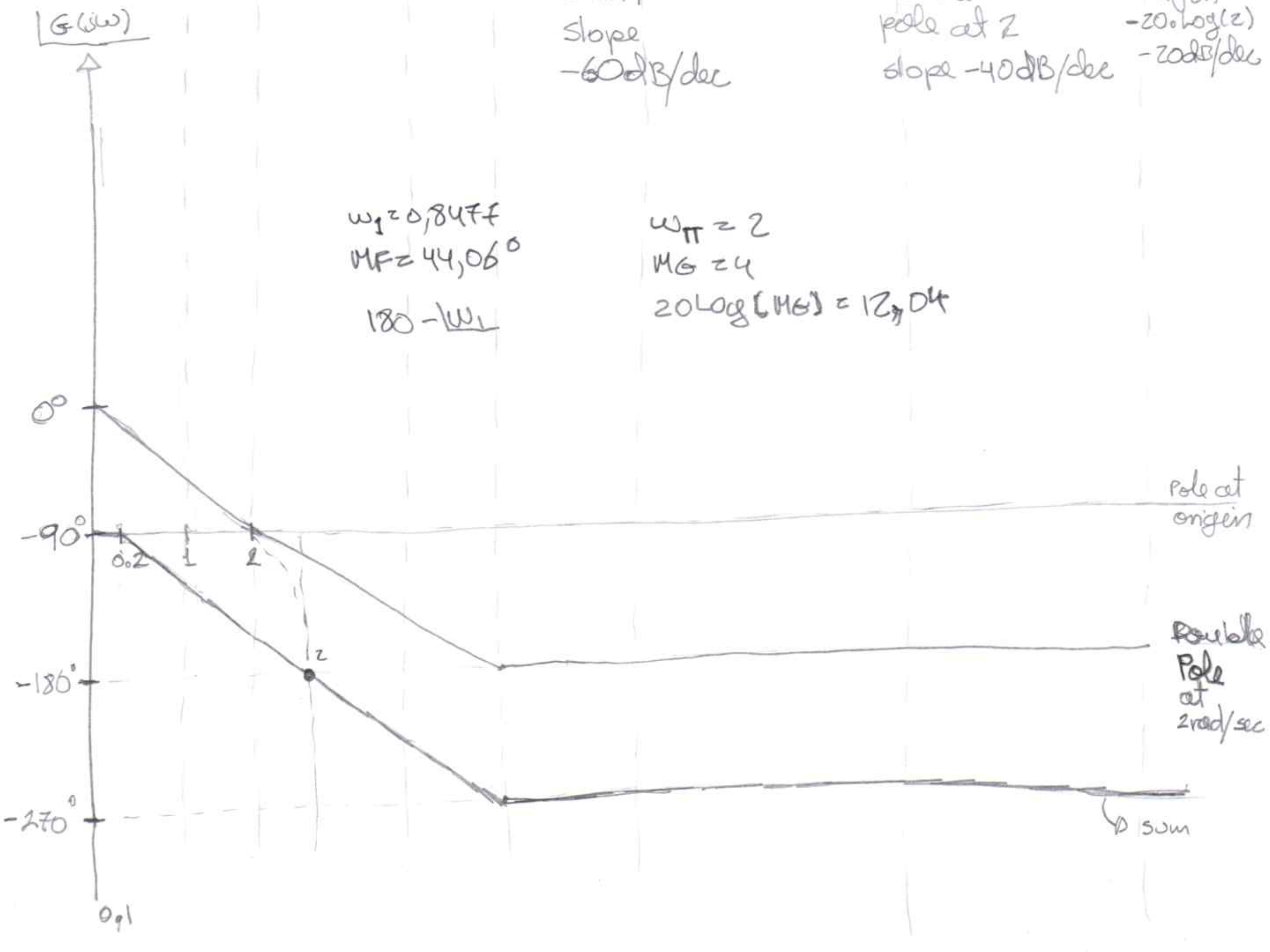
8a) continuacion



$\omega_1 = 0{,}8477$
$MF = 44{,}06^0$

$180 - \underline{|\omega_1}$

$\omega_\pi = 2$
$MG = 4$
$20\log[MG] = 12{,}04$



9/6/2009 teses. teorica

Resposta em frequência

8.

$$G(s) = \frac{1}{s\left(1+\frac{s}{2}\right)^2}$$

a) polos

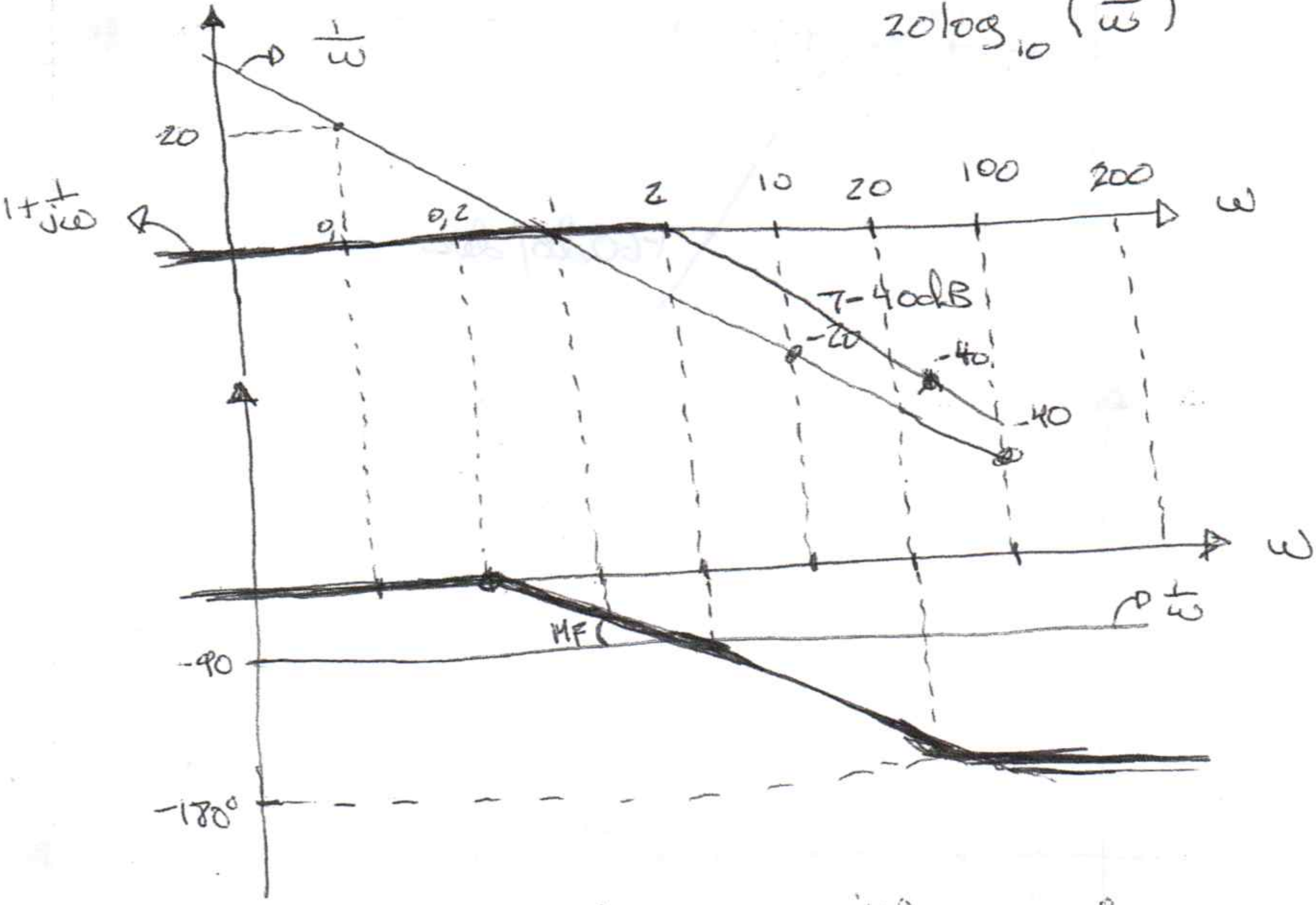
$s=0 \quad \wedge \quad 1+\frac{s}{2}=0$

$s=-2$

$p = 0; -2; -2$

$$G(j\omega) = \frac{1}{j\omega\left(1+\frac{j\omega}{2}\right)^2} \qquad \frac{1}{j\omega} \Rightarrow \left|\frac{1}{j\omega}\right| = \frac{1}{\omega}$$

$20\log_{10}\left(\frac{1}{\omega}\right)$

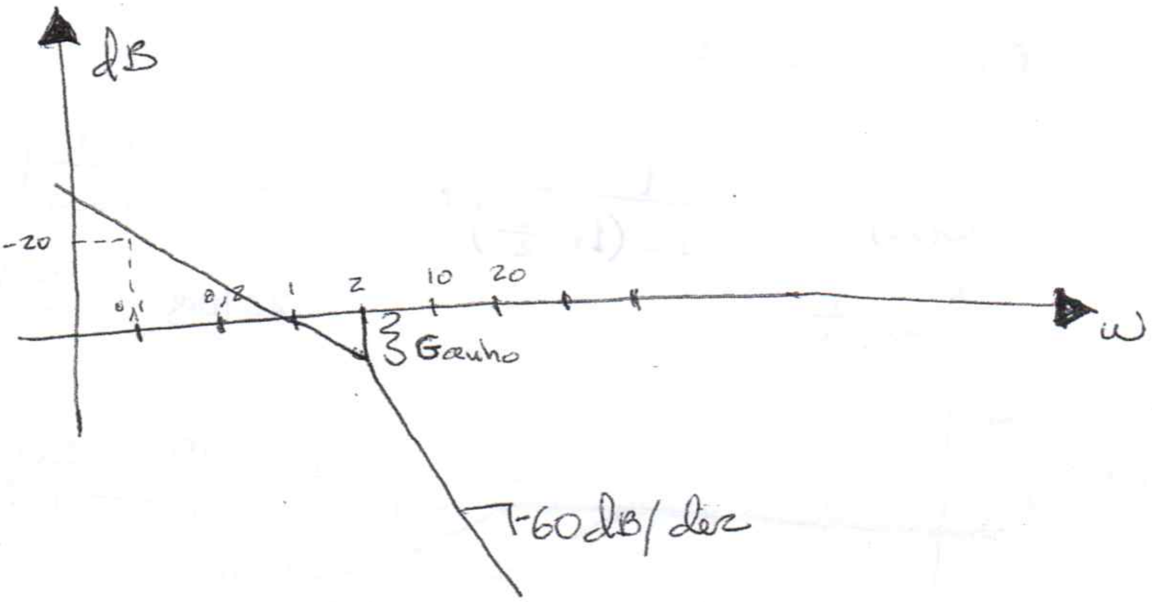


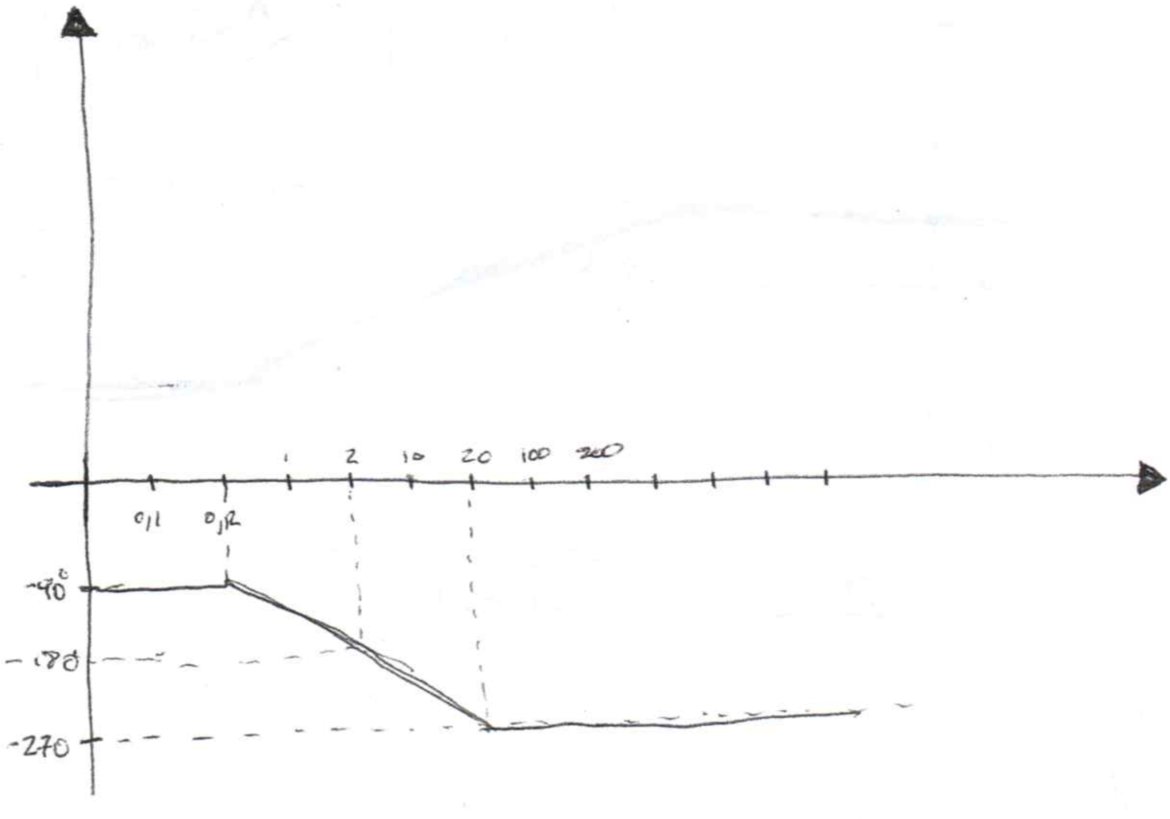
$$\angle\frac{1}{j\omega} = \frac{-j\omega}{(j\omega)(-j\omega)} = \frac{-j\omega}{\omega^2} \Rightarrow 90^\circ$$

$$\left(1+\frac{j\omega}{2}\right) = \begin{cases} \left|1+\frac{j\omega}{2}\right| = 20\log_{10}\sqrt{1^2+\frac{\omega^2}{2^2}} \\ \angle 1+\frac{j\omega}{2} = -\mathrm{arctg}\left(\frac{\omega}{2}\right) \end{cases}$$

$20\log_{10} 1 = 0$

soma





9/6/2009.

tesis teoria

— 11 —

$$\frac{1}{1+\frac{j\omega}{2}}$$

se $\omega \to 0 \to \frac{1}{1} \Rightarrow Mod = 0\,dB$

$\angle 0^\circ$

se $\omega \to \infty \Rightarrow \frac{1}{\frac{j\omega}{2}} \Rightarrow Mod = -20\,dB$

$\angle \frac{2}{j\omega} \Rightarrow -90^\circ$

$MG \cong 10\,dB \qquad \omega_\pi \cong 2\,rad/s$

$MF \cong 45^\circ \qquad \omega_1 \cong 1\,rad/s.$

$$G(j\omega) = \frac{1}{j\omega\left(1+\frac{j\omega}{2}\right)^2}$$

NOTA

$$\angle \frac{1}{j\omega\left(1+\frac{j\omega}{2}\right)^2} = -180^\circ \Rightarrow \omega_\pi$$

[redacted]

$$\Rightarrow \angle \frac{1}{j\omega_\pi} + \angle \frac{1}{1+\frac{j\omega_\pi}{2}} + \angle \frac{1}{1+\frac{j\omega_\pi}{2}} = -180^\circ$$

$$-90 + 2\,\angle \frac{1}{1+\frac{j\omega_\pi}{2}} = -180$$

$$-2\,\angle 1+\frac{j\omega_\pi}{2} = -90^\circ$$

$$\mathrm{artg}\left(\frac{\omega_\pi}{2}\right) = 45^\circ \Leftrightarrow \omega_\pi = 2\,rad/s$$

$$M_G = \left| j\omega\left(1+\frac{j\omega}{2}\right)^2 \right|_{\omega_\pi = 2\,rad/s}$$

$$= |j\omega| \cdot \left|1+\frac{j\omega}{2}\right| \cdot \left|1+\frac{j\omega}{2}\right| = 4 \Rightarrow 20\log_{10}(4)$$

$$= 12{,}04$$

Margem de fase.

$$G(j\omega) = \frac{1}{j\omega\left(1+\frac{j\omega}{2}\right)^2}$$

$$|G(j\omega)| = 1$$

$$\left|\frac{1}{j\omega\left(1+\frac{j\omega}{2}\right)^2}\right| = 1 \Leftrightarrow \left|\frac{1}{\omega\left(\sqrt{1^2+\frac{\omega^2}{2^2}}\right)^2}\right| = 1$$

$$\left|\frac{1}{\omega\left(1+\frac{\omega^2}{4}\right)}\right| = 1 \Leftrightarrow \left|\frac{1}{\omega\left(\frac{4+\omega^2}{4}\right)}\right| = 1$$

$$\Leftrightarrow \left|\frac{4}{4\omega+\omega^3}\right| = 1$$

formulas

MF 1 MG $\qquad \omega_1 = 0{,}847$ rad/s.

$$MF = 180^\circ + \arg[GH(j\omega_1)]$$

$$= 180^\circ + \arg\left[\frac{1}{j0{,}847\left(1+\frac{j0{,}847}{2}\right)^2}\right]$$

$$= 180^\circ - 90^\circ - 2\,\operatorname{arctg}\left(\frac{\frac{0{,}847}{2}}{1}\right)$$

$$= 44{,}09^\circ$$