3. calcular polos ; zeros

resposta ao degrau unitário e gráfico.

a) $G_{(s)} = \dfrac{4}{(s+2)(s+4)}$

$K \Rightarrow \dfrac{4}{2\left(\frac{s}{2}+1\right)4\left(\frac{s}{4}+1\right)}$

$K = \frac{1}{2}$

não tem zeros ;

polos $s=-2 \wedge s=-4$,

$s^2 + 4s + 2s + 8 = s^2 + 6s + 8$

$G_{(s)} = \frac{1}{2} \quad \dfrac{8}{s^2+6s+8}$

$6 = 2\xi\omega_n$

$8 = \omega_n^2 \Rightarrow \omega_n = \sqrt{8}$

$K = \frac{1}{2}$

$6 = 2\xi\sqrt{8} \Rightarrow \xi = \dfrac{6}{2\sqrt{8}} \Rightarrow \underline{\xi > 1}$

a resposta logo é sobreamortecida.

logo se aplicar o degrau unitário $u(t)$

$\Rightarrow \dfrac{1}{s}$

$Y_{(s)} = 4 \times \dfrac{1}{(s+2)(s+4)\,s}$

$\dfrac{1}{(s+2)(s+4)s} = \dfrac{A}{s+2} + \dfrac{B}{s+4} + \dfrac{C}{s}$

$A = \dfrac{1}{(s+4)s}\Bigg|_{-2} = -\dfrac{1}{4} \qquad B = \dfrac{1}{(s+2)s}\Bigg|_{-4} = \dfrac{1}{8} \qquad C = \dfrac{1}{(s+2)(s+4)}\Bigg|_{0} = \dfrac{1}{8}$

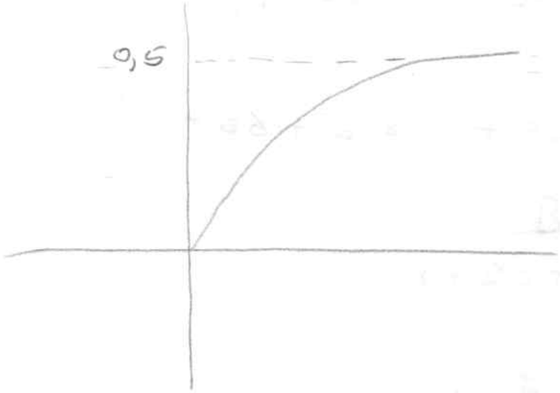
$= \dfrac{-\frac{1}{4}}{s+2} + \dfrac{\frac{1}{8}}{s+4} + \dfrac{\frac{1}{8}}{s}$

$$Y_{(s)} = 4 \times \left[-\frac{\frac{1}{4}}{s+2} + \frac{\frac{1}{8}}{s+4} + \frac{\frac{1}{8}}{s} \right]$$

$$= -\frac{1}{s+2} + \frac{\frac{1}{2}}{s+4} + \frac{\frac{1}{2}}{s}$$

$\therefore$

$$Y(t) = \frac{1}{2} - e^{-2t} + \frac{1}{2} e^{-4t} \qquad t > 0$$



$$M_p = e^{\frac{-\xi \pi}{\sqrt{1-\xi^2}}} \qquad t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$$

3a)

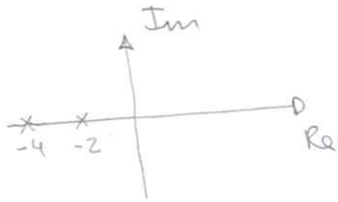
$$G(s) = \frac{4}{(s+2)(s+4)}$$

↖ Roots of the numerators are zeros

↖ Roots of the denominator are poles

- Zeros → no zeros
- Poles: $s+2=0 \;\wedge\; s+4=0$

  $s=-2 \;\wedge\; s=-4$



- Response type:
  - two Distinct Real Poles → $\xi > 1$ (over Damped)
  - the Response increases monotonically

- Considering $G(s) = \frac{Y(s)}{R(s)}$

$$\frac{Y(s)}{R(s)} = \frac{4}{(s+2)(s+4)}$$

- Unit step Response:

$$R(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s} \times \frac{4}{(s+2)(s+4)} = \frac{4}{s(s+2)(s+4)}$$

- Expanding into partial fractions:

$$Y(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$A: \left[\cancel{s} \cdot \frac{4}{\cancel{s}(s+2)(s+4)}\right]_{s=0} = \frac{4}{8} = \frac{1}{2}$$

$$B: \left[\cancel{(s+2)} \cdot \frac{4}{s\cancel{(s+2)}(s+4)}\right]_{s=-2} = -\frac{4}{4} = -1$$

$$C: \left[\cancel{(s+4)} \cdot \frac{4}{s(s+2)\cancel{(s+4)}}\right]_{s=-4} = \frac{4}{8} = \frac{1}{2}$$

$$Y(s) = \frac{1}{2} \cdot \frac{1}{s} - \frac{1}{s+2} + \frac{1}{2} \cdot \frac{1}{s+4}$$

- Applying Inverse Laplace transform:

$$Y(t) = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\}$$

$$Y(t) = \frac{1}{2} - e^{-2t} + \frac{1}{2}\cdot e^{-4t}, \quad t \geq 0$$