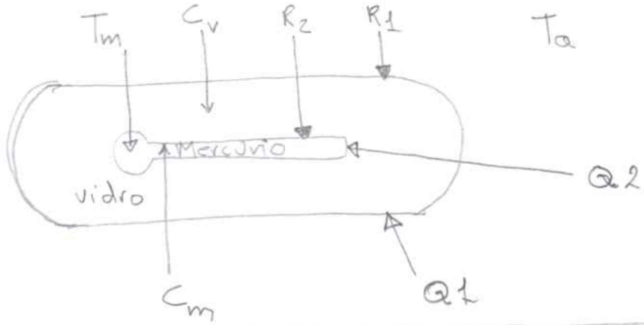
thermal system.

9(a)



Q1 - Q2 - heat energy flow.

C_v - C_m - capacidade calorífica.

$Q = C\,T$

$$\frac{T_m(s)}{T_a(s)} = ?$$

Note: $\boxed{q = \frac{T_1(t) - T_2(t)}{R}}$ • Heat energy flow.

q - heat energy flow

$T_1(t)$ $T_2(t)$ - temperatura

R - resistencia térmico

- Heat spread - A variation in the material lead to an increase in amount of heat stored.

$$\boxed{H.S = C\,[\,T_m(t) - T_m(0)\,]}$$

- Heat Balance equation

theory formulas.

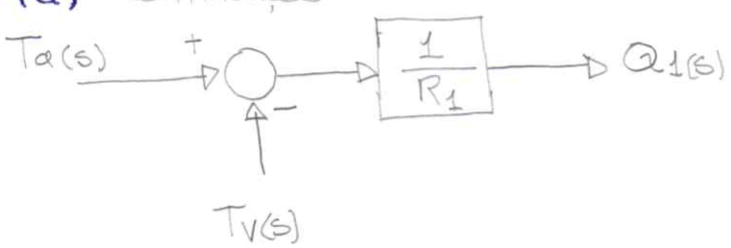
$$\boxed{\frac{T_m(0) - T_m(t)}{R} = C\,\frac{d}{dt}\left(T_m(t) - T_m(0)\right)} \Leftrightarrow \boxed{R.C\,\frac{d}{dt}T_m + T_m = T_0}$$

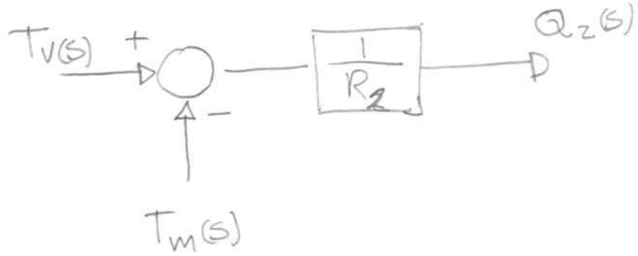
A - Dynamic equation that describes the system behavior:

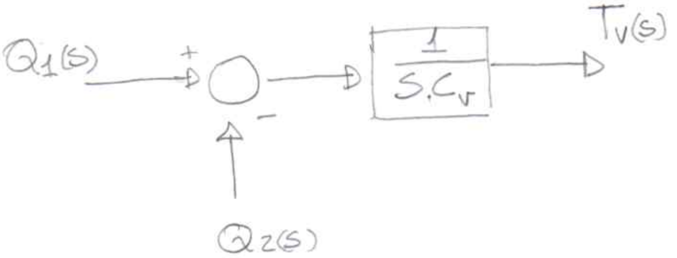
$$\begin{cases} Q_1(t) = \dfrac{T_a(t) - T_v(t)}{R_1} \\ Q_2(t) = \dfrac{T_v(t) - T_m(t)}{R_2} \\ Q_1(t) - Q_2(t) = C_v \cdot \dfrac{d}{dt} T_v(t) \\ Q_2(t) = C_m \cdot \dfrac{d}{dt} T_m(t) \end{cases} \Leftrightarrow \begin{cases} Q_1(s) = \dfrac{T_a(s) - T_v(s)}{R_1} \\ Q_2(s) = \dfrac{T_v(s) - T_m(s)}{R_2} \\ Q_1(s) - Q_2(s) = s\,C_v \cdot T_v(s) \\ Q_2(s) = s\,C_m\,T_m(s) \end{cases}$$

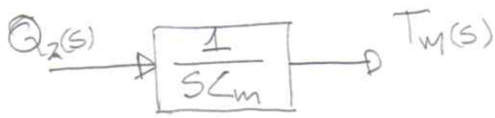
$$C = \frac{q}{m \times \Delta T} = \frac{\text{Joule}}{g \times {}^\circ C}$$

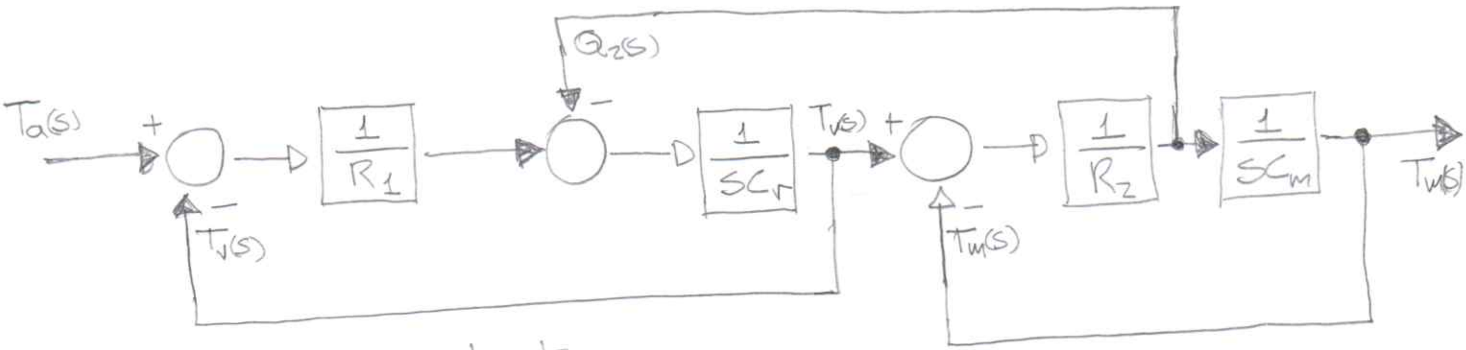
9a) continuação



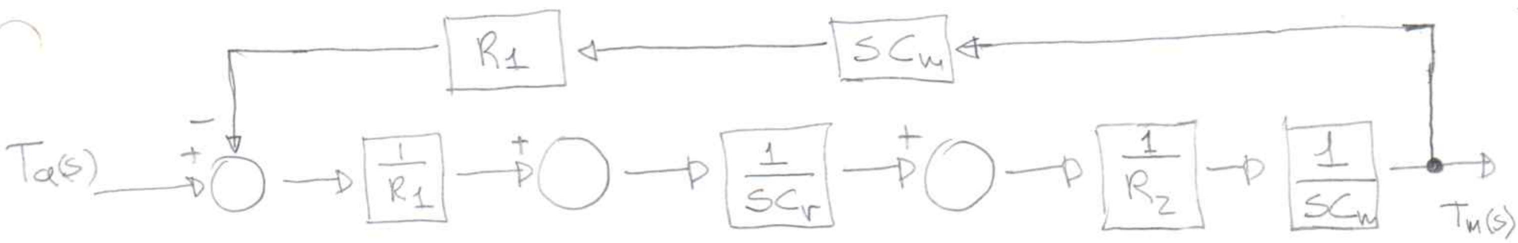


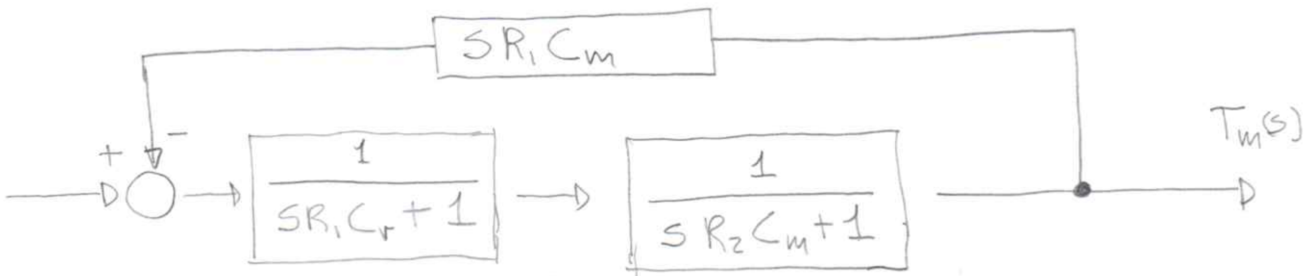






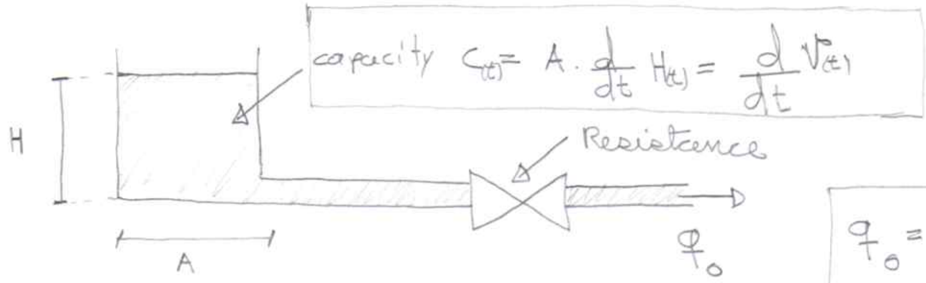
$$c(a) \quad \frac{\frac{1}{R_1}\cdot\frac{1}{sC_r}}{1+\frac{1}{R_1 sC_r}} = \frac{1}{R_1 s C_r + 1}$$





$$\boxed{\frac{T_m(s)}{T_a(s)} = \frac{1}{(1+sR_1C_r)+(1+sR_1C_m)+s.R_1.C_m}}$$

Liquid - level system



theory.

$$\boxed{q_o = \frac{H}{R}} \quad [m^3/s]$$

Law of mass conservation:

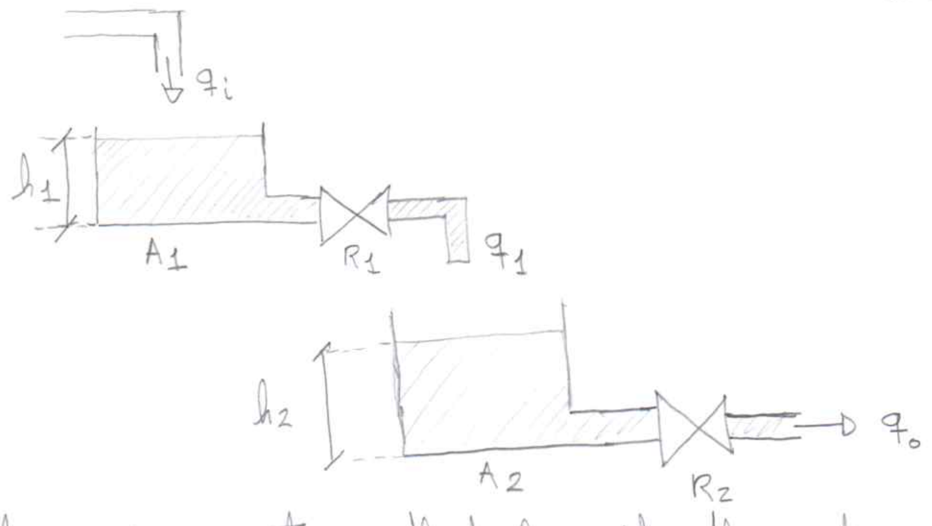
$$\boxed{\frac{d}{dt} V = q_i - q_o} \Rightarrow \boxed{q_i = q_o + \frac{d}{dt} V}$$

—— // ——

Liquid level system

10 a) Determine the transfer function $G(s) = \frac{Q_o(s)}{Q_i(s)}$

$[m^3/s]$



1- Get the dynamic equations that describe the system behavior:

TANK A1:

$$\begin{cases} q_i(t) = q_1(t) + A_1 \frac{d}{dt} h_1(t) \\ q_1(t) = \frac{h_1(t)}{R_1} \end{cases}$$

TANK A2:

$$\begin{cases} q_1(t) = q_o(t) + A_2 \frac{d}{dt} h_2(t) \\ q_o(t) = \frac{h_2(t)}{R_2} \end{cases}$$

10 a) continuação

2- Apply Laplace transform:

① $Q_i(s) = Q_1(s) + s\,A_1\,H_1(s)$

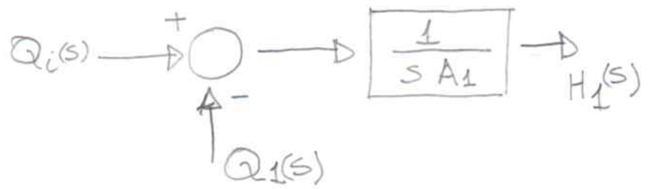
② $Q_1(s) = \dfrac{H_1(s)}{R_1}$

③ $Q_1(s) = Q_o(s) + s\,A_2\,H_2(s)$

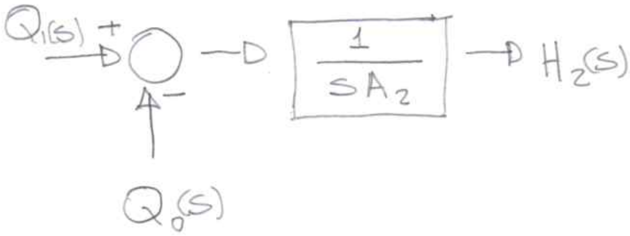
④ $Q_o(s) = \dfrac{H_2(s)}{R_2}$

3- Assemble block Diagram:

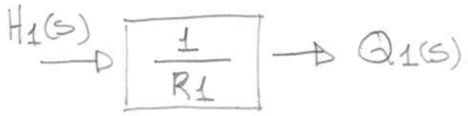
① $H_1(s) = \dfrac{Q_i(s) - Q_1(s)}{s\,A_1}$



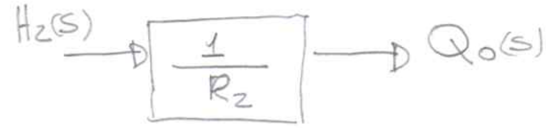
③ $H_2(s) = \dfrac{Q_1(s) - Q_o(s)}{s\,A_2}$



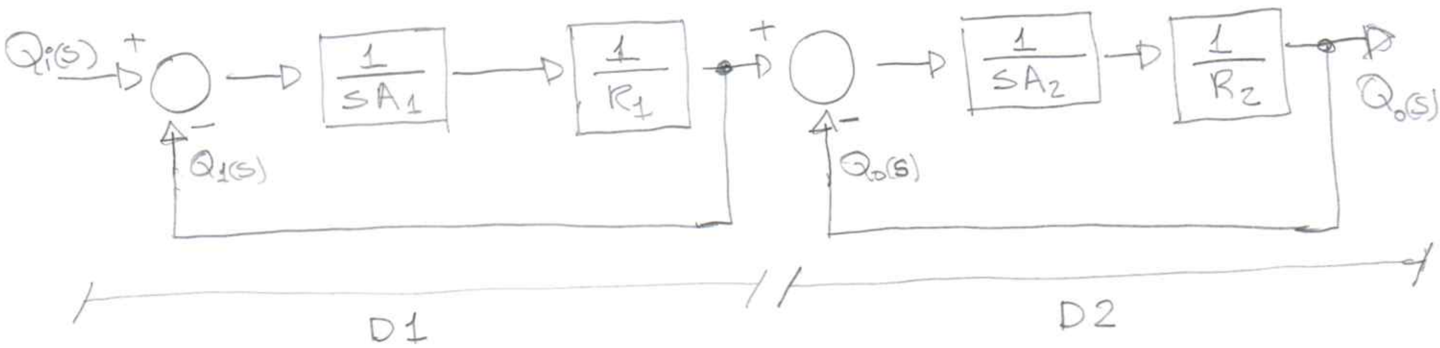
②



④



4- complete Block:



5- simplify the block Diagram:

$$D_1(s) = \frac{\frac{1}{sA_1 \cdot R_1}}{1 + \frac{1}{sA_1 \cdot R_1}} = \frac{1}{(sA_1 \cdot R_1) + 1}$$

$$D_2(s) = \frac{1}{sA_2 \cdot R_2 + 1}$$

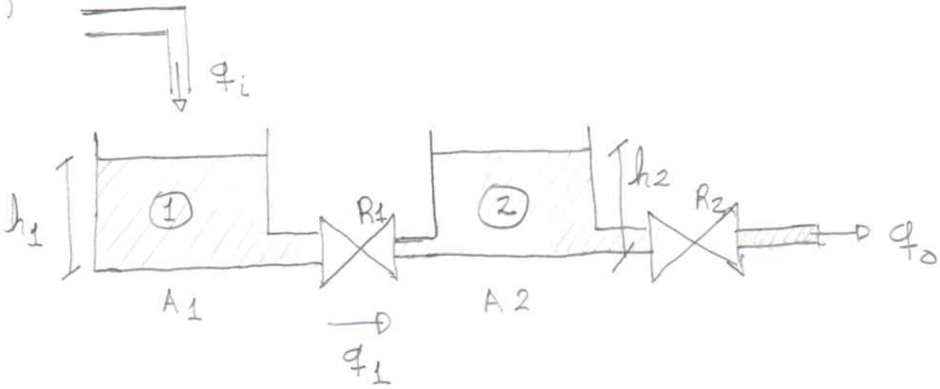
10 a) continuação:

$$\frac{Q_o(s)}{Q_i(s)} = D_1(s) \cdot D_2(s) = \frac{1}{(sA_1R_1+1)\cdot(sA_2R_2+1)}$$

$$\boxed{\frac{Q_o(s)}{Q_i(s)} = \frac{1}{s^2(A_1 \cdot R_1 \cdot A_2 \cdot R_2) + s[(A_1R_1)+(A_2R_2)]+1}}$$

Interconnected tank system

N b)



1- write the dynamic equations that describe the system behaviour:

TANK 1

$$\begin{cases} q_i(t) = q_1(t) + A_1 \frac{d}{dt} h_1(t) \\ q_1(t) = \frac{h_1(t) - h_2(t)}{R_1} \end{cases}$$

TANK 2

$$\begin{cases} q_1(t) = q_o(t) + A_1 \frac{d}{dt} h_2(t) \\ q_o(t) = \frac{h_2(t)}{R_2} \end{cases}$$

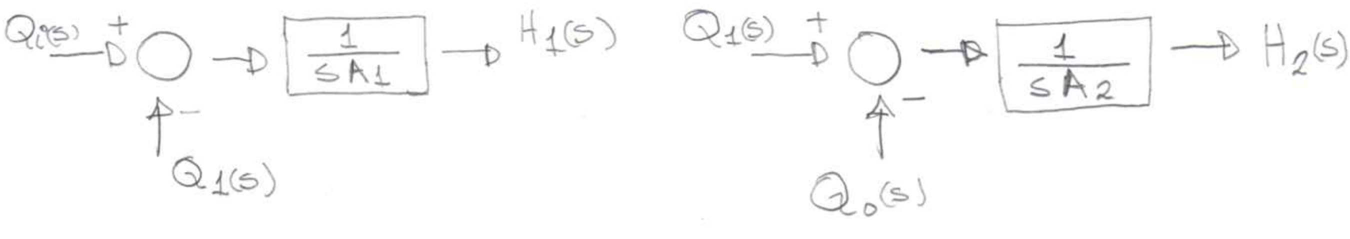
2- Determine the Laplace transform:

TANK 1:

$$\begin{matrix} ① \\ ② \end{matrix} \begin{cases} Q_i(s) = Q_1(s) + s A_1 H_1(s) \\ Q_1(s) = \frac{H_1(s) - H_2(s)}{R_1} \end{cases} \qquad \begin{matrix} ③ \\ ④ \end{matrix} \begin{cases} Q_1(s) = Q_o(s) + s A_2 H_2(s) \\ Q_o(s) = \frac{H_2(s)}{R_2} \end{cases}$$

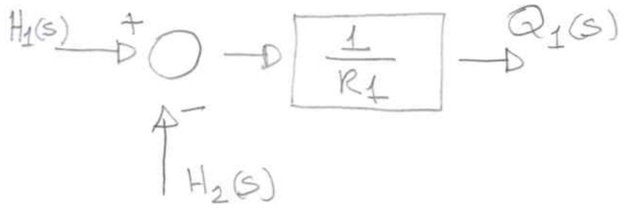
3- Represent each equation in the Laplace Domain individualy in block Diagrams:

① $H_1(s) = \frac{Q_i(s) - Q_1(s)}{s A_1}$ ③ $H_2(s) = \frac{Q_1(s) - Q_o(s)}{s A_2}$

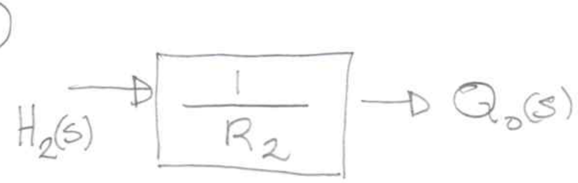


10 b) continue:

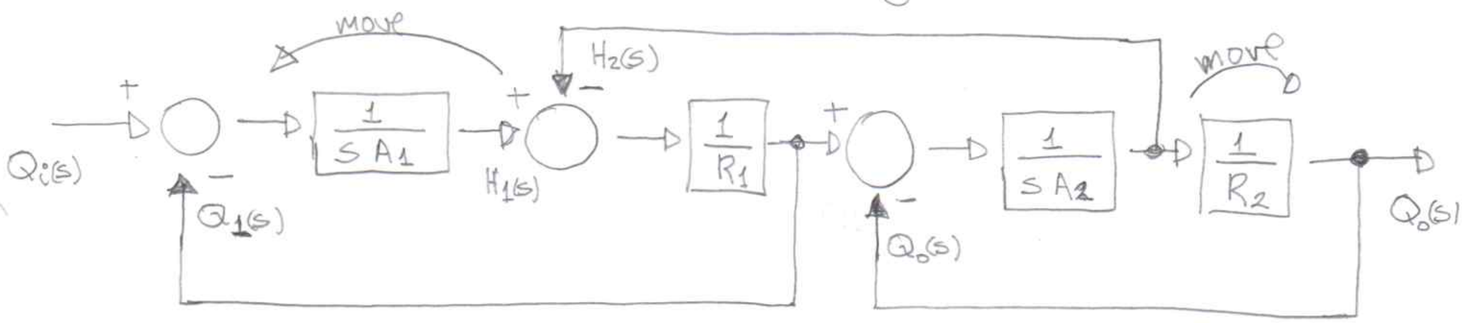
(2)



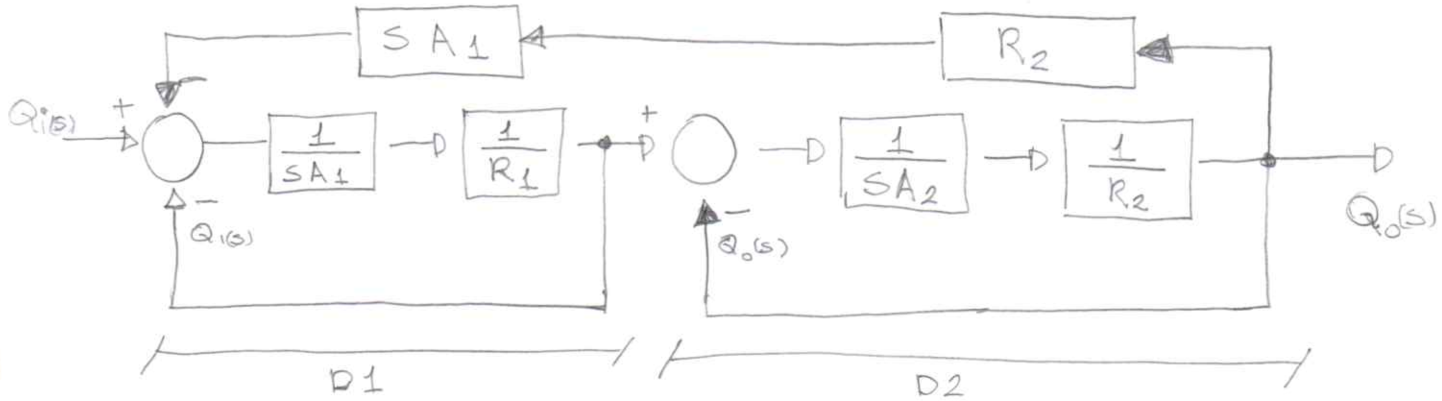
(4)



4- Assemble complete Block Diagram:



5- Simplify the block Diagram:



$$D1 = \frac{1}{sA_1R_1+1} \qquad D2 = \frac{1}{sA_2R_2+1}$$

$$\frac{Q_o(s)}{Q_i(s)} = \frac{D1 \cdot D2}{1 + D1 \cdot D2 \cdot sA_1 \cdot R_2}$$

$$= \frac{\dfrac{1}{s^2(A_1R_1A_2R_2)+s(A_1R_1+A_2R_2)+1}}{1+\dfrac{1}{s^2(A_1R_1 \cdot A_2R_2)+s(A_1R_1+A_2R_2)+1} \cdot sA_1 \cdot R_2}$$

$$\boxed{\frac{Q_o(s)}{Q_i(s)} = \frac{1}{s^2(A_1R_1A_2R_2)+s(A_1R_1+A_2R_2+A_1R_2)+1}}$$