15/4/2020

3 b) $$G(s) = \frac{10(s+7)}{(s+10)(s+20)}$$ ← Zeros → roots of the numerator
← Poles → roots of the denominator

- Zeros:

$s+7=0$

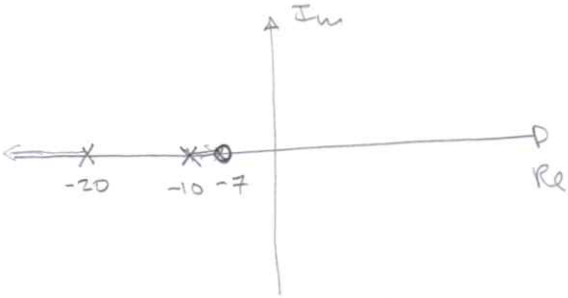
$\boxed{s=-7}$

- Poles:

$s+10=0 \quad \wedge \quad s+20=0$

$\boxed{s=-10 \quad \wedge \quad s=-20}$

complex plane:



- Response type:

two Distinct Real Poles → $\zeta > 1$

(over Damped)

- Unit step Response:

$$R(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{10(s+7)}{s(s+10)(s+20)}$$

- Expand into the partial fractions:

$$G(s) = \frac{10(s+7)}{(s+10)(s+20)} = \frac{10(s+10-3)}{(s+10)(s+20)}$$

$$= 10 \cdot \frac{(s+10)-3}{(s+10)(s+20)} = 10\left[\frac{\cancel{(s+10)}}{\cancel{(s+10)}(s+20)} - \frac{3}{(s+10)(s+20)}\right]$$

$$= \frac{10}{s+20} - \frac{30}{(s+10)(s+20)} = \frac{10}{s+20} - \left[\frac{A}{s+10} + \frac{B}{s+20}\right]$$

$$A: \left[\cancel{(s+10)} \frac{30}{\cancel{(s+10)}(s+20)}\right]_{s=-10} = \frac{30}{10} = 3$$

$$B: \left[\cancel{(s+20)} \frac{30}{(s+10)\cancel{(s+20)}}\right]_{s=-20} = \frac{30}{-10} = -3$$

$$G(s) = \frac{10}{s+20} - \frac{3}{s+10} + \frac{3}{s+20} = \frac{13}{s+20} - \frac{3}{s+10}$$

$$Y(s) = \frac{1}{s} \cdot \left[\frac{13}{s+20} - \frac{3}{s+10}\right] = \frac{13}{s(s+20)} - \frac{3}{s(s+10)}$$

$$= \frac{A}{s} + \frac{B}{s+20} - \left[\frac{C}{s} + \frac{S}{s+10}\right]$$

15/4/2020

3b) continuação

$$A: \left[\cancel{s} \cdot \frac{13}{\cancel{s}(s+20)} \right]_{s=0} = \frac{13}{20}$$

$$B: \left[(s+20) \cdot \frac{13}{s(s+20)} \right]_{s=-20} = -\frac{13}{20}$$

$$C: \left[\cancel{s} \cdot \frac{3}{\cancel{s}(s+10)} \right]_{s=0} = \frac{3}{10}$$

$$D: \left[(\cancel{s+10}) \frac{3}{s(\cancel{s+10})} \right]_{s=-10} = -\frac{3}{10}$$

$$Y_{(s)} = \frac{13}{20} \cdot \frac{1}{s} - \frac{13}{20} \cdot \frac{1}{s+20} - \frac{3}{10} \cdot \frac{1}{s} + \frac{3}{10} \cdot \frac{1}{s+10}$$

$$= \frac{7}{20} \cdot \frac{1}{s} - \frac{13}{20} \cdot \frac{1}{s+20} + \frac{3}{10} \cdot \frac{1}{s+10}$$

$$Y_{(t)} = \frac{7}{20} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{13}{20} \mathcal{L}^{-1}\left\{\frac{1}{s+20}\right\} + \frac{3}{10} \mathcal{L}^{-1}\left\{\frac{1}{s+10}\right\}$$

$$Y_{(t)} = \frac{7}{20} \cdot 1 - \frac{13}{20} \cdot e^{-20t} + \frac{3}{10} \cdot e^{-10t}, \quad t \geq 0$$

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+P

3 b)

$$G(s) = \frac{10\,(s+7)}{(s+10)(s+20)}$$

zeros → Roots of the numerator

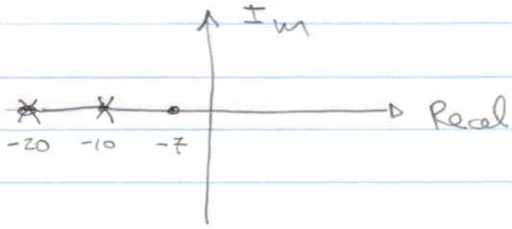
Poles → Roots of the denominator

- Zeros:

$$s+7=0 \Rightarrow s=-7$$

- Poles:

$$s+10=0 \quad \wedge \quad s+20=0$$
$$s=-10 \quad \wedge \quad s=-20$$



- Response type

two Distinct Real Poles ⇒ $\xi > 1$

(over damped)

- Unit step Response:

$$R(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{10(s+7)}{s(s+10)(s+20)}$$

- Expand into partial fractions:

$$G(s) = \frac{10(s+7)}{(s+10)(s+20)} = \frac{10(s+10-3)}{(s+10)(s+20)}$$

$$= 10\cdot\frac{(s+10)-3}{(s+10)(s+20)} = 10\left[\frac{\cancel{(s+10)}}{\cancel{(s+10)}(s+20)} - \frac{3}{(s+10)(s+20)}\right]$$

$$= \frac{10}{(s+20)} - \frac{30}{(s+10)(s+20)} = \frac{10}{s+20} - \left[\frac{A}{s+10} + \frac{B}{s+20}\right]$$

A: $\left[\cancel{(s+10)}\cdot\frac{30}{\cancel{(s+10)}(s+20)}\right]_{s=10} = \frac{30}{10} = 3$

B: $\left[\cancel{(s+20)}\cdot\frac{30}{(s+10)\cancel{(s+20)}}\right]_{s=20} = \frac{30}{-10} = -3$

3b) continua

$$G(s) = \frac{10}{s+20} - \frac{3}{s+10} + \frac{3}{s+20} = \frac{13}{s+20} - \frac{3}{s+10}$$

$$Y(s) = \frac{1}{s} \cdot \left[\frac{13}{s+20} - \frac{3}{s+10} \right]$$

$$= \frac{13}{s(s+20)} - \frac{3}{s(s+10)}$$

$$= \frac{A}{s} + \frac{B}{s+20} - \left[\frac{C}{s} + \frac{D}{s+10} \right]$$

$$A: \left[\cancel{s} \cdot \frac{13}{\cancel{s}(s+20)} \right]_{s=0} = \frac{13}{20}$$

$$B: \left[(s+\cancel{20}) \cdot \frac{13}{s(s+\cancel{20})} \right]_{s=-20} = \frac{13}{-20} = -\frac{13}{20}$$

$$C: \left[\cancel{s} \quad \frac{3}{\cancel{s}(s+10)} \right]_{s=0} = \frac{3}{10}$$

$$D: \left[(s+\cancel{10}) \cdot \frac{3}{s(s+\cancel{10})} \right]_{s=-10} = \frac{3}{-10} = -\frac{3}{10}$$

$$Y(s) = \frac{13}{20} \cdot \frac{1}{s} - \frac{13}{10} \cdot \frac{1}{s+20} - \frac{3}{10} \cdot \frac{1}{s} + \frac{3}{10} \cdot \frac{1}{s+10}$$

$$= \frac{7}{20} \cdot \frac{1}{s} - \frac{13}{20} \cdot \frac{1}{s+20} + \frac{3}{10} \frac{1}{s+10}$$

$$y(t) = \frac{7}{20} \mathcal{L}^{-1}\left\{ \frac{1}{s} \right\} - \frac{13}{20} \mathcal{L}^{-1}\left\{ \frac{1}{s+20} \right\} + \frac{3}{10} \mathcal{L}^{-1}\left\{ \frac{1}{s+10} \right\}$$

$$= \frac{7}{20} \cdot 1 - \frac{13}{20} \cdot e^{-20t} + \frac{3}{10} e^{-10t}, \quad t \geq \emptyset$$

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sistema de 2ª ordem

6

Forma canónica dos sistemas de 2ª ordem:

$$\boxed{\frac{K \quad \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}} \Big|_{K=1}$$

- Se $\xi > 1 \Rightarrow$ sistema sobreamortecido $\Rightarrow$ polos reais distintos
- se $\xi = 1 \Rightarrow$ sistema criticamente Amortecido $\Rightarrow$ polos reais duplos (iguais)
- Se $0 < \xi < 1 \Rightarrow$ sistema subamortecidos $\Rightarrow$ polos complexos conjugados

3 b) $G(s) = \dfrac{10\,(s+7)}{(s+10)(s+20)}$

[redacted]

1- localizar polos e zeros
2- Identificar o tipo de resposta para uma entrada em degrau.

1. Zero $\Rightarrow s = -7$

   Polos $\Rightarrow s = -10$

   $s = -20$

tipo de resposta: Os polos identificados são polos reais distintos
↓

A função G(s) apresenta 2 polos reais e distintos. Portanto o sistema possui uma resposta sobreamortecida ($\xi > 1$).
Dado que G(s) tem um zero é de prever que a resposta possua uma sobrealongação.

- Resposta ao degrau unitário:

$R(s) = \dfrac{1}{s} \Rightarrow Y(s) = \dfrac{10\,(s+7)}{(s+10)(s+20)} \cdot \dfrac{1}{s}$

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3 b) continuação 7

$$Y(s) = \frac{10(s+7)}{s(s+10)(s+20)}$$

$$Y(s) = \frac{a}{s} + \frac{b}{s+10} + \frac{c}{s+20}$$

$$a = \left.\frac{10(s+7)}{(s+10)(s+20)}\right|_{s=0} \Leftrightarrow a = \frac{7}{20}$$

$$b = \left.\frac{10(s+7)}{s(s+20)}\right|_{s=-10} \Leftrightarrow b = \frac{3}{10}$$

$$c = \left.\frac{10(s+7)}{s(s+10)}\right|_{s=-20} \Leftrightarrow c = -\frac{13}{20}$$

$$Y(s) = \frac{7}{20} \cdot \frac{1}{s} + \frac{3}{10} \cdot \frac{1}{s+10} - \frac{13}{20} \cdot \frac{1}{s+20}$$

$$Y(t) = \frac{7}{20} + \frac{3}{10} e^{-10t} - \frac{13}{20} e^{-20t}$$

3

b) $G(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$ two methods.

zeros: -7

polos: -10, -20

$$s^2 + 20s + 10s + 200 = s^2 + 30s + 200$$

$$\begin{cases} 30 = 2\xi\omega_n \\ 200 = \omega_n^2 \end{cases} \qquad \begin{cases} \xi = \dfrac{30}{2 \cdot \sqrt{200}} > 1 \Rightarrow \text{sobreamortecido} \\ \omega_n = \sqrt{200} \end{cases}$$

$$Y(s) = \frac{10(s+7)}{(s+10)(s+20)s} = \frac{a}{s} + \frac{b}{s+10} + \frac{c}{s+20}$$

$$a = \left.\frac{10(s+7)}{(s+10)(s+20)}\right|_0 = \frac{7}{20}$$

$$b = \left.\frac{10(s+7)}{s(s+20)}\right|_{-10} = \frac{3}{10}$$

$$c = \left.\frac{10(s+7)}{s(s+10)}\right|_{-20} = -\frac{13}{20}$$

$$Y(s) = \frac{\frac{7}{20}}{s} + \frac{\frac{3}{10}}{s+10} - \frac{\frac{13}{20}}{s+20}$$

$$Y(t) = \frac{7}{20} + \frac{3}{10}e^{-10t} + \frac{13}{20}e^{-20t}$$

$$\frac{Y(s)}{R(s)} = K \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

K

ξ

ω_n

$$M_p = \frac{V_{pico} - V_{final}}{V_{final}} = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

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good.

notes

teorema canónico dos sistemas de 2º grau

$$\frac{\omega_n^2}{s^2 + 2\,\xi\,\omega_n s + \omega_n^2}$$

- se $\xi > 1 \Rightarrow$ sistema sobreamortecido
  $\Rightarrow$ polos reais distintos
- se $\xi = 1 \Rightarrow$ sistema criticamente amortecido
  $\Rightarrow$ polos reais duplos (iguais)
- se $0 < \xi < 1 \Rightarrow$ sistema subamortecido
  $\Rightarrow$ polo complexos conjugados

—— // ——

3 b)

$$G(s) = \frac{10\,(s+7)}{(s+10)\,(s+20)}$$

$K = \frac{70}{200}$ $\quad s^2+30s+200$ $\quad \left(\frac{s}{10\sqrt{2}}\right)^2 + \frac{30}{200}s + 1$

zeros: -7

polos: -10 ; -20

$\xi > 1$

$$Y(s) = \frac{1}{s} \times \frac{10\,(s+7)}{(s+10)(s+20)}$$

$$= \frac{a}{s} + \frac{b}{s+10} + \frac{c}{s+20}$$

practice fractional equations.

$$y(t) = \frac{7}{20} + \frac{3}{10} e^{-10t} + \frac{13}{20} e^{-20t}$$

4. Formulas

$$M_p = \frac{Y(t_p) - Y(\infty)}{Y(\infty)}$$

$$= e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \rightarrow \omega_n =$$

$$\lim_{s \to 0} \frac{K}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{K}{\omega_n^2}$$

all well explained.

apply definitions.

Proxima semana Pascoa.