9.

$f = 10\,N$ $\qquad$ $M, B \quad K = ?$

$$\Sigma FR = m\,a$$
$$= m\,\ddot{x}$$

$$f(t) - K\,x - B\,\dot{x} = m\,\ddot{x} \; ; \quad m = M$$

$\quad \downarrow \dot{x}$

$$F(s) - K\,X(s) - B\,s\,X(s) = M\,s^2\,X(s)$$

$$F(s) = (K + Bs + Ms^2) \cdot X(s)$$

$$G = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

$$= \frac{\frac{1}{M}}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$

G

$$= \frac{1}{K} \; \frac{\frac{K}{M} \; \omega_n^2}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$

$2\varepsilon\omega_n$

$$2\varepsilon\omega_n = \frac{B}{M} \; ; \quad \omega_n = \frac{K}{M} \; ; \quad G = \frac{1}{K}$$

$M_p = \frac{0{,}003}{0{,}03} = 10\%$

$\Big\} \; \omega_d = \omega_n \cdot \sqrt{1-\varepsilon^2}$

$t_p = 2$

$t_s = 0{,}03$ (é estimação)

$$M_p = e^{\frac{-\varepsilon\pi}{\sqrt{1-\varepsilon^2}}}$$

$\varepsilon = 0{,}5911$

$\omega_n^2 = 225{,}54$

$$\begin{cases} \frac{K}{M} = \omega_n^2 \\ \frac{B}{M} = 2\varepsilon\omega_n \end{cases} \qquad \frac{1}{K} = \frac{0{,}03}{10} \qquad t_p = \frac{\pi}{\omega_n\sqrt{1-\varepsilon^2}}$$

$\downarrow$
$\omega_n = 1{,}9474$

9) $f = 10N$ $\quad M, B$ e $K = ?$

$$f(t) - Kx - B\dot{x} = M\ddot{x}$$

$$F(s) = K\cancel{X(s)} - BS\,X(s) = M s^2 X(s)$$

$$F(s) = (MS^2 + K + BS)\,X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{MS^2 + BS + K}$$

$$= \frac{1}{K} \cdot \frac{\frac{K}{M}}{S^2 + \frac{B}{M}S + \frac{K}{M}}$$

$$\begin{cases} \omega_n^2 = \frac{K}{M} \\ 2\xi\omega_n = \frac{B}{M} \end{cases} \qquad \begin{cases} M = \frac{K}{\omega_n^2} \\ B = 2\xi\omega_n M \end{cases}$$

$$t_p = 2$$

$$M_p = \frac{0{,}03}{0{,}003}$$

$$= 10\,\%$$

$$G = \frac{1}{K} = \frac{0{,}03}{10}$$

$$0{,}1 = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\Downarrow$$

$$\xi = 0{,}5911$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\Downarrow$$

$$\omega_n = 1{,}9474$$

$M = 87{,}66$ kg

$B = 201{,}71$ Ns/m

etc.

9.)

$$\begin{array}{ccccccc} x(\infty) & \leftarrow & \boxed{\text{Graph}} & \rightarrow & t_p & \rightarrow & x(t_p) \\ & & \downarrow & & \downarrow & & \\ & & M_p & & \omega_n & & \\ & & \downarrow & \nearrow & & & \\ & & \xi & & & & \end{array}$$

$x(\infty),\ x(t_p) \rightarrow M_p$

13.) $E(s) = \dfrac{1}{1+G(s)}$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s)$$

$$= \lim_{s\to 0} \frac{s\cdot R(s)}{1+G(s)}$$

se $R(s) = \dfrac{1}{s}$ e tipo 0

↓ polos ξ

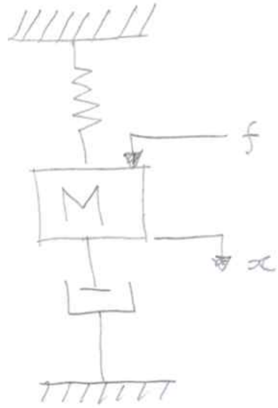
$e_{ss} = \dfrac{1}{K+1}$

tipo 1 1 polo origem

$K_v \Rightarrow$ rampa.

$K_a \rightarrow$ parabola

9) $f(t) = 10\, u(t) \Rightarrow F(s) = \frac{10}{s}$; M, B, K = ?



From the Graphic:

- Peak time $(t_p) = 2$ sec
- Maximum Value of $y(t)$
  $y(t_p) = 0{,}033$
- Final Value $y(\infty) = 0{,}03$

- Obtain the dynamic equations that observes the system behavior:

$$\Sigma F = M \cdot a$$

$$f(t) - K\,x(t) - B\,\dot{x}(t) = M \cdot \ddot{x}(t)$$

$$f(t) = M\,\ddot{x}(t) + B\,\dot{x}(t) + K\,x(t)$$

$\mathcal{L}$

$$F(s) = s^2 M X(s) + s B X(s) + K X(s)$$

$$= (s^2 M + s B + K)\, X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 M + sB + K} = \frac{\frac{1}{M}}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$

$$= \frac{1}{L} \cdot \frac{\frac{L}{M}}{s^2 + \frac{B}{M}s + \frac{K}{M}}$$ $\boxed{L \Rightarrow K}$

- Considering $f(t) = 10\, u(t) \xrightarrow{\mathcal{L}} F(s) = \frac{10}{s}$:

$$X(s) = \frac{1}{\cancel{L}} \cdot \frac{\frac{\cancel{L}}{M}}{s^2 + \frac{B}{M}s + \frac{K}{M}} \cdot \frac{10}{s}$$

$$= \frac{\frac{10}{M}}{s\cdot\left(s^2 + \frac{B}{M}s + \frac{K}{M}\right)}$$

- Applying Laplace final value theorem:

$$x(\infty) = \lim_{t \to \infty} x(t) = \lim_{s \to 0} s \cdot X(s) = \frac{\cancel{s}\,\frac{10}{M}}{\cancel{s}\left(s^2 + \frac{B}{M}s + \frac{K}{M}\right)} = \frac{10}{K}$$

$$x(\infty) = 0{,}03 \Rightarrow \frac{10}{K} = 0{,}03 \Rightarrow K = 333{,}33\ N/m$$

9) continuação

- From the graphic: $t_p = 2$ sec

$$X(t_p) = 0{,}03 + 0{,}003 = 0{,}033$$

$$M_p = \frac{x(t_p) - x(\infty)}{x(\infty)} \times 100\%$$

$$= \frac{0{,}033 - 0{,}03}{0{,}03}$$

$$= \frac{0{,}003}{0{,}03} = 0{,}1$$

$M_p = 10\%$

- From the specification of M_p:

$$M_p = e^{-\pi \xi / \sqrt{1-\xi^2}}$$

$$0{,}1 = e^{-\pi \xi / \sqrt{1-\xi^2}} \iff \xi = 0{,}591$$

- From the specification of t_p:

$$t_p = \frac{\pi}{\omega_n \cdot \sqrt{1-\xi^2}} \iff 2 = \frac{\pi}{\omega_n \cdot \sqrt{1-0{,}591^2}}$$

$$\omega_n = 1{,}95 \text{ rad/sec}$$

- Transfer Function (T.F) second order system:

$$\frac{X(s)}{F(s)} = \frac{\boxed{L}\, \omega_n^2}{s^2 + \underbrace{2\xi\omega_n}\, s + \underbrace{\omega_n^2}}$$

our system:

$$\frac{X(s)}{F(s)} = \boxed{\frac{1}{L}} \cdot \frac{\frac{L}{M}}{s^2 + \boxed{\frac{B}{M}}\, s + \boxed{\frac{K}{M}}}$$

comparing the two equations

$$\begin{cases} \frac{B}{M} = 2\xi\omega_n \\ \frac{K}{M} = \omega_n^2 \end{cases} \iff \begin{cases} - \\ M = \frac{K}{\omega_n^2} \end{cases} \quad \begin{cases} B = 2\xi\omega_n M \\ M = 87{,}66 \text{ Kg} \end{cases}$$

$$\begin{cases} B = 201{,}71 \text{ Ns/m} \\ M = 87{,}66 \text{ Kg} \end{cases}$$