Departamento de Engenharia Electrotécnica
Instituto Superior de Engenharia do Porto

TESIS
Teoria dos Sistemas

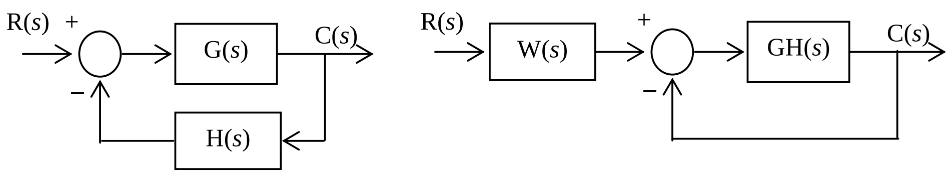
Álgebra dos Diagramas de Blocos

–

Resolução dos Exercícios Propostos

1. Considere os dois sistemas de diagrama de blocos representados nas figuras. Exprima W(*s*) em função de G(*s*) de forma a que a função de transferência dos dois sistemas seja igual.

a)



A função de transferência do diagrama de blocos apresentado no lado esquerdo é:

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}$$

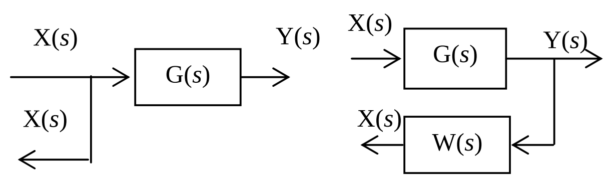
E a função de transferência do diagrama de blocos apresentado no lado direito é:

$$\frac{C(s)}{R(s)}=W(s)\frac{G(s)H(s)}{1+G(s)H(s)}$$

Logo as duas funções de transferência são iguais se:

$$W(s)=\frac{1}{H(s)}$$

b)



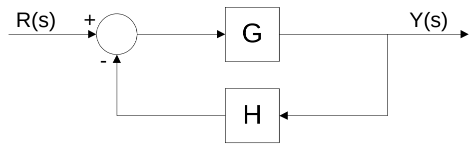
No diagrama de blocos apresentado no lado esquerdo a saída é $Y(s) = G(s)X(s)$ e o sinal realimentado é $X(s)$.

No diagrama de blocos apresentado no lado direito a saída continua a ser $Y(s) = G(s)X(s)$, mas para o sinal realimentado continuar a ser $X(s)$ é necessário que:

$$W(s)=\frac{1}{G(s)}$$

2. Determine a Função de Transferência dos diagramas de blocos representados nas figuras seguintes:

a)



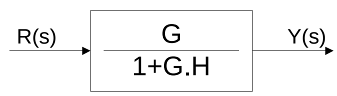
A primeira possibilidade de resolução deste exercício passa pela análise completa das equações representadas no diagrama de blocos. Assim, temos:

$$Y(s) = G.E(s) \Leftrightarrow E(s) = \frac{Y(s)}{G}$$

$$E(s) = R(s) - H.Y(s) \Leftrightarrow \frac{Y(s)}{G} = R(s) - H.Y(s) \Leftrightarrow$$

$$\Leftrightarrow R(s) = \frac{Y(s)}{G} + H.Y(s) \Leftrightarrow R(s) = \left[\frac{1}{G} + H\right].Y(s) \Leftrightarrow \frac{Y(s)}{R(s)} = \frac{G}{1+G.H}$$

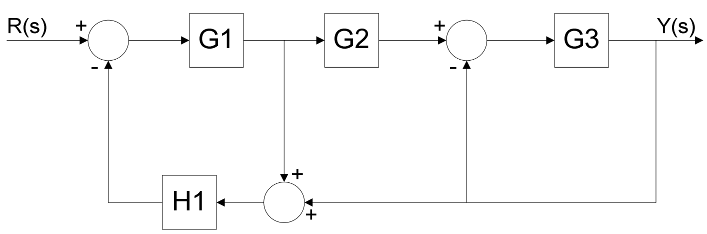
Alternativamente, aplicando as regras da álgebra de blocos, temos:



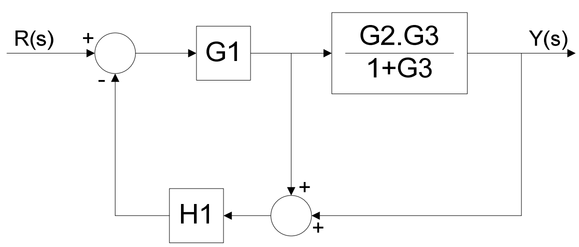
ou seja:

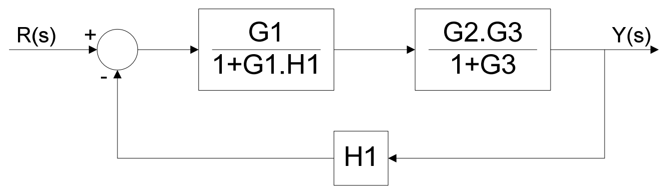
$$\frac{Y(s)}{R(s)} = \frac{G}{1+G.H}$$

b)



Aplicando as regras da álgebra de blocos, temos:

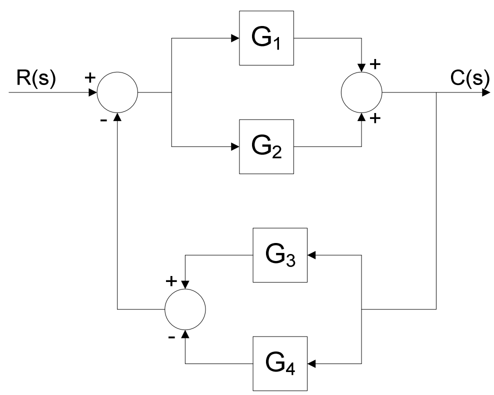




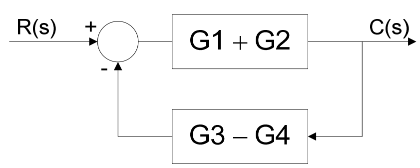
Verifica-se que a Função de Transferência deste sistema é:

$$\frac{Y(s)}{R(s)} = \frac{G1.G2.G3}{(1+G1.H1).(1+G3)+G1.G2.G3.H1}$$

c)



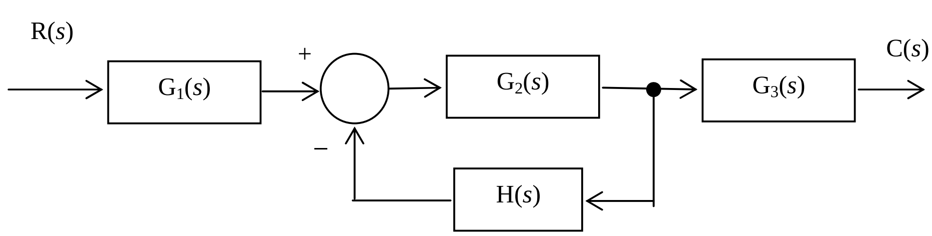
Aplicando as regras da álgebra de blocos, temos:



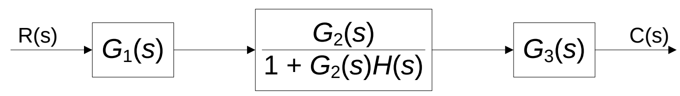
Verifica-se que a Função de Transferência deste sistema é:

$$\frac{C(s)}{R(s)} = \frac{G1+G2}{1+(G1+G2).(G3-G4)}$$

d)



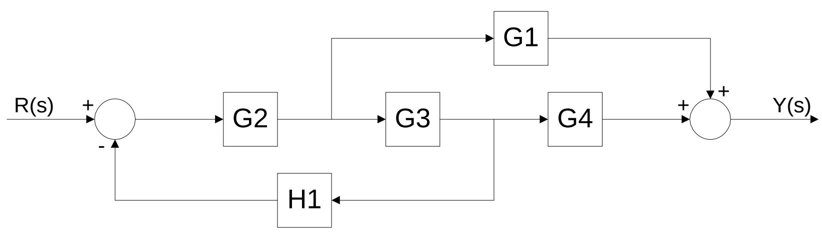
Aplicando as regras da álgebra de blocos, temos:



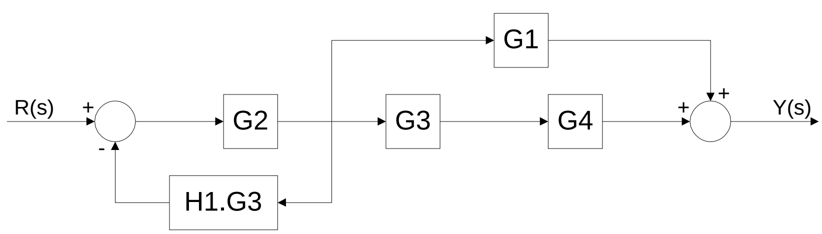
Verifica-se que a Função de Transferência deste sistema é:

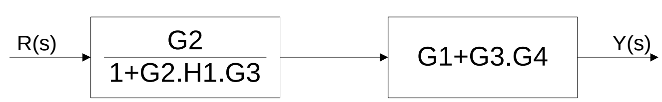
$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)G_3(s)}{1+G_2(s)H(s)}$$

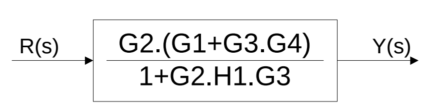
e)



Aplicando as regras da álgebra de blocos, temos:



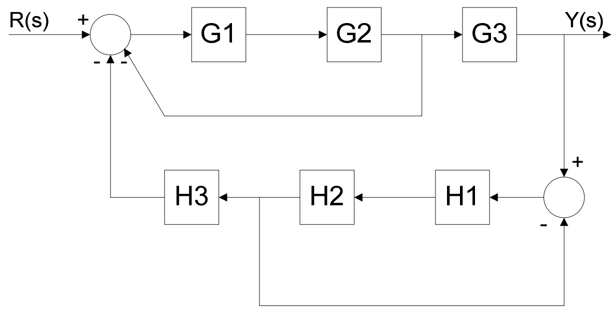




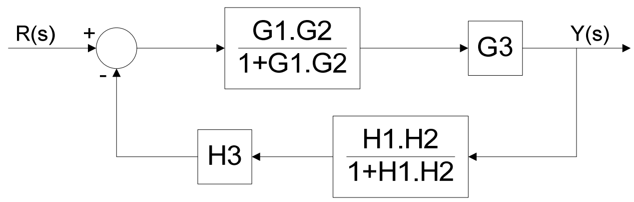
Verifica-se que a Função de Transferência deste sistema é:

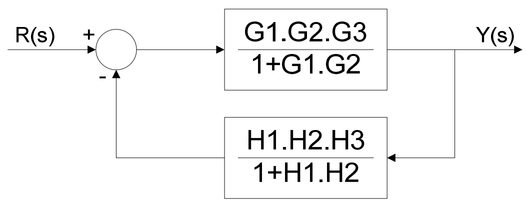
$$\frac{Y(s)}{R(s)} = \frac{G2.(G1+G3.G4)}{1+G2.H1.G3}$$

f)



Aplicando as regras da álgebra de blocos, temos:

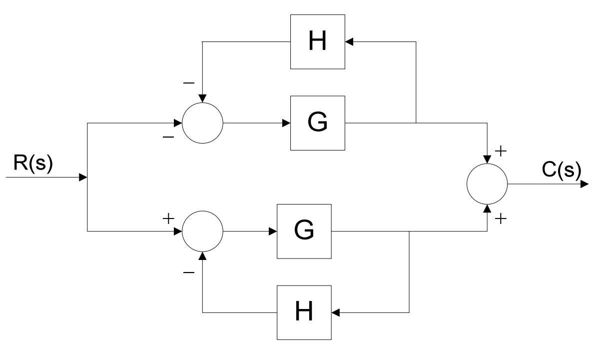




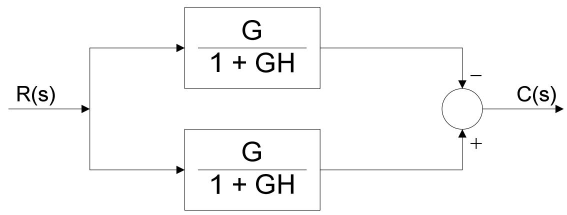
Verifica-se que a Função de Transferência deste sistema é:

$$\frac{Y(s)}{R(s)}=\frac{G1.G2.G3(1+H1.H2)}{(1+G1.G2).(1+H1.H2)+G1.G2.G3.H1.H2.H3}$$

g)



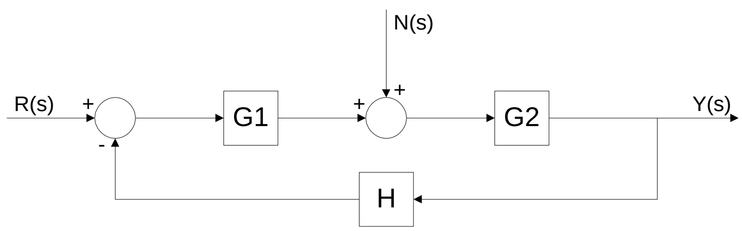
Aplicando as regras da álgebra de blocos, temos:



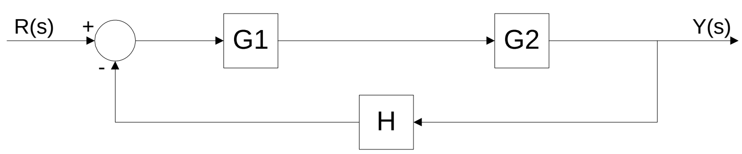
Verifica-se que a Função de Transferência deste sistema é:

$$\frac{C(s)}{R(s)} = 0$$

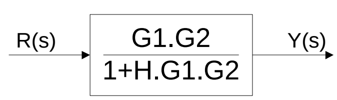
h)



Uma vez que este sistema apresenta duas entradas distintas, R(s) e N(s), devemos aplicar o Teorema da Sobreposição. Assim, considerando N(s)=0, ficamos com o seguinte diagrama de blocos equivalente:



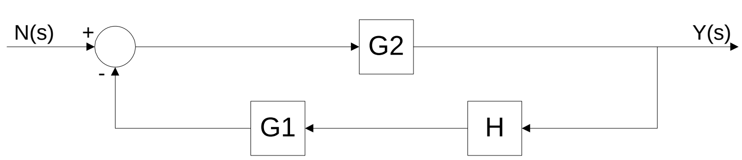
Aplicando as regras da álgebra de blocos, temos:



Logo:

$$\frac{Y(s)}{R(s)} = \frac{G1.G2}{1 + G1.G2.H}$$

Considerando agora R(s)=0, ficamos com o seguinte diagrama de blocos equivalente:



Aplicando as regras da álgebra de blocos, temos:

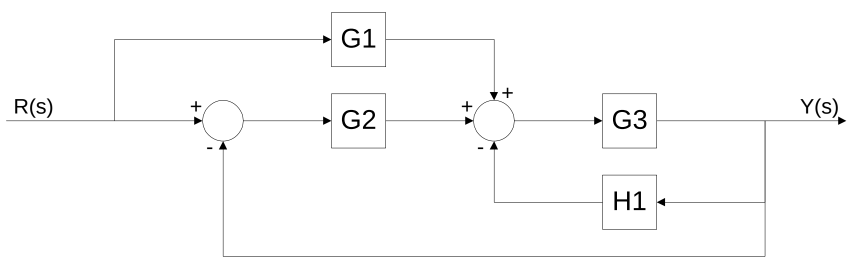
Logo:

$$\frac{Y(s)}{N(s)} = \frac{G2}{1+G1.G2.H}$$

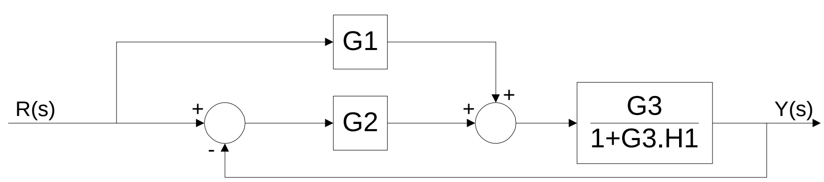
Por aplicação do Teorema da Sobreposição, concluímos que a Função de Transferência deste sistema é:

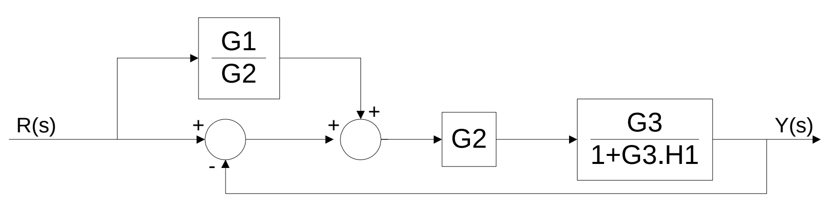
$$Y(s) = \frac{G1.G2}{1+G1.G2.H}.R(s) + \frac{G2}{1+G1.G2.H}.N(s)$$

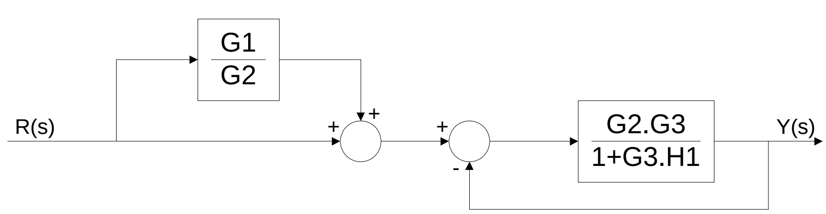
i)

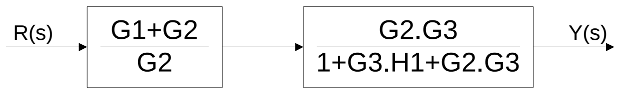


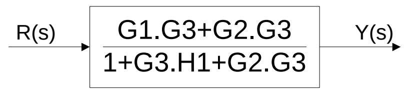
Aplicando as regras da álgebra de blocos, temos:







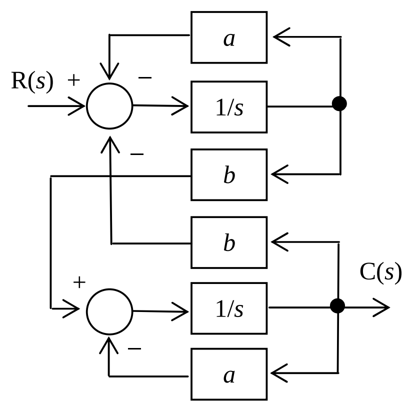




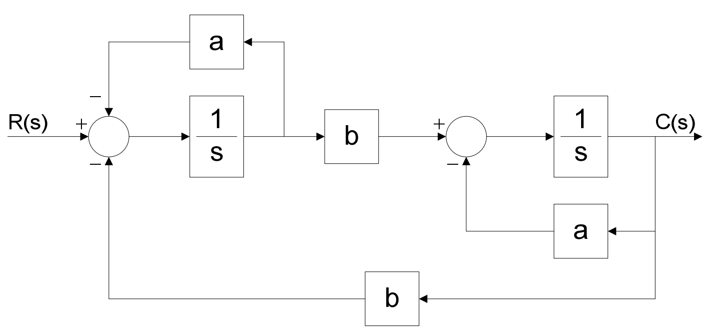
Logo:

$$\frac{Y(s)}{R(s)} = \frac{G1.G3 + G2.G3}{1 + G3.H1 + G2.G3}$$

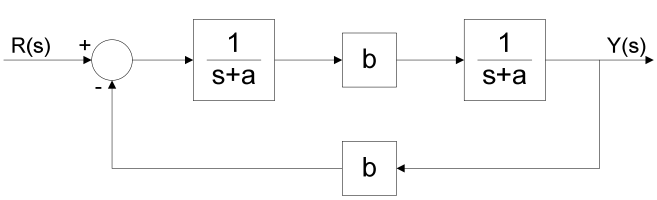
j)

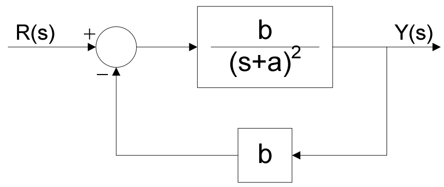


Este diagrama de blocos pode, alternativamente, ser representado da seguinte forma:



Aplicando as regras da álgebra de blocos, temos:

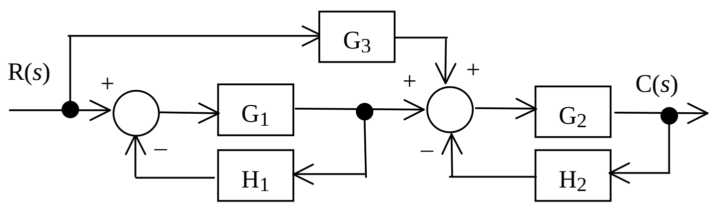




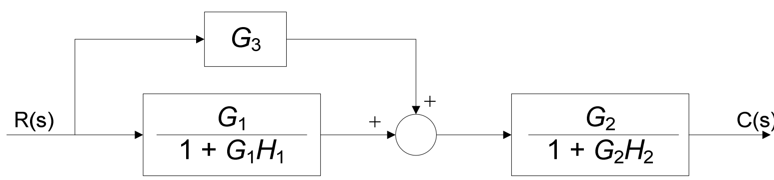
Verifica-se que a Função de Transferência deste sistema é:

$$\frac{C(s)}{R(s)} = \frac{b}{(s+a)^2 + b^2}$$

k)



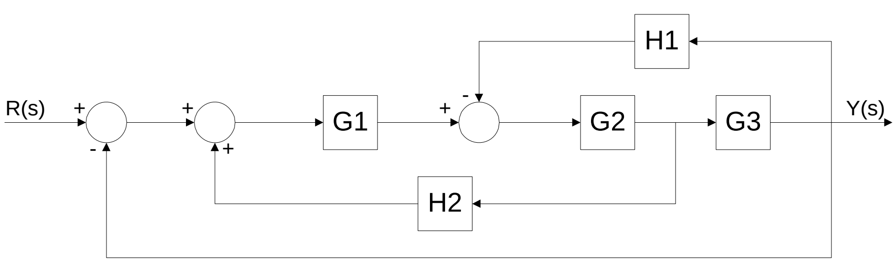
Aplicando as regras da álgebra de blocos, temos:



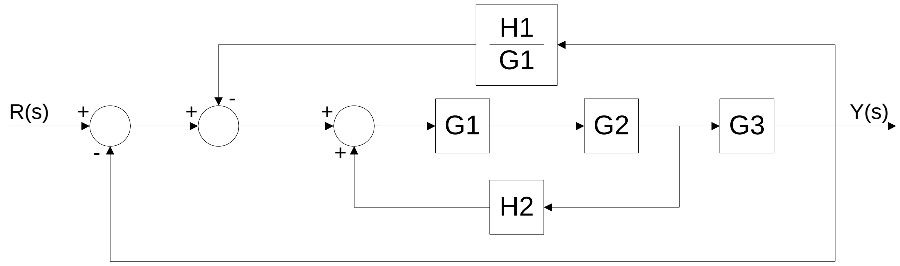
Verifica-se que a Função de Transferência deste sistema é:

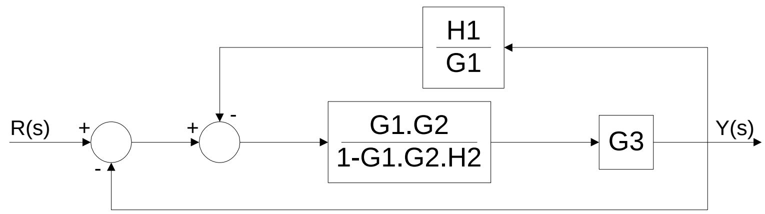
$$\frac{C(s)}{R(s)} = \left(\frac{G_1}{1+G_1H_1} + G_3 \right) \frac{G_2}{1+G_2H_2}$$

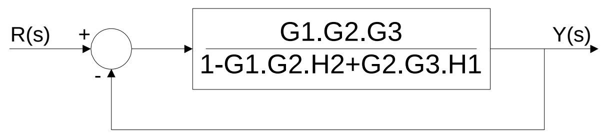
l)

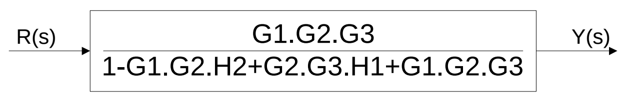


Aplicando as regras da álgebra de blocos, temos:





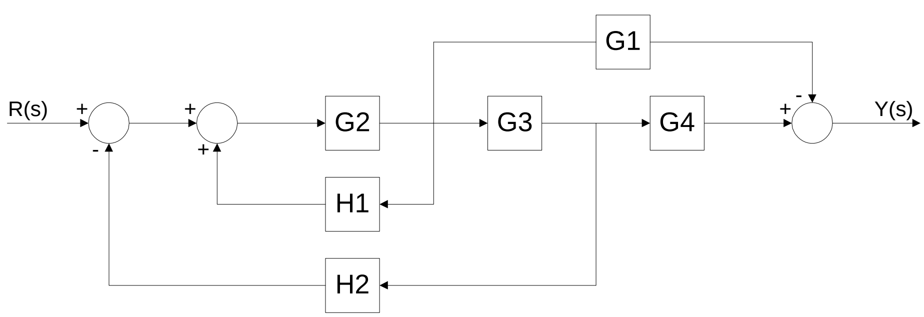




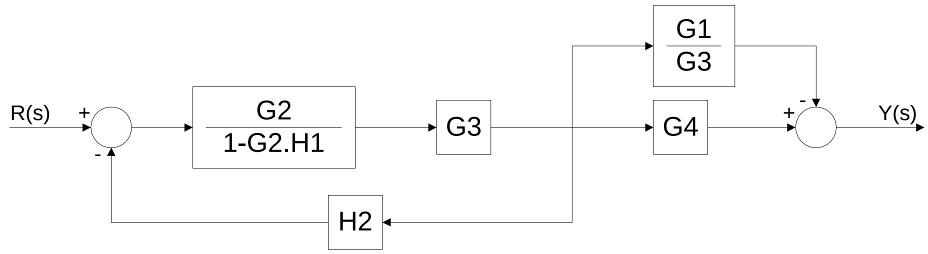
Logo:

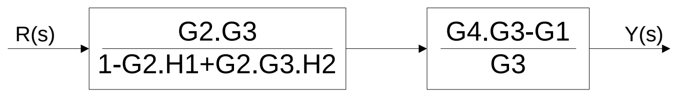
$$\frac{Y(s)}{R(s)} = \frac{G1.G2.G3}{1 - G1.G2.H2 + G2.G3.H1 + G1.G2.G3}$$

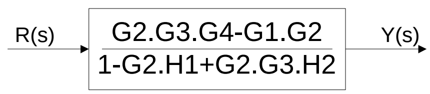
m)



Aplicando as regras da álgebra de blocos, temos:



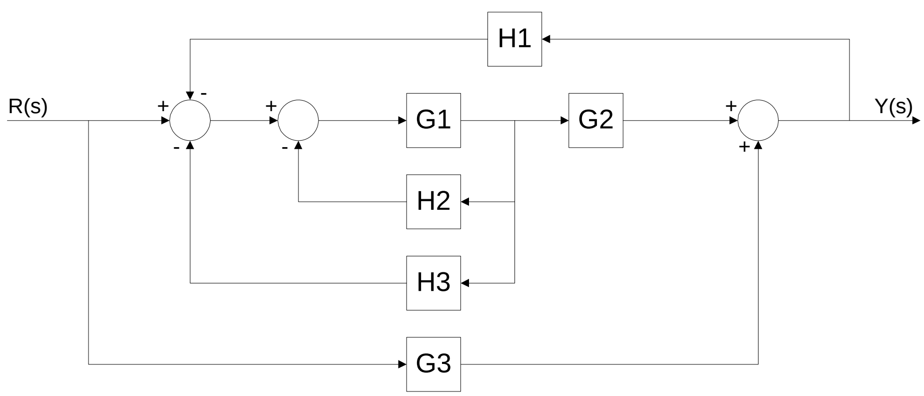




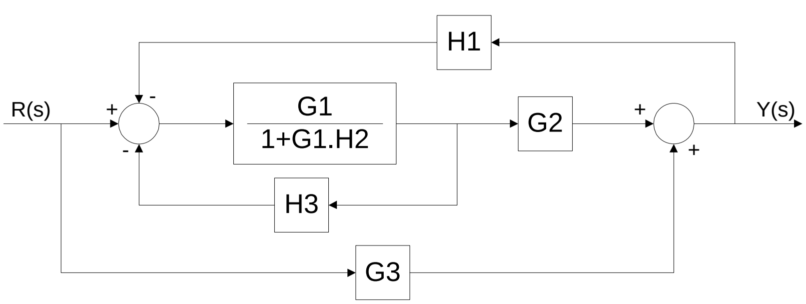
Logo:

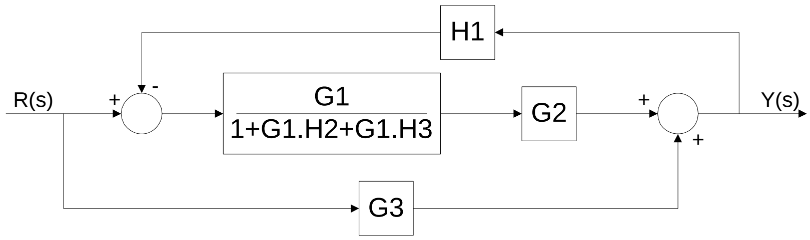
$$\frac{Y(s)}{R(s)} = \frac{G2.G3.G4 - G1.G2}{1 - G2.H1 + G2.G3.H2}$$

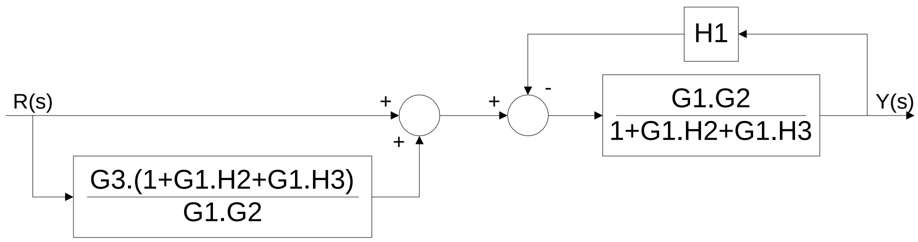
n)

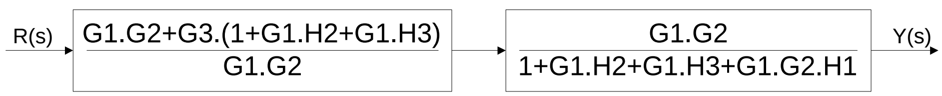


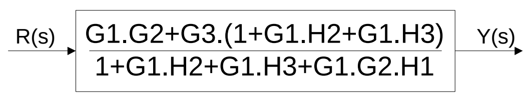
Aplicando as regras da álgebra de blocos, temos:







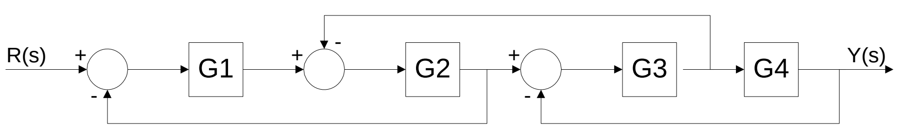




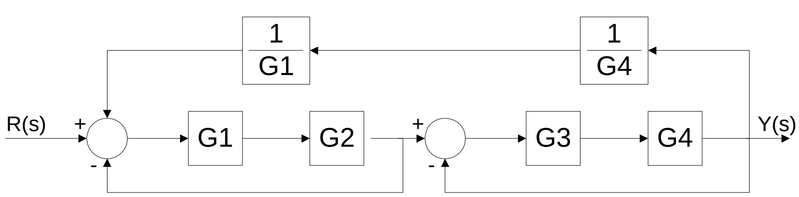
Logo:

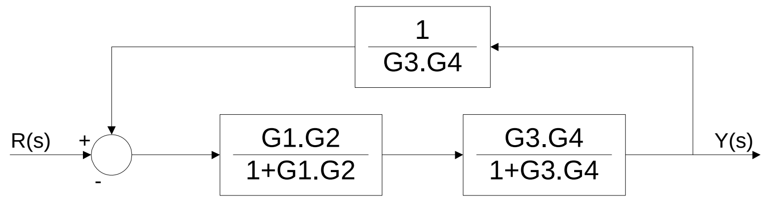
$$\frac{Y(s)}{R(s)} = \frac{G1.G2 + G3.(1 + G1.H2 + G1.H3)}{1 + G1.H2 + G1.H3 + G1.G2.H1}$$

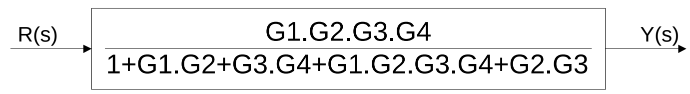
o)



Aplicando as regras da álgebra de blocos, temos:



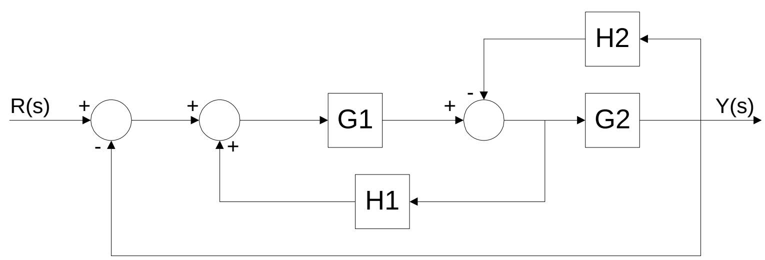




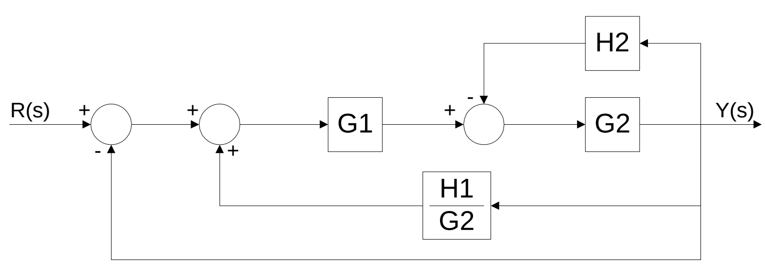
Logo:

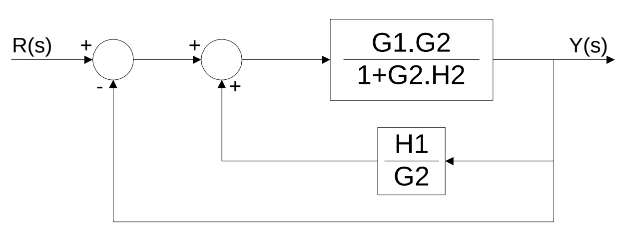
$$\frac{Y(s)}{R(s)} = \frac{G1.G2.G3.G4}{1 + G1.G2 + G3.G4 + G2.G3 + G1.G2.G3.G4}$$

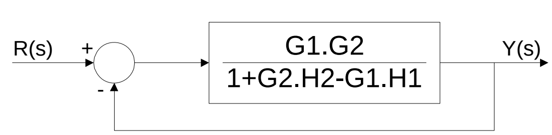
p)

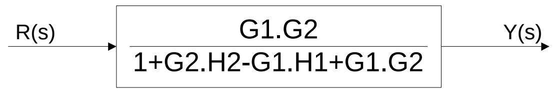


Aplicando as regras da álgebra de blocos, temos:





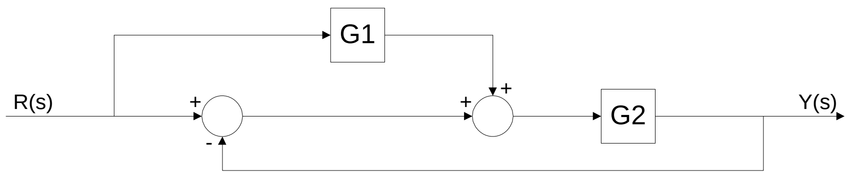




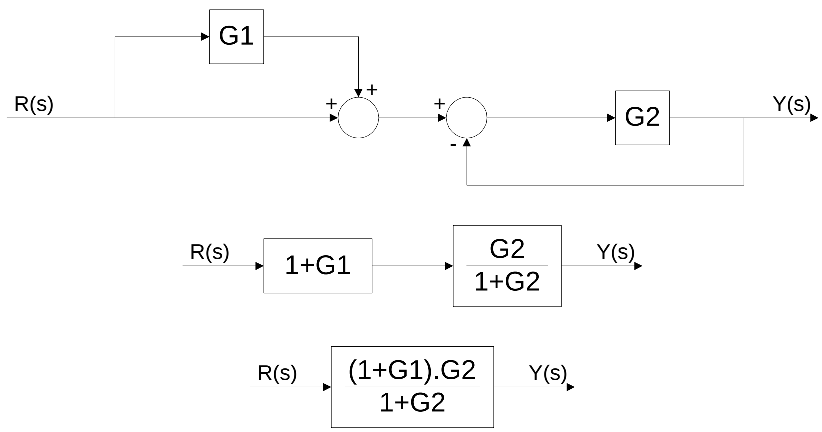
Logo:

$$\frac{Y(s)}{R(s)} = \frac{G1.G2}{1 + G2.H2 - G1.H1 + G1.G2}$$

q)



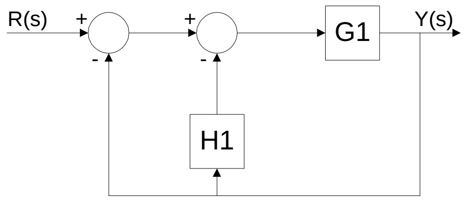
Aplicando as regras da álgebra de blocos, temos:



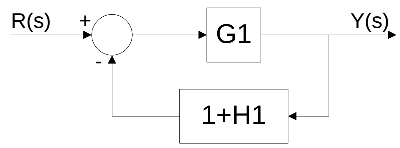
Logo:

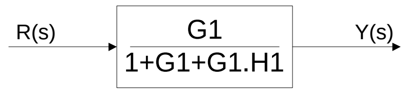
$$\frac{Y(s)}{R(s)} = \frac{(1 + G1).G2}{1 + G2}$$

r)



Aplicando as regras da álgebra de blocos, temos:

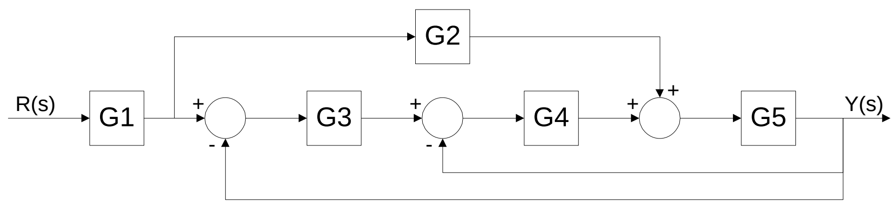




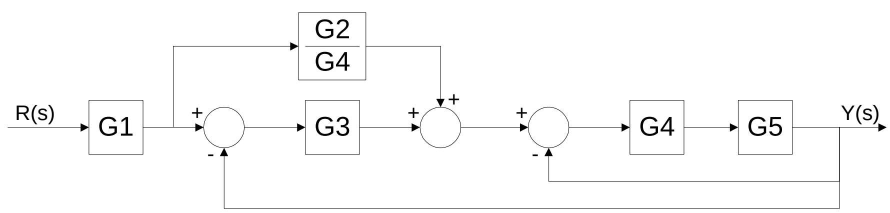
Logo:

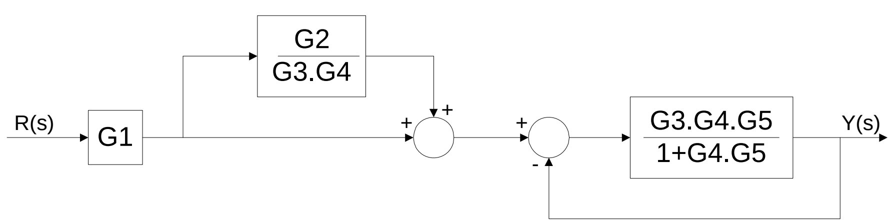
$$\frac{Y(s)}{R(s)} = \frac{G1}{1+G1+G1.H1}$$

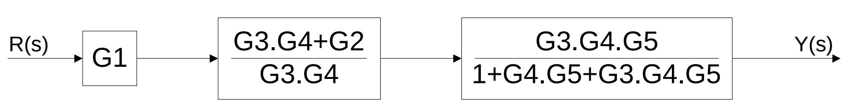
s)

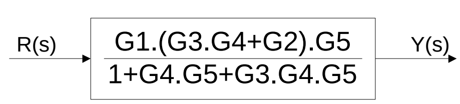


Aplicando as regras da álgebra de blocos, temos:





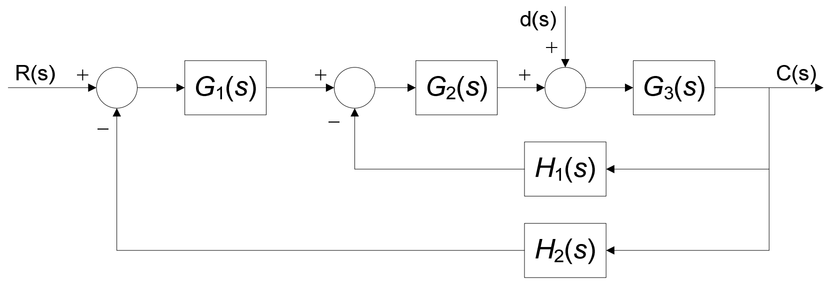




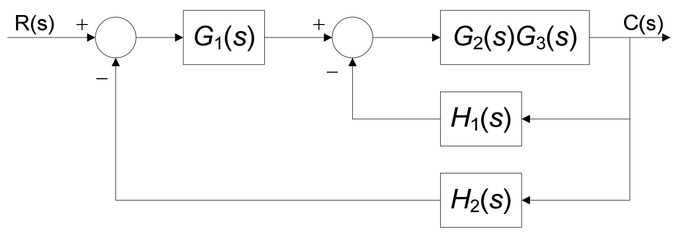
Logo:

$$\frac{Y(s)}{R(s)} = \frac{(G3.G4 + G2).G1.G5}{1 + G4.G5 + G3.G4.G5}$$

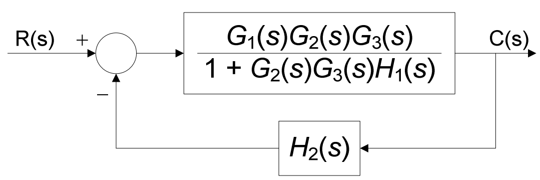
t)



Uma vez que este sistema apresenta duas entradas distintas, $R(s)$ e $d(s)$, devemos aplicar o Teorema da Sobreposição. Assim, considerando $d(s) = 0$, ficamos com o seguinte diagrama de blocos equivalente:



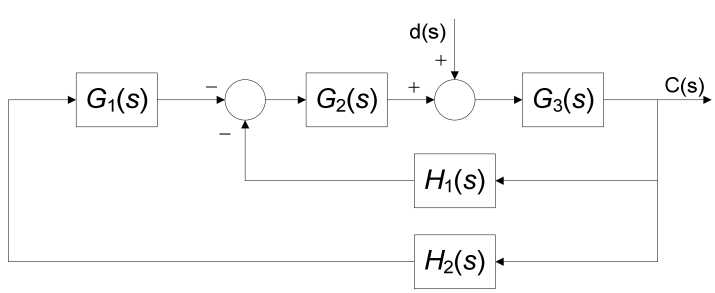
Aplicando as regras da álgebra de blocos, temos:



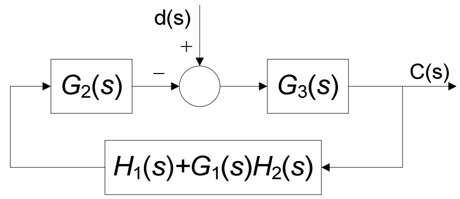
Logo:

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)G_3(s)}{1 + G_2(s)G_3(s)\left[H_1(s) + G_1(s)H_2(s)\right]}$$

Considerando agora $R(s)=0$, ficamos com o seguinte diagrama de blocos equivalente:



Aplicando as regras da álgebra de blocos, temos:



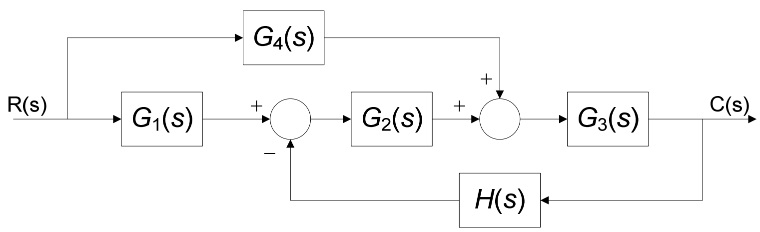
Logo:

$$\frac{C(s)}{d(s)}=\frac{G_3(s)}{1+G_2(s)G_3(s)\left[H_1(s)+G_1(s)H_2(s)\right]}$$

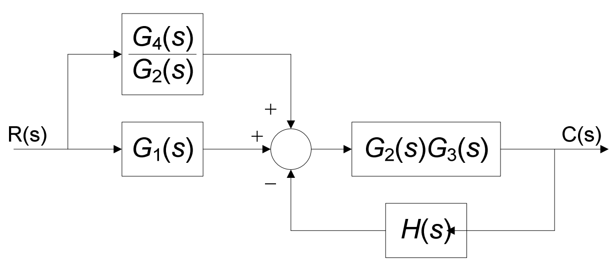
Por aplicação do Teorema da Sobreposição, concluímos que a Função de Transferência deste sistema é:

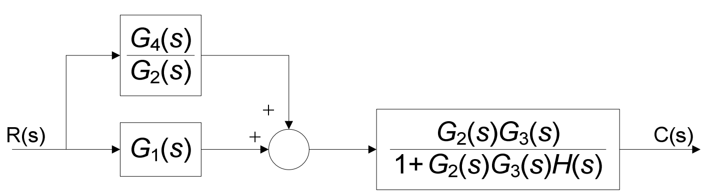
$$C(s)=\frac{G_1(s)G_2(s)G_3(s)}{1+G_2(s)G_3(s)\left[H_1(s)+G_1(s)H_2(s)\right]}R(s)+\frac{G_3(s)}{1+G_2(s)G_3(s)\left[H_1(s)+G_1(s)H_2(s)\right]}d(s)$$

u)



Aplicando as regras da álgebra de blocos, temos:

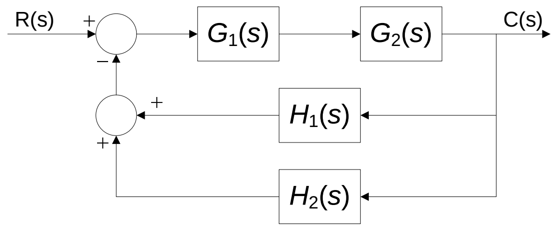




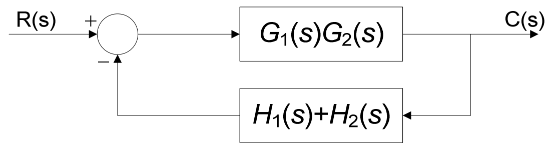
Verifica-se que a Função de Transferência deste sistema é:

$$\frac{C(s)}{R(s)}=\frac{G_3(s)\left[G_1(s)G_2(s)+G_4(s)\right]}{1+G_2(s)G_3(s)H(s)}$$

v)



Aplicando as regras da álgebra de blocos, temos:



Verifica-se que a Função de Transferência deste sistema é:

$$\frac{C(s)}{R(s)}=\frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)\left[H_1(s)+H_2(s)\right]}$$