

1a)

$$G H(s) = \frac{K}{s(s+1)(s+2)} = \text{FTMA}$$

$$\begin{aligned} \text{FTMF} &= \frac{\text{FTMA}}{1 + \text{FTMA}} = \\ &= \frac{\frac{N(s)}{D(s)}}{1 + \frac{N(s)}{D(s)}} = \\ &= \frac{\frac{N(s)}{D(s)}}{\frac{D(s) + N(s)}{D(s)}} = \frac{N(s)}{D(s) + N(s)} = \\ &= \frac{K}{s(s+1)(s+2) + K} \end{aligned}$$

$$\text{FTMA} = \frac{N(s)}{D(s)}$$

$$N(s) = K$$

$$D(s) = s(s+1)(s+2)$$

$$D(s) = s(s+1)(s+2) + K$$

$$= (s^2 + s)(s+2) + K = s^3 + 2s^2 + s + 2s + K =$$

$$D(s) = 0 ; K = ?$$

$$K = -s^3 - 3s^2 - 2s$$

$$\begin{aligned} \frac{d}{ds} K &= \frac{d}{ds} [-s^3 - 3s^2 - 2s] = - \frac{d}{ds} [s^3 + 3s^2 + 2s] \\ &= - [3s^2 + 6s + 2] \end{aligned}$$

$$\frac{d}{ds} K = 0 \mid s = ?$$

$$\begin{aligned} - (3s^2 + 6s + 2) &= 0 \quad (2) \quad 3s^2 + 6s + 2 = 0 \\ s^2 + 2s + \frac{2}{3} &= 0 \end{aligned}$$

$$x = \frac{1}{3}$$

$$P_1 = -0,4226$$

$$P_2 = -1,5773$$

Zeros \rightarrow none

Poles $\rightarrow 0, -1, -2$

calculadora

3 RAMOS

3 Asymptotes

Angulos

$$\rightarrow 180^\circ$$

$$\rightarrow -60^\circ$$

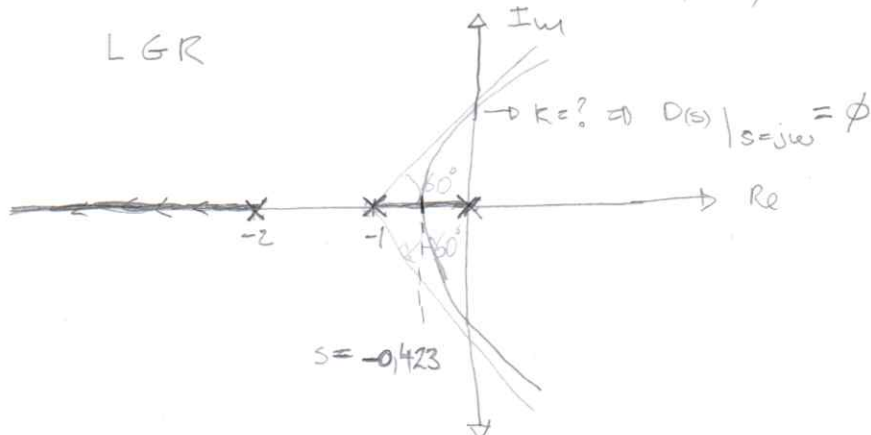
$$\rightarrow +60^\circ$$

Intercecao

Asymptotes

$$= -1$$

LGR



$$1a) \quad s^3 + 2s^2 + s^2 + 2s + K = 0 \quad | \quad s = j\omega \quad \omega = ?$$

$$j\omega^3 - 3\omega^2 + 2j\omega + K = 0 \quad K = ?$$

$$\begin{cases} -j\omega^3 + 2j\omega = 0 & (\times \frac{1}{j}) \\ -3\omega^2 + K = 0 \end{cases} \begin{cases} \omega^3 + 2\omega = 0 & (\times \frac{1}{\omega}) \\ K = 3\omega^2 \end{cases}$$

$$\omega = \pm \sqrt{-2} = \pm j\sqrt{2}$$

$$K = +3 \cdot (\sqrt{2})^2$$

$$= 6$$

$$\begin{cases} \omega(\omega^2 + 2) = 0 \\ K = 3\omega^2 \end{cases} \begin{cases} \omega = 0 \vee \omega = \pm \sqrt{2} \\ K = 6 \end{cases}$$

1. a)

$$G H(s) = \frac{K}{s(s+1)(s+2)}$$

1°

$$\frac{K}{s(s+1)(s+2)} = -1$$

$$K = -s(s+1)(s+2)$$

$$= -s(s^2 + 2s + s + 2)$$

$$= -s^3 - 3s^2 - 2s$$

2°

zeros : none

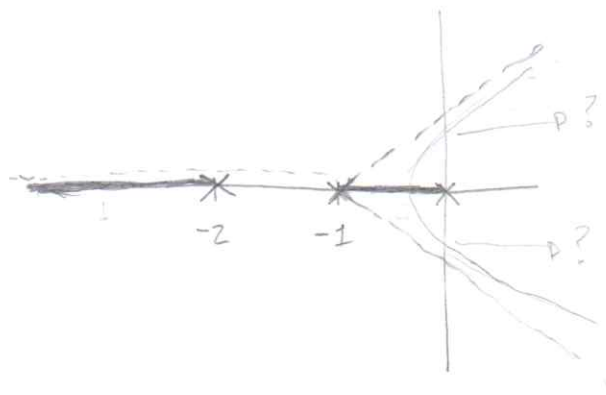
poles : 0, -1, -2

number of branches : 3

number of asymptotes : 3

Asymptotes angles : $180^\circ; \pm 60^\circ$

Asymptotes interception : -1



Exemplo

LGR

$$K W(s) H(s) = K \frac{1}{s(s+1)(s+2)}$$

Z: não existe

$$n = 0$$

P: $0+j0$; $-1+j0$; $-2+j0$

$$d = 3$$

acaba em

$-\infty + j0$,
 $+\infty + j0$
 $+\infty - j0$

$$K \frac{1}{s(s+1)(s+2)} = -1$$

RTMA

$$-\frac{1}{K} = \frac{1}{s(s+1)(s+2)}$$

$$K = -s(s+1)(s+2)$$

$$= -s(s^2 + 3s + 2)$$

$$\frac{dK}{ds} = \frac{d}{ds} (-s^3 - 3s^2 - 2s) = 0$$

$$\begin{cases} s = -1,577 \\ s = -0,423 \end{cases} \Rightarrow -s(s+1)(s+2) \Big|_{s=-0,423}$$

$$K = 0,384$$

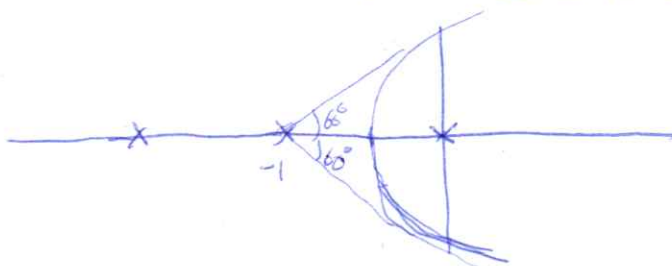
σ - centroid - calculator prog!

Angulo Assumpt - calculator prog!

Routh - Hurwitz

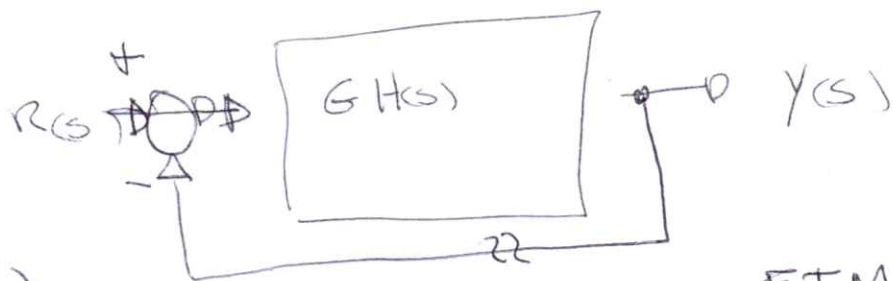
$$s^3 + 3s^2 + 2s + K$$

$$\Rightarrow s = \pm j\sqrt{2}$$



1 a)

LGR



a)

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

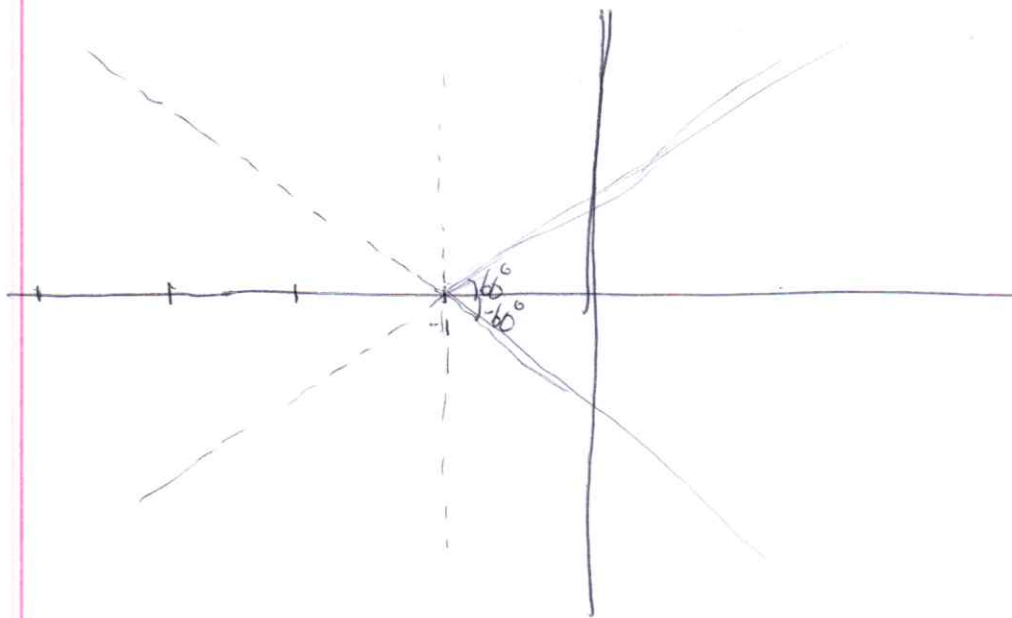
FTMA

zeros: não existe : $n=0$

poles: $0+j0$; $-1+j0$; $-2+j0$: $d=3$

Assíntotas: $\pm 60^\circ$

centroide: -1



\Rightarrow O número de ramos do LGR é igual ao número de polos da FTMA. Ramos $= d = 3$

\Rightarrow O LGR começa nos polos em malha aberta e termina nos zeros em malha aberta ou no infinito.

1a)

LGR

$$\frac{K}{s(s+1)(s+2)} \quad z=1$$

$$R = -s(s+1) \cdot (s+2)$$

$$z = -(s^2 + s)(s+2)$$

$$z = -(s^3 + 2s^2 + s^2 + 2s)$$

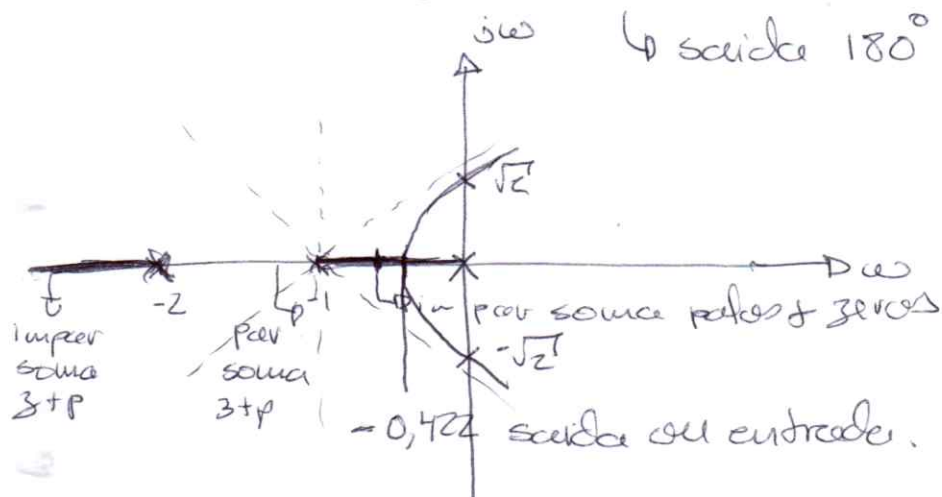
$$z = -s^3 + 3s^2 - 2s$$

$$\frac{dK}{ds} = 0$$

$$z = -3s^2 - 6s - 2 = 0$$

$$(s+1,577)(s+0,422) = 0$$

$$\therefore \begin{cases} s = -1,577 \times \\ s = -0,422 \end{cases}$$



Pontos de interseção eixo imaginário.

$$zD \quad \boxed{1 + G H(s) \Big|_{s=j\omega} = 0}$$

$$1 + \frac{K}{s(s+1)(s+2)} \Big|_{s=j\omega} = 0$$

$$\frac{s(s+1)(s+2) + K}{s(s+1)(s+2)} \Big|_{s=j\omega} = 0$$

1.

$$a) \quad G H(s) = \frac{K}{s(s+1)(s+2)}$$

$\dot{N}^{\circ} \text{ polos} = \dot{N}^{\circ} \text{ ceros.}$

$$\begin{aligned} D &= 3 & \begin{cases} -60^\circ & -180^\circ \\ +60^\circ & +180^\circ \end{cases} \\ N &= 0 \end{aligned}$$

$$\text{formulae} \quad \frac{(2L+1)}{D-N} \cdot 180^\circ \quad \left| \quad L=1, 3, \dots \right.$$

Polos @ 0, -1, -2

zeros: none

$$\phi = \frac{\sum \text{polos} - \sum \text{zeros}}{D - N}$$

$$\frac{K}{s(s+1)(s+2)} = -1$$

$$\begin{aligned} K &= -s \frac{s^2+2s+1}{(s+1)(s+2)} \\ &= -(s^3+3s^2+2s) \end{aligned}$$

$$\boxed{\phi = -1}$$

$$\frac{dK}{ds} = -3s^2 - 6s - 2 = 0$$

valores para $\gamma < 0$

$$\Rightarrow \frac{dK}{ds} = 0 \quad \begin{cases} s = -0,422 \\ s = -1,577 \end{cases} \quad \text{sem significado}$$

$$\left. \frac{K}{s(s+1)(s+2)} + 1 = 0 \right|_{s=j\omega}$$

valores para $\gamma = 0$

$$\left. K + s(s+1)(s+2) = 0 \right|_{s=j\omega}$$

$$\left. K + s^3 + 3s^2 + 2s = 0 \right|_{s=j\omega}$$

$$(j\omega)^3 + 3(j\omega)^2 + 2(j\omega) + K = 0$$

$$j\omega^3 - 3\omega^2 + 2j\omega + K = 0$$

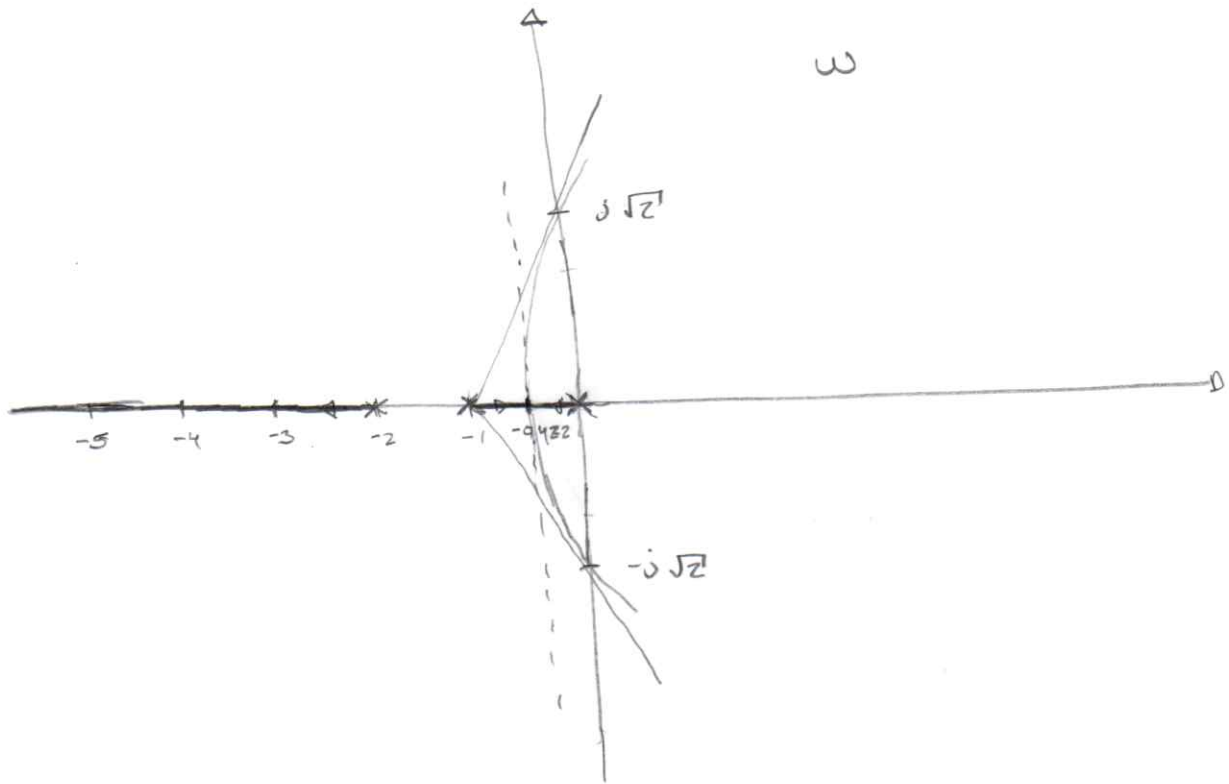
$$\begin{cases} j\omega^3 + 2j\omega = 0 \\ -3\omega^2 + K = 0 \end{cases} \quad \begin{cases} \omega^3 + 2\omega = 0 \\ K = 3\omega^2 \end{cases}$$

$$\begin{cases} \omega(\omega^2 + 2) = 0 \\ \therefore \end{cases} \quad \Rightarrow \omega = \pm \sqrt{2} \quad \vee \omega = 0$$

$$\begin{aligned} K &= 3(\sqrt{2})^2 \\ &= \pm 6 \end{aligned}$$

$$\vee K = 3 \cdot 0 = 0$$

1.



Se $s = -0,422 \Rightarrow K = 1,453$

$K = 60; \omega = \pm \sqrt{2}$

1 a)

LGR → FTMA → FTMF → $P(s) \rightarrow \frac{d}{ds} K$
 ↓
 Poles
 Zeros
 centroid
 Angulo Asympt
 Região LGR
 ↓
 Pólos Quebra
 $P(s) \neq \emptyset$ interceptado eixo imag

$$s(s+1)(s+2) + K = 0$$

$$K = -s(s+1)(s+2)$$

$$z = -(s^2+s)(s+2) = -(s^2+s) \cdot s + (s^2+s) \cdot 2$$

$$z = -(s^3 + s^2 + 2s^2 + 2s)$$

$$z = -(s^3 + 3s^2 + 2s)$$

$$\frac{dK}{ds} = -(3s^2 + 6s + 2)$$

$$-(3s^2 + 6s + 2) = 0$$

$$3s^2 + 6s + 2 = 0$$

$$s = \frac{-3 + \sqrt{3}}{3}$$

$$\wedge s = \frac{-3 - \sqrt{3}}{3}$$

como

$$LGR \ 0 < x < -1$$

$$\frac{-3 + \sqrt{3}}{3} \in LGR$$

— 11 —

$$K + (s^3 + 3s^2 + 2s) = 0$$

$$(j\omega)^3 + 3(j\omega)^2 + 2j\omega + K = 0$$

$$-j\omega^3 - 3\omega^2 + 2j\omega + K = 0$$

$$j \begin{cases} -\omega^3 + 2\omega = 0 \\ -3\omega^2 + K = 0 \end{cases} \begin{cases} \omega = \sqrt{2}; \omega = 0; \omega = -\sqrt{2} \\ K = 3\omega^2 = 3 \cdot \sqrt{2}^2 = 6 \wedge K = -6 \end{cases}$$

$$K = 6 \wedge K = -6$$

∴ interceptado eixo imaginário

$$K = 6 \wedge K = -6 \text{ para } \omega = j\sqrt{2} \wedge \omega = -j\sqrt{2}$$

using
calculator
Equation solver.