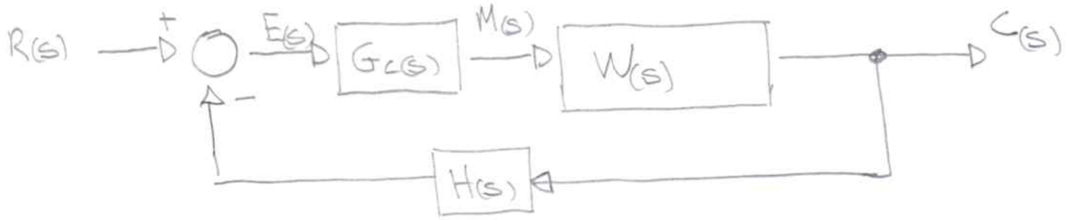
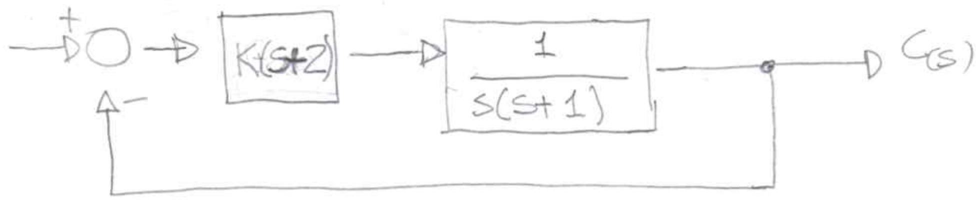
8 a)




$$M(t) = \underbrace{2K\,e(t)}_{\text{Proportional term}} + \underbrace{K \frac{d}{dt} e(t)}_{\text{Derivative term}}$$

∴ Is a PD controller

b) Draw the Root locus and define the behavior zones for multiple values of K considering $H(s) = 1$




$\boxed{R1}$ obtain the characteristic equation in form:

$$GH(s) = -1$$

$$\frac{K(S+2)}{S(S+1)} = -1$$

$\boxed{R2}$ Locate the poles and zeros of the open loop T.F. in the "s" plane

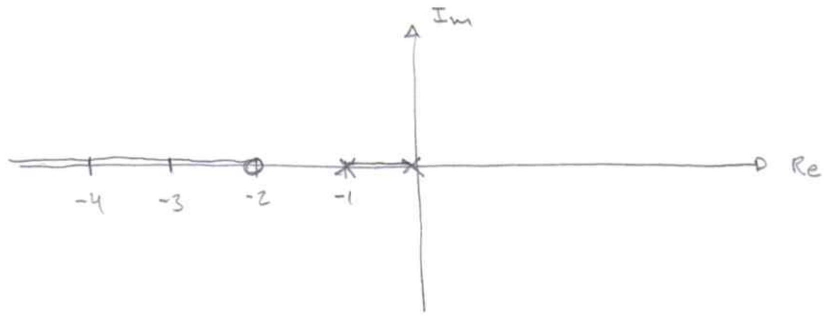
Zeros: $z_1 = -2 \Rightarrow n = 1$

Poles: $\left.\begin{array}{l} P_1 = 0 \\ P_2 = -1 \end{array}\right\}$ $d = 2$ n° of branches

$\boxed{R3}$ Are branches of Root-locus on the Real axis if the total number of Real Poles and Real Zeros to the right is odd.

8 a) continuação

$\boxed{R3}$




$\boxed{R4}$ Determine the assymptote of Root loc:

- Number of distinct assymptotes is $d - n = 2 - 1 = 1$
- Angles of asymptotes $= \dfrac{(1 + 2h) \cdot 180^\circ}{d - n}$

$K = \pi$ In this case the asymptote is the Real Axis, it doesn't make sense to calculate the Real Axis Interception.

$\boxed{R5}$ Find the break away and break-in points:

$$\frac{d}{ds} K = \phi$$

characteristic equation:

$$K \cdot \frac{(s+2)}{s(s+1)} = -1 \Leftrightarrow K = -\frac{s \cdot (s+1)}{(s+2)}$$

$$\frac{d}{ds}\left[-\frac{s \cdot (s+1)}{(s+2)}\right] = \phi \qquad \begin{cases} s_1 = -0{,}59 \\ s_2 = -3{,}42 \end{cases}$$

$\boxed{R6}$ No complex poles

$\boxed{R7}$ Find the points where the Root loc may cross the imaginary axis:

Method $\Rightarrow$ characteristic equation: $1 + GH(s)\Big|_{s=j\omega} = \phi$

$$1 + K \frac{(s+2)}{s(s+1)}\Big|_{s=j\omega} = \phi$$

8 a) continuação

$\boxed{R7}$

$$s.(s+1) + K(s+2)\Big|_{s=j\omega} = \phi$$

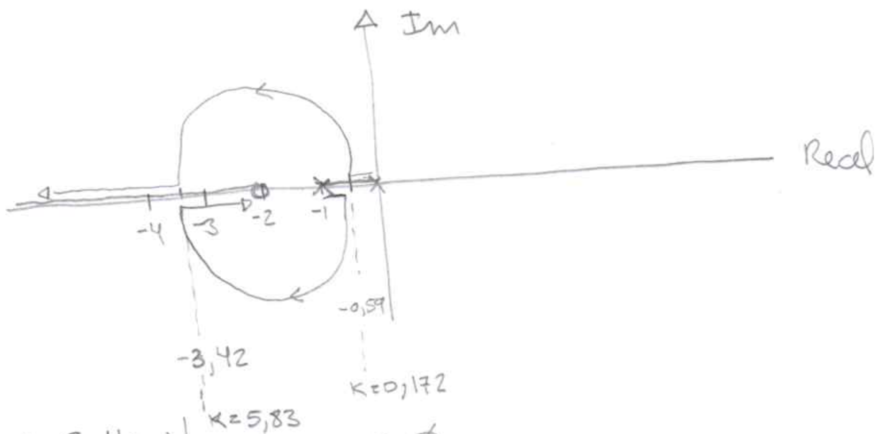
$$s^2 + 1 + Ks + 2K\Big|_{s=j\omega} = \phi$$

$$s^2 + (1+K).s + 2K\Big|_{s=j\omega} = \phi$$

Pole at origin

$$\begin{cases} \omega + 2K = 0 \\ (1+K)\omega = 0 \end{cases} \Leftrightarrow \begin{cases} \omega = \phi \\ K \neq 0 \end{cases} \wedge \cancel{\begin{cases} \omega = 2 \\ K = -1 \end{cases}}$$

Draw the root locus:




$$\Rightarrow 1 + GH(s)\Big|_{s=-0,59} = \phi$$

$$1 + K.\frac{(s+2)}{s(s+1)}\Big|_{s=-0,59} = \phi \quad \Rightarrow K = 0,172$$

$$1 + K.\frac{(s+2)}{s(s+1)}\Big|_{s=-3,42} = \phi \quad \Rightarrow K = 5,83$$

$\Rightarrow$ For $0 < K < 0,172$ and $K > 5,83 \to \xi > 1$
the system is over damped (does not oscilate)

$\Rightarrow$ For $K = 0,172$ and $K = 5,83 \Rightarrow \xi = 1$
the system is critically damped (does not oscilate)

$\Rightarrow$ For $0,172 < K < 5,83 \Rightarrow \xi < 1$
the system is under-damped (oscilates)