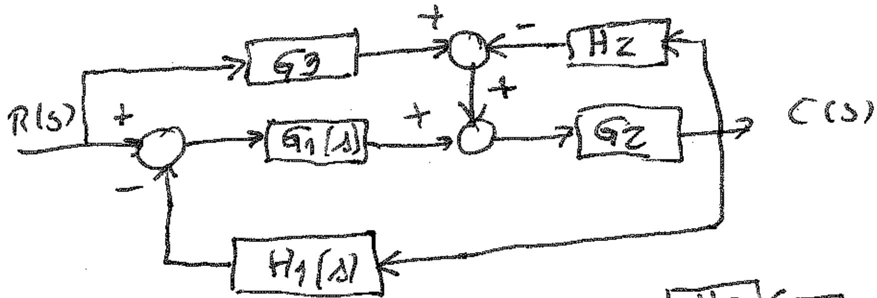
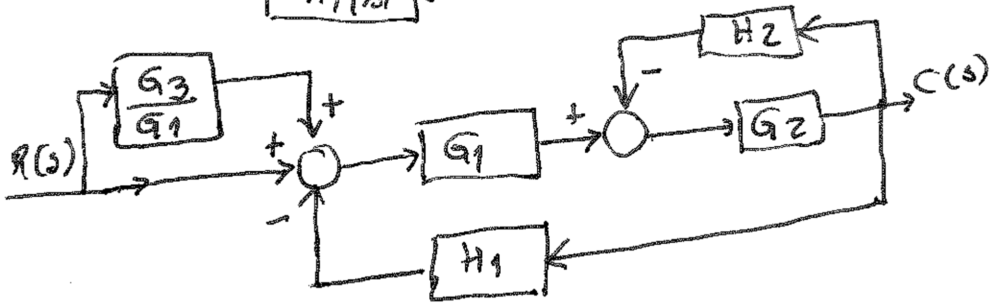
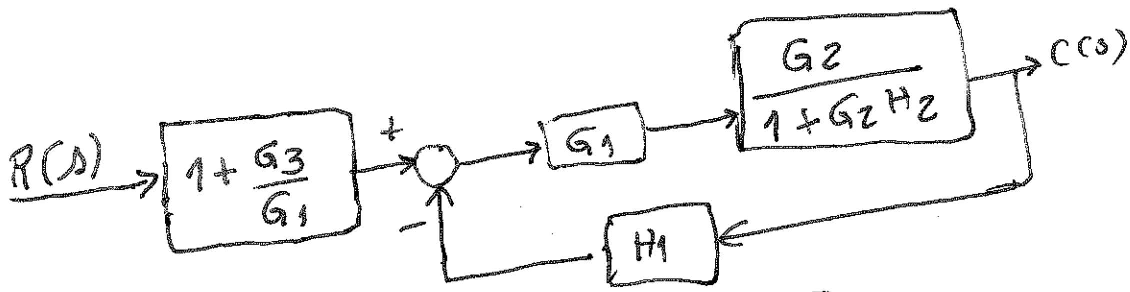
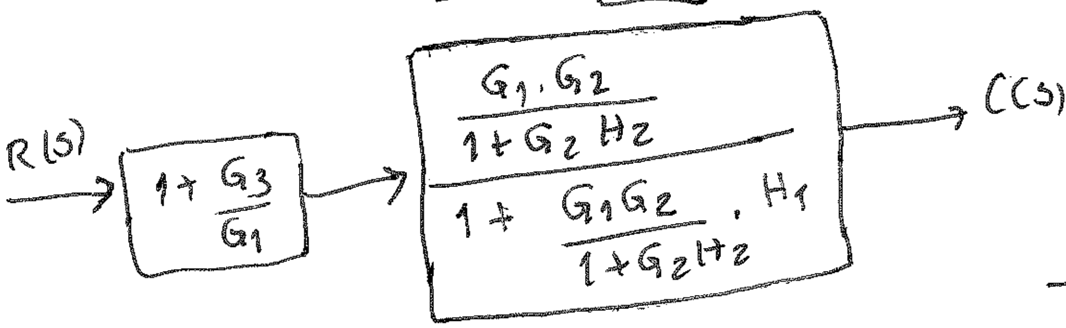
Exame Recurso — Parte 1 — 22 Julho 08

1a)







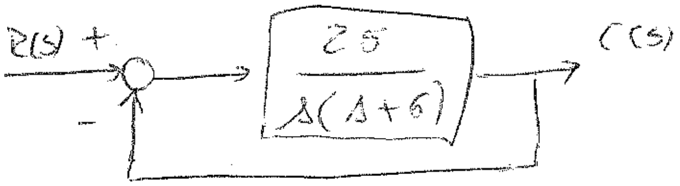


$$\frac{C(s)}{R(s)} = \left(1+\frac{G_3}{\cancel{G_1}}\right)\cdot\left[\frac{(\cancel{G_1}G_2)\cdot\cancel{(1+G_2H_2)}}{\cancel{(1+G_2H_2)}\left[1+G_2H_2+G_1G_2H_1\right]}\right] \Rightarrow$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{(G_1+G_3)\cdot G_2}{1+G_2(H_2+G_1H_1)}}$$

b) $G_1 \cdot G_2$
$G_3 \cdot G_2$

2)



a) t_s, t_p, t_R, M_p

$$\frac{C(s)}{R(s)} = \frac{\frac{25}{s(s+6)}}{1+\frac{25}{s(s+6)}} = \frac{25}{s(s+6)+25} = \frac{25}{s^2+6s+25}$$

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{25}{s^2+6s+25}$$

$$\begin{cases} 2\xi\omega_n = 6 \\ \omega_n^2 = 25 \end{cases} \Rightarrow \begin{cases} \xi = \frac{6}{10} = 0.6 \\ \omega_n = 5 \text{ Rad/seg} \end{cases}$$

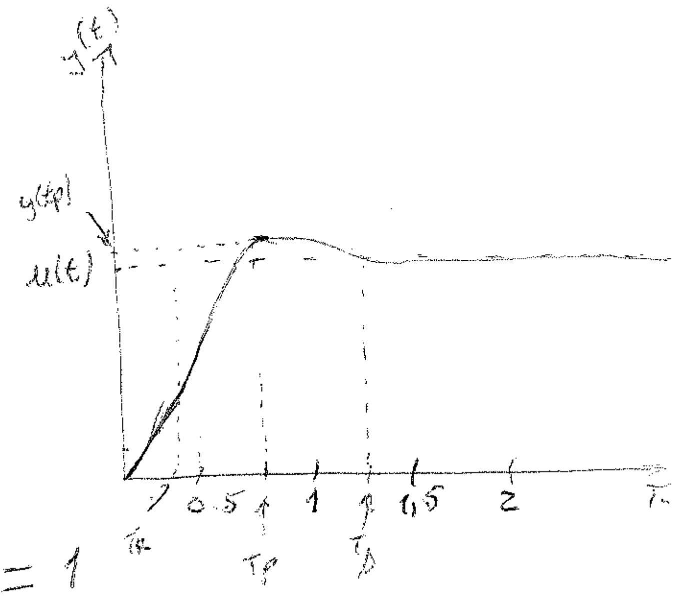
$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 0.095 \Rightarrow M_p\% = 9.5\%$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{3} = 1{,}333 \text{ seg.}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.785 \text{ seg}$$

$$\theta = \arccos(\xi) = 0.927 \text{ Rad/seg}$$

$$t_R = \frac{e^{\theta/tg(\theta)}}{\omega_n} = 0.401 \text{ seg}$$



$$\text{Valor final} = \lim_{t\to\infty} \frac{25}{s^2+6s+25} = 1$$

$$\text{Valor máximo} = 1 + 0.095 = 1.095$$

2b) $r(t) = t^2 \overset{\mathcal{L}}{\rightsquigarrow} R(s) = \frac{1}{s^3}$

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} s^2 \cdot \frac{25}{s(s+6)} = 0$$

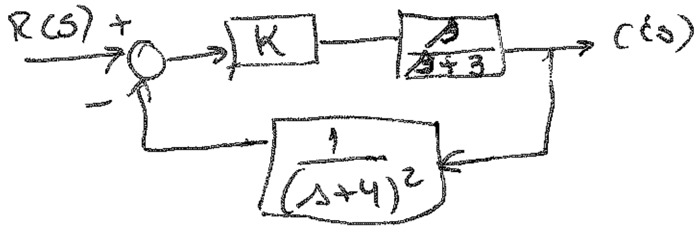
$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

c) A função de transferência tem 2 polos complexos conjugados, pelo que a resposta do sistema é subamortecida $(0 < \xi < 1)$.

$$p_{1,2} = -3 \pm j4$$

Parte 2

3.



a)

$$\frac{C(s)}{R(s)} = \frac{K \cdot \frac{s}{s+3}}{1 + \frac{Ks}{(s+3)} \cdot \frac{1}{(s+4)^2}}$$

EQ. característica : $1 + K\frac{s}{(s+3)(s+4)^2} = 0$

$$\frac{Ks}{(s+3)(s+4)^2} = -1$$

zeros $= n = 1 \quad z = 0$

polos $= d = 3 \quad p_1 = -3 \; ; \; p_{2,3} = -4$

LGR $= d = 3$

Assimptotas $= d - n = 2$

∡ Assimptotas $\Rightarrow \frac{(1+2h) \cdot 180}{d-n}$ $h = 0 \Rightarrow 90°$; $h = 1 \Rightarrow -90°$

centroide $= \frac{-3-4-4-0}{3-1} = \frac{-11}{2} = -5,5$

Pontos quebra!

$$\frac{\partial K}{\partial s} = 0 \Rightarrow \quad K = -\frac{(s+3)(s+4)^2}{s} = \frac{-(s+3)(s^2+8s+16)}{s} \Rightarrow$$

$$\frac{\partial K}{\partial s} = 0 \Rightarrow \frac{-[(s^2+8s+16)+(s+3)(2s+8)]s + (s+3)(s^2+8s+16)}{s^2} = 0$$

$$\Rightarrow (-s^2-8s-16-2s^2-8s-6s-24)s + s^3+8s^2+16s+3s^2+24+48$$

$$+11s^2+16s+72 = 0$$

$$\Rightarrow -3s^3-22s^2-40s+s^3$$

$$= -2s^3-11s^2-24s+72 = 0 \qquad s = -3,531 \pm j\,3,251 \quad = 1,563$$

não há ponto quebra

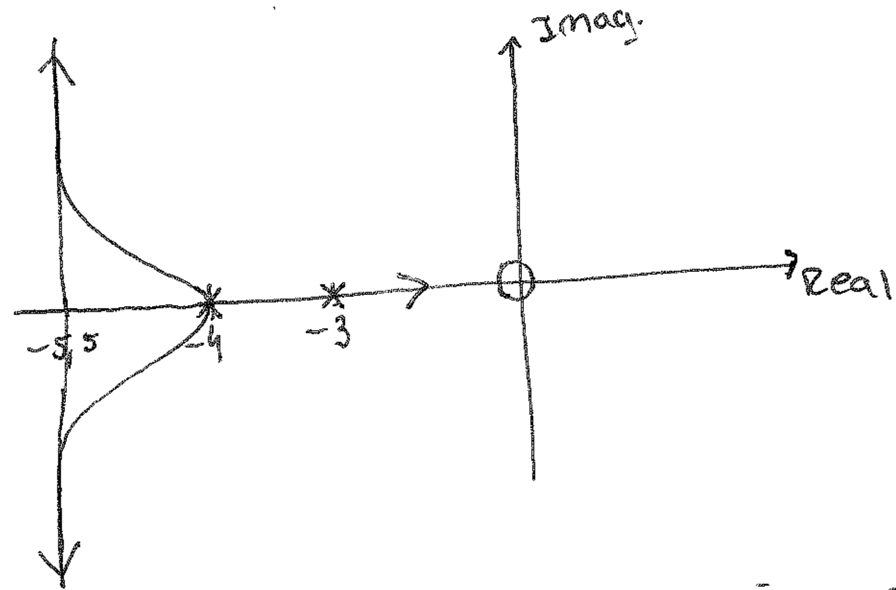
Intersecção eixo imag.

$$1 + K \frac{s}{(s+3)(s+4)^2}\Bigg|_{s=j\omega} = 0 \Rightarrow (s+3)(s+4)^2 + Ks\Big|_{s=j\omega} = 0$$

$$(j\omega+3)(j\omega+4)^2 + Kj\omega = 0 \Rightarrow (j\omega+3)(-\omega^2 + 8j\omega + 16) + Kj\omega = 0$$

$$-j\omega^3 - 8\omega^2 + 16j\omega - 3\omega^2 + 24j\omega + 48 + Kj\omega = 0$$

$$\begin{cases} -\omega^3 + 16\omega + 24\omega + K\omega = 0 \\ -8\omega^2 - 3\omega^2 + 48 = 0 \end{cases} \quad \begin{cases} K = -20,957 \quad \text{Impossível} \\ \omega^2 = \frac{48}{11} = 4.364 \end{cases}$$



b) Para que o sistema não apresente oscilação na saída, os pólos da F.T.M.F. tem que ser reais, logo para ~~não~~ apresentar oscilação as raízes tem que ser complexas.

$$1 + GH(s)\Big|_{s=-4} = 0 \Rightarrow 1 + K\frac{s}{(s+3)(s+4)^2}\Bigg|_{s=-4} = 0 \Rightarrow$$

$$\Rightarrow K = \frac{-(s+3)(s+4)^2}{s}\Bigg|_{s=-4} \Rightarrow K = \frac{-(s^3 + 11s^2 + 40s + 48)}{s}\Bigg|_{s=-4}$$

$K = 0 \Rightarrow$ o que era de esperar já que a partir do polo duplo o LGR sai do eixo Real, e $\boxed{K>0}$ o K no polo é zero.

c) $$\frac{C(s)}{R(s)} = \frac{\dfrac{Ks}{(s+3)}}{\dfrac{(s+3)(s+4)^2+Ks}{(s+3)(s+4)^2}} =$$

$$= \frac{Ks\cancel{(s+3)}(s+4)^2}{\cancel{(s+3)}\left[(s+3)(s+4)^2+Ks\right]} = \frac{Ks(s+4)^2}{(s+3)(s+4)^2+Ks}$$

Eq. característica:

$$(s+3)(s+4)^2+Ks=0 \Rightarrow s^3+11s^2+40s+48+Ks=0$$

$$\Rightarrow s^3+11s^2+(40+K)s+48=0$$

3	1	(40+K)
2	11	48
1	A	
0	B	

$$A = -\frac{\begin{vmatrix} 1 & (40+K) \\ 11 & 48 \end{vmatrix}}{11} = -\frac{\left[48-11(40+K)\right]}{11} = -\frac{(48-440-11K)}{11} = 35{,}64+K$$

$$B = -\frac{\begin{vmatrix} 11 & 48 \\ (35{,}64+K) & 0 \end{vmatrix}}{(35{,}64+K)} = -\frac{(0-48(35{,}64+K))}{(35{,}64+K)} = 48$$

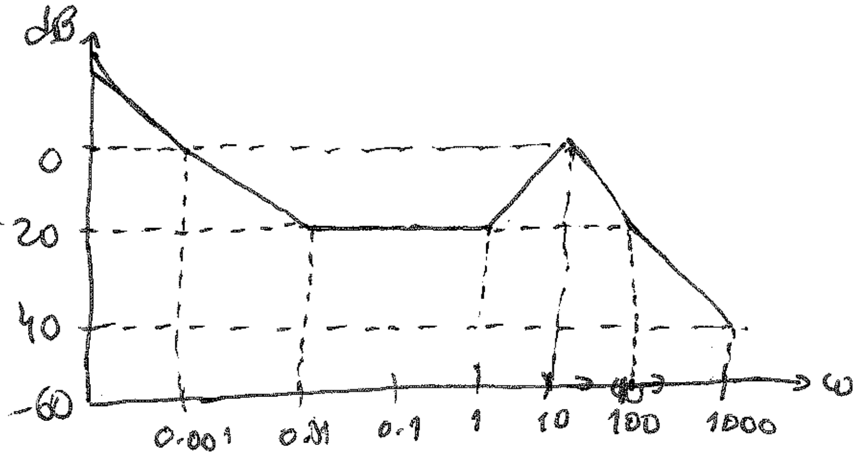
Para o sistema ser estável todas as raízes da Eq. característica têm que estar no semi plano esquerdo, o que corresponde neste critério a não haver trocas de sinal na 1ª coluna. Logo

A >0 e B >0 $\quad 35{,}64+K>0 \Rightarrow K>-35{,}64$

B $=48>0$ $\quad$ logo $K>-35{,}64$ para não haver trocas de sinal

4)

a)



zeros: $\omega_{z_1} = 0.01$ rad/seg $\quad \omega z_2 = 1$ rad/seg.

polos: $\omega_{p_1} =$ na origem $\quad \omega p_{2,3} = 10$ rad/seg

$$G(s) = \frac{1}{s} \cdot \frac{1}{(s+10)^2} \cdot (s+0.01)(s+1) =$$

$$= \underbrace{\frac{0.01}{100}}\ \frac{1}{s} \cdot \frac{1}{\left(\frac{s}{10}+1\right)^2} \cdot \left(\frac{s}{0.01}+1\right)(s+1)$$

$0.0001 \Rightarrow 20\log(0.0001) = -80$ dB

$20\log(10) = 20$ dB

$$G(s) = \frac{10\,(s+0.01)(s+1)}{s(s+10)^2}$$

b)