Shock Waves (1994) 3:327–329

Shock Waves

Technical note

Wave angle for oblique detonation waves

S.A. Ashford, G. Emanuel

The University of Oklahoma, School of Aerospace and Mechanical Engineering, Norman, OK 73019-0601, USA

Received June 8, 1993; accepted December 21, 1993

Abstract. The flow field associated with a steady, planar, oblique detonation wave is discussed. A revision is provided for β-θ diagrams, where β is the wave angle and θ is the ramp angle. A new solution is proposed for weak underdriven detonation waves that does not violate the second law. A Taylor wave, encountered in unsteady detonation waves, is required. Uniqueness and hysteresis effects are also discussed.

Key words: Chapman-Jouguet condition, Oblique detonation wave, Taylor wave, Uniqueness and hysteresis effects

1. Introduction

The flow field of a steady, planar, oblique detonation wave is sketched in Fig. 1a. A ramp with an angle θ causes an attached shock wave in the oncoming supersonic, combustible flow. Immediately downstream of the shock, a very rapid heat addition process is assumed; β provides the orientation of the shock and heat addition. (The states just downstream of the shock and heat addition process are denoted with 2 and 3 subscripts, respectively.) Figure 2 is a conventional β-θ diagram for a perfect gas with $\gamma = 1.4$ and an upstream Mach number M_1 of 5. In the figure, q is the heat addition, per unit mass, due to combustion and is normalized with RT_1, where R is the gas constant and T is the temperature. When $q > 0$, the weak solution lower branch is subdivided into an overdriven regime, where the normal component of the Mach number M_{3n} is subsonic, and an underdriven regime, where M_{3n} is supersonic. The Chapman-Jouguet (CJ) demarcation condition, $M_{3n} = 1$, is indicated with an asterisk in the figures.

Figures equivalent to Fig. 2 can be found in the literature as far back as 1963 (Gross 1963). Many recent examples of this type of figure are also found in the literature, since it is basic to ram accelerators (Hertzberg et al. 1991; Yungster et al. 1991), oblique detonation wave engines (Menees et al. 1992; Ostrander et al. 1987), and detonation wave theory in general (Pratt et al. 1987; Powers and Stewart 1992; Powers and Gonthier 1992; Buckmaster and Lee 1990; Lasseigne and Hussaini 1993). Buckmaster and Lee (1990) point out that a weak underdriven wave violates the second law of thermodynamics. Presumably, a weak solution detonation with a small value for θ should not physically occur.

2. Physical analysis

It is well established that an unsteady, normal detonation wave is frequently accompanied by a rarefaction (Taylor 1950; Courant and Friedricks 1948). This wave occurs immediately downstream of the combustion zone and now is referred to as a Taylor wave. It does not occur when the Mach number M_3', in a shock fixed frame, is less than unity, i.e., the detonation is overdriven. (This type of flow can occur in shock tubes.) It does occur when $M_3' = 1$ and, moreover, the Taylor wave prevents M_3' from exceeding unity. As Taylor's (1950) analysis demonstrates, the presence of the rarefaction wave stems from a global analysis. Its existence has been experimentally verified (Kistiakowsky and Kydd 1955) in a detonation tube experiment, where it always occurs.

The correspondence between an unsteady, one-dimensional flow and a steady, two-dimensional flow is well known. Nevertheless, the need for a Taylor wave in a flow with a steady, oblique detonation has heretofore not been recognized by the aerospace community. In a steady flow, a Taylor wave is just a centered Prandtl-Meyer expansion when a wall has an abrupt turn. It does not occur when $M_{3n} < 1$. Hence, with $q > 0$, it is not present when the detonation is strong or a weak overdriven wave. It first appears at the CJ condition with zero strength. Starting from this condition and letting θ decrease results in a Taylor wave of finite strength, as sketched in Fig. 1b.

The Taylor wave stems from a global requirement; namely, that the velocity tangency condition be satisfied at the wall in region 4 of Fig. 1b. As θ decreases below its CJ value, conditions in regions 1, 2, and 3 remain fixed and β does not change. The flow adjusts by changes in the angular spread and strength of the Taylor wave. In this

Correspondence to: G. Emanuel

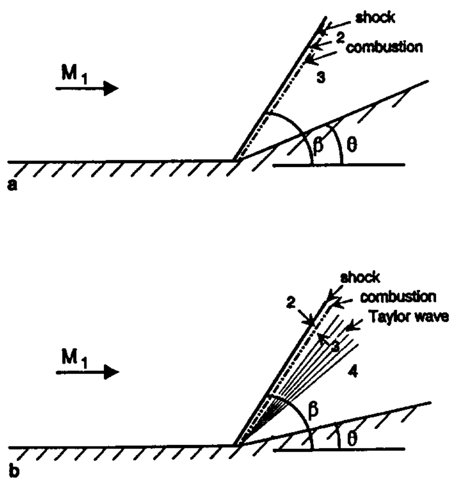

Fig. 1a, b. Schematic of a steady oblique detonation wave; **a** without a Taylor wave, and **b** with a Taylor wave

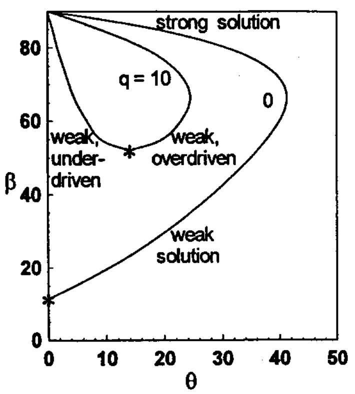

Fig. 2. Conventional β-θ diagram with labelling when $\gamma = 1.4$ and $M_1 = 5$

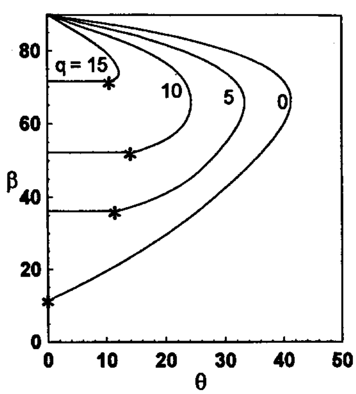

Fig. 3. Revised β-θ diagram that incorporates a Taylor wave when $\gamma = 1.4$ and $M_1 = 5$

regard, the velocity $\mathbf{V}_2$ of state 2 is rotated counterclockwise relative to $\mathbf{V}_1$, i.e., $\mathbf{V}_2$ is rotated toward the shock. One can show that $\mathbf{V}_3$ is rotated clockwise relative to $\mathbf{V}_2$, while $\mathbf{V}_4$ is similarly rotated clockwise relative to $\mathbf{V}_3$ until $\mathbf{V}_4$ is parallel to the downstream wall. Combustion and the Taylor wave jointly rotate the velocity in the same clockwise direction.

A revised β-θ diagram is shown as Fig. 3; only the weak underdriven curves differ from a conventional diagram. Thus, in the weak underdriven regime β is a constant at its CJ value, a viable solution exists, and there is no second law violation.

3. Discussion

There are several interesting peculiarities associated with the solution shown for the weak underdriven regime in Fig. 3. For instance, the $\beta = \beta_{CJ}$ line actually extends to the left of the figure, where θ would be negative. These horizontal lines terminate when $M_4 = \infty$. When $\theta \geq 0$, the $q > 0$ curves are double valued in β, as is the case for the $q = 0$ curve. However, when $q > 0$ and $\theta = 0$ there are three possible solutions; (i) a normal detonation wave with $\beta = \pi/2$; (ii) a detonation with a Taylor wave and $\mu_1 < \beta < \pi/2$, where μ is the Mach angle; (iii) no wave at all. Moreover, this lack of uniqueness may result in hysteresis effects.

For example, suppose an experiment begins with θ negative, an ordinary Prandtl-Meyer expansion, and no detonation wave. With M_1 fixed but θ increasing and becoming positive, the flow would have an ordinary attached shock and not a detonation wave, providing T_2 is below the autoignition temperature. We also require that the combustible mixture not be pre-ignited by free radicals in the upstream flow (Gross and Chinitz 1960). The strength of a weak solution shock, with $q = 0$, may or may not be strong enough to cause ignition before θ reaches its detachment value. If ignition does occur, the solution jumps to a point on a $q > 0$ curve or, if θ is sufficiently large, to a detached wave configuration. On the other hand, consider an experiment with M_1 fixed that starts with an attached detonation in the weak overdriven regime. For a sufficient decrease in θ, the weak underdriven solution shown in Fig. 3 should occur, including one with a negative θ value.

Experimental data for steady, oblique detonation waves is sparse. However, a number of experiments were performed in the 1960's (Gross and Chinitz 1960; Rubins and Cunningham 1965; Rubins and Rhodes 1963). These early experiments were not sophisticated (by current standards) and did not look for either a Taylor wave or the nonexistence of a weak underdriven detonation. Nevertheless, hysteresis was observed (Gross and Chinitz 1960). There is thus a need for basic detonation wave experiments to resolve the issues discussed in this communication.

References

Buckmaster J, Lee CJ (1990) Flow refraction by an uncoupled shock and reaction front. AIAA J 28:1310

Courant R, Friedricks KO (1948) Supersonic flow and shock waves. Interscience Pub., New York, Sect. IIIE

Gross RA, Chinitz W (1960) A study of supersonic combustion. J Aerospace Sci 27:517–524, 534

Gross RA (1963) Oblique detonation waves. AIAA J 1:1225

Hertzberg A, Bruckner AP, Knowlen C (1991) Experimental investigation of ram accelerator propulsion modes. Shock Waves 1:17

Kistiakowsky GB, Kydd PH (1955) Gaseous detonations. VI. The rarefaction wave. J Chem Phys 23:271

Lasseigne DG, Hussaini MY (1993) Interaction of disturbances with an oblique detonation wave attached to a wedge. Phys Fluids A 5:1047

Menees GP, Adelman HG, Cambier JL, Bowles JV (1992) Wave combustors for trans-atmospheric vehicles. J Propulsion and Power 8:709

Ostrander MJ, Hyde JC, Young MF, Kissinger RD, Pratt DT (1987) Standing oblique detonation wave engine performance. AIAA Paper 87-2002

Powers JM, Gonthier KA (1992) Reaction zone structure for strong, weak overdriven, and weak underdriven oblique detonations. Phys Fluids A 4:2082

Powers JM, Stewart DS (1992) Approximate solutions for oblique detonations in the hypersonic limit. AIAA J 30:726

Pratt DT, Humphrey JW, Glenn DE (1987) Morphology of a standing oblique detonation wave. AIAA Paper 87-1785

Rubins PM, Cunningham THM (1965) Shock-induced supersonic combustion in a constant-area duct. J Spacecraft 2:199

Rubins PM, Rhodes Jr RP (1963) Shock-induced combustion with oblique shocks: comparison of experiment and kinetic calculations. AIAA J 1:2778

Taylor G (1950) The dynamics of the combustion products behind plane and spherical detonation fronts in explosives. Proc Roy Soc (Lond) A200:235

Yungster S, Eberhardt S, Bruckner AP (1991) Numerical simulation of hypervelocity projectiles in detonable gases. AIAA J 29:187

This article was processed using Springer-Verlag TEX Shock Waves macro package 1.0 and the AMS fonts, developed by the American Mathematical Society.