

## Scribed Notes-20

Student I'D

202212096

202212097

202212098 (Absent)

202212099

202212100

### Finite Group

- A group  $(G, *)$  is called a **Finite** group if  $G$  is an **Finite** set
- Eg:-  $G = \{1, 2, 3, 4, 5, 6, 7\}$  under multiplication modulo 8 is a finite group as the set  $G$  is a finite set

### Infinite Group

- A group  $(G, *)$  is called a infinite group if  $G$  is an infinite set

Eg: group  $(I, +)$ , where  $I$  is set of integers

### Cyclic Group

- A cyclic group is a group that can be generated by a single element. Every element of a cyclic group is a power of some specific element which is called a generator. A cyclic group can be generated by a generator 'g', such that every other element of the group can be written as a power of the generator 'g'.

### Order of Group:

- The order of the group  $G$  is the number of elements in the group  $G$ . It is denoted by  $|G|$ .  
A group of order 1 has only the identity element, i.e.,  $(\{e\}, *)$ .
- A group of order 2 has two elements, i.e., one identity element and one some other element.

### Order of an Element

- The order of an element in a group is the smallest positive power of the element which gives you the identity element.

$$g \in G; \text{smallest } k > 0 \mid g^k$$

### Proof by contradiction

- If two identity is  $e_1$  and  $e_2$

$$e_1, e_2 \quad e_1 \neq e_2$$

$$e_1 * e_2 = e_2$$

$$e_1 * e_2 = e_1, \text{ so } e_1 = e_2$$

- There is a unique Identity in a group

Suppose for  $x$  there exists two inverse  $y$  &  $z$

$$x * y = e$$

$$x * z = e$$

$$x * y = x * z$$

$$(y * x) * y = (y * x) * z \quad (\text{applying associativity rule})$$

Every element has unique inverse

Every group equation has unique solution

$$g_1 * x = g_2 \quad \text{but } x * g_1 \neq g_2$$

$$g_1 * x = g_2$$

$$(g_1^{-1} * g_1) * x = g_1^{-1} * g_1$$

### Recurrence Relation :

- A Recurrence relation of the sequence  $\{a_n\}$  is a question that expresses  $a_n$  in term of one or more of the previous term of the sequence, namely,  $a_0, a_1, a_2, a_3, \dots, a_{n-1}$  for all integer  $n$  with  $n \geq n_0$ , where  $n_0$  is non negative integer.

### Restriction of Function:

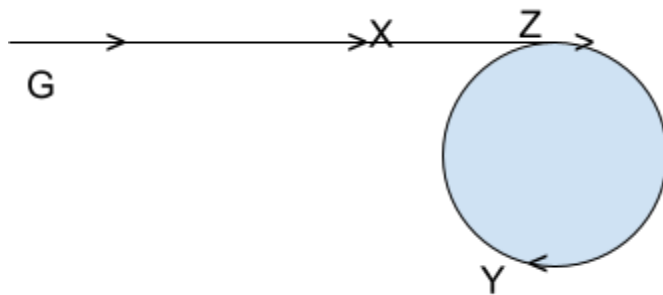
- Let  $f: X \rightarrow Y$  be a function from a set  $X$  to a set  $Y$ . If  $A$  is a subset of  $X$ , then the restriction of  $f$  to  $A$  is the function

$$f|_A: A \rightarrow Y$$

$$x \mapsto f(x).$$

### Injective Function

- $f: G * G \rightarrow G$   
 $\forall g \in G$   
 $F': \{g\} * G \rightarrow G$
- Is an Injective Function  $F^n$   
 $g * x = g * y$   
 $g * (x * g^{-1}) = g * (y * g^{-1})$  (multiply inverse on both sides)  
 $x = y$   
 $F': G * \{g\} \rightarrow G$
- Every row and every column is permutation



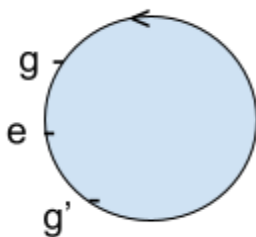
$$x * g = z$$

$$y * g = z$$

$$x * g = y * g$$

$$y = z$$

- So we will always go back to starting point



Because of this the last element=Identity

Second element=Inverse

Cyclic means that Every element will generate the Whole Group

If an element is Generate then its inverse is also a Generator

If no is prime except 0 all number are generate

Every cyclic group is abelian but not all abelian group is cyclic

$$a * b = g^{k_1 + k_2}$$

$$g = g^{k_2} g^{k_1}$$

$$= b^* a$$

So  $\forall$  cyclic are abelian