Tutorial 2

7^{th} August, 2025

1. Let a, b and c be three atomic propositions in propositional logic. Suppose you are told that

   (i) $a \vee (b \wedge c)$ is true, and

   (ii) $(a \vee b) \wedge c$ is true.

   Which of the individual truth values can be inferred from the given information?

2. A boolean function is said to be symmetrical with respect to a propositional variable if substituting true or false for that propositional variable leads to an identical reduced formula in the remaining variables. For the following formulae determine in which variables they are symmetrical.

   (a) $(\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge p_3)$

   (b) $(\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge \neg p_3) \vee (p_1 \wedge \neg p_2 \wedge p_3) \vee (p_1 \wedge p_2 \wedge p_3)$

   (c) $(\neg p_1 \wedge p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (p_1 \wedge \neg p_2 \wedge p_3)$

   (d) $(\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge \neg p_2 \wedge p_3) \vee (\neg p_1 \wedge p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge p_3)$

3. Assume the truth of the statement "Every country has at least one citizen that knows at least one citizen of all other countries." Assume that every country has more than one citizen. Iceland and Norway are countries.

   (a) There is a person in Iceland who knows everyone in Norway.

   (b) There is a person in Iceland who knows no one in Norway.

   (c) There is a person in Iceland who knows someone in Norway.

   (d) Every person in Iceland knows at least one person from Norway.

4. Statement 1: $\exists x$, such that $\forall y$, $x - y = 5$, where $x \in \mathcal{R}$ and $y \in \mathcal{R}$.
   Statement 2: $\forall x$, $\exists y$, such that $x - y = 5$, where $x \in \mathcal{R}$ and $y \in \mathcal{R}$.

   Which of the two statements is correct?

5. (a) Suppose we are given the following:
   - $\exists x, P(x)$
   - $\exists x, Q(x)$

   Can we conclude that $\exists x, P(x) \wedge Q(x)$

   (b) Suppose we are given that $\forall x, (P(x) \vee Q(x))$ Can we conclude that $\exists x, P(x) \wedge Q(x)$

   (c) Suppose we are given that $\forall x, (P(x) \vee Q(x))$ Can we conclude that $(\forall x, P(x)) \vee (\forall x, Q(x))$

6. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \Rightarrow R(x))$ are true then $\forall x(\neg R(x) \Rightarrow P(x))$ is also true.