Discrete Mathematics (Scribed Notes)

ID's:

202212081

202212082

202212083

202212084

202212085

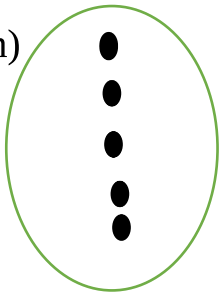
Topic: Functions & Group Theory

- Functions are relations where every element of the domain appears in exactly one ordered pair.

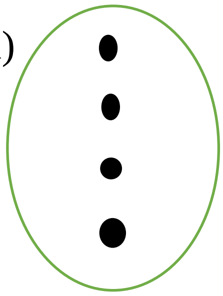
- Conditions for a relation to be a function:
  - Every point of domain has exactly one image.
  - Also, one point of domain cannot have 0, 2 or more images of codomain. It should have exactly one.

Eg. f : m→n

Where m and n are cardinality of domain and codomain respectively.

(m) 

(Domain)

(n) 

(Co-domain)

Total functions in above case is $4^6 = 4096$.

- Formula for finding number of functions on domain m and codomain **n = n^m**

- Standard way to represent all functions from domain(D) to codomain(C) = $\mathbf{C^D}$

Group Theory

A group is a set G along with an underlying operation * satisfying four axioms.

1. Closure
2. Identity
3. Inverse
4. Associativity
5. Commutative (Extra 5th condition, it makes an abelian group).

NOTE:

If the first four axioms are satisfied then it is a group. And if all five axioms are satisfied then it is called a commutative group/abelian group.

1. Closure :

- For all *g1* and *g2* in *G*, g1 * g2 belongs to *G*.
- $\forall\, g_1, g_2 \in G,\ g_1 * g_2 \in G$
- ^ (AND), V (OR) , ⊕ (XOR) is one type of closure axiom.

2.Identity:

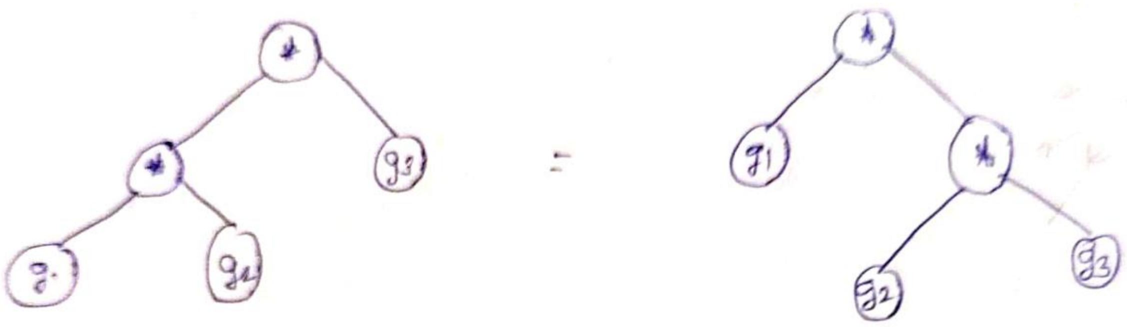
- the operation is written in multiplicative notation, while it is called the zero element or null element if the operation is written in additive notation.
- **∃ e ∈ G | ∀ g ∈ G, e * g = g * e = g**
- ^ (AND), V (OR) , ⊕ (XOR) is one type of identity axiom.

3. Inverse:

- **∀ g , ∃ g’ | g*g’ = g’*g = e**
- ⊕ (XOR) is one type of inverse.

4. Associativity :

- ∀ g1, g2, g3 ∈ G, (g1*g2)*g3 = g1*(g2*g3)
- ^ (AND), V (OR) , ⊕ (XOR) is one type of associative axiom.



$(g_1 * g_2) * g_3$ $\qquad$ $g_1 * (g_2 * g_3)$.

NOTE :

- Implication is not Associative.

- ^ (AND), V (OR) not male group because it violate inverse rule.

- Together all value with ⊕ (XOR) make group

Ex : Below is an example of addition on modulo (%) 5.

(+)%5	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

NOTE :

- **All groups are ternary relation. Groups are important natural examples of ternary relations.**

- **Set of all none-zero real numbers under division (Operation) to devide any eal number other than Zero is not a group because it is not Associative.**