

Discrete Math : Scribed Notes(23/11/22)

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ISOMORPHISM

- A Bijective function to the vertex of one graph to the vertex set of other graph.
- An Isomorphism from $G_1=(V_1,E_1)$ to $G_2=(V_2,E_2)$ is a bijective function $F:V_1 \rightarrow V_2$ such that (x,y) is in $E_1 \iff (f(x), f(y))$ is in E_2 .
- Two graphs are isomorphic if they have at least one isomorphism between them.
- If we define relation on the set of all undirected simple graphs where two graphs are related if and only if they are isomorphic. Then this relation is equivalence relation.

Reflexive:

$$F : u \rightarrow v$$

$$F(u) = v \quad \forall u \in v$$

$$(x, y) \in E \iff (f(x), f(y)) \in E$$

Symmetric:

$$(x, y) \in E_1 \iff (f(x), f(y)) \in E_2$$

$$g(F(x), F(y)) = (x, y) = (x, y)$$

$$F : V_1 \rightarrow V_2 \quad (x, y) \in E$$

$$G : V_2 \rightarrow V_3 \quad (F(x), F(y)) \in E_2$$

Transitive:

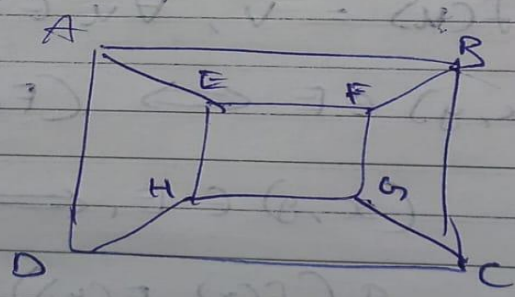
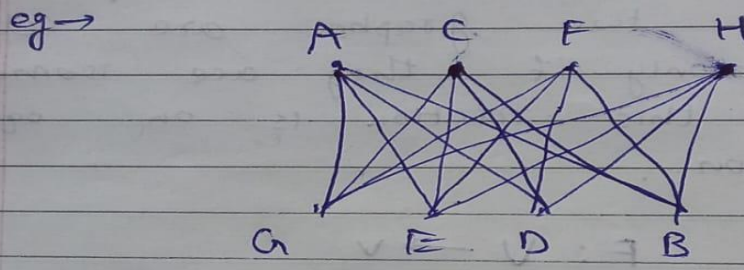
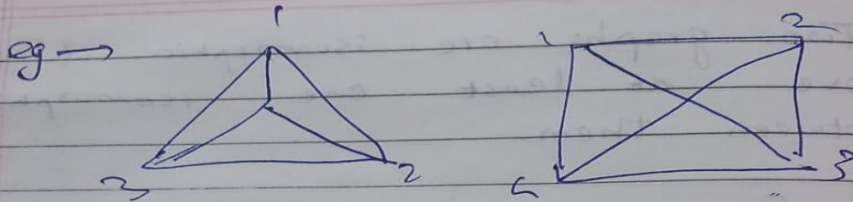
$f : G_1 \rightarrow G_2$

$g : G_2 \rightarrow G_3$

$(x, y) \in E_1 \Leftrightarrow (f(x), f(y)) \in E_2$

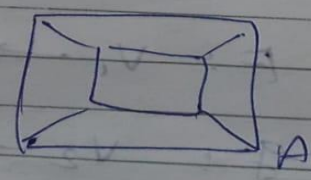
$(a, b) \in E_2 \Leftrightarrow (g(a), g(b)) \in E_3$

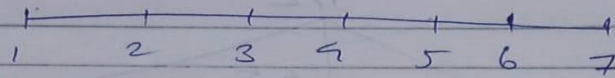
Automorphism is an isomorphism from a graph to itself.



Same.

Automorphism





$$F(1) = 6$$

$$F(2) = \{5, 7\}$$

$\therefore F(2)$ is either 5 or 7

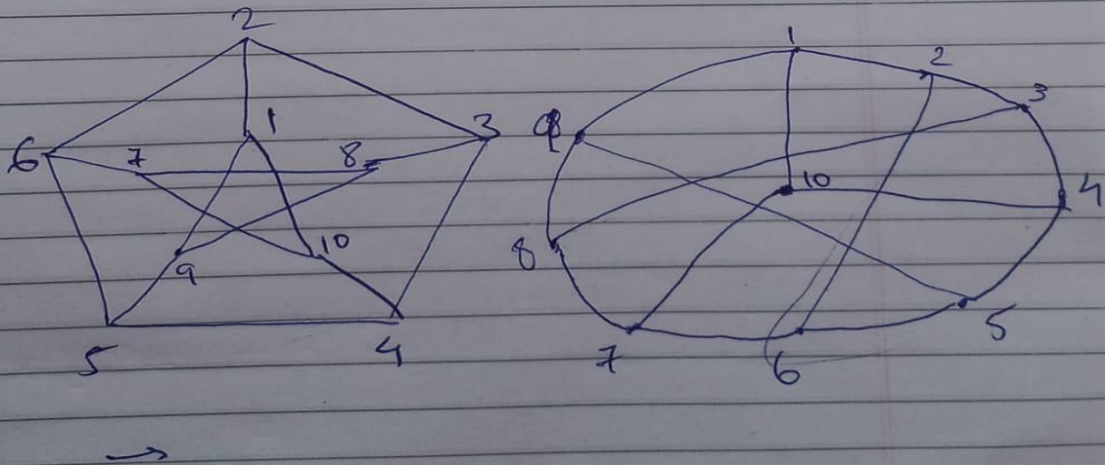
$F(2)$ is mapped to 7.

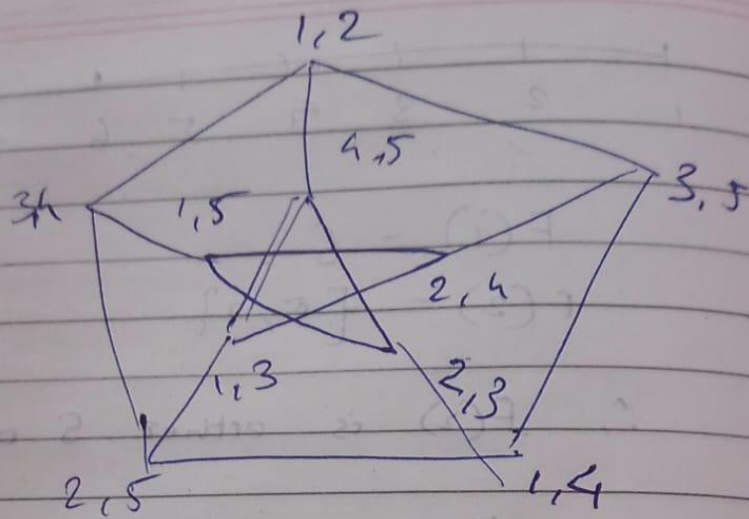
$F(3)$ has to be neighbour of 7 i.e. 6.

but now $F(1) = F(3) = 6$ so
not bijection

\therefore Not an automorphism

Petersen Graph





Algebraic Lefschetz Peterson graph.

eg $\rightarrow G \overset{\text{Isomorphism}}{\cong} H \Leftrightarrow \bar{G} \cong \bar{H}$

