Elementary relation types

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Here, we consider binary relations from a set S onto itself. By binary, we mean we apply the cartesian product only once between S and itself and take the subsets. In general we could apply it several times, giving rise to higher order relations.

A **reflexive relation** R on a set S is one such that $\forall s \in S, (s, s) \in R$.

An **irreflexive relation** R on a set S is one such that $\forall s \in S, (s, s) \notin R$.

As can be observed, there is no single relation that is both reflexive and irreflexive. However, there are relations that fall into neither category.

The counts of these categories is:

- Number of reflexive relations= 2^{n^2-n}
- Number of irreflexive relations= 2^{n^2-n}
- The number of relatios that do not fall into either of the above categories= $2^{n^2} - 2^{n^2-n+1}$

A **symmetric relation** R on a set S is one such that

$$\forall s_1, s_2 \in S, ((((s_1, s_2) \in R) \wedge ((s_2, s_1) \in R)) \vee (((s_1, s_2) \notin R) \wedge ((s_2, s_1) \notin R)))$$

An **anti-symmetric** relation R on a set S is one such that

$$\forall s_1, s_2 \in S, \text{ such that } s_1 \neq s_2, \neg(((s_1, s_2) \in R) \wedge ((s_2, s_1) \in R))$$

Number of symmetric relations $= 2^{n+\binom{n}{2}}$

Number of anti-symmetric relations= $2^n \times 3^{\binom{n}{2}}$

Transitive relations will be covered in the next lecture.