Discrete Mathematics (SC612)
Final Exam
December 24th, 2021

- The exam is closed book and closed notes.

- Duration: 2 hours (10 am- 12 noon). You need to scan and email the answers, to be written on plain paper, immediately after the exam ends.

- There are 7 questions.

- Each quetion is of 20 marks.

- All answers must be backed by detailed reasoning or proof.

1. Define an infinite sequence of propositonal logic formulae, inductively, as follows:

   Base case: $F_2 = p_1 \Rightarrow p_2$

   Induction step:

   $\forall n \geq 3$,

   $F_n = F_{n-1} \Rightarrow p_n$, if n is odd

   $F_n = p_n \Rightarrow F_{n-1}$, if n is even

   Derive a closed form expression for $\#_n$, the number of satisfying assignments of F_n, as a function of n.

2. Consider a set S with n elements. Let R be a binary relation on S, such that it is symmetric and transitive, but not reflexive. Derive an expression for the maximum number of ordered pairs in R, as a function of n.

3. Let the average degree of a tree be t. Derive a formula for the number of vertices of the tree, as a functionof t.

4. For each of the following conditions, give an example of a degree sequence of length 7:

   (a) All graphs with that degree sequence are connected

   (b) All graphs with that degree sequence are disconnected

   (c) Some graphs with that degree sequence are connected and some are disconnected.

5. Refer to the 10 element array shown in the picture. Suppose you are allowed to move the characters around, with the restriction that any character can either be at its initial position or one position to the left or right, No element can be at distance two or more from its original poition. How many configurations are possible respecting this rule?

a	b	c	d	e	f	g	h	i	j

6. Construct a finite state automaton to accept the language of words over $\Sigma = \{a, b, c\}$ that begin with a and end with b.

7. Describe all 2×2 matrices with real number entries such that The matrix multiplied by itself results in the same matrix.