

Scribed Notes-20

Student I'D

202212096

202212097

202212098 (Absent)

202212099

202212100

Finite Group

- A group $(G, *)$ is called a **Finite** group if G is an **Finite** set
- Eg:- $G=\{1,2,3,4,5,6,7\}$ under multiplication modulo 8 is a finite group as the set G is a finite set

Infinite Group

- A group $(G, *)$ is called a infinite group if G is an infinite set

Eg: group $(I, +)$, where I is set of integers

Cyclic Group

- A cyclic group is a group that can be generated by a single element. Every element of a cyclic group is a power of some specific element which is called a generator. A cyclic group can be generated by a generator 'g', such that every other element of the group can be written as a power of the generator 'g'.

Order of Group:

- The order of the group G is the number of elements in the group G . It is denoted by $|G|$. A group of order 1 has only the identity element, i.e., $(\{e\} *)$.
- A group of order 2 has two elements, i.e., one identity element and one some other element.

Order of an Element

- The order of an element in a group is the smallest positive power of the element which gives you the identity element.

$g \in G$; smallest $k > 0$ | g^k

Proof by contradiction

- If two identity is e_1 and e_2

$$e_1, e_2 \quad e_1 \neq e_2$$

$$e_1 * e_2 = e_2$$

$$e_1 * e_2 = e_1, \text{ so } e_1 = e_2$$

- There is a unique Identity in a group

Suppose for x there exists two inverse y & z

$$x * y = e$$

$$x * z = e$$

$$x * y = x * z$$

$$(y * x) * y = (y * x) * z \quad (\text{applying associativity rule})$$

Every element has unique inverse

Every group equation has unique solution

$$g_1 * x = g_2 \quad \text{but } x * g_1 \neq g_2$$

$$g_1 * x = g_2$$

$$(g_1^{-1} * g_1) * x = g_1^{-1} * g_1$$

Recurrence Relation :

- A Recurrence relation of the sequence $\{a_n\}$ is a question that expresses a_n in term of one or more of the previous term of the sequence, namely, $a_0, a_1, a_2, a_3, \dots, a_{n-1}$ for all integer n with $n \geq n_0$, where n_0 is non negative integer.

Restriction of Function:

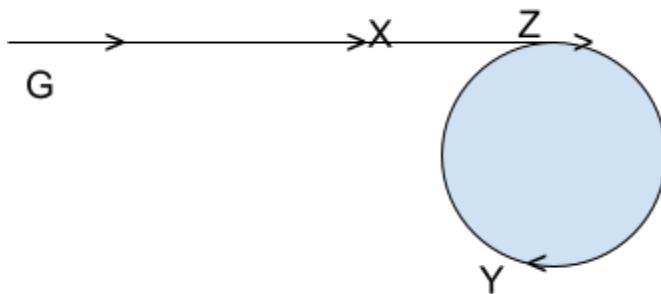
- Let $f: X \rightarrow Y$ be a function from a set X to a set Y . If A is a subset of X , then the restriction of f to A is the function

$f|A:A \rightarrow Y$

$x \mapsto f(x).$

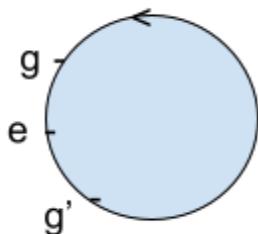
Injective Function

- $f: G^* \rightarrow G$
 $\forall g \in G$
 $F: \{g\}^* \rightarrow G$
- Is an Injective Function F^n
 $g^*x = g^*y$
 $g^*(x^*g^{-1}) = g^*(y^*g^{-1})$ (multiply inverse on both sides)
 $x = y$
 $F: G^*\{g\} \rightarrow G$
- Every row and every column is permutation



$$\begin{aligned} x^*g &= z \\ y^*g &= z \\ x^*g &= y^*g \\ y &= z \end{aligned}$$

- So we will always go back to starting point



Because of this the last element=Identity
Second element=Inverse

Cyclic means that Every element will generate the Whole Group

If an element is Generator then its inverse is also a Generator

If no is prime except 0 all number are generator

Every cyclic group is abelian but not all abelian group is cyclic

$$a^*b = g^{k_1+k_2}$$

$$g = g^k_2 * g^k_1$$

$$= b * a$$

So \forall cyclic are abelian