Transitivity: Its iteration and closure

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In relations, **transitivity** requires:

$$(((a, b) \in R) \wedge ((b, c) \in R)) \Rightarrow ((a, c) \in R)$$

This notion can be iterated. That is it is applied several times. When applying it leads to no further changes, it is called the **transitive closure**. For example the relation $R = \{(a, b), (b, a)\}$ is not transitive. We need to include the pairs (a, a) and (b, b), to render it transitive.

The relation $R = \{(a, b), (b, c), (a, c)\}$ is transitive.

A simple undirected graph can be thought of as a mathematical structure with the following two attributes:

- A finite set V called its set of **vertices** or **nodes**.
- An **edge** set, which is a binary, irreflexive, symmetric relation on the vertex set.

The reachability relation on undirected graphs forms a relation that is

- Reflexive
- Symmetric
- Transitive

A relation of this type (satisfying the above three properties) is called an **equivalence relation**. This is an important class of relatins and we will study them in greater detail.