Discrete Mathematics (SC612)
Insem 1

21^{st} August, 2025

Time: 2 hours
Marks: 20×5

1. Compute the numbr of satisfying assignments of the following formulae:

   (a)

   $$\bigvee_{i=1}^{n-2} \bigvee_{j=i+1}^{n-1} \bigvee_{k=j+1}^{n} (p_i \wedge p_j \wedge p_k)$$

   (b)

   $$\bigwedge_{i=1}^{n-2} \bigwedge_{j=i+1}^{n-1} \bigwedge_{k=j+1}^{n} (p_i \vee p_j \vee p_k)$$

2. (a) Expres the formula ϕ_1 using only $p, q, r. \neg, \oplus$.
   (b) Express the formula ϕ_2 using only $p, q, r, \neg, \Rightarrow$

p	q	r	ϕ_1	ϕ_2
0	0	0	1	1
0	0	1	0	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	0
1	0	1	1	0
1	1	0	1	0
1	1	1	0	0

3. Consider three sets A, B, C such that:

   - $|A| = |B| = |C| = 4$
   - $|A \cap B| = |A \cap C| = |B \cap C| = 2$
   - $|A \cap B \cap C| = 2$

   Calculate $|A \cup B \cup C|$

4. Consider a set A such that:
   $\exists X \subseteq \mathcal{P}(A)$, with $|X| = 125$, and
   $\nexists Y \subseteq \mathcal{P}(A)$ with $|Y| = 140$.
   Calculate the cardinality $|A|$.

5. Express:

   (a) $p \vee q \vee r$
   (b) $p \wedge q \wedge r$

   each using only $\neg$ and $\Rightarrow$ only.