

## **SCRIBED NOTES 25**

**202212121**

**202212122**

**202212123**

**202212124**

### **TODAY'S TOPICS**

- Double counting
- Adjacency matrix
- Incidence matrix
- Degree of vertex
- Handshaking lemma(the first theorem of graph theory)
- Counting by bijection
- Cayley formula
- Unique mapping
- PRÜFER code

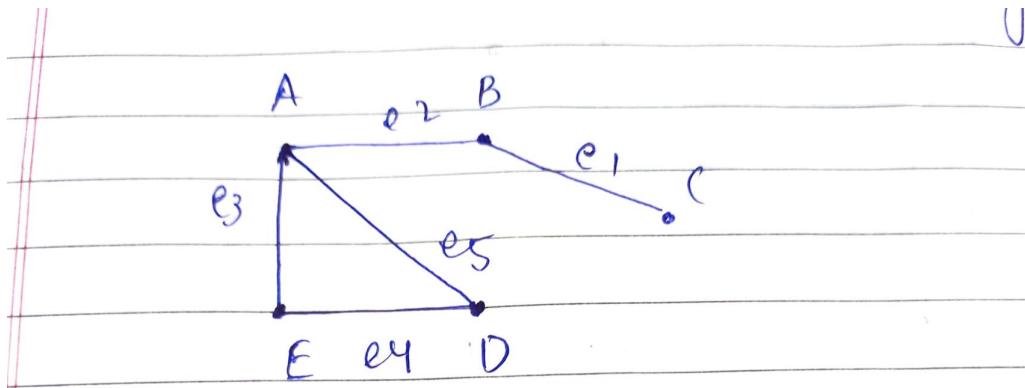
Double counting , symmetry and counting is faster than enumeration .

Adjacency matrix for a graph on n vertices is an  $n \times n$  matrix , where the header row and leading column are labeled by the vertices of the same order and entry  $(i,j)=0$  if there is no edge between  $v_i$  and  $v_j$  and 1 if there is an edge.

$n \times n = n$  number of rows and  $n$  number of columns

## Symmetric matrix for undirected graph

Ex: A graph having 5 vertices and 5 edges



X	A	B	C	D	E
A	0	1	0	1	1
B	1	0	1	0	0
C	0	1	0	0	0
D	1	0	0	0	1
E	1	0	0	1	0

Incidence matrix :  $n \times m$

N = number of vertices

M = number of edges

		EDGES					
		e <sub>1</sub>	e <sub>2</sub>	e <sub>3</sub>	e <sub>4</sub>	e <sub>5</sub>	
A	A	0	1	1	0	1	-3
B	B	1	1	0	0	0	-2
C	C	1	0	0	0	0	-1
D	D	0	0	0	1	1	-2
E	E	0	0	1	1	0	-2

e<sub>1</sub> is the edge between vertices B and C so the B and C would be 1 and rest will be zero same for rest edges

Degree of a vertex is the number of edges incident to it (connected to it).

Number of 1 present in a row is the degree of vertex

For eg: A

Degree of vertex is 3.

For each column number of 1 should be two

**total degree is double of the edge Also Total degree = add degree of all rows And It is always even**

Eg: there are 5 edges so total degree is 10

$$\sum_{v \in V} d(v) = 2|E|$$

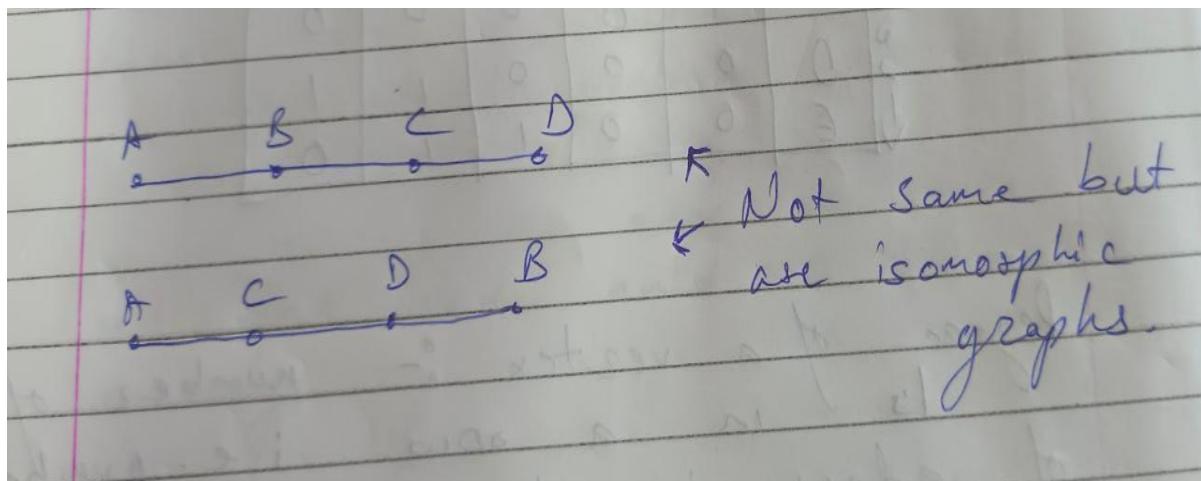
Can't have total degree with odd integer.

It is known as first theorem of graph theory and also known as handshaking lemma.

## COUNTING BY BIJECTION

Cayley formula for number of labeled trees on  $n$  vertices is =  $n^{n-2}$

Isomorphism : Graphs having same structure but different labels.



NOTE : Here we are dealing with unrooted Trees.

Star is a type of graph where we have one vertex at the center and all other vertices are its leaves.

For example if we have tree having 4 vertices,  $4^{4-2} = 16$  i.e. 16 differently labeled trees on 4 vertices.

## PRUFER CODE (Algorithm)

1. List all the leaves.
2. Find lowest index leaf.
3. Delete that leaf.

4. In the string, write down its unique neighbor.
5. Repeat until there are 2 vertices left and the stop.

A tree is a connected acyclic graph.

