

discrete mathematics

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1. Suppose you are given $|\overline{A}| = 70$, $|\overline{B}| = 90$, $|\overline{C}| = 110$, where the complement is with respect to a universal set U of which A, B, C are subsets.
 - (a) Write down the **possible** subset relationships among A, B, C .
 - (b) What is the minimum number of elements in the universal set?
 - (c) What is the maximum number of elements in the universal set U , such that the sets A, B, C are pairwise disjoint.
2. Let ψ be an arbitrary boolean function on three propositional variables (there are 256 of them).
 - (a) Find a boolean function ϕ such that $\psi \oplus \phi = \psi$. The function ϕ should be independent of ψ . That is, the same ϕ works for all choices of ψ .
 - (b) For an arbitrary choice of function, ψ , find a corresponding function ψ' such that $\psi \oplus \psi'$ is equal to the answer obtained in part (a).
 - (c) Find a boolean function α , such that for all possible choices of ψ , $\psi \vee \alpha = \psi$. The function α should be independent of ψ . That is, the same α works for all choices of ψ .
3. Consider the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 - (a) We define a relation R on S , where two numbers are related, if and only if their sum is a prime number (the sum need not belong to the set S). How many ordered pairs does R have?
 - (b) We define a relation R on S , where two numbers are related, if and only if their product is a prime number (the product need not belong to the set S). How many ordered pairs does R have?
4. Suppose there are two players A and B in separate locations who each toss a coin, and cannot see the other's outcome. Suppose A guesses B 's outcome and B guesses A 's outcome using the following rules:
 - A will guess for B the same outcome as A got
 - B will guess for A the opposite outcome as B got.

Consider the following two propositions:

- p : A 's guess is correct
 - q : B 's guess is correct
- (a) Write down the truth table for all possible configurations of p and q (there are four rows) and in the formula column, write 0 if this outcome is not possible in the game, and 1 if this outcome is possible in the game.
- (b) Write this truth table in terms of a syntactic formula using standard logical connectives.