

24th August 2022
Scribed Notes - Lecture 9

Student ID's:

202212041(Absent)

202212042

202212043

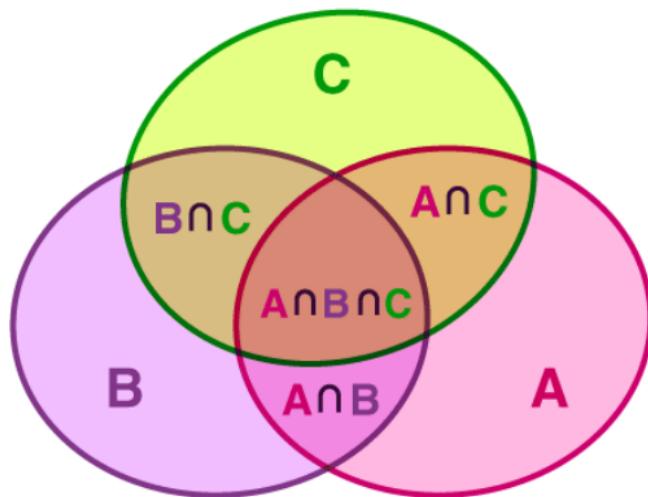
202212044

202212045

Principle of Inclusion and Exclusion

->The Principle of Inclusion and Exclusion, hereafter called Principle of inclusion and Exclusion, gives a formula for the size of the union of n finite sets.

->It is particularly useful in Combinations and Probability problem solving when it is necessary to devise a counting method that ensures an object is not counted twice.



- $|A \cup B| = |A| + |B| - |A \cap B|$
- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

- $|A \cup B \cup C \cup D| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap D| - |A \cap D| - |B \cap D| - |A \cap C| + |A \cap B \cap C| + |A \cap D \cap C| + |A \cap B \cap D| + |D \cap B \cap C| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$

Note: || represent cardinality of sets

Cardinality of subsets can't exceed that of a set

It should not be ambiguous.

What do you mean by Cardinality of set?

The cardinality of a set is the number of elements in a set. Let $A = \{1, 2, 3, 9\}$. The cardinality of set $A = 4$.

PRINCIPLE: Finding union of many sets ensure that every element is counted only once

Assume element i is present in exactly K_i sets $1 \leq K_i \leq n$

Example:

Suppose each student is asked to contribute 1 pen per club and he is a part of 5 clubs, initially, he will contribute 5 pens($5C_1$)

Iteration 1: $\rightarrow 5C_1 (+5)$

Iteration 2: $\leftarrow 5C_2 (-10)$

Iteration 3: $\rightarrow 5C_3 (+10)$

Iteration 4: $\leftarrow 5C_4 (-5)$

Iteration 5: $\rightarrow 5C_5 (1)$

At the end, he will end up contributing just 1 pen.

Formula For Inclusion and Exclusion Principle:

$$\left| \bigcup_{i=1}^n A_i \right| = \sum_{j=1}^n \sum_{\substack{T \subseteq S \\ |T|=j}} (-1)^{j+1} \binom{n}{e_T}$$