

Scribed Notes-19

Student I'D

202212091 (Absent)

202212092

202212093

202212094

202212095

COMPOSITION OF FUNCTIONS AND PERMUTATIONS

Compositions of Functions

Function composition is an operation that takes two functions f and g and produces a function h such that $h(x) = g(f(x))$. In this operation, the function g is applied to the result of applying the function f to x . That is, the functions $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are composed to yield a function that maps x in X to $g(f(x))$ in Z .

Function composition is an associative. If $f(x)=y$ then, $y=f(x)$

Range of the function is subset of second function's domain.

Intuitively, if z is a function of y , and y is a function of x , then z is a function of x . The resulting composite function is denoted $g \circ f : X \rightarrow Z$, defined by $(g \circ f)(x) = g(f(x))$ for all x in X .

→ The composition of two permutations is also a permutation.

Composition of operation on set of permutations from a group.

Set of all permutation under function composition operation form a non-abelian group.

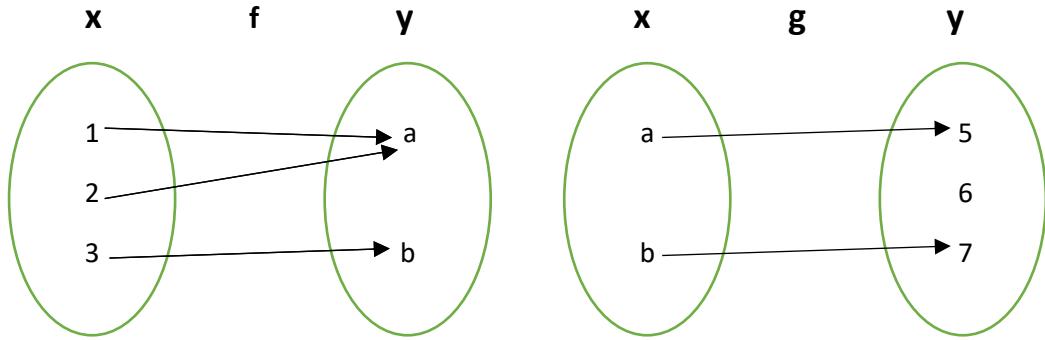
For example,

$f(x)=2x+3$ and $g(x)=2x$ on domain=codomain= All positive integers, then, for $f(g(2))$ firstly we will evaluate $g(2) = 4$ and then replace value of $g(2)$ in $f(g(2))$

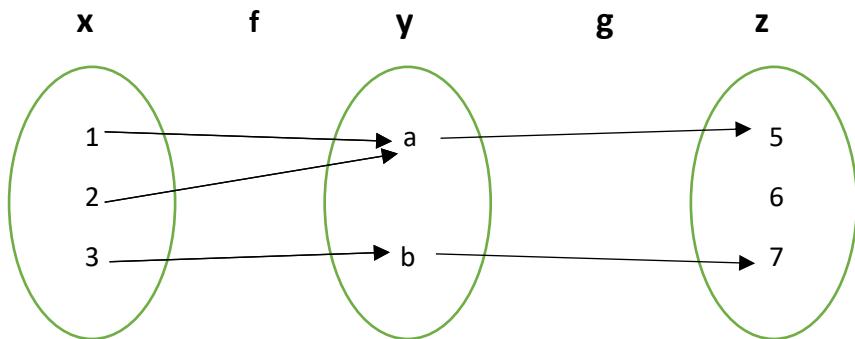
Consider functions, $f: A \rightarrow B$ and $g: B \rightarrow C$. The composition of f with g is a function from A into C defined by $(gof)(x) = g[f(x)]$ and is defined by gof .

To find the composition of f and g , first find the image of x under f and then find the image of $f(x)$ under g .

Eg: Consider the function $f = \{(1, a), (2, a), (3, b)\}$ and $g = \{(a, 5), (b, 7)\}$ as in figure. Find the composition of gof .



Solution: The composition function gof is :



→ Order of permutation

Order of permutation is to reach the identity permutation

$$\left(\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 1 & 6 & 7 \end{array} \right)$$

$$\left(\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 3 & 6 & 5 & 2 & 7 \end{array} \right)$$

Permutation is called cycle if there is only one non trivial orbit.

- **Cycle of Permutation: 3**

$$\begin{array}{ll} 3 \rightarrow 5 & 2 \rightarrow 4 \\ 5 \rightarrow 1 & 4 \rightarrow 6 \\ 1 \rightarrow 3 & 6 \rightarrow 2 \end{array}$$

- Every group can be linked to a permutation group
- Not all cyclic group are abelian group but all abelian group are cyclic group.

x 10	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	7