17th November 2022
Scribed Notes

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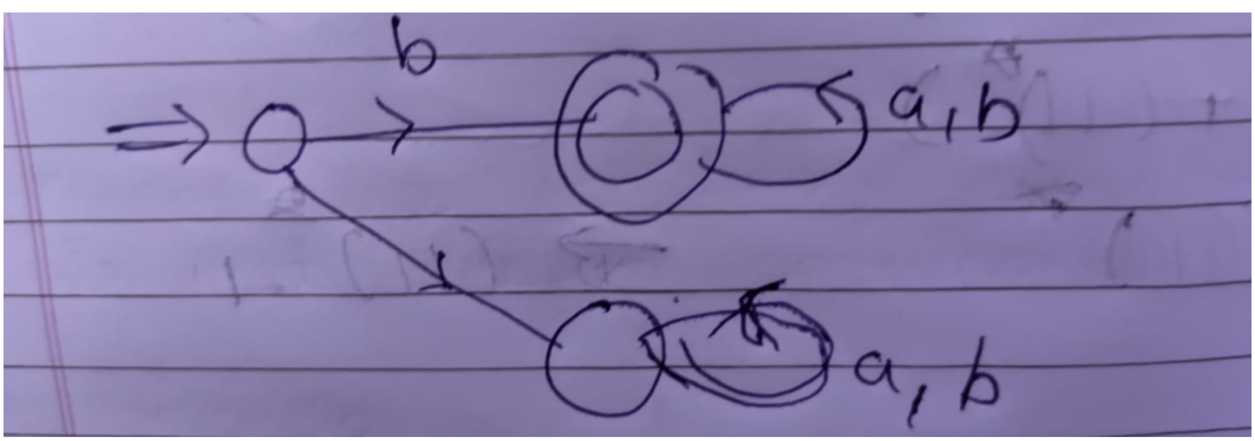
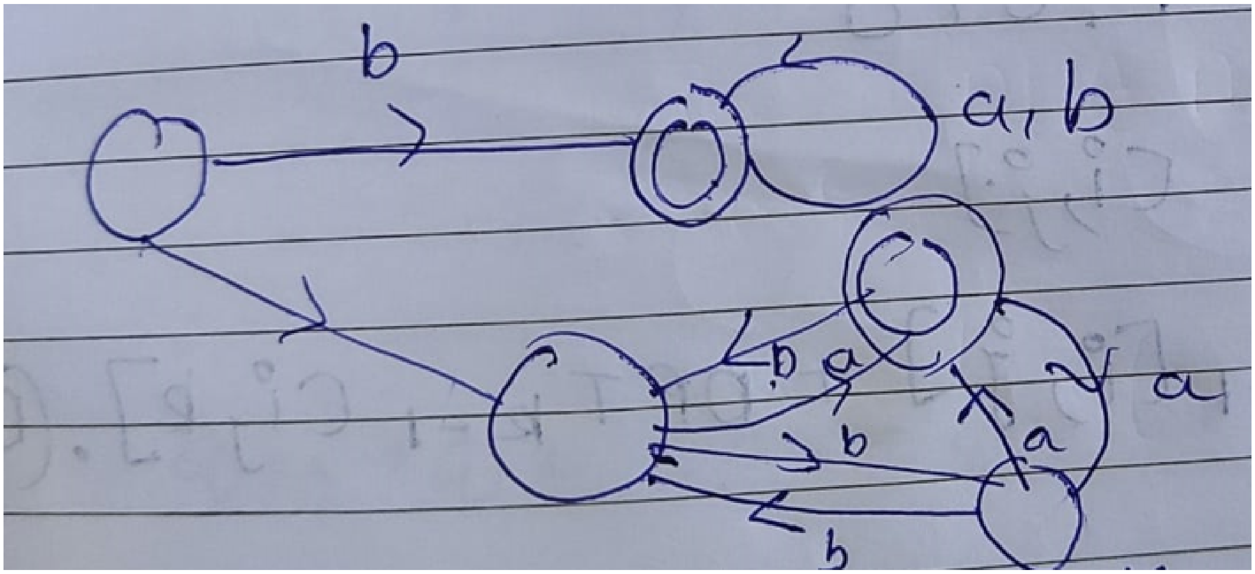
Pumping Lemma

->It gives a method for pumping (generating) many substrings from a given string.

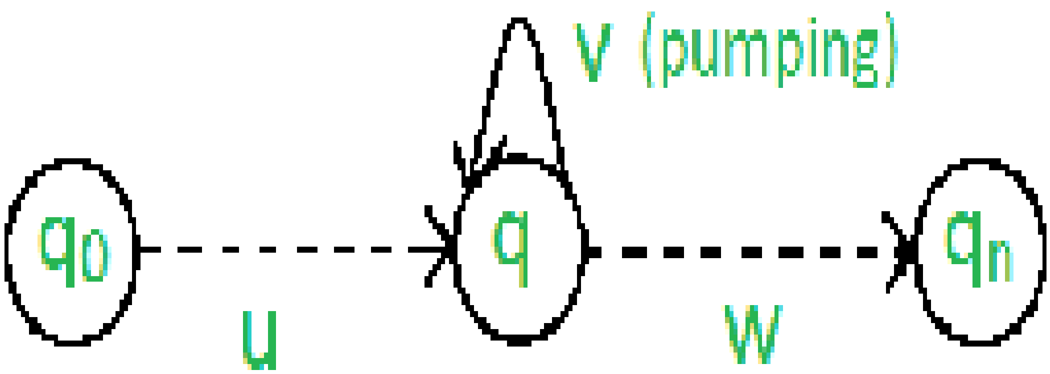
->Automata must have one language 2 or more than 2 automata can have some language but 2 or more than 2 languages can't have the same automata.

->It is used to show that a formal language is not regular.





For any regular language L, there exists an integer n, such that for all x ∈ L with |x| ≥ n, there exists u, v, w ∈ Σ*, such that x = uvw, and (1) |uv| ≤ n (2) |v| ≥ 1 (3) for all i ≥ 0: uviw ∈ L   In simple terms, this means that if a string v is 'pumped



Application of pumping lemma

Pumping lemma is to be applied to show that certain languages are not regular.

It should never be used to show a language is regular.

- If L is regular, it satisfies the Pumping lemma.
- If L does not satisfy the Pumping Lemma, it is not regular.

Idea of Proof By Contradiction

- Start with some assumptions
- A language which is regular has a contradiciton
- Given an automation then it is ambiguous
  But,
  One Automation one language

  One finite automation can not map more than one language

  There exist automation

Proof By Contradiction

1>Assume L is regular
2>Infer that their exist a DFA n(L) with k state ,$k \in N$
3>Pick a suitable word of length k (or more)
4>$w \in L, |w| \geq k$

Generate a false positive

$W=W_1,W_2,W_3$
W_1,W_2
W_1,W_2,W_2,W_3
W_1,W_2,W_2,W_2,W_2,W_3

->A language over A,B

$L=\{a^{k.}b^k\},k \in N \cup \{0\}\}$

->In starting state we have self loop so no. of a and no of b are not equal.

->If language is regular then we have to create automation.if language is not regular then we have to go for pumping lemma.

->If there exist a loop in our FSA (i.e. some states repeat) then we can generate infinite no. of strings by pumping thus making it the fraudulent string that is accepted by automata and hence proving it to be not regular.

Examples of nonregularity proved through the pumping lemma
***Let us illustrate this method for a few example languages. These examples will be**
stated as propositions, with the proofs showing you how the argument works
Example 1:

Let $\Sigma = \{0, 1\}$ be the binary alphabet and define a language over Σ as follows:
SAME $=\{0^M\ 1^m : m \in N\}$

The language SAME is not regular

Example 2:

Use pumping lemma to show that the following language is not regular
$A=\{www|\ W\in\{0,1\}^*\}$
Proof:
1>Take p be the number from pumping lemma and consider
s=www,where $w=0^{p-1}\ 1$.
Hence $|s|=3p>p$

2>Let x,y,z be such that s=xyz ,$|y|>0$, i.e. $y\neq\in$, and $|xy|\leq p$.

3>That means that $y=0^k$,or $y=0^k1$ where $1\leq k\leq p-1$.

4>Now we have either $xy^oz=xy^0tww=^{p-k-1}\ 1ww$,
or $xy^0z=xy^0tww=0^{p-k-1}ww$,but in both cases and $xy^0z\neq www$,
So $xy^0z\in A$!

5>Hence the pumping lemma is not satisfied i.e. A is not regular.

Example 3

$L = \{ 0\ 1 \mid n \geq 0\}$ is not regular
Suppose L were regular. Then let p be the pumping length given by the pumping lemma

Let $s = 0^p\ 1^p$ in L. p p Note that $|s| > p$, so $s = xyz$ with

1> for each $i \geq$ **0,** $xy^i z$ is in L
2> $|y|>0$
3> $|xy| \leq p$

It must be the case that y=0….0,since xy is shorter than p
But then xyyz will have more 0’s than 1’s so it cannot be in L,a contradiction