So far we have,

Type 1 : $\text{Const.}\ n^2$

Type 2 : $T(n) = 3T(\frac{n}{2}) + \text{const.}\ n$

Q1: How to simplify Type 2 like equations / inequalities

Q2: How to compare the running time of two algorithms

To answer we need some terminologies.

Asymptotic notations

Five notations

Big oh 'O'

Small oh 'o'

Big omega Ω

Small omega ω

Theta Θ

Big oh notation 'O' upper bound

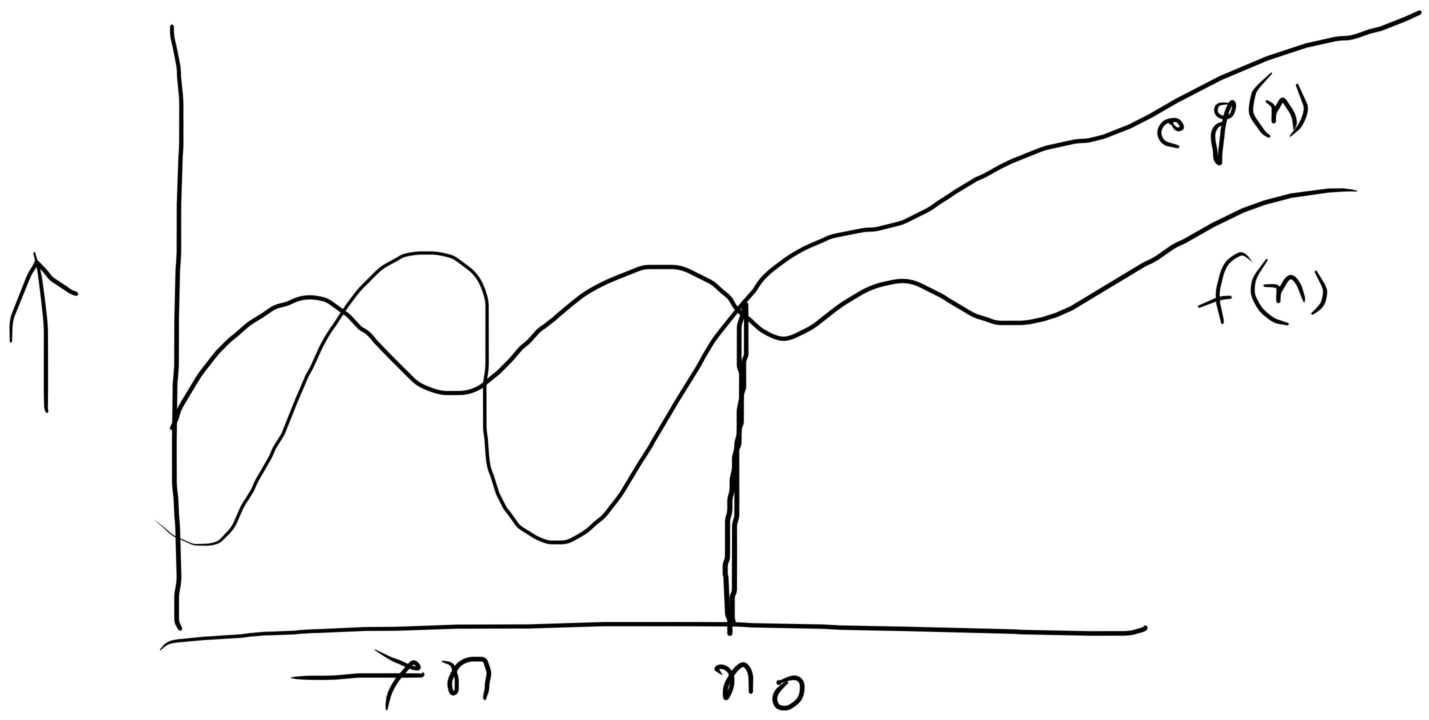
Let $f(n)$ is defined on $1, 2, 3, \ldots$

$f(n) = O(g(n))$ if there exists constants $c > 0$ and $n_0 > 0$ (c, n_0 do not depend on n) such that

$f(n)$ is $O(g(n))$

$f(n) \in O(g(n))$

$$0 \leq f(n) \leq c\,g(n) \quad \forall \; n \geq n_0$$



Ex^m Is $2n^2 = O(n^3)$

$\boxed{\text{Idea: fix } n_0 \text{ then based on this } n_0 \text{ find a suitable } c.}$

find a pair (c, n_0) s.t.

$$2n^2 \leq c \cdot n^3 \quad \forall\ n \geq n_0$$

Assume $n_0 = 2$ then $c \geq 1$

Assume $n_0 = 1$ then $c \geq 2$

$\underline{\underline{Ex^m}}$ Is $f(n) = O(n^2)$ where $f(n) = n^2+2n+1$

$f(n) = n^2+2n+1$

$\leq n^2+2n.n+n^2$

$= 4n^2$

$n \geq 0$

$1 \leq n^2$

$2n \leq 2n^2$

Find (c, n_0) such that

$4n^2 \leq cn^2 \quad \forall n \geq n_0$

Assume $n_0 = 1$ then $c \geq 4$

Ex^n S.T.S. $5n^3 + 2n^2 + 4n + 3 = O(n^5)$

$n_0 = 1, \quad c \geq 14$

$\underline{\underline{Ex^m}}$ Is $n^2 = O(n)$?

find a (c, n_0) s.t.

$$n^2 \le cn \quad \forall n \ge n_0$$

$$\Rightarrow n \le c$$

no matter what c and n_0 can be taken the inequality $n \le c$ can not hold for all $n \ge n_0$

Inparticular when n is larger than $\max\{c, n_0\}$

$\boxed{n^2 = O(n) \text{ is not true}}$

Exm

Is $7n^3+5n^2+2n+3 \doteq O(n^2)$

H.W

Result: $f(n) = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$

where a_i's are real numbers

Then $f(n) = O(n^k)$

Result Find the Big oh estimate of summation of first n positive integers.

$$f(n) = 1+2+\cdots+n = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$O(n^2)$$

HW

First n odd numbers

" even "

" " prime numbers

Result Big oh estimate of $n!$

$$f(n) = n! \approx n^n$$

$$n! = O(n^n)$$

Take log both sides

$$\log(n!) = O(n \log n)$$