Discrete Mathematics (SC612)
Tutorial 11
6^{th} November, 2025

1. Evaluate:
   (a) $5^{984861} \pmod{17}$
   (b) $7^{231987} \pmod{12}$
2. Solve the system of simultaneous congruences given below, for x.

$$x \cong 4 \pmod{7}$$
$$x \cong 2 \pmod{3}$$
$$x \cong 3 \pmod{4}$$
$$x \cong 4 \pmod{5}$$
$$x \cong 9 \pmod{11}$$

3. Construct two 2×2 matrices A and B such that $A \neq B$, but $AC = BC$ where $C \neq 0$, the all 0's matrix.
4. Construct a 2×2 matrix M such that its multiplication with the 2×1 vector with entries 1,4 results in the vector 1,4 itself.

5. (a) Convert 2148 in decimal system to:
    i. Base 2 (binary)
    ii. Base 3 (Ternary)
    iii. Base 7
    iv. Base 8 (Octal)
   (b) Which decimal system integer is represented by the base 7 number 632?
   (c) Which decimal system integer is represented by the base 8 (octal) number 577
   (d) Which decimal system number is represented by the base 3 (ternary) number 21022
   (e) Which decimal system number is represented by the base 2 (binary) number 10101110?

6. (a) Suppose the integers 1 to 10 are randomly written around a circular wheel, show that there will be three successive positions such that the sum of the numbers written there is at least 17.
   (b) If you were to generalise this to integers 1 to n around a wheel, what number would replace 17 as a guaranteed lower bound on maximum value of total of three successive positions?

7. Suppose for positive integers a, b, c, we are told that $a+b+c+ab+bc+ac$ is odd. Is $a + b + c$ even or odd or can take either value?