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Scribed Lecture 6 Notes :-

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Identity Laws :-

In mathematics, an *identity* is a statement true for all possible values of its variables. The algebraic identity of $x + 0 = x$ tells us that anything (x) added to zero equals the original “anything” no matter what value that “anything” (x) may be.

- i) $p \wedge T \equiv p$
- ii) $p \vee F \equiv p$

Domination Laws :-

The complement is used in the operations to form these laws. The idea behind these laws is that if the first number is 1 then the negation of 1 is 0.

- iii) $p \wedge F \equiv F$
- iv) $p \vee T \equiv T$

Idempotent laws:-

Idempotence is the property of certain operations in mathematics and computer science that they can be applied multiple times without changing the result beyond the initial application

- v) $p \vee p \equiv p$
- vi) $p \wedge p \equiv p$

Double Negation :-

In propositional logic, double negation is the theorem that states that "If a statement is true, then it is not the case that the statement is not true." This is expressed by saying that a proposition A is logically equivalent to not (not-A), or by the formula $A \equiv \sim(\sim A)$ where the sign \equiv expresses logical equivalence and the sign \sim expresses negation.

- vii) $\neg\neg p \equiv p$

Associative Laws:-

$$\begin{aligned} \text{viii)} ((p \vee q) \vee r) &\equiv (p \vee (q \vee r)) \\ \text{ix)} ((p \wedge q) \wedge r) &\equiv (p \wedge (q \wedge r)) \end{aligned}$$

Commutative Laws :-

$$\begin{aligned} \text{x)} p \wedge q &\equiv q \wedge p \\ \text{xi)} p \vee q &\equiv q \vee p \end{aligned}$$

Distributive Laws:-

$$\begin{aligned} \text{xii)} p \wedge (q \vee r) &\equiv (p \wedge q) \vee (p \wedge r) \\ \text{xiii)} p \vee (q \wedge r) &\equiv (p \vee q) \wedge (p \vee r) \end{aligned}$$

Negation Laws :-

If either A or B were true, then the disjunction of A and B would be true, making its negation false. Presented in English, this follows the logic that "since two things are both false, it is also false that either of them is true".

$$\begin{aligned} \text{xiv)} p \wedge (\neg p) &\equiv F \text{(called contradiction)} \\ \text{xv)} p \vee (\neg p) &\equiv T \text{ (called tautology/validity)} \end{aligned}$$

De-Morgan's Laws :-

$$\begin{aligned} \text{xvi)} \neg(p \vee q) &\equiv (\neg p) \wedge (\neg q) \\ \text{xvii)} \neg(p \wedge q) &\equiv (\neg p) \vee (\neg q) \end{aligned}$$

Satisfiable:

Boolean Satisfiability or simply SAT is the problem of determining if a Boolean formula is satisfiable or unsatisfiable.

Satisfiable : If the Boolean variables can be assigned values such that the formula turns out to be TRUE, then we say that the formula is satisfiable.

At least one assignment evaluates to true.

Contradiction:

The opposite of a tautology is a contradiction, a formula which is "always false". In other words, a contradiction is false for every assignment of truth values to its simple components.

All evaluate to
False => F

Validity / Tatology:

A tautology is a compound statement which is true for every value of the individual statements. The word tautology is derived from a Greek word where ‘tauto’ means ‘same’ and ‘logy’ means ‘logic’. A compound statement is made with two more simple statements by using some conditional words such as ‘and’, ‘or’, ‘not’, ‘if’, ‘then’, and ‘if and only if’. For example for any two given statements such as x and y , $(x \Rightarrow y) \vee (y \Rightarrow x)$ is a tautology.

A formula in propositional logic in which all evaluations are true.

Absorption laws:-

This law enables a reduction in a complicated expression to a simpler one by absorbing like terms.

$$p \wedge (p \vee q) \equiv p$$

Theorem is something which is always true. If it includes 0, it's not a theorem.

- **Inference Rules:**

P => Q , ~q } Premise
 ~p } Conclusion

$$\frac{P \wedge Q}{Q}$$

$$\frac{P \vee Q}{P}$$