

10th August 2022  
**Scribed Notes - Lecture 5**

**Student Ids :-**

**202212021**

**202212022**

**202212023 (Absent)**

**202212024**

**202212025**

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### **Stability of Boolean functions**

#### **AND**

Unstable if the result evaluates to 1.

If the result evaluates to 1, then it implies a rigid constraint, that all the variables are 1. Hence, change in assignment of any of the variables, will change the result of the expression.

If,  $\mathbf{x_1 \wedge x_2 \wedge x_3 \wedge .... x_k = 1}$ ,  
then, all variables are assigned to 1.

#### **OR**

Unstable if the result evaluates to 0.

If the result evaluates to 0, then it implies a rigid constraint, that all the variables are 0. Hence, change in assignment of any of the variables, will change the result of the expression.

If,  $\mathbf{x_1 \vee x_2 \vee x_3 \vee .... x_k = 0}$ ,  
then, all variables are assigned to 0.

## XOR

Xor function is always unstable, because a change in any of the variables will lead to a change in the result of the expression.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

We can observe from the above XOR truth table that a change in any of the assignments will always change the result.

## IMPLIES

Stability of implies is ambiguous because change in assignment of any variable may or may not change the result. But, in case when the result is 0, it is always unstable.

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

When the result is 0, it will imply a rigid constraint that **p** must be 1, and **q** must be 0. Hence, changing any of the values will change the result.

## Structural inductive definition of binary tree:

There are **2 phases of structural induction**:

1. Base case
2. Induction step

**Formula for finding number of tree can be made by given nodes:**

$$1/(n+1) \binom{2n}{n}.$$

Example if number of nodes - 3

$$=1/(3+1) \binom{6}{3}$$

$$=1/4 \cdot 20$$

$$=5$$

So, 5 numbers of tree

Every truth table (Boolean function) can be represented using conjunctive normal form (CNF) or a disjunctive normal form (DNF)

- CNF is  $\wedge$  of  $\vee$
- DNF is  $\vee$  of  $\wedge$

### From truth table to DNF:

Here, we can select the rows (minterms) where it evaluates to true (1) and form disjunction of those minterms.

Let us consider table with following truth values:

p	q	r	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

F is true for four assignments:

- Row 2: p=0, q=0, r=1, F=1  
 $(\neg p \wedge \neg q \wedge r)$  evaluates to 1
- Row 4: p=0, q=1, r=1, F=1  
 $(\neg p \wedge q \wedge r)$  evaluates to 1
- Row 6: p=1, q=0, r=1, F=1  
 $(p \wedge \neg q \wedge r)$  evaluates to 1
- Row 8: p=1, q=1, r=1, F=1

$(p \wedge q \wedge r)$  evaluates to 1

DNF:  $((\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r))$

### From truth table to CNF:

Here, we can select the rows (minterms) where it evaluates to 0 and form a conjunction of those minterms.

p	q	r	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

F is evaluates to zero four assignments:

- Row 1:  $p=0, q=0, r=0, F=0$

$(p \vee q \vee r)$  evaluates to 0

- Row 3:  $p=0, q=1, r=0, F=0$

$(p \vee \neg q \vee r)$  evaluates to 0

- Row 5:  $p=1, q=0, r=0, F=0$

$(\neg p \vee q \vee r)$  evaluates to 0

- Row 7:  $p=1, q=1, r=0, F=0$

$(\neg p \vee \neg q \vee r)$  evaluates to 0

CNF:  $((p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee r))$