

# Discrete Mathematics

## Tutorial 6

September 18<sup>th</sup>, 2025

1. How many times does a permutation need to be iterated in order to get the identity permutation?
2. Consider the set  $\mathcal{F}$  of all functions  $f : D \rightarrow C$ , where  $|D| = m$  and  $|C| = n$ . Show that  $|\mathcal{F}|$  is exactly the number of ways to partition  $[m]$  into  $n$  parts (here some parts are allowed to be empty).
3. For any integer  $i > 0$ , find the smallest  $n$ , such that there exists a permutation  $\mathcal{P}$  which has to be iterated  $i$  times to get the identity permutation, for the first time.
4. Show that the number of permutations of  $\mathcal{N}$  is uncountably infinite.
5. Count the number of injective, bijective and surjective functions on a set  $S$ , of finite cardinality  $n$ .
6. (a) Consider two function  $f_1 : D_1 \rightarrow C_1$  and  $f_2 : D_2 \rightarrow C_2$ . Give conditions on the sets  $D_1, C_1, D_2, C_2$  for the composition  $g(f())$  to be defined.  
(b) Consider composing two functions where the first function is injective and the second is surjective. What can you say about the composed function?  
(c) Consider composing two functions where the first function is surjective and the second is injective. What can you say about the composed function?