Cartesian Product

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We looked at the operation of **set difference** in the context of sets. Given two sets A and B, the set difference:

$$A \setminus B = \{x | (x \in A) \wedge \neg(x \in B)\}$$

Set difference is a **non-commutative** operation.

The **symmetric difference** of two sets is the union of their set differences. This is denoted by the symbol Δ, usually. Thus $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

It is easy to show that $A \Delta B = \{(x \in A) \oplus (x \in B)\}$.

The **power set** of a set, is a set of all its subsets. Thus if a set is A then its power set is $\mathcal{P}(A)$. It is also often denoted by 2^A. The number of elements in the power set of n elements is 2^n. Example:

$S = \{1, 2, 3\}$

$2^S = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

The **Cartesian Product** of a set S to a set T is:

$$S \times T = \{(x, y) | x \in S, y \in T\}$$

The result of the cartesian product of two sets is again a set. Thus one can apply the cartesian product again using that new set. The cartesian product is a non-commutative operation.

There is no condition on the two sets between which we are applying the cartesian product operation. The cardinality of the result is:

$$|S \times T| = |S|.|T|$$

It is allowed for the two sets to be identical. Thus we can have $S \times S$. If S is a set on n elements then the cartesian product has n^2 elements.

A **relation** is any subset of the cartesian product. Thus, if the sets are distinct, we talk of a relation from S to T. If the two sets are identical then we call it a relation on S.

In general if we define a relation, which is a subset of the cartesian product of two (not necessarily distinct) sets, the first set is called the **domain** of the function and the second set is called the **codomain** of the function.