SC612 Discrete Mathematics Insem

September 2021

Instructions pertaining to the exam

- The exam is closed book and closed notes.
- There are 25 questions, all of them multiple choice.
- There is only one correct option for each question.
- 4 marks are to be awarded for a correct answer and -1 marks for a wrong answer.
- Randomisation of the options is requested, and additionally, if possible, randomisation of the order of the questions themselves.
- Requesting evaluation and totalling be done.
- The correct answer for each question is provided by writing in brackets (correct answer).

This page is for the use of Mettl only, and the actual eam questions begin on the next page. Please erase the highlighted correct answer when rendering the questions for the platform.

1. In propositional logic, we say that a formula ϕ_2 can be inferred from a formula ϕ_1 if for every assignment that makes ϕ_2 true also makes ϕ_1 true. The number of semantically distinct propositional logic formulae (including ψ itself) over the three variables p, q, r, that can be inferred asssuming the formula $\psi = (p \Rightarrow (q \Rightarrow r))$ is:

   (a) 0

   (b) 1

   (c) 2^7 (correct answer)

   (d) 7

2. Let ψ be a formula over variables p_1, p_2, p_3, p_4 and let $\phi = (p_1 \Rightarrow \psi)$. The number of satisfying assignments of this formula ϕ cannot be:

   (a) 6 (correct answer)

   (b) 8

   (c) 10

   (d) 12

3. Let ψ be a formula over variables p_1, p_2, p_3, p_4 and let $\phi = (p_1 \vee \psi)$. The number of satisfying assignments of this formula ϕ cannot be:

   (a) 6 (correct answer)

   (b) 8

   (c) 10

   (d) 12

4. Let ψ be a formula over variables p_1, p_2, p_3, p_4 and let $\phi = (p_1 \wedge \psi)$. The number of satisfying assignments of this formula ϕ can be:

   (a) 6 (correct answer)

   (b) 10

   (c) 12

   (d) 14

5. Let ϕ be a propositional logic formula on 6 variables $p_1, \ldots, p_6$. Suppose ϕ is such that every time we change an assignment by altering the truth of just one propositional variable, the truth value of the formula changes. Given this information, which among the below assignment of truth values makes ϕ evaluate to a different value from the other three?

p_1	p_2	p_3	p_4	p_5	p_6
0	0	1	1	1	1

(a)

p_1	p_2	p_3	p_4	p_5	p_6
1	1	1	1	1	1

(b)

p_1	p_2	p_3	p_4	p_5	p_6
0	0	1	0	0	1

(c)

(d)

p_1	p_2	p_3	p_4	p_5	p_6
0	0	1	0	0	1

6. The boolean function $\phi = (\neg p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge \neg p_3) \vee (p_1 \wedge p_2 \wedge p_3)$ is equivalent to:

   (a) $(p_1 \vee p_2 \vee \neg p_3) \wedge (p_1 \vee \neg p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee \neg p_3) \wedge (\neg p_1 \vee \neg p_2 \vee p_3)$

   (b) $(p_1 \vee p_2 \vee p_3) \wedge (p_1 \vee \neg p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee \neg p_3) \wedge (\neg p_1 \vee \neg p_2 \vee p_3)$

   (c) $(p_1 \vee p_2 \vee \neg p_3) \wedge (p_1 \vee p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee \neg p_3) \wedge (\neg p_1 \vee \neg p_2 \vee p_3)$

   (d) $(p_1 \vee p_2 \vee \neg p_3) \wedge (p_1 \vee \neg p_2 \vee p_3) \wedge (\neg p_1 \vee p_2 \vee \neg p_3) \wedge (p_1 \vee p_2 \vee p_3)$

7. Let A and B be two finite sets. Suppose A has an element x such that $x \notin B$. Then:

   (a) $|A| > |B|$

   (b) $|A| < |B|$

   (c) $|A| = |B|$

   (d) We cannot determine the relative values of $|A|$ and $|B|$ from the information given

8. Which of the following formulae has more thas one satisfying assignment:

   (a) $p_1 \wedge p_2 \wedge p_3$

   (b) $\neg(p_1 \vee p_2 \vee p_3)$

   (c) $(p_1 \vee p_2 \vee p_3) \Rightarrow (\neg p_1 \wedge \neg p_2 \wedge \neg p_3)$

   (d) $(p_1 \Rightarrow (p_2 \Rightarrow p_3))$ (correct answer)

9. The number of assignments of truth values on which the formulae $\phi_1 = ((p_1 \Rightarrow p_2) \Rightarrow p_3)$ and $\phi_2 = (p_1 \Rightarrow (p_2 \Rightarrow p_3))$ evaluate to the same truth value is:

   (a) 3

   (b) 4

   (c) 5

   (d) 6