Discrete Mathematics
Tutorial 1

31^t July, 2025

1. (a) Show that the formula $\phi_n = p_1 \oplus \cdots \oplus p_n$ is associative and commutative, in the sense that one can rearrange the propositions as well as bracket arbitrarily, without altering the final answer.

   (b) Show that the formula is true if and only if the number of propositions getting the truth value T is an odd number.

2. Consider the formulae

   - $\phi_1 = p \vee q \vee r$
   - $\phi_2 = p \wedge q \wedge r$
   - $\phi_3 = p \oplus q \oplus r$
   - $\phi_4 = (p \Rightarrow q) \Rightarrow r$
   - $\phi_5 = p \Rightarrow (q \Rightarrow r)$
   - $\phi_6 = (p \vee q) \wedge r$
   - $\phi_7 = p \vee (q \wedge r)$

   (a) Arrange them in non-decreasing order of satisfying assignments.

   (b) Determine which of them imply the other.

3. Show that any formula in propositional logic can be expressed using only the logical connectives $\Rightarrow$ and $\neg$.

4. (a) Show that all formulae in propositional logic cannot be written only in terms of $\oplus$ and $\neg$.

   (b) Describe, precisely, which formulae in propositional logic can be written using only these two connectives.