Functions & Groups

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A **function** from domain D to codomain C is a special type of relation. The extra condition is that each element of the domain appears exactly once as the first coordinate of an ordered pair.

The number of functions is n^m where $|D| = m$ and $|C| = n$.

A standard notation for the set of all functions from domain D to codomain C is C^D.

A group is a special type of ternary relation on a se. It can also be viewed as a function from $G \times G \to G$.

The basic definition of a group has an underlying set G together with a binary operaion $*$, and these must satisfy the following four axions:

1. **Closure:** $\forall g_1, g_2 \in G, g_1 * g_2 \in G$.
2. **Identity**: $\exists e \in G, \forall g \in G, g * e = e * g = g$.
3. **Inverses**: $\forall g \in G, \exists g' | g * g' = g' * g = e$.
4. **Associativity**: $\forall g_1, g_2, g_3 \in G, (g_1 * g_2) * g_3 = g_1(g_2 * g_3)$.

Extra condition for **Abelian groups**: $\forall g_1, g_2 \in G, g_1 * g_2 = g_2 * g_1$.

Exampls of groups:

1. All Boolean functions over k variables, under the $\oplus$ operation.
2. Integers modulo 5 under addition
3. Integers under addition.

All these are Abelian groups. We will see non-Abelian groups next lecture.

A finite group may be represented as a matrix of $|G| \times |G|$ dimensions. The rows and columns are labelled with the elements and the entry (i, j) is $g_i * g_j$.