22nd September 2022 (Thursday)
Scribed Notes - Lecture 18

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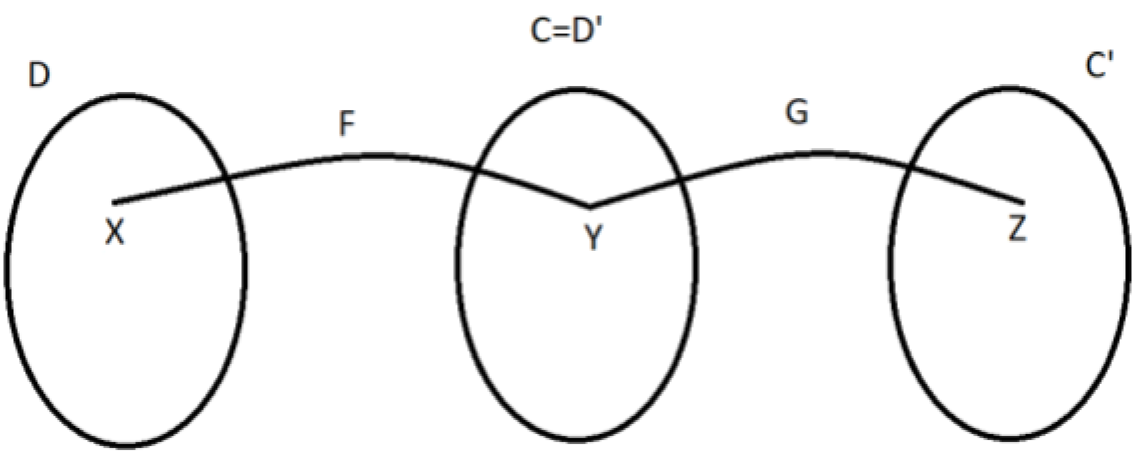
Function composition:

Function composition is an operation that takes two functions f and g and produces a
function h such that h(x) = g(f(x)). In this operation, the function g is applied to the
result of applying the function f to x. That is, the functions f : X → Y and g : Y → Z are
composed to yield a function that maps x in X to g(f(x)) in Z.



Example:

f(x) = 2^{x}

g(x) = x^{3}

f(g(x)) = $2^{x^{3}}$

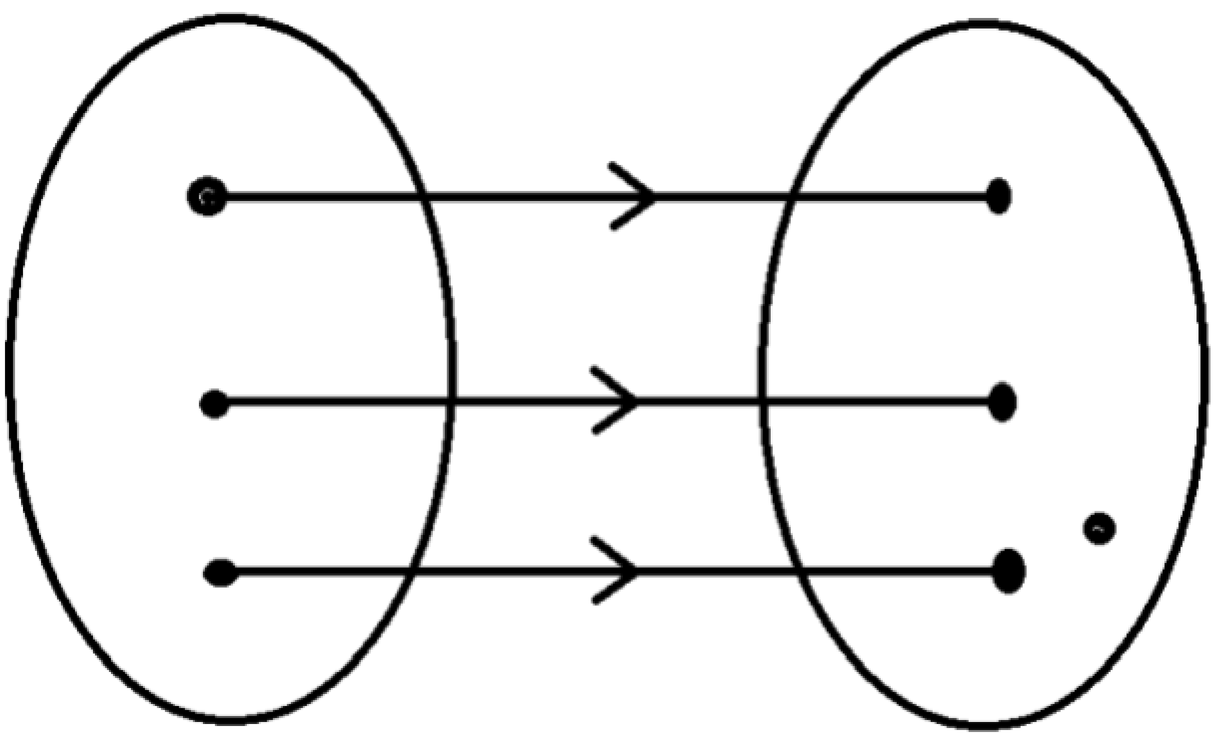
g(f(x)) = 2^{3x}

X = 3
g(f(x)) = 512
f(g(x)) = 512^{3}

So, f(g(x) ≠ g(f(x))
Function composition is not commutative.

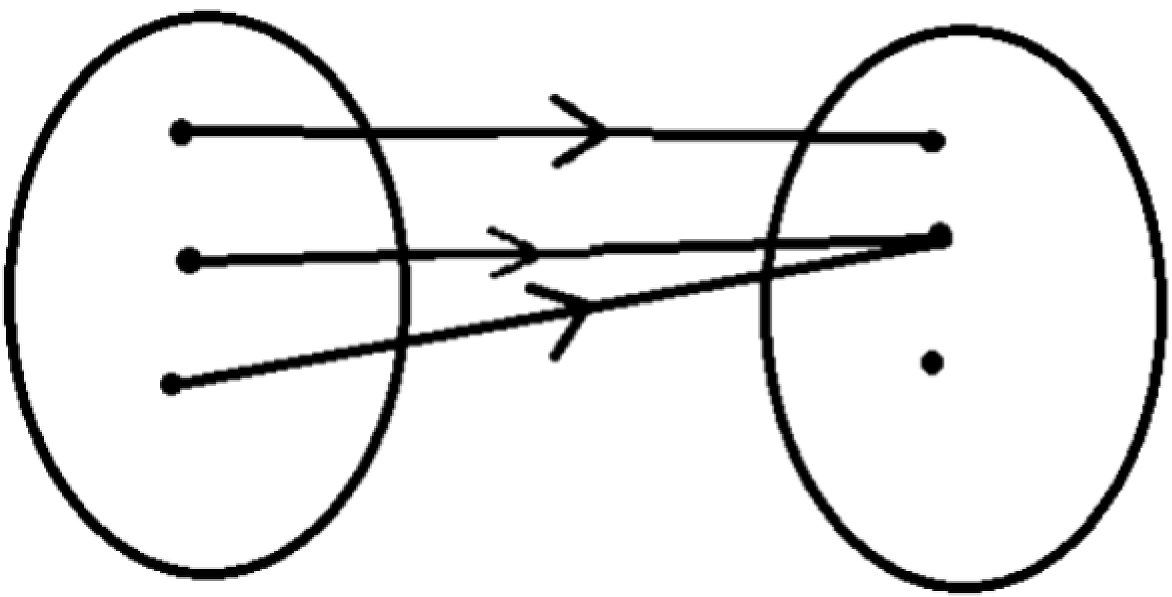
Types of functions:

1. **Injective function:**
   If every point in a domain has a distinct image then it is called an injective function. It is one - one.

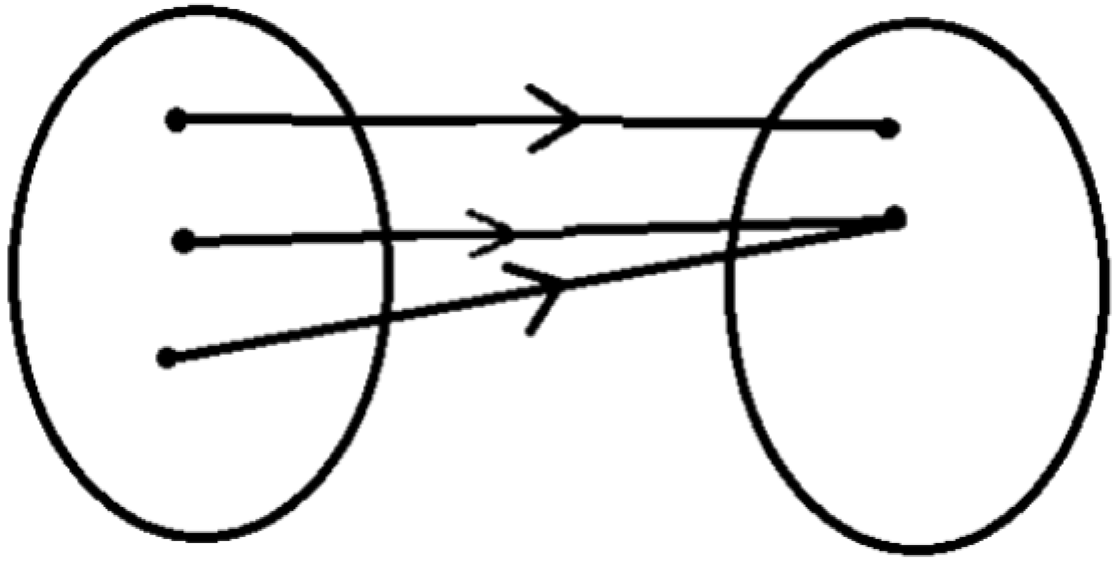


Injective function definition:
$\sim(\exists x, y \in D, x \neq y \mid f(x) = f(y))$
Necessary condition for injective function to construct:
cardinality of codomain must be greater or equal to cardinality of the domain.
|C| >= |D|
It cannot be constructed when the cardinality of domain is greater and cardinality of the codomain is smaller.

Non-Injective: If every point in a domain has no distinct image then it is called an non-injective function.

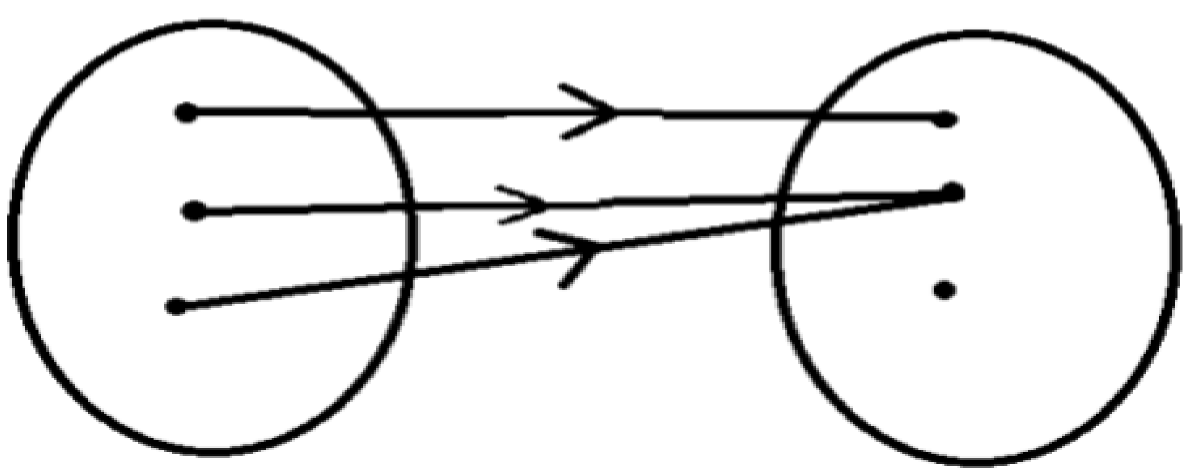


2. **Surjective function:**
   Function is surjective when every point in codomain has atleast one pre-image.
   A function is surjective when its range is equals to codomain.

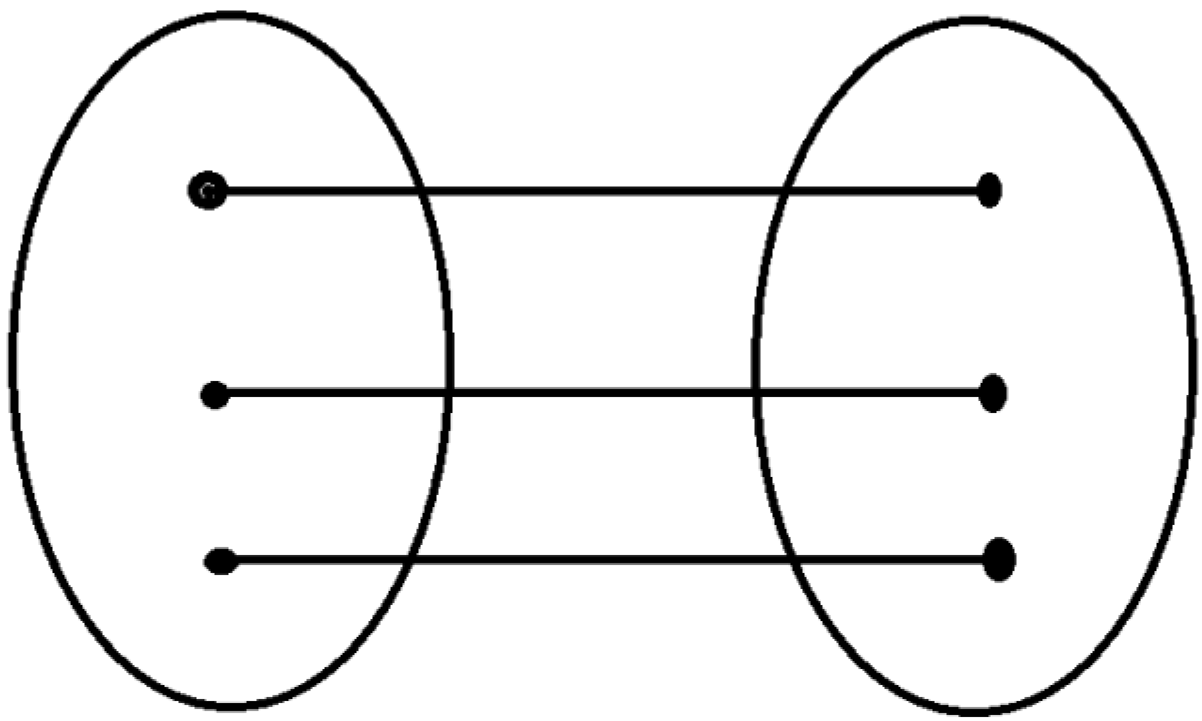


Surjective function definition:
$(\forall y \in C; \exists x \in D \mid f(x) = y)$
Necessary condition for surjective function to construct:
cardinality of domain must be greater than or equal to cardinality of the codomain.
$|D| >= |C|$
It cannot be constructed when the cardinality of domain is smaller and cardinality of the codomain is greater.

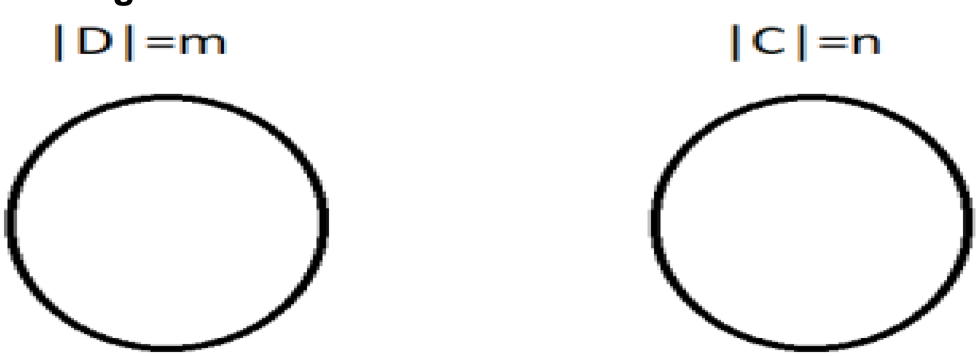
Non-Surjective: There is one point in below figure which is not the image of any elements from domain so it is non-surjective.



3. **Bijective function:**
   If a function is both injective and surjective function then it is called bijective function.
   Necessary condition for bijective function to construct:
   cardinality of domain must be equal to cardinality of the codomain.
   $|D| = |C|$
   It cannot be constructed when the cardinality of domain is not equal to cardinality of the codomain.



Counting a function:



Injective function:
n = terms
Number of injective function: nPm
= n(n - 1)...(n - (m - 1))
= n!/(n-m)!

Surjective function:
It is difficult to calculate.

Bijective function:
= nPn
= n!

Permutation function
A bijective function with same domain and codomain.

x = 1 2 3 4 5 6 7
f(x) = 3 2 6 1 5 4 7
f(1) = 3
f(2) = 2
f(3) = 6
f(4) = 1
f(5) = 5
f(6) = 4
f(7) = 7

σ_1 = 1 2 3 4 5 6 7
3 2 6 1 5 4 7
σ_2 = 1 2 3 4 5 6 7
2 1 3 6 4 5 7
Applying σ_1 and then σ_2
= 1 2 3 4 5 6 7

3 1 5 2 4 5 7

Applying σ_2 and then σ_1

= 1 2 3 4 5 6 7

2 3 6 4 1 5 7

Both results are different. So it is not commutative.