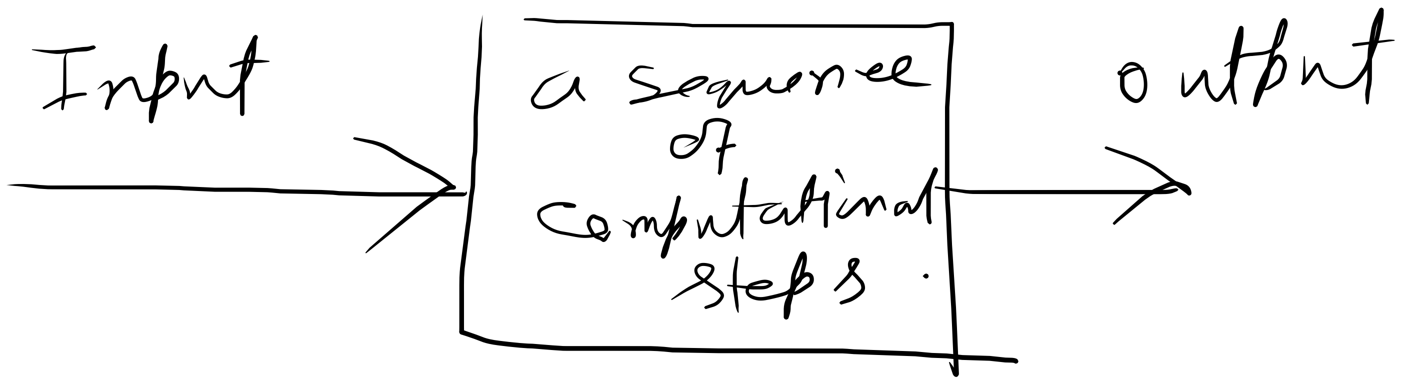
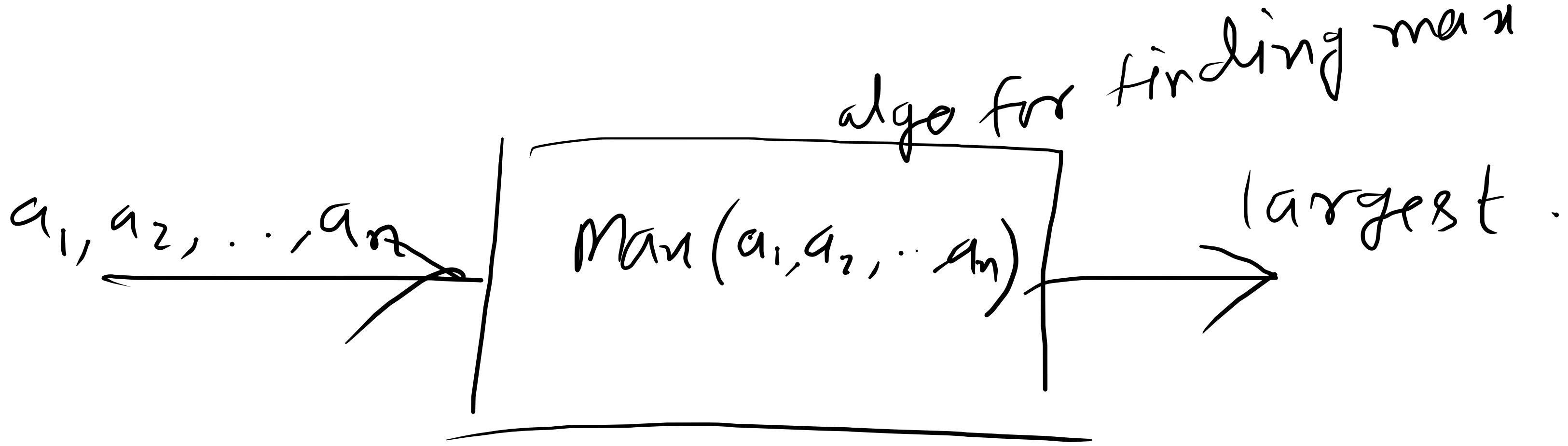
Algorithm

An algorithm is any well defined computational procedure that takes some value/a set of values as input and produces ~~some~~ some value/a set of values as output.



Exm



Max $(a_1, a_2, \ldots, a_n)$

largest $= a_1$ ——— const.

for $i = 2$ to n ——— $n-1$ times.

if $a_i >$ largest ——— const.

largest $= a_i$ — const.

Return largest. ——— const.

Q: count the number of basic operations.

Total # operations: const. + $n-1$ * const. + const. . . .

$= n-1$ * const.

$\approx$ const. n

Integer multiplication

Given two numbers x and y
both are n digits numbers

compute $x * y$

Assumption: allowed only single digit addition or multiplication.

Exm

$x = 5\ 6\ 7\ 8$

$y = 1\ 2\ 3\ 4$

				5	6	7	8
			×	1	2	3	4
			2	2	7	1	2
		1	7	0	3	4	-
	1	1	3	5	6	-	-
	5	6	7	8	-	-	-
	7	0	0	6	6	5	2

3
3
2
2
2
2

Q: How many operations are performed? in terms of n

Each row computation: const. $\times$ n

\# rows : n

$\|$ const $\cdot$ n^2

Adding any two rows : const. $\times$ n

\# pair addition : $n-1$

$\|$ const. $n \times n-1$

$(c \cdot n) \times n-1$

$cn^2 - cn$

$\leq cn^2 - dn^2$

Total time: const. n^2

Q: can we do better?

$10-2 \leq 10-5$

c 8 d 5

A new problem

Integer addition

Given 2 numbers x and y

compute $x + y$

$$
\begin{array}{rcccc}
 & & 1 & 1 & \\
 & 5 & 6 & 7 & 8 \\
+ & 1 & 2 & 3 & 4 \\
\hline
 & 6 & 9 & 1 & 2
\end{array}
$$

Q: How many operations are performed?

Ans constant $* n$

$$x = \underbrace{5\ 6}_{a}\ \underbrace{7\ 8}_{b} \qquad y = \underbrace{1\ 2}_{c}\ \underbrace{3\ 4}_{d}$$

for general n

Step 1: compute $a \cdot c$ —— 672

Step 2: compute $b \cdot d$ —— 2652

Step 3: compute $(a+b) \cdot (c+d)$ —— 6164

Step 4: Step 3 − Step 1 − Step 2 —— 2840

Step 5: 10^{n} step 1 + $10^{n/2}$ step 4 + step 2 ——

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$$x = 5\ 6\ 7\ 8 = 56 \times 10^2 + 78$$

$$x = 10^{n/2} a + b$$

$$y = 10^{n/2} c + d$$

$$x * y = (10^{n/2} a + b)(10^{n/2} c + d)$$

when n is odd

Pseudocode

$T(n)$

Operations

mul (x, y) — const.

if n == 1 return x * y

a, b = first and second half of x — const. n

c, d = " " " " " y — const. n.

tmp1 = mul (a, c) — $T(n/2)$

tmp2 = mul (b, d) — $T(n/2)$

tmp3 = (a+b) — const. n

tmp4 = c+d — const. n

tmp5 = mul (tmp3, tmp4) — $T(n/2)$

tmp6 = tmp5 - tmp1 - tmp2 — const. * n

tmp7 = 10^n tmp1 + $10^{n/2}$ tmp6 + tmp2 — const. * n

return tmp7 — const.

Total time

$$T(n) = 3T(n/2) + \text{const.}\, n$$

Primary school method: $\text{const} \cdot n^2$

This method: $T(n) = 3T(n/2) + \text{const} \cdot n$.

Q: 1 How to simplify 2nd type of equation.

Q. 2 How to compare running time of two different algorithms.