2-D array

Matrix

$$A = \begin{bmatrix} a_{00} & \cdots & a_{0,n-1} \\ a_{10} & \cdots & a_{1,n-1} \\ \vdots & & \\ \vdots & & \\ a_{m+1,1} & \cdots & a_{m-1,n-1} \end{bmatrix} \checkmark$$

Integer matrix:

4 bytes.

Q: How much memory required?

$m \times n \times 4$

Sparse: Most of the element are zero

Dense: opposite of Sparse.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$5 \times 4 \times 4 = 80$$

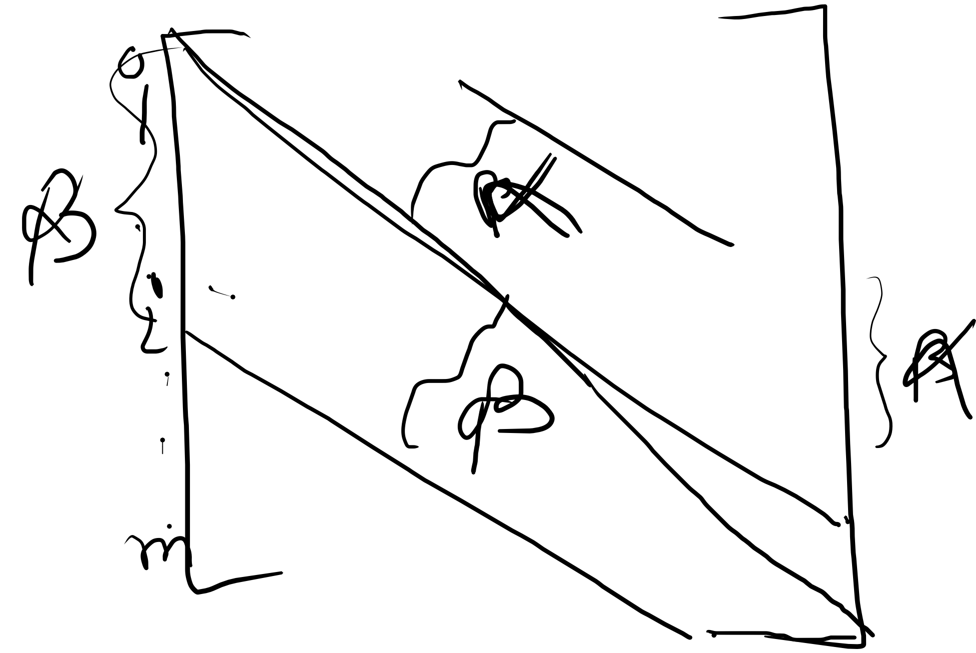
store a sparse matrix by using

(row index, col index, value)

$$\left.\begin{array}{lcl} (0,1,1) & \longrightarrow & 3 \times 4 \\ (0,4,1) & - & 3 \times 4 \\ (1,0,2) & - & 3 \times 4 \\ (2,2,1) & - & 3 \times 4 \end{array}\right\} 48$$

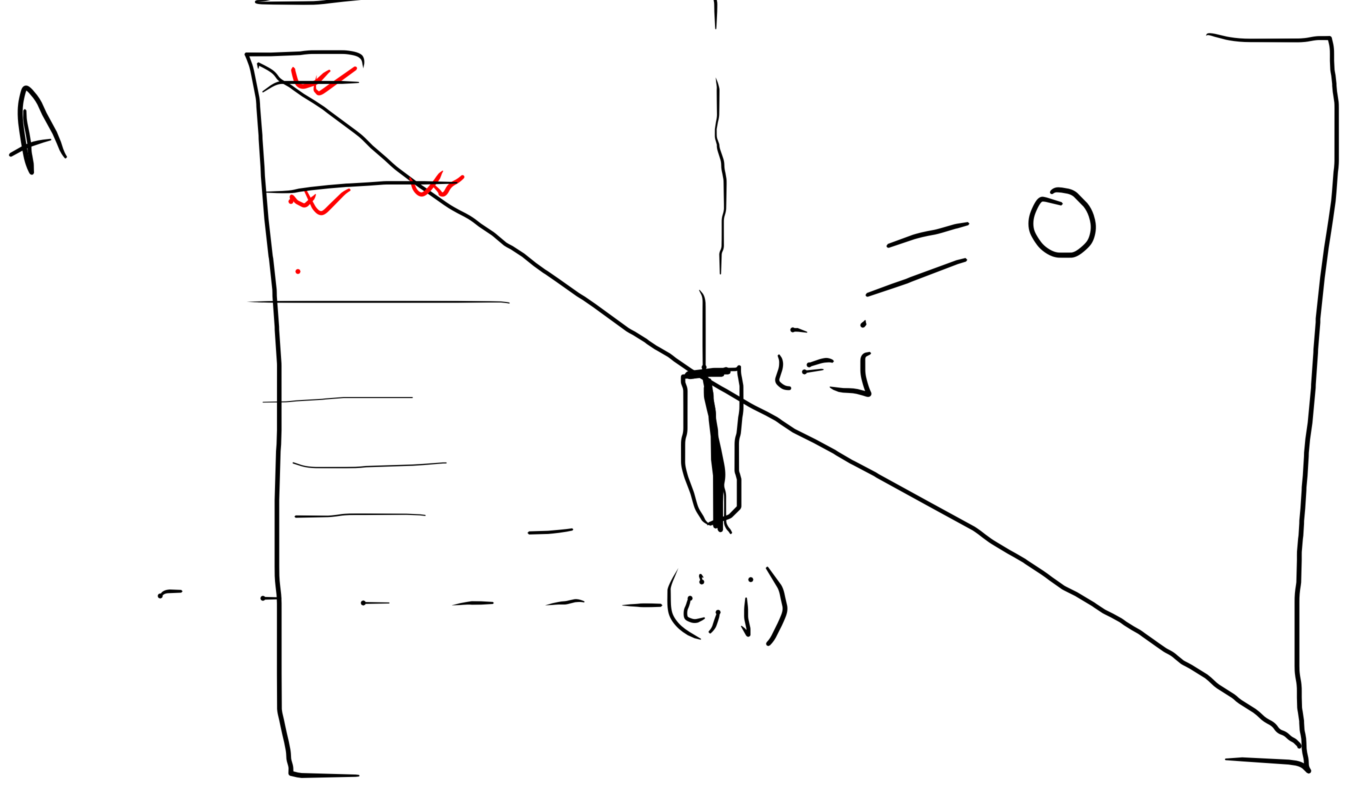
sparse matrices

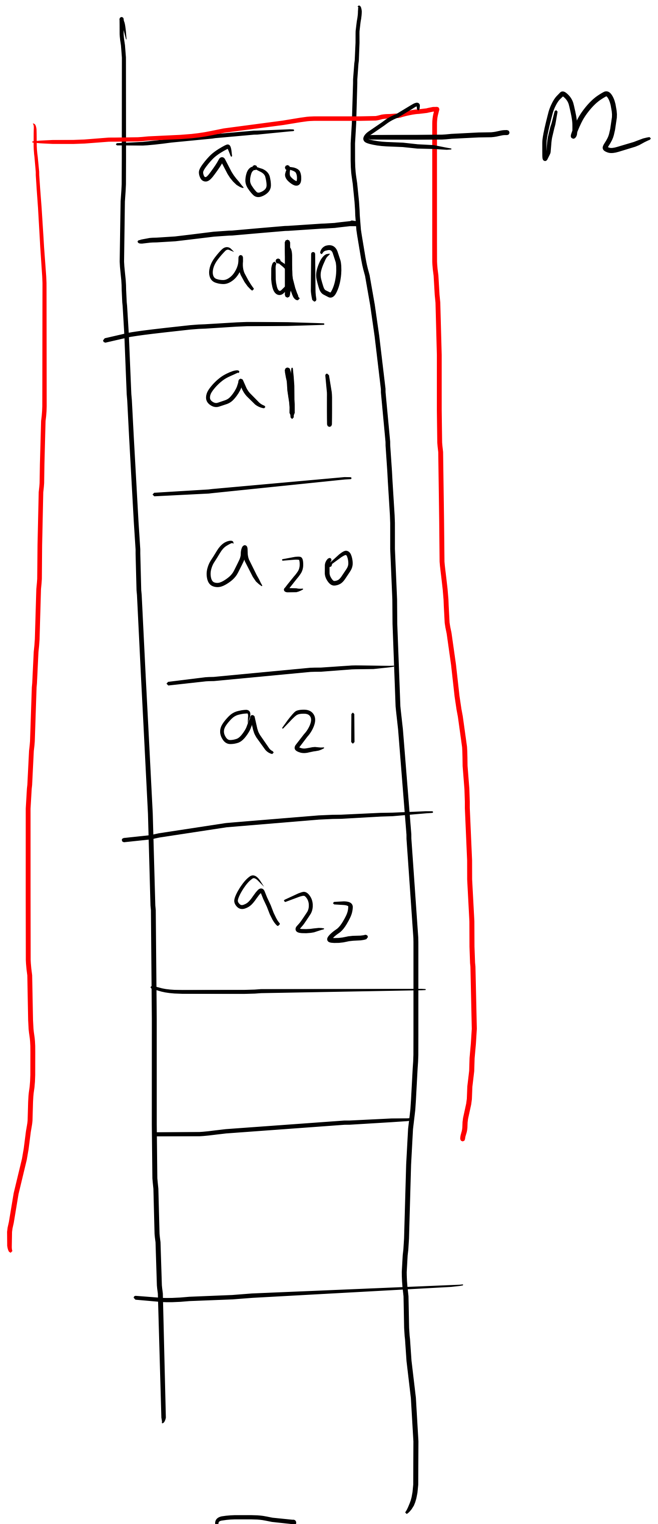
$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$ – diagonal.



- sparse
  - triangular
    - upper
      - left
      - right
    - lower
      - left
      - right.
  - Band
    - αβ-band
      - diagonal
      - bidiagonal.

lower triangular (left)





Address of (i,j)-th element.

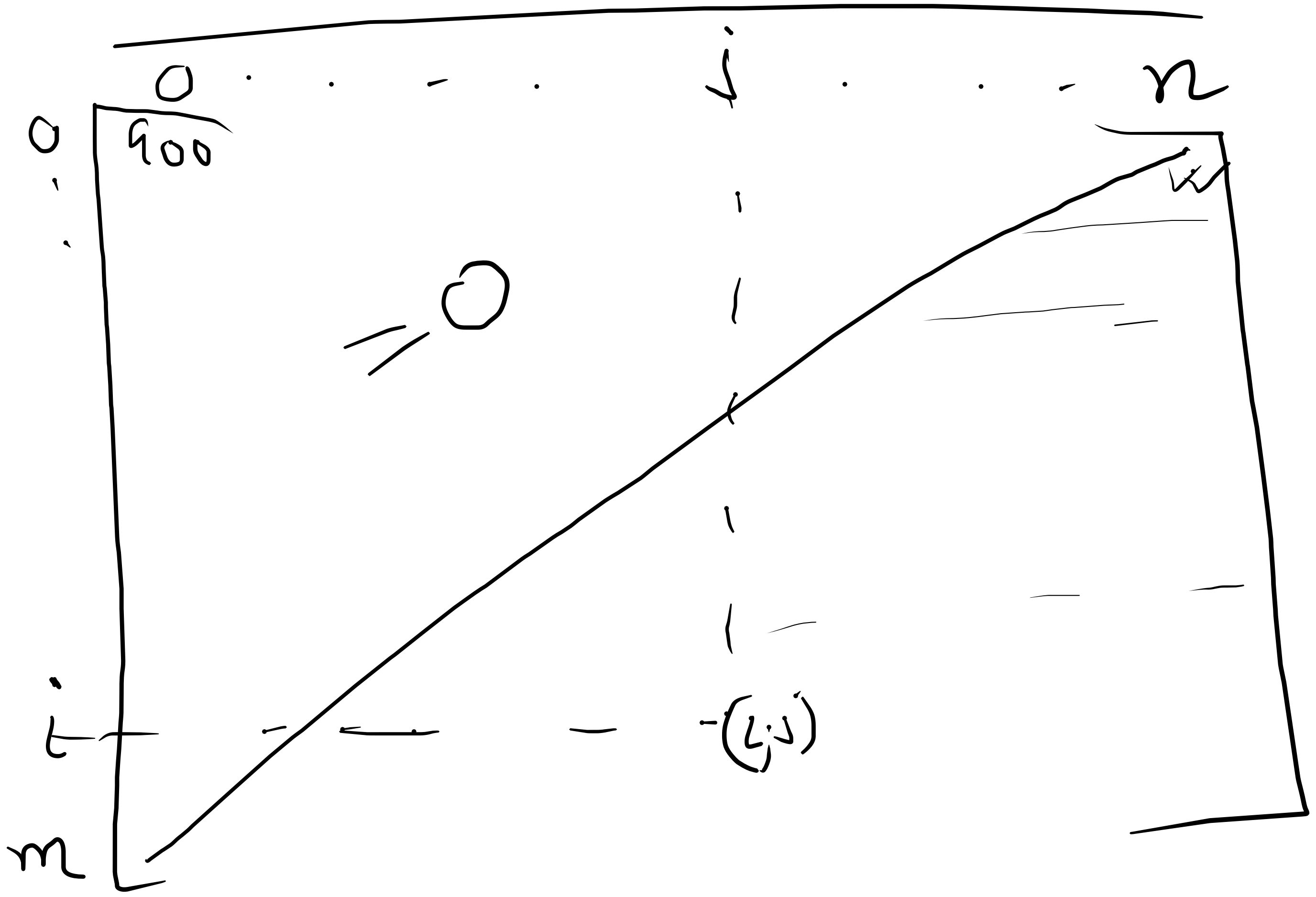
row-major:

$$M + [(1+2+3+\cdots+i)+j] \times w.$$

column-major:

$$M + [(m+(m-1)+(m-2)+\cdots+m-(j-1)) + (j-i)] * w.$$

lower right triangular



row-major:

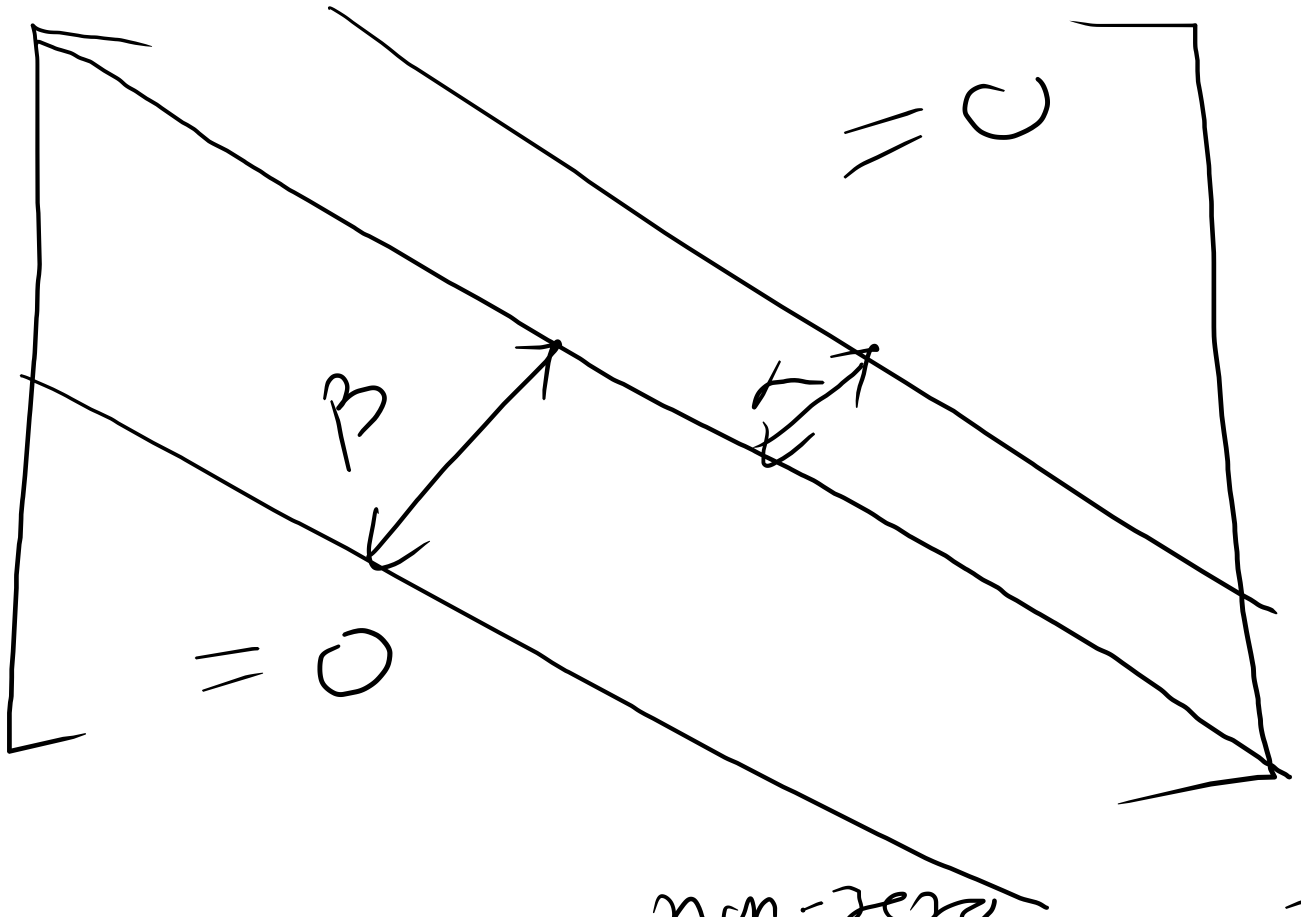
$$\text{Address}(A[i,j]) = M + [(1+2+\cdots+i) + (i-j)] * w$$

column-major: $H \cdot w$

upper triangular

flow

Band matrix



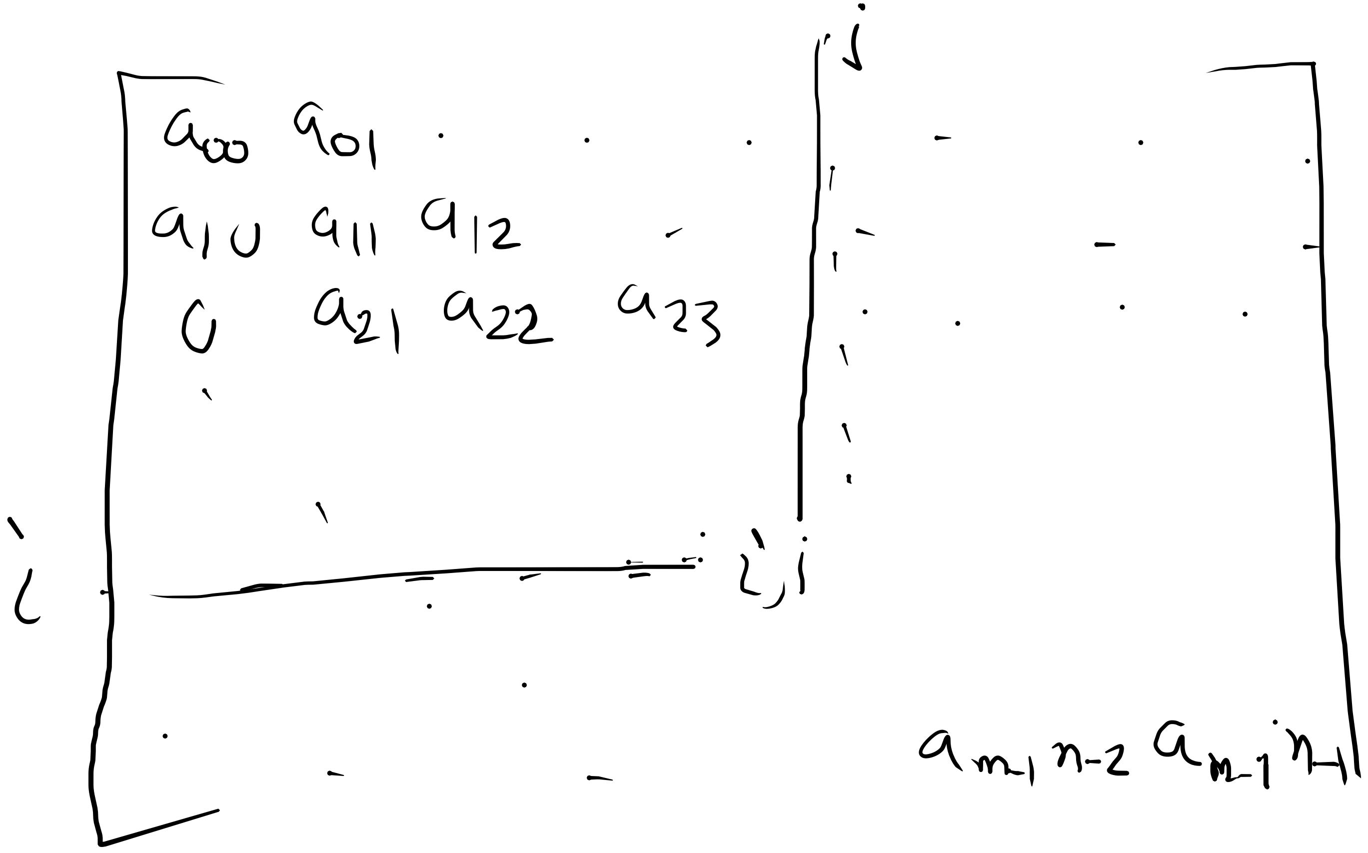
$\alpha = 2$

$$\beta = 3 \quad \begin{bmatrix} 2 & 5 & 9 & 0 & 0 & 0 \\ 6 & 1 & 5 & 2 & 0 & 0 \\ 3 & 1 & 3 & 1 & 6 & 0 \\ 8 & 0 & 1 & 2 & 3 & 5 \\ 0 & 1 & 6 & 5 & 9 & 4 \\ 0 & 0 & 2 & 3 & 4 & 5 \end{bmatrix}$$

	non-zero	zero
upper band :	$j-i \le \alpha$	$j-i > \alpha$
lower band	$i-j \le \beta$	$i-j > \beta$

$\alpha = \beta = 0$ then diagonal

$\alpha = \beta = 1$ then bidiagonal



row major:

$$\text{Address}(A[i][j]) = M + \left[\left(2 + \underbrace{3 + 3 + \cdots + 3}_{i-1 \text{ times}}\right) + \left(j - (i-1)\right)\right] \times w.$$

column-major:

H·W.