Tutorial 4

September 4^{th}, 2025

1. Construct a relation R over $S = \{1, 2, 3, 4, 5\}$, such that it is an equivalence relation as well as a partial order.

2. Let R_e be an equivalence relation on a set S, and let R_p be a partial order on the same set.

   (a) Is the composite relation of applying R_e first and then R_p next, an equivalence relation, a partial order, both or neither?

   (b) Is the composite relation of applying R_p first and then R_e next, an equivalence relation, a partial order, both or neither?

   (c) Is the composite relation of applying R_e twice, in succession, an equivalence relation, a partial order, both or neither?

   (d) Is the composite relation of applying R_p twice, in succession, an equivalence relation, a partial order, both or neither?

3. (a) Consider a relation R over the set $\mathcal{N}$ of positive integers, such that If $\lceil \frac{a}{10} \rceil < \lceil \frac{b}{10} \rceil$, then (a, b) in R and if $\lceil \frac{a}{10} \rceil = \lceil \frac{b}{10} \rceil$, then if $a\%10 > b\%10$, then (a, b) in R. Also, $(a, a) \in R, \forall a \in \mathcal{N}$.

   Is this relation a partial order?

   (b) is the relation defined above an upper lattice, a lower lattice, a lattice or neither?

   (c) If the set on which the above relation is defined is extended to all integers, address the above two questions.

4. Consider a set of sets. We define a relation over this set where two elements are related if and only if their intersection is of size at least 5.

   (a) This relation is reflexive if and only if ____

   (b) Is this relation symmetric, anti-symmetric or neither, in general?

   (c) Is this relation transitive, in general?

5. Consider a generic relation defined over any list of positive integers, which says that $(x, y) \in R$ if and only if,

$$(((x < y) \wedge ((y - x) \leq 200)) \vee ((x - y) \geq 150))$$

This can be instantiated by considering any finite subset of the integers of your choice. When considered over all positive integers determine whether this relation is:

   (a) reflexive/irreflexive or neither
   (b) symmetric/anti-symmetric or neither
   (c) transitive

6. Suppose an equivalence relation over a set S contains exactly 79 ordered pairs. Give the minimum and maximum possible value of $|S|$ and also state which values in this range are possible.

7. We know that a set is a collection of **well defined, distinct** objects, and there is no further restriction. Thus we could have a set of sets (that is the elements of the set are each sets, which are distinct). Consider such a set $\mathcal{A}$ of sets $\{S_1, \ldots, S_n\}$. Let us define a relation R over $\mathcal{A}$ where $(S_i, S_j) \in R$ if and only if $S_i \subseteq S_j$. Is the relation R:

   (a) Reflexive, irreflexive or neither?
   (b) Symmetric, anti-symmetric or neither?
   (c) Transitive?

8. Let $S = \{1, \ldots, 100\}$, the set of the first 100 positive integers. Define a relation R where $(x, y) \in R$ if and only if

$$((x = y) \vee ((|x - y| \leq 15) \wedge (|x - y| \geq 5)))$$

Find the cardinality of the largest subset X of S, such that the relation R restricted to the subset X is an equivalence relation. How many such subsets are there in S?