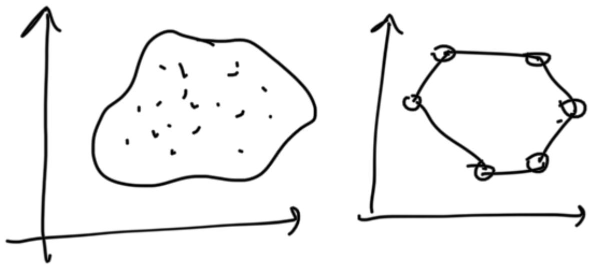
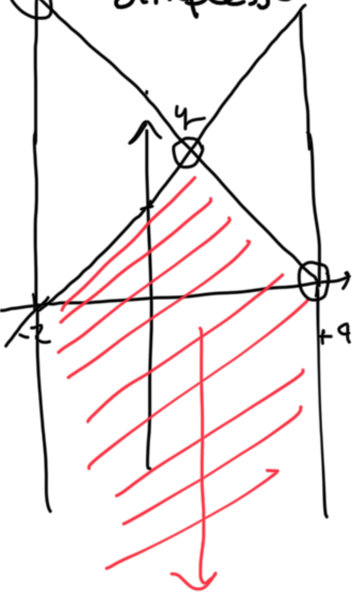
PL	PLI
$\bar{x} \in \mathbb{R}^n$	$\bar{x} \in \mathbb{Z}^n$
Polinomiali	NP-hard
Simplesso	Branch & bound





Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\min 3y$$

$$1 \quad x + 2 \geq 0 \qquad x \leq 4 \quad 2$$

$$3 \quad y \leq x + 2 \qquad y + x \leq 4 \quad 4$$

Si parta dalla base ammissibile corrispondente ai vincoli della seconda riga.

max $c\bar{x}$
s.t. $A\bar{x} \leq \bar{b}$

max $-y$
$-x \leq 2$
$x \leq 4$
$-x + y \leq 2$
$x + y \leq 4$

$c = [0 \ \ -1]$

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 4 \end{bmatrix}$$

1° it.

$B_1 = \{3, 4\}$

$$A_{B_1}^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{-2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\bar{x}_1 = A_{B_1}^{-1} b_{B_1} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad \bar{y}_1 = c A_B^{-1} = [0 \ \ -1] \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = [-\tfrac{1}{2} \ \ -\tfrac{1}{2}]$$

$$h_1 = 3 \qquad \xi_1 = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} \qquad A_{N_1} \xi_1 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$K_1 = 2$

2° it.

$$B_2 = \{2, 4\} \qquad A_{B_2}^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\bar{x}_2 = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \qquad \bar{y}_2 = [0 \ \ -1] \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = [1 \ \ -1] \qquad h_2 = 4$$

$$\xi_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad A_{N_2} \xi_2 = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

problema illimitato

Programma vuoto.
max x
$x \geq 0$
$x \leq -1$

$$\overbrace{A \quad \bar{b} \quad \bar{c}}^{(P)}$$

B è una base per A $\qquad \bar{x}_B = A_B^{-1} \cdot b_B$

$H := \{ i \text{ t.c. } A_i \bar{x}_B \leq b_i \}$

$J := \{ i \text{ t.c. } A_i \bar{x}_B > b_i \}$

$$\begin{cases} B \subseteq H \\ J \cap B = \emptyset \end{cases} \quad \text{ovvie considerazioni}$$

Se $J = \emptyset$ allora B è ammissibile

altrimenti considero il seguente programma lineare:

$$\text{(PA)} \quad \begin{array}{ll} \min \ u\bar{\nu} & u = [1 \dots 1] \\ A_H x \leq b_H & |\bar{\nu}| = |J| \\ A_J x \leq b_J + \cancel{\bar{\nu}} & \\ \bar{\nu} \geq 0 & \end{array}$$

$$\underbrace{x = \bar{x} \quad \bar{\nu} = A_J \bar{x} - b_J}_{\text{è soluzione ammissibile per (PA)}}$$

Risolviamo (PA), due casi:
$\rightarrow$ sol. ottima (x^*, ν^*)

- $\nu^* \neq 0$ allora (P) è vuoto.
- $\nu^* = 0$ allora (P) è non vuoto e x^* è sol. di base ammissibile.

$$B \begin{cases} x + 2 \geq 0 \\ y + x \leq 4 \end{cases} \quad x_B = (-2, 6)$$

$A_B = [\cdots] \quad A_B^{-1} = [\cdot\cdot] \quad x_B = A_B^{-1} b_B$

$H = \{1, 4, 2\}$

$J = \{3\}$

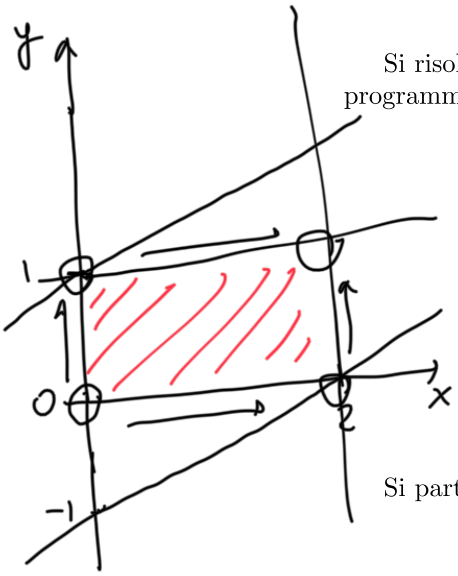
$$\left.\begin{array}{l} \min \ \nu \\ -x \leq 2 \\ x \leq 4 \\ x + y \leq 4 \\ -x + y \leq 2 + \nu \end{array}\right\} \rightarrow \begin{array}{c} x \ \ y \ \ \nu \\ (-2, 0, 0) \end{array}$$

0

1

Da risolvere a casa.

$v \geq 0$



Si risolva, tramite l'algoritmo del simplesso primale, il seguente problema di programmazione lineare:

$$\max x + 2y \qquad C = [1 \quad 2]$$

$$x \geq 0 \qquad y \geq 0$$

$$x - 2 \leq 0 \qquad y - 1 \leq 0$$

$$x + 2 \geq 2y \qquad 2y + 2 \geq x \;\to\; y \geq \tfrac{1}{2}x - 1$$

Si parta dalla base ammissibile corrispondente ai vincoli della prima riga.

$\to y \leq \frac{1}{2}x + 1$

$Ax \leq b$

$$\begin{aligned} -x &\leq 0 \\ -y &\leq 0 \\ x &\leq 2 \\ y &\leq 1 \\ -x + 2y &\leq 2 \\ x - 2y &\leq 2 \end{aligned} \qquad A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ 1 & -2 \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

$$B_1 = \{1, 2\} \qquad A_1^{-1} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad x_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y_1 = c A_{B_1}^{-1} = [1 \quad 2]\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = [-1 \quad -2] \qquad h_1 = 1$$

$$\xi_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad A_{N_1}\xi_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\frac{b_i - A_i \bar{x}}{A_i \xi} \;\to\; \frac{2 - [1 \quad 0]\begin{bmatrix} 0 \\ 0 \end{bmatrix}}{1} = 2 \qquad \frac{2 - [1 \quad -2]\begin{bmatrix} 0 \\ 0 \end{bmatrix}}{1} = 2 \qquad k_1 = 3$$

$$B_2 = \{2, 3\}$$

Da finire a casa.