E9 241 Digital Image Processing
Assignment 03

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Q1. Directional Filtering:

A. Image Generation from Sinusoids:

Results:

- The images x_1 (horizontal stripes), x_2 (vertical stripes), and x_3 (diagonal stripes) were correctly generated based on their respective sinusoidal formulas. The combined image x is a superposition of these three patterns.
- The combined plot of the images x_1, x_2, x_3, x, and the magnitude of the DFT of combined image x is shown below:

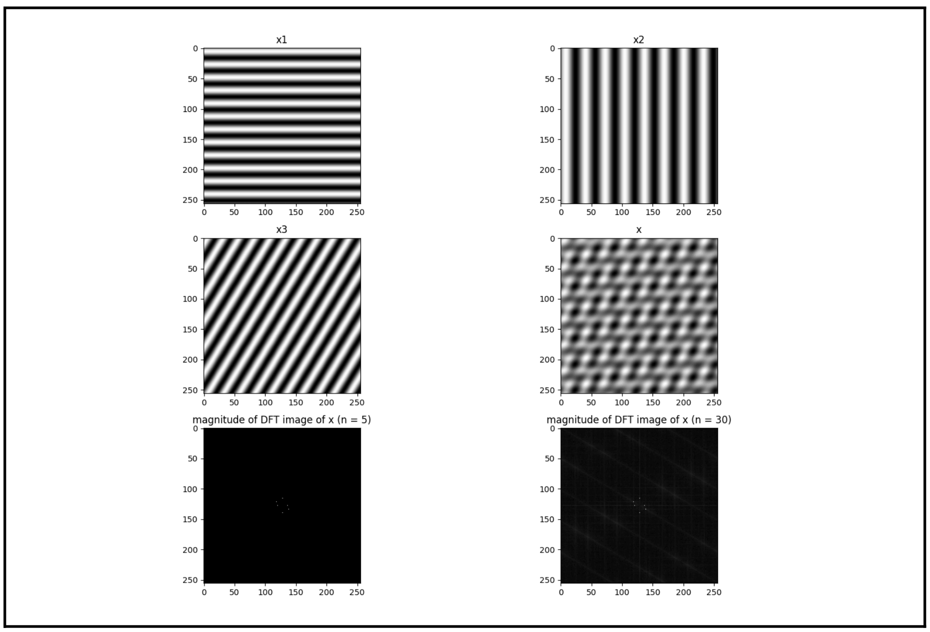

Figure1: Combined plot of the images x_1, x_2, x_3, x, and the magnitude of the combined image x

Inferences:

- The central spot in the 2D DFT image of the magnitude of the combined image is dark. Since, the central spot (DC component) represents the average intensity of the image, the average intensity in the image is dark.

- The 2D DFT decomposes the combined image into its constituent frequencies.
- The horizontal stripes of x1 give rise to the pair of bright spots on the vertical axis, the vertical stripes of x2 give rise to the pair of bright spots on the horizontal axis, and the diagonal stripes of x3 give rise to the pair of bright spots on the diagonal.

B. Directional Filtering Application:

Results:

- For each filter H1, H2, H3, and H4, the magnitude spectrum of the filter, filtered image's magnitude spectrum and the reconstructed image is shown in the figure below.
- The filtered image using H1 filter has only the horizontal stripes, the reconstructed image using the H2 filter has only the vertical stripes, and the reconstructed image using H3 filter only shows the diagonal stripes, while the reconstructed image using combined filter H4 nearly reconstructs the original image.

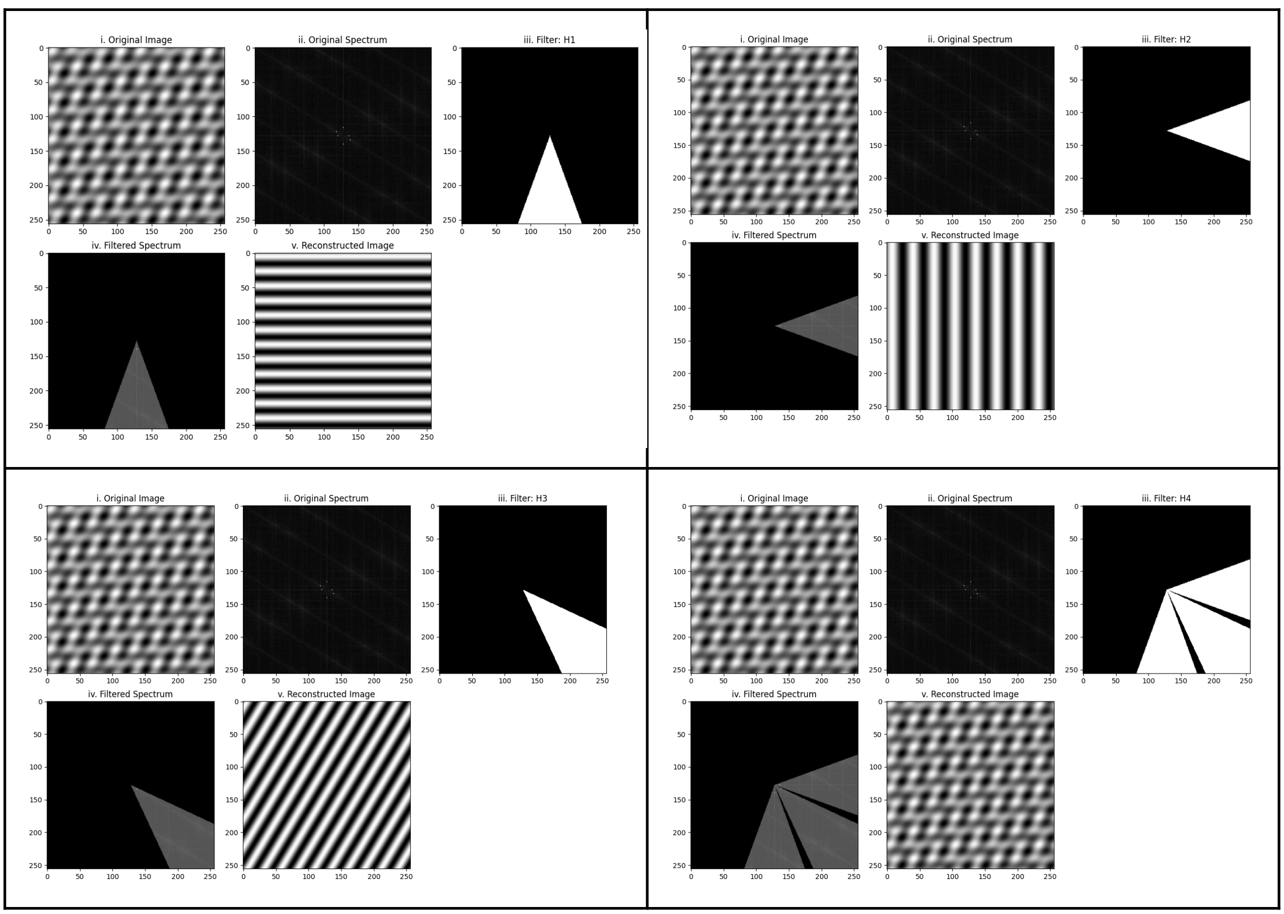

Figure 2: Results of each directional filter H1, H2, H3 and H4

Inferences:

- The filters select only a portion of the frequency spectrum, which enables us to reconstruct the image containing only the desired patterns.
- Since, the filter H4 is a combination of the other three filters, apart from some frequencies it passes almost all the frequency components present in the image and hence resulting in a near-perfect reconstruction of the original image.

C. Mean Squared Errors:

Results:

- The MSE for H1, H2, H3 filters is identical and high: 0.125. And, the MSE for H4 filter is: 0.041.

Inferences:

- The higher values for H1, H2, and H3 confirms that the filters are removing the other two components of the image.
- The very small MSE value for H4 indicates that it is able to pass almost all the frequency components in the original image.

Q2. Gaussian Blurring and Inverse Filtering:

A. Gaussian Blurring:

Results:

- The 13x13 Gaussian kernel was created and then padded to match the size of the building.jpg image. The DFT of the kernel was computed and it was multiplied with the DFT of the image. The inverse of that resulting DFT image was computed to get the blurred image.
- The original image and the image blurred in the Frequency domain using the Gaussian kernel is shown in the figure below:

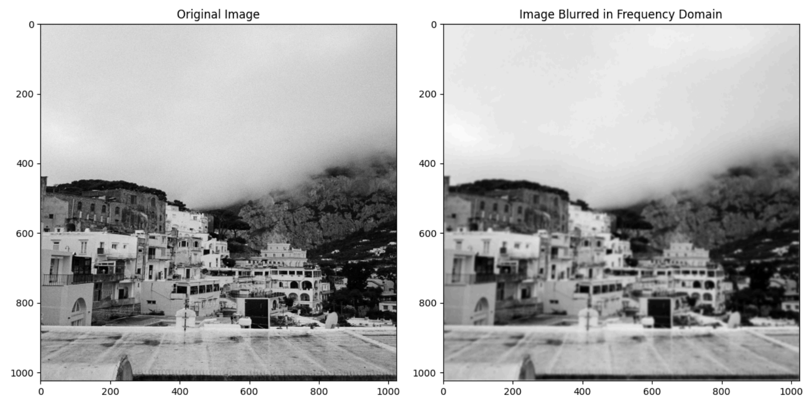

Figure 3: Original image vs Image blurred in the Frequency domain (and then reconstructed using inverse DFT)

Inferences:

- It demonstrates the convolution process. Convolution with a Gaussian kernel in the spatial domain is equivalent to element-wise multiplication in the frequency domain.
- The blurred image shows that some high-frequency details (sharpness, edges, noise) are reduced, which shows that the Gaussian filter is a low pass filter.

B. Analysis of the Gaussian Kernels:

Results:

- The spectrum for the 13x13 Gaussian kernel is blocky.
- The spectrum for the 1036x1036 Gaussian kernel is a smooth and continuous-looking Gaussian function.
- The inverse plots show the corresponding inverse filters of the kernels.

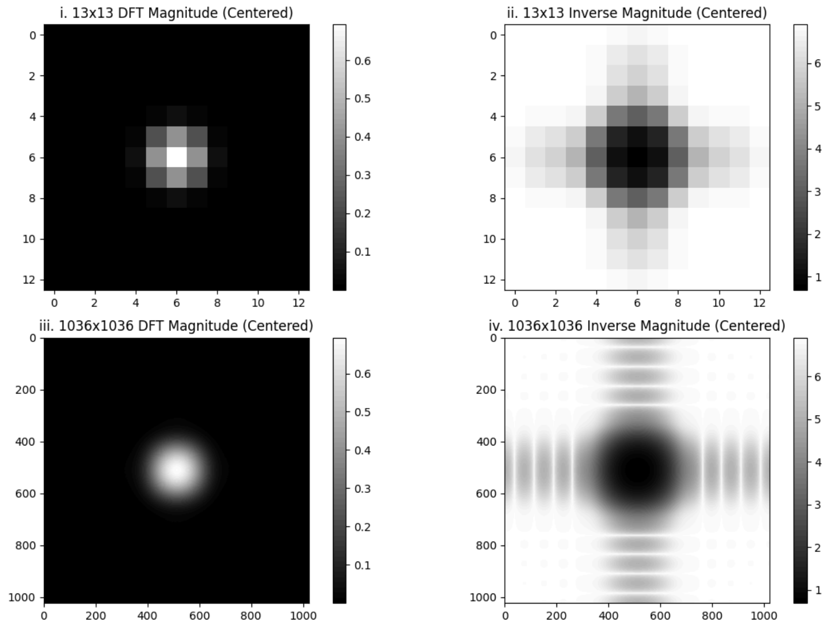

Figure 4: Magnitude spectrum of the two gaussian kernels and their inverse.

Inferences:

- The frequency response of the Gaussian kernel is also a Gaussian which suppresses higher frequencies.
- The inverse kernel spectrum shows that it is a high pass filter that passes the higher frequencies and blocks the lower frequencies. And, it can be used as a high pass filter to sharpen the image in the frequency spectrum.

C. Gaussian Fit:

Results:

- The optimum value of k that minimizes the error is: 0.00011

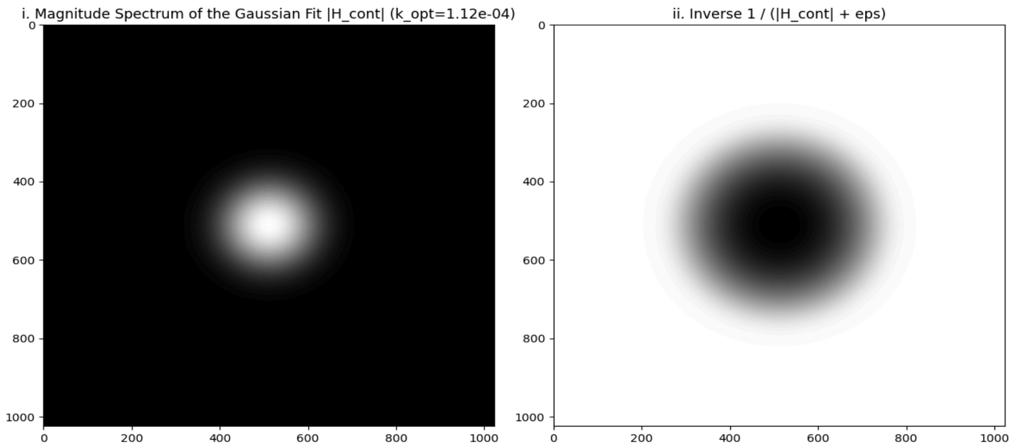

Figure 5: The optimum Gaussian frequency response fit and its inverse.

Inferences:

- The plotted magnitude spectrum of the H_{cont} is similar to the DFT magnitude of the 1036x1036 Gaussian kernel computed in the previous step.
- The close match between the Gaussian fit and the actual padded kernel validates numerical implementation.

D. Inverse Filtering:

Results:

- The blurred image was restored using both the "Direct Kernel Inverse" (from part b) and the "Gaussian Fit Inverse" (from part c).
- The direct kernel inverse produces the closest restoration to the original image
- MSE between the original image and the reconstructed image using the direct kernel inverse: 0.00534
- MSE between the original image and the reconstructed image using Gaussian fit inverse: 0.01393

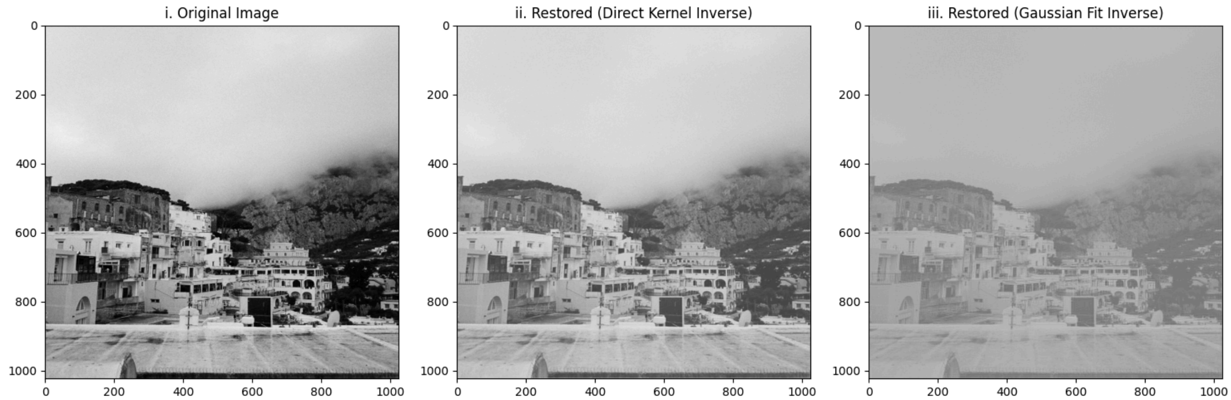

Figure 6: The original image and reconstructed images from the blurred image using both direct kernel inverse and Gaussian fit inverse is shown.

Inferences:

- The MSE values confirm that the Direct Kernel Inverse method gives a significantly better restoration than the Gaussian Fit method.
- The Direct Kernel Inverse used is the exact inverse of the filter that was used to blur the image. But, the Gaussian Fit Inverse is an approximation. Hence the Direct Kernel Inverse method gives a better result.