Lecture 11

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2017-03-29

Load packages

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library(broom)
library(Stat2Data)
data(AutoPollution)
data(CloudSeeding)
data(Hawks)
```

Random number generation

Random number generation

- Reference: Chapter 2 of Robert and Casella (2004), Introducing Monte Carlo Methods with R
  - Online copy available from the library: [http://search.lib.sfu.ca/?q=introducing%20monte%20carlo%20methods%20with%20r]

Why generate random numbers?

- Simulating from a data-generating process allows us to evaluate the performance of methods for making inference from such data.
  - E.G., to evaluate the performance of least squares regression for inference of regression coefficients, we can repeatedly generate datasets, perform inference, and summarize the results (example later).
- Many modern statistical methods are simulation-based; e.g.,
  - permutation tests
  - boostrap tests and confidence intervals
  - cross validation for selecting "tuning" parameters

Random number generation

- We can't generate truly random numbers.
- Instead we generate deterministic sequences or *streams* of pseudo-random numbers.
  - Goal: Every n-tuple from the stream should be statistically indistinguishable from a random sequence of size n drawn from a uniform distribution on (0,1), denoted $U(0,1)$.
- Basic approach is to
  - Generate $U(0,1)$ deviates
  - Transform to a sample from the distribution of interest.
- Generating $U(0,1)$ deviates
  - Several random number generators (RNGs) have been developed
  - No consensus on which is "best"
  - We'll use the default in R

Random number seed

- The current position in the random number stream is called the *seed*.
- Information on the seed is contained in the variable `.Random.seed` in your workspace.
  - If we don't set the seed, R will do it for us.
  - Each time we generate random numbers, the seed is incremented.
  - When we save the workspace, the seed is saved too.
  - **Warning:** If you start R from a previously-saved workspace, you will get the random seed. Starting multiple simulations from the same seed gives the same simulations
- You can set the seed with `set.seed()`.

Setting the seed

```
set.seed(123)
runif(10)
```

```
##  [1] 0.2875775 0.7883051 0.4089769 0.8830174 0.9404673 0.0455565 0.5281055
##  [8] 0.8924190 0.5514350 0.4566147
```

```
set.seed(123)
runif(10)
```

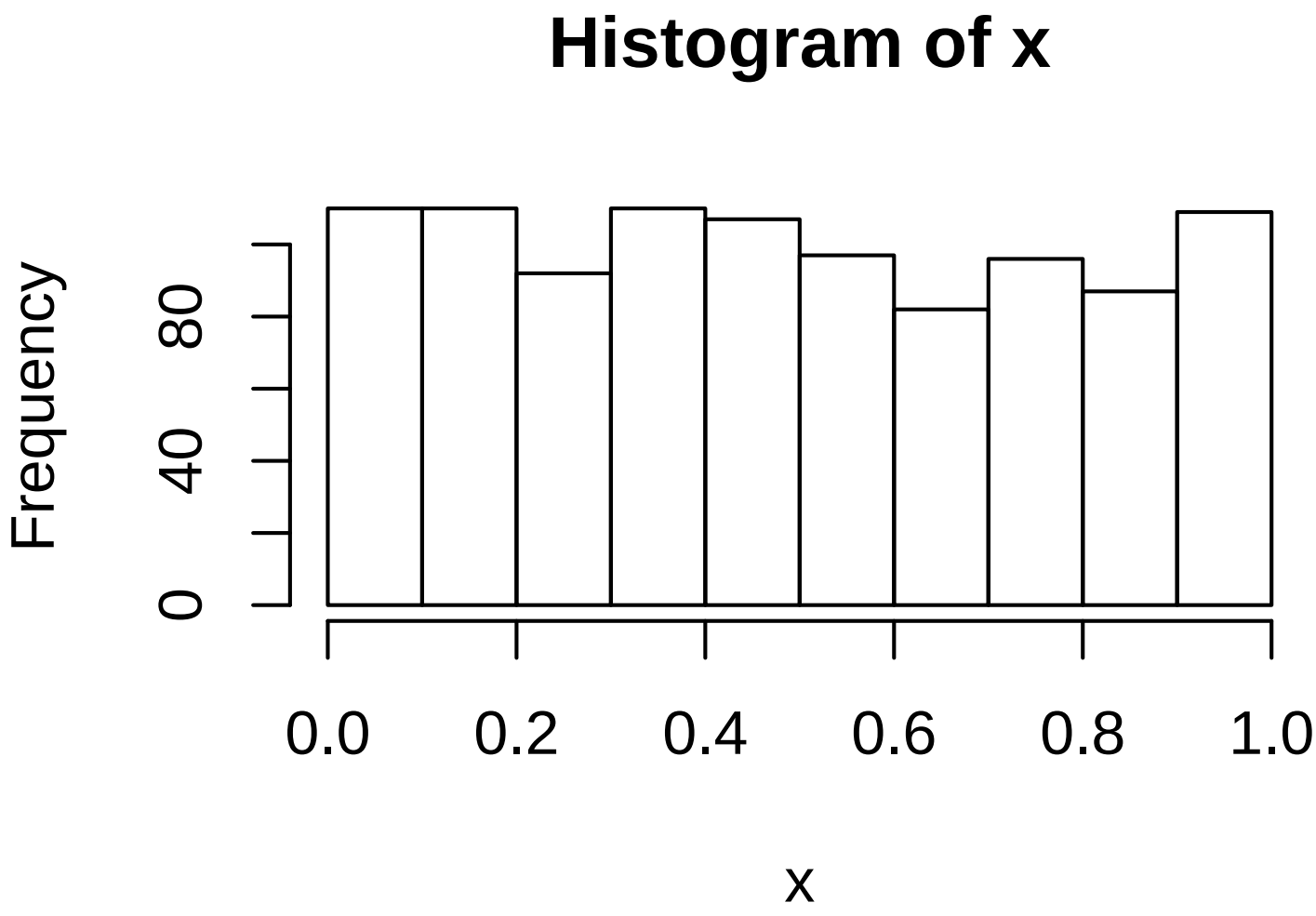
```
##  [1] 0.2875775 0.7883051 0.4089769 0.8830174 0.9404673 0.0455565 0.5281055
##  [8] 0.8924190 0.5514350 0.4566147
```

```
set.seed(42)
runif(10)
```

```
##  [1] 0.9148060 0.9370754 0.2861395 0.8304476 0.6417455 0.5190959 0.7365883
##  [8] 0.1346666 0.6569923 0.7050648
```

Generating uniforms

```
Nsim <- 1000
x <- runif(Nsim)
hist(x)
```



Example: Assessing uniformity with a test

```
ks.test(x,"punif")
```

```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  x
## D = 0.036422, p-value = 0.1408
## alternative hypothesis: two-sided
```

Generating from other distributions

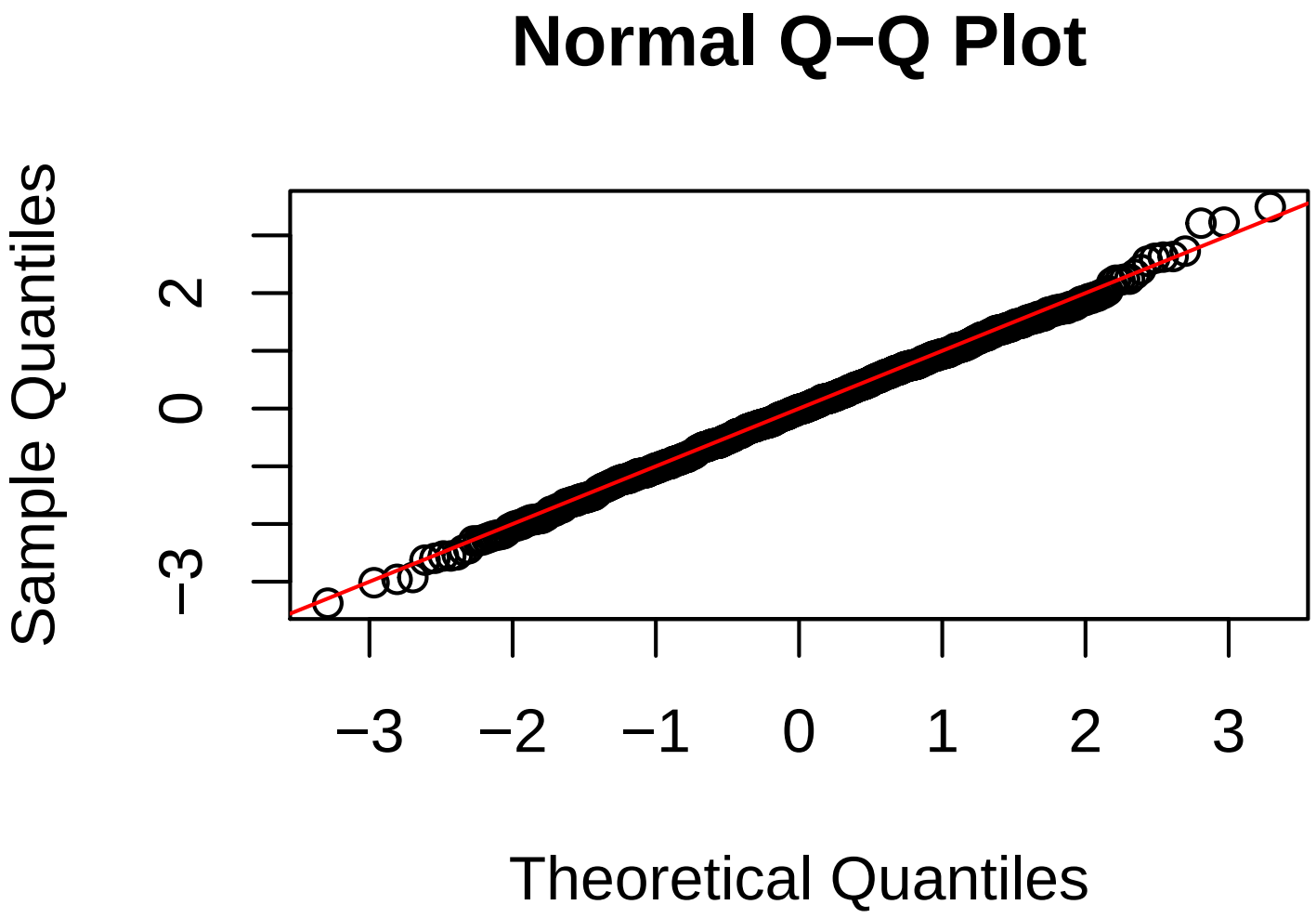
- For common distributions, we don't have to work out the transformations to convert from $U(0, 1)$ deviates to the distribution of interest.
- Built-in functions in R have names like `rnorm()`, `rt()`, `rchisq()`

Example: Generating from a Normal distribution

```
Nsim <- 1000; mu <- 0; sig <- 1
x <- rnorm(Nsim,mean=mu,sd=sig)
```

Example: Assessing normality graphically

```
qqnorm(x); abline(a=0,b=1,col="red")
```



Example: Assessing normality with a test

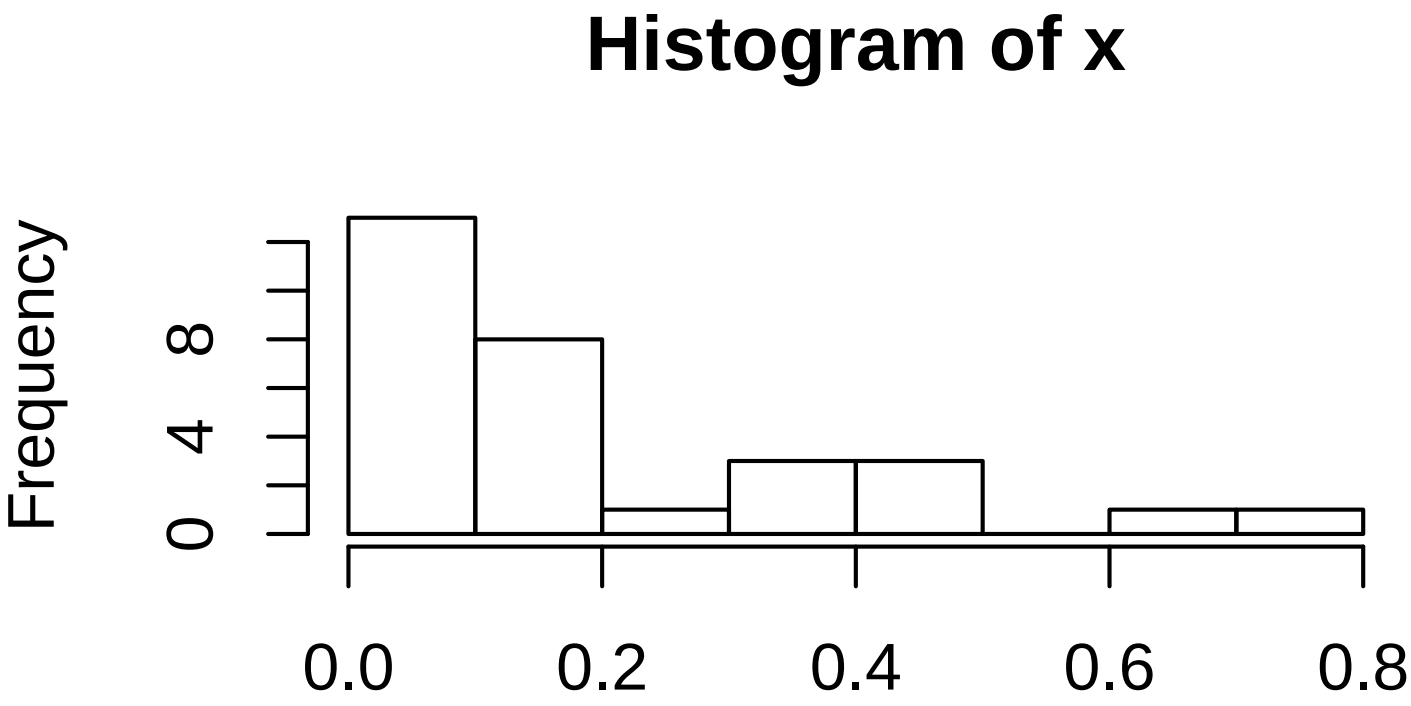
```
ks.test(x,"pnorm")
```

```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  x
## D = 0.019345, p-value = 0.8483
## alternative hypothesis: two-sided
```

Application: Demonstrate the central limit theorem (CLT)

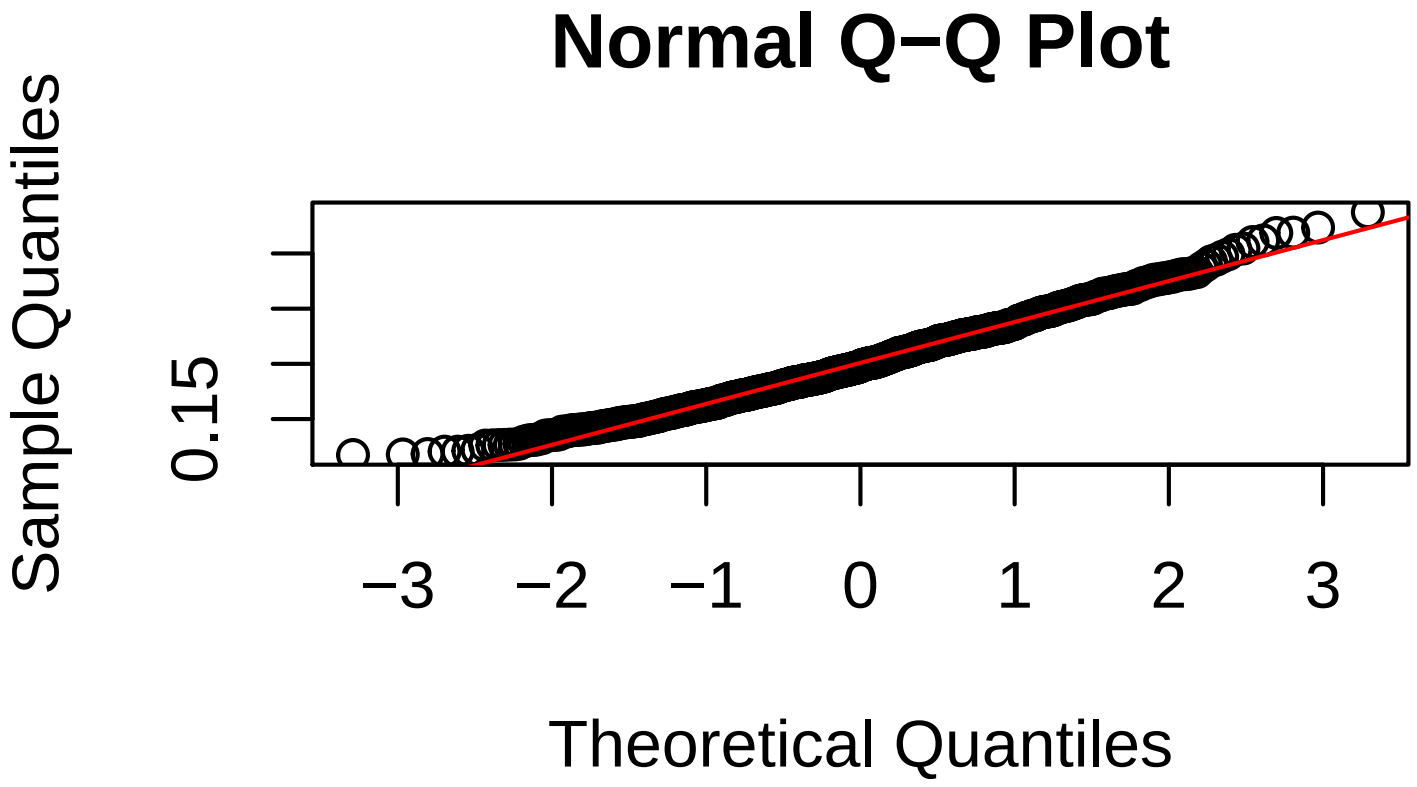
- The CLT says that the sampling distribution of an average is approximately normal for large sample sizes, regardless of the parent population from which the sample is drawn.
- Example data-generating process: Samples of size $n = 30$ from an exponential distribution with rate 5.

```
x <- rexp(n=30,rate=5)
hist(x)
```



CLT: Sampling distribution of averages

```
set.seed(8675309)
xbar <- vector(mode="numeric",length=Nsim)
for(i in 1:Nsim) {
  xbar[i] <- mean(rexp(n=30,rate=5))
}
qqnorm(xbar); abline(a=mean(xbar),b=sd(xbar),col="red")
```



replicate()

- The `replicate()` function can be used for simulation.
- The following is equivalent to the `for` loop in the previous slide.

```
set.seed(8675309)
expMean <- function(n=30,rate=5) { mean(rexp(n=n,rate=rate)) }
xbar2 <- replicate(Nsim,expMean())
all.equal(xbar,xbar2)
```

```
## [1] TRUE
```

Application: Simulation study

- We can conduct a simulation study to evaluate the type 1 error of the t test of the null hypothesis that a regression coefficient is zero.
  - Simulate under the null hypothesis, but with non-normal errors
- Fix
  - sample size `n=30`
  - values of a single covariate `x=(1:n)/n`
- Replicate the following simulation `Nsim` times
  - Simulate responses Y as $Z - 1$ where Z has as exponential distribution with rate 1 (mean 1).
  - Fit the regression model by least squares and return the p-value from the test of association with `x`.
- Calculate the proportion of p-values less than 0.05.
  - Should be 5%

Simulation code: simulating one dataset

```
n <- 30; x <- (1:n)/n
y <- rexp(n)-1
fit <- lm(y~x)
tidy(fit)
```

```
##          term    estimate std.error  statistic   p.value
## 1 (Intercept)  0.05388653 0.3368775  0.1599589 0.8740619
## 2           x -0.11807275 0.5692762 -0.2074085 0.8371919
```

```
tidy(fit) %>% filter(term=="x") %>% select(p.value)
```

```
##     p.value
## 1 0.8371919
```

```
# or
tidy(fit)[2,5]
```

```
## [1] 0.8371919
```

Simulation study

```
Nsim <- 1000
simfunc <- function() {
  y <- rexp(n)-1
  tidy(lm(y~x))[2,5]
}
simout <- replicate(Nsim,simfunc())
alphaNominal <- 0.05
alphahat <- mean(simout<alphaNominal)
alphahat
```

```
## [1] 0.043
```

```
SE <- sqrt(alphahat*(1-alphahat)/Nsim)
c(alphahat - 1.96*SE, alphahat + 1.96*SE)
```

```
## [1] 0.03042679 0.05557321
```

Note on simulation error

- Use the normal approximation to the mean of `Nsim` binary variables.
  - Let α be the true type 1 error of the procedure, and $\hat{\alpha}$ be our simulation-based estimate
  - $\hat{\alpha}$ is a mean and can be shown to be approximately Normal with mean α and SD $\sqrt{\alpha(1-\alpha)/\mathit{Nsim}}$.
  - SE is obtained by plugging in $\hat{\alpha}$
- Can define simulation error as the margin of error in a 95% confidence interval for the parameter being estimated.
  - In our study, the CI covers 0.05, so we say that the empirical type 1 error is within simulation error of the nominal level.

Sampling with and without replacement

- Draw probability weighted samples of size n from a set with `sample()`
  - Sampling can be with replacement or without.

```
myset <- 1:10; nset <- length(myset)
probwts <- rep(1/nset,nset)
n<-8
sample(myset,size = n, replace = TRUE, prob=probwts)
```

```
## [1] 6 3 4 2 8 5 2 2
```

```
sample(myset,size = n, replace = FALSE, prob=probwts)
```

```
## [1]  9  6  1  4  8  7  2 10
```

Notes on sampling with and without replacement

- Weights need not sum to one
  - They will be normalized
- If sampling with replacement, the size of the sample can't exceed the size of the set.
- Sampling *n* without replacement from a set of size *n* is a *permutation* of the set.
  - Using all the defaults of `sample()` gives a permutation.

```
sample(1:10)
```

```
##  [1]  9  6 10  5  1  3  2  4  7  8
```

```
sample(c("cat","dog","fish","zebra"))
```

```
## [1] "zebra" "dog"   "fish"  "cat"
```

Permutation tests

Example

- Makes most sense in the context of a designed experiment.
- Example experiment:
  - 36 cars were randomly assigned to receive one of two types of car muffler.
  - The noise in decibels of each car was measured.
  - Is there a difference in sound level between the two mufflers?
- A standard ANOVA for these data is as follows.

```
mfit <- lm(Noise ~ Type, data=AutoPollution)
tidy(anova(mfit)) # F stat is 1,5 element
```

```
##         term df    sumsq    meansq statistic   p.value
## 1       Type  1  1056.25 1056.2500  1.246181 0.2721107
## 2 Residuals 34 28818.06  847.5899        NA        NA
```

```
Fstat <- tidy(anova(mfit))[1,5]
```

Permutation distribution of F

- Under the null hypothesis the noise of cars doesn't depend on the muffler type, we just have 36 cars assigned randomly to two groups.
  - Cars are said to be *exchangeable* (with repect to noise)
  - The distribution of the F statistic under all possible re-randomizations is the randomization distribution.
- Randomization can be achieved by permuting the `Noise` variable.
  - The randomization distribution can also be called a permutation distribution and the test that compares the observed F to this distribution a permutation test.

Permutation test on example

```
fstatPerm <- function() {
  permdat <- data.frame(Type = AutoPollution$Type,
                        Noise = sample(AutoPollution$Noise))
  mm <- lm(Noise ~ Type, data=permdat)
  tidy(anova(mm))[1,5]
}
pdist <- replicate(1000,fstatPerm())
mean(pdist > Fstat) # permutation p-value
```

```
## [1] 0.291
```

Exchangeability is key assumption

- Responses must be exchangeable under the null hypothesis of no treatment effect
- Cloud seeding data
  - Clouds randomly seeded (S) or not (U)
  - Clouds from several areas of Tasmania; focus on Tasmania East (TE).

```
CloudSeeding %>% group_by(Seeded) %>%
  summarize(n=n(),mean=mean(TE),sd=sd(TE))
```

```
## # A tibble: 2 × 4
##   Seeded     n     mean        sd
##   <fctr> <int>    <dbl>     <dbl>
## 1      S    14 1.090000 0.6707401
## 2      U    14 1.598571 1.6281885
```

- Unseeded clouds are more variable in rainfall, so they are not exchangeable.