# CS 736 : Hybrid ACO K-Means Algorithm

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# Introduction:Image Segmentation

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# Introduction:Image Segmentation

1. Image segmentation is a very widely researched problem, many algorithms have been proposed but no optimal solution has yet been found.
2. Medical images such as MRI, CT, and ultrasound are complex and heterogeneous and contain a large amount of data that needs to be analyzed and interpreted by medical professionals.
3. Image segmentation highlights the relevant part of the image.
4. One instance of segmentation is separating the brain into WM, GM and CSF.

# Existing Approaches

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The shortcomings of the Kmeans algorithm and other existing approaches which our algorithm improves upon are-

1. Sensitive to initialisation.
2. Takes only spectral distance into account, ignores spatial location of pixel
3. Doesn't guarantee convergence to optimal solution.
4. Several methods to decide choice of initialisation of centroids have been proposed. KMeans++ and Farthest Point clustering are a few to mention. This still doesn't guarantee convergence.

# ACO: Algorithm

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1. Pheromone is a bio-communicator used by ants. Ants deposit pheromones on paths where they find food. Initially there is no pheromone content in the environment.
2. Ants choose a path with probability proportional to the pheromone content on that path.
3. Moreover, due to evaporation, the pheromone concentration in the longer path reduces, decreasing the probability of selection of this path in further stages.
4. Ultimately, all ants move along the shorter path.

# ACO: Algorithm

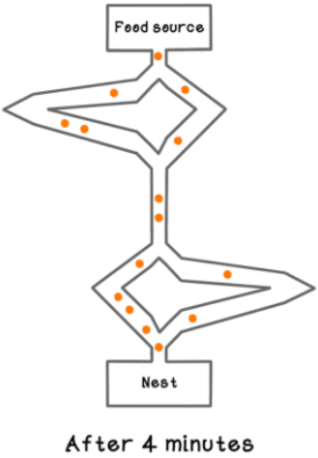


Figure: ACO in Action

# ACO: Algorithm

Let us denote the pheromone levels by $\rho$ and the lengths of paths by $l$. The probability of selecting the $i^{th}$ path is

$$P_i = \frac{\rho_i}{\rho_1 + \rho_2}$$

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The pheromone levels are updated as-

$$\rho_i = \rho_i(1 - \mu) + \frac{K}{l_i}$$

K is some parameter and $\mu$ is the evaporation rate which is also a parameter.

# Motivation

- Desiring convergence of K-Means to the global optimum necessitates a **brute-force approach**, which in turn requires an exponential amount of time.
- We also want to address all the shortcomings listed previously
- Hybrid Ant Colony Optimisation over K-Means addresses this issue in **polynomial time!**. Heavy math is involved here, more details are in the paper.

# HyperParameter Tuning

- **K**: Number of Clusters
- **M**: Number of Ants
- **max_iter**: Limit on number of iterations of CFS
- **max_iter_ant**: Limit on number of iterations of IFS
- $\alpha, \beta$: Heuristic Coefficients of Probability Function
- **Q**: Amount of pheromone deposited per ant
- $\rho$: Evaporation coefficient

# Hybrid ACO KMeans Optimisation

Proposed by S.A. El-Khatiba, Y.A. Skobtsovb, S.I. Rodzin

- We consider both spatial centroids and spectral centroids.
- We use the notion of pheromone and its evaporation in order to reduce the impact of the previously made choices which are of lower priority.

# Objective functions

The objective functions are f1,f2 and f3 (equations 3,4,5).

1. f1 measures the spectral distance between clusters
2. f2 measures the spatial compactness of all clusters
3. f3 measures the sum spectral compactness of all clusters

We can take the target function to be some addition of normalized version of f1,-f2 and -f3.
Ideally we would keep iterating until the target function stops increasing at some point.

# Objective functions

$$f_1(m) = \sum_{k=1}^{K-1} \sum_{j=k+1}^{K} CDist(C_k, C_j) \tag{1}$$

$$f_2(m) = \sum_{k=1}^{K} \sum_{p \in S_k} PDist(C_k, X_p) \tag{2}$$

$$f_3(m) = \sum_{k=1}^{K} \sum_{p \in S_k} CDist(C_k, X_p) \tag{3}$$

# HACOK - Initialisation

**Initialisation:**

- Random spatial centroid initialisation, and cluster centroid values taken as the pixel values at these spatial centroids.
- All pheromone values $\tau_j(X)$ initialised to 1. $\tau_j(X)$ denotes pheromone level associated with pixel X and cluster j

# HACOK - Updates

**Updates:**

- Outer loop is **Community Food Search**, Inner loop is **Individual Food Search**.
- In every CFS iteration, we run IFS for all ants, and choose the **best ant**, and make necessary updates before next iteration of CFS

# HACOK - IFS

In an IFS iteration(for ant a):

1. Update probabilities $P_k(X)$ of pixel X lying in $k^{th}$ cluster, using Probability function of ACO, which depends on pheromone $\tau_i(X)$ and heuristic function $\eta_i(X)$, with tunable heuristic parameters $\alpha$ & $\beta$.
2. Update memberships of pixels. Assign cluster label k to pixel X with probability $P_k(X)$.
3. Update cluster centers(spatial and central) with euclidean and spectral mean respectively.
4. **Repeat** until cluster centers do not vary.

# HACOK - CFS

In a CFS iteration:

1. Run IFS for every ant $a_i$, and choose the best ant based on improvement of objective funtions $f_1(a_i), f_2(a_i), f_3(a_i)$.
2. Update pheromone values considering the solution of the best ant, tunable with parameters evaporation rate $\rho$ and pheromone deposit $\mathbf{Q}$
3. **Repeat** until objective functions do not improve.

# Equations

$CC_i$ denotes the $i^{th}$spectral cluster center and $PC_i$ denotes the $i^{th}$ spatial cluster center.
Here $PDist(i, j)$ denotes the euclidean distance between points i and j and $CDist(i, j)$ denotes the spectral distance between points i and j
$\tau_i(X_n)$ denotes the pheromone level of the pixel $X_n$ towards the $i^t h$ cluster.
$\eta_i(X_n)$ is the heuristic information of pixel $X_n$ belonging to cluster i it depends on the spatial and spectral distance, both.

$$P_i(X_n) = \frac{[\tau_i(X_n)]^\alpha [\eta_i(X_n)]^\beta}{\sum_{j=0}^{K} [\tau_i(X_n)]^\alpha [\eta_i(X_n)]^\beta} \tag{4}$$

$$\eta_i(X_n) = \frac{k}{CDist(X_n, CC_i) * PDist(X_n, CC_i)} \tag{5}$$

# Equations

$$\tau_i(X_n) = (1-\rho)\tau_i(X_n) + \Delta\tau_i(X_n) \tag{6}$$

$$\Delta\tau_i(X_n) = \begin{cases} \frac{QMin(k')}{AvgCDist(k',i)AvgPDist(k',i)}, & X_n \in \text{Cluster i} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

# Results - 1

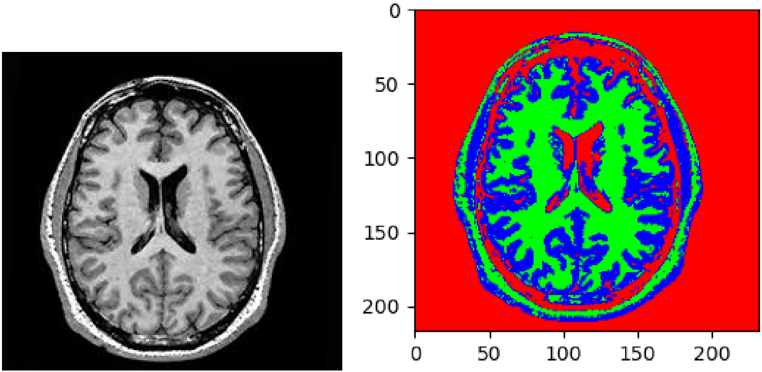



Figure: Original and Segmented Image

K = 3, M = 3, $\alpha = 2$, $\beta = 5$, C=10, max_iter=6, max_iter_ant = 4, Q = 100, $\rho = 0.2$

# Results - 1

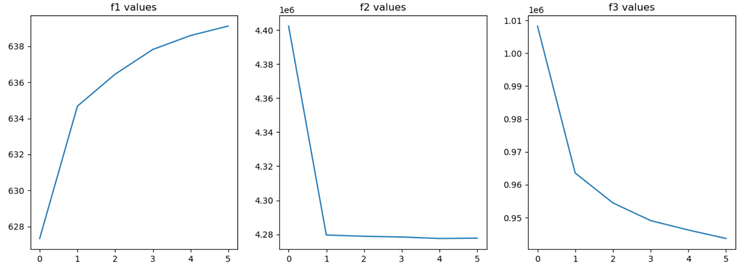



Figure: f1 f2 and f3 values across iterations

# Results - 2

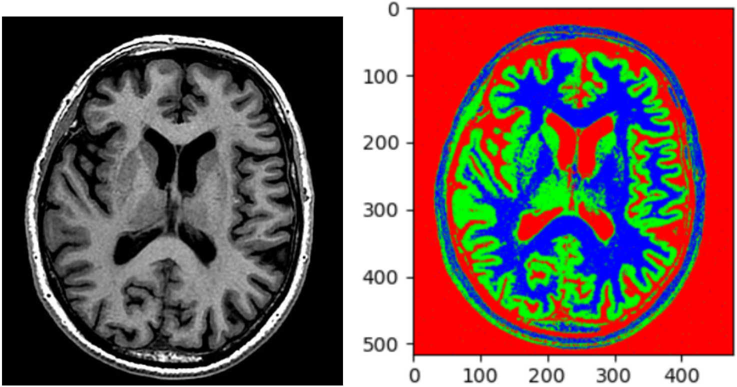
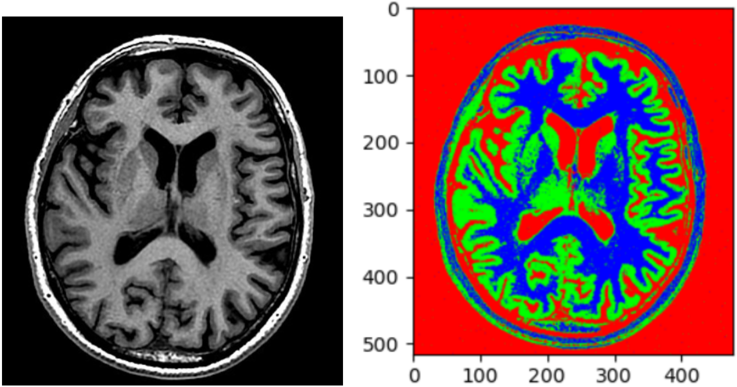



Figure: Original and Segmented Image

K $= 3$, M $= 6$, $\alpha = 1.2$, $\beta = 1.5$, max_iter=10, max_iter_ant $= 10$, Q $= 150$, $\rho = 0.2$

# Results - 2

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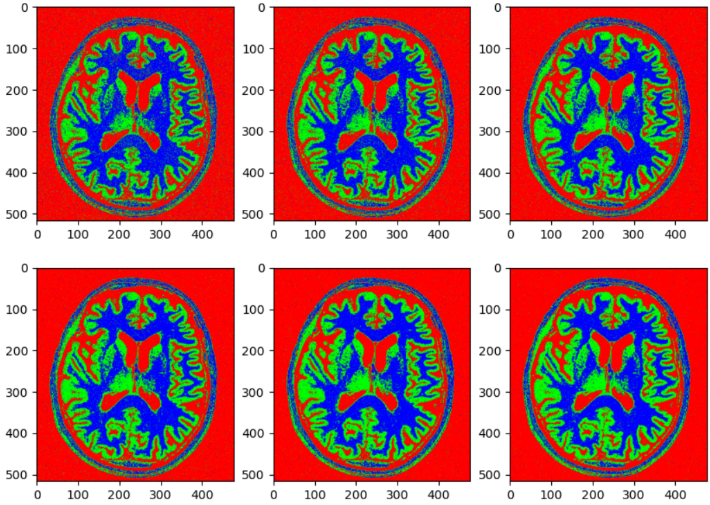


Figure: Improvements over CFS iterations

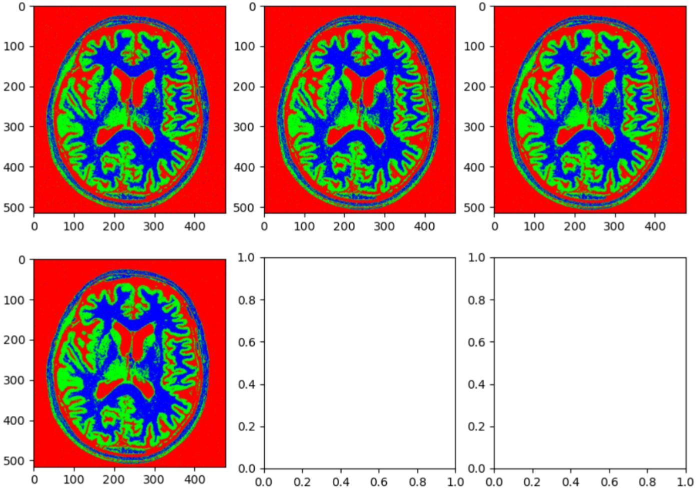


Figure: Improvements over CFS iterations

# Further Ideas

1. This algorithm also has a fuzzy variant Hybrid Ant Fuzzy Algorithm (HAFA).
2. Similar to this, spatial distance can also be incorporated to K-means to improve clustering.
3. This algorithm may be extended to different classes, but different classes of problems(eg- lung,heart etc) require different values of the hyperparameters.

# References

1. Link to the paper-https://www.sciencedirect.com/science/article/pii/S1877050921009686
2. Reference on ACO- https://www.geeksforgeeks.org/introduction-to-ant-colony-optimization/
3. Link to the paper on soft clustering alternative- https://link.springer.com/chapter/10.1007/978-3-030-20915-5_12
4. Link to the paper with theoritical analysis- https://www.sciencedirect.com/science/article/pii/S1877050919304053