Nonlinear State Estimation Techniques with Examples

Marcus Krogh Nielsen

Technical University of Denmark

mkrni@dtu.dk
mkn@unibiogroup.com

October 1, 2020

Overview

1. System Model
2. Estimation Model
3. Example

System Model

Consider a system governed by the dynamics

$$\begin{aligned} d\boldsymbol{x}(t) &= f(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta)dt + \sigma(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta)d\boldsymbol{\omega}(t), \\ \boldsymbol{z}(t) &= g(t, \boldsymbol{x}(t), \theta), \\ \boldsymbol{y}(t_k) &= h(t_k, \boldsymbol{x}(t_k), \theta) + \boldsymbol{v}(t_k), \end{aligned}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the input vector, $\boldsymbol{d}(t) \in \mathbb{R}^{n_d}$ is the disturbance vector, $\boldsymbol{z}(t) \in \mathbb{R}^{n_z}$ is the output vector, and $\boldsymbol{y}(t_k) \in \mathbb{R}^{n_y}$ is the measurement vector. In the state equation, the diffusion is described by the Wiener process $\boldsymbol{\omega}(t)$, i.e., $d\boldsymbol{\omega}(t) \sim \mathcal{N}(0, Idt)$. In the measurement equation, the noise is normally distributed as $\boldsymbol{v}(t_k) \sim \mathcal{N}(0, R(\theta))$.

The disturbances to the system, $\boldsymbol{d}(t)$, are unmeasured.

Estimation Model

For such a system, we define the estimation (or control) model

$$
\begin{aligned}
d\boldsymbol{x}(t) &= f(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta)dt + \sigma(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta)d\boldsymbol{\omega}(t), \\
d\boldsymbol{d}(t) &= \sigma_d(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta)d\boldsymbol{\omega}_d(t), \\
\boldsymbol{z}(t) &= g(t, \boldsymbol{x}(t), \theta), \\
\boldsymbol{y}(t_k) &= h(t_k, \boldsymbol{x}(t_k), \theta) + \boldsymbol{v}(t_k),
\end{aligned}
$$

where the variables and parameters are defined similarly to the system model, and the unmeasured disturbances are governed by the Wiener process $\boldsymbol{\omega}_d(t)$, i.e., $d\boldsymbol{\omega}_d(t) \sim \mathcal{N}(0, Idt)$.

Estimation Model

From the estimation model, we define the augmented variables, as

$$\bar{\boldsymbol{x}}(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{d}(t) \end{bmatrix}, \qquad d\bar{\boldsymbol{\omega}}(t) = \begin{bmatrix} d\boldsymbol{\omega}(t) \\ d\boldsymbol{\omega}_d(t) \end{bmatrix},$$

the augmented functions, as

$$\bar{f}(t, \bar{\boldsymbol{x}}(t), u(t), \theta) = \begin{bmatrix} f(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta) \\ 0 \end{bmatrix},$$
$$\bar{\sigma}(t, \bar{\boldsymbol{x}}(t), u(t), \theta) = \begin{bmatrix} \sigma(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta) & 0 \\ 0 & \sigma_d(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta) \end{bmatrix},$$

and the overloaded output and measurement functions, as

$$\bar{g}(t, \bar{\boldsymbol{x}}(t), u(t), \theta) = g(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta),$$
$$\bar{h}(t, \bar{\boldsymbol{x}}(t), u(t), \theta) = h(t, \boldsymbol{x}(t), u(t), \boldsymbol{d}(t), \theta).$$

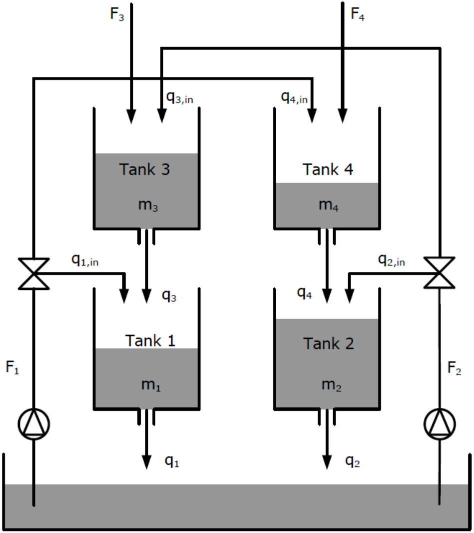
Estimation Model

For such a system, we define the estimation (or control) model

$$\begin{aligned} d\bar{\boldsymbol{x}}(t) &= \bar{f}(t, \bar{\boldsymbol{x}}(t), u(t), \theta)dt + \bar{\sigma}(t, \bar{\boldsymbol{x}}(t), u(t), \theta)d\bar{\boldsymbol{\omega}}(t), \\ \boldsymbol{z}(t) &= \bar{g}(t, \bar{\boldsymbol{x}}(t), \theta), \\ \boldsymbol{y}(t_k) &= \bar{h}(t_k, \bar{\boldsymbol{x}}(t_k), \theta) + \boldsymbol{v}(t_k), \end{aligned}$$

where the diffusion $\bar{\boldsymbol{\omega}}(t)$ is still a Wiener process, i.e., $d\bar{\boldsymbol{\omega}}(t) \sim \mathcal{N}(0, Idt)$.

Modified Four Tank System - Illustration

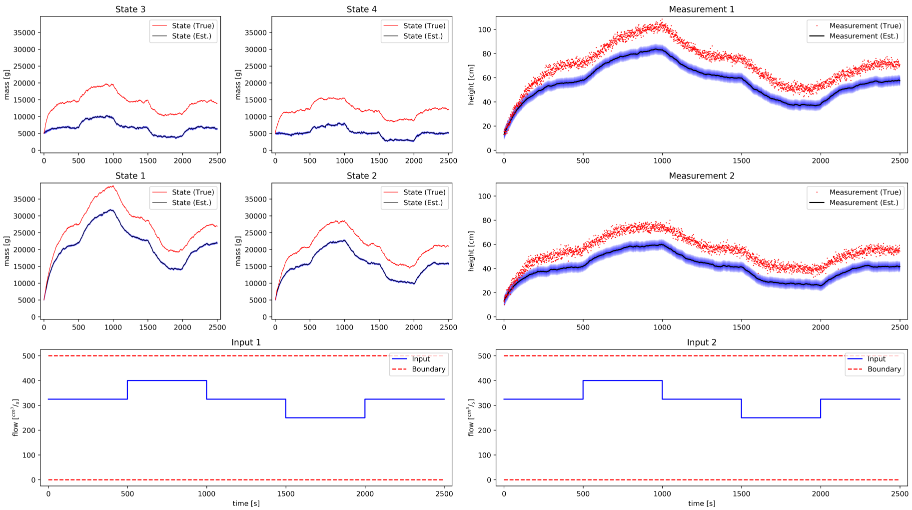


Modified Four Tank System - Model

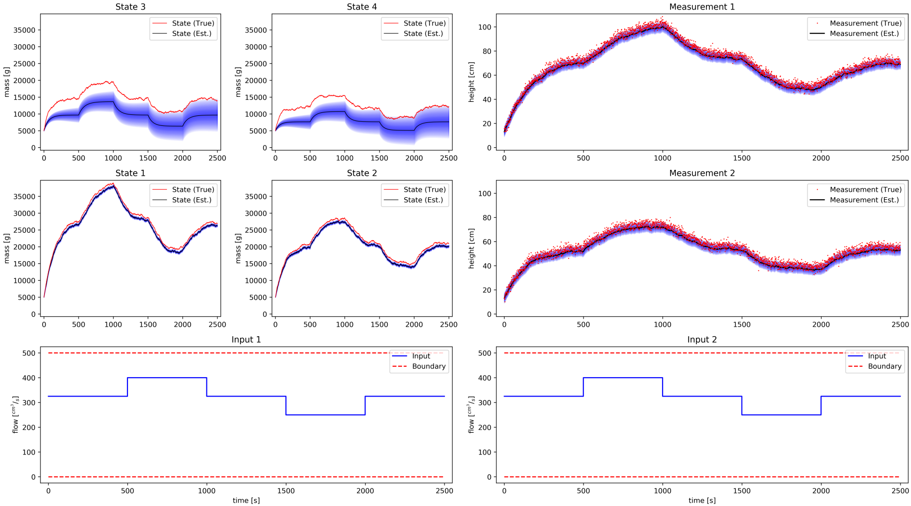
In this test example, we define the Modified Four Tank System. This system is governed by the same stochastic differential equation model as define in the system model slide.

Write out model equations for modified four tank system?

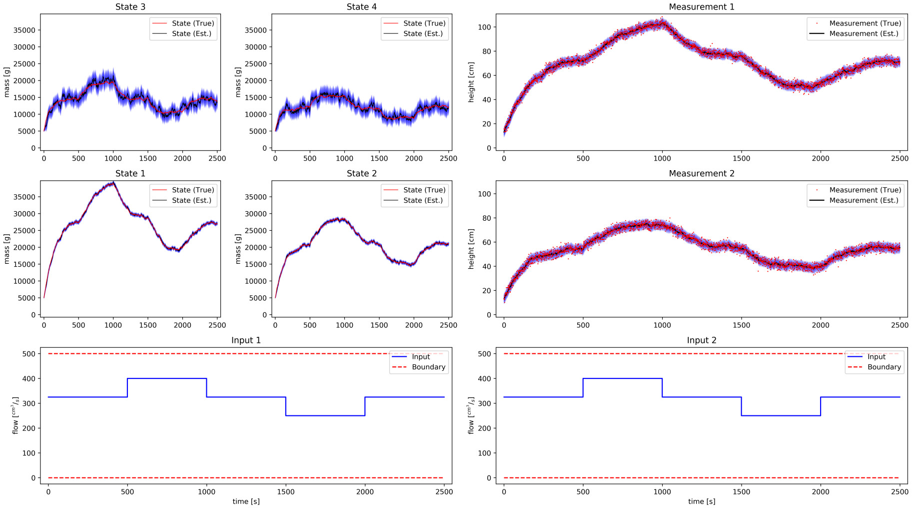
Modified Four Tank System - Simulated Example (CD-EKF [No augmentation])



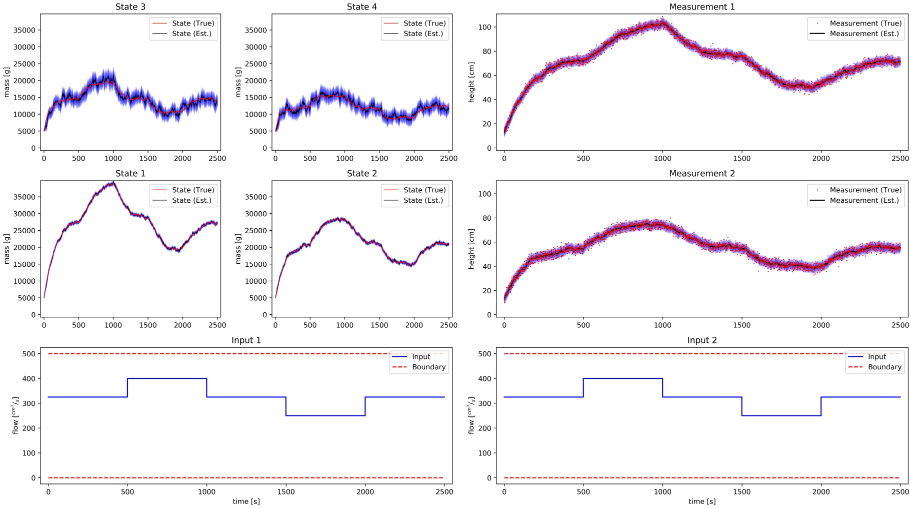
Modified Four Tank System - Simulated Example (CD-EKF)



Modified Four Tank System - Simulated Example (CD-UKF)



Modified Four Tank System - Simulated Example (CD-EnKF)



Modified Four Tank System - Simulated Example (CD-PF)